# Observation of spin polarization in $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$ at BESIII 

Cui Li

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## Outline

- Electromagnetic Form Factors
- The BESIII experiment
- Observation of spin polarization in $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$ at BESIII
$>\Lambda \bar{\Lambda}$ decay asymmetry parameters with $J / \psi$
$>$ Time-like $\Lambda$ electromagnetic form factors with scan data at 2.396 GeV

Summary

## Electromagnetic form factors

- Assuming one-photon exchange ( $e^{+} e^{-} \rightarrow \gamma^{*} \rightarrow B \bar{B}$ ), the Born cross section of spin $1 / 2$ baryon-antibaryon pair production can be parameterized in terms of $G_{E}$ and $G_{M}$ :

$$
\begin{equation*}
\sigma_{B \bar{B}}(s)=\frac{4 \pi \alpha^{2} \beta}{3 s}\left[\left|G_{M}(s)\right|^{2}+\frac{1}{2 \tau}\left|G_{E}(s)\right|^{2}\right] . \tag{1}
\end{equation*}
$$

$\alpha=1 / 137.036$ the fine-structure constant, $\beta=\sqrt{1-4 m_{B}^{2} c^{4} / s}$ the velocity of the produced baryon, $c$ the speed of light, $s$ the square of the c.m. energy, $m_{B}$ the mass of the baryon and $\tau=s /\left(4 m_{B}^{2}\right)$.
$\square$ The effective form factor is defined as

$$
\begin{equation*}
|G(s)| \equiv \sqrt{\frac{2 \tau\left|G_{M}(s)\right|^{2}+\left|G_{E}(s)\right|^{2}}{2 \tau+1}} \tag{2}
\end{equation*}
$$

## Electromagnetic form factors

$\square$ The $|G(s)|$ can be straightly calculated:

$$
\begin{equation*}
|G(s)|=\sqrt{\frac{\sigma_{B \bar{B}}(s)}{\left(1+\frac{1}{2 \tau}\right)\left(\frac{4 \pi \alpha^{2} \beta}{3 s}\right)}} \tag{3}
\end{equation*}
$$

$G_{E}(s)$ and $G_{M}(s)$ can be expressed in terms of the effective form factor $G(s)$ and the ratio $R=\left|G_{E}(s) / G_{M}(s)\right|$ :

$$
\begin{equation*}
\left|G_{M}(s)\right|^{2}=\frac{2 \tau+1}{2 \tau+R^{2}}|G(s)|^{2}, \quad\left|G_{E}(s)\right|^{2}=R^{2} \frac{2 \tau+1}{2 \tau+R^{2}}|G(s)|^{2} \tag{4}
\end{equation*}
$$

$\square$ The ratio $R$ can be extracted from the scattering angle of the baryon:

$$
\begin{equation*}
\frac{d \sigma_{\text {Born }}(s)}{d \cos \theta}=\frac{2 \pi \alpha^{2} \beta}{4 s}\left[\left(1+\cos ^{2} \theta\right)+\frac{1}{\tau} R^{2}\left(\sin ^{2} \theta\right)\right] \tag{5}
\end{equation*}
$$

## Electromagnetic form factors

- This can be expressed in a simpler form, linear in $\cos ^{2} \theta$

$$
\begin{equation*}
\frac{d \sigma}{d \cos \theta} \propto 1+\eta \cos ^{2} \theta \tag{6}
\end{equation*}
$$

where $\eta$ is the slope and satisfies $-1<\eta<1$.
$\square R$ can then be extracted from $\eta$

$$
\begin{equation*}
R=\sqrt{\tau} \sqrt{\frac{1-\eta}{1+\eta}} \tag{7}
\end{equation*}
$$

Conversely $\eta$ can be expressed in terms of $R$ as:

$$
\begin{equation*}
\eta=\frac{\tau-R^{2}}{\tau+R^{2}} \tag{8}
\end{equation*}
$$

## Space-like versus Time-like Electromagnetic Form Factors



- Spin $1 / 2$ baryons have magnetic $G_{M}$ and electric $G_{E}$ form factors
- Space-like $\left(q^{2}<0\right) G_{E}$ and $G_{M}$ real numbers
- Time-like $\left(q^{2} \geq 4 M_{B}^{2}>0\right)$
$>G_{E}(s)=\left|G_{E}(s)\right| e^{i \Phi_{E}}, G_{M}(s)=\left|G_{M}(s)\right| e^{i \Phi_{M}}$
$>$ Relative phase: $\Delta \Phi=\Phi_{E}-\Phi_{M}$


## Experimental status

- Babar Collaboration:
- $e^{+} e^{-} \rightarrow \gamma \Lambda \bar{\Lambda}$

- Based on very little data.

Total error on the $G_{E} / G_{M}$ ratio $33-100 \%$.


$\square$ polarization: $-0.22<P_{n}<0.28$
$\square$ Relative phase: $-0.76<\sin \phi<0.98$

## Experimental status

- CLEO-c: very few hyperon events
@3.773GeV (15 ~ 105 events)
Phy. Lett. B739(2014)

- Far from threshold $=$ low cross sections $=$ small data samples $=$ large uncertainties.
$\square$ No angular distributions $\rightarrow$ no ratio $R=G_{E} / G_{M}$ extracted $\rightarrow$ the EMFFs calculated assuming $\mathrm{R}=1$ or 0 .


## Experimental status

with data at $2.2324 \mathrm{GeV}, 2.4 \mathrm{GeV}, 2.8 \mathrm{GeV}$ and 3.065 GeV collected by BESIII in 2011-2012

- The 2.2324 GeV is the closest to the $\Lambda \bar{\Lambda}$ threshold so far.
$\square$ The cross section of $319.5 \pm 57.6 \mathrm{pb}$ which is larger than theoretical predictions.


See Liang Yan's talk

## Beijing Electron Positron Collider (BEPC)

- Founded: 1984

$$
\mathrm{Ecm}=2-4.6 \mathrm{GeV}
$$

- 1989-2005 (BEPC):

$$
\mathrm{L}_{\text {peak }}=1.0 \times 10^{31} / \mathrm{cm}^{2} \mathrm{~s}
$$

- 2008-now (BEPCII):
$\mathrm{L}_{\text {peak }}=1.0 \times 10^{33} / \mathrm{cm}^{2} \mathrm{~s}$
(Apr. 5, 2016)
 positron collider)


## BEijing Spectrometer (BES)



## Data collected at BESIII



World largest data sample of $J / \psi, \psi(2 S)$ and $\psi(3770)$
Unique data sample at XYZ (charmonium-like resonances) region

- Can cover 0-4.6 GeV from annihilation or ISR


## Energy scan 2014-2015 at BESIII



World leading scan data between 2.0 GeV and 3.08 GeV

- Nucleon and hyperon EMFFs available

$$
e^{+} e^{-} \rightarrow \gamma^{*} \rightarrow J / \psi \rightarrow \Lambda \bar{\Lambda}
$$

$e^{+} e^{-} \rightarrow \gamma^{*} \rightarrow J / \psi \rightarrow \Lambda \bar{\Lambda}$


- Process described by two complex numbers: magnetic $G_{M}$ and electric $G_{E}$ form factors.
$\square$ Two real parameters:
$>\alpha_{\psi}$ angular distribution
$>\Delta \Phi=\arg \left(G_{E} / G_{M}\right)$ the phase between the two form factors

Dubnickova, Dubnicka, Rekalo Nuovo Cim. A109 (1996) 241
Gakh, Tomasi-Gustafsson NPA771 (2006) 169
Czyz, Grzelinska, Kuhn PRD75 (2007) 074026
Fäldt EPJ A51 (2015) 74; EPJ A52 (2016)141
Fäldt, Kupsc PLB772 (2017) 16

- $\alpha_{\psi}$ well known
$\square d \Gamma / d \Omega \propto$
$1+\alpha_{\psi} \cos ^{2} \theta$


BESIII,PRD 95, 052003 (2017)
$\square \Delta$ never considered before

## Baryon polarization in $e^{+} e^{-}$annihilation


$e^{+} e^{-} \rightarrow\left(\Lambda \rightarrow p \pi^{-}\right) \bar{\Lambda}$

$\square$ Hyperon polarization can be determined using the angular distribution of the daughter particle.

Exclusive decay distributions for

$$
e^{+} e^{-} \rightarrow\left(\Lambda \rightarrow p \pi^{-}\right)\left(\bar{\Lambda} \rightarrow \bar{p} \pi^{+}\right) \quad e^{+} e^{-} \rightarrow\left(\Lambda \rightarrow p \pi^{-}\right)\left(\bar{\Lambda} \rightarrow \bar{n} \pi^{0}\right)
$$

$$
\begin{aligned}
& d \Gamma \propto \mathcal{W}(\boldsymbol{\xi}) d \boldsymbol{\xi}=\mathcal{W}(\boldsymbol{\xi}) d \cos \theta_{\Lambda} d \Omega_{1} d \Omega_{2} \quad \xi:\left(\cos \theta_{\Lambda}, \Omega_{1}, \Omega_{2}\right) \\
& \Lambda \rightarrow p \pi^{-}: \Omega_{1}=\left(\cos \theta_{1}, \phi_{1}\right) \quad \alpha_{1} \rightarrow \alpha_{-} \\
& \bar{\Lambda} \rightarrow \bar{p} \pi^{+}\left(o r \bar{n} \pi^{0}\right): \Omega_{2}=\left(\cos \theta_{2}, \phi_{2}\right) \\
& \quad \bar{\Lambda} \rightarrow \bar{n} \pi^{0}: \alpha_{2} \rightarrow \bar{\alpha}_{0} \quad \bar{\Lambda} \rightarrow \bar{p} \pi^{+}: \alpha_{2} \rightarrow \alpha_{+}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) & =1+\alpha_{\psi} \cos ^{2} \theta_{\Lambda} \\
& +\alpha_{1} \alpha_{2} \sqrt{\left(\sin ^{2} \theta_{\Lambda} \sin \theta_{1} \sin \theta_{2} \cos \phi_{1} \cos \phi_{2}+\cos ^{2} \theta_{\Lambda} \cos \theta_{1} \cos \theta_{2}\right)} \\
& +\alpha_{1} \alpha_{2} \sqrt{1-\alpha_{\psi}^{2}} \cos (\Delta \Phi)\left\{\sin \theta_{\Lambda} \cos \theta_{\Lambda}\left(\sin \theta_{1} \cos \theta_{2} \cos \phi_{1}+\cos \theta_{1} \sin \theta_{2} \cos \phi_{2}\right)\right\} \\
& +\alpha_{1} \alpha_{2} \sqrt[\alpha_{\psi}\left(\cos \theta_{1} \cos \theta_{2}-\sin ^{2} \theta_{\Lambda} \sin \theta_{1} \sin \theta_{2} \sin \phi_{1} \sin \phi_{2}\right)]{ }
\end{aligned}
$$

Fäldt, Kupsc PLB772 (2017) 16

$$
J / \psi \rightarrow\left(\Lambda \rightarrow p \pi^{-}\right)\left(\bar{\Lambda} \rightarrow \bar{p} \pi^{+} / \bar{n} \pi^{0}\right)
$$



A simultaneous maximum likelihood fit is performed to two data sets.

- Background events subtracted.


## Fit results



| Parameters | This work | Previous results |  |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{\alpha}_{\psi}$ | $\mathbf{0 . 4 6 1} \pm \mathbf{0 . 0 0 6} \pm \mathbf{0 . 0 0 7}$ | $0.469 \pm 0.027$ BESIII |  |
| $\Delta \Phi(\mathrm{rad})$ | $\mathbf{0 . 7 4 0} \pm \mathbf{0 . 0 1 0} \pm \mathbf{0 . 0 0 8}$ | - |  |
| $\alpha_{-}$ | $\mathbf{0 . 7 5 0} \pm \mathbf{0 . 0 0 9} \pm \mathbf{0 . 0 0 4}$ | $0.642 \pm 0.013$ | PDG |
| $\alpha_{+}$ | $-\mathbf{0 . 7 5 8} \pm \mathbf{0} .010$ | $\mathbf{0 . 0 0 7}$ | $-0.71 \pm 0.08$ |
| $\bar{\alpha}_{0}$ | $-\mathbf{6 9 2}+\mathbf{0 . 0 1 6} \pm \mathbf{0 . 0 0 6}$ | PDG |  |
| $\boldsymbol{A}_{C P}$ | $-\mathbf{0 . 0 0 6} \pm \mathbf{0 . 0 1 2} \pm \mathbf{0 . 0 0 7}$ | $0.006 \pm 0.021$ | PDG |
| $\bar{\alpha}_{0} / \alpha_{+}$ | $\mathbf{0 . 9 1 3} \pm \mathbf{0 . 0 2 8} \pm \mathbf{0 . 0 1 2}$ | - |  |

CP asymmetry:

$$
A_{C P}=\frac{\alpha_{-}+\alpha_{+}}{\alpha_{-}-\alpha_{+}}
$$

The result of $\alpha_{\psi}$ is consistent with previous BESIII measurement.

- Spin polarization of $\Lambda$ and $\bar{\Lambda}$ are observed.
$\square$ The result of $\alpha_{-}$is $\sim 5 \sigma$ larger than the PDG value.

$$
e^{+} e^{-} \rightarrow \gamma^{*} \rightarrow\left(\Lambda \rightarrow p \pi^{-}\right)\left(\bar{\Lambda} \rightarrow \bar{p} \pi^{+}\right) @ 2.396 \mathrm{GeV}
$$

## The $\bar{p} p \rightarrow \bar{\Lambda} \Lambda$ at the PS185



Fig. 4.30. Polarisation for $\overline{\mathrm{p}} \rightarrow \bar{\Lambda} \Lambda$ at various energies.


Fig. 4.31. Spin observables for $\overline{\mathrm{p} p} \rightarrow \bar{\Lambda} \Lambda$ at 1546 and $1695 \mathrm{MeV} / c$.

P Polarization and spin-correlation are observed. (Phys Rep 368 (2002) 119)
Theoretical model of meson-exchange describes PS185 data well. (PRC 45, 931(1992); PRC46, 2158(1992))

## Theoretical prediction of Time-like $\Lambda$ form factors

Time-like $\Lambda$ EMFFs studied by Haidenbauer and Meissner (PLB 761 (2016) 456-461)




- Restrict to one-photon exchange
$\square$ PS185 data $p \bar{p} \rightarrow \Lambda \bar{\Lambda}$ used as input to fit $\Lambda \bar{\Lambda}$ potentials (Phys Rep 368 (2002) 119)
$\square$ The ratio $R$ and the phase $\Delta \Phi$ are model dependent
- Inconclusive BaBar results (PRD 76 (2007) 092006)


## Event selection of $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$

- Track level
$>$ Polar angle: $|\cos \theta|<0.93$
$>$ Momentum should be less than 0.5 GeV
$>$ At least 4 tracks
Tracks with $p<0.2 \mathrm{GeV}$ are assigned to be pions, $p>0.2 \mathrm{GeV} p \bar{p}$
- Secondary vertex fit to reconstruct $\Lambda$ and $\bar{\Lambda}$
- Four constraint Kinematic fit to $\Lambda$ and $\bar{\Lambda}$


## Momentum distribution of final states


$25 / 33$

## Distribution of $\chi^{2}(4 C)$

- $\chi^{2}(4 C)<50$

Optimized by the value of figure of merit (FOM), $\frac{S}{\sqrt{S+B}}$
$\square S$ is signal MC sample, $S+B$ is data
The FOM is not sensitive to the EMFFs and phase.




## Invariant mass of $p \pi^{-}$and $\bar{p} \pi^{+}$






- $\left|M\left(p \pi^{-} / \bar{p} \pi^{+}\right)-m_{\Lambda}\right|<0.006 \mathrm{GeV}, m_{\Lambda}$ is the mass of $\Lambda$ from PDG.


## Background study by sideband method




- $N_{B G}=\frac{1}{2} N_{\text {blue }}-\frac{1}{4} N_{\text {green }}=14 \pm 4$
$\square$ Background level is $2.5 \%$.


## Decay distribution

$\square$ Assume CP symmetry in this case $\alpha_{\Lambda}=-\alpha_{\bar{\Lambda}}$
The decay distribution described in a simpler form

$$
\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) & =\mathcal{T}_{0}(\boldsymbol{\xi})+\eta \mathcal{T}_{5}(\boldsymbol{\xi}) \\
& -\alpha_{\Lambda}^{2}\left(\mathcal{T}_{1}(\boldsymbol{\xi})+\sqrt{1-\eta^{2}} \cos (\Delta \Phi) \mathcal{T}_{2}(\boldsymbol{\xi})+\eta \mathcal{T}_{6}(\boldsymbol{\xi})\right) \\
& +\alpha_{\Lambda} \sqrt{1-\eta^{2}} \sin (\Delta \Phi)\left(\mathcal{T}_{3}(\boldsymbol{\xi})-\mathcal{T}_{4}(\boldsymbol{\xi})\right)
\end{aligned}
$$

$\mathcal{T}_{i}$ are known functions of the five-dimensional $\boldsymbol{\xi}\left(\theta, \Omega_{1}\left(\theta_{1}, \phi_{1}\right), \Omega_{2}\left(\theta_{2}, \phi_{2}\right)\right)$
$R=\left|G_{E} / G_{M}\right|$
$\eta=\frac{\tau-R^{2}}{\tau+R^{2}}$
Fäldt, Kupsc PLB772 (2017) 16

## Fit results

A A maximum likelihood fit is performed to the data set.

- With PDG value $\alpha_{\Lambda}=0.642$

$$
\begin{aligned}
& >R=\left|G_{E} / G_{M}\right|=0.94 \pm 0.16 \pm 0.03 \\
& >\Delta \Phi=42^{\circ} \pm 16^{\circ} \pm 8^{\circ} .
\end{aligned}
$$

- With BESIII value $\alpha_{\Lambda}=0.75 \pm 0.01$

$$
\begin{aligned}
& >R=0.96 \pm 0.14 \pm 0.02 \\
& >\Delta \Phi=37^{\circ} \pm 12^{\circ} \pm 6^{\circ}
\end{aligned}
$$




- Spin polarization of $\Lambda$ and $\bar{\Lambda}$ are observed.


## Comparison of $\left|G_{E} / G_{M}\right|$ and $\Delta \Phi$


$\square$ Results of data support the $\Lambda \bar{\Lambda}$ model I (Red line) PRC 45, 931(1992)

## Results of the cross section and effective EMFFs

$\square$ The cross section $\sigma=\frac{N_{\text {signal }}}{\operatorname{L\epsilon }(1+\delta) \operatorname{Br}\left(\Lambda \rightarrow p \pi^{-}\right) \operatorname{Br}\left(\bar{\Lambda} \rightarrow \bar{p} \pi^{+}\right)}$
$>$ ISR and vacuum polarization factor $1+\delta$ is from ConExc
$>\epsilon$ is the detection efficiency, $L$ is the luminosity
$>\sigma=119.0 \pm 5.3$ (stat.) $\pm 5.1$ (sys.) pb
Effective form factors are refated to $\sigma,\left|G\left(q^{2}\right)\right|=\sqrt{\frac{\sigma\left(q^{2}\right)}{\left(1+\frac{1}{2 \tau}\right)\left(\frac{4 \pi \alpha^{2} \beta}{3 q^{2}}\right)}}$
$>|G|=0.123 \pm 0.003$ (stat.) $\pm 0.003$ (sys.)
$\alpha \approx \frac{1}{137}$ is the fine structure constant,
$\beta=\sqrt{1-\frac{1}{\tau}}$ is the velocity, $\tau=\frac{q^{2}}{4 m_{\Lambda}^{2}}$.
Previous measurements

|  | $\sigma(\mathrm{pb})$ | $\|G\|$ | Reference |
| :---: | :---: | :---: | :---: |
| BESIII $\sqrt{s}=2.40 \mathrm{GeV}$ | $128 \pm 19 \pm 18$ | $0.127 \pm 0.009 \pm 0.009$ | Phys. Rev. D 97, 032013 (2018) |
| BaBar $\sqrt{s}=2.35-2.40 \mathrm{GeV}$ | $176 \pm 34$ | $0.152 \pm 0.016$ | Phys. Rev. D 76, 092006 (2007) |

## Summary

- Hyperon spin polarization is observed in $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$
$\square$ The phase is measured for the first time.
- With J/ $\psi$
$>$ The phase determined to be $42.3^{\circ} \pm 0.62^{\circ} \pm 0.5^{\circ}$
$>$ Decay asymmetry parameter of $\Lambda \rightarrow p \pi^{-}$obtained to be $0.750 \pm 0.009 \pm 0.004$
$>$ The $C P$ odd observable $A_{C P}=-0.006 \pm 0.012 \pm 0.007$
- With scan data at 2.396 GeV

$$
\begin{array}{ll}
\text { PDG value } \alpha_{\Lambda}=0.642 & \text { BESIII value } \alpha_{\Lambda}=0.75 \pm 0.01 \\
>R=\left|G_{E} / G_{M}\right|=0.94 \pm 0.16 \pm 0.03 & >R=\left|G_{E} / G_{M}\right|=0.96 \pm 0.14 \pm 0.02 \\
>\Delta \Phi=42^{\circ} \pm 16^{\circ} \pm 8^{\circ} . & >\Delta \Phi=37^{\circ} \pm 12^{\circ} \pm 6^{\circ}
\end{array}
$$

## Thank you for your attention!

## Check contribution from two-photon exchange

Significant involvement of two-photon exchange would result in an asymmetric $\Lambda$ angular distribution(PLB 659 197-200 (2008)):

$$
\begin{equation*}
A=\frac{N(\cos \theta>0)-N(\cos \theta<0)}{N(\cos \theta>0)+N(\cos \theta<0)} \tag{9}
\end{equation*}
$$

- In the formula of decay distribution, an additional term $\kappa \cos \theta \sin ^{2} \theta$ due to interference of the single and two photon amplitudes.
$\square$ The asymmetry $A$ can be determined from $\kappa$ in the following way:

$$
\begin{equation*}
A=\frac{3}{4} \frac{\kappa}{3+\eta} . \tag{10}
\end{equation*}
$$

$\square$ The asymmetry is found to be $A=0.001 \pm 0.037$
$\square$ This result indicates a negligible contribution from two-photon exchange.

## The method of Maximum Log Likelihood

A simpler form of $\frac{d \sigma}{d \cos \theta} \propto 1+\eta \cos ^{2} \theta$

- $R$ can then be extracted by $R=\sqrt{\tau} \sqrt{\frac{1-\eta}{1+\eta}}$
$\square$ The decay distribution could be expressed as:

$$
\begin{align*}
& \qquad \mathcal{W}(\boldsymbol{\xi})=\mathcal{T}_{0}(\boldsymbol{\xi})+\eta \mathcal{T}_{5}(\boldsymbol{\xi}) \\
& -\alpha_{\Lambda}^{2}\left(\mathcal{T}_{1}(\boldsymbol{\xi})+\sqrt{1-\eta^{2}} \cos (\Delta \Phi) \mathcal{T}_{2}(\boldsymbol{\xi})+\eta \mathcal{T}_{6}(\boldsymbol{\xi})\right)  \tag{11}\\
& +\alpha_{\Lambda} \sqrt{1-\eta^{2}} \sin (\Delta \Phi)\left(\mathcal{T}_{3}(\boldsymbol{\xi})-\mathcal{T}_{4}(\boldsymbol{\xi})\right) \cdot \boldsymbol{T}_{p}^{\hat{x}}\left(\theta_{1}, \varphi_{1}\right) \\
& \mathcal{T}_{i} \text { are known functions as vector } \boldsymbol{\xi} \\
& \text { five-dimensional } \boldsymbol{\xi}\left(\theta, \Omega_{1}\left(\theta_{1}, \phi_{1}\right), \Omega_{2}\left(\theta_{2}, \phi_{2}\right)\right)
\end{align*}
$$

- $\mathcal{T}_{i}$ are known functions as vector $\boldsymbol{\xi}$


## The method of Maximum Log Likelihood

- The probability distribution function of the $i$ :th event by the vector $\boldsymbol{\xi}\left(\theta, \Omega_{1}, \Omega_{2}\right)$ :

$$
\begin{equation*}
\mathcal{P}\left(\xi_{i} ; \eta, \Delta \Phi\right)=\mathcal{W}\left(\xi_{i} ; \eta, \Delta \Phi\right) \epsilon\left(\xi_{i}\right) / \mathcal{N}(\eta, \Delta \Phi) \tag{12}
\end{equation*}
$$

where $\epsilon(\boldsymbol{\xi})$ is the efficiency.
$\square$ The joint probability density for observing the $N$ events in the data sample is:

$$
\begin{equation*}
\mathcal{P}\left(\boldsymbol{\xi}_{1}, \boldsymbol{\xi}_{2}, \ldots, \boldsymbol{\xi}_{N} ; \eta, \Delta \Phi\right)=\prod_{i=1}^{N} \mathcal{P}\left(\boldsymbol{\xi}_{i} ; \eta, \Delta \Phi\right)=\prod_{i=1}^{N} \frac{\mathcal{W}\left(\boldsymbol{\xi}_{i} ; \eta, \Delta \Phi\right) \epsilon\left(\boldsymbol{\xi}_{i}\right)}{\mathcal{N}(\eta, \Delta \Phi)} \tag{13}
\end{equation*}
$$

$\mathcal{N}(\eta, \Delta \Phi)$ is calculated with a PHSP MC sample

## The method of Maximum Log Likelihood

By taking the natural logarithm of the joint probability density, the efficiency function can be separated

$$
\begin{equation*}
\ln \mathcal{P}\left(\xi_{1}, \boldsymbol{\xi}_{2}, \ldots, \boldsymbol{\xi}_{N} ; \eta, \Delta \Phi\right)=\sum_{i=1}^{N} \ln \frac{\mathcal{W}\left(\boldsymbol{\xi}_{i} ; \eta, \Delta \Phi\right)}{\mathcal{N}(\eta, \Delta \Phi)}+\sum_{i=1}^{N} \ln \epsilon\left(\xi_{i}\right) \tag{14}
\end{equation*}
$$

Technically, minimization $\mathcal{S}=-\ln \mathcal{L}$ instead of maximization is performed using MINUIT.

$$
\begin{equation*}
\mathcal{S}=-\ln \mathcal{L}=-\sum_{i=1}^{N} \ln \frac{\mathcal{W}\left(\xi_{i} ; \eta, \Delta \Phi\right)}{\mathcal{N}(\eta, \Delta \Phi)} \tag{15}
\end{equation*}
$$

$\square$ By a fit, $\eta$ and $\Delta \Phi$ are given.

