

# Relative phase of $\Lambda$ form factors

Rinaldo Baldini Ferroli and Simone Pacetti

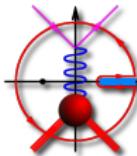


## Electromagnetic Structure of Strange Baryons

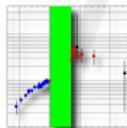
GSI Helmholtzzentrum für Schwerionenforschung GmbH

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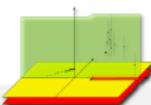
# Agenda



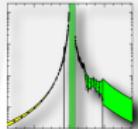
Baryon form factors and dispersion relations



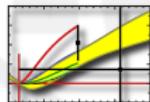
Space-like and time-like data on  $G_E/G_M$



Space-like and time-like  $G_E/G_M$  via DR's



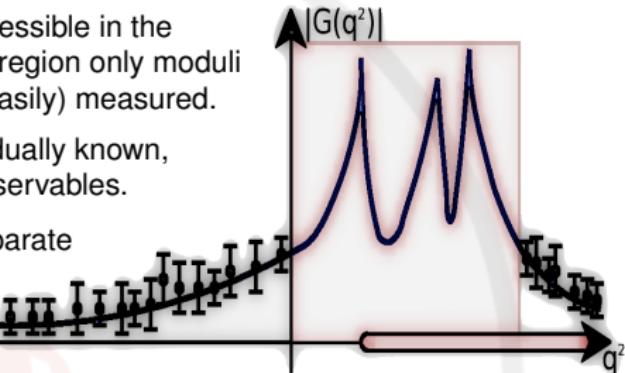
Asymptotic  $G_M$  from a DR sum rule



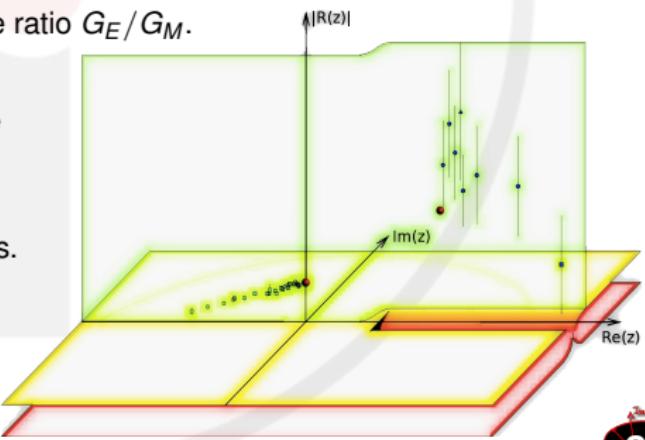
Hints for the ratio of  $\Lambda$  form factors

# About proton form factors

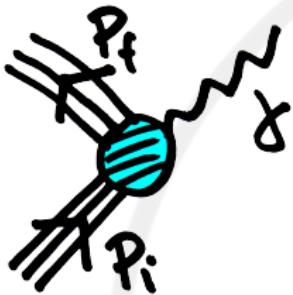
- \* Proton form factors are completely accessible in the space-like region while in the time-like region only moduli above the physical threshold can be (easily) measured.
- ⚠ In the space-like region they are individually known, especially by means of polarization observables.
- ❖ Only recently, attempts to measure separate value of moduli in the time-like region have become stronger.



- 
- ⌚ So far, the better known quantity is the ratio  $G_E/G_M$ .
  - \* Using dispersion relations (analyticity) space-like and time-like values can be exploited to extract information on phase and asymptotic behavior.
  - \* Analyticity imposes serious constraints. A space-like zero for the ratio  $G_E/G_M$  does require an asymptotic phase of 180 degrees.



# Proton-photon vertex



Nucleon electromagnetic four-current ( $q = p_f - p_i$ )

$$\langle P_f | J_{\text{EM}}^\mu(0) | P_i \rangle = e \bar{u}(p_f) \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} F_2(q^2) \right] u(p_i)$$

$F_1(q^2)$  and  $F_2(q^2)$  are the Dirac and Pauli form factors

$$F_1(0) = Q_p$$

$$F_2(0) = \kappa_p$$

$Q_p$  = electric charge

$\kappa_p$  = anomalous magnetic moment

Breit frame

$$p_f = (E, \vec{q}/2)$$

$$p_i = (E, -\vec{q}/2)$$

$$q = (0, \vec{q})$$

$$\langle P_f | J_{\text{EM}}^\mu(0) | P_i \rangle \equiv J_{\text{EM}}^\mu = (J_{\text{EM}}^0, \vec{J}_{\text{EM}})$$

$$\circlearrowleft J_{\text{EM}}^0 = e \left( F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2) \right)$$

$$\diamondsuit \quad \vec{J}_{\text{EM}} = e \bar{u}(p_f) \vec{\gamma} u(p_i) (F_1(q^2) + F_2(q^2))$$

Sachs form factors

$$\circlearrowleft G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2)$$

$$\diamondsuit \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

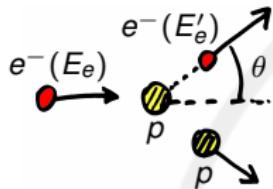
Normalizations

$$\circlearrowleft G_E(0) = Q_p$$

$$\diamondsuit \quad G_M(0) = \mu_p = \kappa_p + Q_p$$

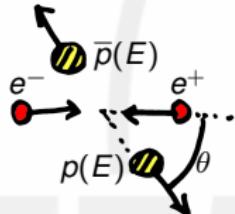
$\mu_p$  = total magnetic moment

# Cross sections and Coulomb correction



Elastic scattering cross section (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2\left(\frac{\theta}{2}\right)}{4E_e^3 \sin^4\left(\frac{\theta}{2}\right)} \left[ G_E^2 - \tau \left( 1 + 2(1-\tau) \tan^2\left(\frac{\theta}{2}\right) \right) G_M^2 \right] \frac{1}{1-\tau}$$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \textcolor{blue}{C}}{16E^2} \left[ (1 + \cos^2(\theta)) |G_M|^2 + \frac{1}{\tau} \sin^2(\theta) |G_E|^2 \right]$$

$$\tau = E^2/M_p^2$$

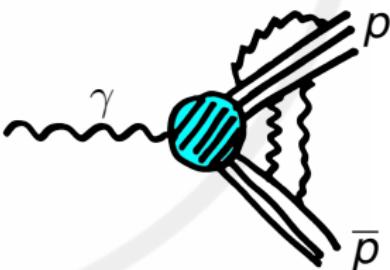
$$\beta = \sqrt{1 - 1/\tau}$$

Coulomb correction

$$C = \frac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}}$$

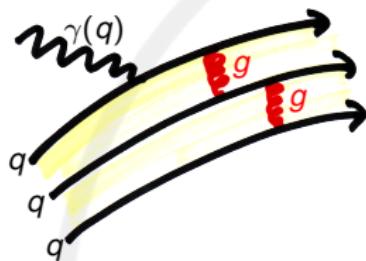
Ⓐ  $p\bar{p}$  Coulomb interaction as FSI

Ⓑ Only S-wave



# pQCD asymptotic behavior Space-like region

V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze,  
LNC7 (1973) 719  
S. J. Brodsky, G. R. Farrar, PRL31 (1973) 1153  
M. V. Galynsky, E. A. Kuraev JETPL96 (2012) 6



- ▲ pQCD: as  $q^2 \rightarrow -\infty$ ,  $F_1, F_2, G_E, G_M$  follow power laws driven by counting rules
- Valence quarks exchange gluons to distribute the photon momentum transfer  $q$

## Non-helicity-flip current $J^{\lambda, \lambda}(q^2)$

- ▲  $J^{\lambda, \lambda}(q^2) \propto G_M(q^2)$
- ◆ Two gluon propagators
- $G_M(q^2)_{q^2 \rightarrow -\infty} \sim (q^2)^{-2}$

## Helicity-flip current $J^{\lambda, -\lambda}(q^2)$

- ▲  $J^{\lambda, -\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$
- ◆ [Two gluon propagators]/ $\sqrt{-q^2}$
- $G_E(q^2)_{q^2 \rightarrow -\infty} \sim (q^2)^{-2}$

## Dirac and Pauli form factors

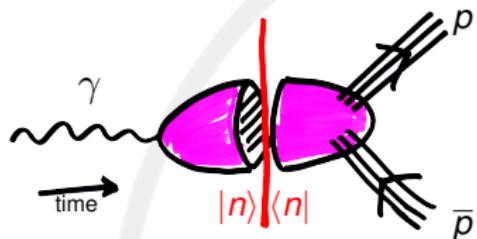
- ◆  $F_1(q^2)_{q^2 \rightarrow -\infty} \sim (q^2)^{-2}$
- $F_2(q^2)_{q^2 \rightarrow -\infty} \sim (q^2)^{-3}$

## Ratio of Sachs form factors

- ▲  $\frac{G_E(q^2)}{G_M(q^2)}_{q^2 \rightarrow -\infty} \sim \text{constant}$

# Nucleon form factors

## Time-like region ( $q^2 > 0$ )



◆ Crossing symmetry:

$$\langle P(p')|j^\mu|P(p)\rangle \rightarrow \langle \bar{P}(p')P(p)|j^\mu|0\rangle$$

◆ Form factors are complex functions of  $q^2$

### Optical theorem

$$\text{Im} \langle \bar{P}(p')P(p)|j^\mu|0\rangle \sim \sum_n \langle \bar{P}(p')P(p)|j^\mu|n\rangle \langle n|j^\mu|0\rangle \implies \begin{cases} \text{Im} F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$  are on-shell intermediate states:  $2\pi, 3\pi, 4\pi, \dots$

### Time-like asymptotic behavior

#### Phragmèn-Lindelöf theorem

If  $f(z) \rightarrow a$  as  $z \rightarrow \infty$  along a straight line, and  $f(z) \rightarrow b$  as  $z \rightarrow \infty$  along another straight line, and  $f(z)$  is regular and bounded in the angle between, then  $a = b$  and  $f(z) \rightarrow a$  uniformly in this angle.

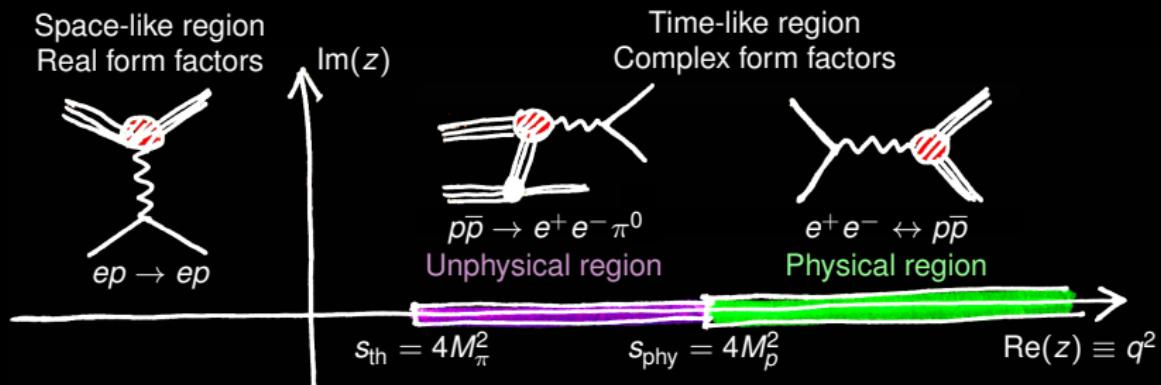
\*  $\underbrace{\lim_{q^2 \rightarrow -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \rightarrow +\infty} G_{E,M}(q^2)}_{\text{time-like}}$

▲  $G_{E,M} \underset{q^2 \rightarrow +\infty}{\sim} (q^2)^{-2}$

Must be real

# Analyticity of form factors

$q^2$ -complex plane



Only the real axis of the  $q^2$ -complex plane is experimentally accessible

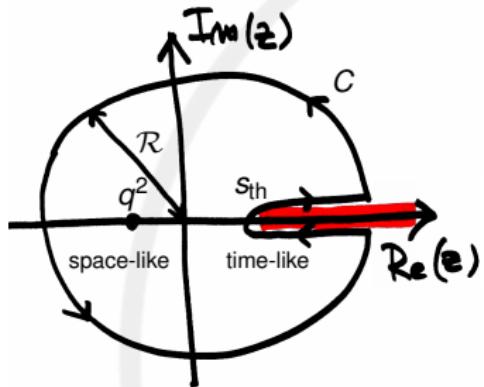
Space-like region $q^2 < 0$	Time-like region* $s_{\text{th}} < q^2 \leq s_{\text{phy}}$	Time-like region $q^2 \geq s_{\text{phy}}$	
$ep \rightarrow ep$	$p\bar{p} \rightarrow e^+ e^- \pi^0$	$e^+ e^- \leftrightarrow p\bar{p}$	$e^+ e^- \leftrightarrow p\bar{p}$ (pol.)
$G_E, G_M$	$ G_E ,  G_M $	$ G_E ,  G_M $	$ G_E ,  G_M , \arg(G_E/G_M)$

\* C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC75, 045205

E. A. Kuraev et al., JETP115, 93

G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeissi, A.G. Gakh PRC86, 025204

# Dispersion relations



- \* The form factors are **analytic** on the  $q^2$ -plane with a **multiple cut** ( $s_{\text{th}} = 4M_\pi^2, \infty$ )
- \* Dispersion relation for the **imaginary part** ( $q^2 < 0$ )

$$G(q^2) = \lim_{R \rightarrow \infty} \frac{1}{2\pi i} \oint_C \frac{G(z) dz}{z - q^2} = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\text{Im} G(s) ds}{s - q^2}$$

- \* Dispersion relation for the **logarithm** ( $q^2 < 0$ )  
B.V. Geshkenbein, *Yad. Fiz.* 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{\text{th}} - q^2}}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\ln |G(s)| ds}{(s - q^2)\sqrt{s - s_{\text{th}}}}$$

## Experimental inputs

- ⌚ Space-like data on the **real values** of form factors from:  $ep \rightarrow ep$  and  $e^\uparrow p \rightarrow e^- p^\uparrow$ , with polarization
- ⌚ Time-like data on form factor **moduli** from:  $e^+ e^- \leftrightarrow p\bar{p}$
- ⌚ Time-like data on  **$G_E/G_M$  phase** from:  $e^+ e^- \leftrightarrow p^\uparrow \bar{p}$  (pol.)

## Theoretical ingredients

- ⌚ Analyticity  $\Rightarrow$  convergence relations
- ❖ Normalization and threshold values
- ▲ Asymptotic behavior  $\Rightarrow$  super-convergence relations

# Advantages and drawbacks of dispersive approaches

## Advantages



DR's are based on unitarity and analyticity  $\Rightarrow$  model-independent approach



DR's relate data from different processes in different energy regions

$$\left[ \begin{array}{c} \text{space-like} \\ \text{form factor} \\ e^+ p \rightarrow e^- p \end{array} \right] = \int_{S_{\text{th}}}^{\infty} \left[ \begin{array}{c} \text{Im(form factor) or } \ln|\text{form factor}| \\ \text{over the time-like cut } (s_{\text{th}}, \infty) \\ e^+ e^- \rightarrow p\bar{p} + \text{theory} \end{array} \right]$$



Normalizations and theoretical constraints can be directly implemented



Form factors can be computed in the whole  $q^2$ -complex plane

## Drawbacks



Very long-range integration

### Remedy #1

pQCD power laws

### Remedy #2

Subtracted DR's



No data in the unphysical region, crucial in dispersive analyses

# A DISPERSEIVE APPROACH FOR $G_E/G_M$

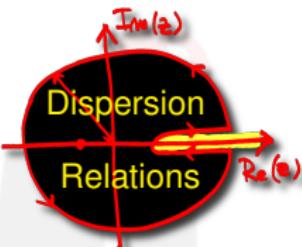


# Dispersive approach for the ratio $R = \mu_p G_E / G_M$

We start from the imaginary part of the ratio  $R(q^2)$ , written in the most general and model-independent way as

$$I(q^2) \equiv \text{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Theoretical constraints can be applied directly on this function  $I(q^2)$



The function  $R(q^2)$  is reconstructed in time and space-like regions

Additional theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of  $R(q^2)$

# Parametrization for $G_E/G_M$

The imaginary part of  $R(q^2)$  is parametrized by two series of orthogonal polynomials

$$\text{Im} [R(q^2)] \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\text{phy}} - s_{\text{th}}}{s_{\text{phy}} - s_{\text{th}}} \quad s_{\text{th}} \leq q^2 \leq s_{\text{phy}} \\ \sum_j D_j T_j(x') & x' = \frac{2s_{\text{phy}}}{q^2} - 1 \quad q^2 > s_{\text{phy}} \end{cases}$$

## Theoretical conditions on $\text{Im} [R(q^2)]$

- ⌚  $R(4M_\pi^2)$  is real  $\implies I(4M_\pi^2) = 0$
- ⌚  $R(4M_p^2)$  is real  $\implies I(4M_p^2) = 0$
- ⌚  $R(\infty)$  is real  $\implies I(\infty) = 0$

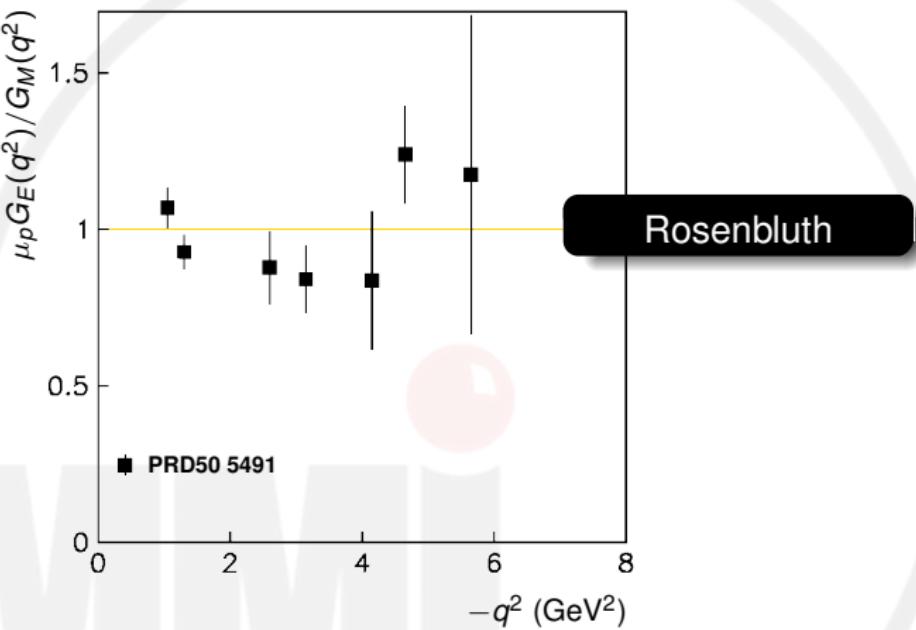
## Theoretical conditions on $R(q^2)$

- ❖ Continuity at  $q^2 = 4M_\pi^2$
- ❖  $R(4M_p^2)$  is real and  $\text{Re} [R(4M_p^2)] = \mu_p$

## Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- ⚠ Space-like region ( $q^2 < 0$ ) data for  $R$  from JLab and MIT-Bates
- ⚠ Time-like region ( $q^2 \geq 4M_p^2$ ) data for  $|R|$  from FENICE+DM2, *BABAR*, and E835

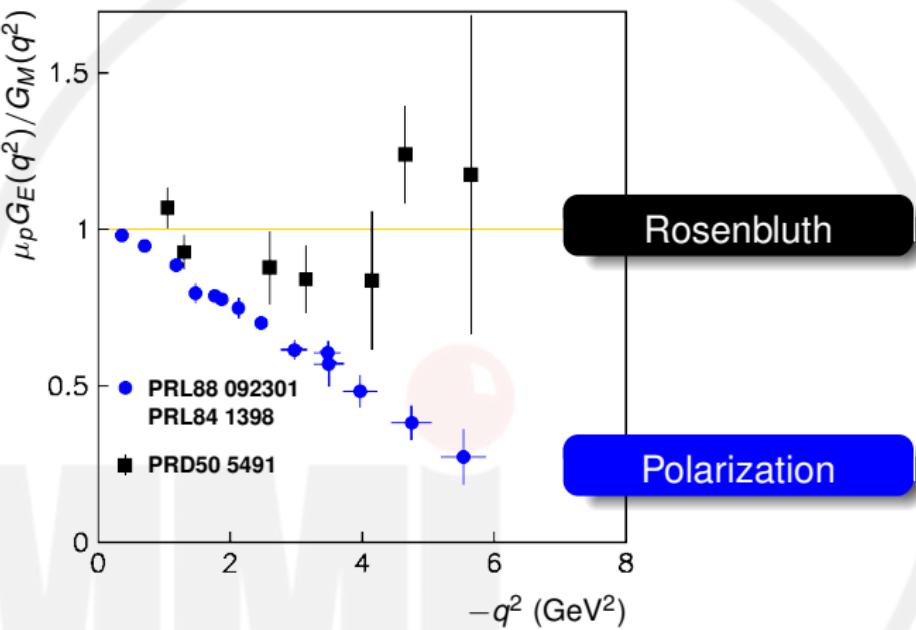
# Space-like data on $G_E/G_M$



Radiative corrections of  
polarization technique

Radiative corrections in  
Rosenbluth method

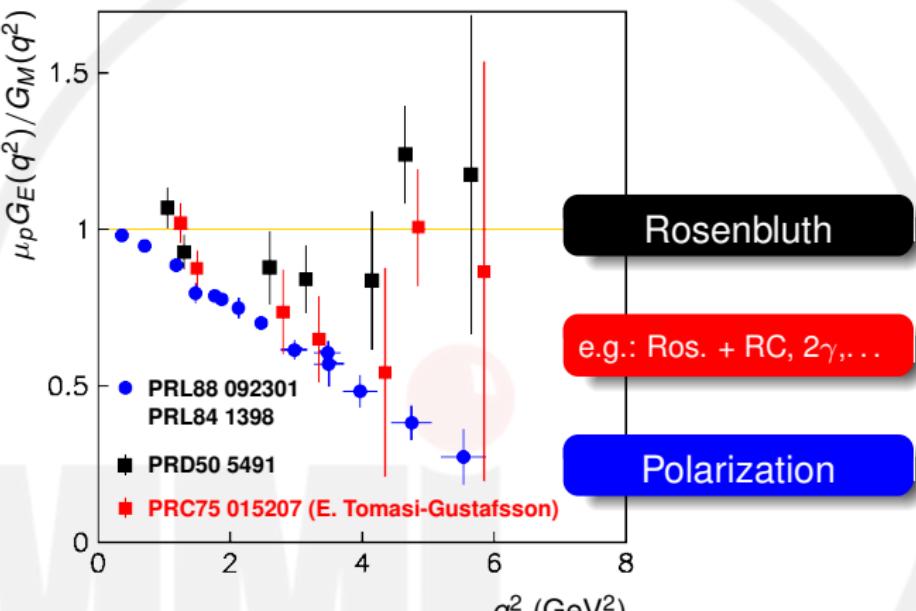
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# Space-like data on $G_E/G_M$



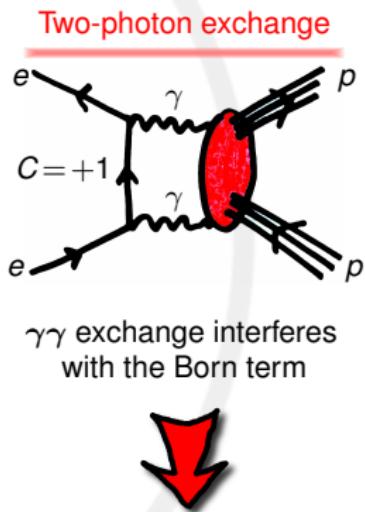
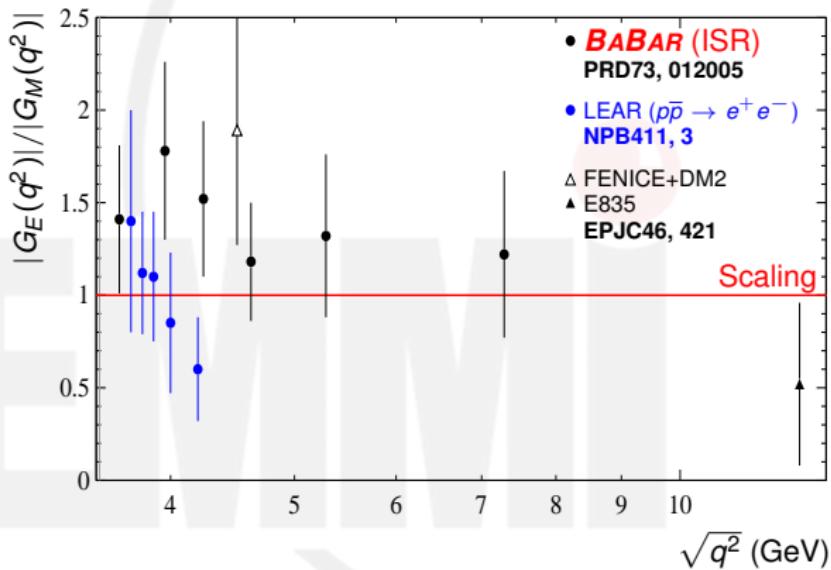
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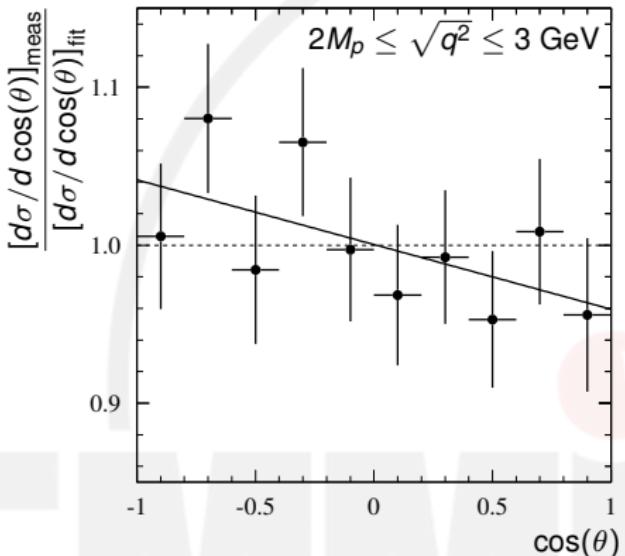
# Time-like data on $|G_E/G_M|$

$$\frac{d\sigma}{d \cos(\theta)} = \frac{\pi \alpha^2 \beta C}{2q^2} |G_M|^2 \left[ (1 + \cos^2(\theta)) + \frac{4M_p^2}{q^2} \sin^2(\theta) \left| \frac{G_E}{G_M} \right|^2 \right]$$



# Time-like two-photon contribution?

Phys. Lett. B659 (2008) 197  
arXiv:1302.0055



Integrated over the  $p\bar{p}$  - CM energy  
from threshold up to 3 GeV

The MC-fit assumes  
one-photon exchange

Slope =  $-0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

$$\langle A \rangle_{\cos \theta_p} = \frac{\sigma(\cos \theta_p > 0) - \sigma(\cos \theta_p < 0)}{\sigma(\cos \theta_p > 0) + \sigma(\cos \theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos \theta_p \geq 0)$  is the cross section integrated with  $\sqrt{q^2} \leq 3 \text{ GeV}$  and  $\cos \theta_p \geq 0$

# The result for $R = \mu_p G_E / G_M$

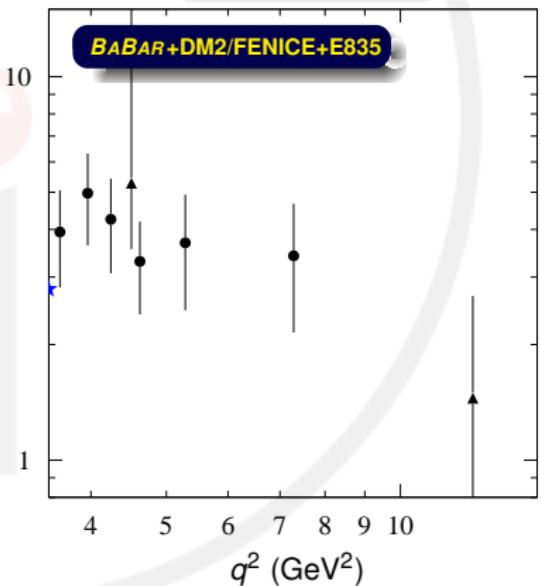
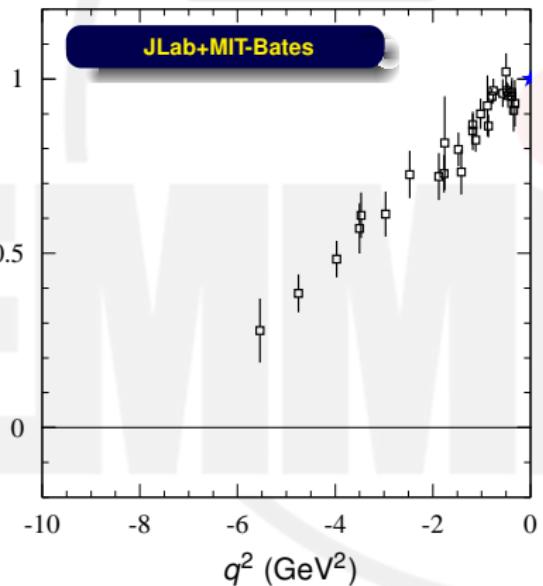
EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

Re( $q^2$ )

$R(q^2)$  space-like

$|R(q^2)|$  time-like



# The result for $R = \mu_p G_E / G_M$

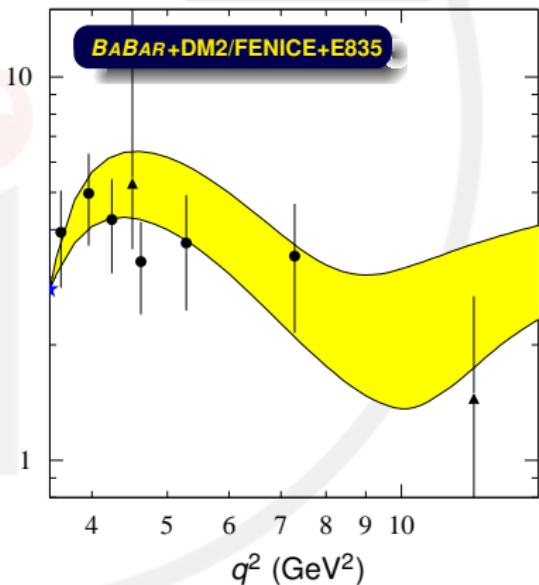
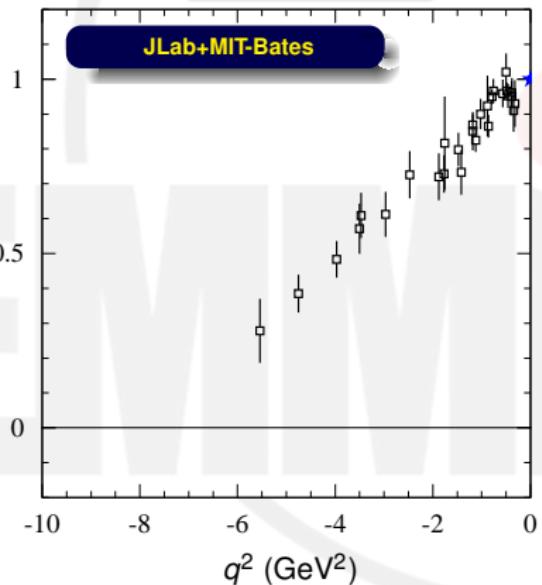
EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

$\text{Re } q^2$

$R(q^2)$  space-like

$|R(q^2)|$  time-like



# The result for $R = \mu_p G_E / G_M$

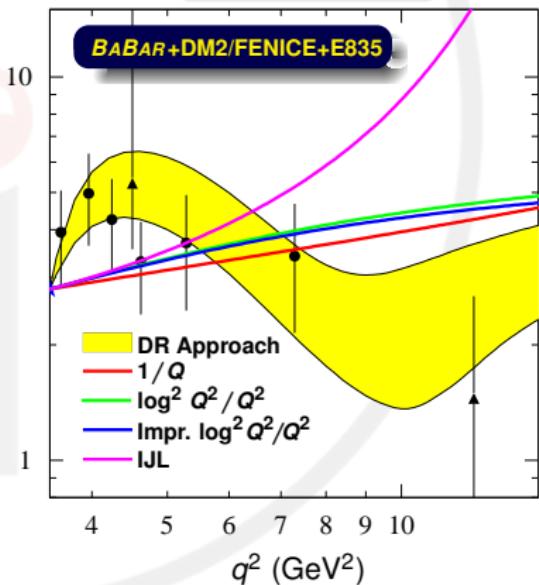
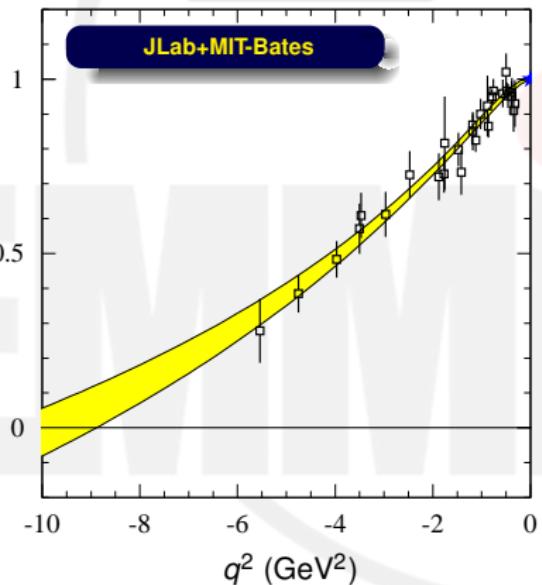
EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

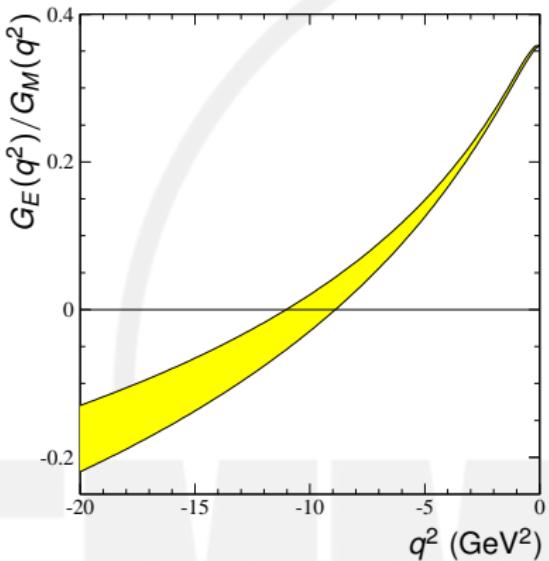
$\text{Re } q^2$

$R(q^2)$  space-like

$|R(q^2)|$  time-like

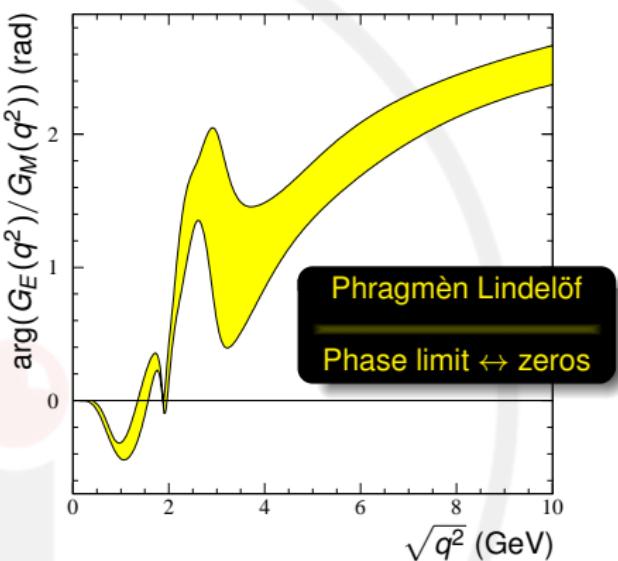


# Space-like zero and phase



Space-like zero

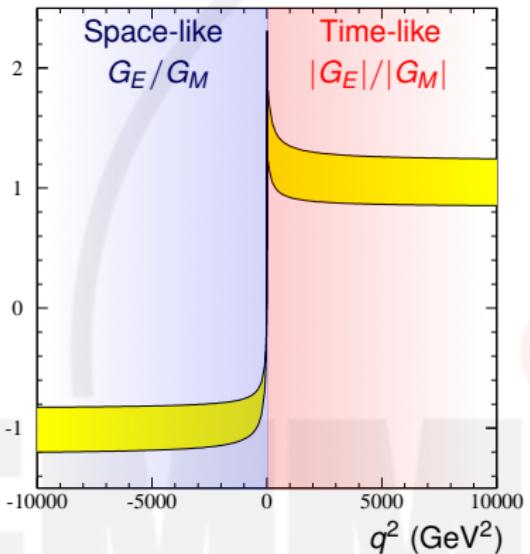
$$t_0^{\text{BABAR}} = (-10 \pm 1) \text{ GeV}^2$$



Phase from dispersion relations

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{\text{th}}}}{\pi} \operatorname{Pr} \int_{s_{\text{th}}}^{\infty} \frac{\ln |R(s)| ds}{\sqrt{s - s_{\text{th}}}(s - q^2)}$$

# Asymptotic $G_E^p/G_M^p$



Real asymptotic values for  $G_E/G_M$

◆  $\frac{G_E}{G_M} \xrightarrow{|q^2| \rightarrow \infty} -1.0 \pm 0.2$

Asymptotic behaviour of  $F_2/F_1$

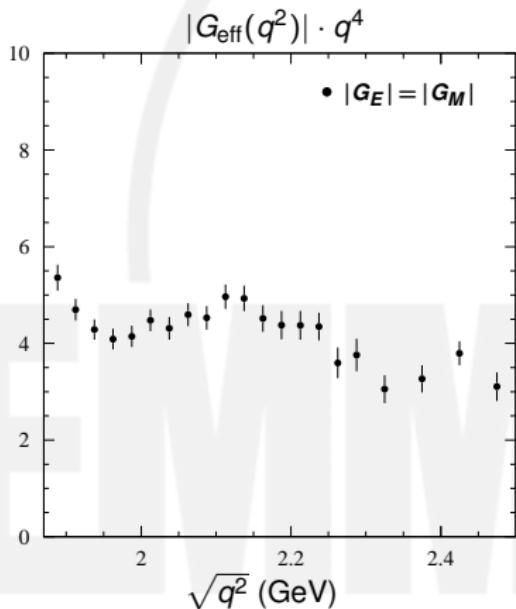
◎  $\left| \frac{q^2}{4M_N^2} \frac{F_2}{F_1} \right| \xrightarrow{|q^2| \rightarrow \infty} \left| \frac{G_E}{G_M} - 1 \right| = 2.0 \pm 0.2$

pQCD prediction

$$\left| \frac{G_E(q^2)}{G_M(q^2)} \right| \xrightarrow{|q^2| \rightarrow \infty} 1$$

# $|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421

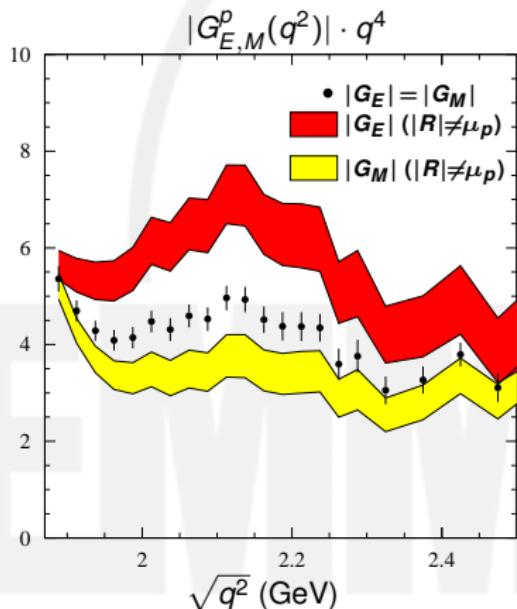


$$|G_{\text{eff}}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{1}{2\tau}\right)^{-1}$$

- ◆ Usually what is extracted from the cross section  $\sigma(e^+e^- \rightarrow p\bar{p})$  is the effective time-like form factor  $|G_{\text{eff}}^p|$  obtained assuming  $|G_E^p| = |G_M^p|$  i.e.  $|R| = \mu_p$

# $|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421



$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- ❖ Usually what is extracted from the cross section  $\sigma(e^+e^- \rightarrow p\bar{p})$  is the effective time-like form factor  $|G_{\text{eff}}^p|$  obtained assuming  $|G_E^p| = |G_M^p|$  i.e.  $|R| = \mu_p$
- \* Using our parametrization for  $R$  and the **BABAR** data on  $\sigma(e^+e^- \rightarrow p\bar{p})$ ,  $|G_E^p|$  and  $|G_M^p|$  may be disentangled

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# A SUM RULE FOR $G_M$

$\uparrow \text{Im}(z)$

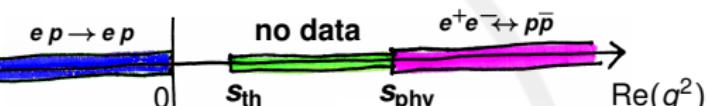
$\rightarrow \text{Re}(z)$

# Dispersion relations and sum rules

Geshkenbein, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- \* DR's connect space and time values of a form factor  $G(q^2)$

$$G(q^2) = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\text{Im} G(s) ds}{s - q^2}$$



## Drawbacks

- \* The imaginary part is not experimentally accessible
- \* There are no data in the unphysical region  $[s_{\text{th}}, s_{\text{phy}}]$
- \* We need to know the asymptotic behavior

- \* They applied the DR for the imaginary part to the function

$$\phi(z) = f(z) \frac{\ln G(z)}{z \sqrt{s_{\text{th}} - z}} \quad \text{with} \quad \int_0^{s_{\text{phy}}} f^2(z) dz \ll 1$$

## Advantages

- (\*) The DR integral contains the modulus  $|G(s)|$
- (\*) The unphysical region contribution is suppressed

## Drawback

- (\*) Zeros of  $G(z)$  are poles for  $\phi(z)$

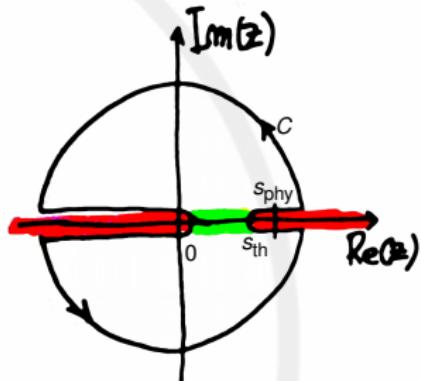
# Attenuated DR and sum rule

YF20 128

Assuming  $G(q^2) \neq 0$  and using the Cauchy theorem, we have the new DR

$$\oint_C \phi(z) dz = 0$$

$$\underbrace{- \int_{-\infty}^0 \frac{\text{Im}[f(t)] \ln G(t)}{t \sqrt{s_{\text{th}} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{\text{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s \sqrt{s - s_{\text{th}}}} ds}_{\text{Time-like}}$$



Convergence relation to find the asymptotic power-law behavior of  $G_M$

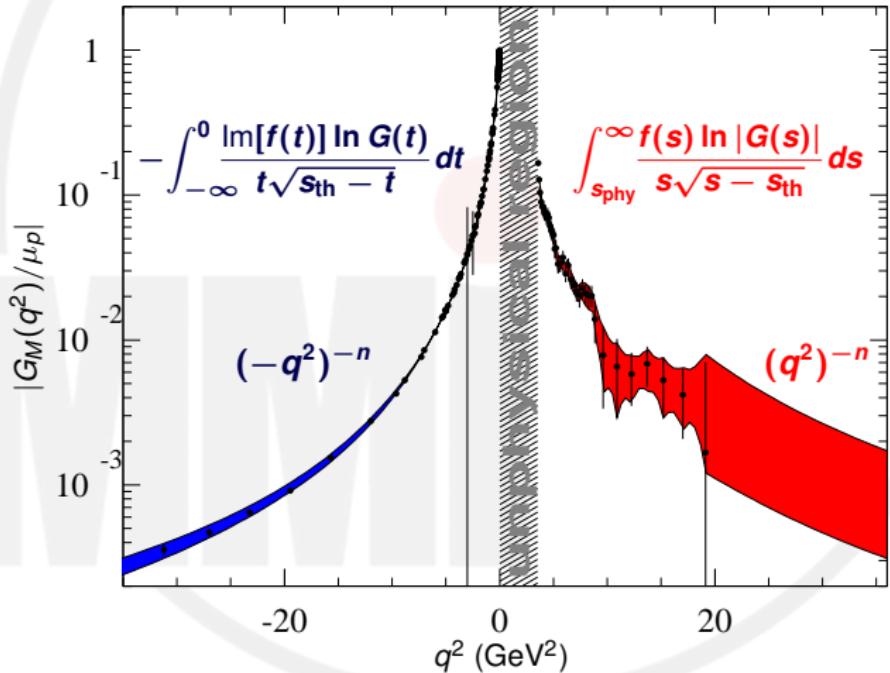
$$\underbrace{- \int_{-\infty}^0 \frac{\text{Im}[f(t)] \ln G(t)}{t \sqrt{s_{\text{th}} - t}} dt}_{\text{Space-like data} + (-t)^{-n}} = \int_{s_{\text{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s \sqrt{s - s_{\text{th}}}} ds \approx \underbrace{\int_{s_{\text{phy}}}^{\infty} \frac{f(s) \ln |G(s)|}{s \sqrt{s - s_{\text{th}}}} ds}_{\text{Time-like data} + s^{-n}}$$

**$n$  is the only free parameter**

# Sum rule: result for $G_M$

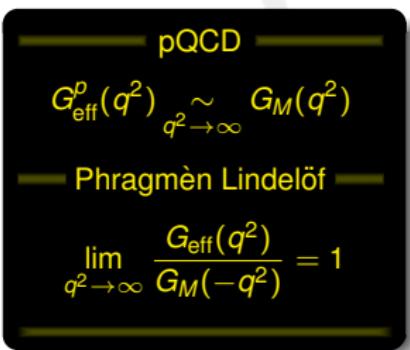
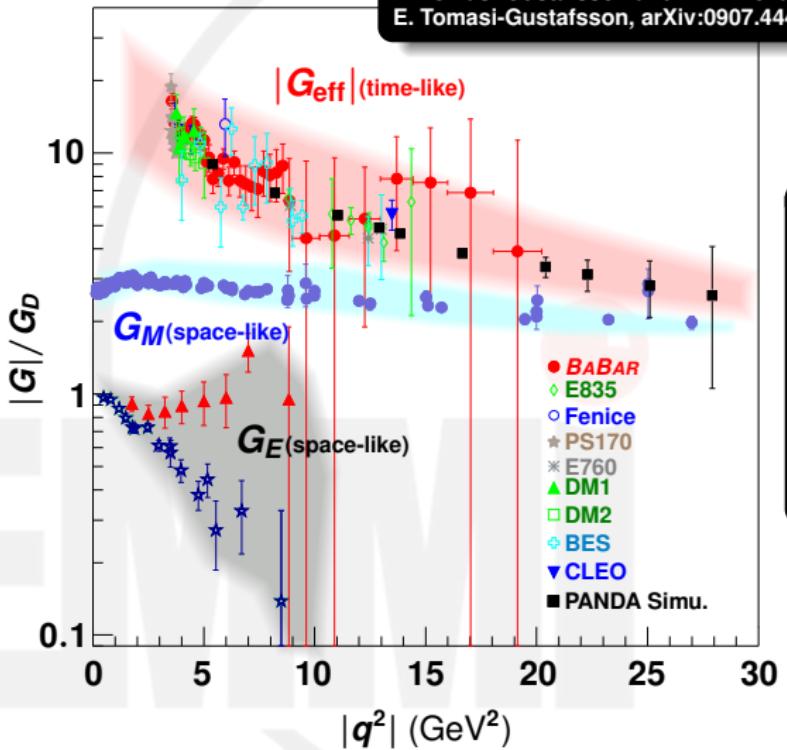
ChPhysC34 874

$$G_M(q^2) \underset{|q^2| \rightarrow \infty}{\propto} |q^2|^{-(2.27 \pm 0.36)}$$



# Asymptotic behaviors

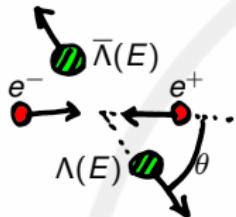
E. Tomasi-Gustafsson and M. P. Rekalo, PLB504, 291  
E. Tomasi-Gustafsson, arXiv:0907.4442



# PHASE AND MODULUS OF $G_E^1/G_M^1$

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# $\Lambda$ form factors



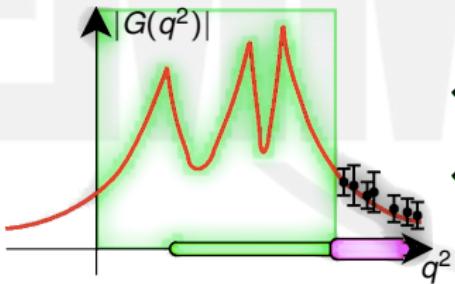
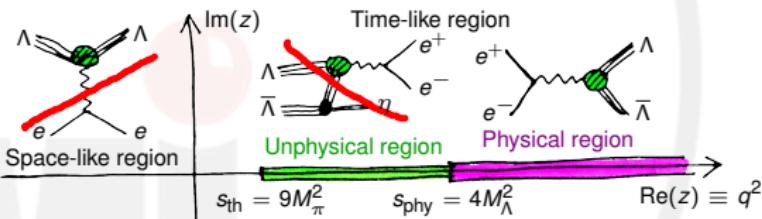
Same definitions, but for labels and Coulomb factor...  
 Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{16E^2} \left[ \left( 1 + \cos^2(\theta) \right) |G_M^\Lambda|^2 + \frac{1}{\tau} \sin^2(\theta) |G_E^\Lambda|^2 \right]$$

$$\tau = E^2 / M_\Lambda^2$$

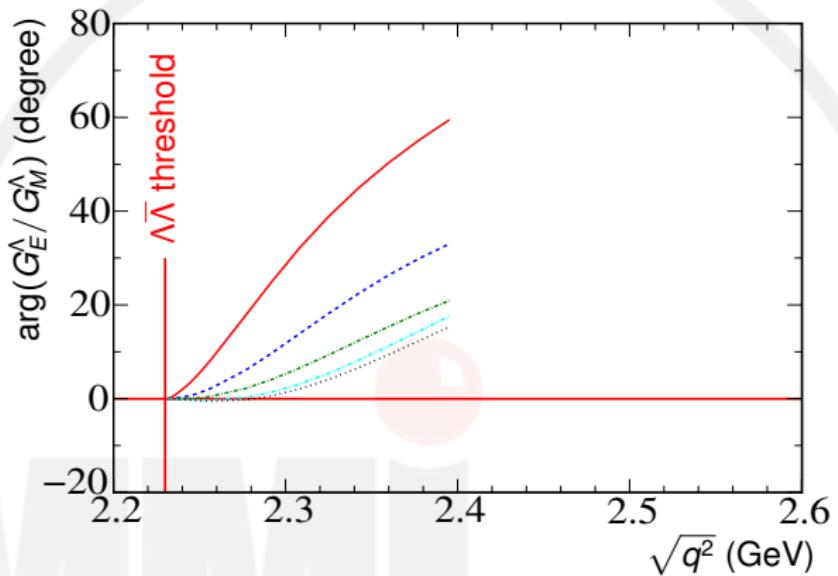
$$\beta = \sqrt{1 - 1/\tau}$$

- \* Same analyticity as for nucleons.
- \* Difficult to measure in space-like and unphysical regions.
- \* Relative phase from weak decay.



- ◆ Same unitarity and intermediate states contributions, but for the isospin.
- ◆ Form factors have not vanishing imaginary part above the theoretical threshold.

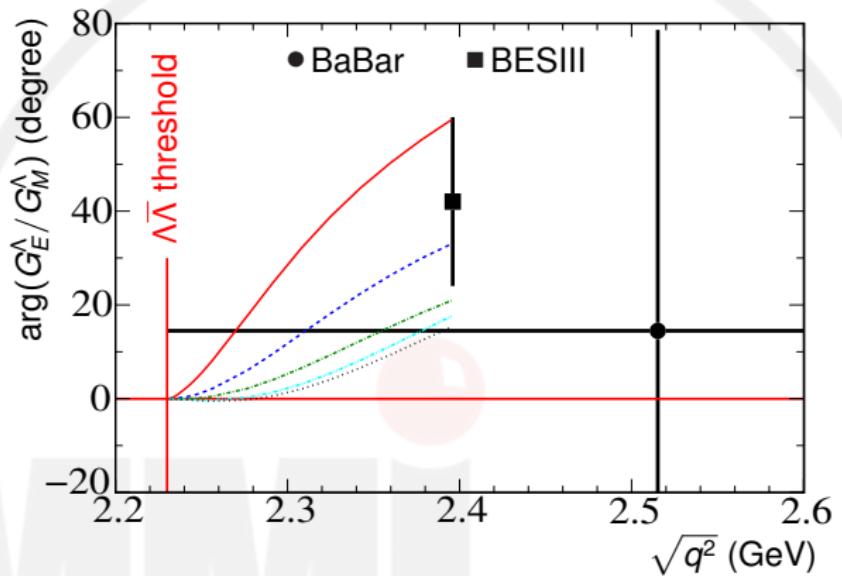
# Phase of $G_E^\Lambda/G_M^\Lambda$



▲ Theoretical prediction based considering only  $\Lambda\bar{\Lambda}$  FSI

[J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]

# Phase of $G_E^\Lambda/G_M^\Lambda$



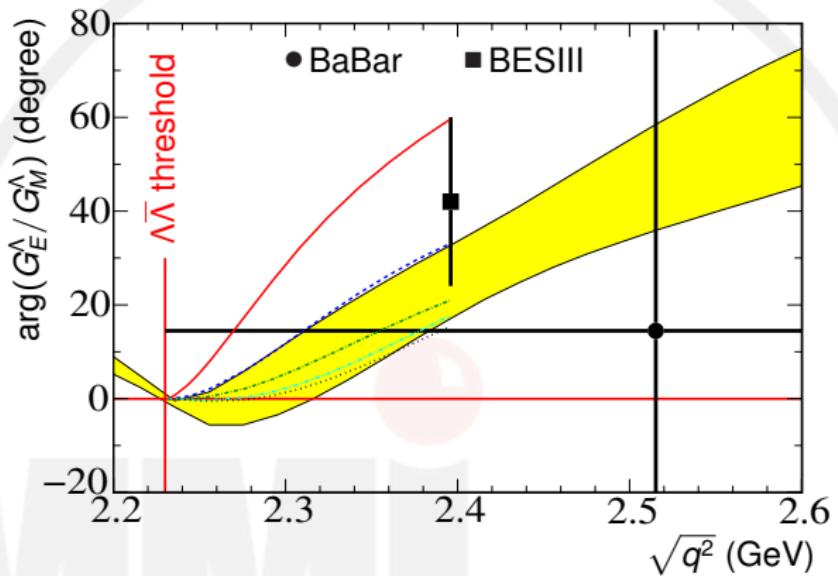
▲ Theoretical prediction based considering only  $\Lambda\bar{\Lambda}$  FSI

[J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]

◎ Data from BaBar and BESIII (preliminary)

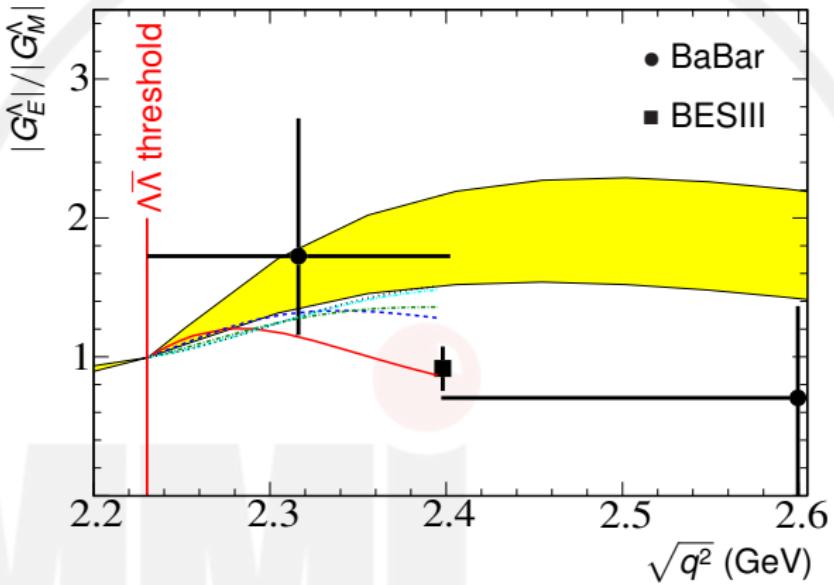
[PRD 76 (2007) 092006, K. Schönning, 668.WE-Heraeus-Seminar, 2018, Bad Honnef (Germany)]

# Phase of $G_E^\Lambda/G_M^\Lambda$



- ▲ Theoretical prediction based considering only  $\Lambda\bar{\Lambda}$  FSI  
[J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]
- ⌚ Data from BaBar and BESIII (preliminary)  
[PRD 76 (2007) 092006, K. Schönning, 668.WE-Heraeus-Seminar, 2018, Bad Honnef (Germany)]
- \* "Lambdization" of proton, i.e., proton results with  $\sqrt{q^2} \rightarrow \sqrt{q^2} + (M_\Lambda - M_p)$   
[EPJA32 421]

# Modulus of $G_E^\Lambda / G_M^\Lambda$



⚠ Theoretical prediction based considering only  $\Lambda\bar{\Lambda}$  FSI

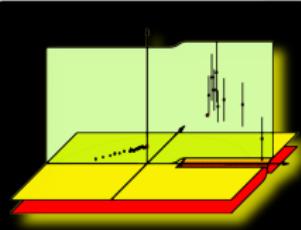
[J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]

🌀 Data from BaBar and BESIII (preliminary)

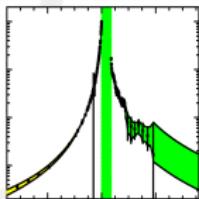
[PRD 76 (2007) 092006, K. Schönning, 668.WE-Heraeus-Seminar, 2018, Bad Honnef (Germany)]

\* "Lambdization" of proton, i.e., proton results with  $\sqrt{q^2} \rightarrow \sqrt{q^2} + (M_\Lambda - M_p)$   
[EPJA32 421]

# Final considerations



- ❖ Space-like zero for  $G_E$
- ❖ Time-like phase of  $G_E/G_M$  goes to  $180^\circ$
- \* Time-like form factors separation

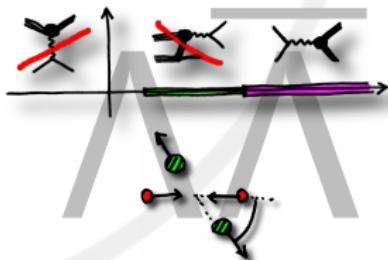


Space-like and time-like "fixed" data on  $|G_M^p|$  and analyticity

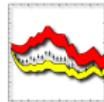


Confirmation of the pQCD asymptotic behavior

- ⌚ Relative phase and modulus for the ratio  $G_E^\Lambda/G_M^\Lambda$  agree with (only) FSI interaction
- ⌚ It gives information about space-like behavior only if the complex structure is due the intrinsic nature of the baryon-photon vertex
- ⌚ An asymptotic relative phase of  $180^\circ$  would imply a space-like zero for  $G_E^\Lambda$

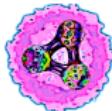


# “To do” list



**Time-like  $|G_E|$ - $|G_M|$  separation:**

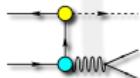
DR and data



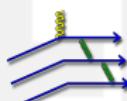
**Understanding threshold effect(s):**



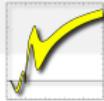
Dispersive analyses: integral equation, sum rule,...



Experimental observation in  $p\bar{p} \rightarrow \pi^0 I^+ I^-$   
[PRC75,045205(07)]

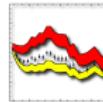


**Asymptotic behavior: DR and data for the phase**



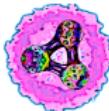
**Zeros  $\leftrightarrow$  phases: DR and data**

# "To do" list



Time-like  $|G_E|$ - $|G_M|$  separation:

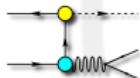
DR and data



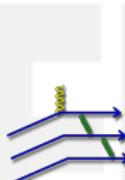
Understanding threshold effect(s):



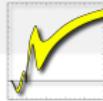
Dispersive analyses: integral equation, sum rule,...



Experimental observation in  $p\bar{p} \rightarrow \pi^0 I^+ I^-$   
[PRC75,045205(07)]



Asymptotic behavior: DR and data for the phase



Zeros  $\leftrightarrow$  phases: DR and data



Dalitz decays  $B^* \rightarrow B e^+ e^-$ :

- importance?
- interpretation?



Thank you