Quark and baryon number fluctuations from Dyson-Schwinger equations

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FAIR next generation scientists workshop; 6th edition
Grand Hotel Arenzano, Arenzano (Genoa), Italy
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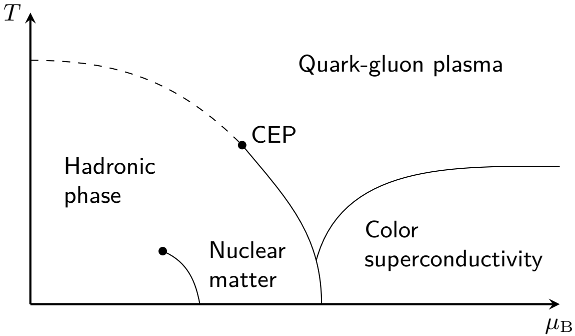
Theoretical approaches

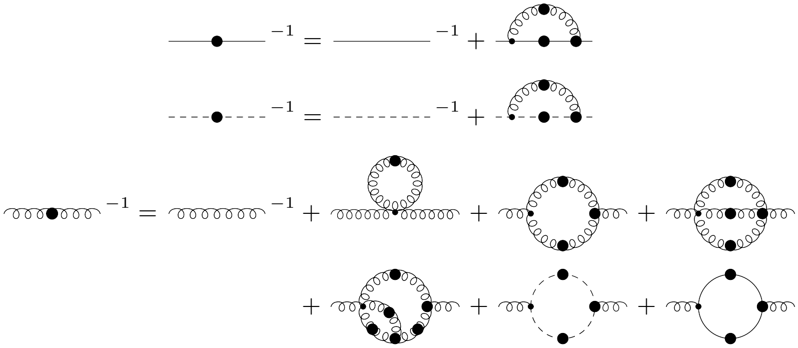
- Lattice QCD . . . limited to $\mu_B/T \lesssim 3$ due to sign problem
- Effective models . . . generalizable?
- Functional methods . . . all QCD degrees of freedom & no sign problem (but truncations are necessary)



Master equation

$$0 = \int \mathcal{D}\vec{\varphi}\, \frac{\delta}{\delta\varphi_i} \exp\left(-S_{\mathrm{E}}[\vec{\varphi}] + \int \mathrm{d}^4x\, \vec{J}\cdot\vec{\varphi} \right)$$

DSEs for propagators of QCD:



How to truncate?

Quark DSE

Main focus on quark propagator:

- Source for order parameters (chiral symmetry, confinement)
- Starting point for fluctuations

Dressed quark-gluon vertex:

- Studied in vacuum

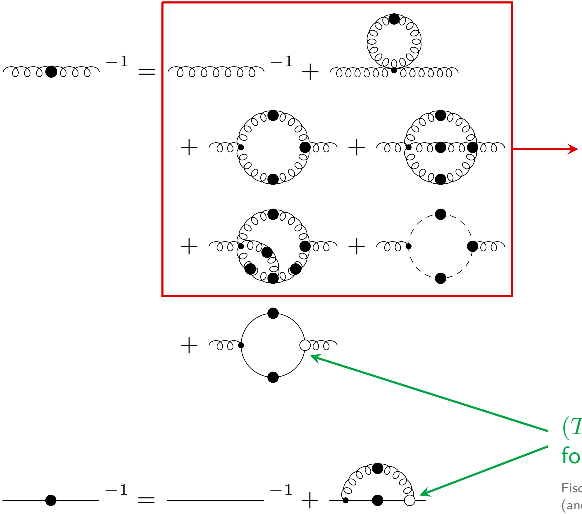
  Fischer, Williams; PRL 103, 122001 (2009)
  Mitter, Pawlowski, Strodthoff; PRD 91, 054035 (2015)
  Williams; EPJA 51, 53 (2015)
  Williams, Fischer, Heupel; PRD 93, 034026 (2016)
  Sternbeck *et al.*; PoS (LATTICE2016) 349
- $T \neq 0$: Ansatz based on STI and known perturbative behavior

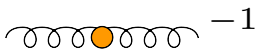
Dressed gluon propagator:

- Two strategies:
  - Model gluon propagator

    Qin, Chang, Chen, Liu, Roberts; PRL 106, 172301 (2011)
    Gao, Liu; PRD 94, 076009 (2016)
  - Explicit treatment of gluonic sector
- Here: Use the latter; consistent mass and flavor dependencies

How to truncate?





quenched, T-dependent
lattice gluon propagator

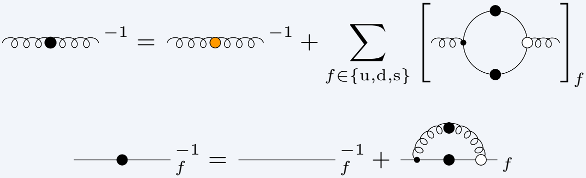
Fischer, Maas, Müller; EPJC 68, 165 (2010)
Maas, Pawlowski, von Smekal, Spielmann; PRD 85, 034037 (2012)

(T, μ)-dependent ansatz
for quark-gluon vertex

Fischer, Luecker, Welzbacher; PRD 90, 034022 (2014)
(and references therein)

Three-flavor QCD with Dyson-Schwinger equations

Final set of truncated DSEs



- Quenched lattice gluon propagator as input & unquenching via quark loops
- Non-trivial coupling between different quark flavors
- Vertex ansatz built along STI and perturbation theory

Result: Dressed (i.e. non-perturbative) quark and unquenched gluon propagators

Phase structure: $N_\mathrm{f} = 2 + 1$ at physical point

$S_f^{-1} =$ 

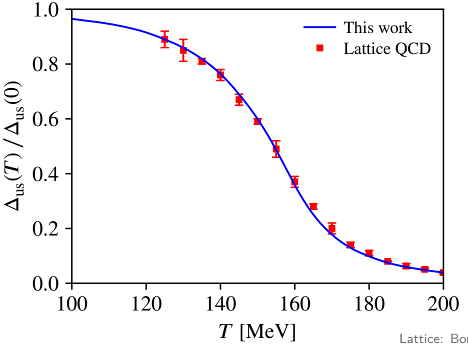
Chiral order parameter:

Quark condensate

$$\langle \bar{\psi}\psi \rangle_f = -Z_2^f Z_m^f \, \mathrm{Tr}[S_f]$$

Subtracted condensate

$$\Delta_{ff'} = \langle \bar{\psi}\psi \rangle_f - \frac{m_f}{m_{f'}} \langle \bar{\psi}\psi \rangle_{f'}$$



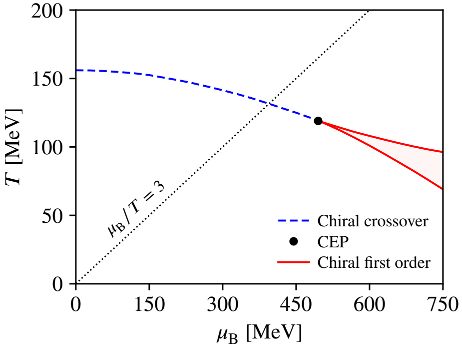
Lattice: Borsányi *et al.*; JHEP 2010, 09:073

Phase structure: $N_\mathrm{f} = 2+1$ at physical point

Main result

Second order CEP at large chemical potential:

$$\mu_\mathrm{B}^\mathrm{CEP} = 495\,\mathrm{MeV}\,, \quad T^\mathrm{CEP} = 119\,\mathrm{MeV}$$



- Ratio: $\mu_\mathrm{B}^\mathrm{CEP}/T^\mathrm{CEP} \approx 4.16$
- Crossover temperature: $T_\mathrm{c}^{(\mu=0)} = 156\,\mathrm{MeV}$

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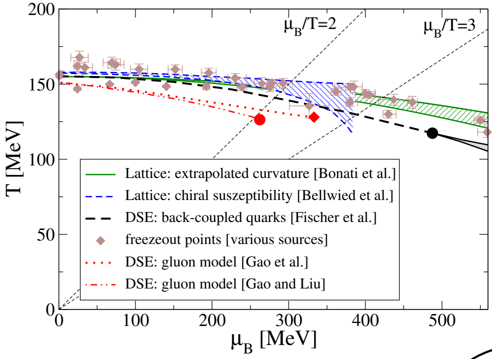

Figure taken from:
Fischer; PPNP 105, 1 (2019)

- Ratio: $\mu_\mathrm{B}^\mathrm{CEP} / T^\mathrm{CEP} \approx 4.16$
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backcoupling important!

Contact to experiment: Fluctuations

Fluctuations from QCD's grand-canonical potential

$$\chi_{ijk}^{\mathrm{uds}} = -\frac{1}{T^{4-(i+j+k)}} \frac{\partial^{i+j+k}\,\Omega}{\partial\mu_{\mathrm{u}}^{i}\,\partial\mu_{\mathrm{d}}^{j}\,\partial\mu_{\mathrm{s}}^{k}}$$

Relation to conserved charges:
("quark basis $\leftrightarrow$ phenomenological basis")

$$\mu_{\mathrm{u}} = \mu_{\mathrm{B}}/3 + 2\mu_{\mathrm{Q}}/3$$

$$\mu_{\mathrm{d}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3$$

$$\mu_{\mathrm{s}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3 - \mu_{\mathrm{S}}$$

Ratios related to experimental quantities, e.g.:

$$\frac{\chi_3^{\mathrm{B}}}{\chi_1^{\mathrm{B}}} = \frac{S_{\mathrm{B}}\,\sigma_{\mathrm{B}}^3}{M_{\mathrm{B}}}\,, \qquad \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}} = K_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2$$

Sensitive to phase structure: $\chi_2^{\mathrm{B}} \sim \xi^c$ (with $c > 0$) and $\xi \to \infty$ at CEP

Review: Luo, Xu; Nucl. Sci. Tech. 28, 112 (2017)

Fluctuations from DSEs: Formalism

Grand-canonical potential from 2PI formalism:

Cornwall, Jackiw, Tomboulis; PRD 10, 2428 (1979)

$$\Omega = -\frac{T}{V}\left(\mathrm{Tr}\log\frac{S^{-1}}{T} - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}S\right] + \Phi_{\mathrm{int}}[S]\right) + \Omega_{\mathrm{YM}}$$

First order fluctuation (density):

$$\rho_f = T^3\chi_1^f = -\partial\Omega/\partial\mu_f = -Z_2^f\,\mathrm{Tr}[\gamma_4 S_f]$$

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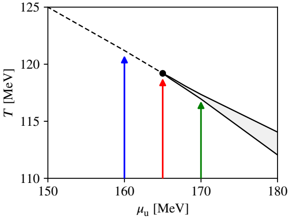
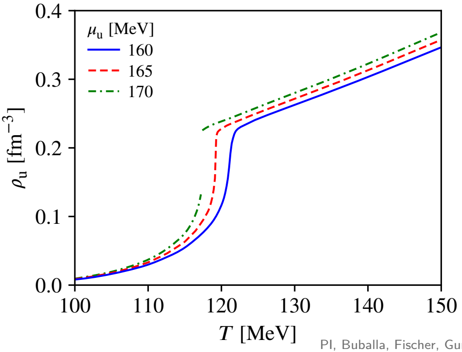
Behavior around CEP:
Consistent with effective models

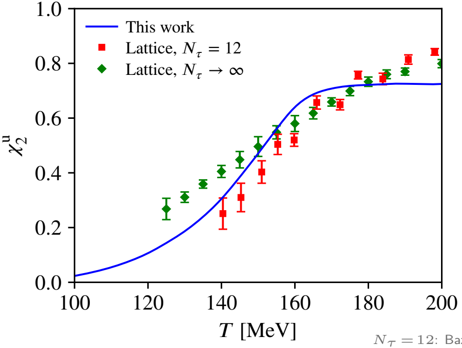
e.g. Schaefer, Wambach; PRD 75, 085015 (2007)
Buballa; Phys. Rep. 407, 205 (2005)





PI, Buballa, Fischer, Gunkel; in prep.

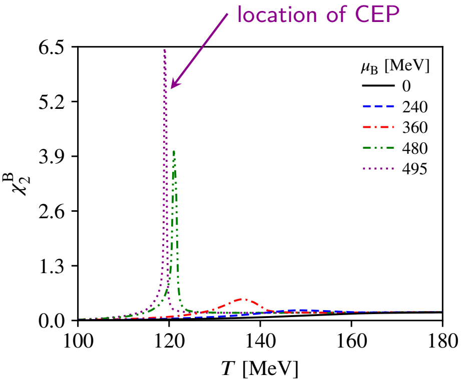
Fluctuations from DSEs: Comparison with lattice

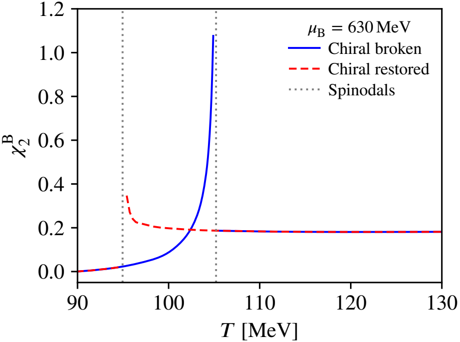


$N_\tau = 12$: Bazavov *et al.*; PRD 85, 054503 (2012)
$N_\tau \to \infty$: Borsányi *et al.*; JHEP 2012, 01:138

- Same ballpark as lattice results
- Saturation below Stefan-Boltzmann limit: Truncation artifact

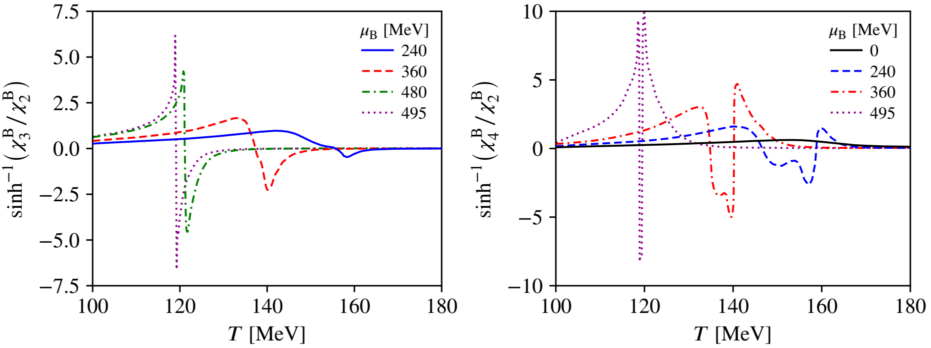
PI, Buballa, Fischer, Gunkel; in prep.





- Divergence of fluctuation indicates CEP
- Jump at phase boundary in first order region

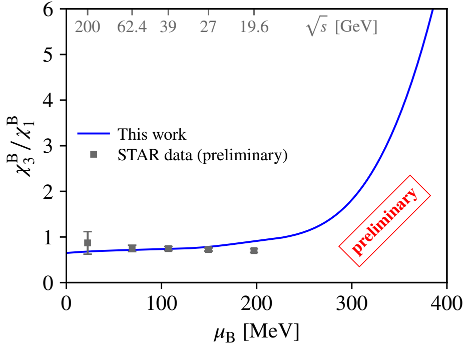
PI, Buballa, Fischer, Gunkel; in prep.



- Approach to CEP is clearly visible in ratios
- Higher order fluctuations more sensitive to phase structure and CEP
- Qualitative agreement with effective models

e.g. Fu, Wu; PRD 82, 074013 (2010)

Fluctuations from DSEs: Comparison with experiment



Data: Luo, PoS CPOD2014, 019 (2014)
Thäder, NPA 956, 320 (2016)
$0 \sim 5\%$ centrality; $0.4 < p_t/\mathrm{GeV} < 2.0$; $|y| < 0.5$

- Same ballpark as data from STAR at small μ_B
- Increasing μ_B: Ratio χ_3^B/χ_1^B rises significantly due to the CEP
- Beware: Finite volume effects? Non-equilibrium effects? ...

Summary and outlook

QCD phase diagram with DSEs ($2+1$ flavors at physical point):

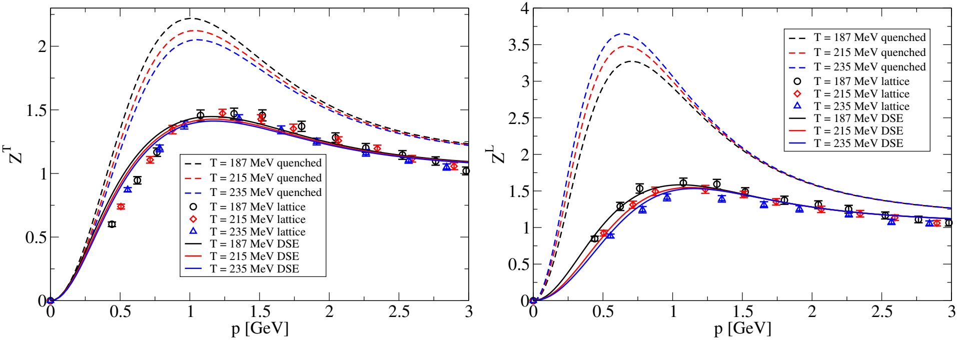
- Backcoupling of quarks onto gluons important
- CEP at large chemical potential: $\mu_{\mathrm{B}}^{\mathrm{CEP}}/T^{\mathrm{CEP}} \approx 4.16$
- Combined result from functional methods (DSE + FRG):
  No CEP at $\mu_{\mathrm{B}}/T < 3$

Fluctuations with DSEs:

- First calculation in a truncation with reasonable CEP location
- $\mu = 0$: Same ballpark as lattice results
- $\mu \neq 0$: Qualitative agreement with effective models
- Critical behavior clearly seen
- Track singularity behavior ⇔ Map phase diagram & locate CEP

Backup slides

Gluon at non-zero temperature



- Very good agreement of DSE and lattice results
- DSE prediction verified by lattice

Figures taken from: Fischer; PPNP 105, 1 (2019)
DSE results: Fischer, Luecker; PLB 718, 1036 (2013)
Lattice results: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck; PRD 87, 114502 (2013)

Ansatz for quark-gluon vertex

$$S_f^{-1}(p) = \mathrm{i}\tilde{p}_4\gamma_4 C_f(p) + \mathrm{i}\vec{\not{p}} A_f(p) + B_f(p)$$

Vertex ansatz:

$$\Gamma_\nu^f(q,p,k) = \widetilde{Z}_3\,\Gamma(k^2)\,\gamma_\nu \left(\delta_{4\nu}\,\frac{C_f(q)+C_f(p)}{2} + (1-\delta_{4\nu})\,\frac{A_f(q)+A_f(p)}{2} \right)$$

Phenomenological dressing function:

$$\Gamma(k^2) = \frac{d_1}{d_2+k^2} + \frac{1}{1+k^2/\Lambda^2}\left(\frac{\alpha_s\beta_0}{4\pi}\log\left(1+k^2/\Lambda^2\right)\right)^{2\delta}$$

- Abelian STI (leading term of Ball-Chiu vertex)
  Ball, Chiu; PRD 22, 2542 (1980)
- Perturbative running in the ultraviolet
- Ansatz for IR
  - d_1 fixed via T_c
  - d_2 fixed to match scale of quenched lattice gluon data