Nuclear Pasta Matter in the Inner Crust of Neutron Stars

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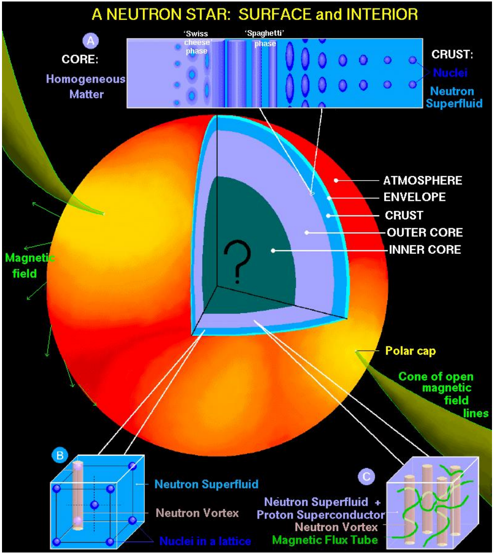
Outline

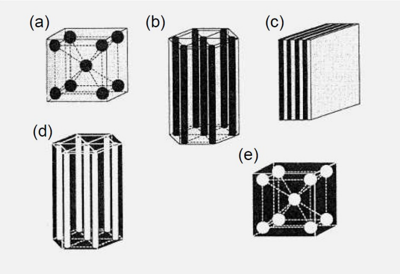
Nuclear Pasta Matter inside a Neutron Star



Structure of nuclear pasta important for e.g.:

- Neutrino opacity (neutron star cooling)
- Transport properties (electrons)
- Tidal deformability
- Rotation of crust and core (shear viscosity)

Nuclear Pasta Matter

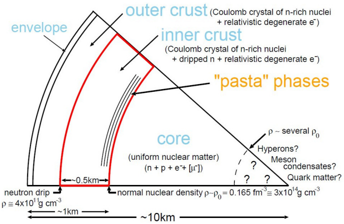


Basic nuclear pasta shapes (from Oyamatsu)

- Nuclear pasta appears at approx. $\rho = 1/8\,\rho_0\,, \quad \rho_0 \approx 0.16\,\mathrm{fm}^{-3}$.
- Pasta shapes appear due to the competition of Coulomb and nuclear surface force (leads to periodic structures).
- Several shapes appear like “spaghetti” (rod-like) shapes (b) or “lasagna” (slab-like) shapes (c).
- At higher densities inverted pasta appears (holes have rod-like or sphere-like structure) (d and e).

Sites for Nuclear Pasta

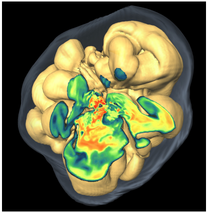
Neutron star



schematic picture of a neutron star (Watanabe et. al.)

- In inner crust of neutron stars.
- Proton fraction of $X_p \lesssim 0.1$.
- Thickness of the layer is about 100 m.

Core Collapse Supernovae

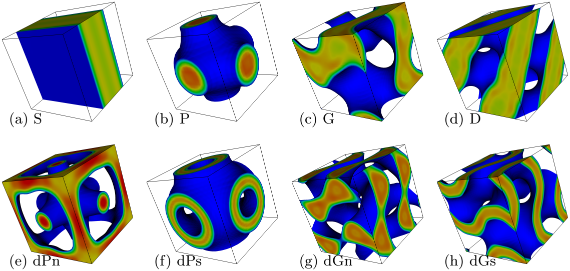


Supernova calculation (Janka et al.)

- After 100 msec central density reaches ρ_0.
- Core reaches temperatures of several MeV.
- Total amount ∼20% of total mass.

→ periodic boundary conditions

Minimal Surface Pasta Candidates



Single minimal surface (top) and double minimal surface configurations under study.

- Basic Slab (a) and three triply periodic minimal surface configurations.
- Shapes occupying roughly half the space.

- Minimal surfaces divide space into two halves:

  Single: One half-space is filled with matter.

  Double: Either both half-spaces are partly filled (network-like), or region around dividing surface (surface-like) is filled with matter.

The Hartree-Fock Code Sky3D

- Computational box on equidistant lattice with usually about 1 fm lattice spacing.
- Full 3d, no symmetry assumptions.
- Using FFT for derivatives.
- State of the art Skyrme forces.
- Exact treatment of Coulomb force assuming a constant electron background.
- Code is now parallelized and good scaling has been accomplished with up to $\approx$ 1500 cores for large problems. *Afibuzzaman, BS, Aktulga, CPC 223, 34-44*

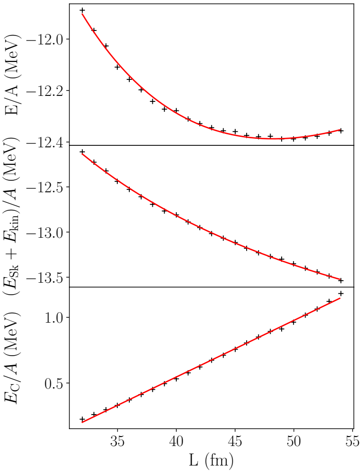
Static Iterations

Solving the Schrödinger equations for a single Slater determinant in the mean-field approximation.

$$\hat{h}\psi_\alpha(\boldsymbol{r}) = \epsilon_\alpha \psi_\alpha(\boldsymbol{r})$$

Energy Minimization

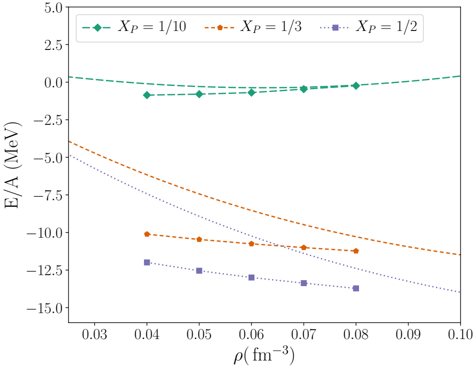
- Plane waves as initial state
- Imposing configuration by evolving wave functions in guiding potential
- Guiding potential has form of nodal approximation.
- Only for the first 200 iterations.
- Afterwards self-consistent DFT.
- Minimizing energy w.r.t. periodic length.



Double Gyroid (dGn) for $X_P = 1/2$, $\rho = 0.05\,\mathrm{fm}^{-3}$

Nuclear Slabs vs. Uniform Matter

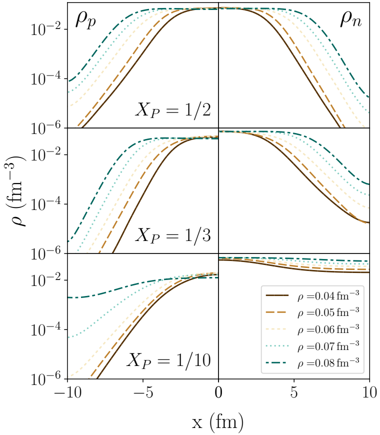
- Energy of slab is lower than for uniform matter.
- Difference is up to 4 MeV for $X_P = 1/2$ and $1/3$ but small for $X_P = 1/10$.
- Energies converge at high densities.



Energy per nucleon for uniform matter (lines) and slabs (lines with points) with the same Skyrme interaction TOV-min.

Slab Densities

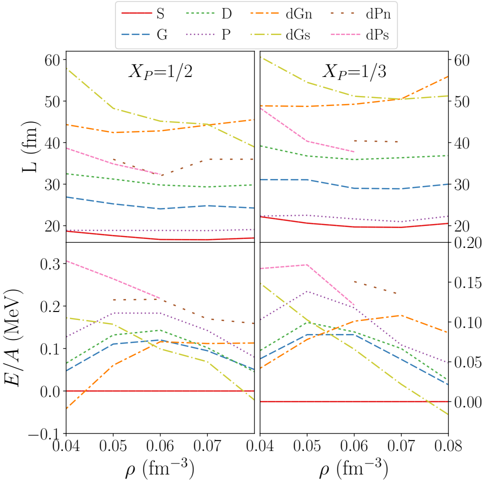
- Nucleon background is largest for highest density.
- Neutron background increases for smaller X_P.
- For $X_P = 1/10$ and $\rho = 0.08\,\mathrm{fm}^{-3}$ density is almost uniform.



Density profiles of slab configurations.

Optimal Box Lengths and Energies

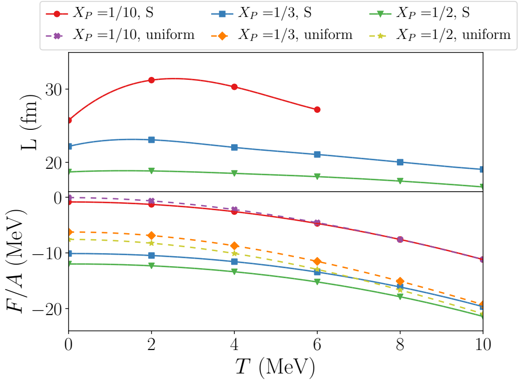
- Box (periodic) length smaller for single structures.
- Double Gyroid with up to $L \approx 60\,\mathrm{fm}$.
- Double P is not stable for the whole density range.
- Energies are very close to slab $\Delta E < 0.3\,\mathrm{MeV}$
- Double gyroids can be even lower in energy than slab, but other configurations not studied here, can take over.



Optimal box lengths (top) and energy per nucleon (bottom) for $X_P = 1/2$ (left) and $1/3$ (right).

Optimal Box Lengths and Free Energy

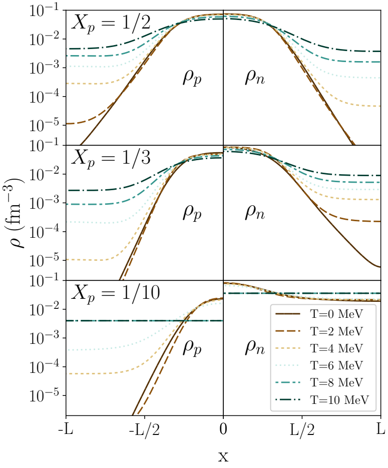
- Minimizing Free energy: $F = E - TS$
- Box lengths increases for moderate temperature (about $2\,\mathrm{MeV}$) and then decrease for larger temperatures.
- Free energy of slab approaches uniform matter for high temperatures.



Finite temperature optimal box lengths and free energy per nucleon for slab and uniform matter.

Slab Densities

- With increasing temperature background density becomes larger, maximum density lower.
- Densities approach uniform distribution.
- For larger X_P configuration is still different from uniform matter, even at $T = 10\,\mathrm{MeV}$.
- For $X_P = 1/10$ density is uniform for $T > 6\,\mathrm{MeV}$.



Slab density profiles for finite temperature at $\rho = 0.04\,\mathrm{fm}^{-3}$.

Neutrino Scattering Cross Section

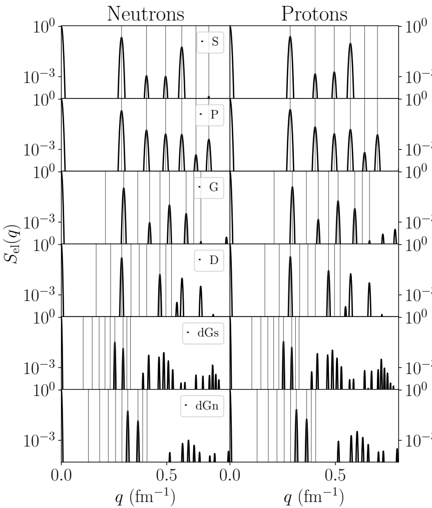
- Neutrino cross section per nucleon can be described as

$$\frac{1}{N}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = S(\boldsymbol{q})\frac{G_F^2 E_\nu^2}{4\pi^2}\left[c_a^2(3-\cos\theta)+c_v^2(1+\cos\theta)\right]$$

- Neutrino scattering is proportional to the structure factor $S(\boldsymbol{q})$.
- Structure factor is the only ingredient from nuclear matter.
- Structure factor depends on the reaction (elastic/inelastic) (charge neutral/charge current).

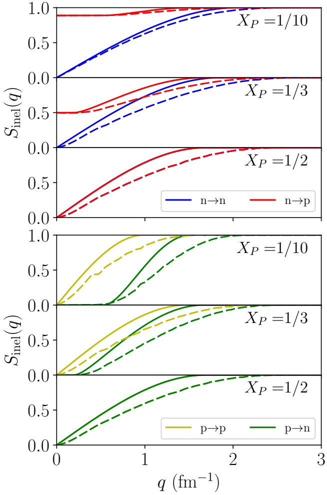
Elastic Structure Factors (Preliminary)

- $S_{\mathrm{el}}(\mathbf{q}) = \frac{1}{N^2}|F(\boldsymbol{q})|^2$
  F is the form factor (Fourier transform of density).
- While Form factors have different structures, the first peak is at very similar position (around $q = 0.35\,\mathrm{fm}^{-1}$).
- Rest of the spectrum depends on exact shape.
- Double configurations show a second dominant peak right after the first.



Inelastic Structure Factors (Preliminary)

- $S_{\mathrm{inel}} \sim \int f_{\mathrm{i}}(\boldsymbol{k}) \cdot (1 - f_{\mathrm{f}}(\boldsymbol{k} + \boldsymbol{q}))\mathrm{d}^3 k$
- Measure for the phase space available for scattering.
- Uniform: solid lines.
  Slab: dashed lines.
  $\rho = 0.04\,\mathrm{fm}^{-3}$
- Pasta decreases structure factor by up to 30%.



Conclusion

- Studied a variety of pasta configurations around mid density.
- Pasta lowers energy w.r.t. uniform matter.
- All pasta shapes are rather close in energy.
- Finite temperature changes periodic length of pasta, but pasta can exist up to high temperatures.
- While elastic structure factors vary in its structure significantly, the first dominant peaks are very close to each other.
- Inelastic structure factors are reduced up to 30% for nuclear pasta.

Thank you for your attention

Collaborators:

- Gabriel Martinez-Pinedo (GSI / TU Darmstadt)
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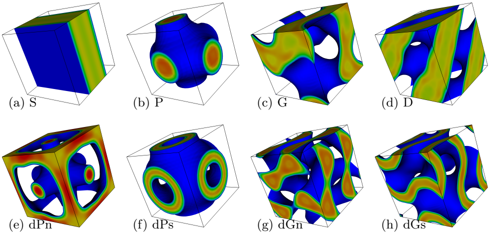
Minimal Surface Parametrization



Nodal approximations:

$$\phi_S = \cos X$$

$$\phi_P = \cos X + \cos Y + \cos Z$$

$$\phi_G = \cos X \sin Y + \cos Y \sin Z + \cos Z \sin X$$

$$\phi_D = \cos X \cos Y \cos Z + \cos X \sin Y \sin Z + \sin X \cos Y \cos Z + \sin X \sin Y \cos Z$$

- Dividing surface: $\phi_i = 0$.
- Single domain: $\phi_i < 0$.
- Double surface-like domain: $|\phi_i| < t$.
- Double network-like domain: $|\phi_i| > t$.

Finite Temperature Calculations

At finite temperature single-particle states are occupied according to the Fermi distribution.

$$f_{\alpha,q} = \left[\exp\left(\frac{E-\mu}{T}\right)+1\right]^{-1}$$

Density can now be derived as

$$\rho_q(\boldsymbol{r}) = \sum_{\alpha\in q} f_{\alpha,q}|\psi_\alpha(\boldsymbol{r})|^2 \quad .$$

Instead of internal energy, at finite temperature the Free energy is minimized.

$$F = E - TS$$

The entropy can be obtained from

$$S = -\sum_{\alpha}\left[f_{\alpha,q}\ln(f_{\alpha,q}) + (1-f_{\alpha,q})\ln(1-f_{\alpha,q})\right] \quad .$$

Electron Screening

Until now we considered a uniform electron background. However, free electrons can screen the protons.
The Poisson equation changes to

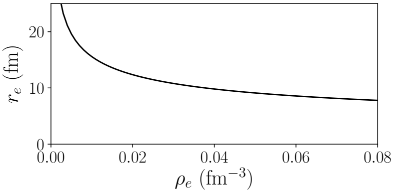
$$\left(\boldsymbol{\nabla}^2 + k_{\mathrm{TF}}^2\right) \phi(\boldsymbol{r}) = -4\pi \rho_p(\boldsymbol{r}) \quad ,$$

and can be solved in Fourier space more easily:

$$\tilde{\phi}(\boldsymbol{k}) = \frac{-4\pi \tilde{\rho}_p(\boldsymbol{k})}{(-\boldsymbol{k}^2 + k_{\mathrm{TF}}^2)} \quad .$$

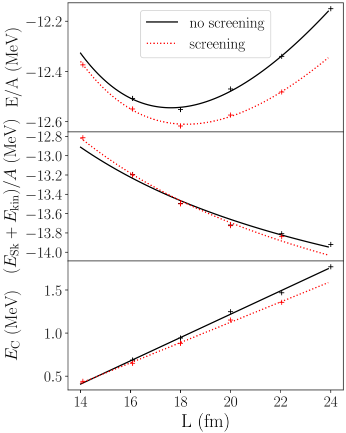
For a point charge the solution is

$$\phi(r) = \frac{Q}{r} \exp(-r/r_e)$$



Electron Screening: Slabs

- Potential and kinetic energy do not change drastically.
- Coulomb energy is reduced.
- The result is a larger optimal box length and a slightly lower energy.
- $\frac{\Delta E}{E} < 1.0\%$
- $\frac{\Delta L}{L} < 5.0\%$

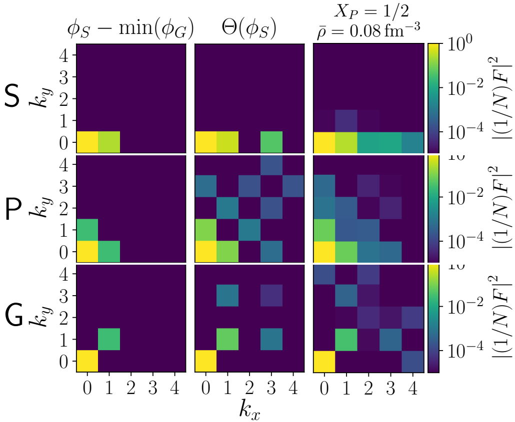


Slab at $X_P = 1/2$ and $\rho = 0.05\,\mathrm{fm}^{-3}$.

TPMS Form Factor Systematics (Preliminary)

- $F(k) = \mathrm{FT}(\rho(r))$.
- First dominant peak determined from nodal approximation of configuration.
- Other structure determined from details of density distribution.
- First dominant peak shifts for double structures to $\boldsymbol{k}_{\mathrm{double}} = 2\boldsymbol{k}_{\mathrm{single}}$

$\phi_S = \cos X$
$\phi_P = \cos X + \cos Y + \cos Z$
$\phi_G = \cos X \sin Y + \cos Y \sin Z + \cos Z \sin X$.



One quadrant of Fourier components of density of Gyroid. Soft model (left), hard model (center), and real Pasta calculation (right).

Structure Factors for Finite Temperature