The gravitational wave frequencies and damping timescales of f-modes in neutron stars

Universal and empirical relations

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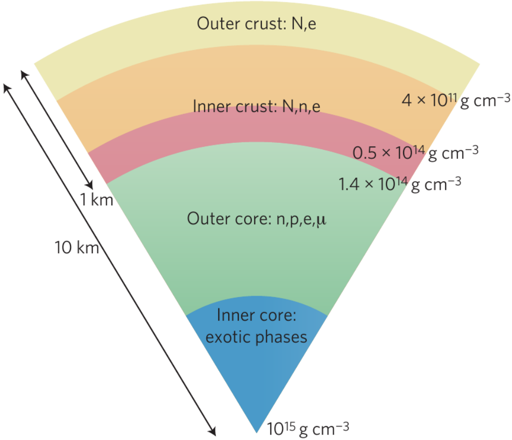
Table of Contents

Neutron stars

- Evolutionary scenario of massive stars, typically $M \sim 8 - 20\ M_\odot$
- Central densities typically a few times nuclear saturation density & masses $1 - 3\ M_\odot$
- Compact stars, GR needed to describe them



Newton 2013

Motivation

- Perturbed neutron stars have modes which can be damped through gravitational waves (GWs).
- Relevant/promising systems:
  - Binary neutron star mergers ($t_{\text{lifetime}} > \tau_{GW}$)
  - Core-collapse supernovae
  - Rotating deformed stars
  - Oscillations/instabilities in mature stars
- The f-mode should be the most efficient emitter in many cases.
- Based on Gen. Rel. Gravit. (2018) 50:12 (GL & Stergioulas)

Modes in Newtonian theory - I

Main equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{(mass conservation)}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = -\frac{\nabla p}{\rho} - \nabla \Phi \qquad \text{(momentum conservation)}$$

$$\nabla^2 \Phi = 4\pi G \rho \qquad \text{(Poisson equation)}$$

where ρ the density, p the pressure, $\mathbf{v}$ the velocity and Φ the gravitational potential.

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Simplest case $\rightarrow$ radial mode:

- Small Lagrangian perturbations $\delta r(r,t),\ \delta\rho(r,t)$
- Boundary conditions: $\delta r = 0|_{r=0}$ and $\Delta p = 0|_{\text{surface}}$

Modes in Newtonian theory - II

Non-radial modes: Additional θ, ϕ dependence. The displacement vector is decomposed into spherical harmonics $Y^{lm}(\theta, \phi)$ and perturbations look like

$$\delta\rho(r, \theta, \phi, t) = \delta\rho(r)\ Y^{lm}(\theta, \phi)\ e^{i\omega t}.$$

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Non-radial mode families:

- $p-$modes: restored by pressure
- $f-$mode: first/nodeless $p-$mode
  For a non-rotating uniform density neutron star scales as

$$\omega \sim \sqrt{M/R^3}.$$

- $g-$modes: driven by buoyancy in non-isentropic stars (gradients)
- Other important families: r-modes, w-modes, etc

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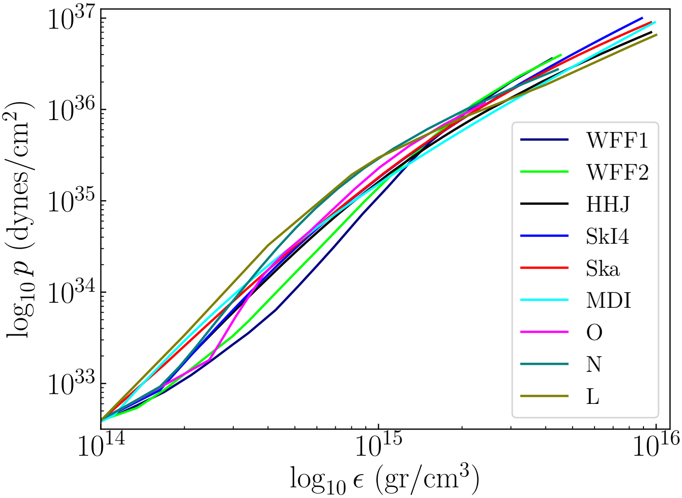
Picture in General Relativity

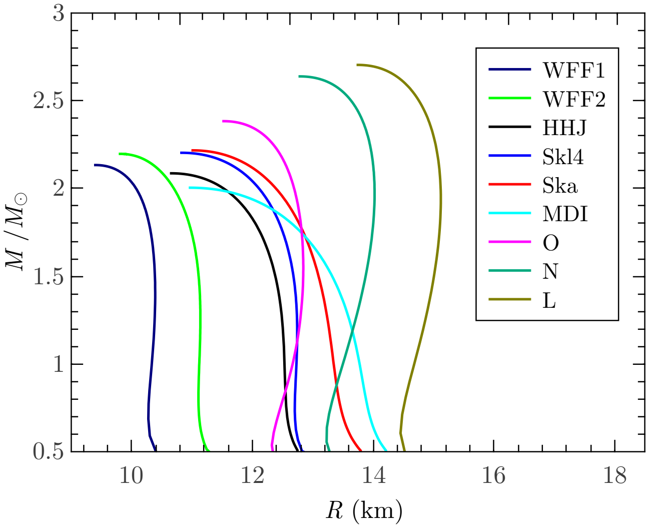
- Background:
  - Metric: $ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$.
  - Perfect fluid: $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p g_{\mu\nu}$
  - TOV system to determine mass $m(r)$, pressure $p(r)$, relativistic energy density $\epsilon(r)$, etc.
  - Equation of State (EoS) to close the system

We took 9 different EoS into account and investigate the $(l, m) = (2, 0)$ case.



Mass-Radius diagram

The M-R diagrams produced by those EoS.



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  - TOV system to determine mass $m(r)$, pressure $p(r)$, relativistic energy density $\epsilon(r)$, etc.
  - Equation of State (EoS) to close the system
- Perturbations:
  - Expand everything in spherical harmonics e.g.

$$\delta\epsilon = r^l \delta\epsilon^{lm} Y_{lm} e^{i\omega t}.$$

  - For $l > 1$ non radial oscillations emit gravitational waves.
  - The modes are quasi-normal $\omega = \sigma + \frac{i}{\tau_{\mathrm{GW}}}$, where $\sigma = 2\pi f$.
  - Note:

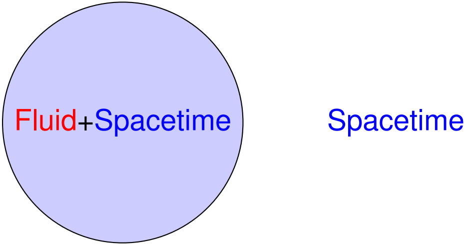
$$e^{i\omega t} = e^{i\sigma t} e^{-t/\tau_{\mathrm{GW}}}.$$

Problem outline

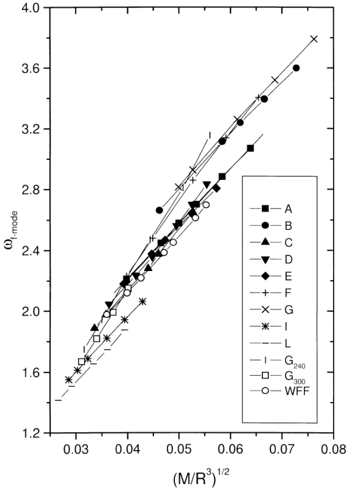
Main equations:

$$\delta(\nabla_\mu T^{\mu\nu}) = 0 \qquad \text{(Fluid)}$$
$$\delta G^{\mu\nu} = 8\pi \delta T^{\mu\nu} \qquad \text{(Spacetime)}$$

In order to determine $(\delta p, \delta\epsilon, \delta u^\mu, \delta g_{\mu\nu})$ a total of $2 + 2$ functions are needed.



Solutions for a discrete set of ω's [$\rightarrow$ $(f, \tau_{\rm GW})$] for purely outgoing waves.



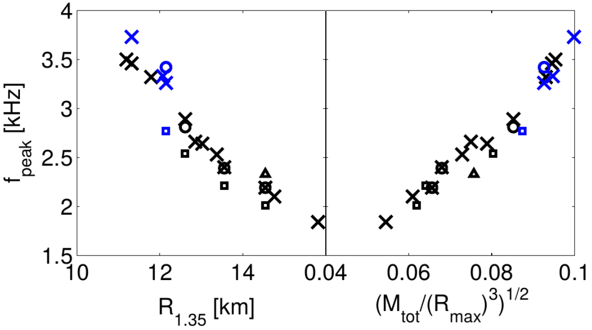
Andersson & Kokkotas 1998

Here:

$$f(\text{kHz}) \approx 0.78 + 1.635 \left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2},$$

where $\bar{M} = \dfrac{M}{1.4\, M_\odot}$ and $\bar{R} = \dfrac{R}{10\text{ km}}$.

Empirical relations for BNS



Bauswein & Janka 2012

E.g. for the left panel:

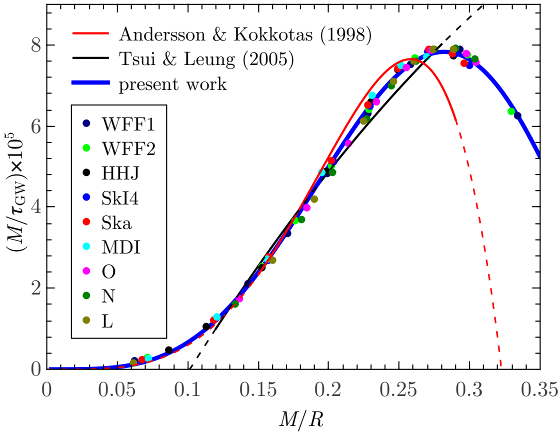
$$R_{1.35} = 21.28(f_{peak})^{-0.5276} + 0.3394.$$

Universal relation for τ_{GW} of the $f-$mode

The gravitational damping timescale satisfies the empirical relation

$$M/\tau_{\rm GW} = 0.112(M/R)^4 - 0.53(M/R)^5 + 0.628(M/R)^6,$$

which is accurate in the whole range $0 < M/R < 0.33$.



Conclusions

- An introduction to the physics of the $f-$mode was made.
- We discussed empirical relations between the frequency of the $f-$mode and the mean density of the corresponding configuration.
- We presented a universal empirical relation relating τ_{GW} scaled by M^3/R^4 to the compactness M/R, which is accurate in the range $0 < M/R < 0.33$.

Thank you for your attention!