Inhomogeneous phases of the 2+1-dimensional Gross-Neveu model in the limit of infinite flavors

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Motivation

- Phase diagram of QCD still not fully explored
- No first principle approach
  (e.g. perturbation theory only in the high energy regime, lattice QCD only at vanishing μ due to sign problem etc.)
- Theorists: Application of effective models with several similarities to QCD

Motivation

- Aim: Computation of phase diagrams in QCD inspired models in the large-N limit and exploration of chiral symmetry breaking
- Path to go: Develop numerical methods on the analytically solved Gross-Neveu model in 1+1 dimensions
- Actual work: Inhomogeneous phases & phase diagram of 2+1-dimensional GN model

Outline

1. The Gross-Neveu model in the large-N limit
   - Properties
   - Chiral symmetry & Phases
2. Numerics
3. Inhomogeneous phases in 2+1 dimensions

Outline

Gross-Neveu model

- Why is it interesting for us ?
  - Consists of interacting fermions
  - Discrete chiral symmetry
  - Spontaneous symmetry breaking
- Action

$$\mathrm{S}[\bar{\psi}, \psi] = \int d^{\mathrm{d}} x \left(\sum_{f=1}^{N} \bar{\psi}_f \left(\gamma^{\mu} \partial_{\mu} + \gamma_0 \mu \right) \psi^f - \frac{g^2}{2} \left(\sum_{f=1}^{N} \bar{\psi}_f \psi^f \right)^2 \right)$$

- Bosonization with scalar field $\sigma := -g^2 \sum_{j=1}^{N} \bar{\psi}_j \psi_j$

$$\mathrm{S'} = \int \mathrm{d}^d x \left(\frac{1}{2g^2} \sigma^2 + \sum_{j=1}^{N} \bar{\psi}_j \underbrace{\left(\gamma^{\mu} \partial_{\mu} + \gamma^0 \mu + \sigma \right)}_{Q:=} \psi_j \right)$$

Bosonization

- Integrating out the fermion integrals (Gaussian integral over two Grassman variables) with $\lambda := \mathrm{N} \cdot g^2$

$$\mathrm{S}_{\mathrm{eff}} = \mathrm{N}\left(\frac{1}{2\lambda}\int \mathrm{d}^d x \sigma^2 - \ln\left(\det(Q)\right)\right)$$

$$Z = \int \mathcal{D}\sigma e^{-S_{\mathrm{eff}}}$$

- Limit $N \to \infty$ suppresses bosonic fluctuations, i.e only the minimum of $\mathrm{S}_{\mathrm{eff}}$ in σ represents physical configuration

Outline

Chiral symmetry

- Action of GN invariant under discrete symmetry transformation

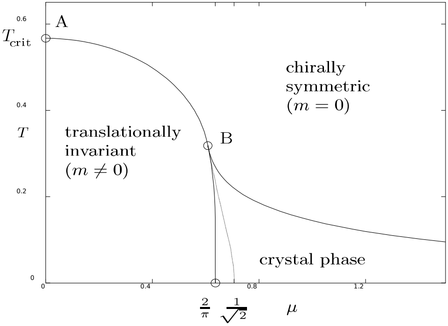
$$\psi \to \gamma_5 \psi, \ \bar{\psi} \to -\bar{\psi}\gamma_5.$$

- Chiral symmetry transformation for effective action

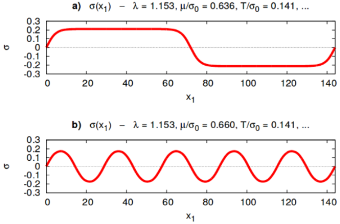
$$\sigma \to -\sigma$$

- Characterization of phase diagram with order parameter σ:
  - Homogenous restored phase: $\sigma = 0$, chiral symmetry restored
  - Homogenous broken phase: $\sigma = const. \neq 0$, broken chiral symmetry
  - Inhomogeneous phase: $\sigma(x)$, spatial dependence of the chiral condensate

Analytical solution for 1+1 dim.



(a) Phase diagram of 1+1 dim. GN model
{M. Thies, K. Urlichs, Phys. Rev. D67, 125015(2003)}



(b) Structur of chiral condensate in crystal phase for different μ
{M. Wagner, PoS LATTICE2007, 339 (2007)}

Outline

Detection of phase boundaries

- Homogeneous second order boundary via determination of sign change in $\frac{\partial^2}{\partial\sigma^2} S_{\text{eff}}|_{\sigma=0}$, then test for first order boundary
- Inhomogeneous boundary: $\sigma(x) \to \sigma_i$ on the lattice
- Stability analysis via Hessian: $(H)_{ik} = \frac{\partial^2}{\partial\sigma_i \partial\sigma_k} S_{\text{eff}}|_{\vec{\sigma}=0}$
  $\to$ H positiv definit $\leftrightarrow$ $\vec{\sigma} = 0$ is stable minimum, symmetry restored
  $\to$ H indefinit $\leftrightarrow$ chiral symmetry broken
  $\to$ find boundary from inhomogeneous to restored phase
- Eigenvectors corresponding to negative eigenvalues are directions in σ-space that lead to a lower action
- Multidimensional minimization in $\sigma_i \ \forall i$ to find physical configuration for given (μ, T)

1+1 dim. homogeneous phase diagram

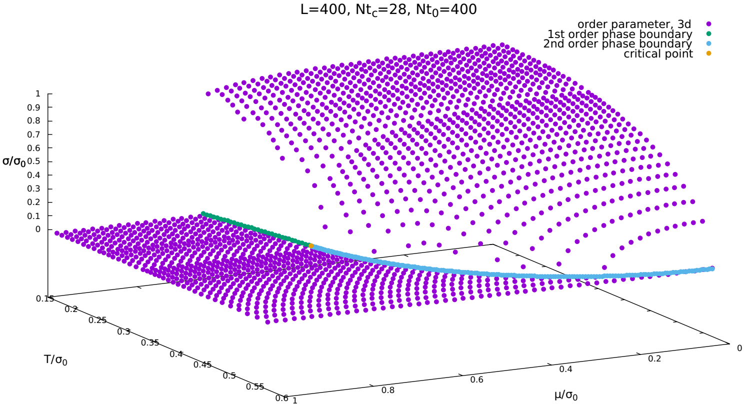
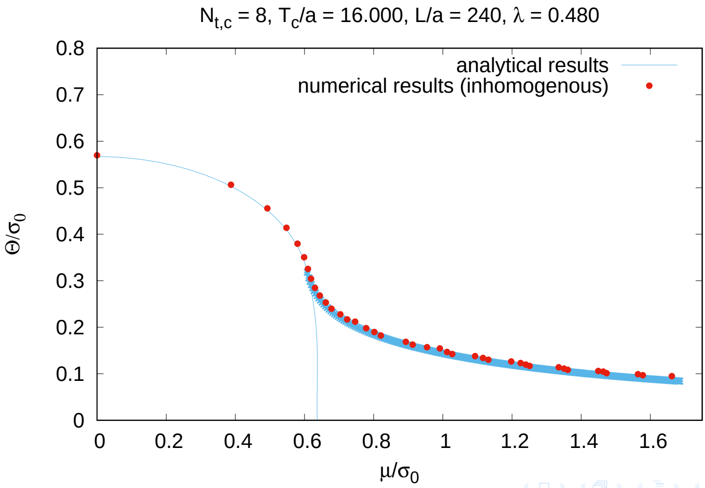


Figure: Homogeneous phase diagram with values of order parameter σ

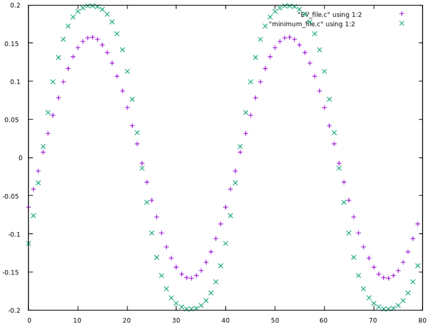
1+1 dim. Phase diagram

- Test with analytical results on homogeneous boundary as well as on inhomogeneous boundary, small finite volume effects remaining

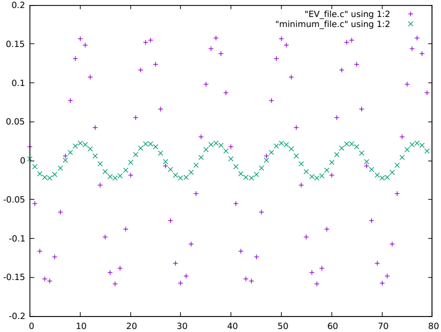


Inhomogeneous condensate

- Value of the chiral condensate against x_i
- purple points are from eigenvectors, green points after minimization

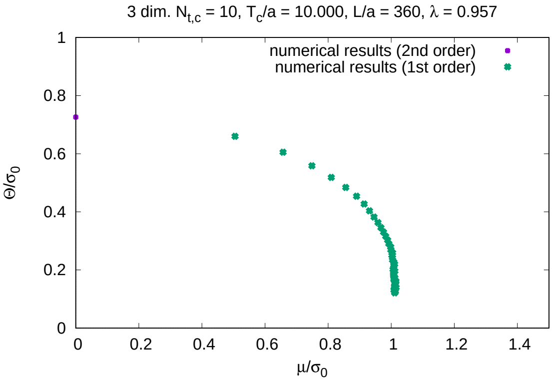


$T/\sigma_0 = 0.14, \mu/\sigma_0 = 0.64$



$T/\sigma_0 = 0.14, \mu/\sigma_0 = 1.13$

Will inhomogeneous phases survive in the $2 + 1$ dimensional Gross Neveu model?



(a) Homogeneous phase diagram, matches with analytical solution
Konrad Urlichs, "Das Phasendiagram des Gross-Neveu-Modells", Diploma thesis(2003)

- Eigenvectors for $\sigma = \sigma(x, y)$
- Phase diagram suffers from huge lattice artifacts

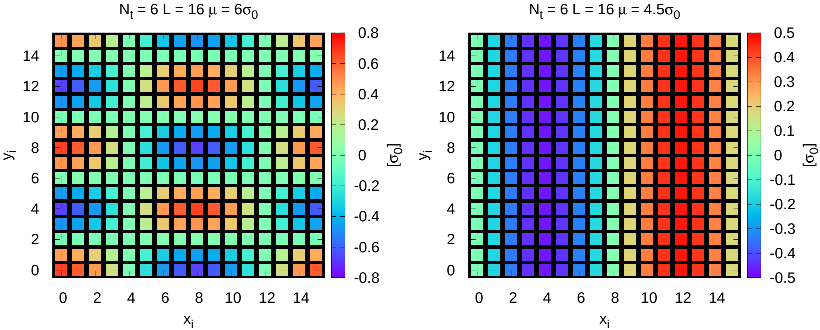


Figure: Eigenvectors of Hessian in inhomogeneous phase of 2+1 dimensional GN
[M.Winstel, "Phase diagrams of QCD-inspired models in the large-N limit", Bachelor thesis, GU Frankfurt, 2018]

Computational costs

- Homogeneous boundary efficient due to analytic simplifications
- Inhomogeneous boundary difficult:
  - Evaluation of $\ln(\det Q)$
  - Computation of Hessian's eigenvalues & eigenvectors
  - Minimization in σ_i for full phase diagram

T/σ_0	Spatial extent $L[a]$	Computation time
0.23	8	$\sim$ 13min
0.23	16	$\sim$ 86h
0.46	16	$\sim$ 43h
0.23	20	$\sim$ 20d

Table: Runtime for μ of phase transition for fixed a

- $\rightarrow$ Restriction to $\sigma = \sigma(x)$ instead of full degrees of freedom

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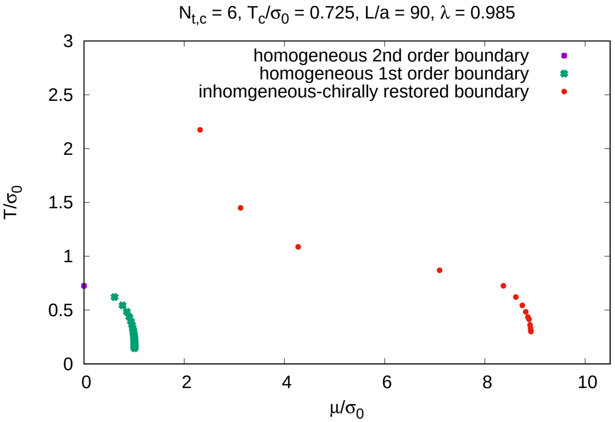
2+1 dimensional GN model



- Preliminary: Numerical phase diagram of $2 + 1$-dimensional Gross-Neveu model, allowing only $\sigma = \sigma(x)$

2+1 dimensional GN model

Current challenges:

- At high μ and low T some eigenvectors (with $\lambda < 0$) do not lower action $\rightarrow$ Mistake in stability analysis? Wrong phase boundary?
- On the other side: Minimization algorithm finds inhomogeneous minima in this region
- Checkup and further investigations necessary

Shape of chiral condensate

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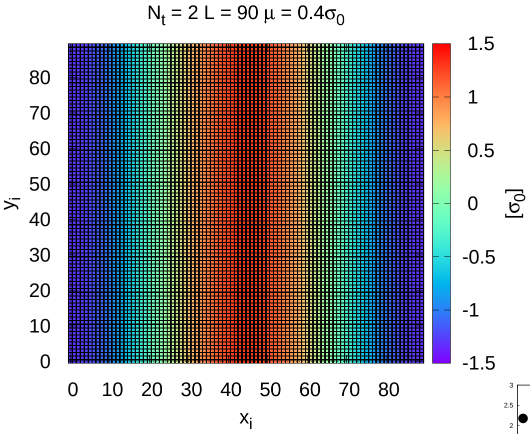
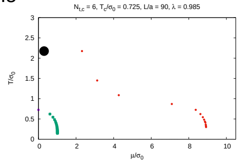
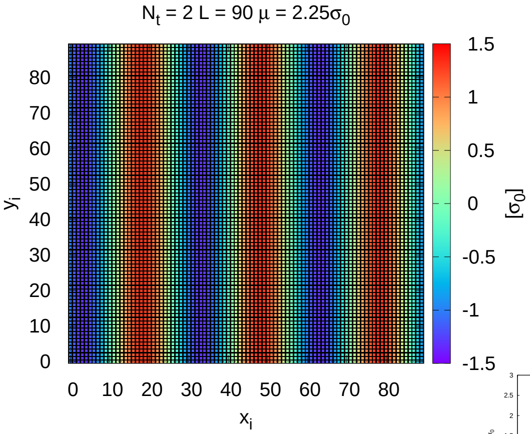
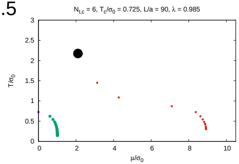




- Eigenvectors indicate the shape of chiral condensate, that leads to a lower action. Wavelength of stripe depends on chemical potential.

Shape of chiral condensate

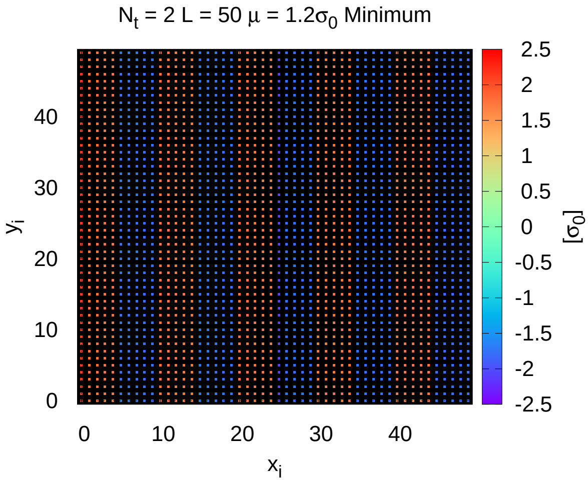




- At lower chemical potential a wide range of wavelengths occurs, increasing μ leads to the disappearance of larger wavelengths as expected from $1+1$ dimensions

Shape of chiral condensate

- Minimum with minimization algorithm



Summary & Outlook

- First numerical results indicate a large inhomogeneous phase
- Crystal structure found by multidimensional minimization algorithm
- Numerical problem at small T and large $\mu \rightarrow$ rigorous checkups and cross checks necessary
- Improvement of numerical methods
- Outlook
  - Full phase diagram via minimization algorithm (computer time issue)
  - Minimization for $\sigma = \sigma(x, y)$ at certain (μ, T)
  - Extend to models that are closer to QCD, e.g NJL