

Quarks & mesons at finite chemical potential

Pascal Gunkel

in collaboration with Christian S. Fischer and Philipp Isserstedt

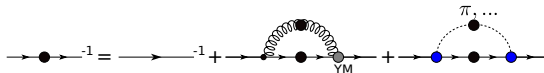
Institute for Theoretical Physics
Justus-Liebig-University Gießen

6th FAIRNESS
May 22, 2019



HGS-HIRe *for FAIR*
Helmholtz Graduate School for Hadron and Ion Research

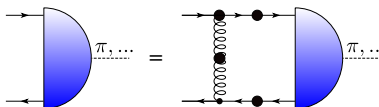
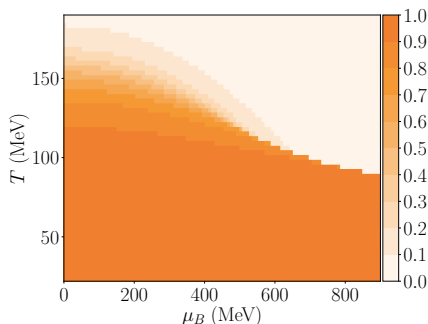
Outline



1. Introduction and motivation

2. QCD phase diagram

3. Mesons at finite chem. pot.



Phase structure of QCD

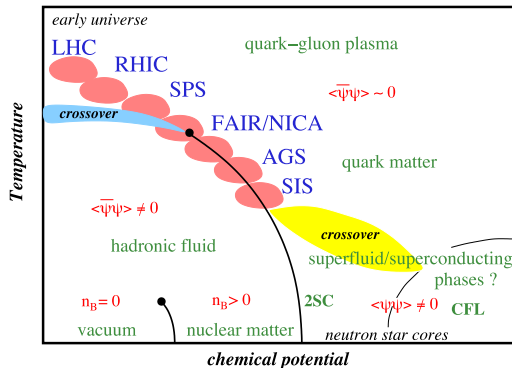


Figure: Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

Previous studies of our group:

QCD phase diagram with $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ quark flavors:

- Luecker and Fischer, Prog.Part.Nucl.Phys. 67(2), 200–205 (2012)
- Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3), 34022 (2014)

Baryon effects on the location of QCD's CEP:

- Eichmann, Fischer and Welzbacher, Phys.Rev. D 93, 034013 (2016)

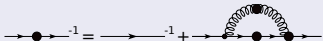
Mesonic back-coupling effects in vacuum and finite T:

- Fischer, Nickel and Wambach Phys.Rev. D 76, 094009 (2007)
- Fischer and Williams Phys.Rev. D 78, 074006 (2008)
- Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

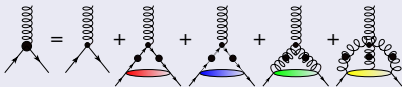
Dyson-Schwinger approach

Coupled set of Dyson-Schwinger equations (DSE):

Quark DSE:

$$\text{quark line}^{-1} = \text{quark line}^{-1} + \text{quark line}^{-1} \text{ gluon loop} \text{ quark line}$$


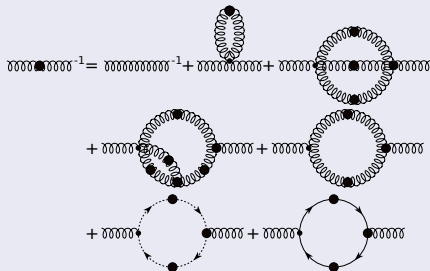
Quark-Gluon-Vertex DSE:

$$\text{quark-gluon vertex} = \text{bare vertex} + \text{quark loop} + \text{gluon loop} + \text{ghost loop} + \text{four-point vertex}$$


Further vertex and ghost DSEs:

...

Gluon DSE:

$$\text{gluon line}^{-1} = \text{gluon line}^{-1} + \text{gluon loop} + \text{quark loop} + \text{ghost loop}$$


Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step I):

Quark DSE:

$$\text{quark line}^{-1} = \text{quark line}^{-1} + \text{quark line}^{-1} \text{ (with gluon loop) } \text{quark line}$$

Quark-Gluon-Vertex DSE:

$$\text{quark-gluon vertex} = \text{tree-level vertex} + \text{gluon loop} + \text{ghost loop} + \text{quark loop} + \text{quark-gluon loop} + \text{quark-ghost loop}$$

Skeleton expansion:

$$\text{quark-gluon vertex} = \text{tree-level vertex} + \text{gluon loop} + \text{ghost loop} + \text{quark loop} + \dots$$

Gluon DSE:

$$\text{gluon line}^{-1} = \text{gluon line}^{-1} + \text{gluon loop} + \text{ghost loop} + \text{quark loop} + \text{quark-ghost loop}$$

Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step I):

Quark DSE:

$$\text{quark}^{-1} = \text{quark}^{-1} + \text{quark}^{-1} \text{ gluon quark}$$

Quark-Gluon-Vertex DSE:

$$\text{quark-gluon vertex} = \text{bare vertex} + \text{gluon loop} + \text{ghost loop} + \text{quark loop} + \text{mixed loops} + \dots$$

Skeleton expansion:

$$\text{quark-gluon vertex} = \text{bare vertex} + \text{meson insertions} + \text{nucleon insertions} + \dots$$

Gluon DSE:

$$\text{gluon}^{-1} = \text{gluon}^{-1} + \text{ghost loop} + \text{quark loop} + \text{gluon loop} + \text{mixed loops}$$

Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step II):

Quark DSE:

$$\text{quark line}^{-1} = \text{quark line}^{-1} + \text{quark line}^{-1} \text{ gluon loop} \text{ quark line}$$

Quark-Gluon-Vertex DSE:

$$\text{quark-gluon vertex} = \text{quark-gluon vertex}_{\text{YM}} + \text{quark-gluon vertex}_{\pi, \dots} + \text{quark-gluon vertex}_{N, \dots}$$

Gluon DSE:

$$\text{gluon line}^{-1} = \text{gluon line}^{-1} + \text{gluon line}^{-1} \text{ gluon loop} \text{ gluon line} + \text{gluon line}^{-1} \text{ quark loop} \text{ gluon line} + \text{gluon line}^{-1} \text{ ghost loop} \text{ gluon line}$$

Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step II):

Quark DSE:

$$\text{quark}^{-1} = \text{quark}^{-1} + \text{quark}^{-1} \text{ gluon quark}$$

Quark-Gluon-Vertex DSE:

$$\text{quark-gluon vertex} = \text{YM} + \pi, \dots + \cancel{N, \dots}$$

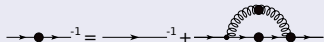
Gluon DSE:

$$\text{gluon}^{-1} = \text{gluon}^{-1} + \text{gluon}^{-1} \text{ gluon gluon} + \text{gluon}^{-1} \text{ quark quark} + \text{gluon}^{-1} \text{ ghost ghost}$$

Truncation

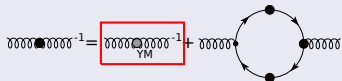
Coupled set of **truncated** Dyson-Schwinger equations (Step III):

Quark DSE:



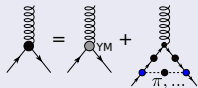
The diagram shows the Dyson-Schwinger equation for the quark propagator. On the left, a horizontal line with a black dot in the middle is followed by a superscript $^{-1}$. This is set equal to a horizontal line with a superscript $^{-1}$ plus a term consisting of a horizontal line with a black dot in the middle, followed by a loop of two wavy lines (gluons) connected by two black dots, and then another horizontal line with a black dot in the middle.

Gluon DSE:



The diagram shows the Dyson-Schwinger equation for the gluon propagator. On the left, a wavy line with a black dot in the middle is followed by a superscript $^{-1}$. This is set equal to a term consisting of a wavy line with a black dot in the middle, followed by a box labeled 'YM' containing a wavy line with a black dot in the middle and a superscript $^{-1}$, and then another wavy line with a black dot in the middle. This is plus a term consisting of a wavy line with a black dot in the middle, followed by a circular loop of four wavy lines connected by four black dots, and then another wavy line with a black dot in the middle.

Quark-Gluon-Vertex DSE:



The diagram shows the Dyson-Schwinger equation for the quark-gluon vertex. On the left, a wavy line with a black dot in the middle is connected to two horizontal lines with black dots in the middle. This is set equal to a term consisting of a wavy line with a black dot in the middle connected to two horizontal lines with black dots in the middle, with a 'YM' label below the vertex, plus a term consisting of a wavy line with a black dot in the middle connected to two horizontal lines with black dots in the middle, with a dashed line and a black dot in the middle between the two horizontal lines, and a label $\pi, \dots$ below the dashed line.

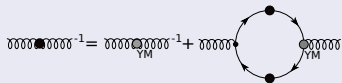
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step IV):

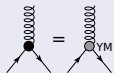
Quark DSE:



Gluon DSE:



Quark-Gluon-Vertex DSE:



Lattice data for quenched gluon:

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Maas, Pawłowski, von Smekal, Spielmann, Phys.Rev.D 85 (2012) 034037

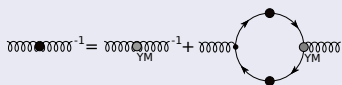
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (DSE):

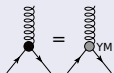
Quark DSE:



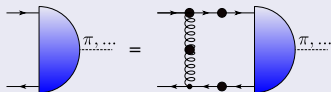
Gluon DSE:



Quark-Gluon-Vertex DSE:



Homogeneous BSE:



Lattice data for quenched gluon:

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Maas, Pawłowski, von Smekal, Spielmann, Phys.Rev.D 85 (2012) 034037

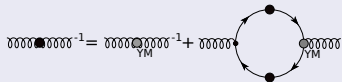
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (DSE):

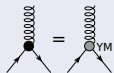
Quark DSE:



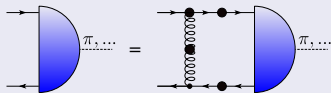
Gluon DSE:



Quark-Gluon-Vertex DSE:



Homogeneous BSE:



Lattice data for quenched gluon:

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Maas, Pawłowski, von Smekal, Spielmann, Phys.Rev.D 85 (2012) 034037

Present QCD phase diagram

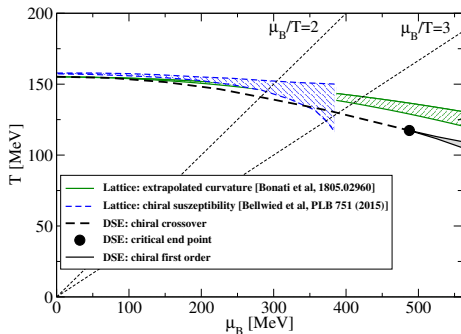
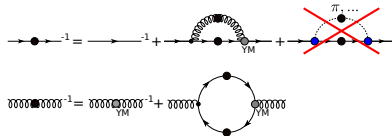


Figure: Fischer, PNP 105 (2019) 1

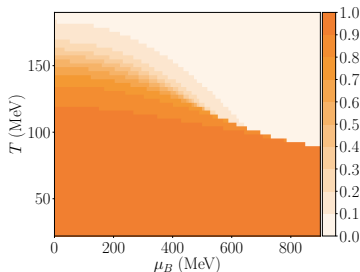
CEP at large μ_q :

- Fischer, Luecker, PLB 718 (2013) 1036
- Fischer, Fister, Luecker, Pawłowski, PLB 732 (2014) 273
- Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022



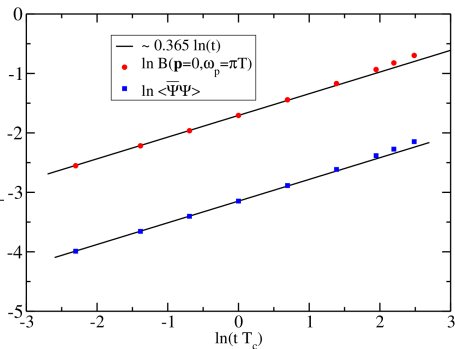
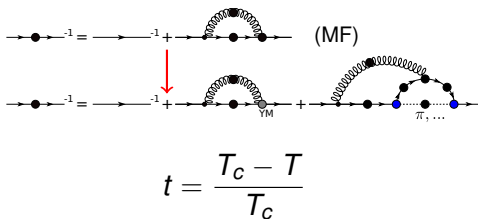
→ Details: see talk by Philipp Isserstedt (later today)

Chiral order parameter:



Universality class ($N_f = 2$, chiral limit)

Importance of mesonic back-coupling effects:



$T = 0$: Meson corrections in order of $\sim 10 - 20\%$

$T = T_c$: Meson corrections dominant $\longrightarrow$ critical scaling ($m_q \rightarrow 0$):

$$\langle \bar{\Psi} \Psi \rangle(t) \sim t^{\nu/(2-\eta)} \sim t^{\nu/2} \sim t^{\beta}$$

$$\beta = 0.365, \nu = 0.73$$

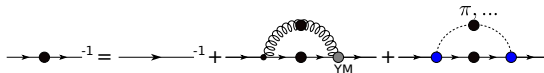
(Heisenberg class)

$T = 0$: Fischer, Williams, Phys.Rev. D 78, 074006 (2008)

$T = T_c$: Fischer, Mueller, Phys.Rev. D 84, 054013 (2011)

Connection to mesons at finite chemical potential

Wanted: Influence of Meson back-coupling onto QCD phase diagram and CEP



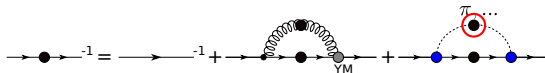
Needed: Meson propagator in medium

Meson Bethe-Salpeter amplitude in medium

First step: Investigate Meson amplitudes and properties at finite chemical potential

Connection to mesons at finite chemical potential

Wanted: Influence of Meson back-coupling onto QCD phase diagram and CEP



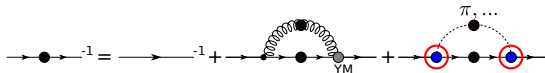
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Connection to mesons at finite chemical potential

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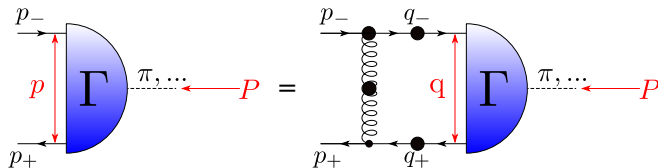


Needed: Meson propagator in medium

Meson Bethe-Salpeter amplitude in medium

First step: Investigate Meson amplitudes and properties at finite chemical potential

Homogeneous Bethe-Salpether equation (BSE)



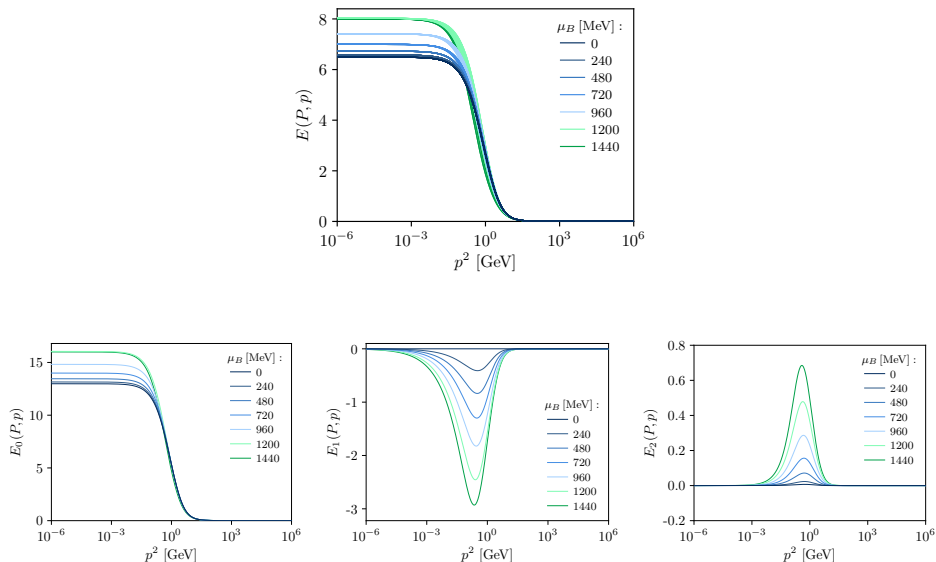
Pseudoscalar amplitude in vacuum

$$\Gamma_P(P, p) = \gamma_5 \left[E(P, p) - i \not{P} F(P, p) - i \not{p} (Pp) G(P, p) + [\not{P}, \not{p}] H(P, p) \right]$$

Pseudoscalar amplitude in medium

$$\Gamma_P(P, p) = \gamma_5 \left[E(P, p) - i P_4 \gamma_4 K(P, p) - i \vec{P} \vec{\gamma} F_s(P, p) + \dots \right]$$

Pion Bethe-Salpeter amplitude

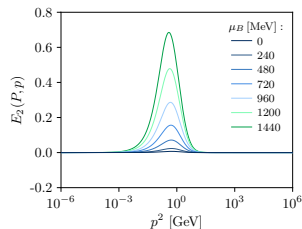
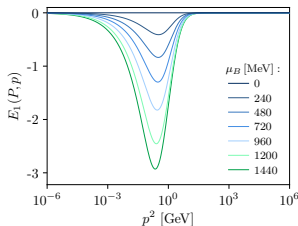
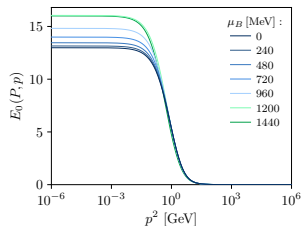
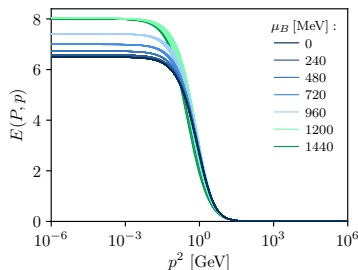


Pion Bethe-Salpeter amplitude

**Chebyshev
expansion:**

$$E(P^2, p^2, z) \approx \sum_j E_j(P^2, p^2) T_j(z)$$

$$z = \sphericalangle(P, p)$$



Pion Bethe-Salpeter amplitude

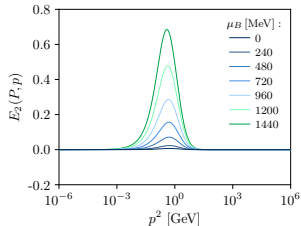
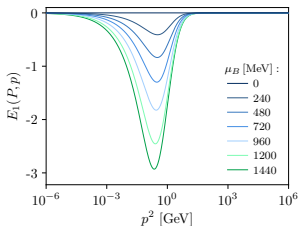
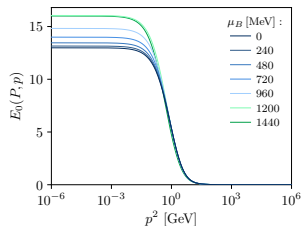
**Chebyshev
expansion:**

$$E(P^2, p^2, z) \approx \sum_j E_j(P^2, p^2) T_j(z)$$

**Charge-conjugated
pion amplitude:**

$$\bar{\Gamma}_\pi(P, p) = [C \Gamma_\pi(P, -p) C^{-1}]^T$$

$$z = \sphericalangle(P, p)$$



Pion Bethe-Salpeter amplitude

**Chebyshev
expansion:**

$$E(P^2, p^2, z) \approx \sum_j E_j(P^2, p^2) T_j(z)$$

$$z = \sphericalangle(P, p)$$

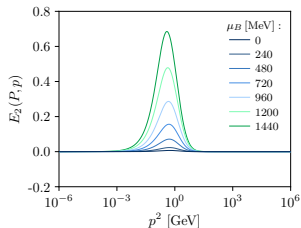
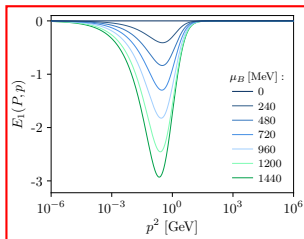
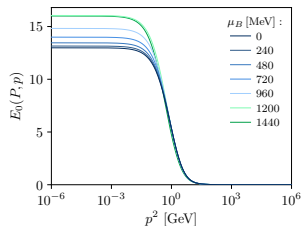
**Charge-conjugated
pion amplitude:**

$$\bar{\Gamma}_\pi(P, p) = [C\Gamma_\pi(P, -p)C^{-1}]^T$$

Pion: $J^{PC} = 0^{-+}$

→ odd Chebyshev
coefficients vanish

→ μ_B breaks C-Parity



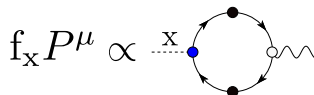
Meson properties

Meson mass:

$$K(P)\Gamma_x(P) = \lambda_x(P)\Gamma_x(P), \quad \lambda_x(P_{\text{os}}) = 1 \quad \Leftrightarrow \quad P_{\text{os}}^2 = -m_x^2$$

Meson decay constants:

$$X = \pi, \dots$$



From vacuum to medium:

$$f_x P^\mu \xrightarrow{\text{medium}} \left[f_x^t \mathcal{P}_{\mu\nu}^t(v) + f_x^s \mathcal{P}_{\mu\nu}^s(v) \right] P^\nu$$

Long. $\mathcal{P}_{\mu\nu}^{\mathcal{L}}(v)$ and trans. proj. $\mathcal{P}_{\mu\nu}^{\mathcal{T}}(v)$ with assigned direction $v = (\vec{0}, 1)$

Meson propagator

Meson velocity:

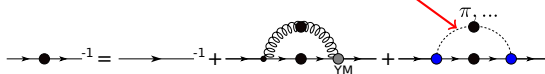
$$u_x = \frac{f_x^s}{f_x^t}$$

Meson dispersion relation:

$$\omega^2 = u^2 (\vec{P}^2 + m_x^2)$$

Meson propagator:

$$D_x(P) = \frac{1}{P_4^2 + u_x^2 (\vec{P}^2 + m_x^2)}$$



Son and Stephanov Phys.Rev. D, 66(7) (2002)
Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

$X = \pi, \dots$

Definition:

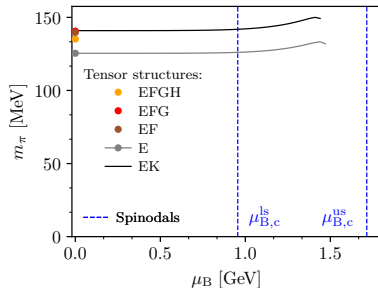
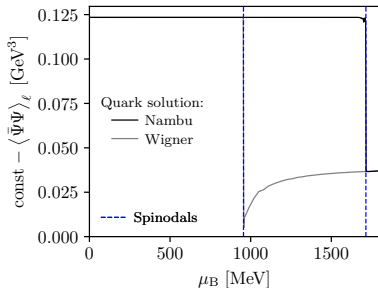
T. D. Cohen, Phys. Rev. Lett. 91 , 222001 (2003)
T. D. Cohen, arXiv:hep-ph/0405043 (2004)

$\mu_B < \text{mass gap of the system } \delta \text{ and } T = 0$

$\Rightarrow$ Partition function and observables independent from μ_B

$\delta = m_B = \text{lightest baryon mass in medium}$

Pion properties results at finite chemical potential



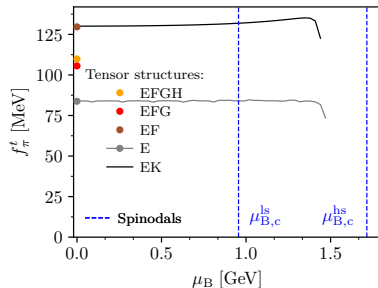
- Silver Blaze property fulfilled
- No pion solution above $\mu_{B,c}^{\text{pion}}$ (eff. int)

Qualitative agreement with simpler truncation schemes:

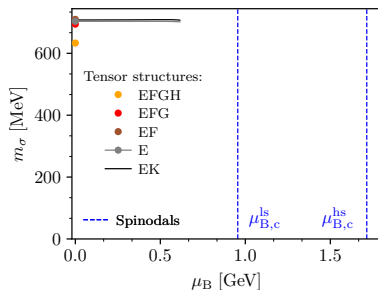
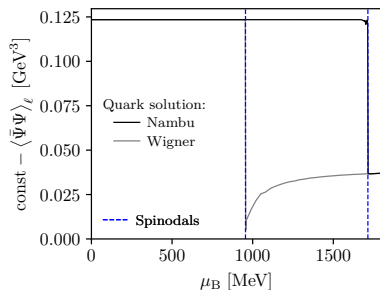
Roberts, Phys.Part.Nucl. 30:223-257 (1999)

Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)

Jiang, Shi, Li, Sun, and Zong, PRD 78, 116005



Sigma properties results at finite chemical potential



- Silver Blaze property fulfilled
- No pion solution above $\mu_{B,c}^{\text{pion}}$ (eff. int)

Qualitative agreement with simpler truncation schemes:

Roberts, Phys.Part.Nucl. 30:223-257 (1999)

Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)

Jiang, Shi, Li, Sun, and Zong, PRD 78, 116005

Summary

- Successfully implemented quark back coupling onto gluon
- Present QCD phase diagram for $N_f = 2 + 1$ quark flavors
 - good curvature agreement
 - CEP beyond lattice chiral susceptibility exclusion area
- Meson properties (m_π, f_π, m_σ) at $\mu_B \neq 0$, up to two tensor structures
 - $\mu_B \neq 0$ breaks C-Parity of mesons ✓
 - m_π, f_π, m_σ fulfill Silver Blaze property ✓
 - No pion solution above certain μ_B^c (eff. int.)

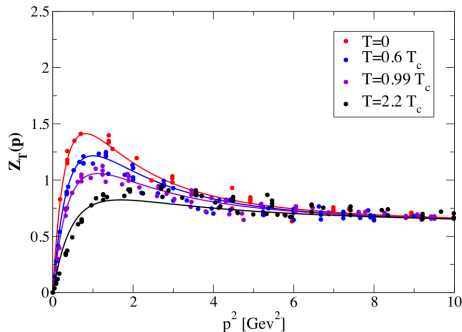
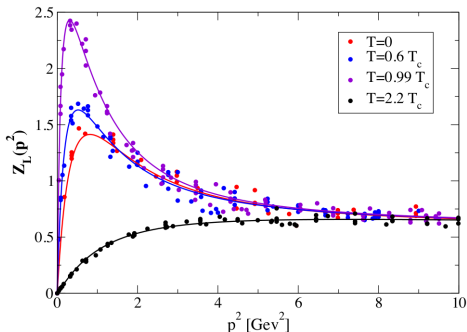
1. Publish present results
2. meson properties at finite chem. pot. & temp.
3. meson back coupling (finite chem. pot. & temp.)
→ influence on CEP

Thank you for your attention!

Backup slides

Truncation scheme properties I

T-dependent gluon propagator from quenched lattice simulations:



Crucial difference between transversal and longitudinal gluon

($T_c = 277$ MeV)

Cucchieri, Maas, Mendes, PRD 75 (2007)

CF, Maas, Mueller, EPJC 68 (2010)

Aouane et al., PRD 85 (2012) 034501

Cucchieri, Mendes, PoS FACESQCD 007 (2010)

Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503

FRG: Fister, Pawłowski, arXiv:1112.5440

Truncation scheme properties II

Vertex truncation: STI and perturbative behavior at large momenta constrain vertex

$$\Gamma_{\nu}^f(p, q, k) = \Gamma(k^2) \gamma_{\nu} (\delta_{\nu,s} \Sigma_A + \delta_{\nu,4} \Sigma_C)$$

$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{k^2}{\Lambda^2 + k^2} \left(\frac{\beta_0 \alpha(\mu'') \ln[k^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta}$$

With first Ball-Chiu structure: $\Sigma_X = \frac{X(\vec{p}^2, \omega_p) + X(\vec{q}^2, \omega_q)}{2}$, $X \in \{A, C\}$

Abelian WTI: from approximated STI

Perturbation theory

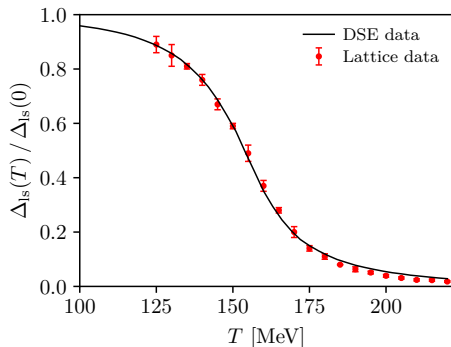
Infrared ansatz: d_2 fixed to match gluon input, d_1 fixed via quark condensate

Fischer and Mueller, Phys.Rev. D 80, 074029 (2009)

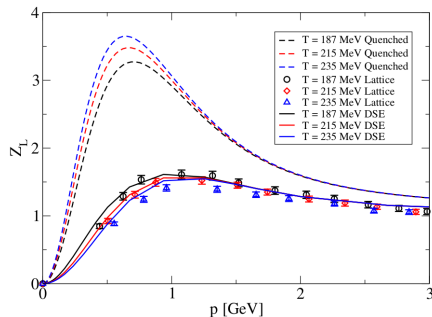
Fischer, Phys.Rev.Lett. 103, 052003 (2009)

Truncation scheme properties III

Determination of d_1 and prediction for unquenched gluon:



Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073



Lattice: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck, PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

Quantitative agreement: DSE results verified by lattice

Order parameter

Chirality:

$$\rightarrow \bullet \rightarrow^{-1} = \rightarrow \rightarrow^{-1} + \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow$$

Quark condensate

$$\langle \bar{\Psi} \Psi \rangle^f = -Z_m Z_2 \int_q \text{tr}_{DC} [S^f(p)]$$

Regularized quark condensate

$$\Delta_{f'f} = \langle \bar{\Psi} \Psi \rangle^{f'} - \frac{m_B^{f'}}{m_B^f} \langle \bar{\Psi} \Psi \rangle^f$$

Deconfinement:

$$\langle L[A] \rangle \propto e^{-\frac{F_q}{T}}, \quad \text{static quark free energy } F_q$$

Dressed Polyakov loop¹

$$\Sigma = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\Psi} \Psi \rangle_\varphi$$

Polyakov loop potential²

$$L[A] := \frac{1}{N_c} \text{tr}_C \left(\mathcal{P} e^{i \int d\tau A_0(\vec{x}, \tau)} \right)$$

1:

Synatschke, Wipf, Wozar,
PRD 75, 114003 (2007)

Bilgici, Bruckmann,
Gattringer, Hagen, PRD
77 094007 (2008)

Fischer, PRL 103 052003
(2009)

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \left(\text{diagram 1} - \text{diagram 2} - \text{diagram 3} - \frac{1}{6} \text{diagram 4} + \text{diagram 5} \right)$$

2:

Braun, Gies, Pawłowski,
PLB 684, 262 (2010)

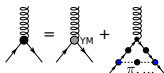
Braun, Haas, Marhauser,
Pawłowski, PRL 106
(2011)

Fister, Pawłowski, PRD
88 045010 (2013)

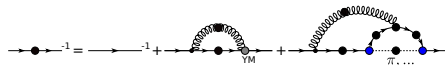
Fischer, Fister, Luecker,
Pawłowski, PLB 732

Skeleton expansion

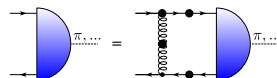
Only mesonic contributions:



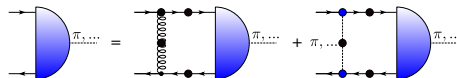
Inserting vertex into quark:



BSE:



Assumption: Only Yang-Mills part present in BSE $\Rightarrow$ rewrite Quark DSE by inserting DSE into second diagram



Approximation justified if BSE vertex function with and without pion interaction term do not differ strongly

Fischer, Nickel and Wambach, Phys.Rev. D 76(9) (2007)

Definition:

first coefficient (κ) in taylor series expansion of the transition line in terms of $\frac{\mu_q}{T_c(0)}$

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_q}{T_c(0)} \right)^2 + O \left[\left(\frac{\mu_q}{T_c(0)} \right)^4 \right]$$

Remark:

Curvature depends on choice of pseudo-critical temperature definition in crossover region

Silver Blaze property example

Definition:

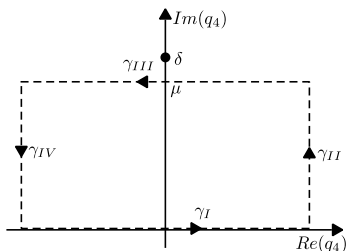
T. D. Cohen, Phys. Rev. Lett. 91 , 222001 (2003)
T. D. Cohen, arXiv:hep-ph/0405043 (2004)

$\mu_B < \text{mass gap of the system } \delta \text{ and } T = 0$

$\Rightarrow$ Partition function and observables independent from μ_B

$$\delta = m_B = \text{lightest baryon}$$

Example:



Substitution:

$$\langle \bar{\Psi} \Psi \rangle \sim \int_q S(\vec{q}^2, q_4 + i\mu_q)$$

$$q_4 \rightarrow \underline{\underline{q_4 + i\mu_q}} \int_q S(\vec{q}^2, q_4) \sim \langle \bar{\Psi} \Psi \rangle_{vac}$$

Condition: No singularity in γ