Exploring the partonic phase at finite chemical potential within a selfconsistent off-shell transport approach

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FAIRness

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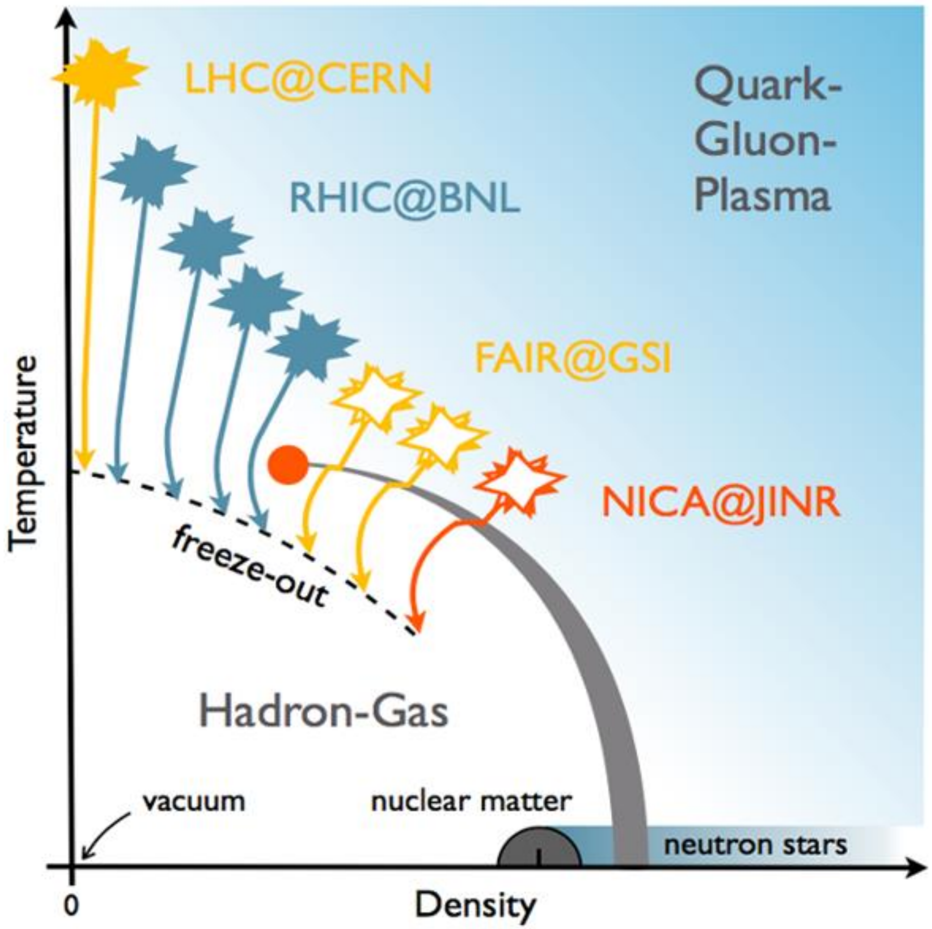
Outline

- Introduction / motivations
- The Dynamical QuasiParticle model (DQPM)
- Implementation of the (T, μ_B)-dependent EoS in the PHSD
- Results for Heavy-Ion Collisions
- Summary

Motivation

- **Explore the QCD phase diagram at finite temperature and chemical potential through heavy-ion collisions**
- **Available information:**
  - **Experimental data at SPS, BES at RHIC**
  - **Lattice QCD calculations**

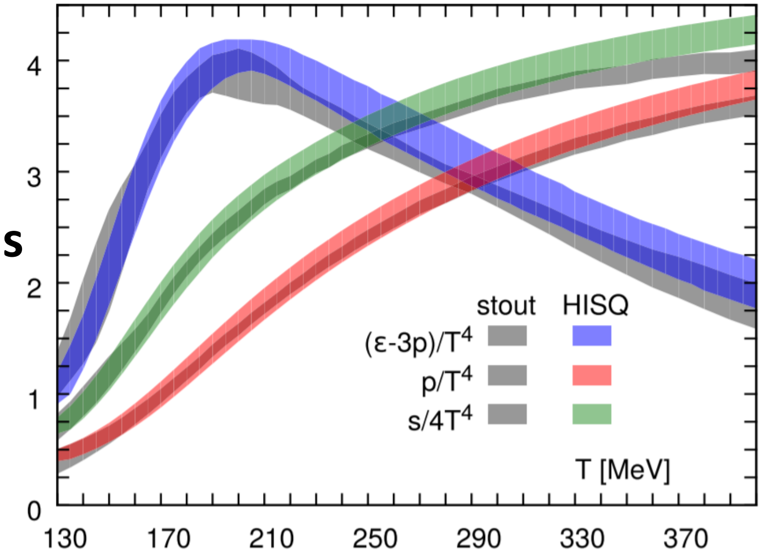
Probes of the QGP at finite (T, μ_B)



Lattice EoS for $\mu_B = 0$ and $\neq 0$

- **Lattice QCD data: well known at $\mu_B = 0$**
- **Crossover from hadron gas to the QGP**
- **Results available at finite μ_B from analytical continuation or from a series expansion in terms of the susceptibilities**
- **A lot of information to constrain effective models for the QGP:**

! need to be interpreted in terms of degrees-of-freedom



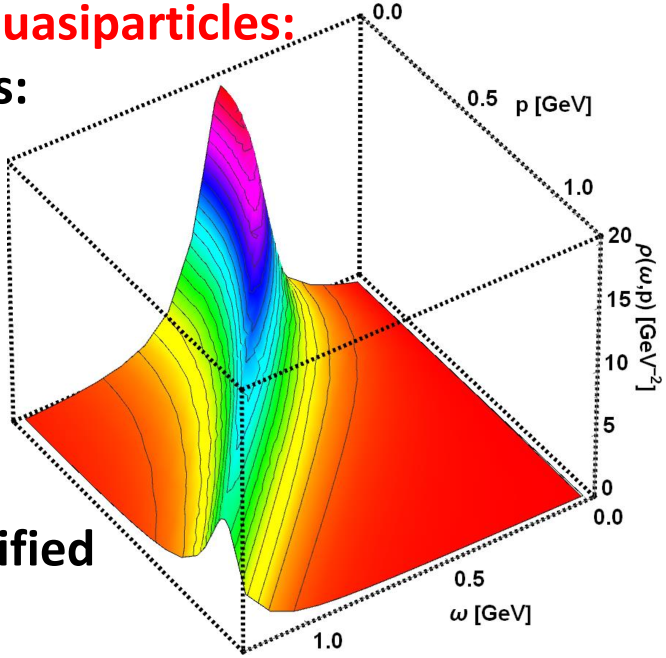
See talk of Dr. Jana N. Guenther

Lattice results from: Phys.Rev. D90 (2014) 094503; PoS CPOD2017 (2018) 032

❖ **How to learn about degrees-of-freedom of QGP ? ➔ HIC experiments**

Dynamical QuasiParticle Model (DQPM)

- The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:



$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$
$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$

- Resummed properties of the quasiparticles are specified by scalar complex self-energies:

gluon propagator: $\Delta^{-1} = P^2 - \Pi$	&	quark propagator $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2g_g\omega$	&	quark self-energy: $\Sigma_q = M_q^2 - i2g_q\omega$

- Real part of the self-energy: thermal mass (M_g, M_q)
- Imaginary part of the self-energy: interaction width of partons (γ_g, γ_q)

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Parton properties

- **Modeling of the quark/gluon masses and widths (inspired by HTL calculations)**

$$M_g^2(T,\mu_B) = \frac{g^2(T,\mu_B)}{6}\left(\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$M_{q(\bar{q})}^2(T,\mu_B) = \frac{N_c^2-1}{8N_c}g^2(T,\mu_B)\left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q(\bar{q})}(T,\mu_B) = \frac{1}{3}\frac{N_c^2-1}{2N_c}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

$$\gamma_g(T,\mu_B) = \frac{1}{3}N_c\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

- **Only one parameter (c = 14.4) + (T,μ_B)- dependent coupling constant to determine from lattice results**

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM coupling constant

- **Input: entropy density as a function of temperature for $\mu_B = 0$**

$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$

$$s_{SB}^{QCD} = 19/9\pi^2 T^3$$

- **Scaling hypothesis at finite $\mu_B \approx 3\mu_q$**

$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right)$$

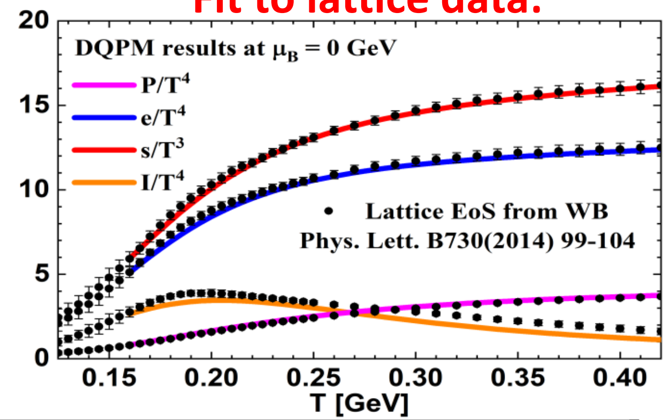
with the effective temperature

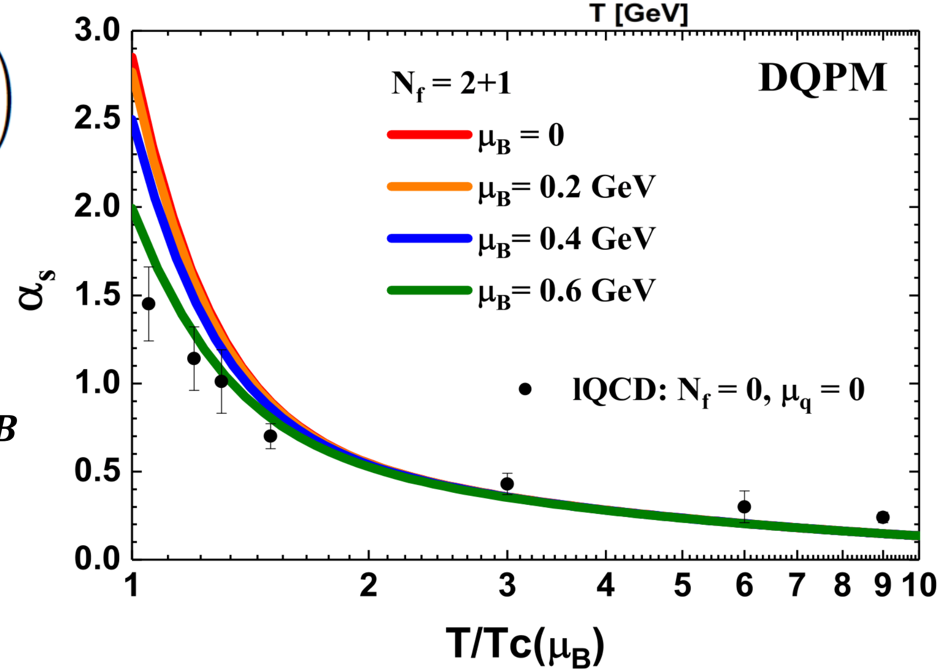
$$T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$

and the critical temperature at finite μ_B

$$T_c(\mu_B) = T_c\sqrt{1 - \alpha\mu_B^2}$$

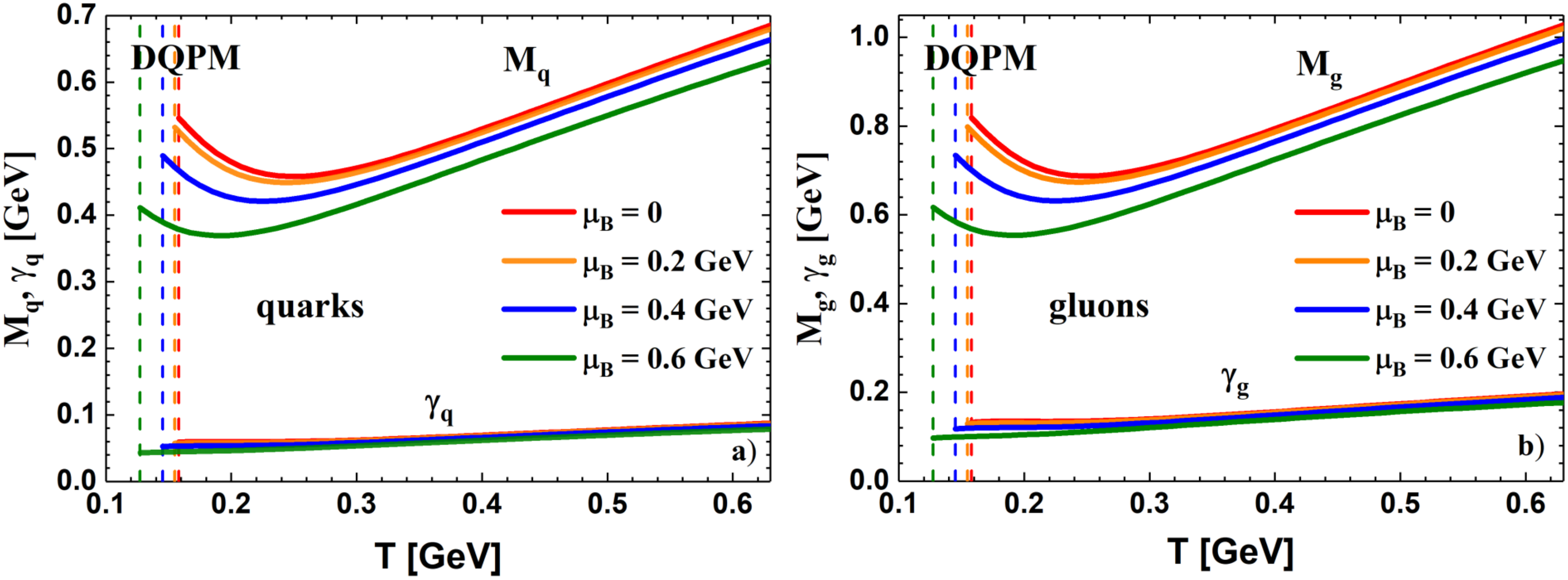
Fit to lattice data:





DQPM : parton properties

- **DQPM masses and widths as a function of (T, μ_B)**



DQPM : Thermodynamics

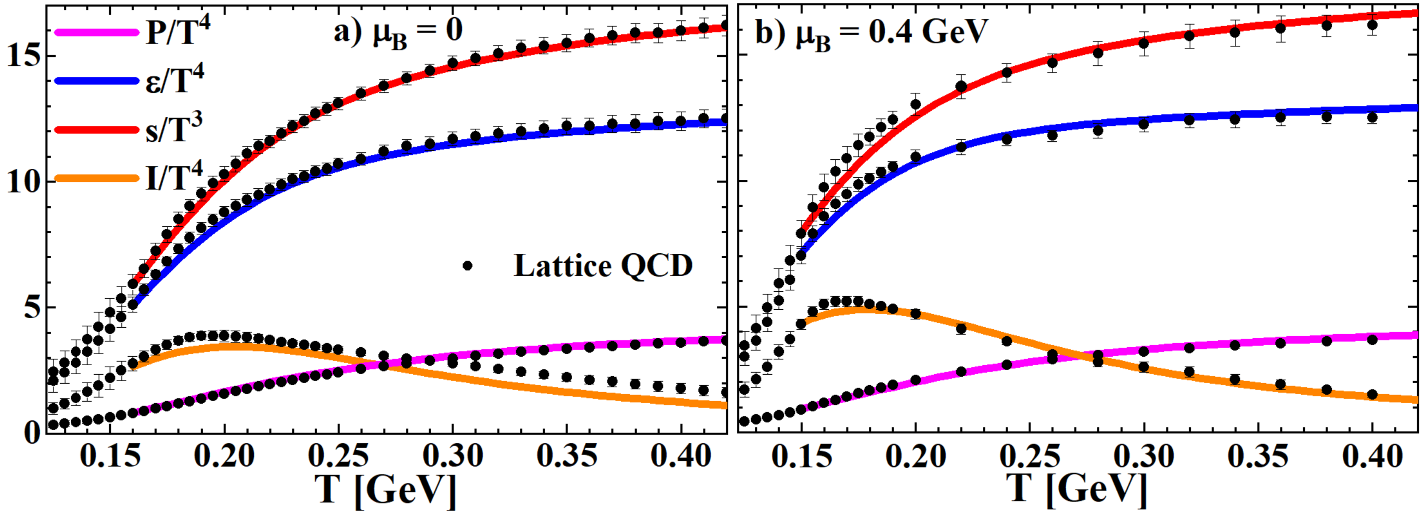
- **Entropy and baryon density**

in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[d_g \frac{\partial n_B}{\partial T} \left(\mathrm{Im}(\ln -\Delta^{-1}) + \mathrm{Im}\,\Pi\, \mathrm{Re}\,\Delta \right) + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} \left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\, \mathrm{Re}\,S_q \right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} \left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\, \mathrm{Re}\,S_{\bar{q}} \right) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[\sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} \left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\, \mathrm{Re}\,S_q \right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} \left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\, \mathrm{Re}\,S_{\bar{q}} \right) \right]$$

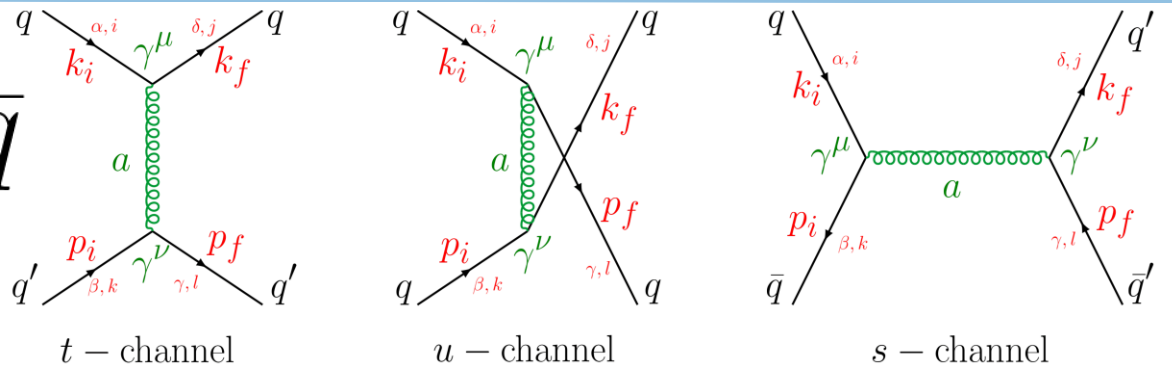
Input:
lattice EoS
$\mu_B = 0$



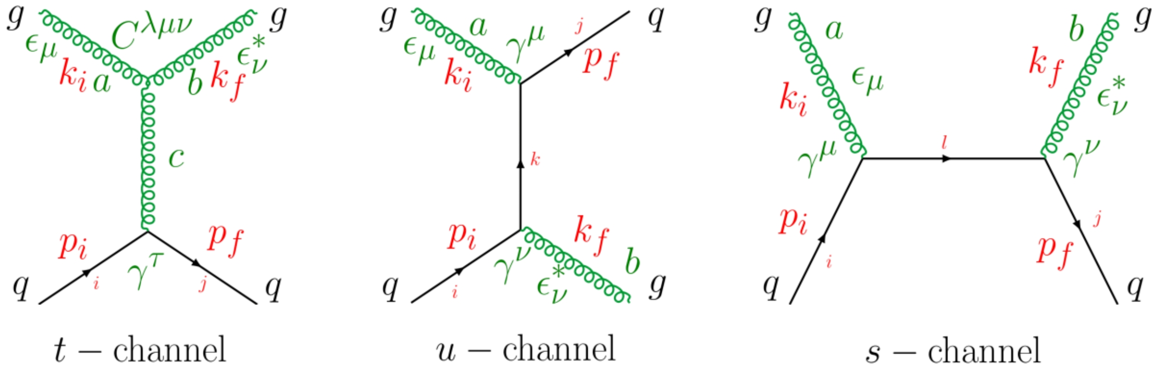
Output:
DQPM EoS
$\mu_B > 0$

Partonic interactions: matrix elements

$qq', \; q\bar{q}$



gq



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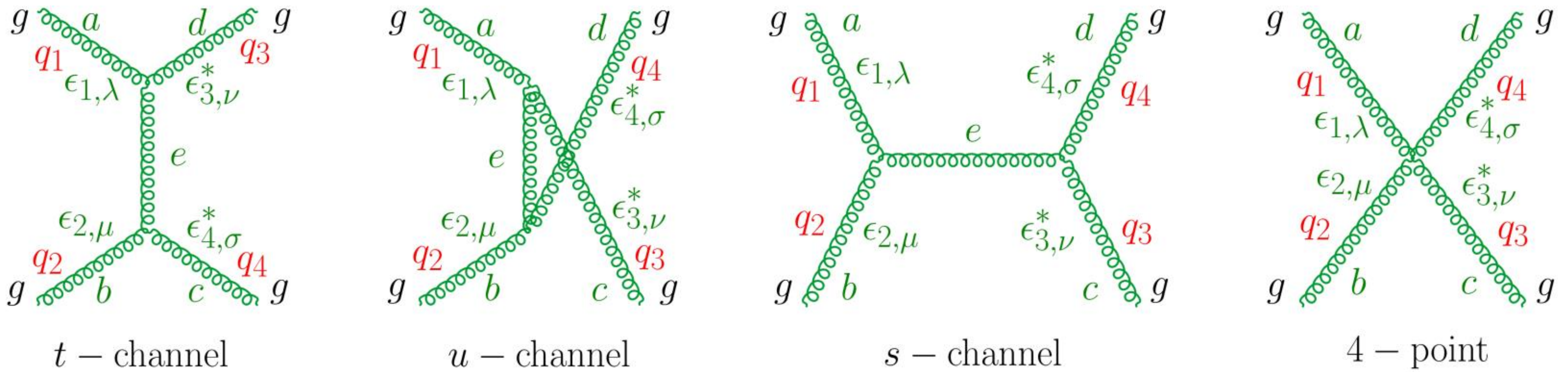
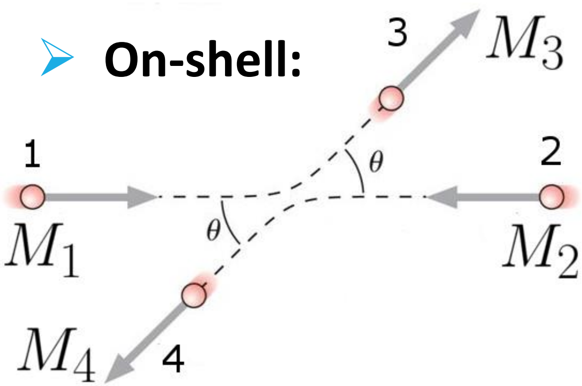
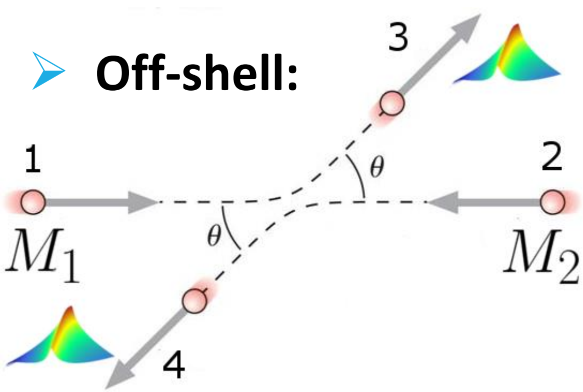


Differential cross section

- **On-shell:**

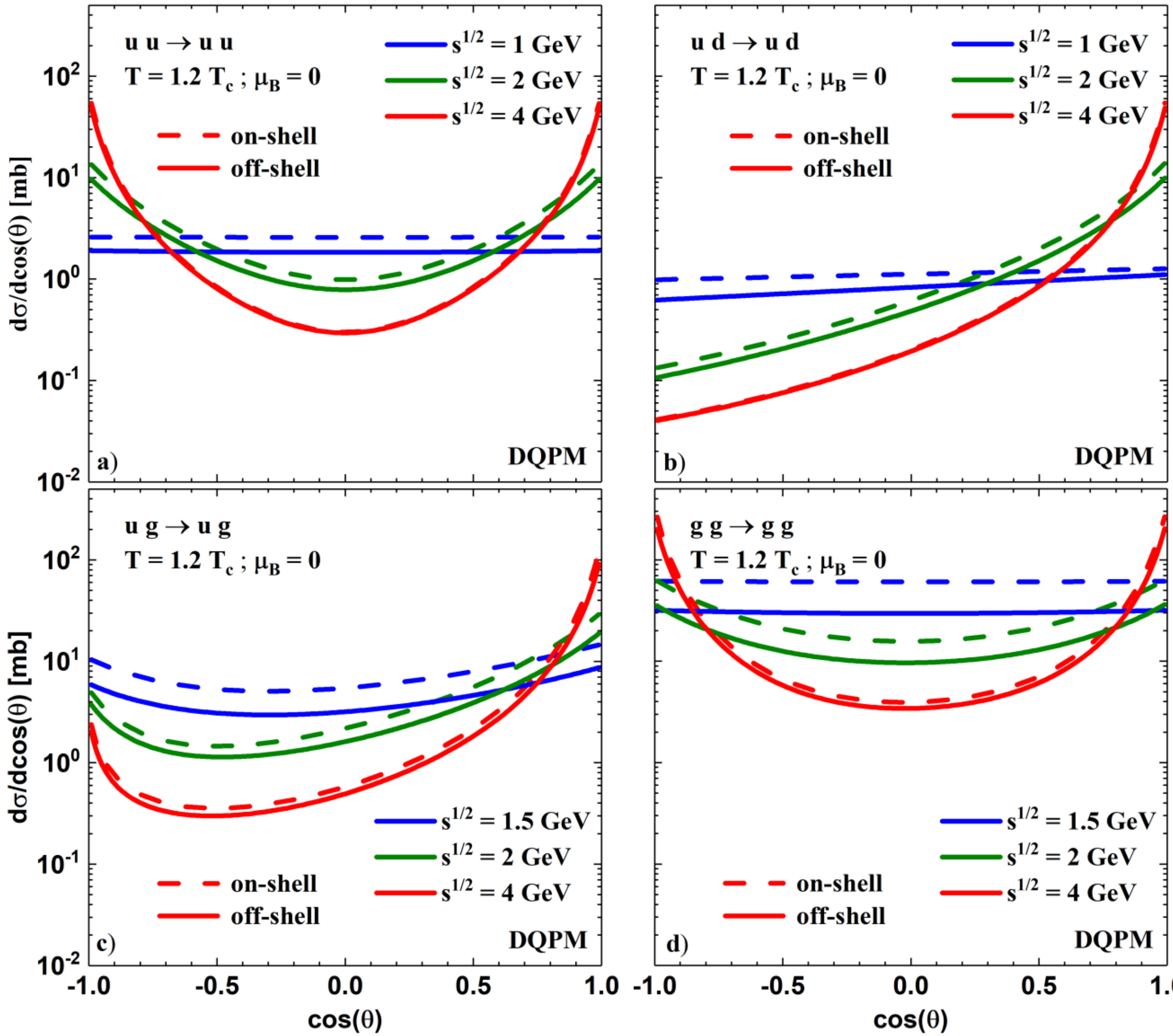


- **Off-shell:**



- **Off-shell σ < on-shell σ**

  since $\omega_3 + \omega_4 < \sqrt{s}$



Collisional widths

$$\Gamma_i^{\rm off}(T,\mu_q) = \frac{d_i}{n_i^{\rm off}(T,\mu_q)} \int \frac{d^4p_i}{(2\pi)^4}\ \theta(\omega_i)\ \tilde\rho_i\ f_i(\omega_i,T,\mu_q)$$
$$\times \sum_{j=q,\bar q,g} \int \frac{d^4p_j}{(2\pi)^4}\ \theta(\omega_j)\ d_j\ \tilde\rho_j\ f_j$$
$$\times \int \frac{d^4p_3}{(2\pi)^4}\ \theta(\omega_3)\ \tilde\rho_3 \int \frac{d^4p_4}{(2\pi)^4}\ \theta(\omega_4)\ \tilde\rho_4 (1\pm f_3)(1\pm f_4)$$
$$\times |\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\ (2\pi)^4\delta^{(4)}\left(p_i+p_j-p_3-p_4\right),$$

- **off-shell density**

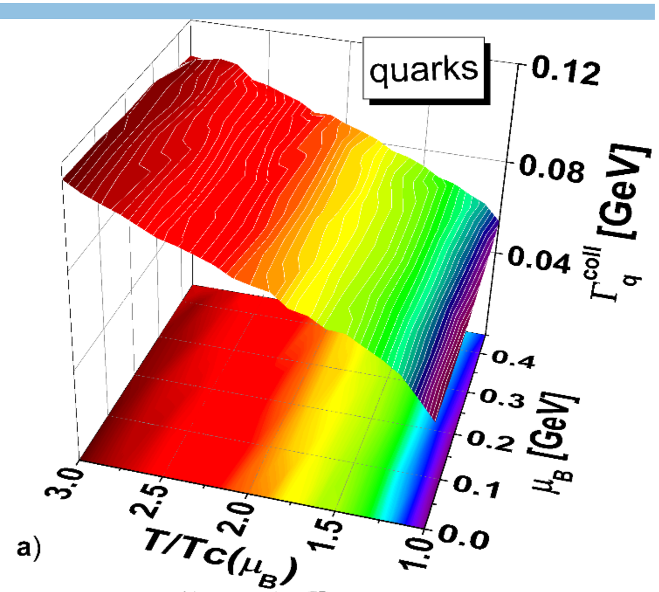
$$n_i^{\rm off}(T,\mu_q) = d_i \int \frac{d^4p_i}{(2\pi)^4}\ \theta(\omega_i)\ 2\omega_i\ \tilde\rho_i\ f_i(T,\mu_q)$$

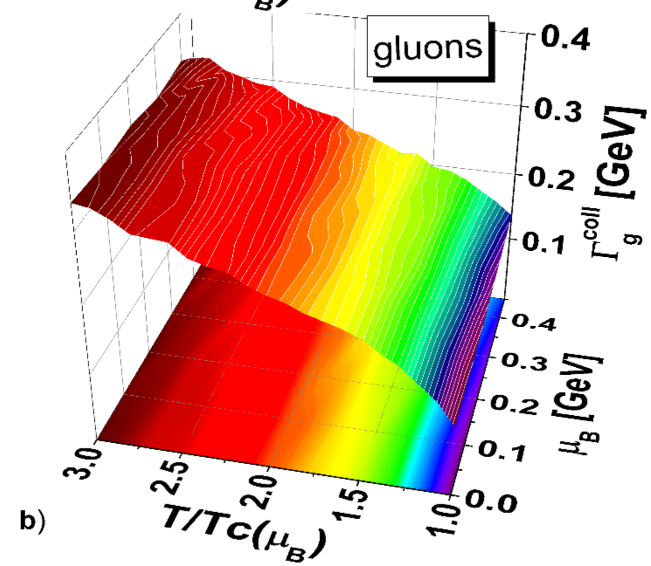
- **renormalized spectral-function for the time-like sector**

$$\tilde\rho_j(\omega_j,\mathbf{p}_j) = \frac{\rho(\omega_j,\mathbf{p}_j)\ \theta(p_j^2)}{\int_0^\infty \frac{d\omega_j}{(2\pi)}\ 2\omega_j\ \rho(\omega_j,\mathbf{p}_j)\ \theta(p_j^2)}$$

normalized to 1 and

$$\lim_{\gamma_j\to 0} \rho_j(\omega,\mathbf{p}) = 2\pi\ \delta(\omega^2-\mathbf{p}^2-M_j^2)$$





Transport coefficients: shear viscosity

- **Kubo formalism**

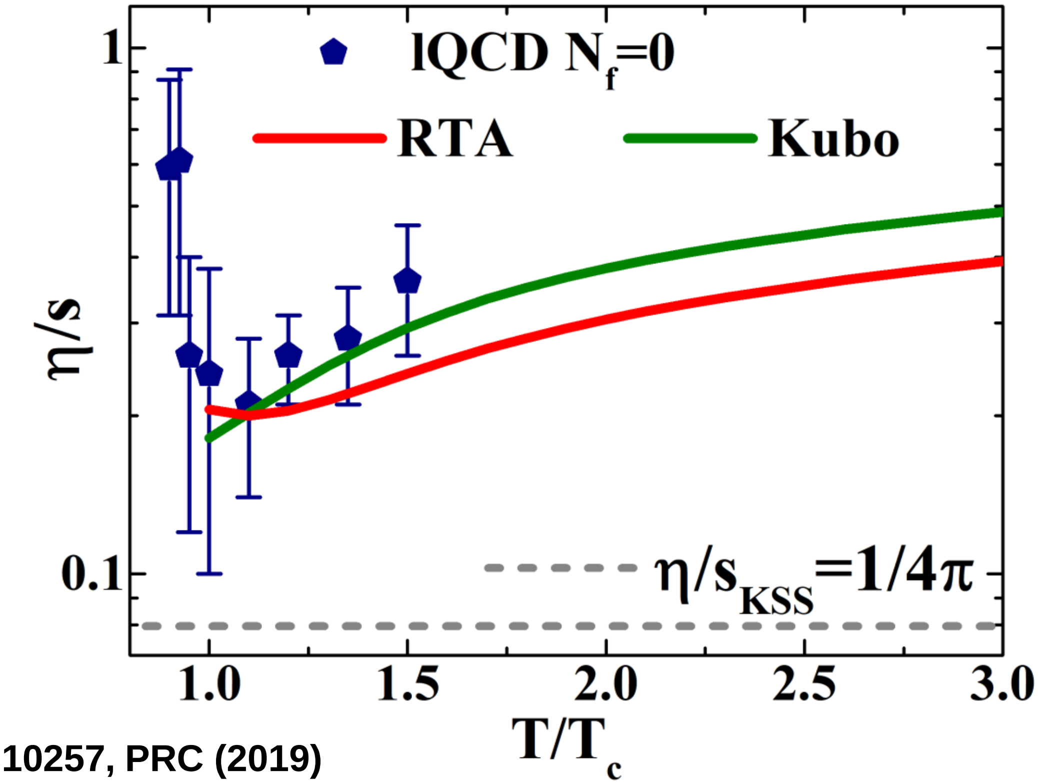
$$\eta^{\text{Kubo}}(T,\mu_q) = -\int \frac{d^4p}{(2\pi)^4}\, p_x^2 p_y^2 \sum_{i=q,\bar{q},g} d_i \,\frac{\partial f_i(\omega)}{\partial \omega}\, \rho_i(\omega,\mathbf{p})^2$$

$$= \frac{1}{15T}\int \frac{d^4p}{(2\pi)^4}\, \mathbf{p}^4 \sum_{i=q,\bar{q},g} d_i \left((1 \pm f_i(\omega)) f_i(\omega)\right) \rho_i(\omega,\mathbf{p})^2$$

- **Relaxation Time Approximation**

$$\eta^{\text{RTA}}(T,\mu_q) = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3} \sum_{i=q,\bar{q},g} \left(\frac{\mathbf{p}^4}{E_i^2\, \Gamma_i(\mathbf{p}_i,T,\mu_q)}\, d_i \left((1 \pm f_i(E_i)) f_i(E_i)\right)\right)$$

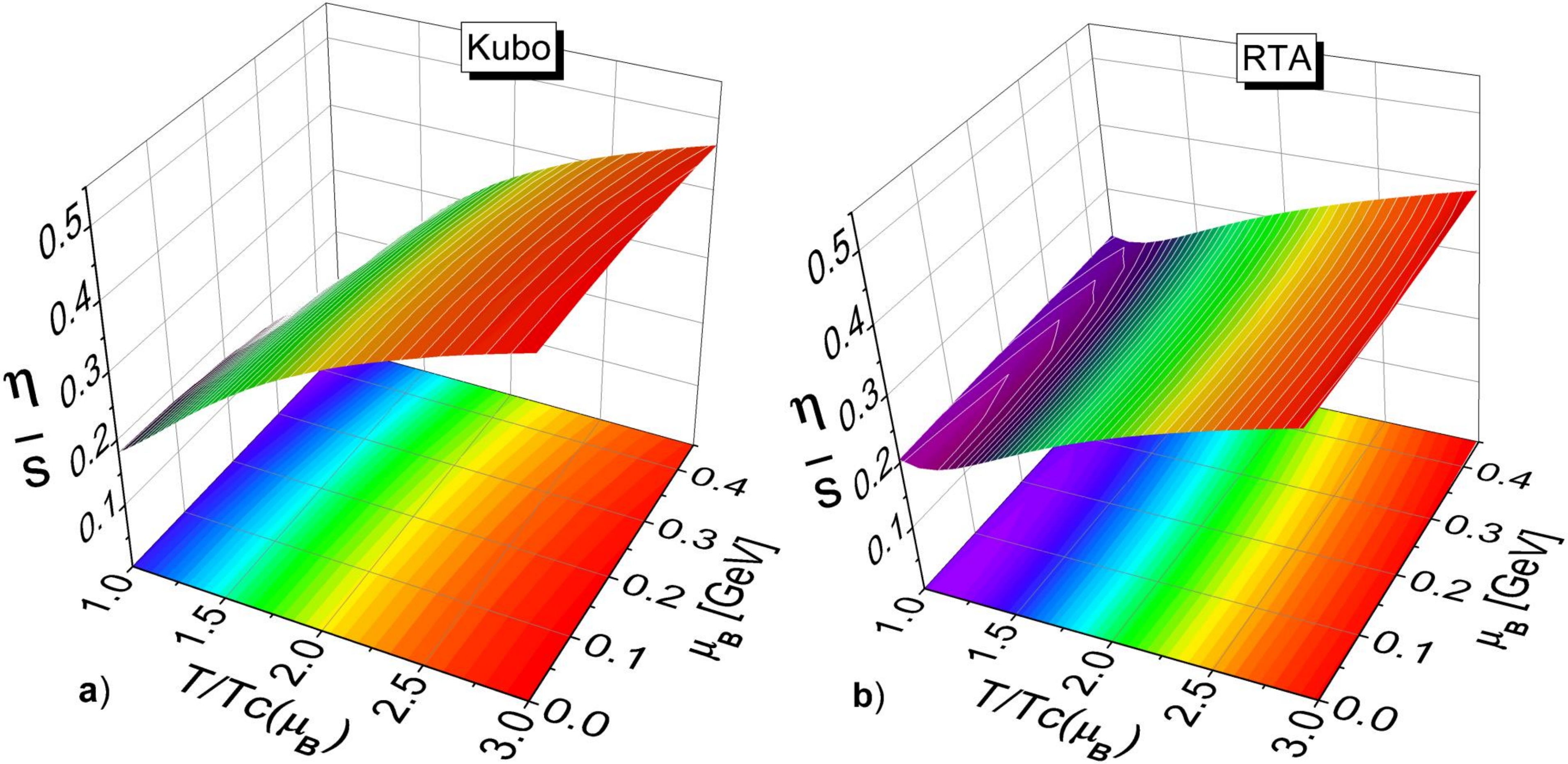
Collisional widths



P. Moreau, O.Soloveva, L.Oliva et al., arXiv:1903.10257, PRC (2019)

Transport coefficients: shear viscosity

- Very weak μ_B dependence



Transport coefficients: bulk viscosity

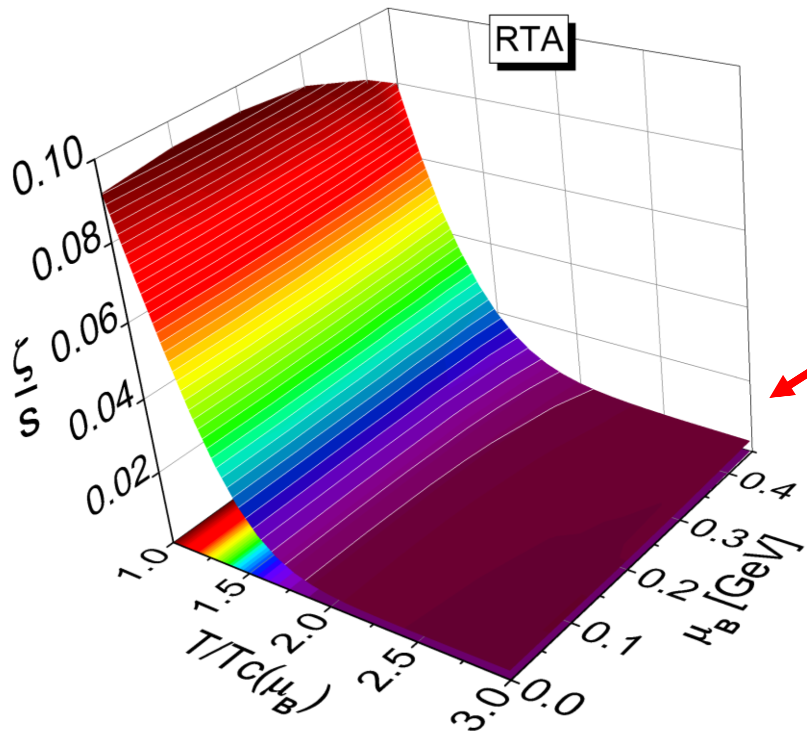
- **Relaxation Time Approximation**

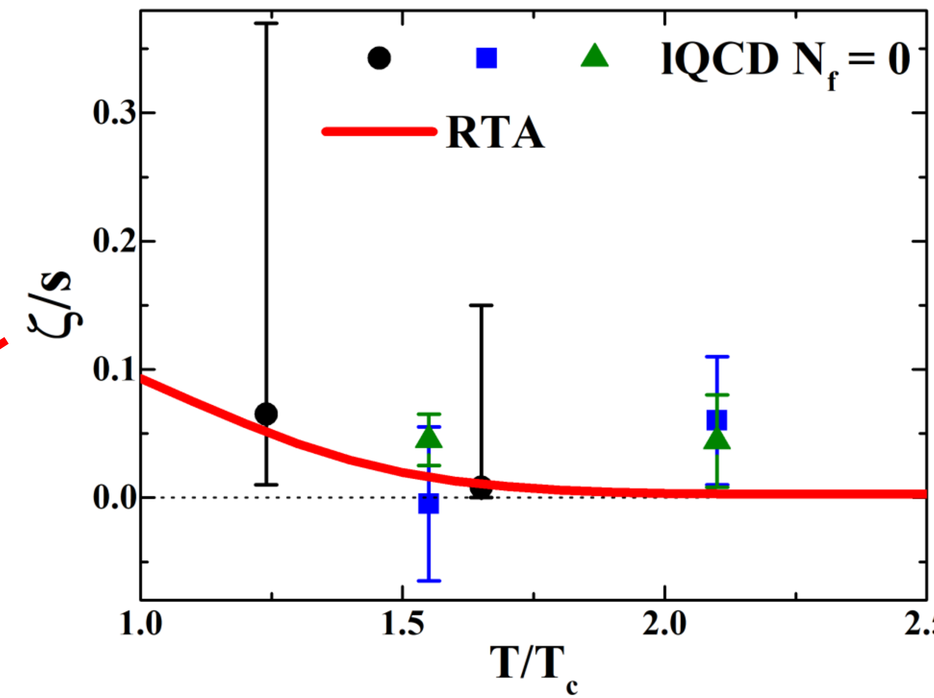
$$\zeta^{\mathrm{RTA}}(T,\mu_q) = \frac{1}{9T}\int \frac{d^3p}{(2\pi)^3}\sum_{i=q,\bar{q}}$$

$$\left(\frac{\mathbf{p}^4}{E_i^2\,\Gamma_i(\mathbf{p}_i,T,\mu_q)}\, d_i\left((1\pm f_i(E_i))f_i(E_i)\right)\right)[\mathbf{p}^2-3c_s^2(E_i^2-T^2\frac{dm_q^2}{dT^2})]^2$$

From DQPM parametrization

Collisional widths



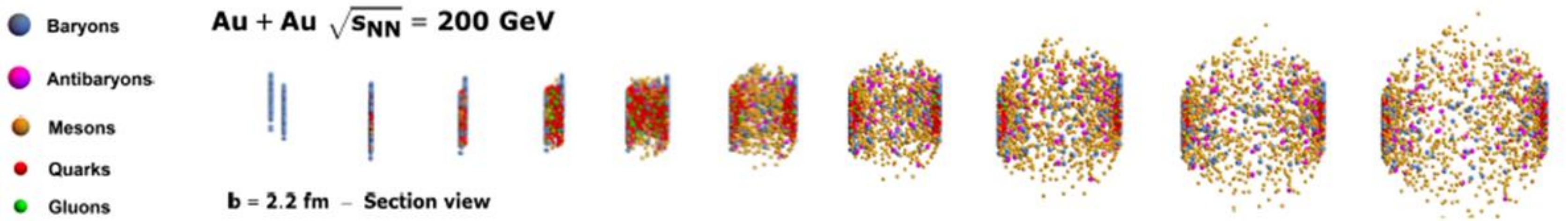


PHSD

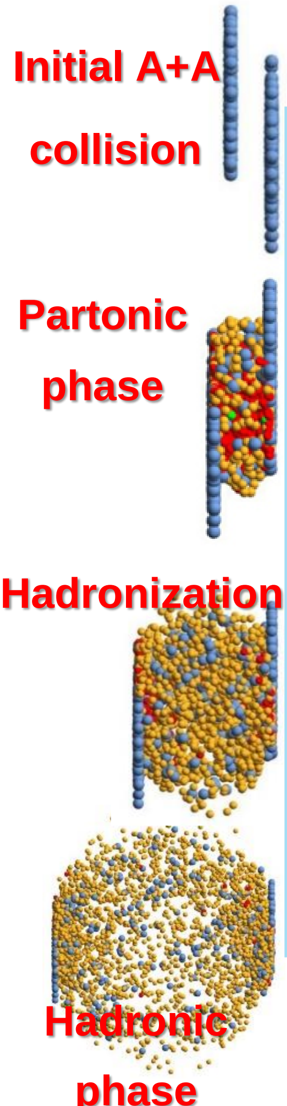
- **Goal:** Study the properties of strongly interacting matter under extreme conditions from a microscopic point of view
- **Realization:** dynamical many-body transport approach

Parton-Hadron-String-Dynamics (PHSD)

- Explicit parton-parton interactions, explicit transiton from hadronic to partonic degrees of freedom
- **Transport theory:** off-shell transport equations in phase-space representation based on Kadanoff-Baym equations for the partonic and hadronic phase

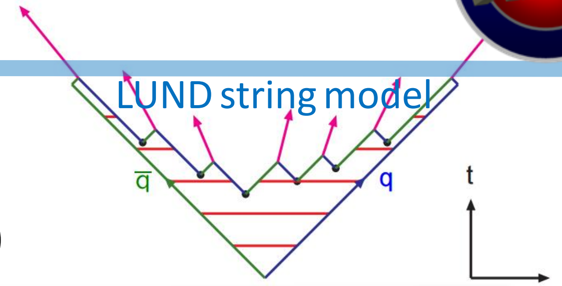


Stages of a collision in the PHSD



- **String formation** in primary NN collisions

  → **decays** to pre-hadrons (baryons and mesons)



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**

  Dissolution of pre-hadrons → DQPM

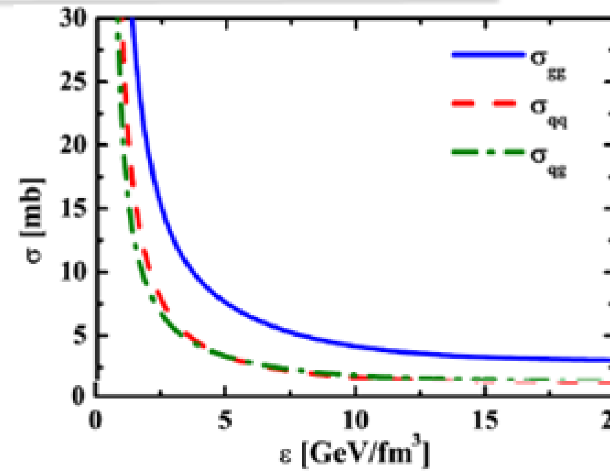
  → massive quarks/gluons and mean-field energy

  (quasi-)elastic collisions :

  $$q+q \to q+q \quad g+q \to g+q$$
  $$q+\bar{q} \to q+\bar{q} \quad g+\bar{q} \to g+\bar{q}$$
  $$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \quad g+g \to g+g$$

  inelastic collisions:

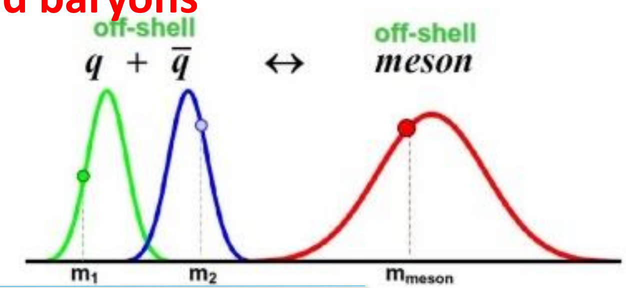
  $$q+\bar{q} \to g$$
  $$g \to q+\bar{q}$$



- **Hadronization to colorless off-shell mesons and baryons**

  $$g \to q+\bar{q}, \quad q+\bar{q} \leftrightarrow meson \ ('string')$$
  $$q+q+q \leftrightarrow baryon \ ('string')$$

  Strict 4-momentum and quantum number conservation



- **Hadron-string interactions – off-shell HSD**

Extraction of (T, μ_B) in PHSD

- In each space-time cell of the PHSD, the **energy-momentum tensor** is calculated by the formula: $$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$$
- Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in **the local rest frame** (LRF)

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} \Rightarrow \begin{pmatrix} \epsilon^{LRF} & 0 & 0 & 0 \\ 0 & P_x^{LRF} & 0 & 0 \\ 0 & 0 & P_y^{LRF} & 0 \\ 0 & 0 & 0 & P_z^{LRF} \end{pmatrix}$$

Xu et al., Phys.Rev. C96 (2017), 024902

For **each space-time cell** of the PHSD:

- Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}
- use DQPM relation: $$\frac{n_B}{T^3} \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right) + \ldots$$

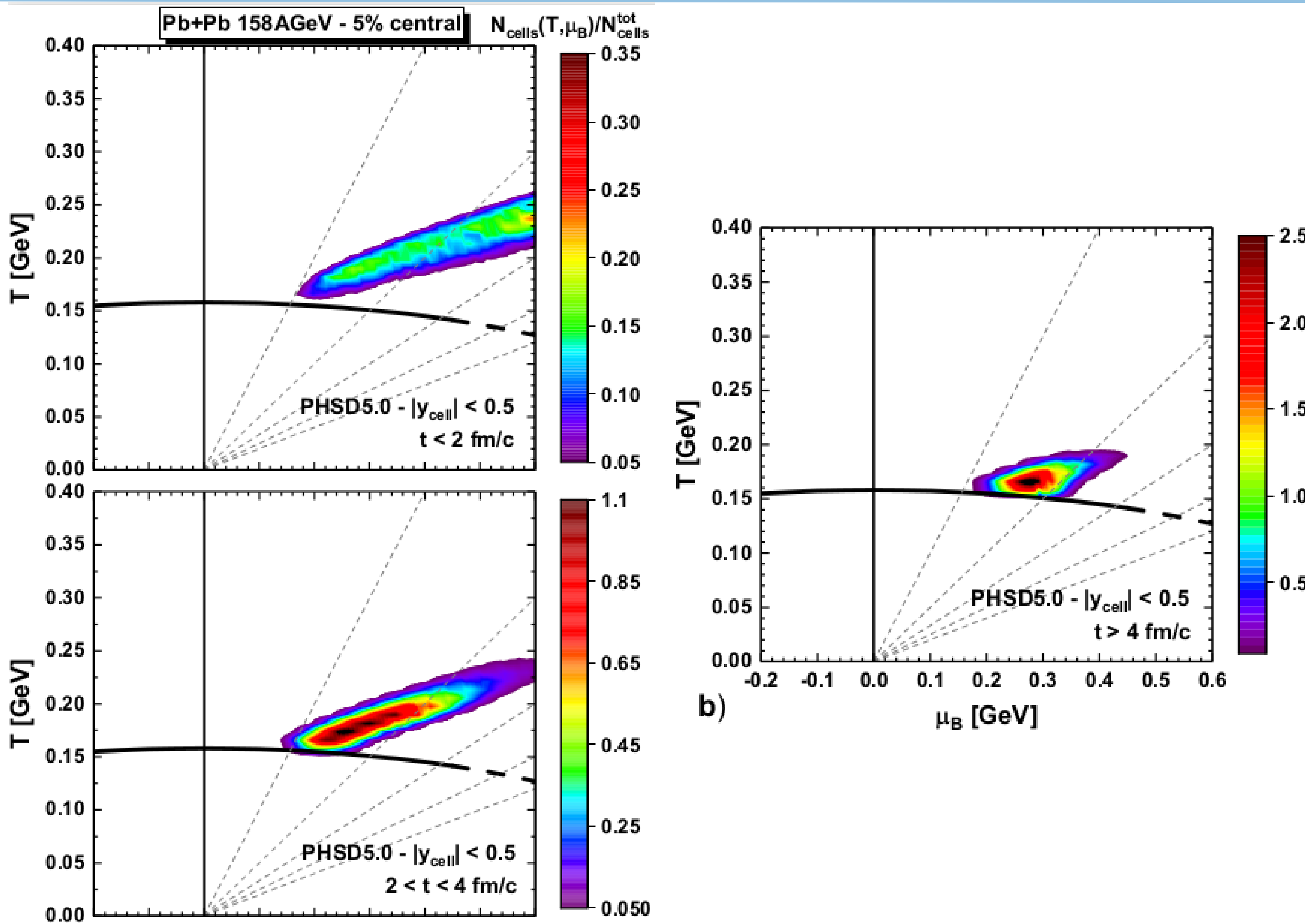
See talk of Dr. Jana N. Guenther

$$\Delta\epsilon/T^4 \approx \frac{1}{2}\left(T\frac{\partial\chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2 + \ldots$$

➔ obtain (T, μ_B) by solving the system of coupled equations using ε^{PHSD} and n_B^{PHSD}

P. Moreau, O.Soloveva, L.Oliva et al., arXiv:1903.10257, PRC (2019)

Illustration for HIC ($\sqrt{s_{NN}} = 17$ GeV)

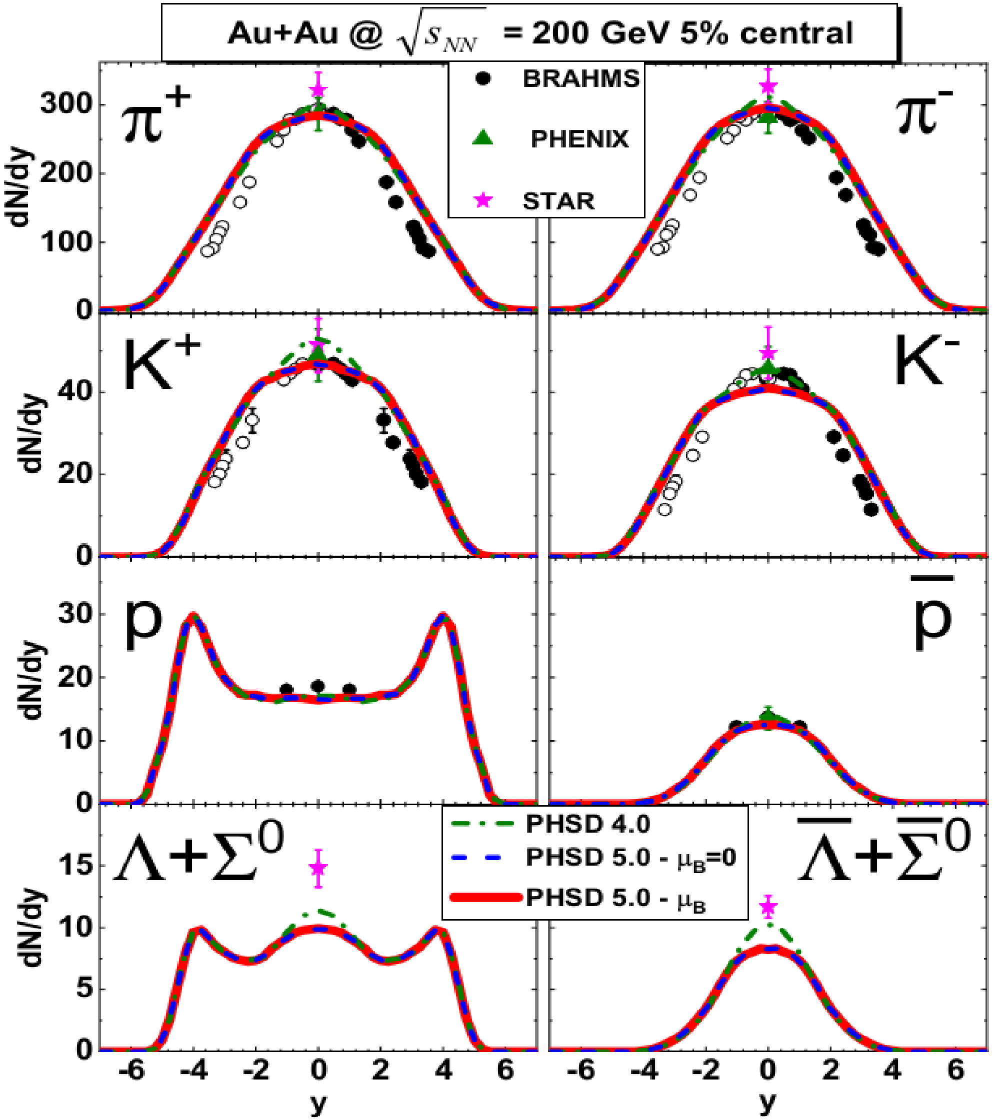


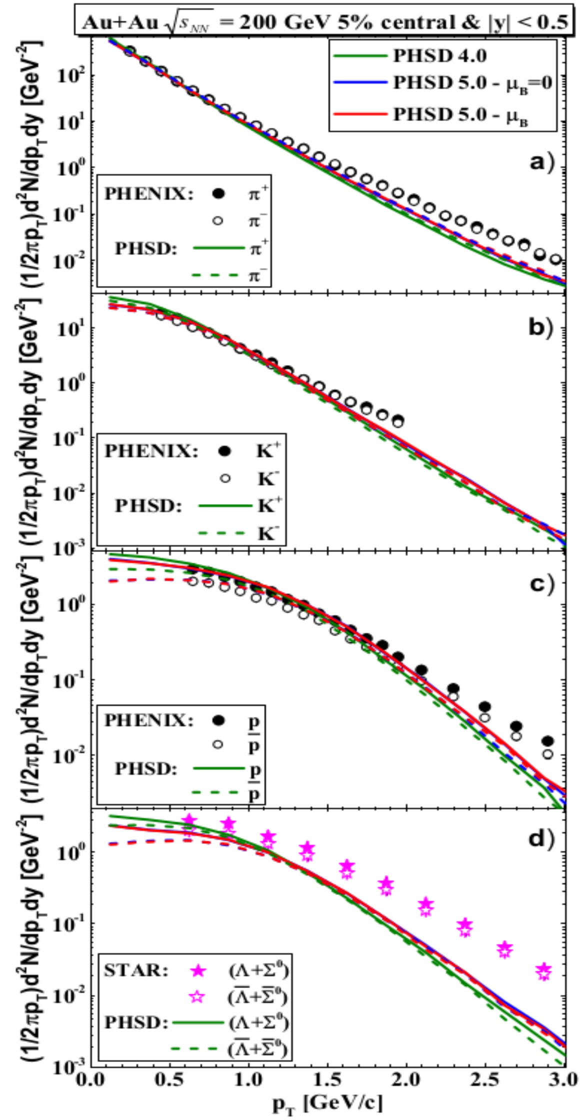
Results for HIC

- **Comparison between three different results:**

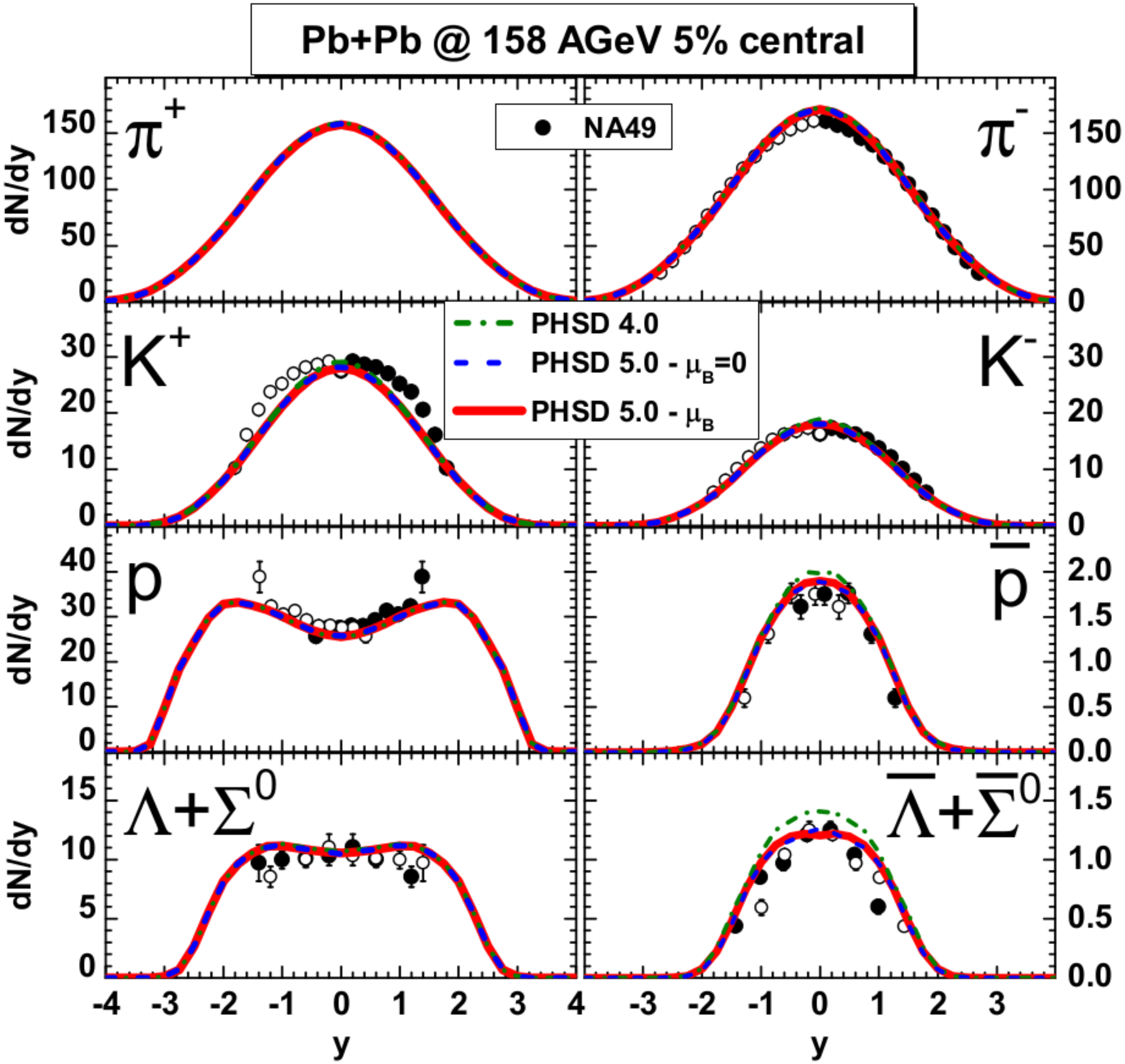
  1) PHSD 4.0 : only $\sigma(T)$ and $M(T)$
  2) PHSD 5.0 : with $\sigma\left(\sqrt{s}, T, \mu_B = 0\right)$ and $M(T, \mu_B = 0)$
  3) PHSD 5.0 : with $\sigma\left(\sqrt{s}, T, \mu_B\right)$ and $M(T, \mu_B)$

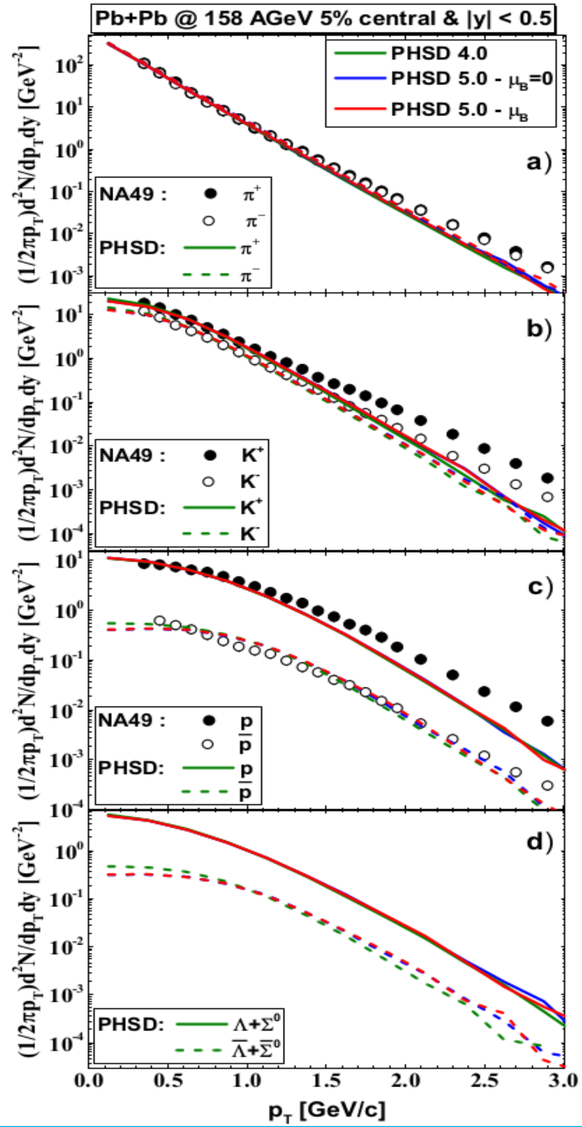
Results for HIC ($\sqrt{s_{NN}} = 200$ GeV)



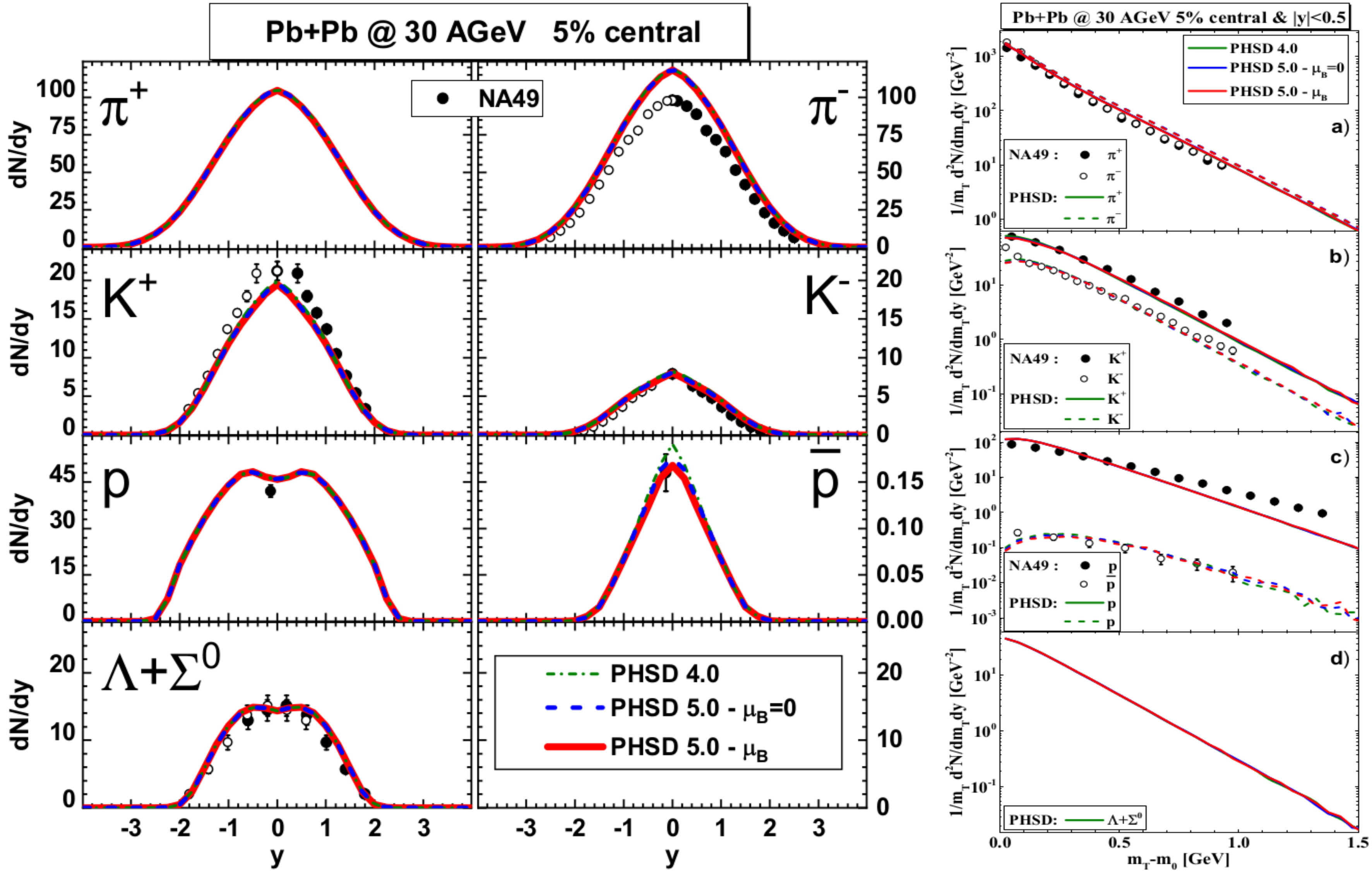


Results for HIC ($\sqrt{s_{NN}} = 17$ GeV)





Results for HIC ($\sqrt{s_{NN}} = 7.6$ GeV)



Summary / Outlook

- (T, μ_B)-dependent cross sections and masses have been implemented in PHSD
- High-μ_B regions are probed at low $\sqrt{s_{NN}}$ or high rapidity regions
- But, QGP fraction is small at low $\sqrt{s_{NN}}$:

  no effects seen in bulk observables

  P. Moreau, O.Soloveva, L.Oliva et al., arXiv:1903.10257, PRC (2019)

- **Outlook:**
  - Study more sensitive probes to finite-μ_B dynamics
  - More precise EoS finite/large μ_B
  - Possible 1st order phase transition at large μ_B?!

Summary / Outlook

Thank you for your attention!

Find out more on the PHSD web-site=)

DQPM EoS at finite (T, μ_B)

- **Taylor series of thermodynamic quantities in terms of (μ_B/T)**
- **With the 6[nd] order susceptibility. Example 2[nd] order:**

$$\Delta P/T^4 = \frac{P(T,\mu_B) - P(T,0)}{T^4} \approx \frac{1}{2}\chi_2^B(T)\left(\frac{\mu_B}{T}\right)^2$$

$$\frac{n_B}{T^3} = \left.\frac{\partial(P/T^4)}{\partial(\mu_B/T)}\right|_T \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right)$$

$$\Delta s/T^3 = \frac{s(T,\mu_B) - s(T,0)}{T^3} = \frac{1}{T^3}\left.\frac{\partial \Delta P}{\partial T}\right|_{\mu_B}$$

$$= T\left.\frac{\partial(\Delta P/T^4)}{\partial T}\right|_{\mu_B} + 4(\Delta P/T^4) \approx \frac{1}{2}\left(T\frac{\partial \chi_2^B(T)}{\partial T} + 2\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2$$

$$\Delta\epsilon/T^4 = \frac{\epsilon(T,\mu_B) - \epsilon(T,0)}{T^4}$$

$$= \Delta s/T^3 - \Delta P/T^4 + \left(\frac{\mu_B}{T}\right)\frac{n_B}{T^3} \approx \frac{1}{2}\left(T\frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2$$

A. Bazavov, Phys. Rev. D 96, 054504(2017)

Isentropic trajectories for (T, μ_B)

- Correspondance $s/n_B \leftrightarrow$ collisional energy

s/n_B = 420 ↔ 200 GeV

= 144 ↔ 62.4 GeV

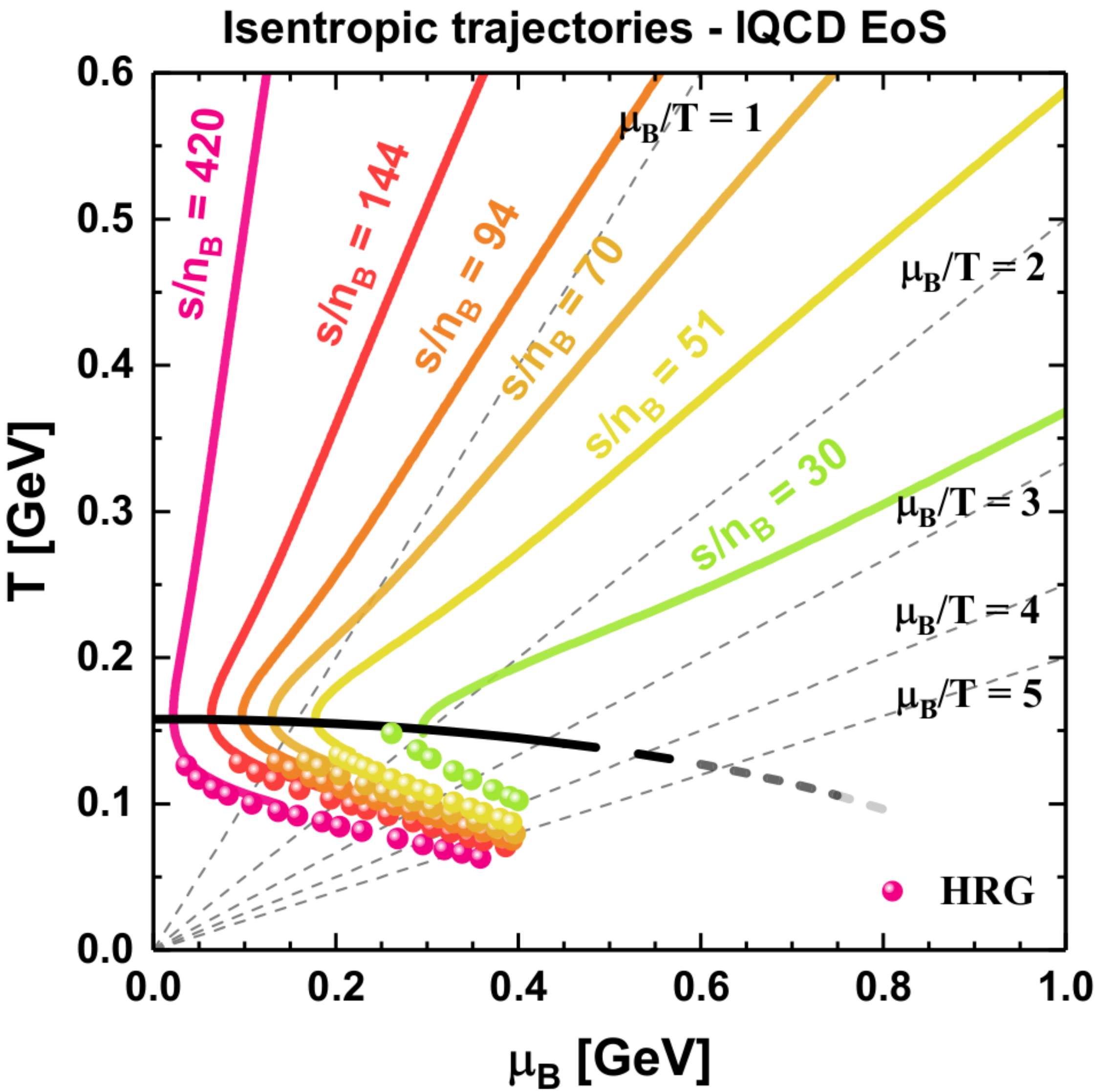
= 94 ↔ 39 GeV

= 70 ↔ 25 GeV

= 51 ↔ 19.6 GeV

= 30 ↔ 14.5 GeV

- Safe for $(\mu_B/T) < 2$



Energy-momentum tensor in PHSD

- **Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in the local rest frame (LRF)**

$$T^{\mu\nu}\ (x_\nu)_i = \lambda_i\ (x^\mu)_i = \lambda_i\ g^{\mu\nu}\ (x_\nu)_i$$

- **Landau-matching condition:** Xu et al., Phys.Rev. C96 (2017), 024902

$$T^{\mu\nu} u_\nu = \epsilon u^\mu = (\epsilon g^{\mu\nu}) u_\nu$$

- **Evaluation of the characteristic polynomial:**

$$P(\lambda) = \begin{vmatrix} T^{00} - \lambda & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} + \lambda & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} + \lambda & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} + \lambda \end{vmatrix}$$

- **The four solutions λ_i are identified to $(e, -P_1, -P_2, -P_3)$**

The pressure components P_i do not necessarily correspond to (P_x, P_y, P_z)

Extraction of (T, μ_B) in PHSD

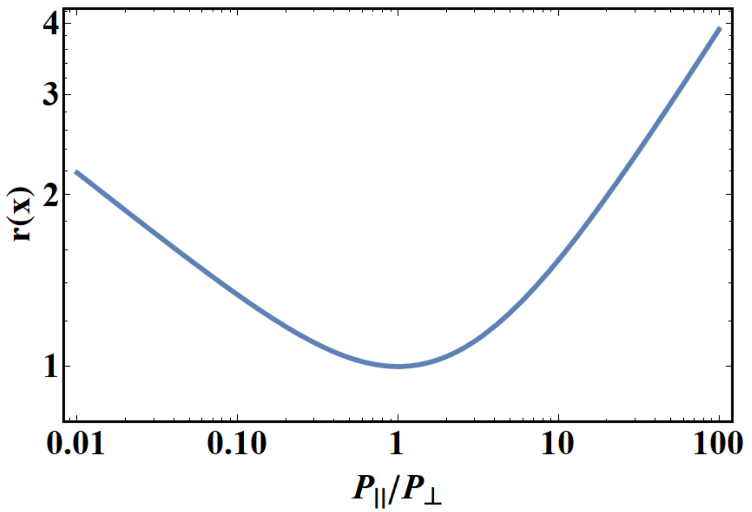
- **Correction for the medium anisotropy to extract values for (T, μ_B)**

$$\epsilon^{\rm anis} = \epsilon^{\rm EoS} \; r(x)$$

$$P_\perp = P^{\rm EoS} \; [r(x) + 3x r'(x)]$$

$$P_{||} = P^{\rm EoS} \; [r(x) - 6x r'(x)]$$

$$r(x) = \begin{cases} \dfrac{x^{-1/3}}{2}\left[1 + \dfrac{x \;\mathrm{arctanh}\sqrt{1-x}}{\sqrt{1-x}}\right] & \text{for } x \le 1 \\ \dfrac{x^{-1/3}}{2}\left[1 + \dfrac{x \arctan\sqrt{x-1}}{\sqrt{x-1}}\right] & \text{for } x \ge 1 \end{cases}$$



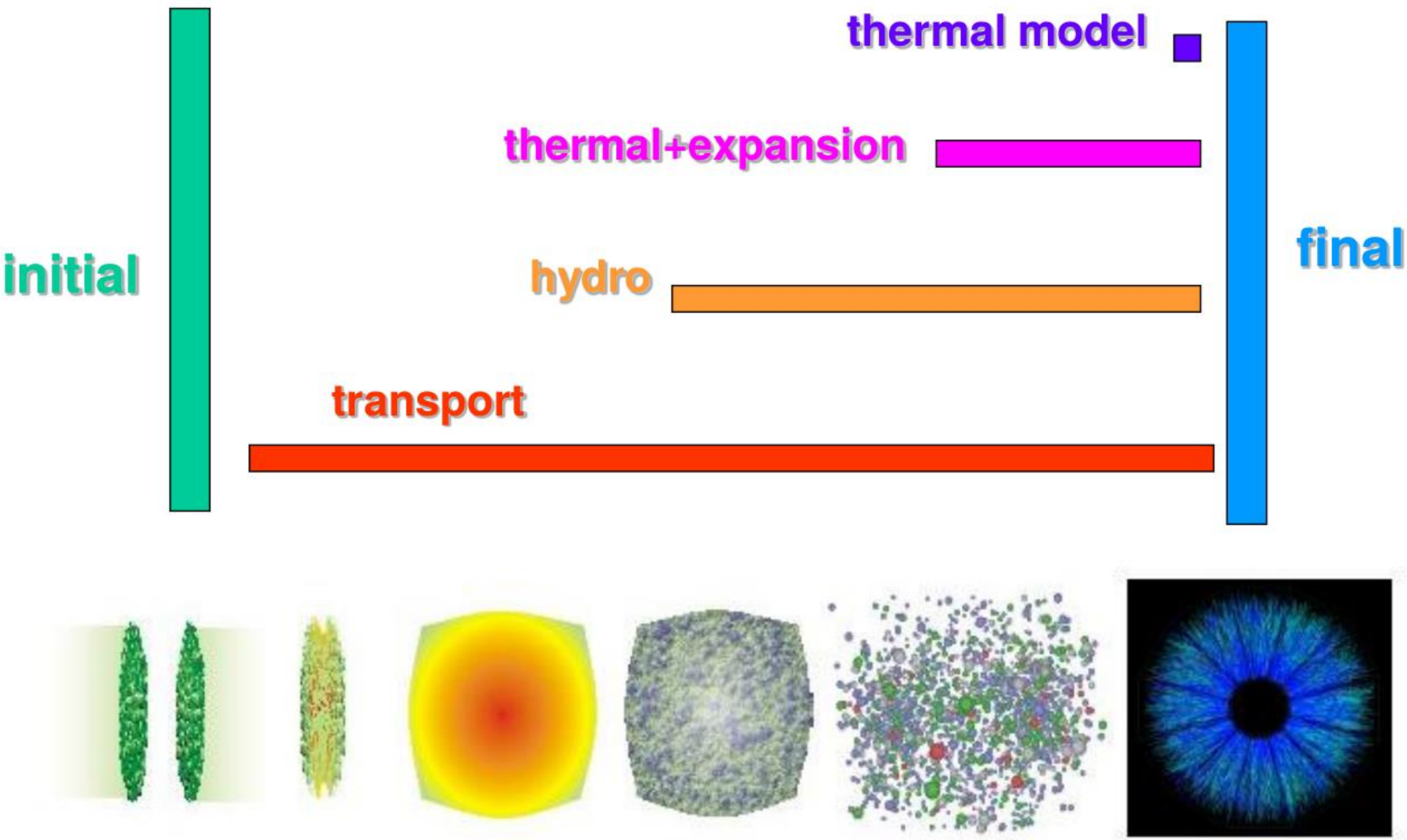
Ryblewski, Florkowski, Phys.Rev. C85 (2012) 064901

- **We have to solve the following system in PHSD:**

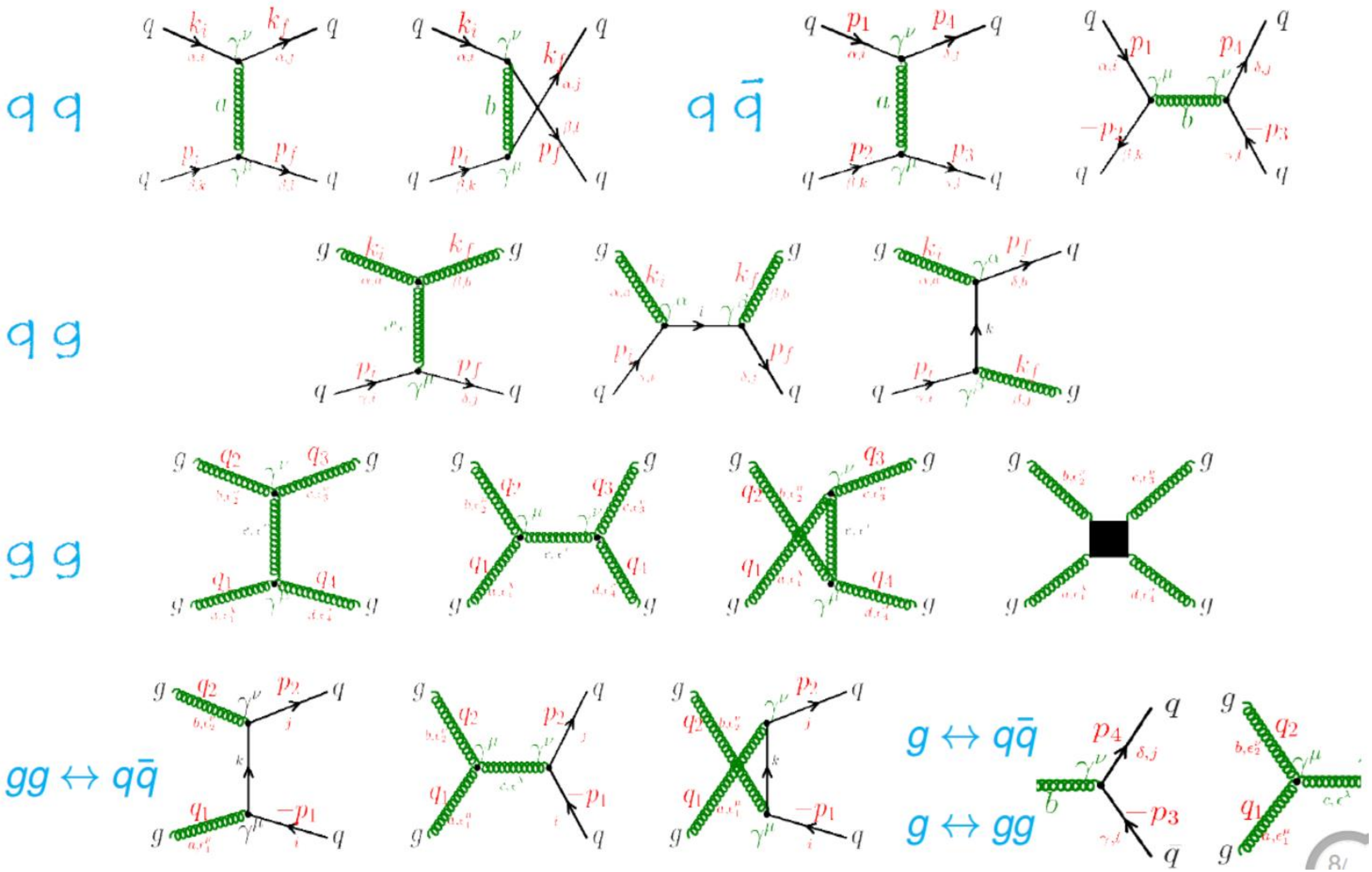
$$\begin{cases} \epsilon^{\rm EoS}(T, \mu_B) = \epsilon^{\rm PHSD}/r(x) \\ n_B^{\rm EoS}(T, \mu_B) = n_B^{\rm PHSD} \end{cases}$$

Done by Newton-Raphson method

Models of Heavy-Ion Collisions



DQPM: q, qbar, g elastic/inelastic scattering (leading order)



Time-like and space-like quantities

Separate time-like and space-like single-particle quantities by Θ(+P²), Θ(-P²):

$$\tilde{\mathrm{Tr}}_{\mathbf{g}}^{\pm} \cdots = \mathbf{d_g} \int \frac{\mathbf{d}\omega}{2\pi} \frac{\mathbf{d^3p}}{(2\pi)^3} \, 2\omega \, \rho_{\mathbf{g}}(\omega) \, \Theta(\omega) \, \mathbf{n_B}(\omega/\mathbf{T}) \, \underline{\Theta(\pm \mathbf{P^2})} \cdots$$ gluons

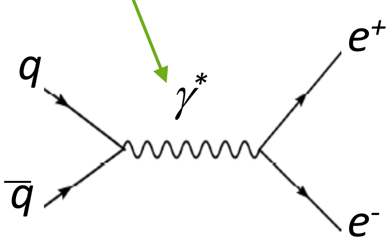
$$\tilde{\mathrm{Tr}}_{q}^{\pm} \cdots = d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \, 2\omega \, \rho_q(\omega) \, \Theta(\omega) \, n_F((\omega - \mu_q)/T) \, \underline{\Theta(\pm P^2)} \cdots$$ quarks

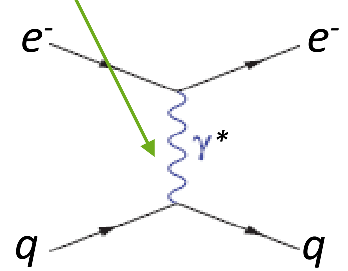
$$\tilde{\mathrm{Tr}}_{\bar{q}}^{\pm} \cdots = d_{\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \, 2\omega \, \rho_{\bar{q}}(\omega) \, \Theta(\omega) \, n_F((\omega + \mu_q)/T) \, \underline{\Theta(\pm P^2)} \cdots$$ antiquarks

Time-like: Θ(+P²): particles may decay to real particles or interact

Space-like: Θ(-P²): particles are virtuell and appear as exchange quanta in interaction processes of real particles

Examples:





Cassing, NPA 791 (2007) 365: NPA 793 (2007)