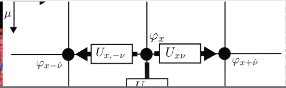
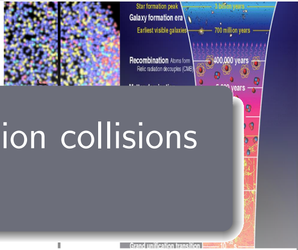
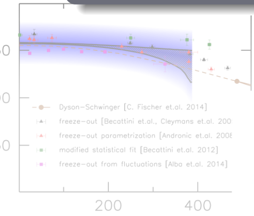
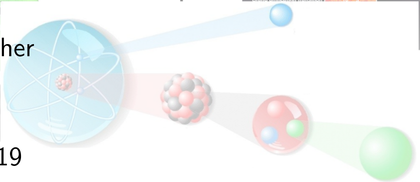




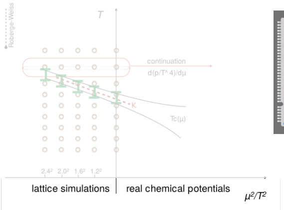
The QCD phase diagram and heavy ion collisions from lattice QCD





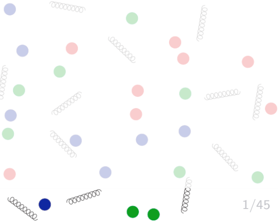
Jana N. Guenther

May 22nd 2019



SFB TR55

WB collaboration



The QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a,\mu\nu} + \overline{\psi} \left(\mathrm{i}\gamma_{\mu} D^{\mu} - m \right) \psi$$

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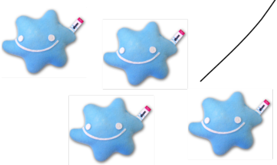
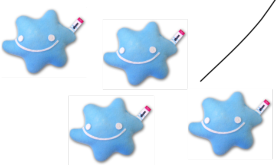


pictures: particlezoo.net

The QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \overline{\psi}\,(\mathrm{i}\gamma_\mu D^\mu - m)\,\psi$$



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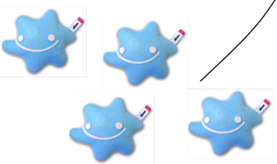


pictures: particlezoo.net

Why LQCD?

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \overline{\psi}\left(\mathrm{i}\gamma_\mu D^\mu - m\right)\psi$$

- Because of the strong coupling and the self interaction of gluons perturbation theory is not feasible
- Path integral quantization:

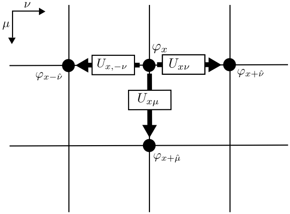
$$\langle 0 | T \hat{\phi}_1 \ldots \hat{\phi}_n | 0 \rangle = \frac{\int D\phi\, \hat{\phi}_1 \ldots \hat{\phi}_n e^{\mathrm{i} \int \mathrm{d}x\, \mathcal{L}}}{\int D\phi\, e^{\mathrm{i} \int \mathrm{d}x\, \mathcal{L}}}$$

First problem: There are many points in space-time $D\phi = \prod_i \mathrm{d}\phi(x_i)$

Solution: Replace continuous space by a discrete 4d lattice

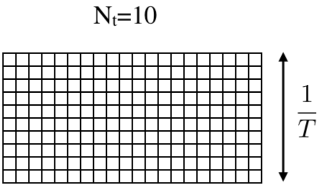
How do we do that?

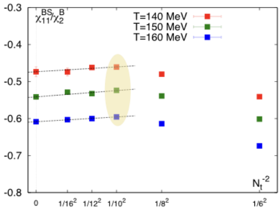
- We look at a 4d space-time lattice with size $N_s^3 \times N_t$
- Fermions live on the lattice sites gauge fields live on the links
- We use Euclidean space-time: $t \longrightarrow i\tau$



- We can do Monte-Carlo-Simulations (with importance sampling) to solve our integrals
- Everything is determined in terms of our lattice spacing a
- a has to be determined by comparison with physical observables (for example $a = \frac{(am_p)_{\text{lattice}}}{938 \text{ MeV}}$)
- We have to take the limits $a \longrightarrow 0$, $N_s \longrightarrow \infty$
- To do thermodynamics: $T = \frac{1}{aN_t}$

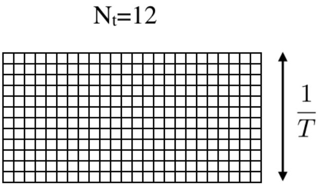
The continuum limit

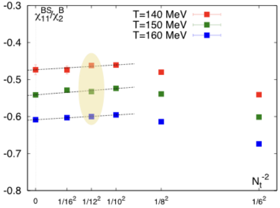


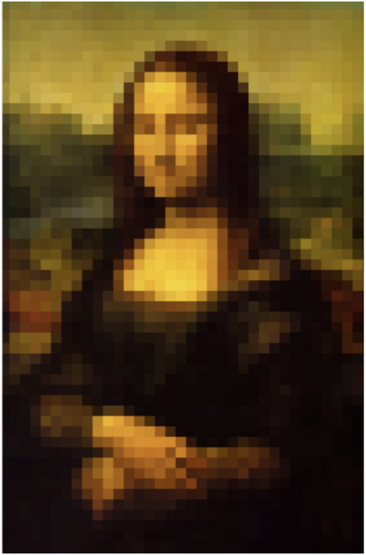


picture by Claudia Ratti; slide by Szabolcs Borsanyi

The continuum limit







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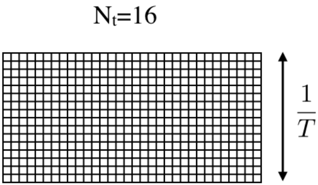
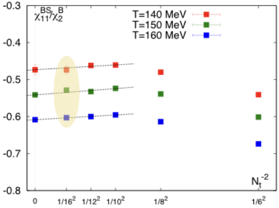
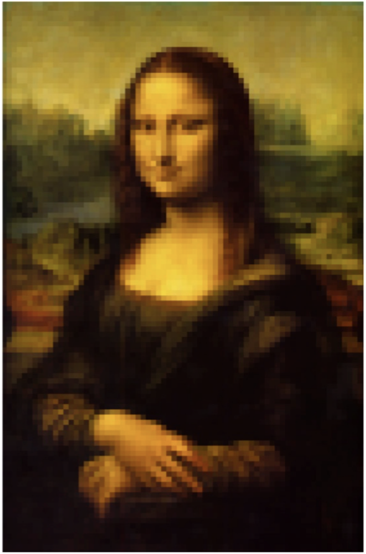
The continuum limit







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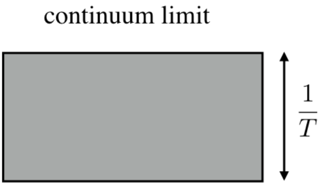
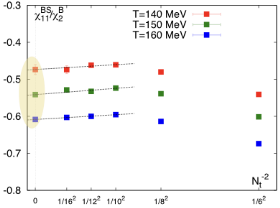
The continuum limit





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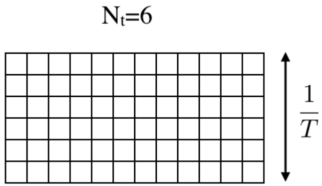

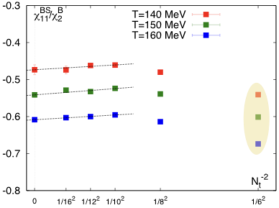

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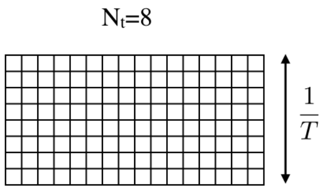
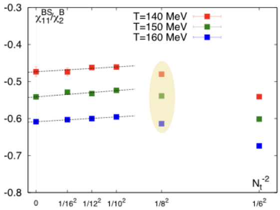
The continuum limit

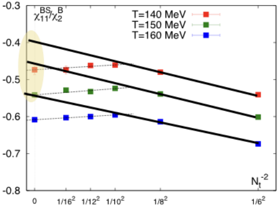




picture by Claudia Ratti; slide by Szabolcs Borsanyi

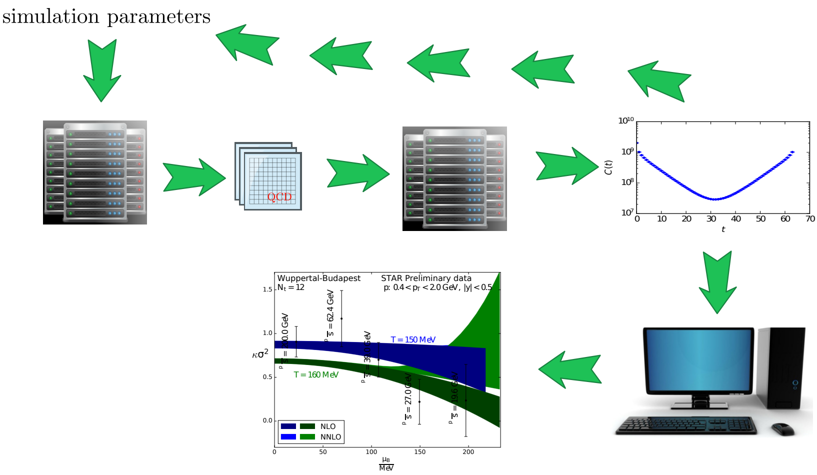
The continuum limit

False continuum limit



picture by Claudia Ratti; slide by Szabolcs Borsanyi

The work flow



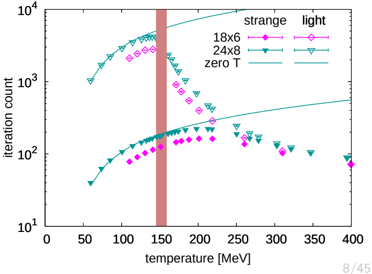
picture by Lukas Varnhorst

Why aren't we finished yet?

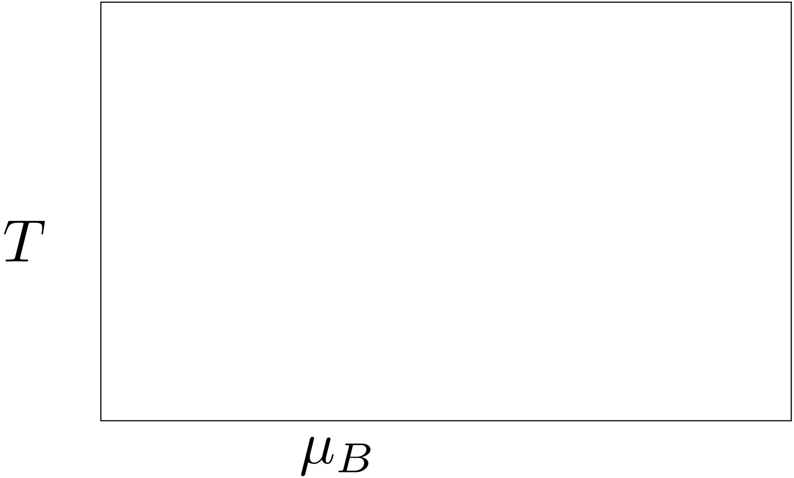
- Simulations take a lot of computer time
- Not everything can be calculated directly. For example:
  - Only observables that can be calculated in Euclidean space
  - Only thermal equilibrium
  - Only simulations at

    $\mu_B = 0 \Rightarrow \langle n_B \rangle = 0$ ↯
    heavy ion collision experiments

1000 configurations on a $64^3 \times 16$ lattice cost about 1 million core hours

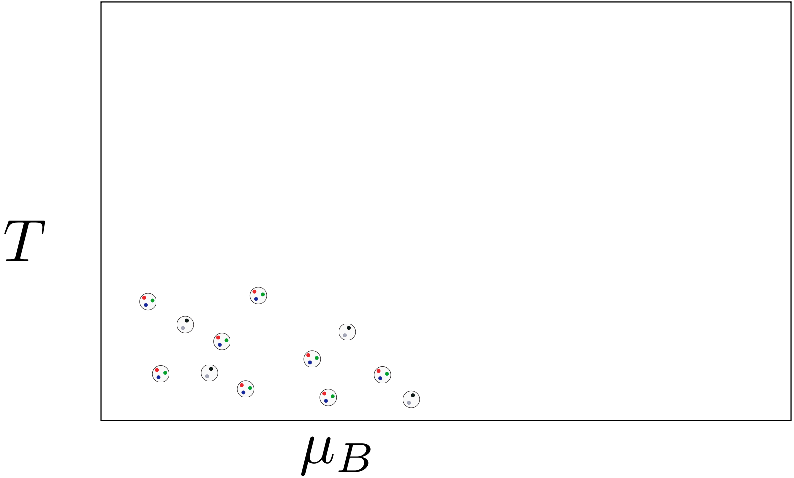


The (T, μ_B)-phase diagram of QCD



Our observables:
T_c, Equation of state, Fluctuations

The (T, μ_B)-phase diagram of QCD



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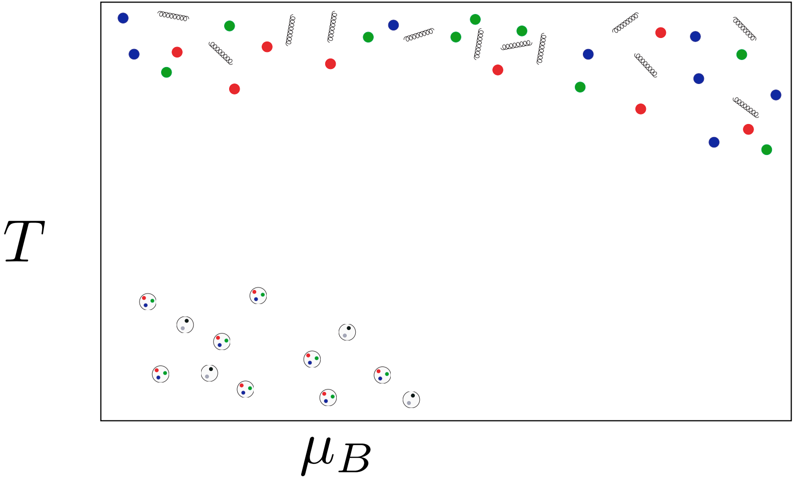

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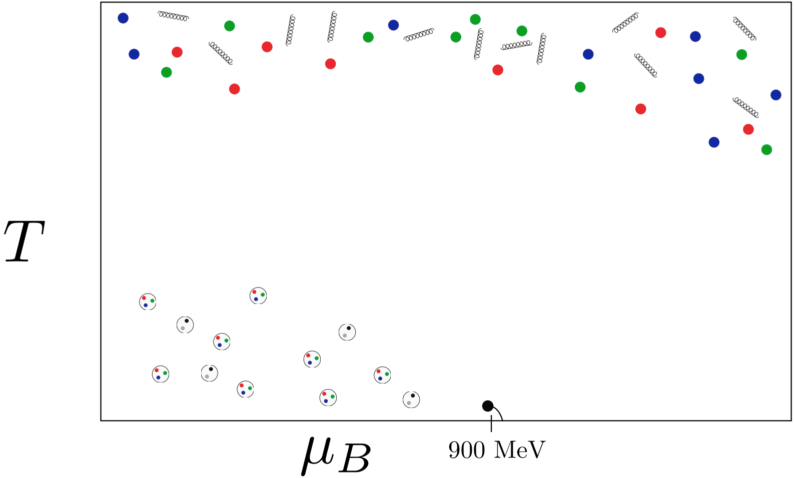

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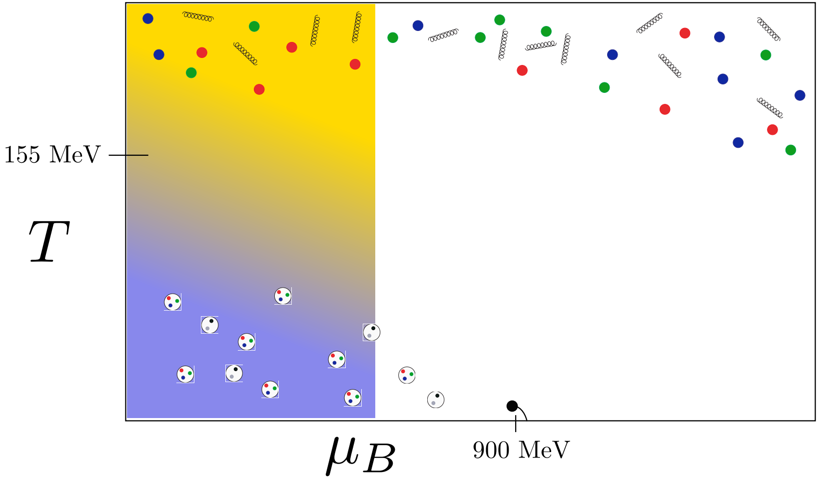

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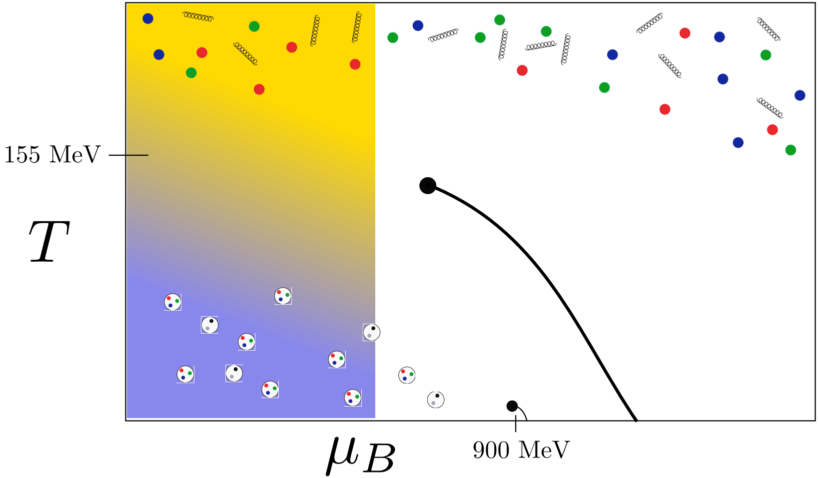

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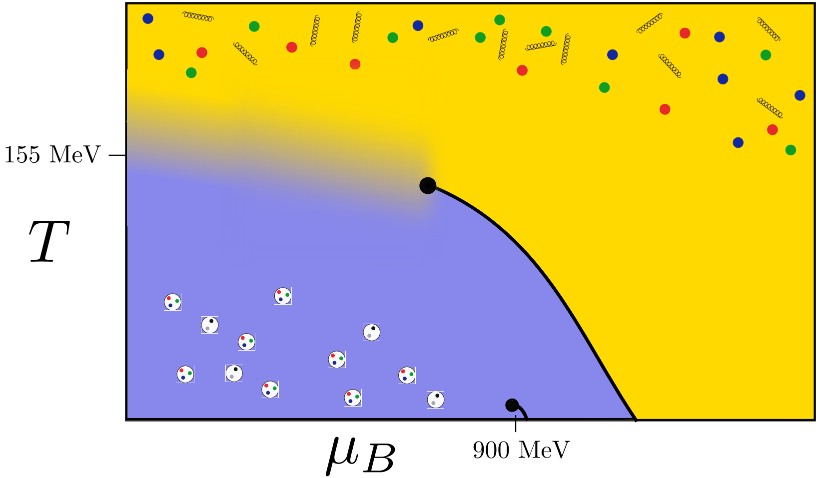

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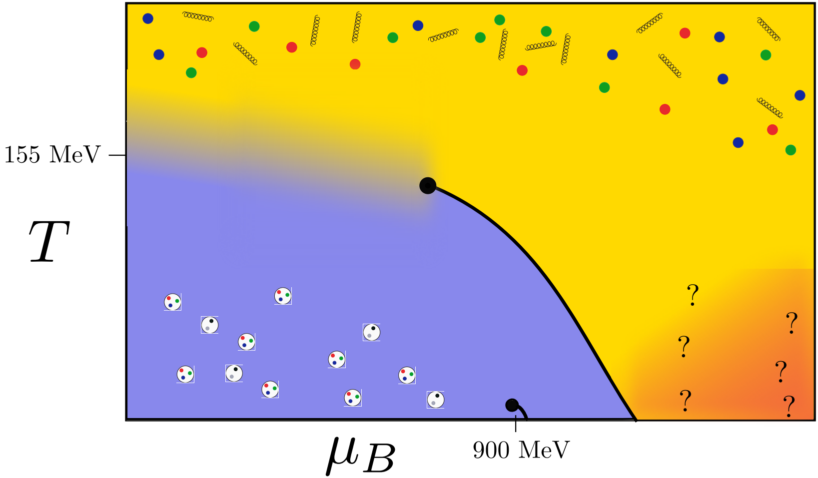

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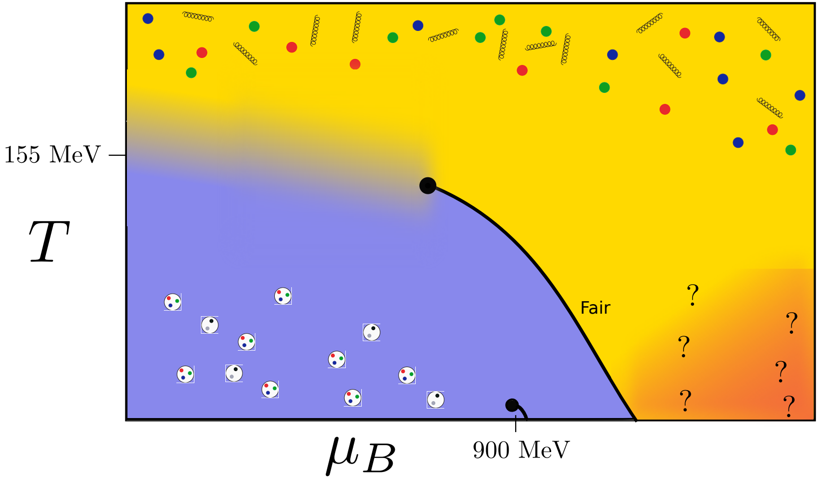

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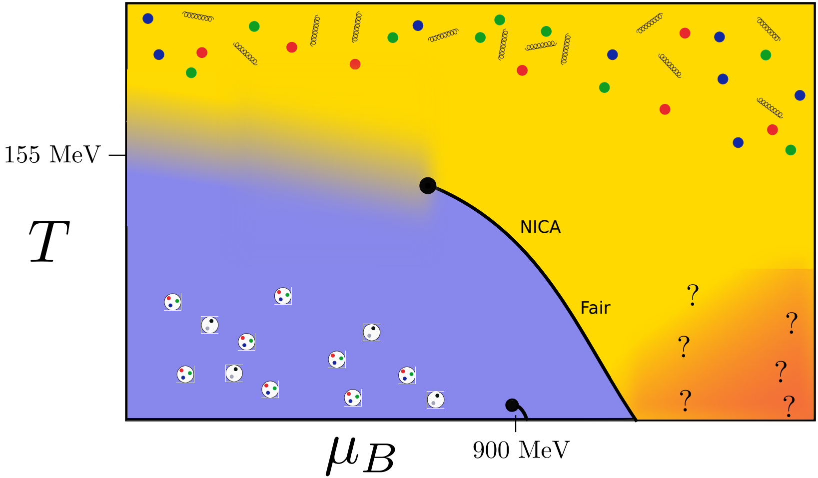

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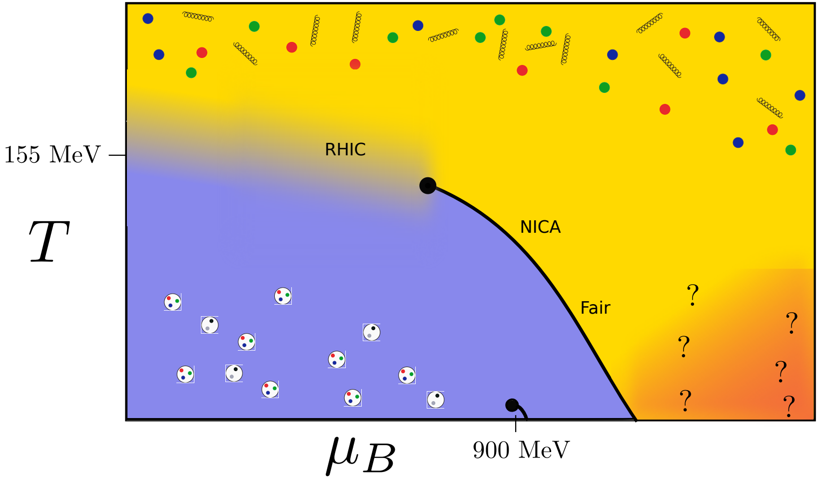

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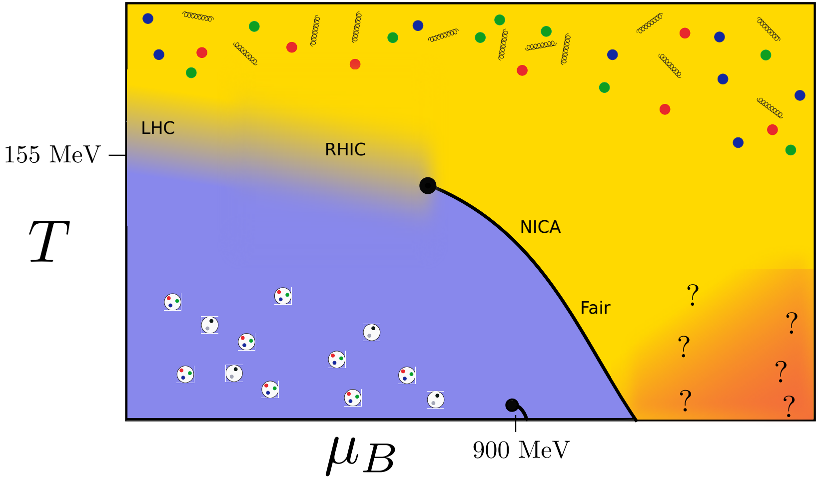


Our observables:

T_c, Equation of state, Fluctuations

The sign problem

The QCD partition function:

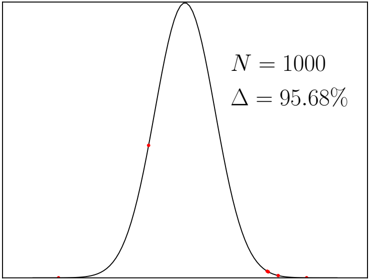
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \;\det M(U) e^{-\beta S_G(U)}$$

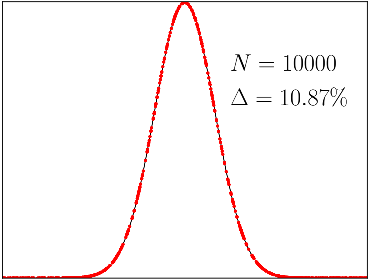
- For Monte Carlo simulations $\det M(U) e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}} \cdot \frac{200}{N}$$

The x_i are drawn from a uniform distribution in the interval $[-100, 100]$

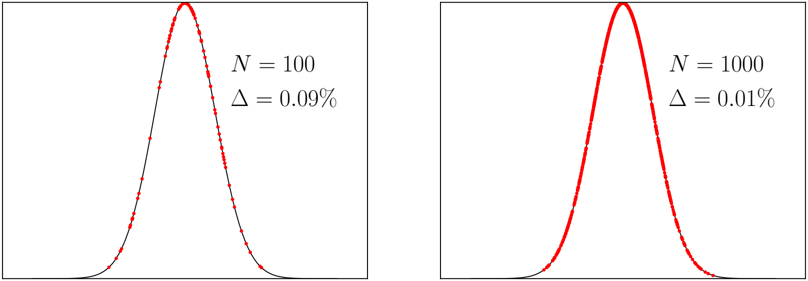




Importance sampling

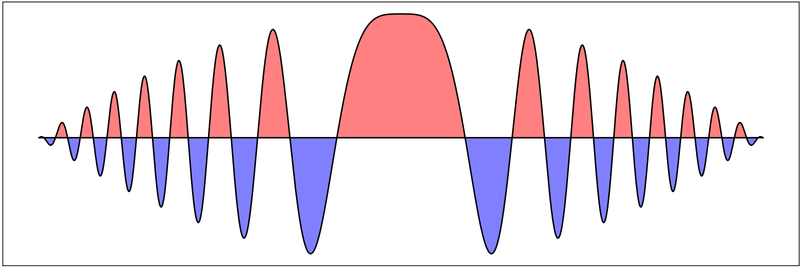
$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\cdot\frac{1}{N}$$

The x_i are drawn from a normal distribution



The sign problem

$$\int_{-\infty}^{\infty} (100 - x^2) \frac{e^{-\frac{i}{2}x^2}}{\sqrt{2\pi}}$$

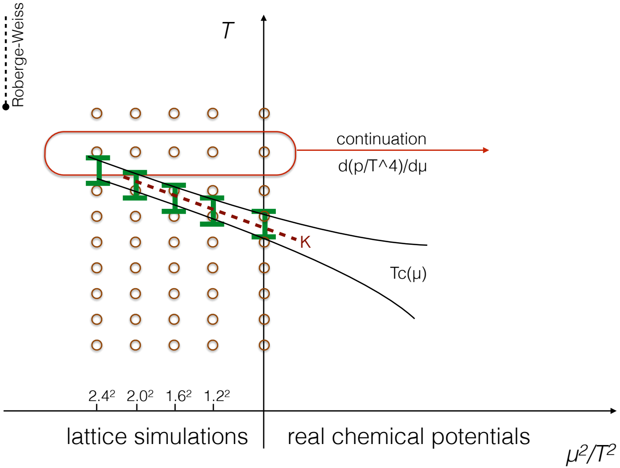


Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Complex Langevin
- Lefshetz Thimble
- Density of state methods
- Dual variables
- ...

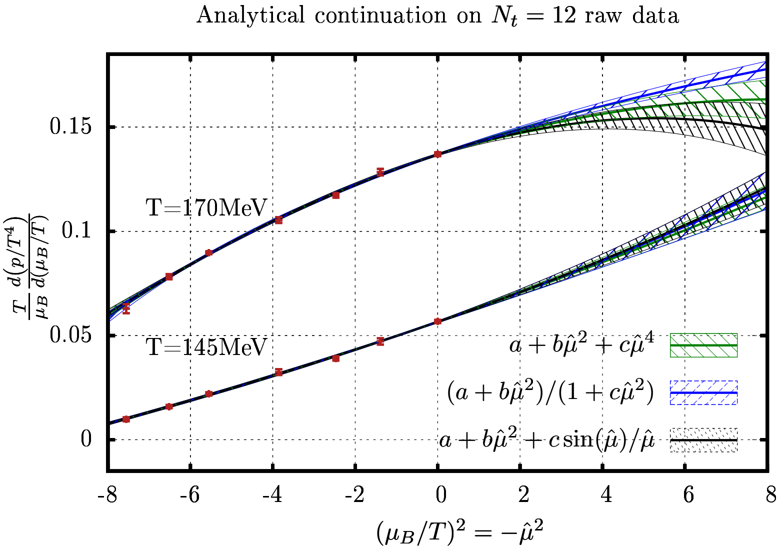
- Taylor expansion $\longrightarrow$ [Bazavov et al., Bazavov:2017dus], [Bonati et al., Bonati:2018nut]
- Imaginary μ

Analytic continuation



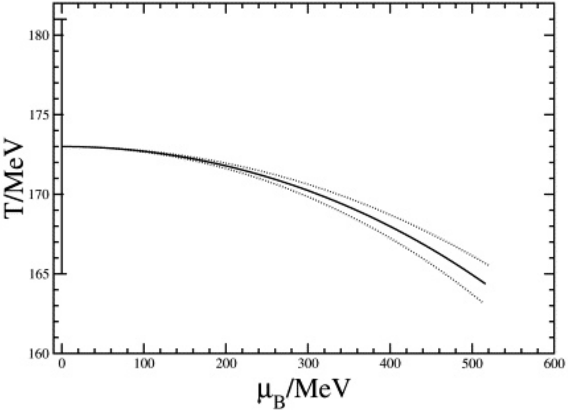
Common technique: [de Forcrand, Philipsen, deForcrand:2002hgr], [Bonati et al., Bonati:2015bha], [Cea et al., Cea:2015cya], [D'Elia et al., DElia:2016jqh], [Bonati et al., Bonati:2018nut] . . .

Different functions



First time

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$



[de Forcrand, Philipsen, deForcrand:2002hgr]

Curvature of the transition temperature

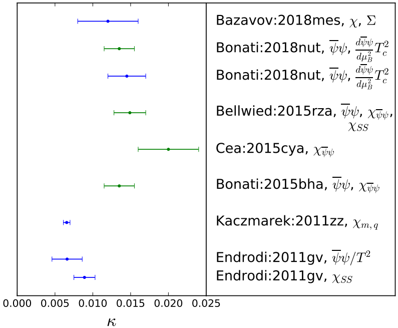
Curvature function:

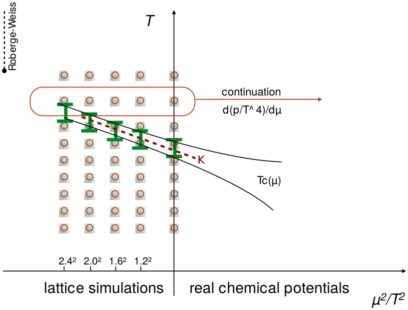
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1+ax}{1+bx}$$

$$C_3(x) = \frac{1}{1+ax+bx^2}$$





Continuum limit in action

Continuum extrapolation:
$\kappa = \kappa^{\mathrm{c}} + A\left(\frac{1}{N_t}\right)^2$
Combined curvature fit and continuum extrapolation with:
$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{\mathrm{c}} + c_1 \frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$

Continuum limit in action

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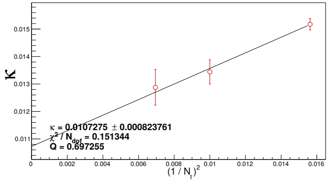
Extrap. with $Nt = 8, 10, 12$



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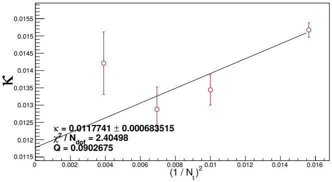
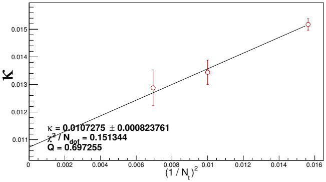
Extrap. with $Nt = 8, 10, 12, 16$



Extrap. with $Nt = 8, 10, 12$



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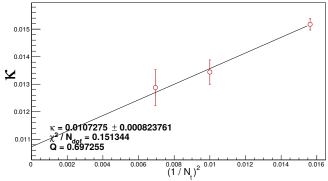
Continuum extrapolation:

$$\kappa = \kappa^{\mathrm{c}} + A \left(\frac{1}{N_t} \right)^2$$

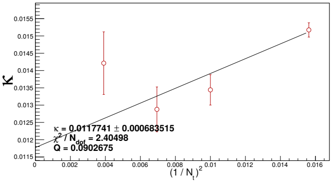
Combined curvature fit and continuum extrapolation with:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{\mathrm{c}} + c_1 \frac{1}{N_t^2} \right) \left(\frac{\mu_B}{T_c} \right)^2$$

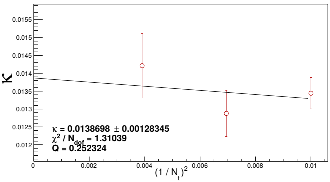
Extrap. with $Nt = 8,\ 10,\ 12$



Extrap. with $Nt = 8,\ 10,\ 12,\ 16$



Extrap. with $Nt = 10,\ 12,\ 16$



Curvature of the transition temperature

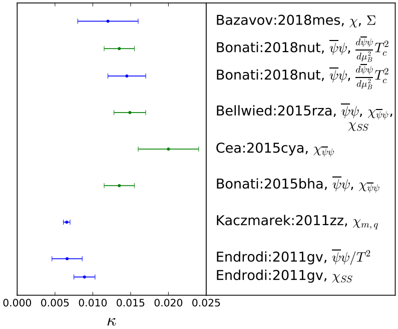
Curvature function:

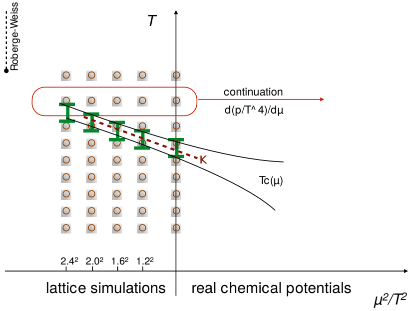
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1+ax}{1+bx}$$

$$C_3(x) = \frac{1}{1+ax+bx^2}$$





Curvature of the transition temperature

Curvature function:

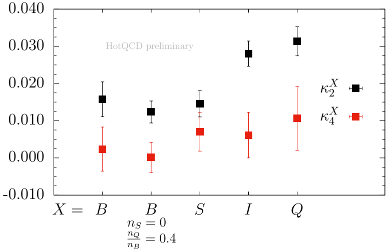
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

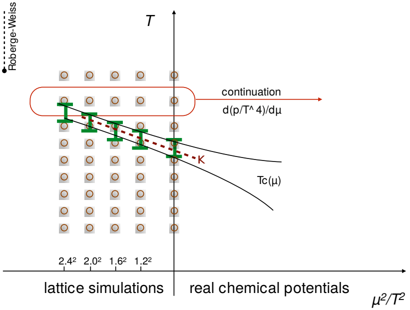
$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1+ax}{1+bx}$$

$$C_3(x) = \frac{1}{1+ax+bx^2}$$

[Steinbrecher:2018phh]





Curvature of the transition temperature

Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

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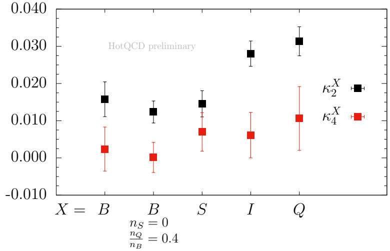
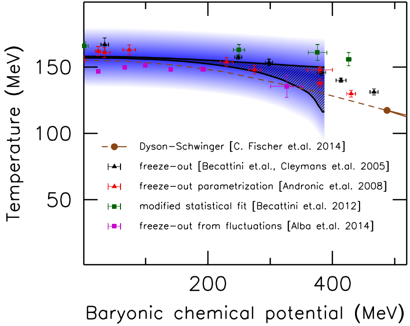
[Steinbrecher:2018phh]





Curvature of the transition temperature

Curvature function:

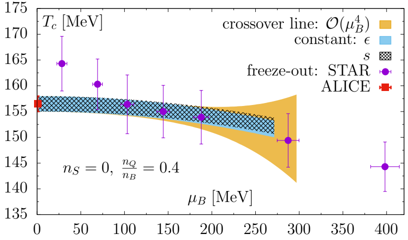
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

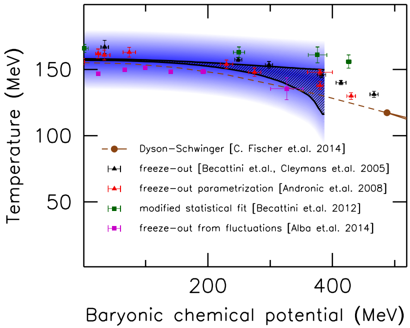
$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

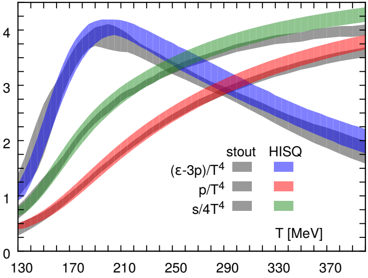
[Bazavov:2018mes]

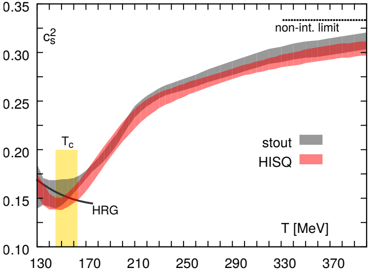




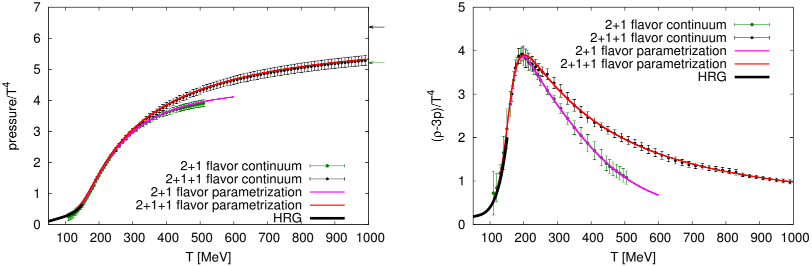
$\mu_B = 0$

Equation of state with 2+1 quarks
stout: [Borsanyi:2013bia]
HISQ: [Bazavov:2014pvz]





high temperatures

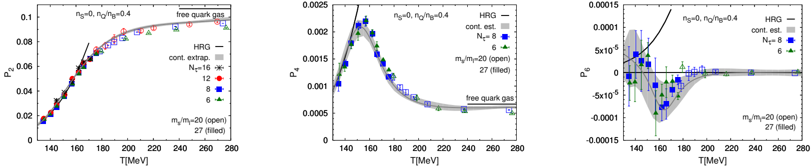


[Borsanyi:2016ksw]

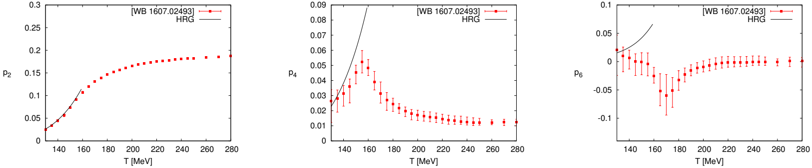
The influence of the charm quark shows up around 300 MeV.

$\mu_B > 0$

$$\frac{P}{T^4} = p_0 + p_2 \frac{\mu_B^2}{T^2} + p_4 \frac{\mu_B^4}{T^4} + p_6 \frac{\mu_B^6}{T^6} + \ldots$$



[Bazavov:2017dus]



[Gunther:2016vcp]

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The fit function

$$\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}, \quad \hat{\mu} = \frac{\mu}{T}$$

$$\frac{p}{T^4} = \chi_0^B + \frac{1}{2!}\chi_2^B\hat{\mu}_B^2 + \frac{1}{4!}\chi_4^B\hat{\mu}_B^4 + \frac{1}{6!}\chi_6^B\hat{\mu}_B^6 + \frac{1}{8!}\chi_8^B\hat{\mu}_B^8 + \frac{1}{10!}\chi_{10}^B\hat{\mu}_B^{10}$$

From this we can calculate the derivatives that we can measure on the lattice:

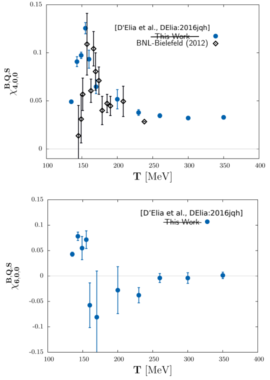
$$\chi_1^B(\hat{\mu}_B) = \frac{\partial}{\partial\hat{\mu}_B}\left(\frac{p}{T^4}\right) = \chi_2^B\hat{\mu}_B + \frac{1}{3!}\chi_4^B\hat{\mu}_B^3 + \frac{1}{5!}\chi_6^B\hat{\mu}_B^5 + \frac{1}{7!}\chi_8^B\hat{\mu}_B^7 + \frac{1}{9!}\chi_{10}^B\hat{\mu}_B^9$$

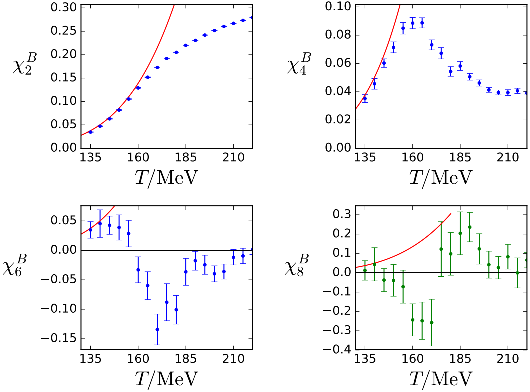
$$\chi_2^B(\hat{\mu}_B) = \frac{\partial^2}{\partial\hat{\mu}_B^2}\left(\frac{p}{T^4}\right) = \chi_2^B + \frac{1}{2!}\chi_4^B\hat{\mu}_B^2 + \frac{1}{4!}\chi_6^B\hat{\mu}_B^4 + \frac{1}{6!}\chi_8^B\hat{\mu}_B^6 + \frac{1}{8!}\chi_{10}^B\hat{\mu}_B^8$$

$$\chi_3^B(\hat{\mu}_B) = \frac{\partial^3}{\partial\hat{\mu}_B^3}\left(\frac{p}{T^4}\right) = \chi_4^B\hat{\mu}_B + \frac{1}{3!}\chi_6^B\hat{\mu}_B^3 + \frac{1}{5!}\chi_8^B\hat{\mu}_B^5 + \frac{1}{7!}\chi_{10}^B\hat{\mu}_B^7$$

$$\chi_4^B(\hat{\mu}_B) = \frac{\partial^4}{\partial\hat{\mu}_B^4}\left(\frac{p}{T^4}\right) = \chi_4^B + \frac{1}{2!}\chi_6^B\hat{\mu}_B^2 + \frac{1}{4!}\chi_8^B\hat{\mu}_B^4 + \frac{1}{6!}\chi_{10}^B\hat{\mu}_B^6$$

χ_2^B, χ_4^B, χ_6^B and χ_8^B

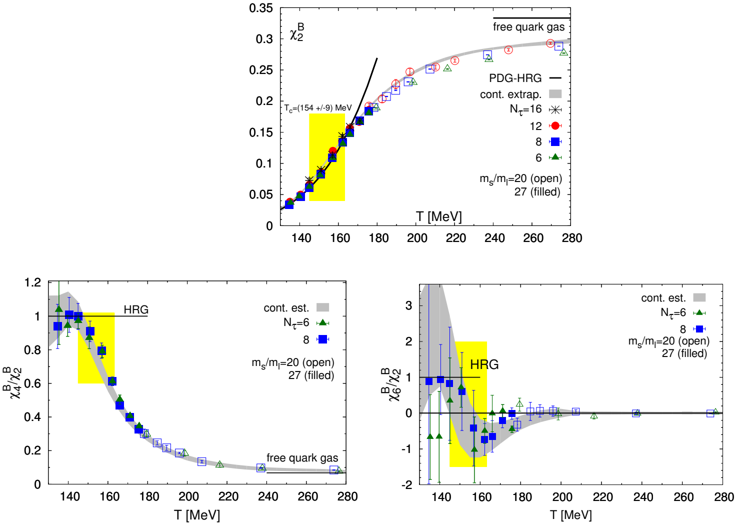




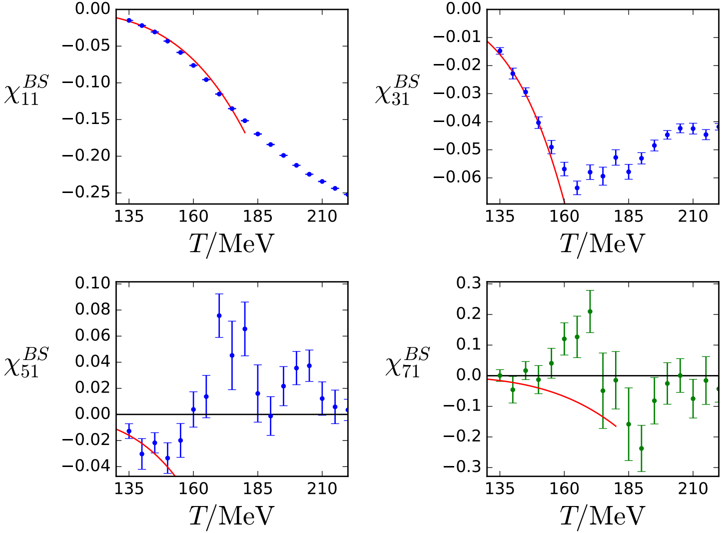
[D'Elia et al., DElia:2016jqh], see also [Bazavov et al., Bazavov:2017dus]

χ_2^B, χ_4^B and χ_6^B

[Bazavov et al., Bazavov:2017dus]



χ_{11}^{BS}, χ_{31}^{BS}, χ_{51}^{BS} and χ_{71}^{BS}



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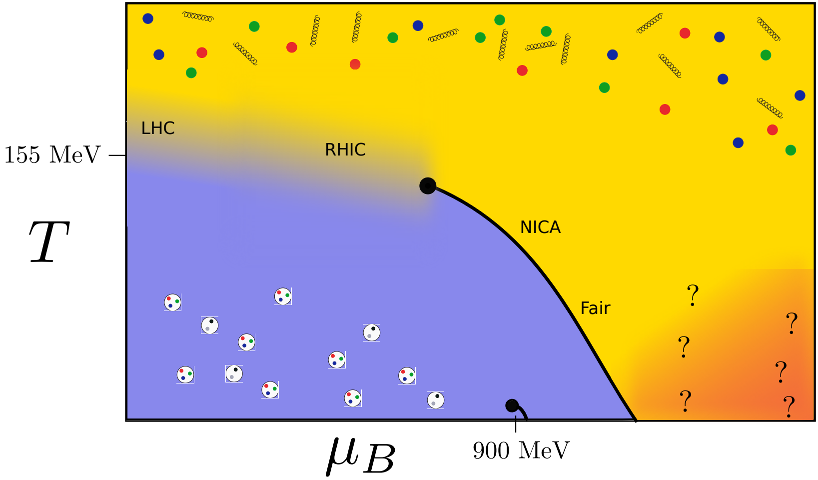
The (T, μ_B)-phase diagram of QCD



Our observables:
T_c, Equation of state, Fluctuations

Convergence radius estimators

1. Ratio test for the pressure:

$$p(\mu) = p_0 + p_2\hat{\mu}^2 + p_4\hat{\mu}^4 + p_6\hat{\mu}^6 + \dots$$

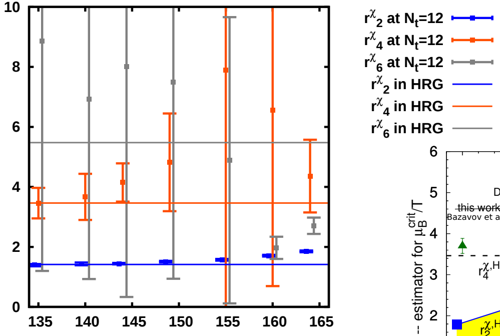
Ratio test $\rightarrow r_{2n}^{p} = \sqrt{\frac{p_{2n}}{p_{2n+2}}}$

2. Ratio test for the susceptibility:

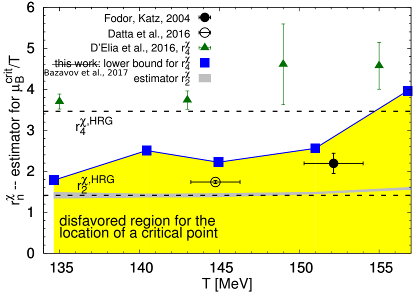
$$\chi_2(\mu) = 2p_2 + 12p_4\hat{\mu}^2 + 30p_6\hat{\mu}^4 + \dots$$

Ratio test $\rightarrow r_{2n}^{\chi} = \sqrt{\frac{2n(2n-1)}{(2n+1)(2n+2)}}\, r_{2n}^{p}$

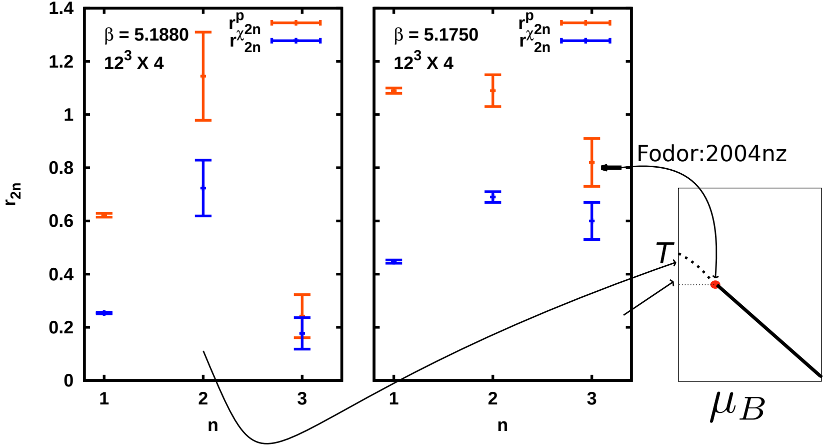
Ratios for the radius of convergence



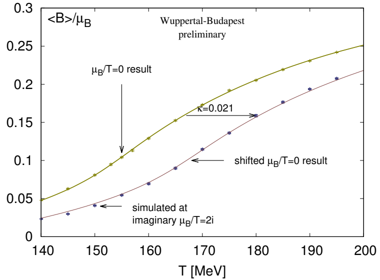
[Bazavov et al., Bazavov:2017dus]



$N_t = 4$ Toy model with critical endpoint



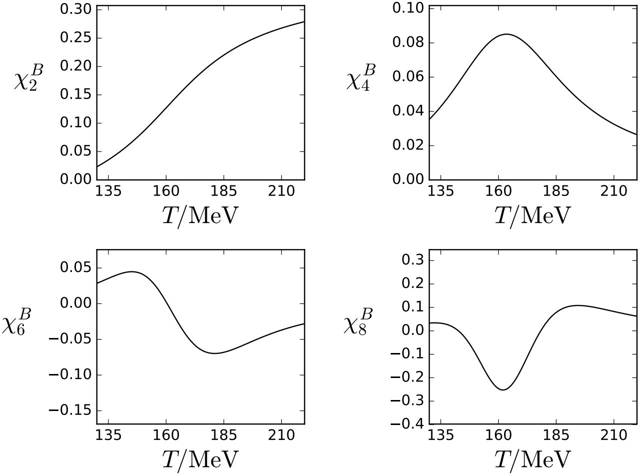
Toy model without critical endpoint



- Start with some parametrization of the curve χ_1^B/μ_B at $\mu = 0$
- Assume that the only difference in the physics at finite μ is a shift in this curve
- The inflection point of this curve is one possible definition of T_c, so shift the curve by using the κ values found in the literature
- You now have a model prediction of χ_1^B for any finite μ, differentiate it a few times at $\mu = 0$ to get estimates of χ_4^B, χ_6^B and χ_8^B

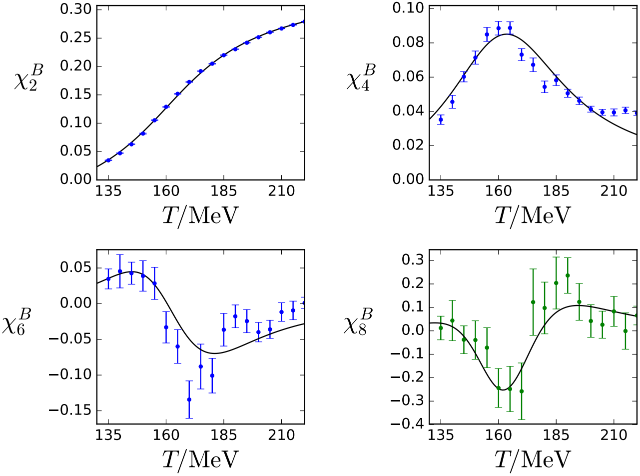
NOTE: The model assumes no criticality

Fluctuations in the toy model



CURVE: The simple model described in the previous slide without criticality

Fluctuations in the toy model



CURVE: The simple model described in the previous slide without criticality

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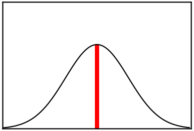
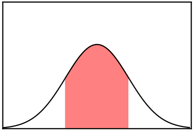
Observables

Cumulants of the net baryon number distributions:

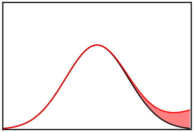
- mean M_B



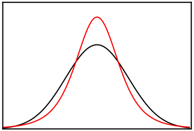
- variance σ_B^2



- skewness S_B: asymmetry of the distribution

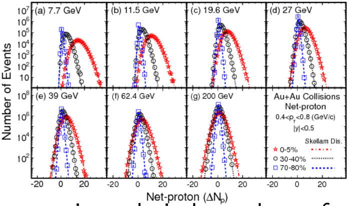


- kurtosis κ_B: "tailedness" of the distribution



Calculating observables

We have derivatives with respect to $\hat{\mu}_B$, $\hat{\mu}_Q$ and $\hat{\mu}_S$ of the pressure at $\mu_S = \mu_Q = 0$. Notation:



$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k},$$

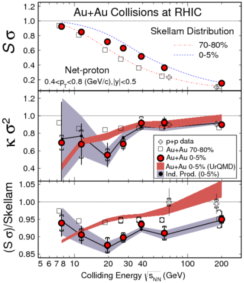
with $\hat{\mu}_i = \mu/T$.

We want ratios of the cumulants that are approximately independent of the volume at $\mu_B > 0$, $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$:

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \ldots$$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \ldots$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$



[Bazavov:2017dus], [Karsch:2017zzw]
plots: [STAR, Adamczyk:2013dal]

Measured observables

On each ensemble we measure the $\chi^{BQS}_{i,j,k}$ up to the fourth derivative:

- χ^{B}_{1}
- χ^{B}_{2}
- χ^{B}_{3}
- χ^{B}_{4}
- χ^{Q}_{1}
- $\chi^{BQ}_{1,1}$
- $\chi^{BQ}_{1,2}$
- $\chi^{BQ}_{1,3}$
- χ^{S}_{1}
- $\chi^{BS}_{1,1}$
- $\chi^{BS}_{1,2}$
- $\chi^{BS}_{1,3}$

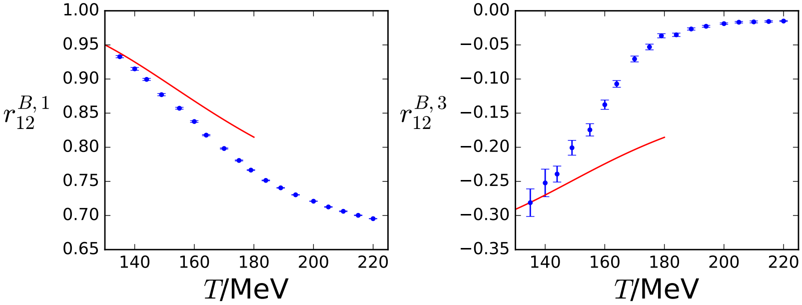
- χ^{Q}_{2}
- $\chi^{BQ}_{2,1}$
- $\chi^{BQ}_{2,2}$
- χ^{S}_{2}
- $\chi^{BS}_{2,1}$
- $\chi^{BS}_{2,2}$
- $\chi^{QS}_{1,1}$
- $\chi^{BQS}_{1,1,1}$
- $\chi^{BQS}_{2,1,1}$

- χ^{Q}_{3}
- $\chi^{BQ}_{3,1}$
- χ^{S}_{3}
- $\chi^{BS}_{3,1}$
- $\chi^{QS}_{2,1}$
- $\chi^{BQS}_{1,2,1}$
- $\chi^{QS}_{1,2}$
- $\chi^{BQS}_{1,1,2}$

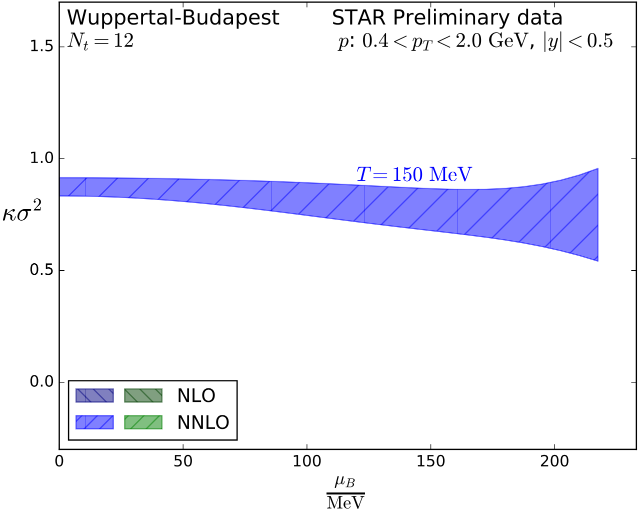
- χ^{Q}_{4}
- χ^{S}_{4}
- $\chi^{QS}_{3,1}$
- $\chi^{QS}_{2,2}$
- $\chi^{QS}_{1,3}$

M_B/σ_B^2

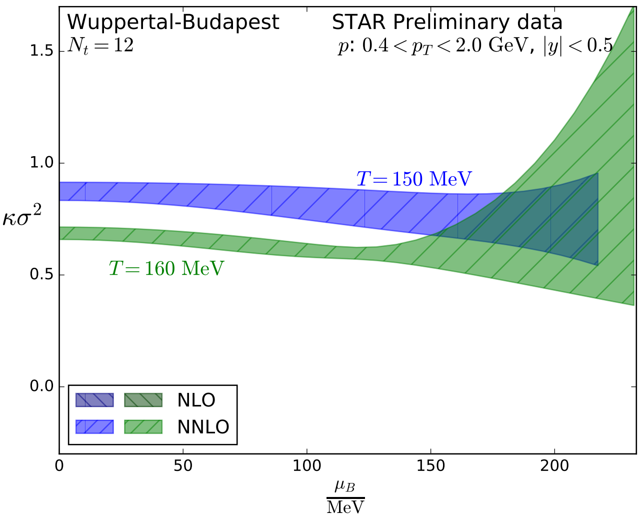
$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \dots$$



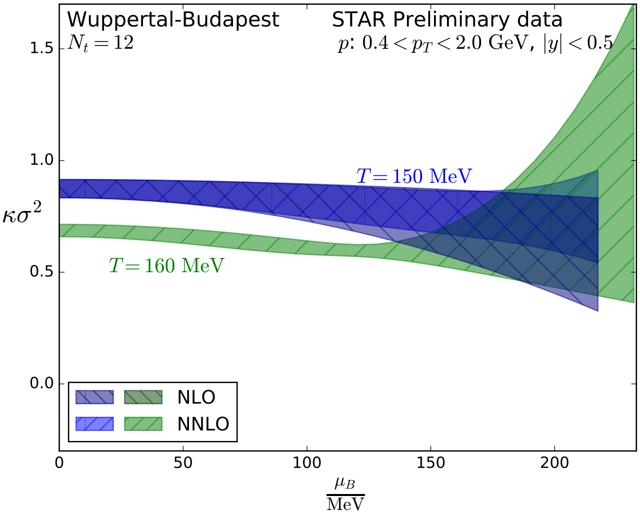
Extrapolation $\kappa\sigma^2$



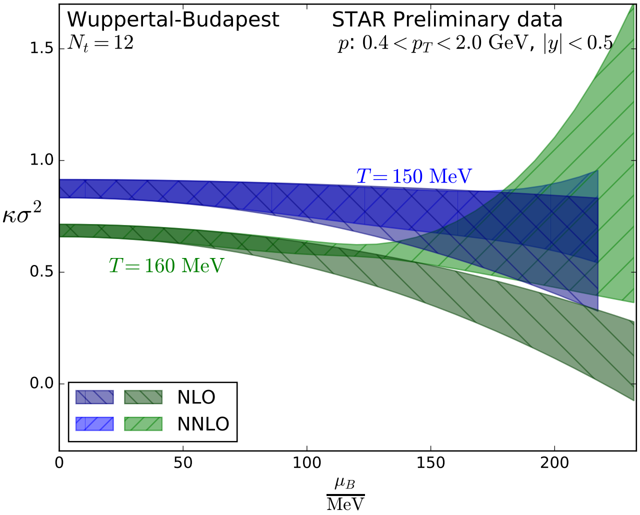
Extrapolation $\kappa\sigma^2$



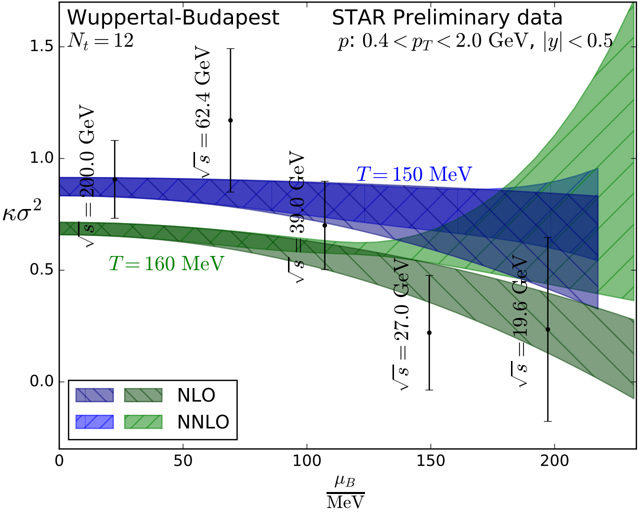
Extrapolation $\kappa\sigma^2$



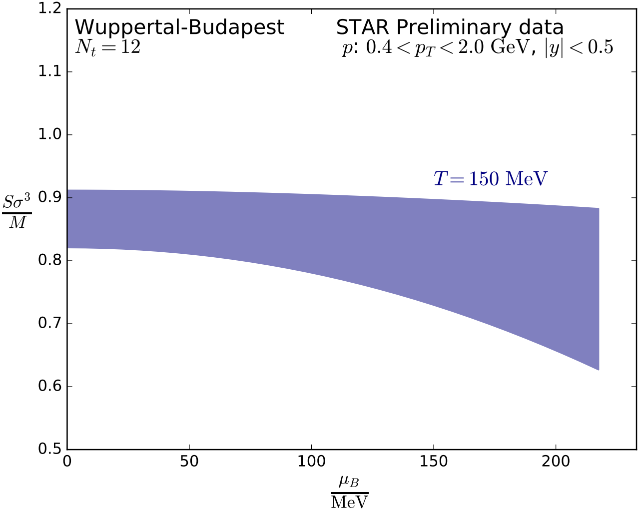
Extrapolation $\kappa\sigma^2$



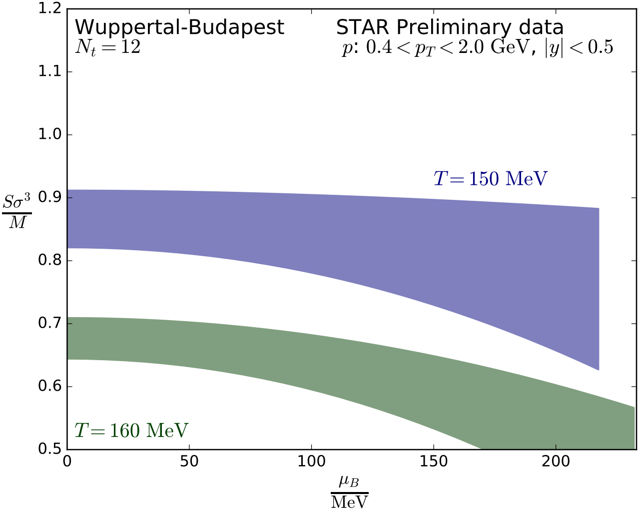
Extrapolation $\kappa\sigma^2$



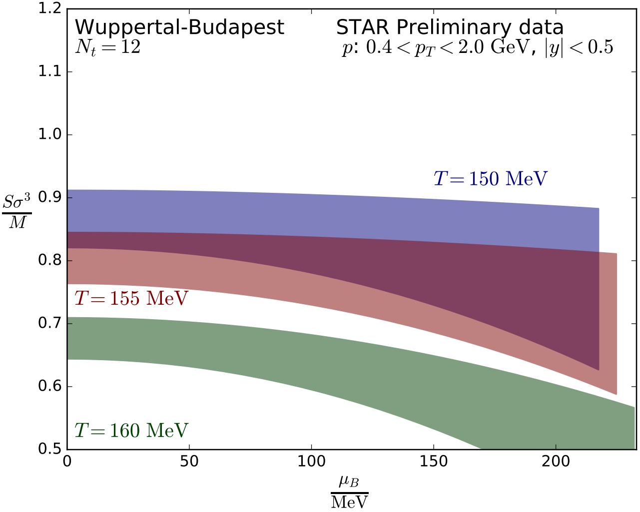
Extrapolation $\frac{S\sigma^3}{M}$



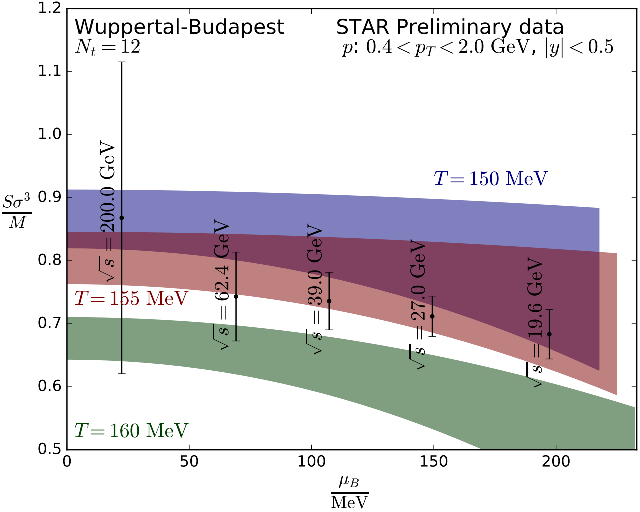
Extrapolation $\frac{S\sigma^3}{M}$



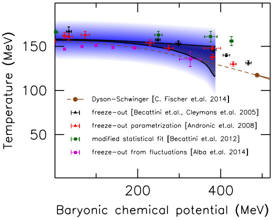
Extrapolation $\frac{S\sigma^3}{M}$

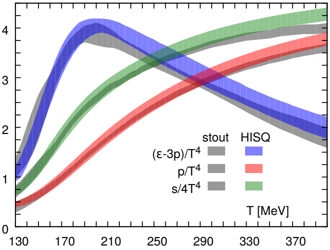


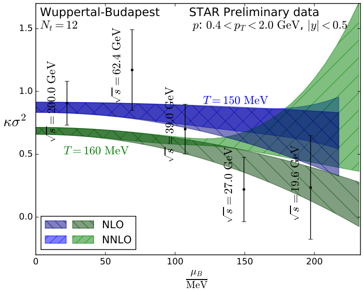
Extrapolation $\frac{S\sigma^3}{M}$

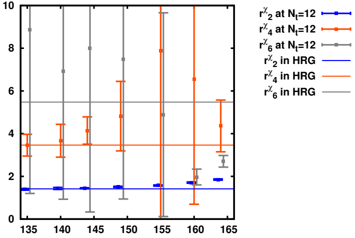


Summary


Temperature (MeV)
Baryonic chemical potential (MeV)
Dyson−Schwinger [C. Fischer et.al. 2014]
freeze−out [Becattini et.al., Cleymans et.al. 2005]
freeze−out parametrization [Andronic et.al. 2008]
modified statistical fit [Becattini et.al. 2012]
freeze−out from fluctuations [Alba et.al. 2014]


stout HISQ
(ε-3p)/T4
p/T4
s/4T4
T [MeV]


Wuppertal-Budapest
$N_t = 12$
STAR Preliminary data
p: $0.4 < p_T < 2.0$ GeV, $|y| < 0.5$
$\sqrt{s} = 200.0$ GeV
$\sqrt{s} = 62.4$ GeV
$\sqrt{s} = 39.0$ GeV
$\sqrt{s} = 27.0$ GeV
$\sqrt{s} = 19.6$ GeV
$T = 150$ MeV
$T = 160$ MeV
$\kappa\sigma^2$
NLO
NNLO
$\frac{\mu_B}{\mathrm{MeV}}$


r^χ_2 at N_t=12
r^χ_4 at N_t=12
r^χ_6 at N_t=12
r^χ_2 in HRG
r^χ_4 in HRG
r^χ_6 in HRG