

Quarkonium hadronic transitions

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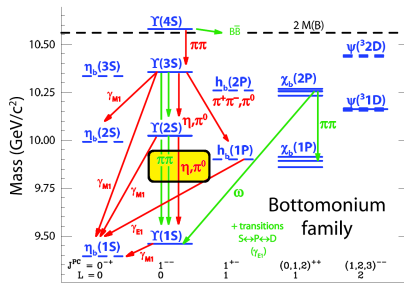
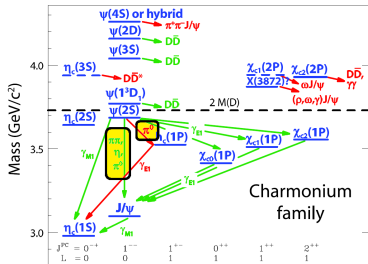
with: Antonio Pineda

based on: [arXiv:1905.03794](https://arxiv.org/abs/1905.03794)

FAIRNESS2019, 20-24 May 2019.



Introduction and Motivation

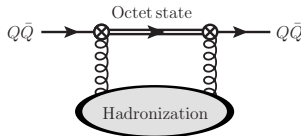


• Chiral Symmetry

- ▶ Allowed because the energy release is small.
- ▶ Parametrizes the decays in terms of a few Low-energy constants. Brown, Cahn
Phys.rev.Lett.35 (1975); Mannel, Urech Z.Phys.C73 (1997); Casalbuoni et al Phys.Lett.B309 (1993)
- ▶ Good descriptions of the processes.
- ▶ However lacks predictive power.

Introduction and Motivation

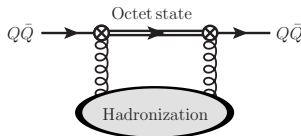
- Adding microscopic information to constrain the chiral description:
 - ▶ Heavy quark mass $1/m_Q$ expansion: heavy quark spin symmetry.
 - ▶ Two step process. [Gottfried Phys.Rev.Lett.40 \(1978\)](#)



- ▶ Multipole expansion:
 - * $\mathbf{r} \cdot \mathbf{E}$ as the leading heavy-quark-gluon interaction.
 - * Requires $m_Q v \gg \Lambda_{QCD}$
- ▶ OPE of the octet propagator. [Voloshin Nucl.Phys.B154 \(1979\)](#)
 - * Write the amplitude in terms of time-local gluonic operators.
 - * Requires $\Lambda_{QCD} \ll m_Q v^2$ and the pion energy $E \ll m_Q v^2$.
- ▶ The local gluonic operators can be hadronized using the axial and scale anomalies.

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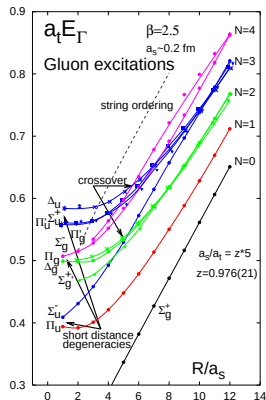


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Not well justified for transitions between different principal quantum number: $E \sim m_Q v^2$

Luty, Sundrum Phys.Lett.B312 (1993)

Our approach



- Quenched lattice NRQCD.

Juge, Kuti, Morningstar Phys.Rev.Lett.90
(2003)

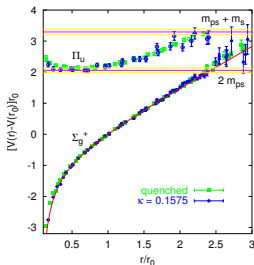
- We propose an alternative way to compute the amplitudes:

- ▶ Work in an EFT framework.
- ▶ $1/m_Q$ expansion.
- ▶ Still use the multipole expansion.
- ▶ Take advantage that the **spectrum of the color-octet sector** corresponds to the **quarkonium hybrid** one (in the quenched approximation).
- ▶ Organize the computation within a $1/N_c$ expansion.

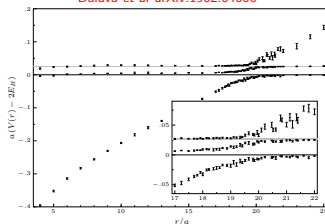
Our approach: what about light-quark states?

- How does the spectrum of static states change in **unquenched** case?

SESAM/TCL Col. G. Bali et al. Phys.Rev.D62 (2000)



Bulava et al arXiv:1902.04006



- What we know:
 - Σ_g^+ and Π_u^- do not change much below string breaking distance.
 - No information about other Λ_{η}^{σ} , but there is no reason to expect a different behavior.
 - One should consider $Q\bar{q}-\bar{Q}q$ and possible tetraquarks states.
 - For transitions **below threshold** the couplings of these states to quarkonium are $1/N_c$ **suppressed**.

0 EFT incorporating $1/m_Q$ and multipole expansions: weakly-coupled pNRQCD.

Pineda, Soto Nucl.Phys.Suppl 64 (1998); Brambilla, Pineda, Soto, Vairo Nucl.Phys.B566 (2000)

1 Write the most general hadronic version of pNRQCD.

- * Singlet, hybrid and pion fields as d.o.f.
- * Chiral, multipole, $1/m_Q$, $1/N_c$ expansions.

2 Match the hadronic and partonic versions of the theory.

- * No d.o.f integrated out.
- * Dynamical energy scale $E \sim \Lambda_{QCD}$.

3 Compute transition amplitudes in the hadronic EFT.

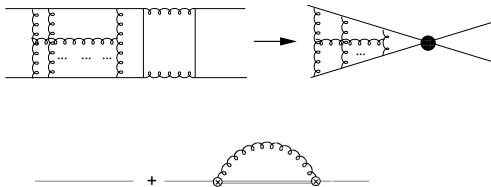
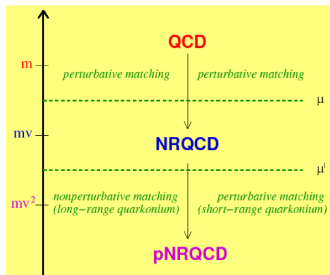
Potential Non-Relativistic QCD

Motivation

- ▶ Quarkonium systems are non-relativistic bound states.
- ▶ **Multiscale system:** $m_Q \gg m_Q v \gg m_Q v^2$, and Λ_{QCD} . m is the heavy-quark mass, $v \ll 1$ the heavy quark velocity.
- ▶ We can exploit the **scale hierarchies** by building an **Effective Field Theory (EFT)**.

Matching procedure

- ▶ Integrating out the m_Q scale leads to the well known NRQCD. Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995. Since $m_Q \gg \Lambda_{QCD}$ the matching is perturbative.
- ▶ The degrees of freedom in pNRQCD are a **color singlet** (S) and **octet fields** (O^a) and the ultrasoft gluons.
- ▶ R is the CoM coordinate and r the relative coordinate of the quark pair.
- ▶ **Multipole expansion:** Since $r \sim 1/m_Q v$, integrating out $m_Q v$ implies a multipole expansion of the gluon fields.



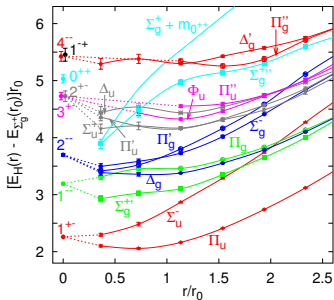
- At $E \sim m_Q v^2$ Quarkonium is described by pNRQCD, which takes a Schrödinger-like form with a potential interactions, but also (ultrasoft) gluon effects.

$$\mathcal{L} = \Phi(\mathbf{r})^\dagger (i\partial_0 - \mathbf{p}^2/2m_Q - V(r) + \text{int. with low-energy d.o.f}) \Phi(\mathbf{r})$$

- When matching NRQCD to pNRQCD there are two possibilities:
- Weak-coupling: $m_Q v \gg \Lambda_{\text{QCD}} \Rightarrow$ Perturbative matching.
 - Strong coupling: $m_Q v \sim \Lambda_{\text{QCD}} \gg m_Q v^2 \Rightarrow$ Non-perturbative matching.

Hybrid states in pNRQCD

- In the short distance limit the static energies are characterized by $O(3) \times C$ instead of $D_{\infty h}$.



Foster, Michael Phys.Rev.D59 (1999)

Gluelumps

- $G_k^{i a}$ create a basis of color-octet eigenstates of $h_0(\mathbf{R})$ in the presence of a static, local, color-octet source O^a .

$$h_0(\mathbf{R}) G_k^{i a}(\mathbf{R}) |0\rangle = \Lambda_k G_k^{i a}(\mathbf{R}) |0\rangle$$

- The gluon and light-quark Hamiltonian density leading order in the multipole expansion.

$$h_0 = \frac{1}{2} (\mathbf{E}^2 - \mathbf{B}^2) - \sum_{j=1}^{n_f} \bar{q}_j [i \mathbf{D} \cdot \boldsymbol{\gamma} - m_j] q_j$$

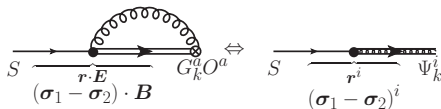
- States are constrained to satisfy the Gauss law.

- We project the pNRQCD Lagrangian to the Fock subspace spanned by

$$\int d^3 r d^3 R \sum_k P_{k\lambda}^i O^{a\dagger}(\mathbf{r}, \mathbf{R}) G_k^{i a}(\mathbf{R}) |0\rangle \Psi_{k\lambda}(t, \mathbf{r}, \mathbf{R})$$

- $P_{k\lambda}^i$ projects $G_k^{i a}$ into a representation of $D_{\infty h}$.

Mixing terms Matching



- Matching the partonic and hadronic versions of pNRQCD:

$$i \sqrt{\frac{T_F}{N_c}} \hat{r}_\lambda^{\dagger i} r^j \langle 0 | G_{1--}^{ai\dagger}(R) g E^{aj}(R) | 0 \rangle = i r \cdot \hat{r}_\lambda^{\dagger} t^{(r1--)}$$

$$i \frac{g C_F}{2m_Q} \sqrt{\frac{T_F}{N_c}} (\sigma_1 - \sigma_2)^i \hat{r}_\lambda^{\dagger i} \langle 0 | G_{1+-}^{ai\dagger}(R) B^{aj}(R) | 0 \rangle = i (\sigma_1 - \sigma_2) \cdot \hat{r}_\lambda^{\dagger} t^{(S1+-)}$$

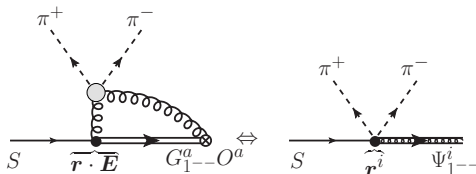
- If the gluelump is dominated by the lowest dimension gluonic operator

$$G_{1--}^a = Z_E^{-1/2} g E^a + Z_{D \times B}^{-1/2} (D \times g B)^a + \dots$$

$$G_{1+-}^a = Z_B^{-1/2} g B^a + Z_{D \times E}^{-1/2} (D \times g E)^a + \dots$$

$$\Rightarrow t^{(r1--)} = \sqrt{\frac{T_F Z_E}{N_c}}, \quad t^{(S1+-)} = \frac{C_F}{2m_Q} \sqrt{\frac{T_F Z_B}{N_c}}$$

$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^+\pi^-$: Matching



- Partonic pNRQCD

$$ig\sqrt{\frac{T_F}{Z_EN_c}}\hat{r}_\lambda\cdot\mathbf{r}\langle\pi^+(p_+)\pi^-(p_-)|\mathbf{E}^a\cdot\mathbf{E}^a|0\rangle$$

- Hadronic pNRQCD

$$i4\mathbf{r}\cdot\hat{\mathbf{r}}_\lambda\left(-t_{d0}^{(r1--)}E_+E_-+t_{di}^{(r1--)}\mathbf{p}_+\cdot\mathbf{p}_--t_m^{(r1--)}m_\pi^2\right)$$

- The divergence on the scale current allows to determine the matrix element up to an unknown constant κ Voloshin, Zakharov Phys.Rev.Lett 45 (1980); Novikov, Shifman Z.Phys.C8 (1981); Chivukula *et al* Annals Phys.192 (1989)

$$\langle\pi^+(p_+)\pi^-(p_-)|\frac{\beta_0\alpha_s}{2\pi}\mathbf{E}^2|0\rangle=\left(2-\frac{9}{2}\kappa\right)E_+E_- - \left(2+\frac{3}{2}\kappa\right)\mathbf{p}_+\cdot\mathbf{p}_-+3m_\pi^2$$

- Subject to potentially large $\mathcal{O}(\alpha_s)$ corrections and neglected anomalous dimension of the $q\bar{q}$ operator.

$$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^+\pi^-$$



- Chiral amplitude up to $\mathcal{O}(p^2)$

$$\mathcal{A}_\chi = -a_1 p_+^0 p_-^0 + a_2 \mathbf{p}_+ \cdot \mathbf{p}_- - a_3 m_\pi^2$$

- In our approach

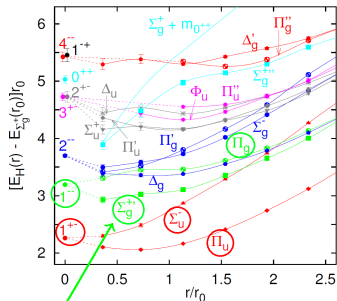
$$a_1 = -\frac{8\pi^2 T_F}{3\beta_0 N_c} \beta_r^{(12)} \left(2 - \frac{9}{2} \kappa \right)$$

$$a_2 = -\frac{8\pi^2 T_F}{3\beta_0 N_c} \beta_r^{(12)} \left(2 + \frac{3}{2} \kappa \right)$$

$$a_3 = -\frac{8\pi^2 T_F}{\beta_0 N_c} \beta_r^{(12)}$$

- The sum over hybrid intermediate states

$$\beta_r^{(n'n)} = \sum_m \langle S_{n'} | \hat{\mathbf{r}}_\lambda^\dagger \cdot \mathbf{r} | \Psi_m \rangle \left(\frac{1}{m_n - m_m} + \frac{1}{m_{n'} - m_m} \right) \langle \Psi_m | \hat{\mathbf{r}}_\lambda^\dagger \cdot \mathbf{r} | S_n \rangle$$

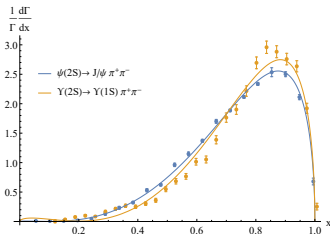
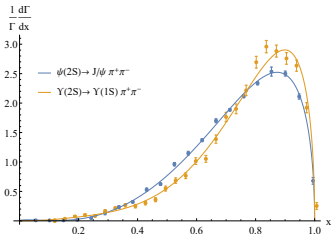


- See [M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 \(2015\); R. Onocala, J. Soto Phys.Rev.D96 \(2017\)](#) for the details on how to obtain the hybrid spectrum and wavefunctions.

$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^+\pi^-$: Normalized Line-shape and decay widths

• Chiral EFT

• Hadronic pNRQCD



$$x = (m_{\pi\pi} - 2m_{\pi}) / (m_{2S} - m_{1S} - 2m_{\pi})$$

$$a_1^c/a_2^c = -0.20^{+3.93}_{-0.98}, \quad a_3^c/a_2^c = 3.12^{+3.78}_{-15.05}, \quad \chi^2_{\text{d.o.f}} = 0.13,$$

$$a_1^b/a_2^b = 4.41^{+1.93}_{-3.07}, \quad a_3^b/a_2^b = -15.05^{+13.06}_{-7.50}, \quad \chi^2_{\text{d.o.f}} = 0.29,$$

$$\kappa_c = 0.277 \pm 0.015, \quad \chi^2_{\text{d.o.f}} = 0.17,$$

$$\kappa_b = 0.342^{+0.015}_{-0.017}, \quad \chi^2_{\text{d.o.f}} = 1.25,$$

$$\boxed{\kappa = 0.301(31)}$$

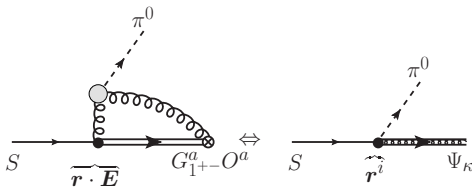
* Data: ATLAS JHEP 1701 (2017) (charm) Belle Phys.Rev. D96 (2017) (bottom).

• Decay widths:

$$\Gamma_{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-} = 38.0^{(-8.5)}_{(+13.0)} \Lambda_1^{(-4.1)}_{(+4.5)} \kappa (\pm 17.4)_{\text{s.p.}} \text{ keV}, \quad \Gamma^{\text{exp}} = 102.1(2.9) \text{ keV},$$

$$\Gamma_{Y(2S) \rightarrow Y(1S) \pi^+ \pi^-} = 2.51^{(-0.47)}_{(+0.66)} \Lambda_1^{(-0.28)}_{(+0.31)} \kappa (\pm 1.21)_{\text{s.p.}} \text{ keV}, \quad \Gamma^{\text{exp}} = 5.71(48) \text{ keV}.$$

$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1P)\pi^0$: Matching



- Partonic pNRQCD

$$\frac{i}{3} \sqrt{\frac{T_F}{N_c Z_B}} \hat{r}_\lambda \cdot \mathbf{r} \langle \pi^0 | g^2 \mathbf{E} \cdot \mathbf{B} | 0 \rangle$$

- Hadronic pNRQCD: We need **isospin breaking** operators

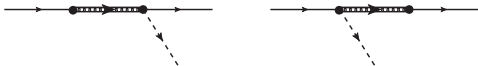
$$-it^{(r1^{+-})} \mathbf{r} \cdot \hat{\mathbf{r}}_\lambda 2m_\pi^2 \frac{m_d - m_u}{m_u + m_d}$$

- From the axial anomaly Gross, Treiman, Wilczek Phys.RevD19 (1979); Novikov, Shifman, Vainshtein, Zakharov Nucl.PhysB165 (1980):

$$\langle \pi^0 | \frac{\alpha_s}{\pi} \mathbf{E} \cdot \mathbf{B} | 0 \rangle = \frac{m_d - m_u}{m_u + m_d} F_\pi m_{\pi^0}^2$$

$$\Rightarrow t^{(r1^{+-})} = -\frac{2\pi^2}{3} F_\pi \sqrt{\frac{T_F}{N_c Z_B}}$$

$$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1P)\pi^0$$



► Amplitude:

$$i\mathcal{A} = -F_\pi m_\pi^2 \frac{2\pi^2 c_F}{3m_Q} \frac{T_F}{N_c} \frac{m_d - m_u}{m_u + m_d} \beta_\sigma^{(12)}$$

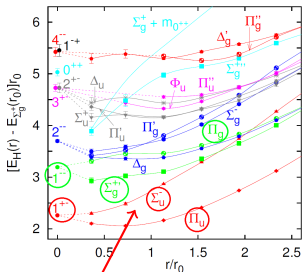
► The sum over hybrid intermediate states

$$\beta_\sigma^{(12)} \equiv \sum_m \left(\langle h_c(1^1P_1) | \hat{r}_\lambda \cdot \mathbf{r} | \Psi_m \rangle \frac{i}{m_{h_c} - m_m} \langle \Psi_m | \hat{r}_\lambda^\dagger \cdot (\sigma_1 - \sigma_2) | \psi(2^3S_1) \rangle \right. \\ \left. + \langle h_c(1^1P_1) | \hat{r}_\lambda \cdot (\sigma_1 - \sigma_2) | \Psi_m \rangle \frac{i}{m_{\psi(2S)} - m_m} \langle \Psi_m | \hat{r}_\lambda^\dagger \cdot \mathbf{r} | \psi(2^3S_1) \rangle \right).$$

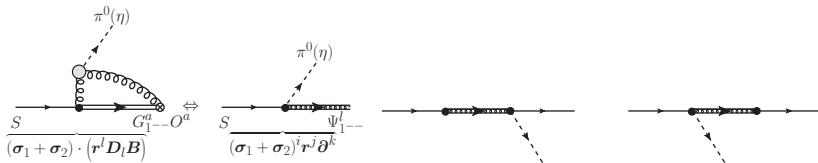
► Spectrum & wave functions, see: Berwein, Brambilla, JTC, Vairo. Phys.Rev.D92 (2015); Oncala, Soto Phys.Rev.D96 (2017)

► Total width:

$$\Gamma_{\psi(2S) \rightarrow h_c(1P)\pi^0} = 104_{+80}^{(-35)} \Lambda_1 (\pm 21)_{\text{l.q.}} (\pm 1)_{\text{s.p.}} \text{ eV}, \quad \Gamma^{\text{exp}} = 255(39) \text{ eV}$$



$$Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^0(\eta)$$



► Amplitudes (including $\pi^0 - \eta$ mixing)

$$i\mathcal{A}_{\pi^0} = -i \frac{T_F}{N_c} \frac{c_F}{m_Q} \frac{8\pi^2}{45} \frac{m_d - m_u}{m_d + m_u} F_\pi m_\pi^2 \left(\epsilon_{1S}^* \times \epsilon_{2S} \right) \cdot \mathbf{p} \beta_r^{(12)}$$

$$i\mathcal{A}_\eta = -i \frac{T_F}{N_c} \frac{c_F}{m_Q} \frac{8\pi^2}{45\sqrt{3}} \left(m_\eta^2 - m_\pi^2 \right) F_\pi \left(\epsilon_{1S}^* \times \epsilon_{2S} \right) \cdot \mathbf{p} \beta_r^{(21)}$$

► Total widths:

$$\Gamma_{\psi(2S) \rightarrow J/\psi \pi^0} = 40_{-14}^{+9} \Lambda_1 (\pm 8)_{\text{l.q.}} (\pm 18)_{\text{s.p.}} \text{ eV},$$

$$\Gamma^{\text{exp}} = 373(14) \text{ eV},$$

$$\Gamma_{\psi(2S) \rightarrow J/\psi \eta} = 1.19_{+0.41}^{-0.27} \Lambda_1 (\pm 0.5)_{\text{s.p.}} \text{ keV},$$

$$\Gamma^{\text{exp}} = 9.91(30) \text{ keV},$$

$$\Gamma_{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^0} = 0.21_{+0.06}^{-0.04} \Lambda_1 (\pm 0.05)_{\text{l.q.}} (\pm 0.10)_{\text{s.p.}} \text{ eV},$$

$$\Gamma^{\text{exp}} < 1.28 \text{ eV},$$

$$\Gamma_{\Upsilon(2S) \rightarrow \Upsilon(1S) \eta} = 1.58_{+0.42}^{-0.80} \Lambda_1 (\pm 0.76)_{\text{s.p.}} \text{ eV},$$

$$\Gamma^{\text{exp}} = 9.3(1.5) \text{ eV}.$$

► Chiral Symmetry check:

$$R \left(\frac{\psi(2S) \rightarrow J/\psi \pi^0}{\psi(2S) \rightarrow J/\psi \eta} \right) = 3.34(\pm 0.39)_{\text{l.q.}} \cdot 10^{-2}, \quad R^{\text{exp}} = 3.76(25) \cdot 10^{-2},$$

$$R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^0}{\Upsilon(2S) \rightarrow \Upsilon(1S) \eta} \right) = 13.5(\pm 2.8)_{\text{l.q.}} \cdot 10^{-2}, \quad R^{\text{exp}} < 13.8 \cdot 10^{-2}.$$

► Uncertainties: $\mathcal{O}(\alpha_s)$ corrections to hadronization, hybrid interpolating operators, NLO chiral:

$$R_{c,\pi} \equiv R \left(\frac{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-}{\psi(2S) \rightarrow J/\psi \pi^0} \right) = 955(\mp 191)_{\text{l.q.}} \left(\begin{smallmatrix} -103 \\ +114 \end{smallmatrix} \right)_{\kappa}, \quad R_{c,\pi}^{\text{exp}} = 274(13),$$

$$R_{c,\eta} \equiv R \left(\frac{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-}{\psi(2S) \rightarrow J/\psi \eta} \right) = 31.9 \left(\begin{smallmatrix} -3.5 \\ +3.8 \end{smallmatrix} \right)_{\kappa}, \quad R_{c,\eta}^{\text{exp}} = 10.3(4),$$

$$R_{b,\pi} \equiv R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^+ \pi^-}{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^0} \right) = 11.8(\mp 2.4)_{\text{l.q.}} \left(\begin{smallmatrix} -1.3 \\ +1.5 \end{smallmatrix} \right)_{\kappa} \times 10^3, \quad R_{b,\pi}^{\text{exp}} > 4.5(4) \times 10^3,$$

$$R_{b,\eta} \equiv R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^+ \pi^-}{\Upsilon(2S) \rightarrow \Upsilon(1S) \eta} \right) = 1.59 \left(\begin{smallmatrix} -0.18 \\ +0.20 \end{smallmatrix} \right)_{\kappa} \times 10^3, \quad R_{b,\eta}^{\text{exp}} = 0.61(11) \times 10^3.$$

- Measure of $\beta_{r,b}^{(21)} / \beta_{r,c}^{(21)}$, most uncertainties cancel or are reduced.

$$R_{bc,\pi\pi} \equiv R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-}{\psi(2S) \rightarrow J/\psi\pi^+\pi^-} \right) = 6.60_{-0.38}^{+0.30} \Lambda_1 (\pm 0.03)_{\kappa} (\pm 0.31)_{\text{s.p.}} \times 10^{-2},$$

$$R_{bc,\pi\pi}^{\text{exp}} = 5.59(0.50) \times 10^{-2}$$

$$R_{bc,\pi} \equiv R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^0}{\psi(2S) \rightarrow J/\psi\pi^0} \right) = 5.3_{-0.3}^{+0.2} \Lambda_1 (\pm 0.25)_{\text{s.p.}} \times 10^{-3},$$

$$R_{bc,\pi}^{\text{exp}} < 3.4(1) \times 10^{-3},$$

$$R_{bc,\eta} \equiv R \left(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S)\eta}{\psi(2S) \rightarrow J/\psi\eta} \right) = 1.33_{-0.08}^{+0.06} \Lambda_1 (\pm 0.06)_{\text{s.p.}} \times 10^{-3},$$

$$R_{bc,\eta}^{\text{exp}} = 0.94(15) \times 10^{-3}.$$

- Uncertainties: Hybrid interpolating operators. Test of the computational scheme.

$$\frac{R_{bc,\pi}}{R_{bc,\pi\pi}} = 0.08, \quad \frac{R_{bc,\pi}^{\text{exp}}}{R_{bc,\pi\pi}^{\text{exp}}} < 0.06(1), \quad \frac{R_{bc,\eta}}{R_{bc,\pi\pi}} = 2.0 \times 10^{-2}, \quad \frac{R_{bc,\eta}^{\text{exp}}}{R_{bc,\pi\pi}^{\text{exp}}} = 1.7(3) \times 10^{-2}$$

Outlook and Conclusions

- ▶ Details and further discussion in [arXiv:1905.03794](https://arxiv.org/abs/1905.03794)
- ▶ Approach summary:
 - * We have studied one-pion or eta and two-pion transition between quarkonium states below threshold.
 - * Previous approaches using the twist expansion are not well justified.
 - * EFT incorporating $1/m_Q$, multipole, chiral and $1/N_c$ expansions systematically.
 - * The intermediate octet states in the transitions correspond to the hybrid spectrum.
- ▶ $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^+\pi^-$
 - * Good description of the decay width spectrum, depending on one parameter.
 - * Large uncertainties on the decay widths: sensitivity to the long distance part of the potentials, $\mathcal{O}(\alpha_s)$ corrections to the gluon hadronization.
- ▶ $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1P)\pi^0$ Predictions compatible with experiments
- ▶ $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^0$ Predictions smaller than experimental values.
- ▶ Uncertainties cancel in particular ratios of the decay widths
 - * We can assess the importance of different sources of uncertainties.
 - * Clean(er) tests of the of our approach and comparison with the twist expansion.

Thank you for your attention