Exploration of the phase diagram and the thermodynamic properties of QCD at finite temperature and chemical potential with PNJL effective model

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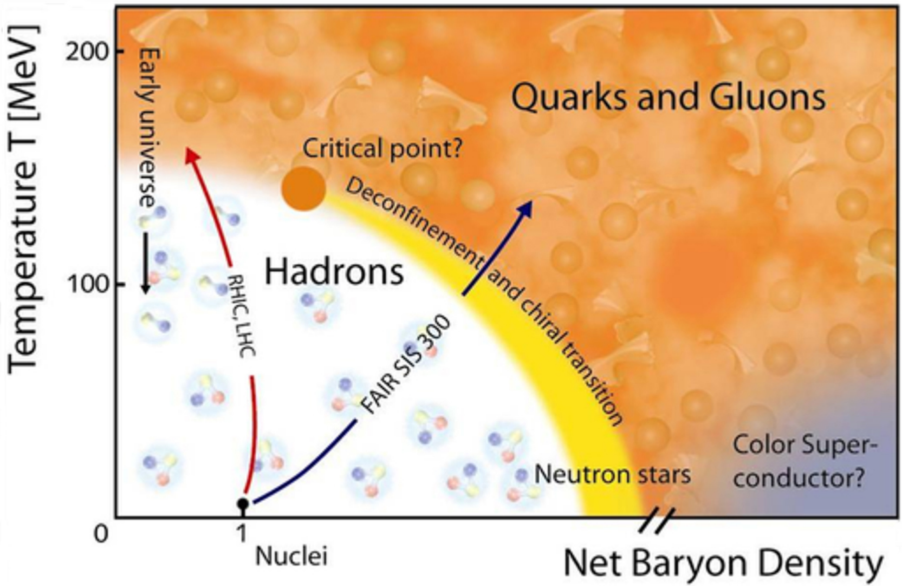
Phase diagram



PNJL 2+1 flavour Lagrangian

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma^\mu D_\mu - \hat{m})q + G[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2] + \\ + K\{det[\bar{q}(1+\gamma^5)q] + det[\bar{q}(1-\gamma^5)q]\} - \mathcal{U}(\Phi[A], \bar{\Phi}[A], T) \tag{1}$$

Here $D^\mu = \partial^\mu - iA^\mu$, $A^\mu = \delta^\mu_0 A^0$, the fields Φ and $\bar{\Phi}$ are Polyakov fields defined as:

$$\Phi \equiv \frac{1}{N_c} Tr\langle\langle L \rangle\rangle \qquad \bar{\Phi} \equiv \frac{1}{N_c} Tr\langle\langle L^\dagger \rangle\rangle \tag{2}$$

Where L is the Polyakov loop defined in terms of the gauge field A_4,after Wick rotation:

$$L(\vec{x}) \equiv \mathcal{P}\Big\{ i \int_0^\beta d\tau A_4(\tau, \vec{x}) \Big\} \tag{3}$$

Below I indicate the quark chiral condensate as:

$$\langle \bar{q}_i q_i \rangle \equiv \varphi_i \quad i = u, d, s \tag{4}$$

Thermodynamic Potential

To study the model in the case of non-vanishing chemical potential one introduces a modified Lagrangian:

$$\mathcal{L}'_{PNJL} = \mathcal{L}_{PNJL} + \mu\bar{q}\gamma^0 q \tag{5}$$

From this Lagrangian one obtains the Thermodynamic Potential per unit volume($\omega = \Omega/V$):

$$\begin{aligned}\omega(\Phi,\bar{\Phi},T,M_i,\mu_i) =& \,\mathcal{U}(\Phi,\bar{\Phi},T) + G\sum_{i=u,d,s}\varphi_i^2 + K\varphi_u\varphi_d\varphi_s + \\ & -2N_c\sum_{i=u,d,s}\int^\Lambda \frac{d^3p}{(2\pi)^3}E_i - 2T\int^\Lambda \frac{d^3p}{(2\pi)^3}\{z_\Phi^{i+}(E_i,\mu) + z_\Phi^{i-}(E_i,\mu)\}\end{aligned} \tag{6}$$

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$$z_\Phi^{i+}(E_i,\mu_i) \equiv \ln[1 + N_c(\Phi + \bar{\Phi}\,\mathrm{e}^{-\beta(E_i-\mu_i)})\,\mathrm{e}^{-\beta(E_i-\mu_i)} + \mathrm{e}^{-3\beta(E_i-\mu_i)}] \tag{7}$$

$$z_\Phi^{i-}(E_i,\mu_i) \equiv \ln[1 + N_c(\bar{\Phi} + \Phi\,\mathrm{e}^{-\beta(E_i+\mu_i)})\,\mathrm{e}^{-\beta(E_i+\mu_i)} + \mathrm{e}^{-3\beta(E_i+\mu_i)}] \tag{8}$$

MFE and Thermodynamics quantities

Minimizing the thermodynamic potential one gets the mean field equations:

$$\frac{\partial \omega}{\partial \varphi_i} = 0 \qquad \frac{\partial \omega}{\partial \Phi} = 0 \qquad \frac{\partial \omega}{\partial \bar{\Phi}} = 0 \tag{9}$$

and all the thermodynamic quantities of interest:

$$P(T, \mu_i) = -\omega(T, \mu_i) \tag{10}$$

The number density of particle of type i is:

$$n_i(T, \mu_i) \equiv -\frac{\partial \omega(T, \mu_i)}{\partial \mu_i}\Big|_T \tag{11}$$

The entropy density of system is:

$$s(T, \mu_i) = -\frac{\partial \omega(T, \mu_i)}{\partial T}\Big|_{\mu_i} \tag{12}$$

Mass Gap equation and Chiral Symmetry

The PNJL Lagrangian is chiral symmetric if $m_i = 0$. For non vanishing current mass chiral symmetry is explicitly broken. Moreover, at low temperature and chemical potential, chiral symmetry is also dynamically broken by self-interaction of quarks: the chiral condensate is negative and large.

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The mass gap equation for the quark of species i reads:

$$M_i = m_i - 2G\varphi_i - 2K\varphi_j\varphi_k \qquad i \neq j \neq k \tag{13}$$

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The second term of the RHS of the equation is due to the 4-fermion interaction vertex and the third term is due to the 6-fermion interaction vertex. This vertex mixes the chiral condensates one with an others.

Fluctuations

In many different fields the study of fluctuations can provide physical insights into the underlying microscopic physics. The fluctuations can become invaluable physical observable in spite of their difficult character. Some examples of this fact are:

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My work is focused on fluctuations of conserved charges in the QGP (B, Q, S) explored through their cumulants. [1]

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Cumulants

One can consider a probability distribution $P(x)$ and define the generating function of its moments:

$$G(\theta) = \int P(x) e^{\theta x} dx \tag{14}$$

The moments of $P(x)$ are:

$$\langle x^n \rangle = \left. \frac{d^n G(\theta)}{d\theta^n} \right|_{\theta=0} \tag{15}$$

For many practical purposes, it is more convenient to use cumulants rather than moments for characterizing a probability distribution. One can define the cumulant generating function as

$$K(\theta) = \log(G(\theta)) \tag{16}$$

and the cumulants become:

$$\langle x^n \rangle_c = \left. \frac{d^n K(\theta)}{d\theta^n} \right|_{\theta=0} \tag{17}$$

Cumulants and Moments

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In my work I consider the follow combinations of cumulants:

$$\kappa\sigma^2 = \frac{\langle x^4 \rangle_c}{\langle x^2 \rangle_c} \qquad \frac{\gamma\sigma^3}{M} = \frac{\langle x^3 \rangle_c}{\langle x \rangle_c} \tag{19}$$

Quark susceptibilities

In the Gran canonical ensemble it is possible to define cumulants as:

$$\langle \hat{N}^n \rangle_c = \frac{\partial^n(-\Omega/T^4)}{\partial(\mu/T)^n} \tag{20}$$

The cumulants of $\hat{N}$ are extensive variables. Starting from:

$$\Omega = \omega V \tag{21}$$

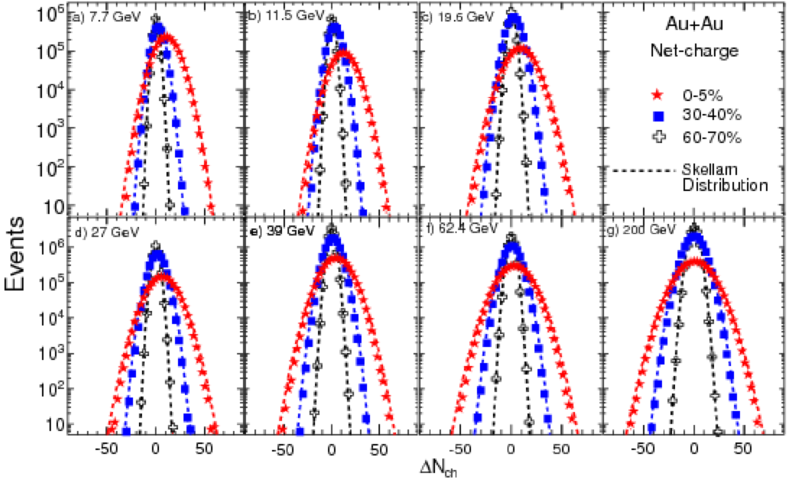
it is more convenient to define the generalized susceptibilities as;

$$\chi^n \equiv \frac{\partial^n(-\omega/T^4)}{\partial(\mu/T)^n} = \frac{\langle \hat{N}^n \rangle_c}{V} \tag{22}$$

then

$$\kappa\sigma^2 = \frac{\chi^4}{\chi^2} \qquad \frac{\gamma\sigma^3}{M} = \frac{\chi^3}{\chi^1} \tag{23}$$

HIC measurements



Nihar Ranjan Sahoo and the Star Collaboration 2014 J. Phys.: Conf. Ser. 535 012007

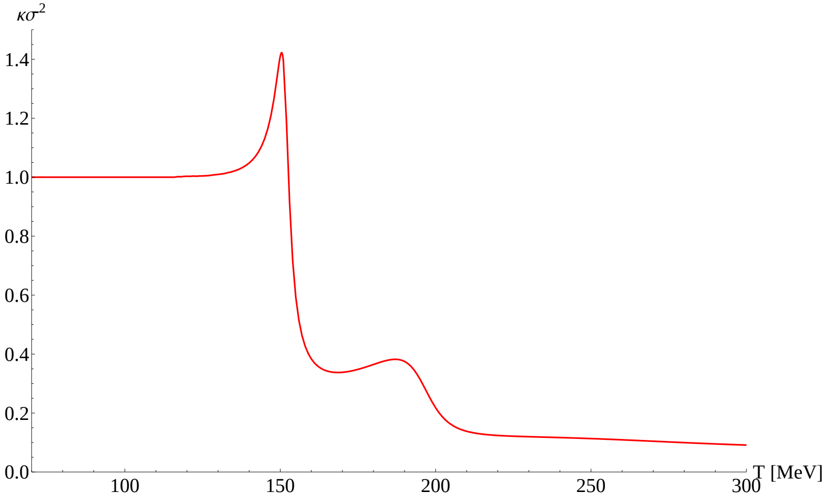
Possible scenarios

From a general point of view the phase diagram of QCD is a 4-dimension space: 3 dimensions for the quark chemical potentials and 1 for temperature. Then it is possible to choose a particular relation between the chemical potentials for reducing the phase diagram of QCD to 2 dimensions. I perform my calculations in the following scenarios:

- Symmetric chemical potential: $\mu_u = \mu_d = \mu_s = \frac{1}{3}\mu_B$
- (Quasi-)Neutral Strangeness: $\mu_u = \mu_d = \frac{1}{3}\mu_B$, $\mu_s = 0$
- HIC: $\frac{n_Q}{n_B} = 0.4$, $n_s = 0$

The last scenario is the one experimental available today

Kurtosis at null chemical potential



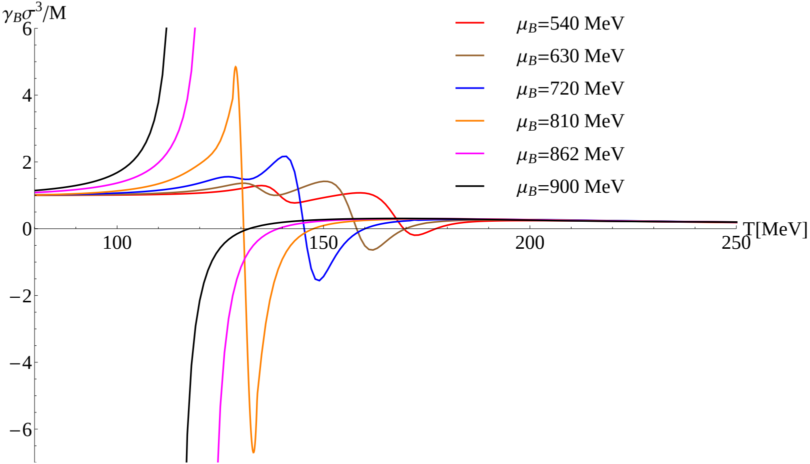
$$\kappa\sigma^2 \sim B^2$$

Kurtosis at null chemical potential

Kurtosis at null chemical potential as a function of temperature shows two peaks, the first one is due to the deconfinement transition and the other one is due to chiral symmetry restoration. It means that PNJL predicts two different temperature for chiral and deconfinement transitions at $\mu_i = 0$. This feature does not agree with lattice calculations. The split of transitions depends from the value of parameters of Polyakov Fields Potential $\mathcal{U}$.

The value of Kurtosis at low temperature is close to unity, after the transition, the value of kurtosis is close to $1/9$. Indeed the kurtosis is proportional to the square of baryon charge of the elementary degree of freedom. In the confined phase the elementary dof are hadrons (B=1) and in the deconfined phase are quarks (B=1/3)

Baryon-Skewness in the symmetric chemical potential scenario



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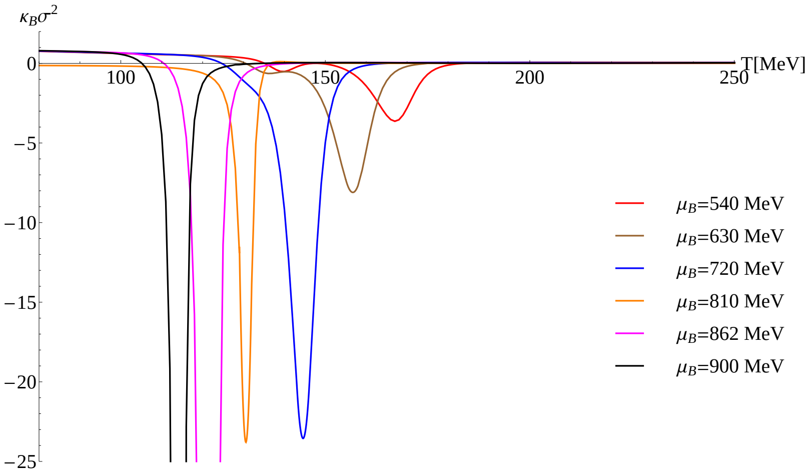
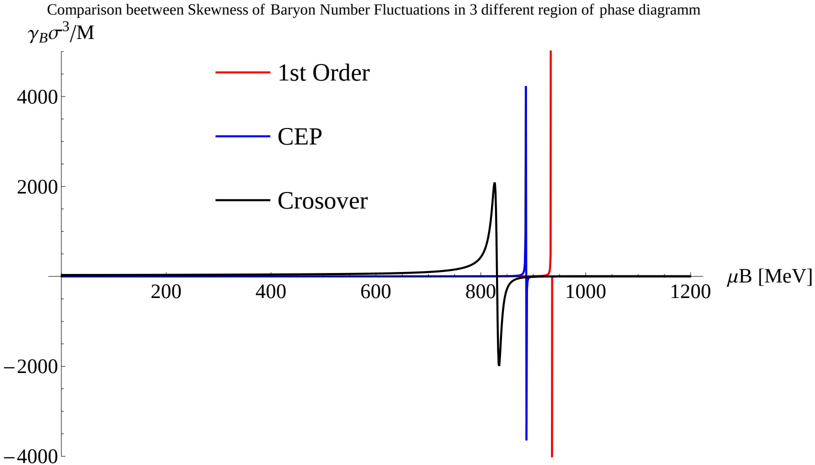


Skewness in the (Quasi-)Neutral Strangeness scenario



T=142.9 MeV (Black line), T=132.9 (Blue line), T=122.9 (Red line)

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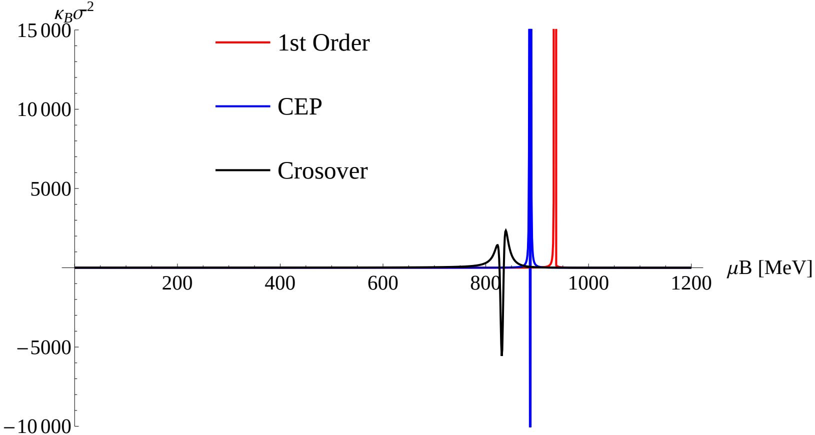

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metastable region

Near the 1^{st}-order transition line, the Gran Canonical Potential has 2 minima and 1 maximum. The system admit a metastable region.
On the first order line, the two minima are degenerate. It means that crossing the first order line the order parameters have a jump discontinuity.

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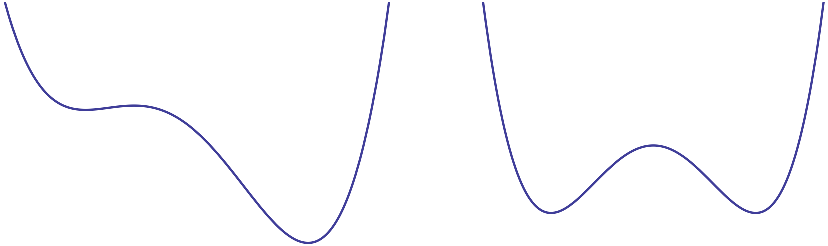

Figura: Shape of Grand Canonical Potential in metastable region

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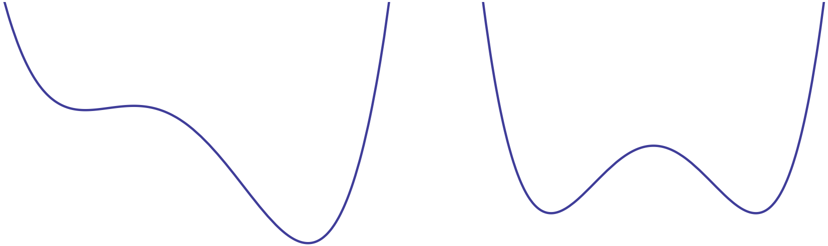

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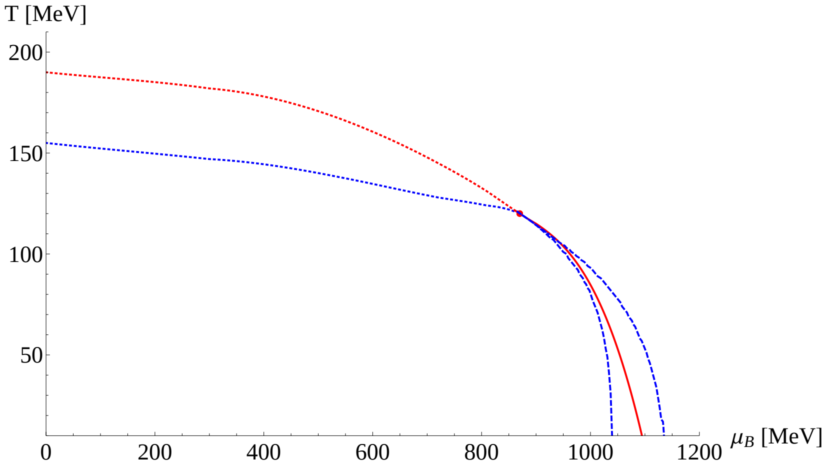
I perform the calculation on the symmetric chemical potential scenario to find the metastable region

Phase Diagram in symmetric chemical potential scenario



Conclusion and outlook

Conclusion:

- PNJL model provides a good qualitative and semi-quantitative guidance to describe the chiral and deconfinement QCD transition
- Nature of the active degrees of freedom is displayed by high order cumulants (e.g. kurtosis)

outlooks:

- Calculation in the HIC scenarios are in progress and others thermodynamics quantities are currently under investigation as isentropic lines
- I'm going to perform the calculation of mixed susceptibility for the comparison with Lattice QCD and experimental results.

Thank you for your attention!

Backup slide

Polyakov Fields

The value of Polyakov Fields is connected to the energy for produce a free quark from the vacuum:

$$\Phi \sim e^{-\beta F_q} \tag{24}$$

In the confined region, where it is not possible to create a sigle quarks from vacuum, $F_q \to +\infty$ then $\Phi \to 0$.
In the deconfined region F_q is finite and $\Phi \neq 0$.
At extremely high temperature $\Phi \to 1$. In any case Φ is smaller than unity

Polyakov Potential

The effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$ must be real (because it will appear in the Gran Canonical Potential), it must be singular in $\Phi = \bar{\Phi} \to 1$ and it depends explicitly by temperature. The potential proposed by C.Ratti et all in 2006 satisfy the requests.

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}; T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln[1 - 6\bar{\Phi}\Phi + 4(\Phi^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2] \quad (25)$$

where

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \qquad b(T) = b_3\left(\frac{T_0}{T}\right)^3 \quad (26)$$

a_0	a_1	a_2	b_3
3.51	-2.47	15.22	-1.75

2

[2] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D73, 014019 (2006).