An Application of the AMPT Model for SIS100 / FAIR Energies

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Outline

- A Multi-Phase Transport (AMPT) model
- Incorporation of finite nuclear thickness to string melting AMPT
- Analytical understanding by extending the Bjorken ε formula
  to lower energies such as SIS100 / FAIR energies
- Comparisons of extended Bjorken formula with AMPT results
- Summary

A Multi-Phase Transport (AMPT) Model

AMPT aims to provide a self-contained kinetic description
of essential stages of high energy heavy ion collisions:

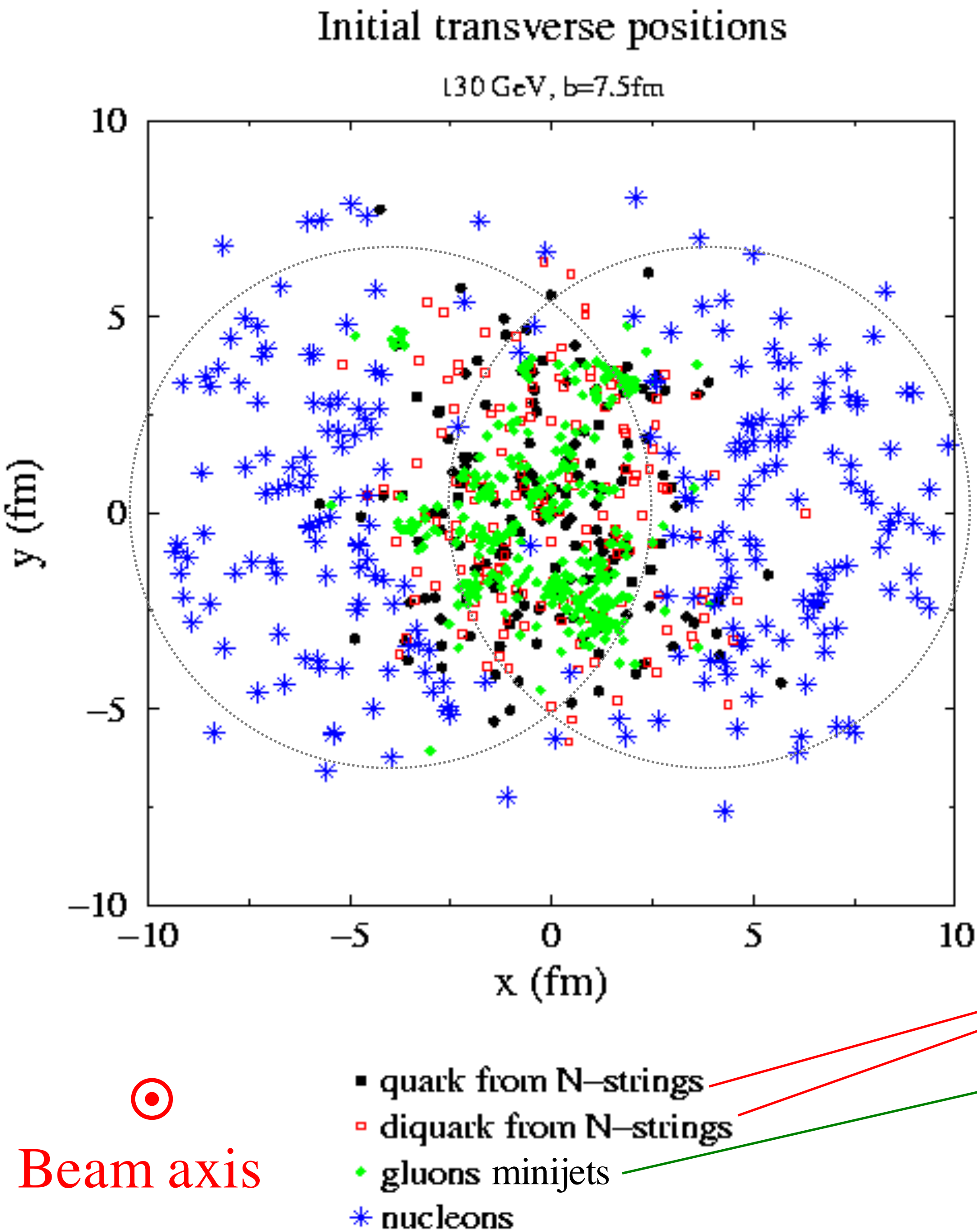
- Event-by-event from initial condition to final observables
- Can address non-equilibrium dynamics
  (e.g. partial equilibration and thermalization, initial flow)
- Self-consistent Chemical and kinetic freeze-out

- Publicly available since 2004 and often updated:
  source codes at *http://myweb.ecu.edu/linz/ampt/*

It is also a test-bed of different ideas & may lead to new discoveries:

- the discovery of v_3 by Alver & Roland
- v_2 & v_3 may be dominated by anisotropic parton escape
  instead of hydrodynamics flow, due to low/modest opacity

String melting version of AMPT



String Melting AMPT:
we convert strings into partonic matter;
should be more realistic at high energies;
this enabled AMPT to produced enough v2 at high energies using pQCD-like small parton cross section.

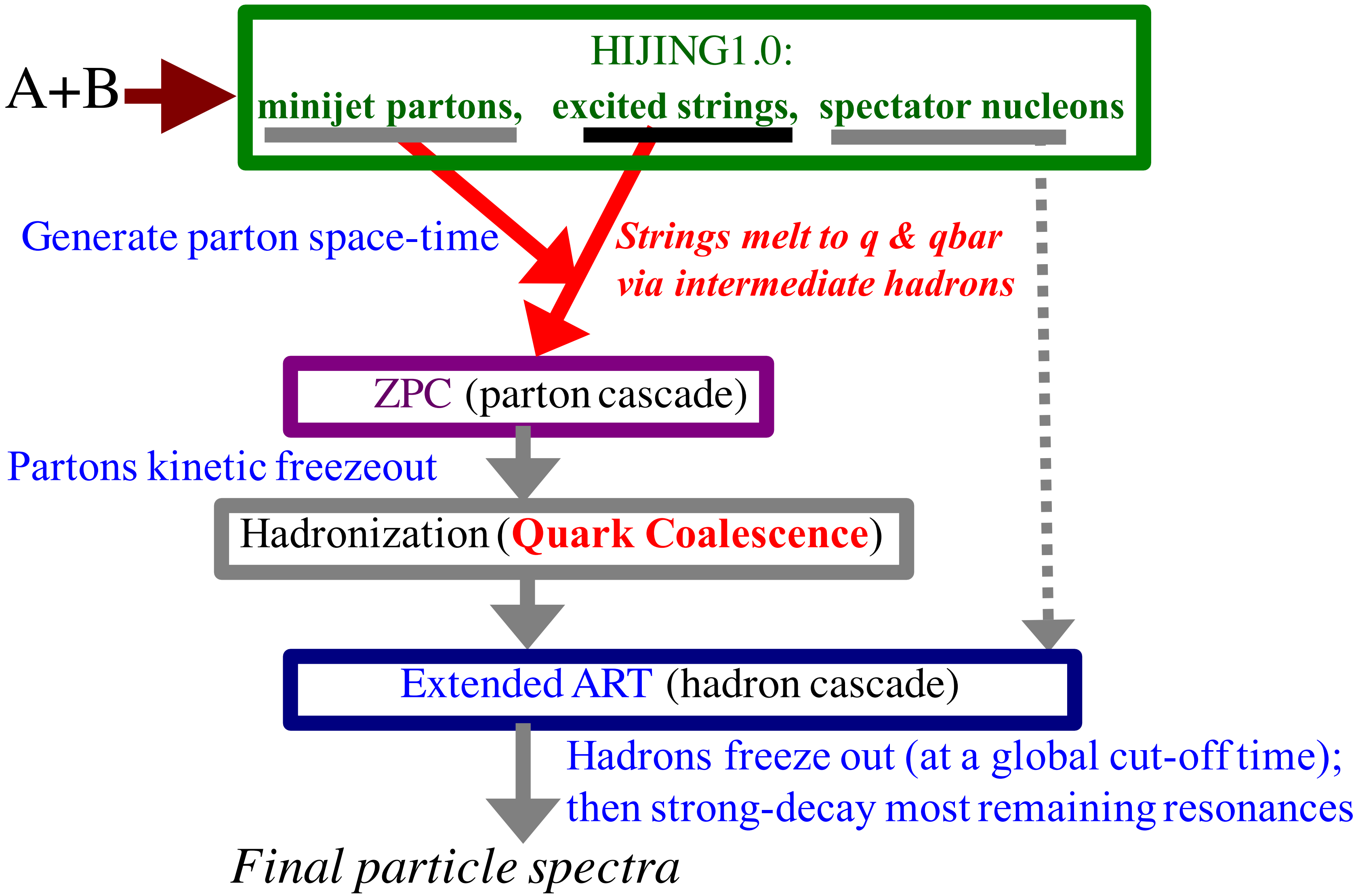
ZWL and Ko, PRC 65 (2002)

Initial condition in default AMPT:
soft (strings) & *hard (minijets)*

⊙
Beam axis

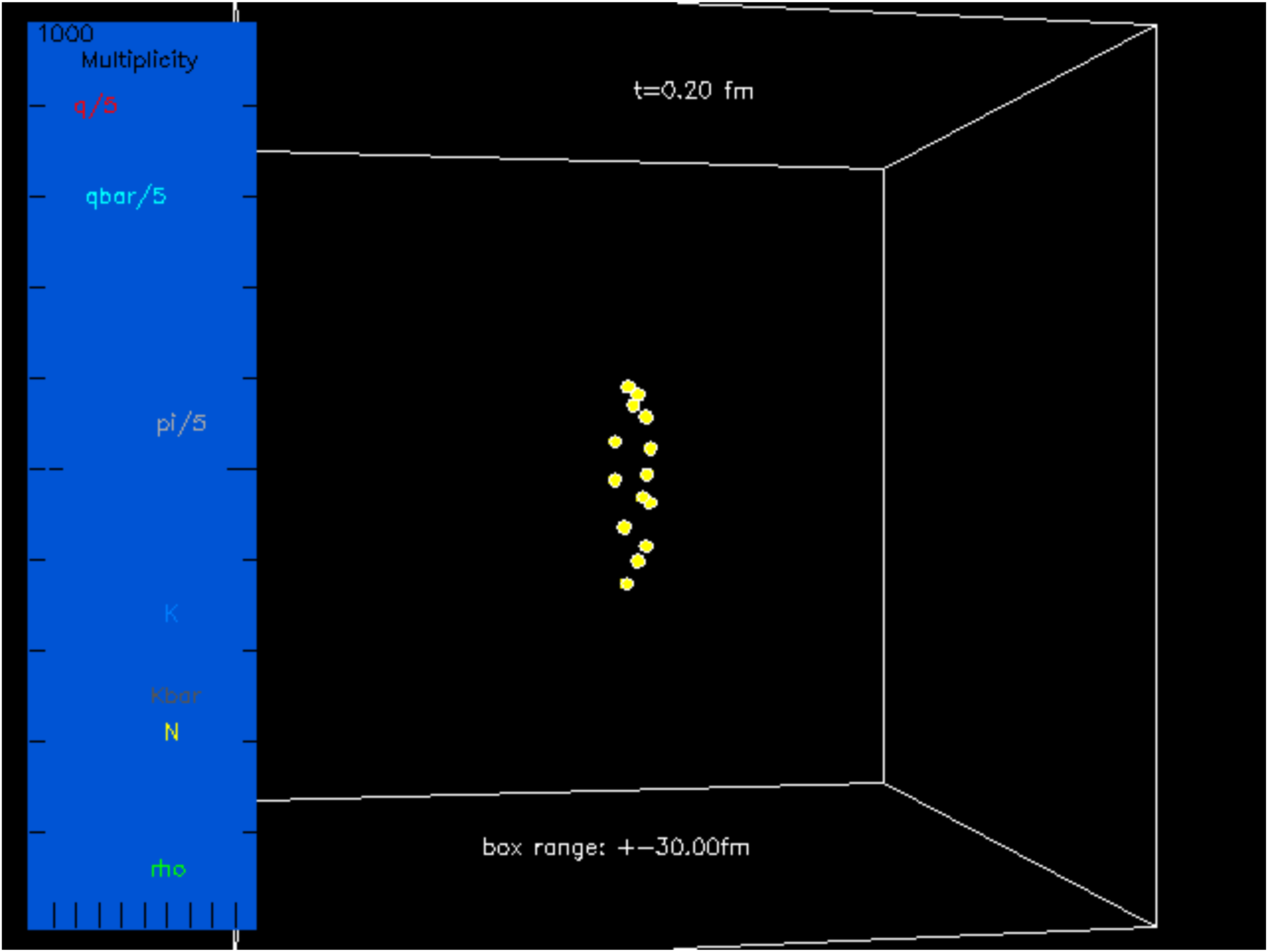
Strings are in high density overlap area,
but not in parton cascade.

Structure of String Melting AMPT



ZWL et al. PRC72 (2005)

String melting AMPT : 1 central Au+Au event at 200AGeV



Side view:

Beam axes

AMPT: default (Def) versus string melting (SM)

	AMPT-Def [1]	**AMPT-SM [2]**	**AMPT-SM in [3]**	**AMPT-SM in [4]**
Lund string ***a***	2.2	2.2	0.5	0.55 for RHIC, 0.30 for LHC
Lund string ***b***(GeV^{-2})	0.5	0.5	0.9	0.15, also limit $P(s)/P(q) \leq 0.4$
α_s in parton cascade	0.47	0.47	0.33	0.33
Parton cross section	~3 mb	~ 6 mb	1.5 mb	3 mb
Model describes	dN/dy, p_T not v2 or HBT	v2 & HBT not dN/dy or p_T	dN/dy, v2 (LHC) not p_T	dN/dy, p_T & v2 (π,K@RHIC, LHC)

[1] ZWL et al. PRC64 (2001).

[2] ZWL and Ko, PRC 65 (2002); ZWL et al. PRC 72 (2005).

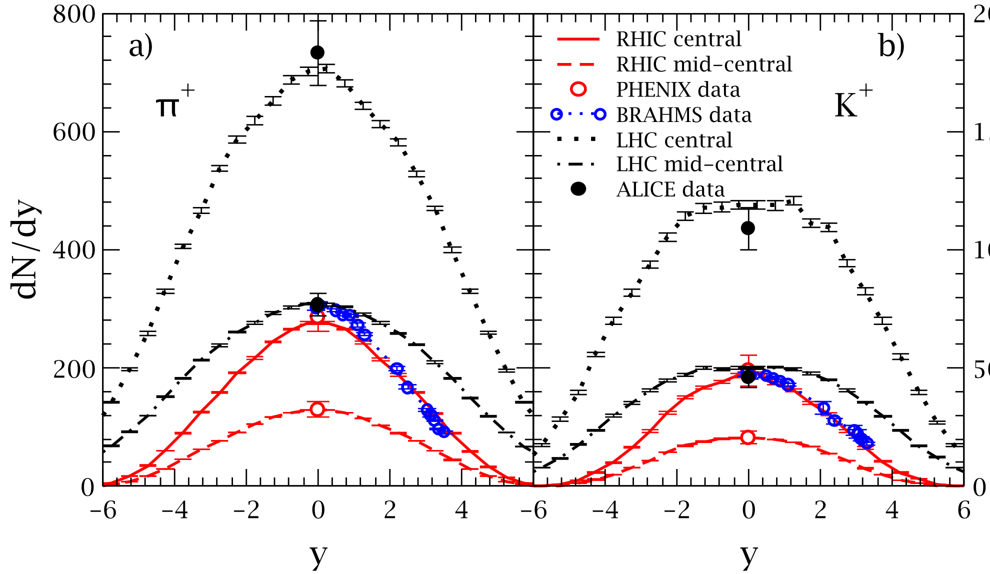
[3] Xu and Ko, PRC 83 (2011).

[4] ZWL, PRC 90 (2014): AMPT-SM can be tuned to reasonably reproduce simultaneously *dN/dy, p_T–spectra & v2* of low-p_T (<2GeV/c) π & K data
for central (0-5%) and mid-central (20-30%) 200AGeV Au+Au collisions (RHIC) or 2.76AGeV Pb+Pb collisions (LHC).

Predictions for 5.02ATeV Pb+Pb collisions in Ma and Lin, PRC(2016)

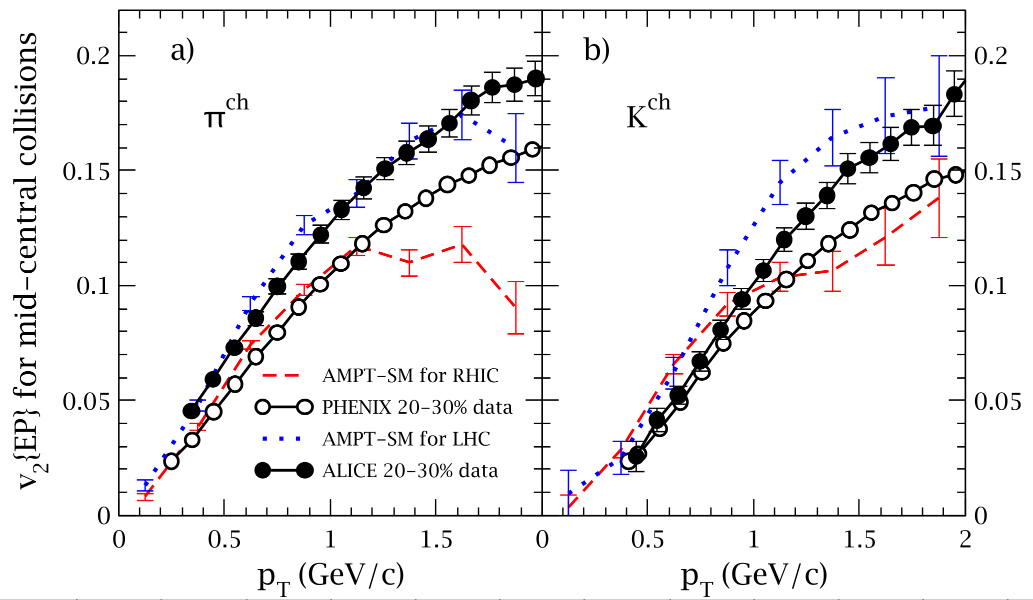
String melting version of AMPT at RHIC/LHC energies

dN/dy of π & K:

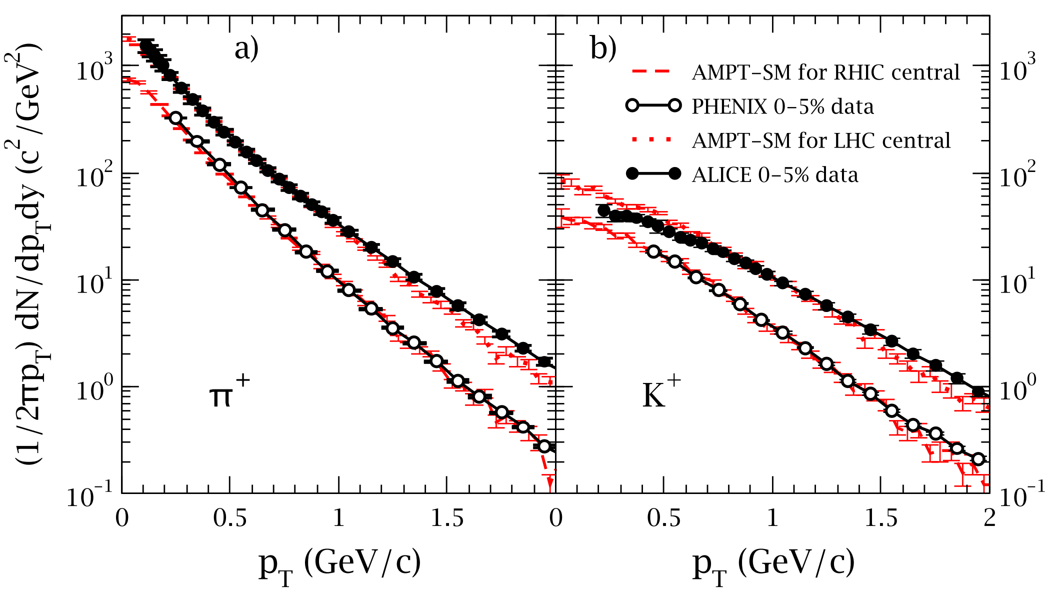


ZWL, PRC 90 (2014)

v2 of π & K (mid-central collisions):

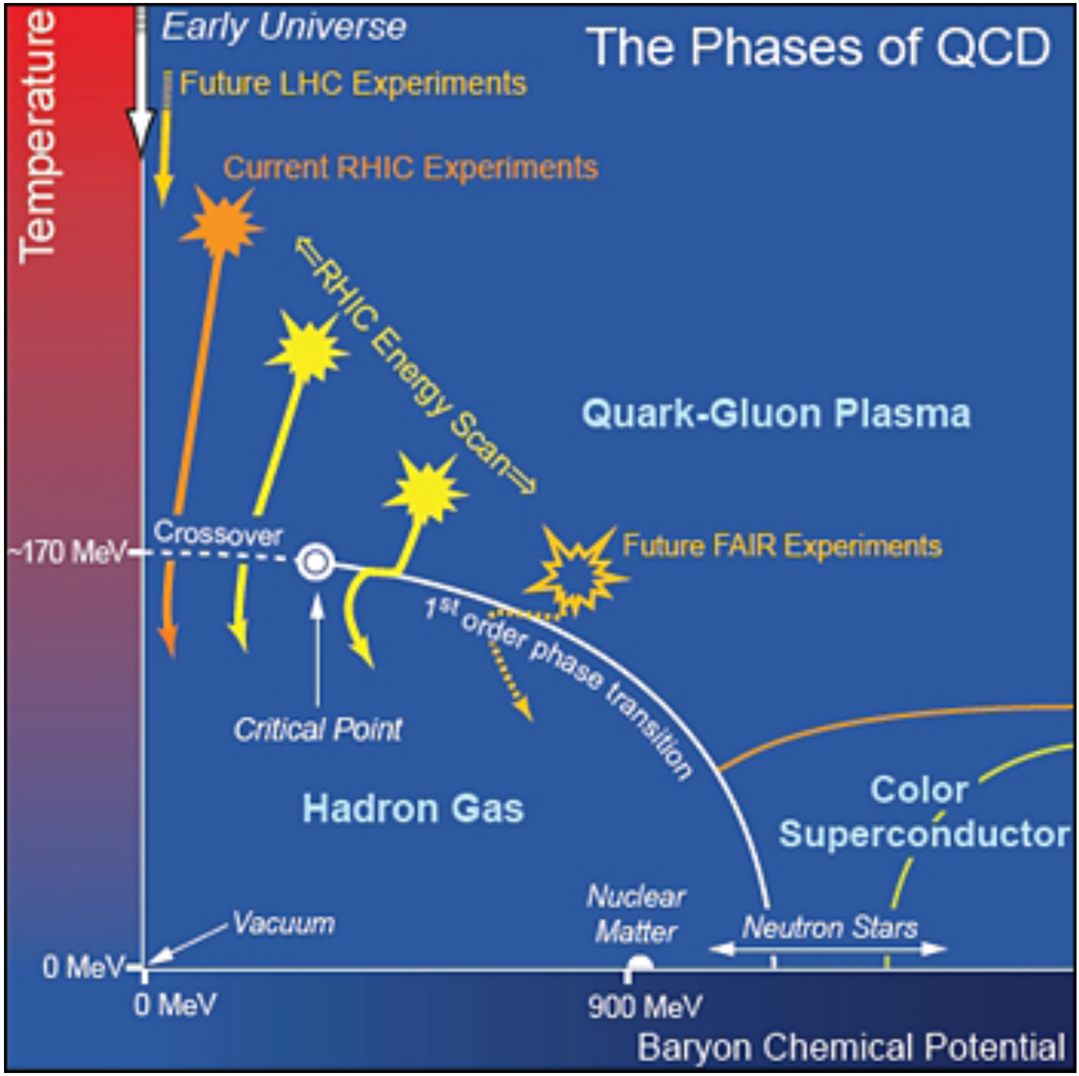


p_T -spectra of π & K (central collisions):

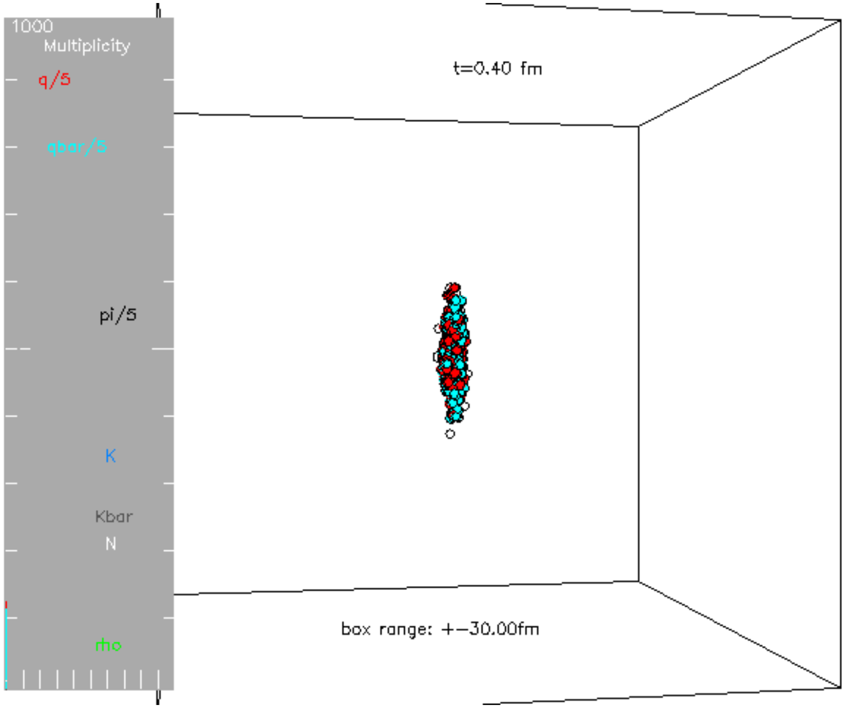


Application of string melting AMPT to lower energies

- At lower energies, trajectory of nuclear collisions is important for potential effects from the QCD critical point.
- Trajectory depends on the time evolution of energy density ε or T & net-baryon density n_B or μ_B
- Before studying these effects, the model first needs to describe the initial densities, including the peak value and time dependence: ε^{max}, $\varepsilon(t)$, …



from bnl.gov

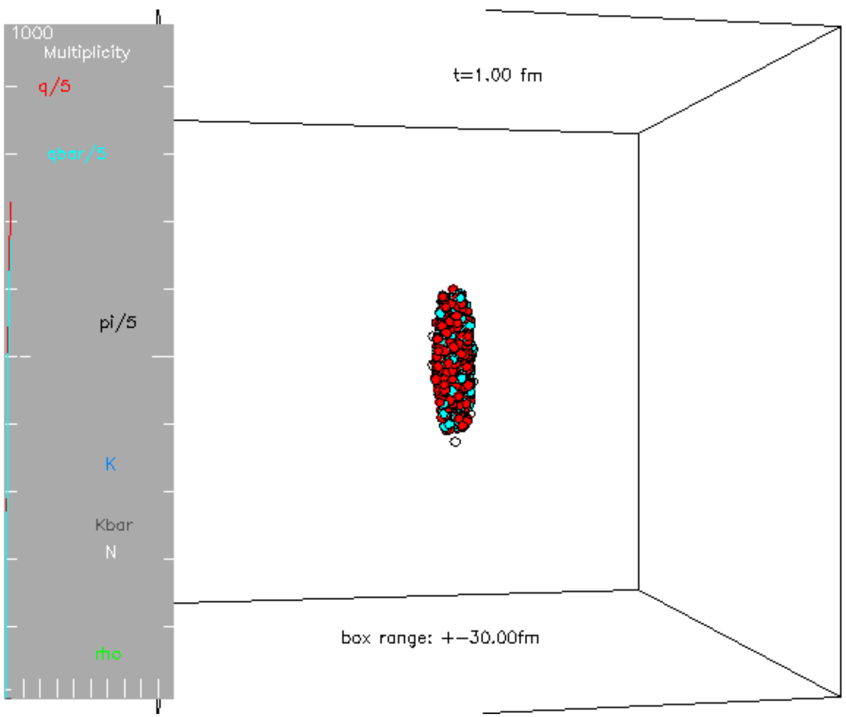


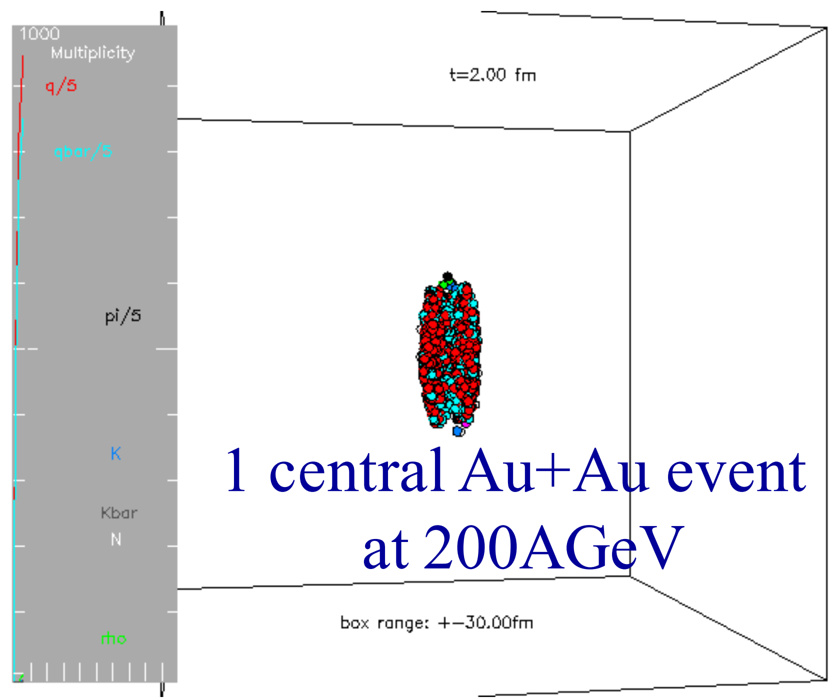
String melting AMPT
was implemented for high energies:
finite nucleus width was neglected.

At lower energies,
finite width may have important effects.

So we have recently included finite width
to string melting AMPT.

ZWL & Y. He, in progress



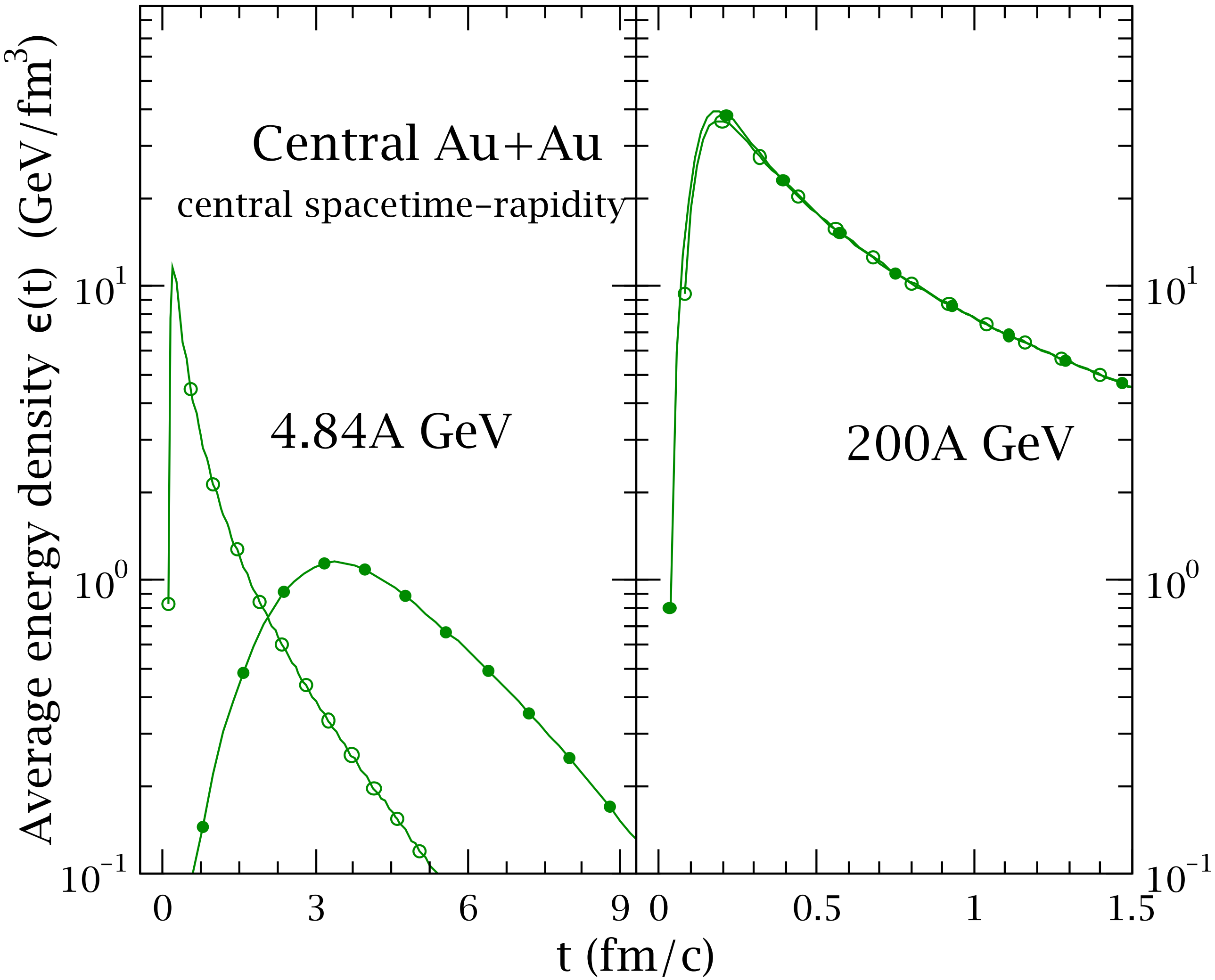


1 central Au+Au event
at 200AGeV

Incorporation of finite nuclear thickness for string melting AMPT

Effect of finite thickness (filled circles):

- is large at low energy, gives much lower ε^{max} and different shape
- small effect at high energy *as expected*



What about analytical understanding?

→ extension of the Bjorken ε formula to lower energies

ZWL, arXiv:1704.08418v2/PRC(2018)

Extension of the Bjorken ε formula

A common model is the Bjorken formula:

$$\epsilon(\tau) = \frac{1}{\tau\, A_T} \frac{dE_T(\tau)}{dy}$$

At high energies, initial particles are produced from a pancake (at z=0) at t=0.

For partons in a thin slab of thickness **-d<z<d** in central rapidity (y~0) at time **t** :

$$v_z = |\tanh(y)| \approx |y| < \frac{d}{t}.$$

Energy within the slab is then

$$E = N \frac{d\langle E\rangle}{dy} \Delta y = N \frac{d\langle E\rangle}{dy} \frac{1}{\cancel{2}} \left[\frac{2d}{t} \right] . \qquad (3)$$

It follows that the central energy density ϵ is

$$\epsilon \approx \frac{N}{\mathscr{A}} \frac{d\langle E\rangle}{dy} \frac{1}{\cancel{2}t} . \qquad (4)$$

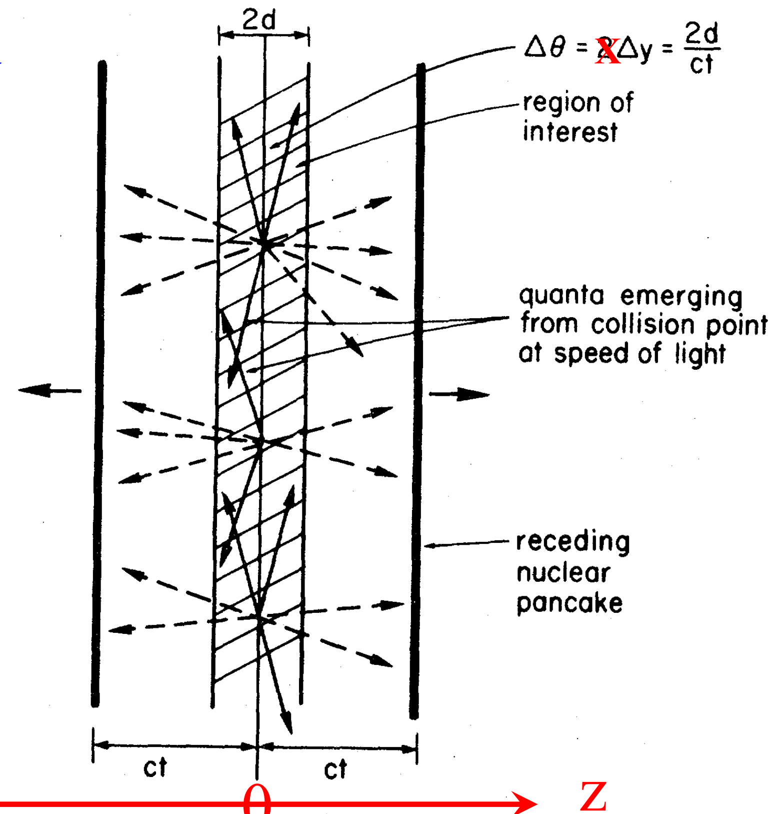


FIG. 2. Geometry for the initial state of centrally produced plasma in nucleus-nucleus collisions.

Bjorken, PRD 27 (1983)

Extension of the Bjorken ε formula

Bjorken, PRD 27 (1983)

In spite of Fig.1,
the Bjorken formula neglects
finite thickness of (boosted) nuclei
→ it is only valid at high energies
where crossing time << τ_F

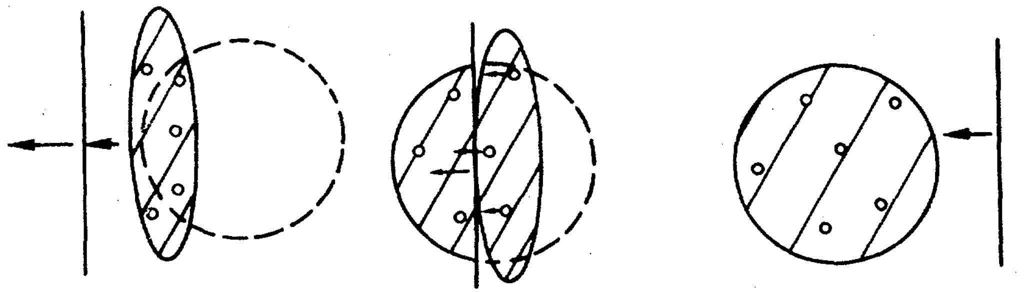


FIG. 1. Schematic of the evolution of a compressed "baryon fireball" in nucleus-nucleus collisions, according to the mechanism of Anishetty, Koehler, and McLerran (Ref. 8).

From PHENIX NPA757 (2005):

Eq. (5) here is essentially identical[5] to Eq. (4) of Bjorken's result [74], and so is usually referred to as the *Bjorken energy density* $\varepsilon_{\mathrm{Bj}}$. It should be valid as a measure of peak energy density in created particles, on very general grounds and in all frames, as long as two conditions are satisfied: (1) A finite formation time τ_{Form} can meaningfully be defined for the created secondaries; and (2) The thickness/"crossing time" of the source disk is small compared to τ_{Form}, that is, $\tau_{\mathrm{Form}} \gg 2R/\gamma$. In particular, the validity of Eq. (5) is completely independent of the shape of the $dE_T(\tau_{\mathrm{Form}})/dy$ distribution to the extent that

[5] A (well-known) factor of 2 error appears in the original.

Extension of the Bjorken ε formula

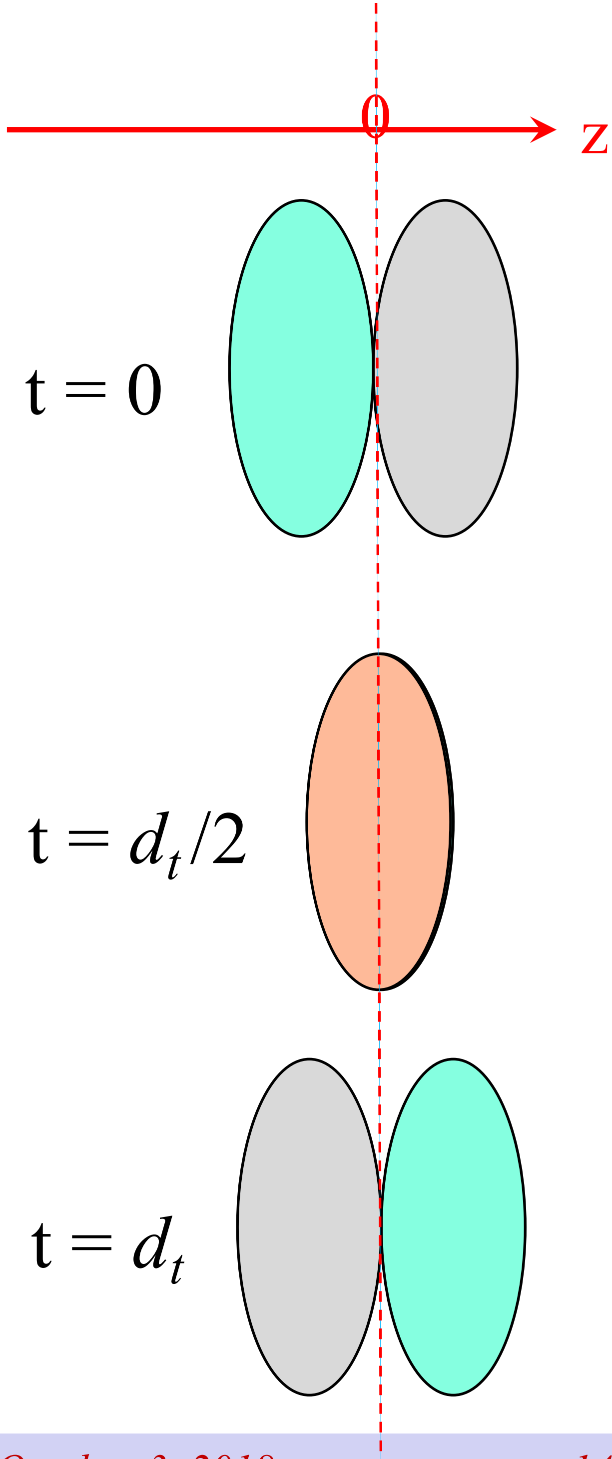
Considering central A+A collisions
in the center-of-mass frame
& using the hard sphere model for nucleus:
crossing time

$$d_t = \frac{2R_A}{\sinh y_{CM}} = \frac{2R_A}{\gamma\,\beta}$$

For central Au+Au collisions:

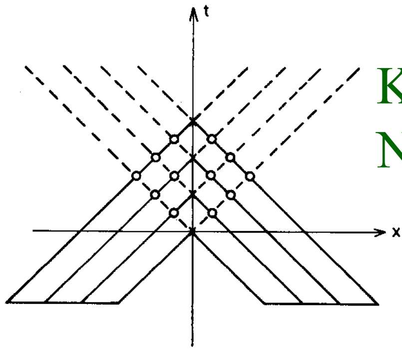
$\sqrt{s_{NN}}$ (GeV)	3	5	11.5	27	50	200
$d_t\ (fm/c)$	10.5	5.3	2.2	0.91	0.49	0.12

Need crossing time << τ_F
→ the Bjorken formula is only valid for
$\sqrt{s_{NN}}$ > ~50 GeV *for* $\tau_F = 0.5\ fm/c$.



Extension of the Bjorken ε formula

Goal: fix this problem
& derive a Bjorken-type formula
that's also valid at lower energies
($\sqrt{s_{NN}}$< ~50 GeV).



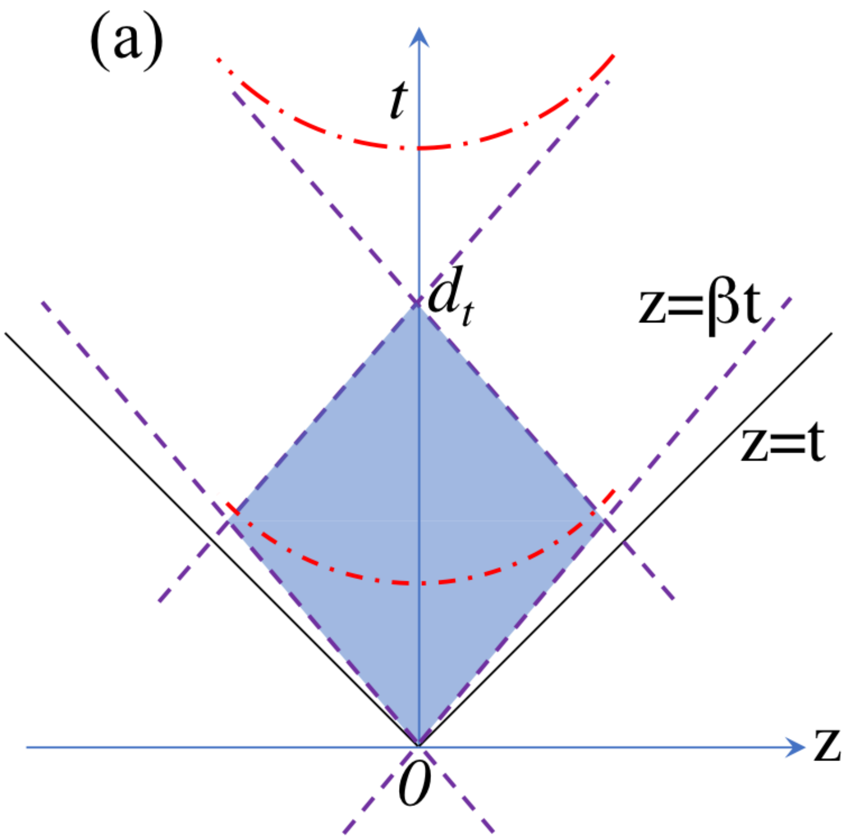
Kajantie et al.
NPB (1983)

Fig. 5. An alternative description of the A + A collision. In addition to the pairwise N + N collisions on the time axis (crosses), the secondaries may further interact with the incoming nucleons (circles). This would enhance the energy density in the central region.

Consider a schematic picture:
two nuclei come into contact at time *0*
and pass each other at time d_t .

The shaded area
is the primary collision region,
so initial energy production takes place
over a finite duration of t & z.

We shall neglect secondary scatterings
& only consider the central region (η_s ~0)

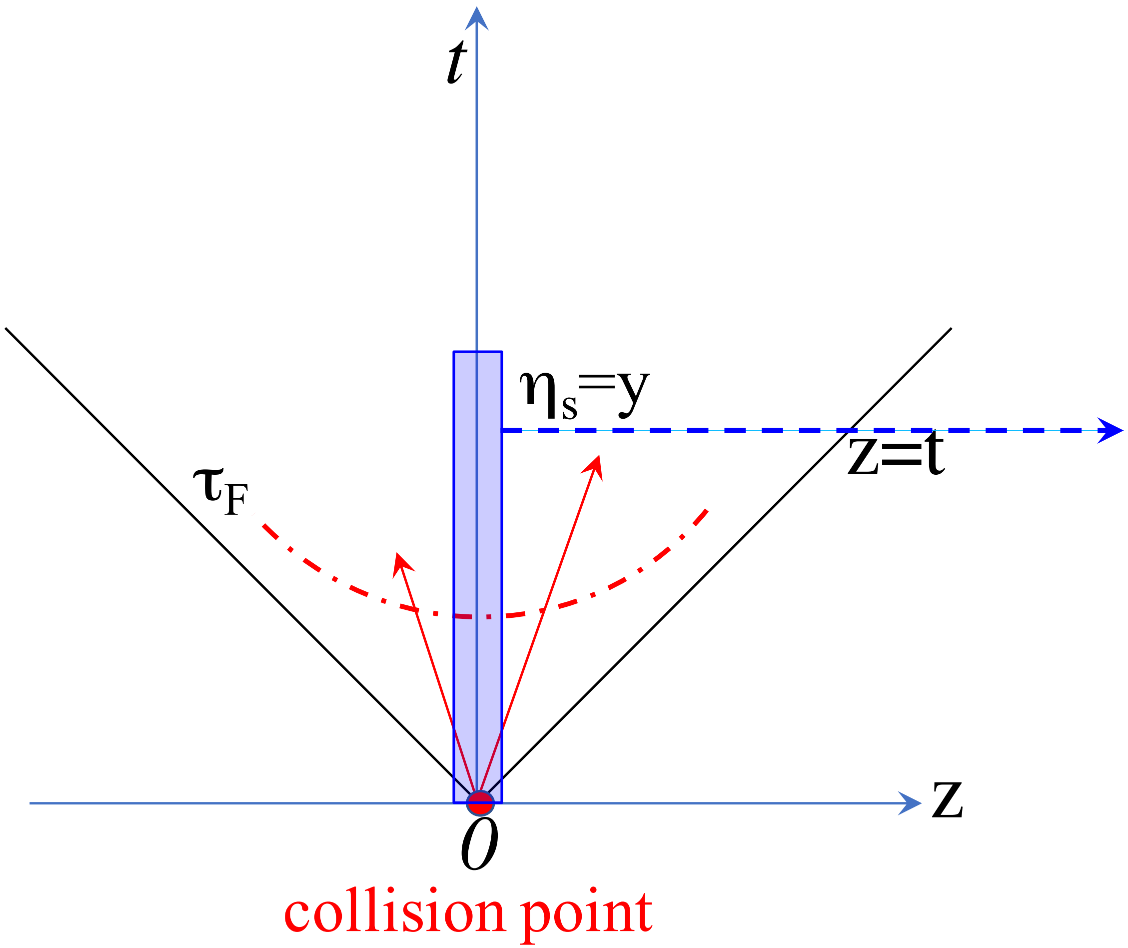


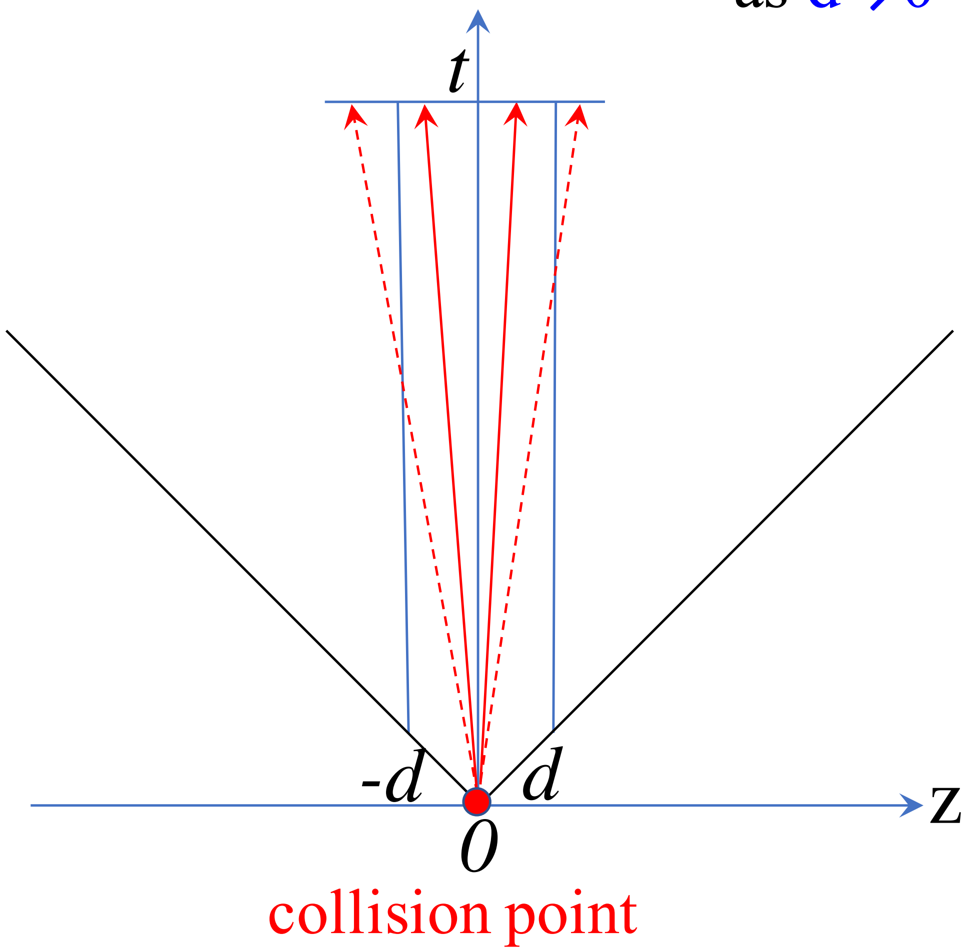
Extension of the Bjorken ε formula

Picture for the Bjorken formula:

(a) for all rapidities:

(b) for centrality rapidity η_s=y~0:
as d→0



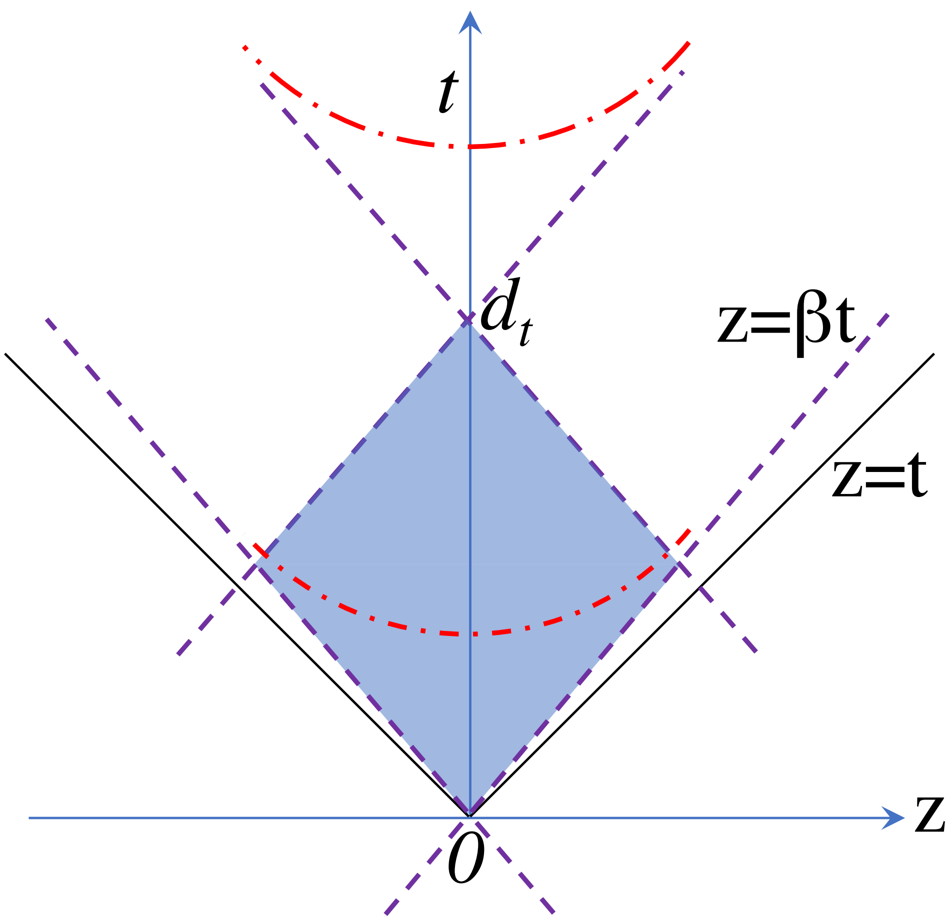


Extension of the Bjorken ε formula

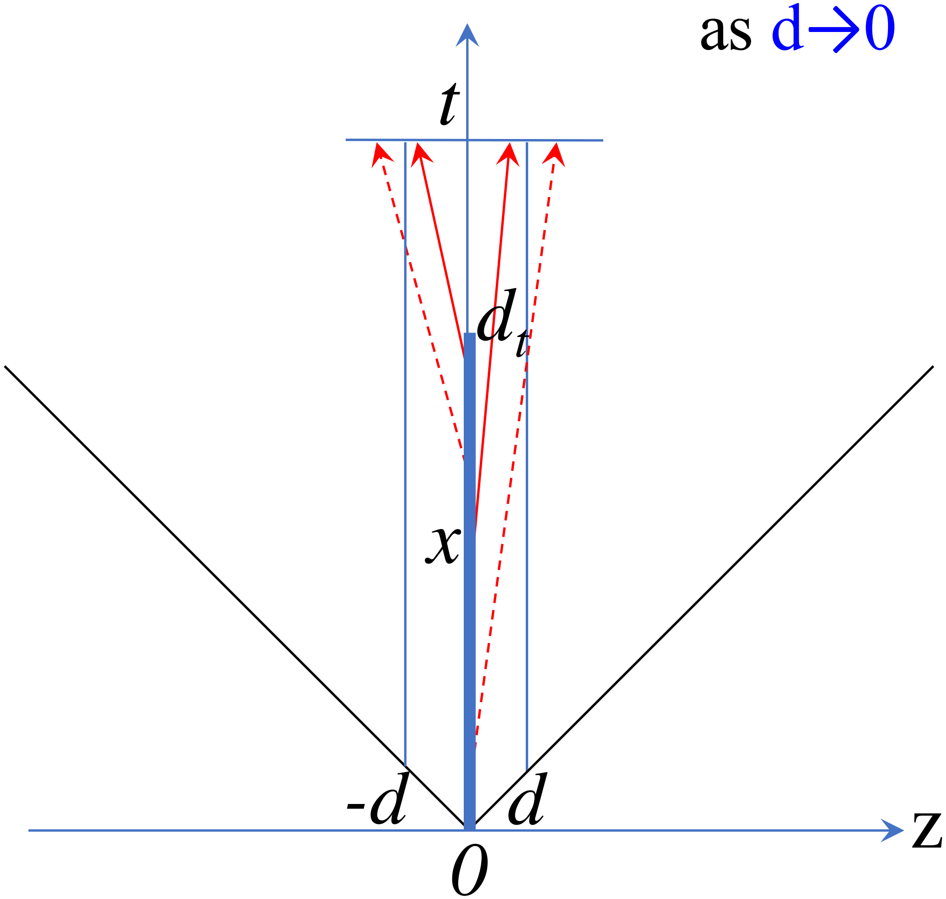
Picture with finite thickness:

Method:
introduce the finite time duration in the initial energy production
(*but neglect the finite z-width*)

(a) for all rapidities:

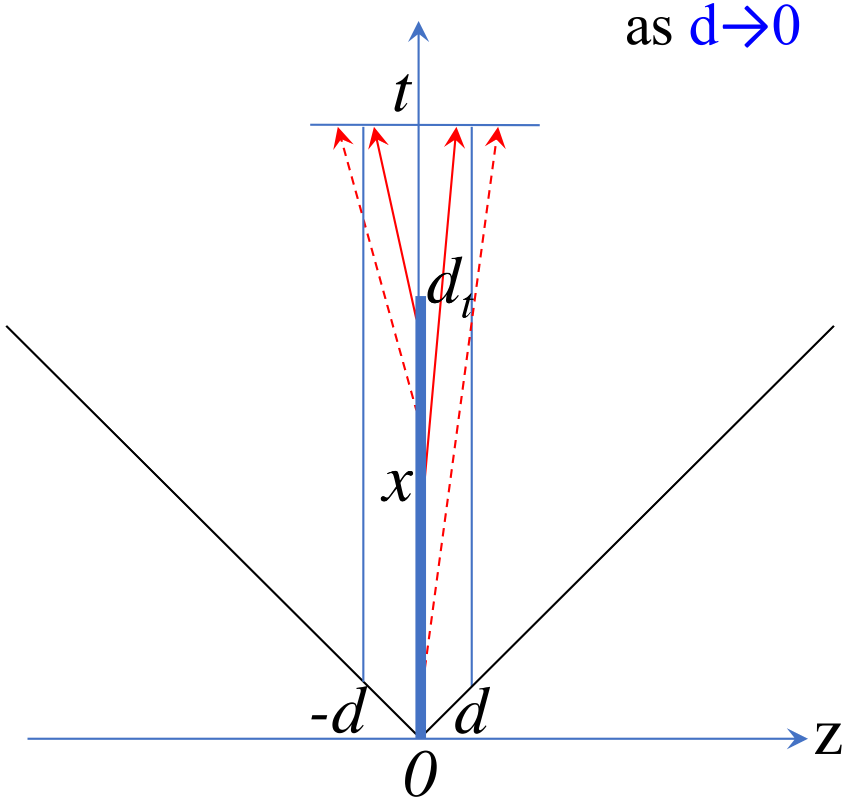


(b) for centrality rapidity $\eta_s=y\sim0$:
as $d\to0$



Extension of the Bjorken ε formula

(b) for centrality rapidity $\eta_s = y \sim 0$:
as $d \to 0$



Average energy density ε within the slab diverges as $t \to 0$, like the Bjorken formula.

So we assume a finite formation time τ_F for initial particles, then at any time $t \geq \tau_F$:

$$\varepsilon(t) = \frac{1}{A_T} \int_0^{t-\tau_F} \frac{d^2 E_T}{dy\, dx} \frac{dx}{(t-x)}.$$

This applies even during the crossing time.

To proceed, we now take a specific form for the time profile $\frac{d^2 E_T}{dy\, dx}$.

Extension of the Bjorken ε formula: the uniform profile

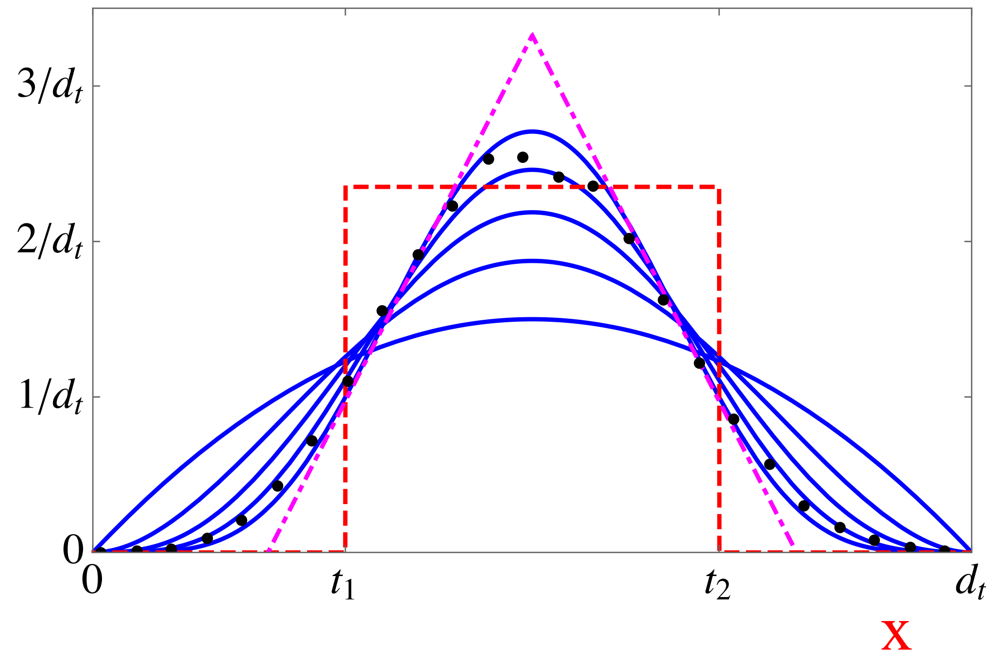
$$\varepsilon(t) = \frac{1}{A_T}\int_0^{t-\tau_F} \frac{d^2E_T}{dy\,dx}\frac{dx}{(t-x)}$$

The simplest (uniform) profile:

initial energy (at y~0) is produced uniformly from time t_1 to t_2 :

$$\frac{d^2E_T}{dy\,dx} = \frac{1}{t_{21}}\frac{dE_T}{dy}$$

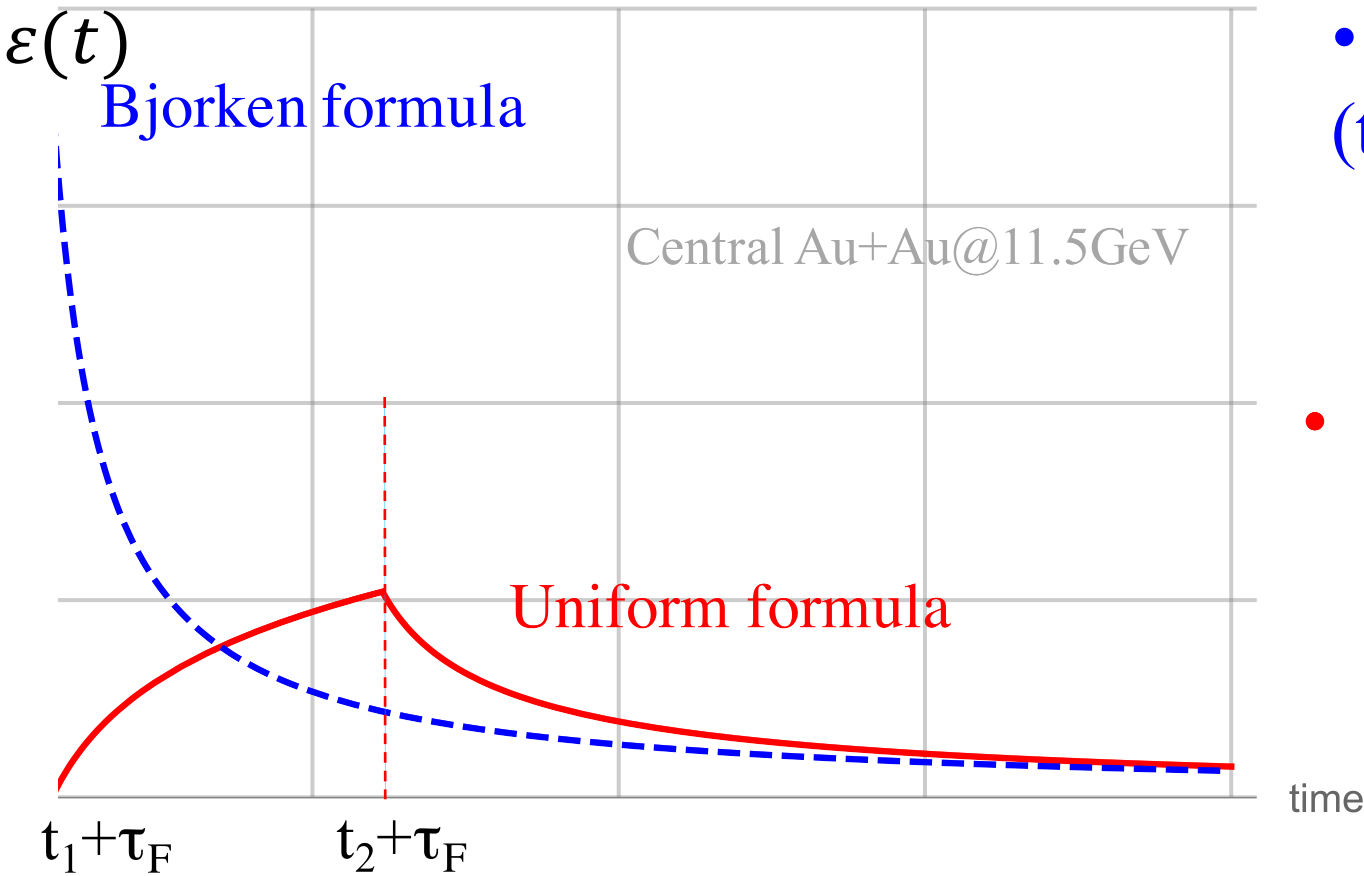
for $x \in [t_1, t_2]$,
with $t_{21} \equiv t_2 - t_1$



Extension of the Bjorken ε formula: the uniform profile

→ solution: $$\epsilon_{\rm uni}(t) = \frac{1}{A_{\rm T}\, t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{\tau_{\rm F}}\right), \text{if } t \in [t_1+\tau_{\rm F}, t_2+\tau_{\rm F}];$$
$$= \frac{1}{A_{\rm T}\, t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{t-t_2}\right), \text{if } t \geq t_2+\tau_{\rm F}.$$

ZWL, arXiv:1704.08418v2/PRC(2018)



- *At high energies:* (thin nuclei, $t_{21}/\tau_F \to 0$): $\varepsilon_{uni}(t) \to \varepsilon_{Bj}(t)$ *analytically*
- *At lower energies:* very different from Bjorken

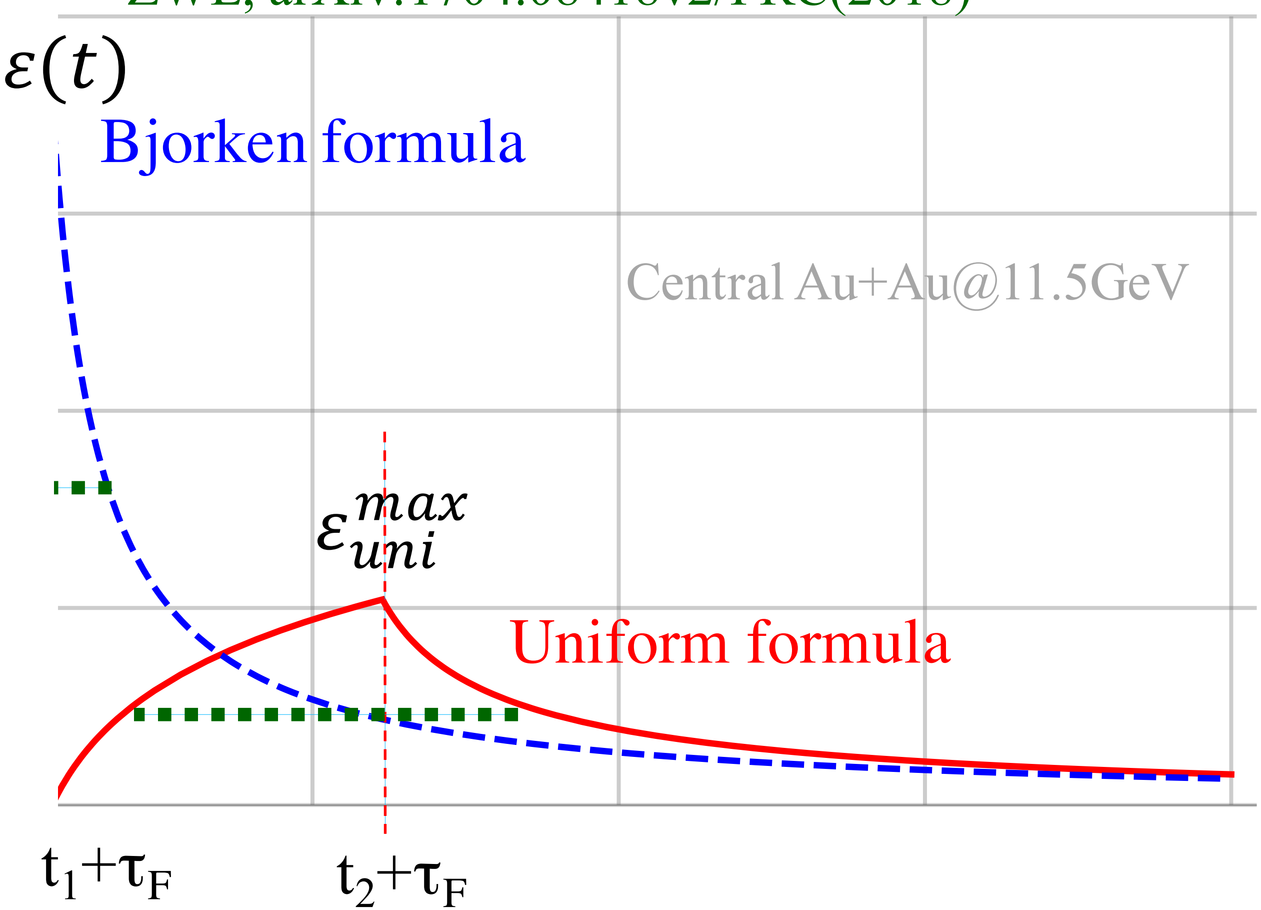
dE_T/dy parameterization from PHENIX PRC 71 (2005)

Extension of the Bjorken ε formula: the uniform profile

Peak energy density: $\epsilon_{\rm uni}^{max} = \epsilon_{\rm uni}(t_2+\tau_{\rm F}) = \dfrac{1}{A_{\rm T} t_{21}} \dfrac{dE_{\rm T}}{dy} \ln\left(1+\dfrac{t_{21}}{\tau_{\rm F}}\right)$

→ ratio over Bjorken: $\dfrac{\epsilon_{\rm uni}^{max}}{\epsilon_{\rm Bj}(\tau_{\rm F})} = \dfrac{\tau_{\rm F}}{t_{21}} \ln\left(1+\dfrac{t_{21}}{\tau_{\rm F}}\right).$ ≤ 1 always.

ZWL, arXiv:1704.08418v2/PRC(2018)



1) For $t_{21}/\tau_F \to 0$ (high energy):
ratio → 1 (→ *Bjorken*)

2) For $t_{21}/\tau_F >> 1$ (low energy):
ratio → 0;

$\varepsilon_{uni}^{max} \propto \ln\left(\frac{1}{\tau_F}\right)$, not $\frac{1}{\tau_F}$,

so the peak energy density

- << Bjorken value
- much less sensitive to τ_F
- FWHM width in t >> Bjorken

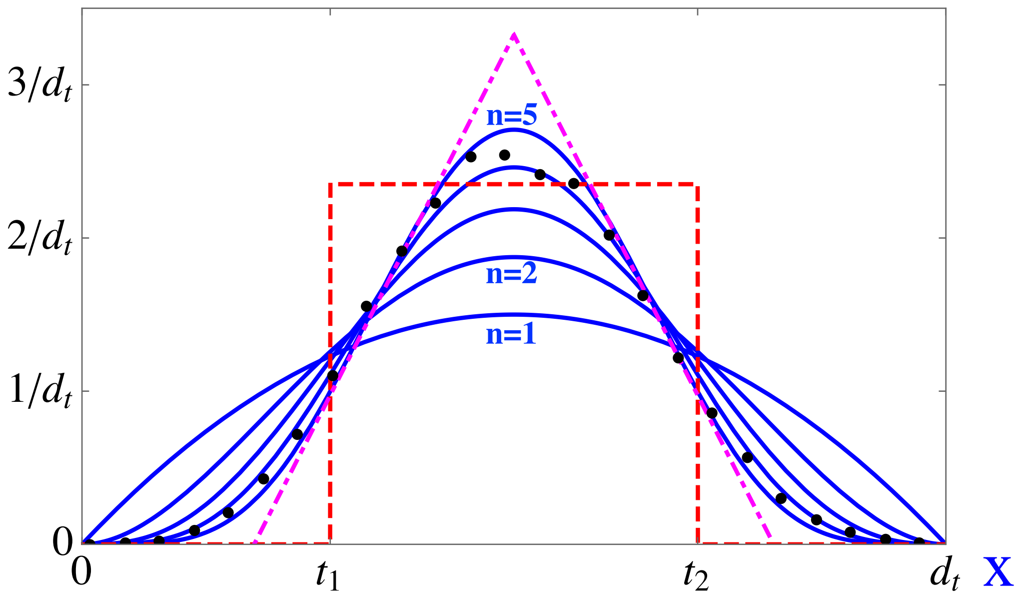
Extension of the Bjorken ε formula: beta or triangular profiles

A more realistic profile:

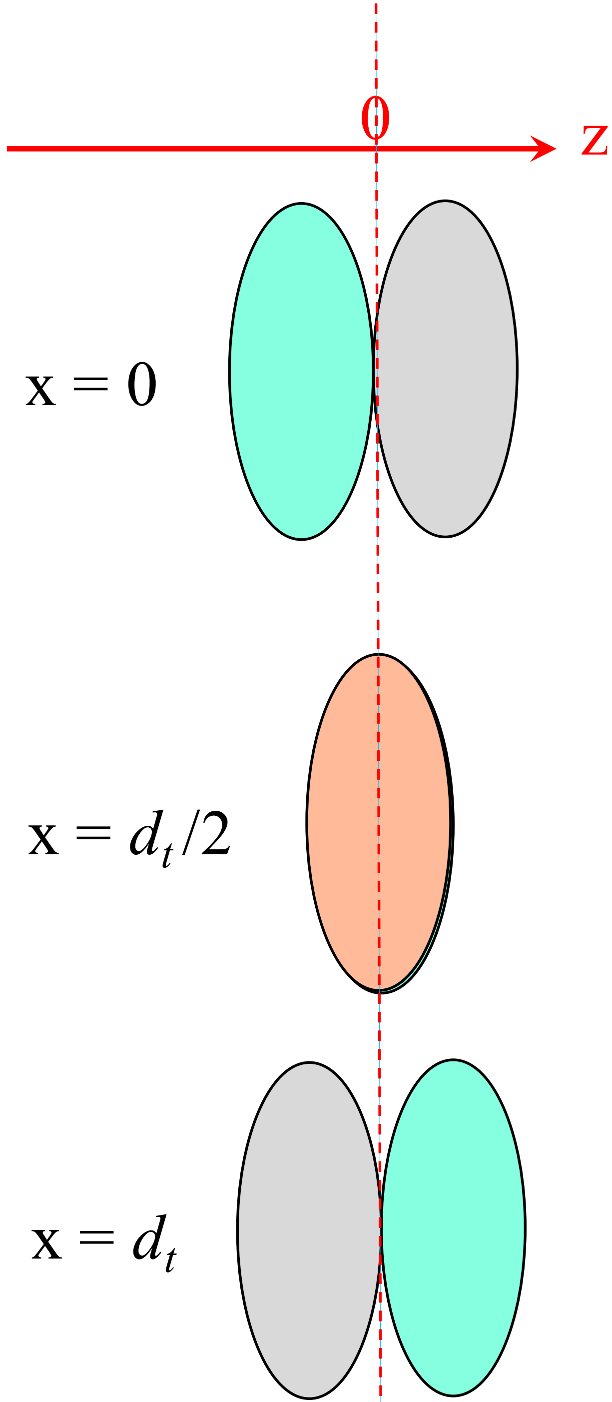
~0 energy is produced at $x = 0$ & d_t ,
most energy is produced around $x = d_t/2$:

$$\frac{d^2E_T}{dy\,dx} = a_n[x(d_t - x)]^n \frac{dE_T}{dy} \quad \text{(beta profile)}$$

or a symmetric triangular profile

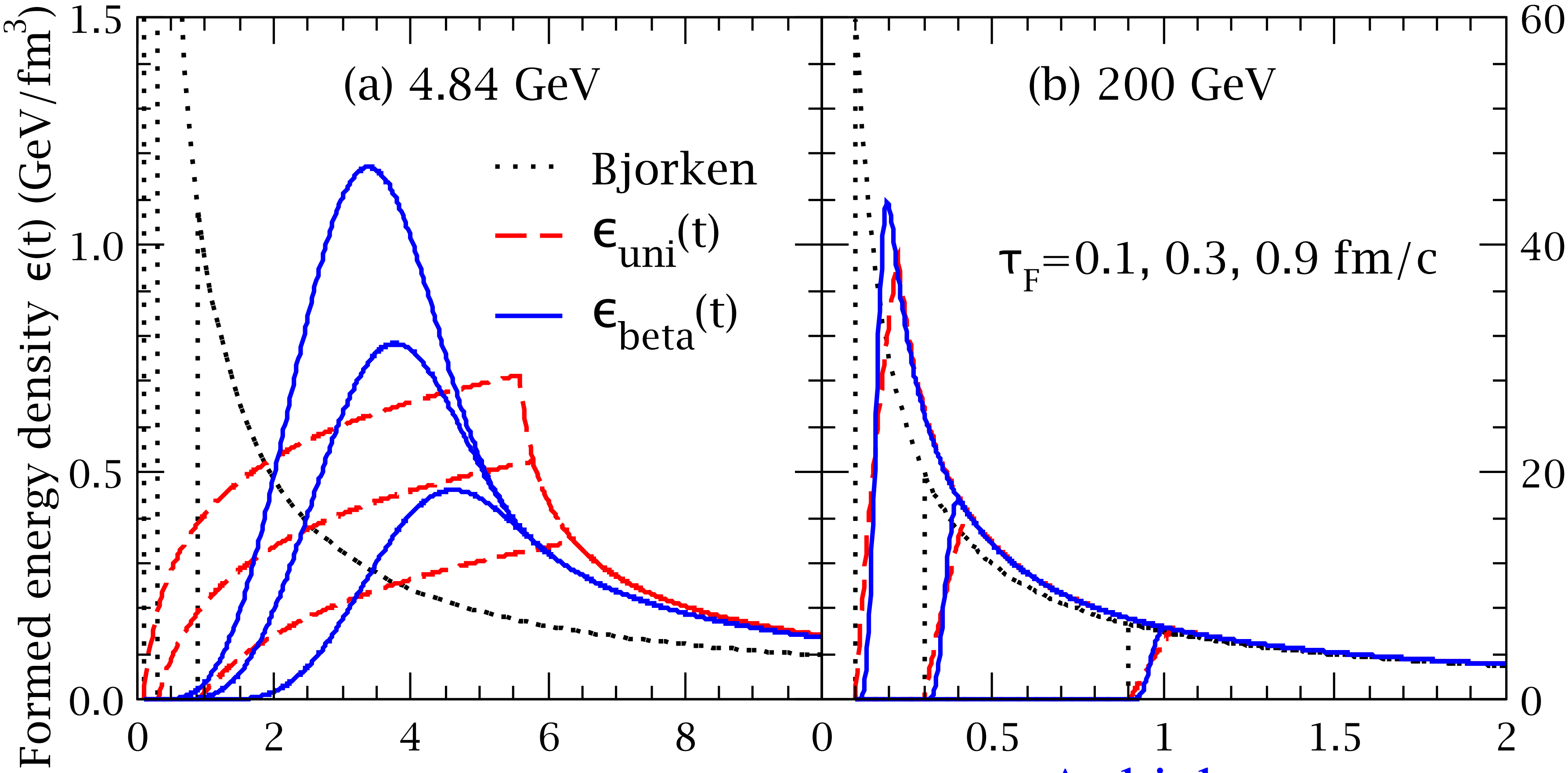


Circles: *time profile of initial partons within mid-η_s from string melting AMPT for central Au+Au @11.5 GeV.*



Applying extended formula to central Au+Au collisions

We compare 1) the uniform time profile (with $t_1 = 0$ & $t_2 = d_t$),
2) the beta time profile (n = 4). 3) the Bjorken formula:



At low energy:
ε^{max} << Bjorken value,
is much less sensitive to τ_F :

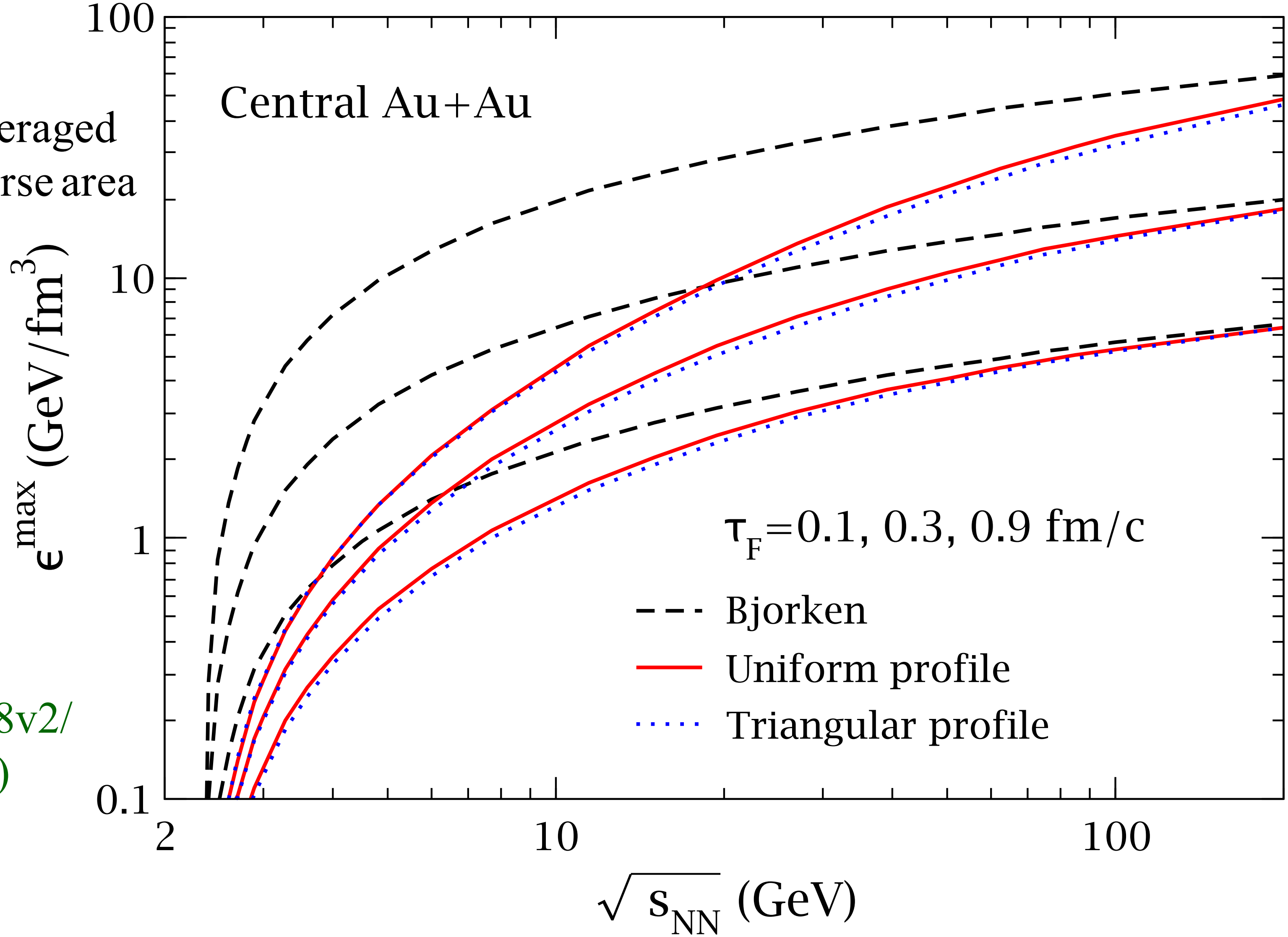
factor of 2.1 or 2.5 change (not factor of 9) when τ_F changes from 0.1 to 0.9 fm/c.

At high energy,
solution ~ Bjorken.

Applying extended formula to central Au+Au collisions

At lower energies:
ε^{max} << Bjorken value (at the same τ_F),
but increases with $\sqrt{s_{NN}}$ much faster than the Bjorken formula

Peak energy density averaged over the nucleus transverse area



ZWL, arXiv:1704.08418v2/ PRC(2018)

Comparison of extended Bjorken formula with AMPT results

Overall:

- AMPT with F.T. (filled circles)

~ our extension

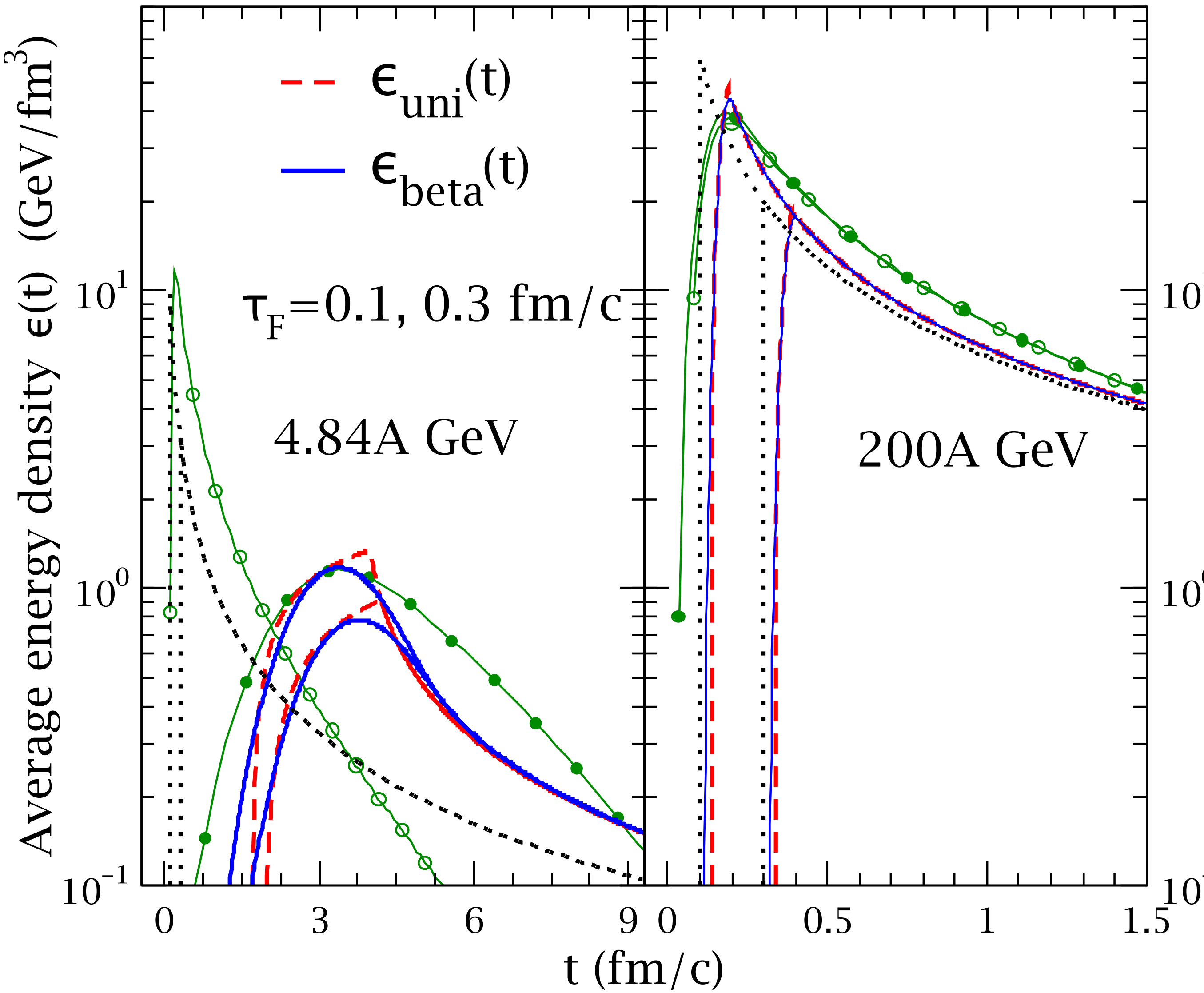
- AMPT w/o F.T. (open circles)

~ Bjorken formula,

- *Small effect*

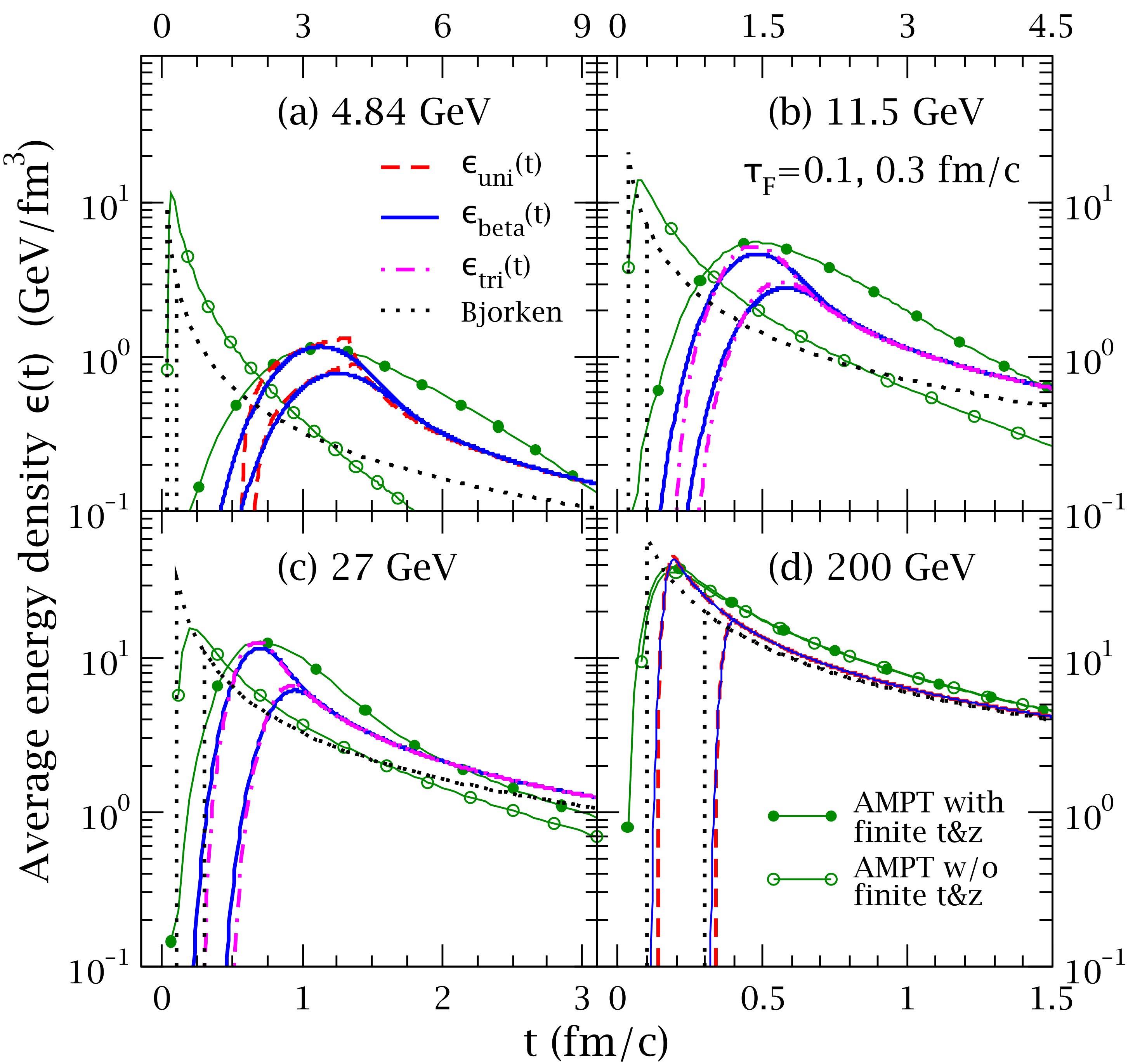
of finite thickness at 200 GeV.

F.T.=finite thickness



$\varepsilon_{uni}(t) \sim \varepsilon_{beta}(t)$, *since here we set t_1 & t_2 of the uniform profile so that it has the same mean & standard deviation as the beta profile.*

Comparison of extended Bjorken formula with AMPT results

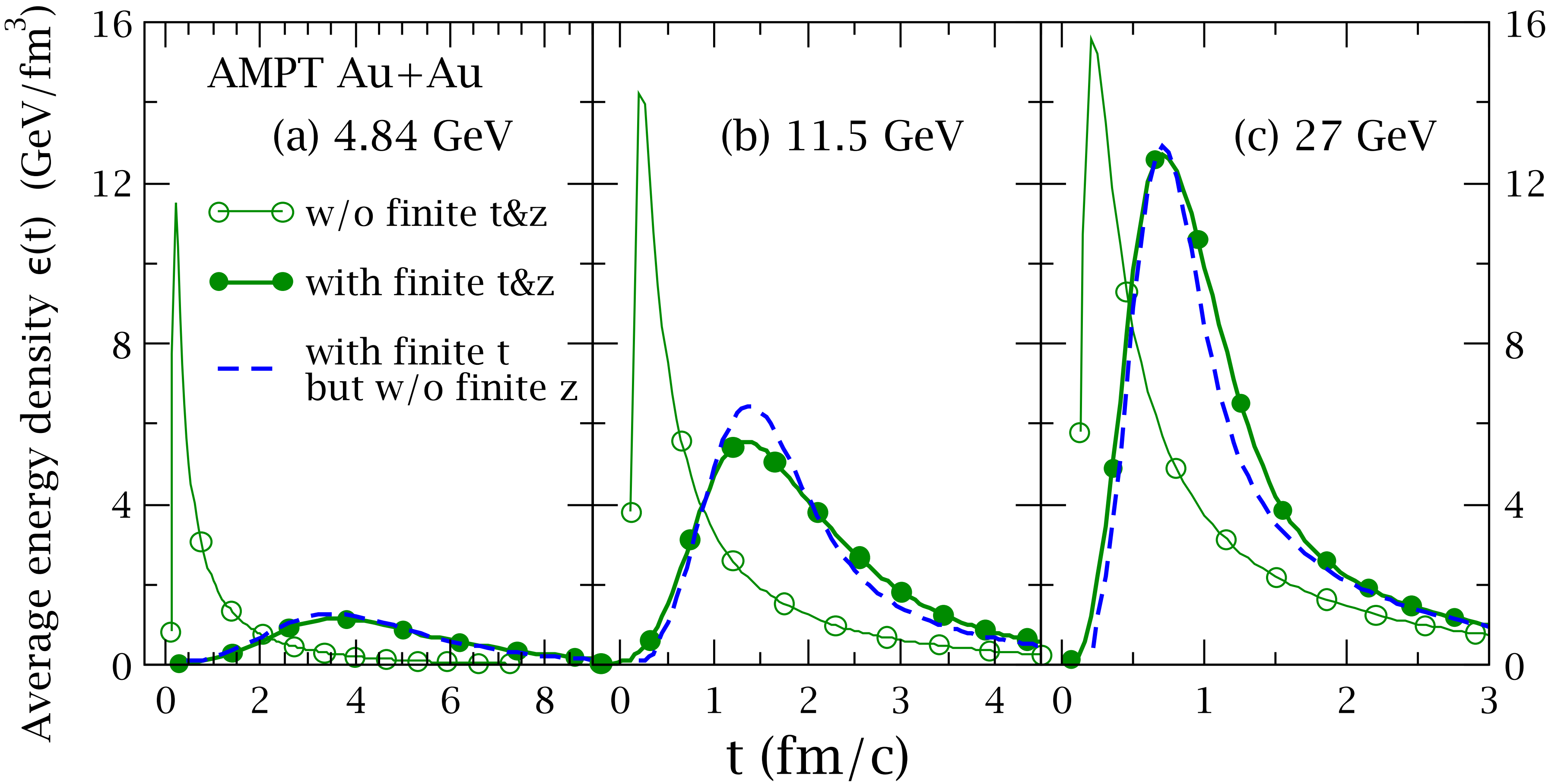


Note: AMPT has *variable τ_F, Woods-Saxon, secondary scatterings, transverse expansion, finite width in z.*

Here we set t_1 & t_2 of the uniform profile and triangular profile so that they each have the same mean & standard deviation as the beta profile (n=4).

Results from string melting AMPT

Our analytical results include finite width in **t** but not the finite width in **z**.



AMPT-SM results show:

- Effect of finite z-width is small, once finite t-width is included.
- Effect of finite t-width is very important at low energies
- Peak energy density ε^{max} increases with $\sqrt{s_{NN}}$ much faster than Bjorken.

Summary

Thank you!

- Effect of finite nuclear thickness is important at lower energies

- We have incorporated finite nuclear thickness into string melting AMPT, to lay a better foundation for further studies of dense matter effects when parton matter is expected to be formed.

- We have analytically extended the Bjorken ε formula: now valid at low energies (as well as high energies)

- AMPT results confirm key features of the extended formula.
  At low energies (*compared to the Bjorken formula*):
  - the maximum energy density ε^{max}
    is much lower,
    but increases with $\sqrt{s_{NN}}$ much faster,
    is much less sensitive to the formation time τ_F.
  - the initial energy density $\varepsilon(t)$ decreases much slower with time.