The nuclear pairing functional and charge radii

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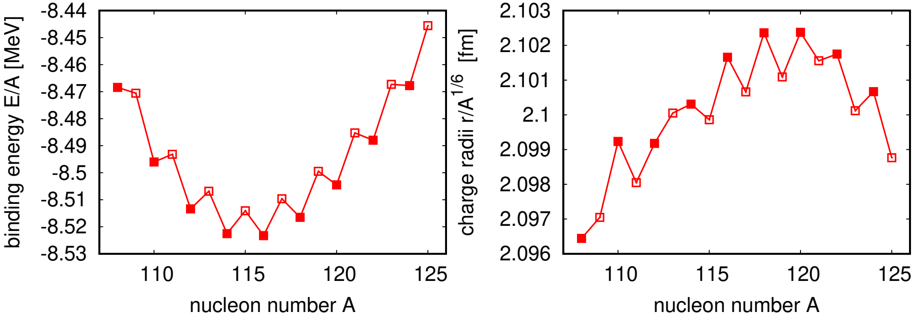
Outline

1) The physics case: Pairing in nuclei

Indicators for nuclear pairing: odd-even effects in nuclei

trend of experimental energies and charge r.m.s. radii along Sn isotopes



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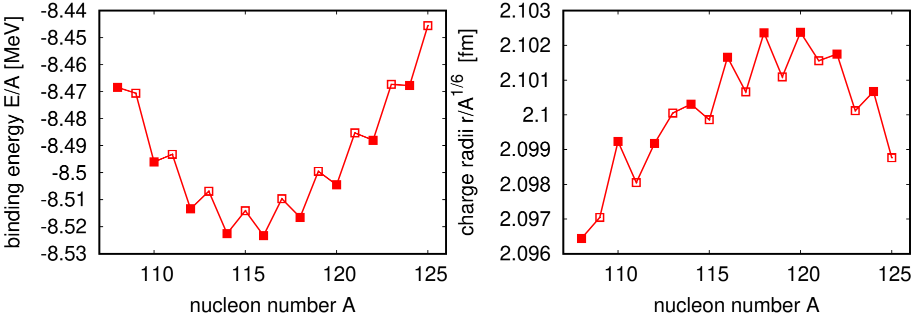

can be described by a nuclear pairing functional:

$$E_{\rm pair} = V_{\rm pair} \int d^3 r \chi^*(\mathbf{r})\chi(\mathbf{r}) \qquad , \qquad \chi(\mathbf{r}) = \sum_\alpha u_\alpha v_\alpha |\varphi_\alpha(\mathbf{r})|^2$$

χ → pair density

$u_\alpha v_\alpha$ → pairing amplitudes

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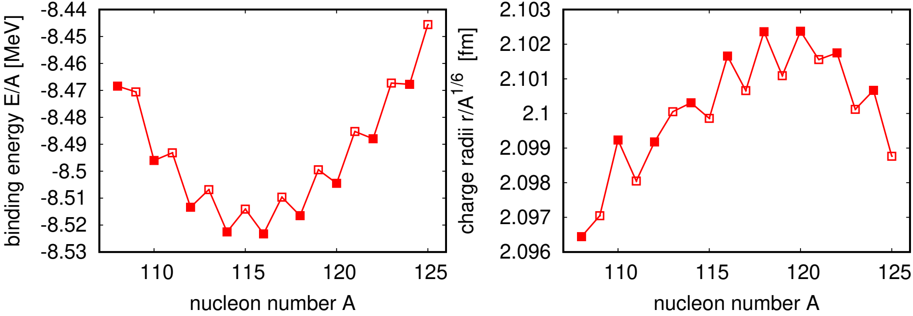

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$$E_{\text{pair}} = V_{\text{pair}} \int d^3r \chi^*(\mathbf{r})\chi(\mathbf{r})\, f\big(\rho(\mathbf{r})\big) \quad , \quad \chi(\mathbf{r}) = \sum_\alpha u_\alpha v_\alpha |\varphi_\alpha(\mathbf{r})|^2$$

pairing amplitudes

pair density

density dependence $\leftrightarrow$ trend with A

Two nuclear energy-density functionals

mean field part — pairing part

Skyrme: $E_{\mathrm{Sk,mf}}[\rho,\tau,\mathcal{J}] + \sum_q V_{q,\mathrm{pair}} \int d^3r\, \chi_q^*(\mathbf{r})\chi_q(\mathbf{r}) \left(1 + C_\chi \rho(\mathbf{r})\right)$ **Sk**

Fayans: $E_{\mathrm{Fy,mf}}[\rho,\tau,\mathcal{J}] + \sum_q V_{q,\mathrm{pair}} \int d^3r\, \chi_q^*(\mathbf{r})\chi_q(\mathbf{r}) \left(1 + C_\chi \rho(\mathbf{r}) + C_\chi^\nabla (\nabla\rho(\mathbf{r}))^2\right)$ **Fy**

$[\rho,\tau,\mathcal{J}]$ → density, kinetic dens., spin-orbit dens.

$C_\chi^\nabla (\nabla\rho(\mathbf{r}))^2$ → crucial term in **Fy !**

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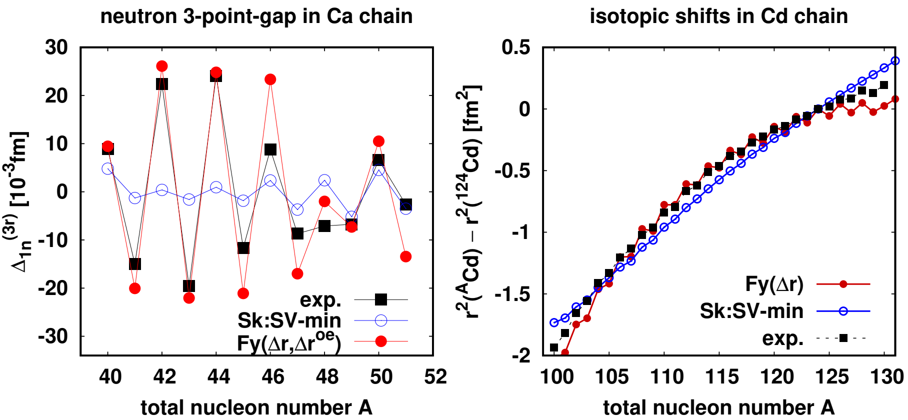
calibrated to ???? $\Longrightarrow$
what are the best **“pairing observables” ?**

calibrated by χ^2-fit to empirical data in selected semi-magic nuclei:
binding energies E_B,
r.m.s. radii, diffraction radii, surface thickness
selected spin-orbit splittings

Fayans-pairing and trend of r.m.s.charge radii



$\Longrightarrow$: Sk-pairing unable to reproduce trend of r.m.s. radii in Ca and Cd chains

Fy-pairing can be tuned to accomodate that $\longleftrightarrow$ gradient term in E_{pair}

Pairing(-gap) observables (from ground-state properties)

spectral gap (proton or neutron):

$\overline{\Delta} = \sum_\alpha u_\alpha v_\alpha \Delta_\alpha$ only theoretically

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focuses on pairing

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traditional fits: $\overline{\Delta}_n(\mathrm{theory}) \equiv \Delta^{(5)}_{1,n}(\mathrm{exp})$

2) Calibration & statistical analysis

Calibration by χ^2 fit, correlations etc

model parameters p

minimal $\chi^2 \longleftrightarrow$ **p**

$$\chi^2(\mathbf{p}) = \sum_{\mathcal{O}\in\{\text{pool}\}} \frac{\left(\mathcal{O}(\mathbf{p}) - \mathcal{O}^{(\text{exp})}\right)^2}{\Delta\mathcal{O}^2}$$

quality measure – to be minimized

$\mathcal{O}(\mathbf{p})$ = computed observable

$\mathcal{O}^{(\text{exp})}$ = experimental value for $\mathcal{O}$

$\Delta\mathcal{O}$ = adopted uncertainty for $\mathcal{O}$

chosen pool of data

bulk properties of finite nuclei:

energy E_B,

radii R_c, r_c, surface th. σ_c,

l·s splitting $\Delta\varepsilon_{ls}$,

odd-even staggering of en. Δ_{gap}

predicted observables & extrapolation uncertainties

statistical correlations (alignment) between observables (CoD)

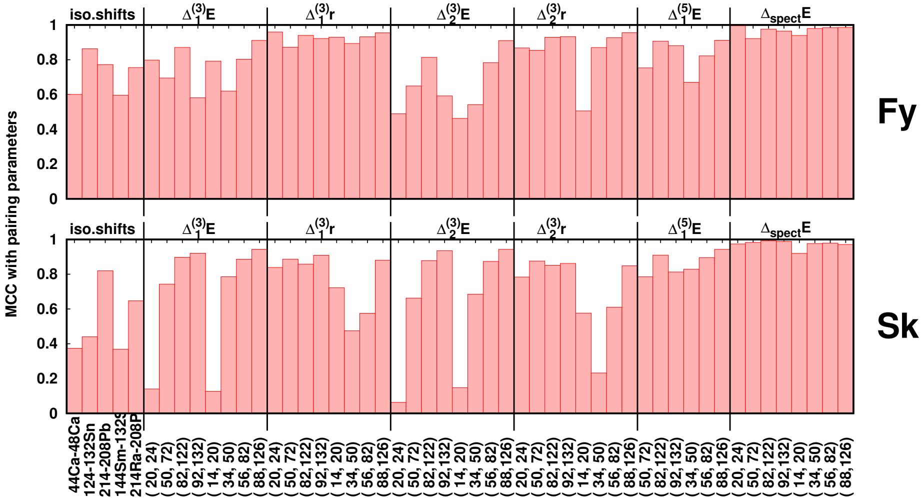
correlation between a group of observables with another observable (MCC)

3) Exploring the information content of pairing observables

Strategies for testing pairing observables

- **Sensitivity to pairing functional:**
  Look at multiple-correlation-coefficients (MCC) of observable with group of pairing parameters.
  $\leftrightarrow$ fit Fy&Sk to standard data without any pairing observables
- **Quality of mean-field description:**
  Check impact of collective ground-state correlations from lowest quadrupole mode.
  $\leftrightarrow$ take existing results for standard Skyrme forces as typical reference
- **Experimental precision:**
  No problem for simple ground-state observables.
  But may become critical for differences of observables.

MCC between pairing observables and group of pairing parameters

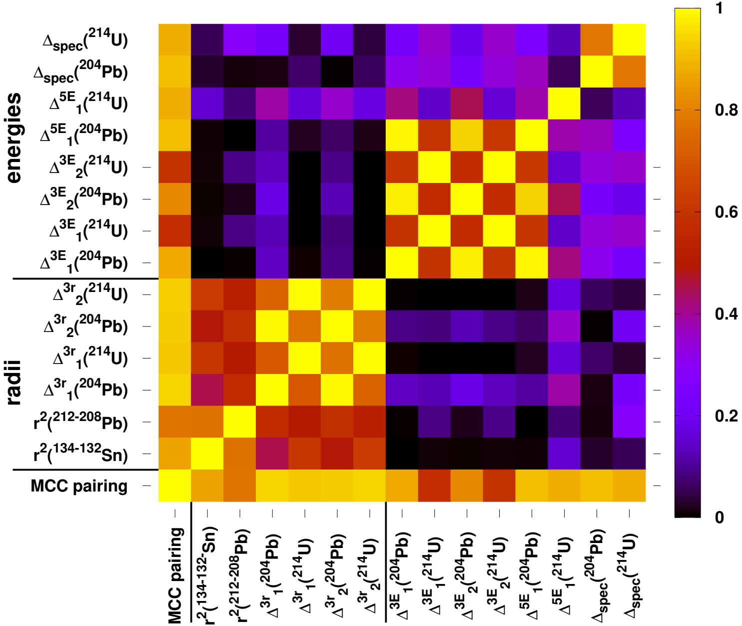


results depend somewhat on functional – nonetheless comparable general trends

all pairing observables are strongly correlated with pairing funct. – occasioanl exceptions

in the average holds: $\overline{\Delta}_{\rm spect}$ closest; $\Delta_1^{(5)}$, $\Delta_1^{(3r)}$, $\Delta_2^{(3r)}$ next; $\Delta_1^{(3)}$, $\Delta_2^{(3)}$, &iso.shifts least.

Matrix of CoD/MCC - example Fy (fit without pairing obs.)



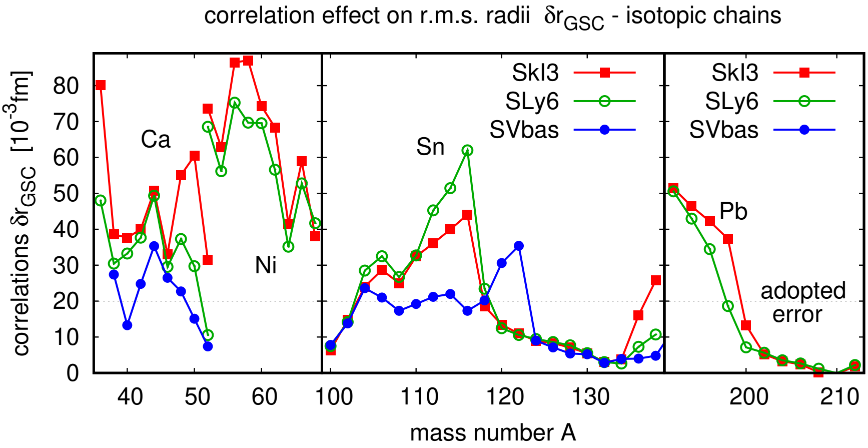
energy info

independent from

radius info

both carry pairing info

Effect of ground-state correlations (GSC) on r.m.s. charge radii



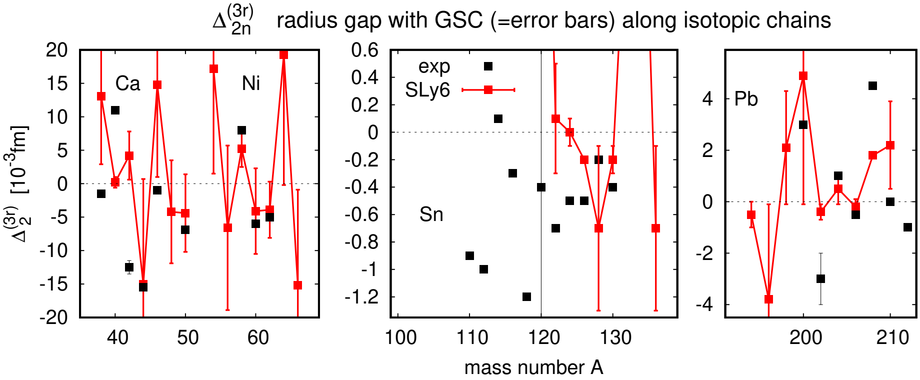
trends and size of δr_{GSC} similar for different parametrizations

lowest δr_{GSC} for semi-magic nuclei near doubly-magic ones

GSC smaller for heavier nuclei, rather large for small nuclei

adopted error for charge r.m.s. radii = max(20 mfm, δr_{GSC})

Even-even gaps of r.m.s. charge radii



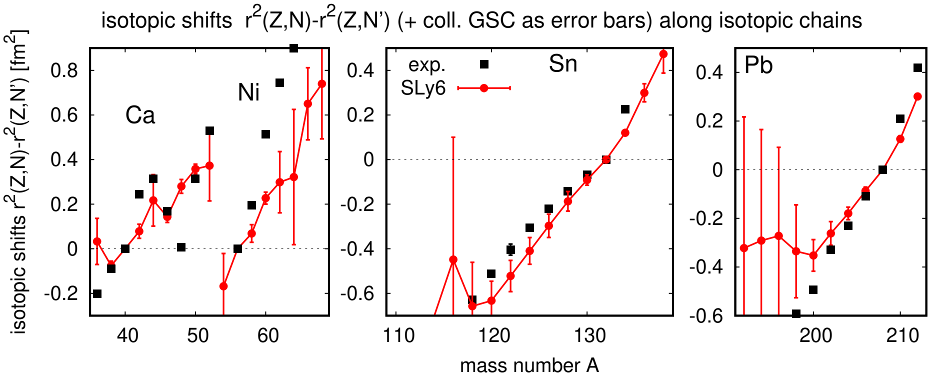
subtractions in $\Delta_{2n}^{(3r}$ produce often very small GSC, smaller than for radii

subtractions in $\Delta_{2n}^{(3r}$ produce often very small values for $\Delta_{2n}^{(3r}$

typical experimental errors 10mfm systematic (cancel in $\Delta_{2n}^{(3r}$), 1 mfm statistical

$\Longrightarrow$ too uncertain for Sn region, useful info for Ca/Ni region and some Pb

Correlation effects on isotopic shifts



GSC again reduced by difference $r^2(Z,N) - r^2(Z,N')$

but larger trends and fluctuations remain, well above typical error 10 mfm

$\Longrightarrow$ promising data point for all elements

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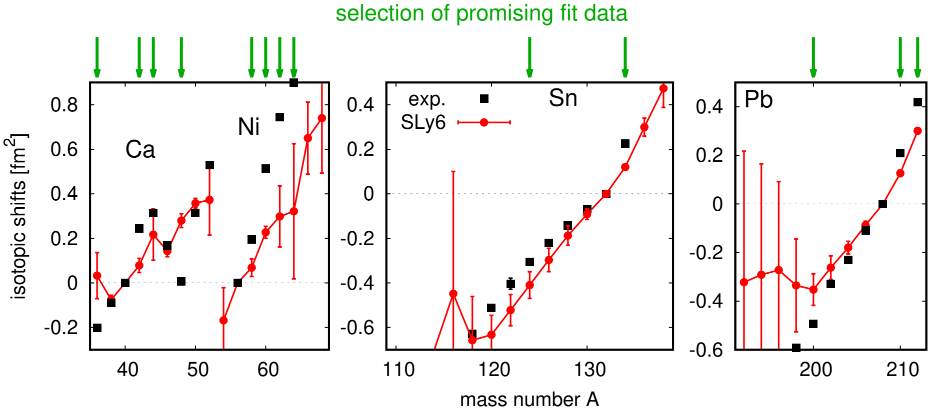

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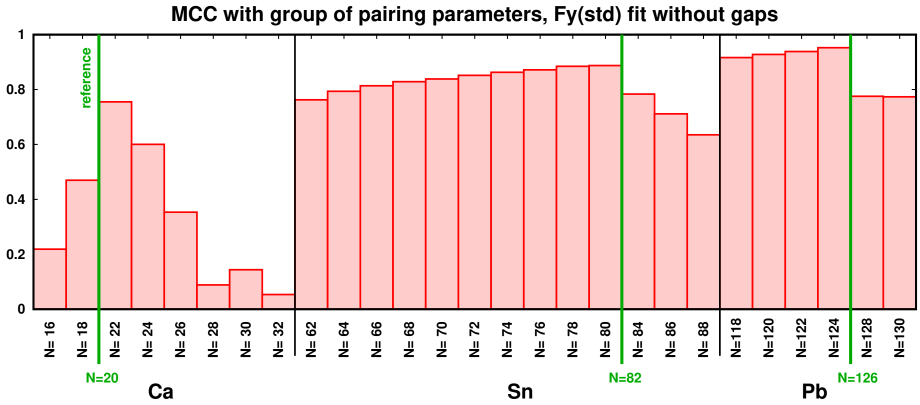
many useful data in light nuclei, a couple of crucial points in heavier elements

MCC of isotopic shifts with group of pairing parameters – Fy



correlations with pairing stronger for heavier nuclei (fewer points, more impact)

light nuclei provide many test cases, sensitivity mixed with shell properties

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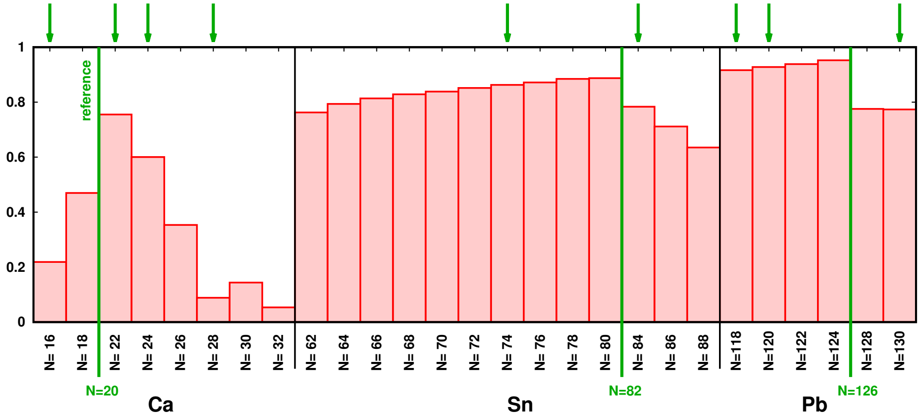

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light nuclei provide many test cases, sensitivity mixed with shell properties
selection has some points with pairing bias, others mixed mean-field info

4) Conclusions

combination of statistical analysis (impact obs.) & physics (validity mean field)

$\Longrightarrow$

radius information ($\Delta^{3r}_{2n/2p}$, iso-shifts) cannot replace $\Delta^{(3)}_{1/2}$ of energies,

but provides invaluable additional benchmarks:

many data points in small A nuclei ($Z, N = 20, 28$),

selected iso-shifts in heavier chains

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possible observables for pairing properties:

$\Delta^{(3)}_{1/2}E$, $\Delta^{(3)}_{1/2}r$, $\Delta^{(5)}_{1/2}E$, isotopic/isotonic shifts $r^2(A) - r^2(A')$

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collective ground-state correlations:
small correlations for semi-magic nuclei near doubly-magic nuclei $\Rightarrow$ fit nuclei
$\Delta^{(3r)}_{1/2}$: small correlation effects (cancel by subtractions),
sigificant and improtant data for small A,
but for large A the values are too small ($<$ exp. error),
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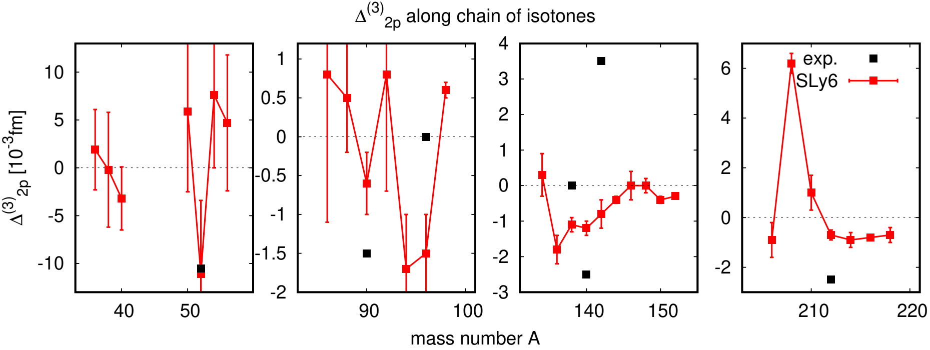
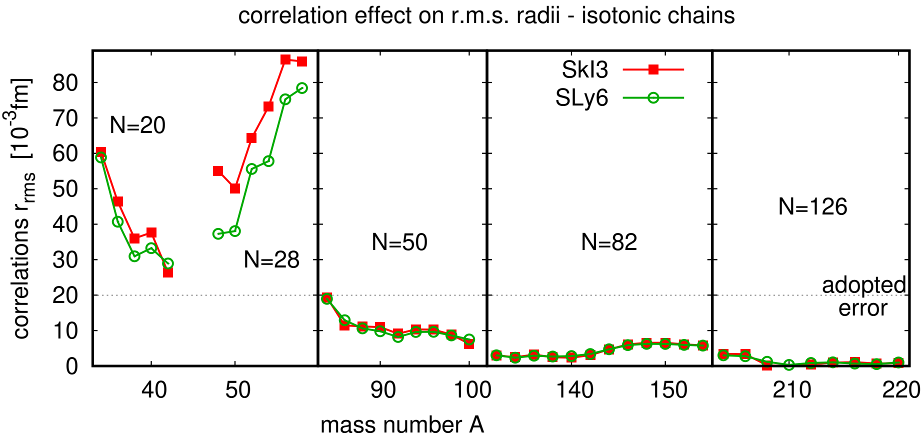
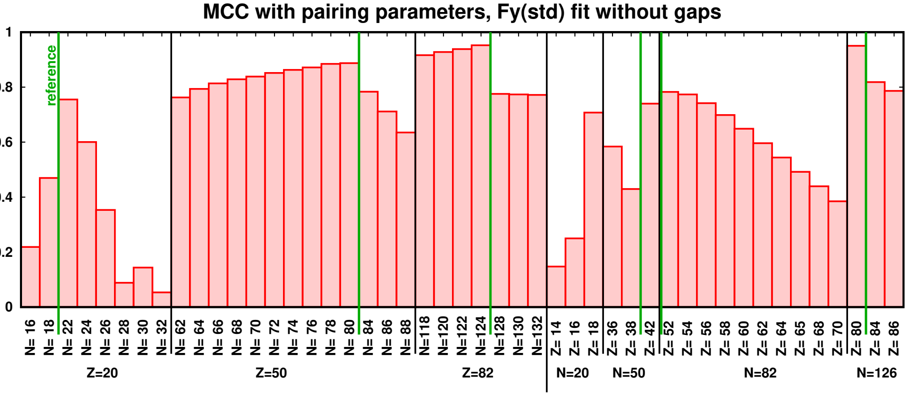


Effect of ground-state correlations on r.m.s. charge radii

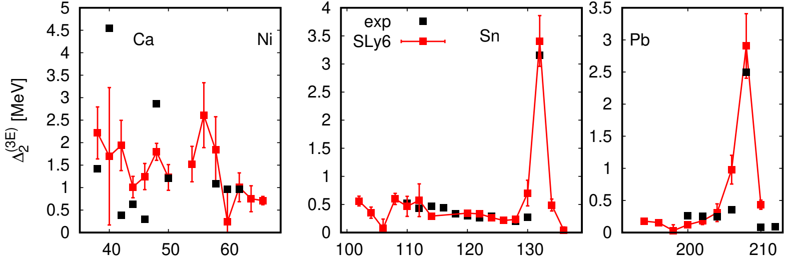


MCC of isotopic shifts with group of pairing parameters – Fy



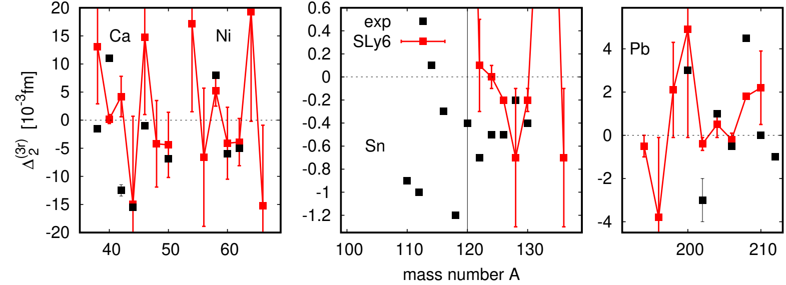
Even-even gaps - energies and radii in comparison

$\Delta_{2n}^{(3E)}$ energy gap (GSC as error bars) along isotopic chains



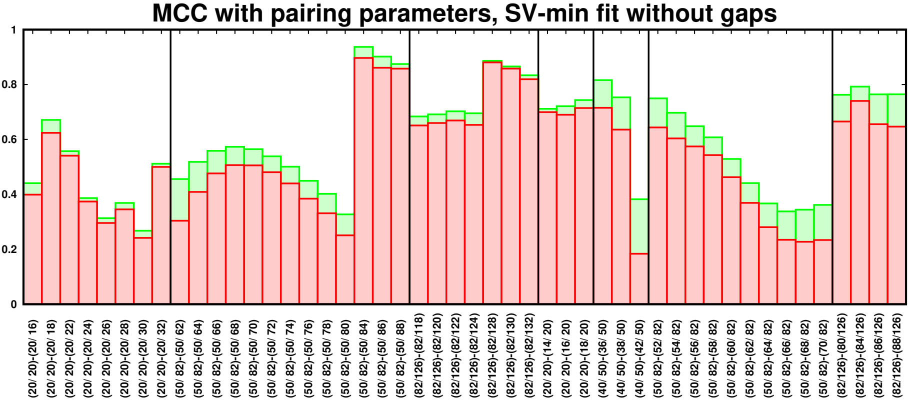
correlation effect $\Delta_{2n}^{(3E)}$ $\approx$ 0.05-0.1MeV
$\leftrightarrow$
value $\approx$ 0.3-1MeV
$\Rightarrow$ robust

$\Delta_{2n}^{(3r)}$ radius gap with GSC (=error bars) along isotopic chains



correlation effect $\Delta_{2n}^{(3r)} \approx$ value
&
exp. error $\approx$ value
$\Rightarrow$ limited use

MCC of isotopic shifts with group of pairing parameters – SV



All on r.m.s. radii