Quantifying Errors from Chiral Effective Field Theory

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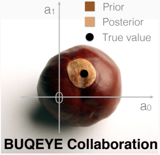
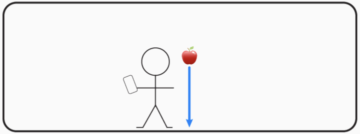


Based on:
- S. Wesolowski, R. J. Furnstahl, J. A. Melendez, D. Phillips, arXiv:1808.08211; &
- J. A. Melendez, S. Wesolowski, and R. J. Furnstahl Phys. Rev. C **96**, 024003, Editors' Suggestion

Physical Motivation

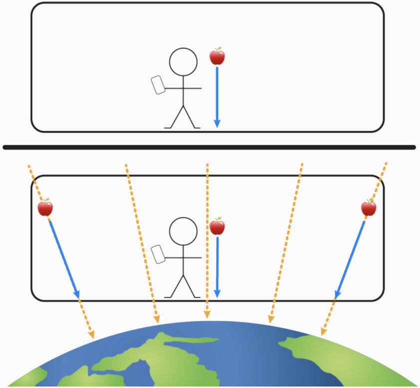
Scales in Physics



Grav. force (short distances):

$$F = -mg$$

Scales in Physics



Grav. force (short distances):

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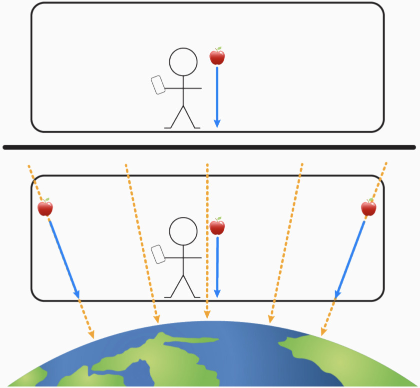
Grav. force (large distances):

$$F = -\frac{GMm}{r^2}$$

The laws look quite different!

Scales in Physics



Grav. force (short distances):

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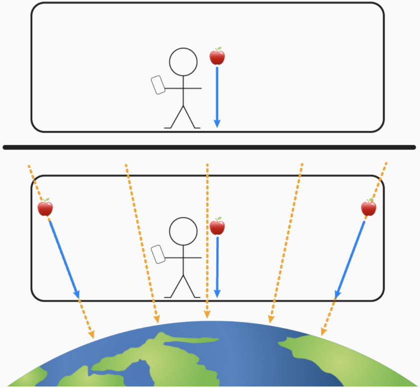
Connected via series expansion about radius of Earth R:

$$F \approx -mg + 2mg\left(\frac{r-R}{R}\right) - 3mg\left(\frac{r-R}{R}\right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R}\right)^3\right]$$

Scales in Physics



Grav. force (short distances):

$$F = -mg$$

Grav. force (large distances):

$$F = -\frac{GMm}{r^2}$$

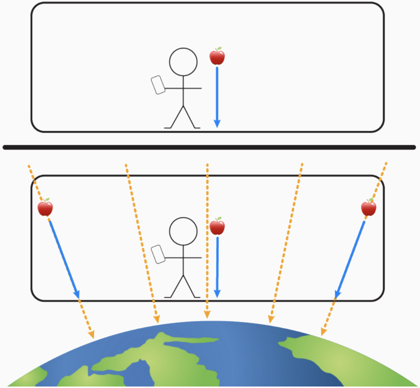
???

The laws look quite different!

Can fit unknown parameters to data $\Rightarrow$ inverse problem!

$$F \approx a_0 + a_1 \left(\frac{r-R}{R}\right) + a_2 \left(\frac{r-R}{R}\right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R}\right)^3\right]$$

Scales in Physics



Grav. force (short distances):

$$F = -mg$$

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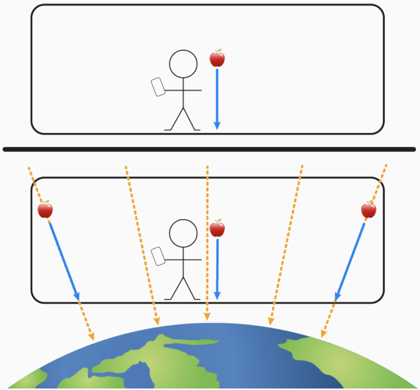
$$F = -\frac{GMm}{r^2} \quad ???$$

The laws look quite different!

Use prior info from physics:

$$F \approx mg \left\{ a_0' + a_1' \left(\frac{r-R}{R} \right) + a_2' \left(\frac{r-R}{R} \right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R} \right)^3 \right] \right\}$$

Scales in Physics



Grav. force (short distances):

$$F = -mg$$

Grav. force (large distances):

$$F = ??? \; -\frac{GMm}{r^2}$$

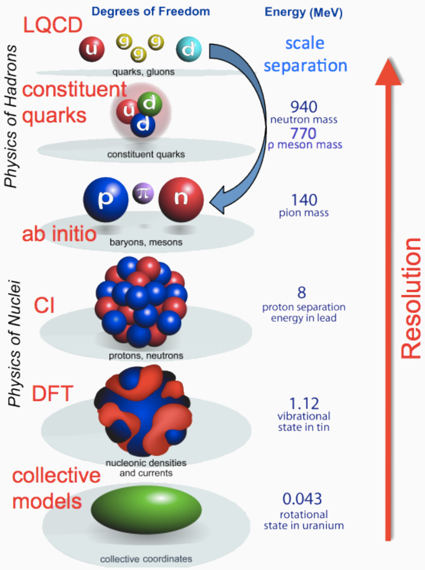
The laws look quite different!

Propagate full uncertainty

$$F \approx mg\left\{a_0' + a_1'\left(\frac{r-R}{R}\right) + a_2'\left(\frac{r-R}{R}\right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R}\right)^3\right]\right\}$$

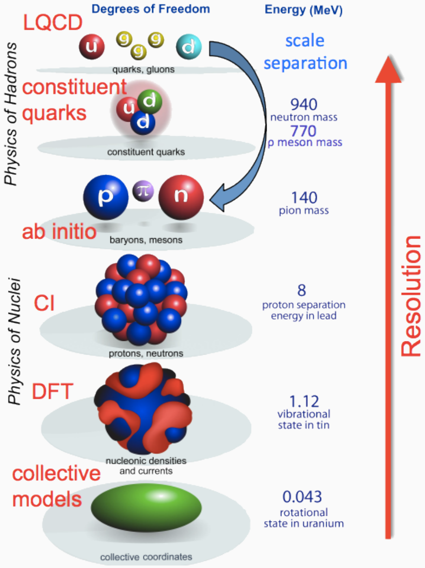
Predictions in Low-Energy Nuclear Physics

- There is interesting physics at all scales



Predictions in Low-Energy Nuclear Physics

- There is interesting physics at all scales
- Nuclear physics spans lengths from 10^{-15}–10^{9} m



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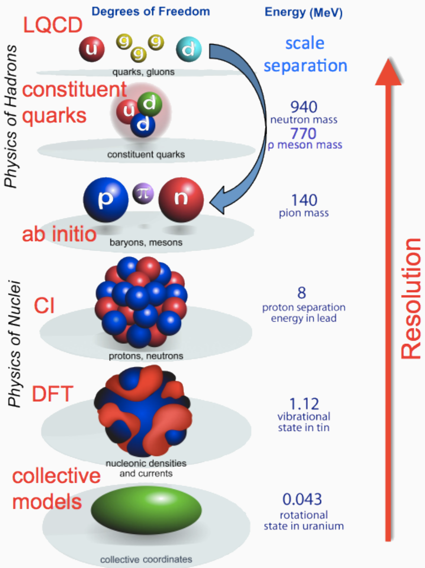
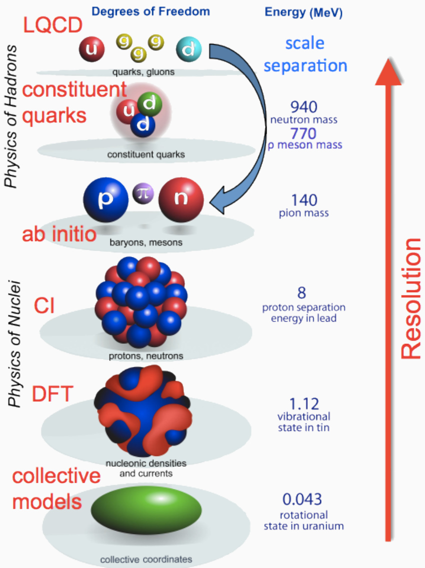
- Fine details at one level of analysis do not affect the physics at a coarser level of analysis

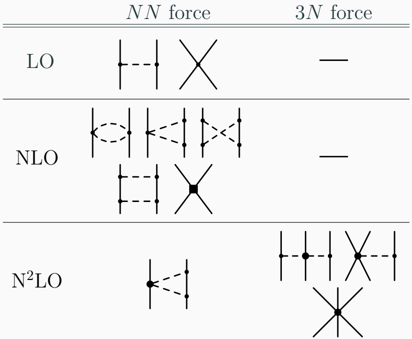


Predictions in Low-Energy Nuclear Physics

- There is interesting physics at all scales
- Nuclear physics spans lengths from 10^{-15}–10^{9} m
- Fine details at one level of analysis do not affect the physics at a coarser level of analysis
- Start simple → add corrections to reach desired precision.

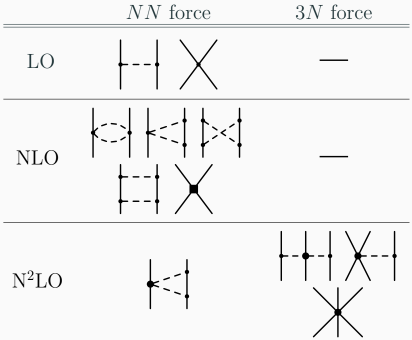


Chiral EFT



- An expansion in the nuclear force
- Ordered by increasing factors of small parameter Q
- Truncation $\to$ main source of uncertainty
- Force convergence $\neq$ prediction convergence
- The debate on the “best” expansion is ongoing

Chiral EFT



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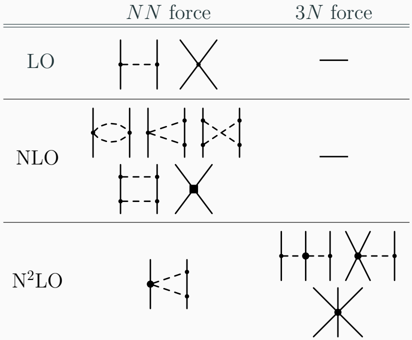
We want to

- Fit unknown parameters $\vec{a}$, or low-energy constants, with discrepancy δy_{th}

Chiral EFT



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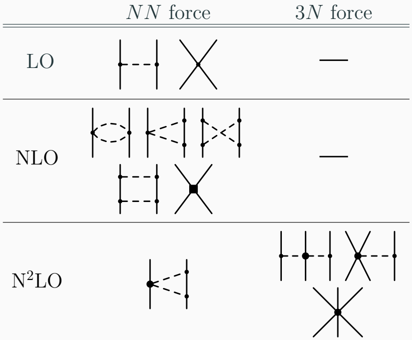
- Quantify uncertainty in predictions (aka observables) y_{th}

Chiral EFT



- An expansion in the nuclear force
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We want to

- Fit unknown parameters $\vec{a}$, or low-energy constants, with discrepancy δy_{th}
- Quantify uncertainty in predictions (aka observables) y_{th}
- Test existing EFTs, uncover physics

$$y_{\mathrm{exp}}(x) = y_{\mathrm{th}}(x, \vec{a}) + \delta y_{\mathrm{th}}(x) + \delta y_{\mathrm{exp}}$$

To theorists, magic

$$y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}}$$

Parameters

Discrepancy

χ^2 fit

$$y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) \qquad + \delta y_{\text{exp}}$$

rigorous fit

$$y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}}$$

$$\underbrace{}_{}\overbrace{y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x)}^{\text{Full Prediction}}$$

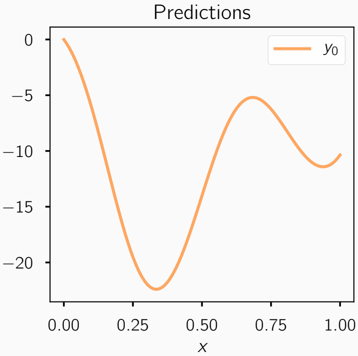
$$y_{\text{exp}}(x) = \overbrace{y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x)}^{\text{Full Prediction}} + \delta y_{\text{exp}}$$

$$y_{\mathrm{exp}}(x) = y_{\mathrm{th}}(x, \vec{a}) + \underbrace{\delta y_{\mathrm{th}}(x)}_{\substack{\text{Can we build this?} \\ \text{Can we use it?}}} + \delta y_{\mathrm{exp}}$$

Toy Predictions

- Theoretical predictions could look like the following

$$\{y_0\}$$

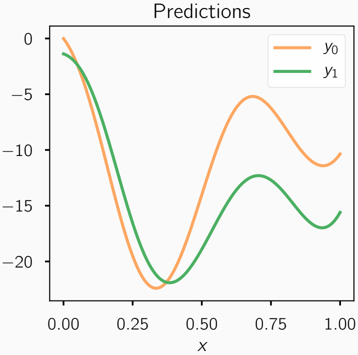


$$y_0 \to \text{LO}$$

Toy Predictions

- Theoretical predictions could look like the following

$$\{y_0, y_1\}$$



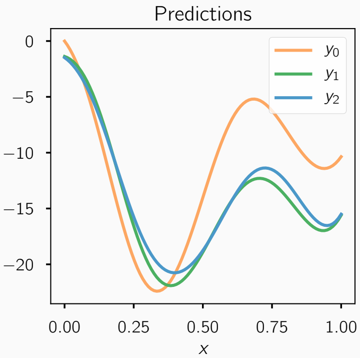
$y_0 \rightarrow$ LO

$y_1 \rightarrow$ NLO

Toy Predictions

- Theoretical predictions could look like the following

$$\{y_0, y_1, y_2\}$$

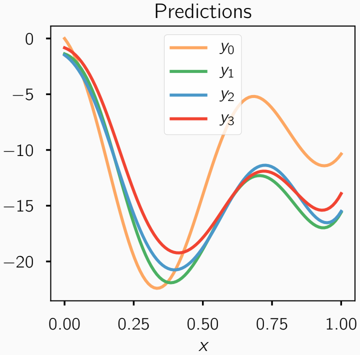


$$y_0 \rightarrow \text{LO}$$
$$y_1 \rightarrow \text{NLO}$$
$$y_2 \rightarrow \text{N}^2\text{LO}$$

Toy Predictions

- Theoretical predictions could look like the following

$$\{y_0, y_1, y_2, y_3\}$$

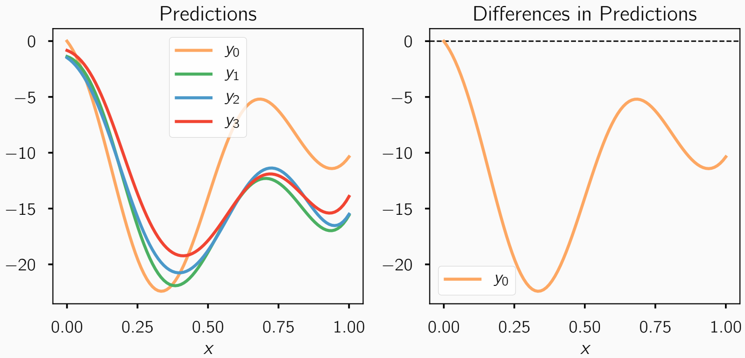


$$y_0 \to \text{LO}$$
$$y_1 \to \text{NLO}$$
$$y_2 \to \text{N}^2\text{LO}$$
$$\vdots$$
$$y_k \to \text{N}^k\text{LO}$$

Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.

$$y_0 = y_0$$



Toy Predictions

- Theoretical predictions could look like the following
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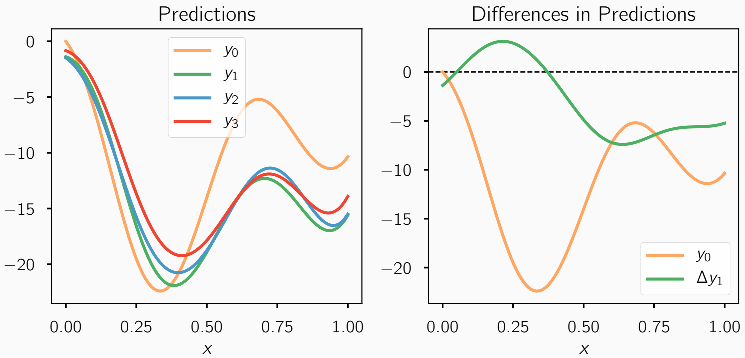
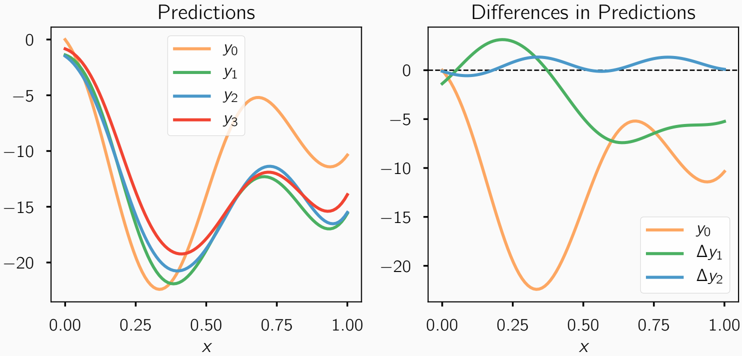
$$y_1 = y_0 + \Delta y_1$$



Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.

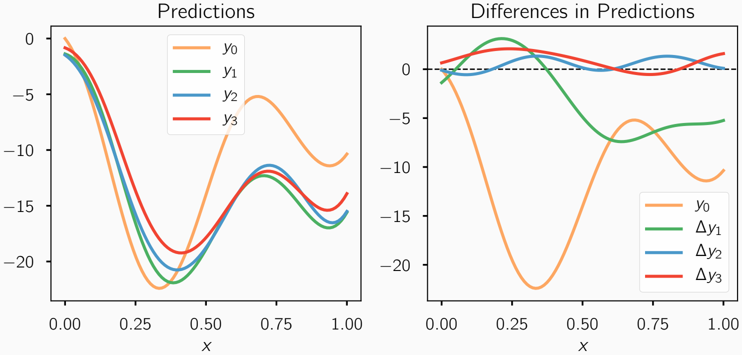
$$y_2 = y_0 + \Delta y_1 + \Delta y_2$$



Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.
- $\Delta y_n = y_{\text{ref}} c_n Q^n$

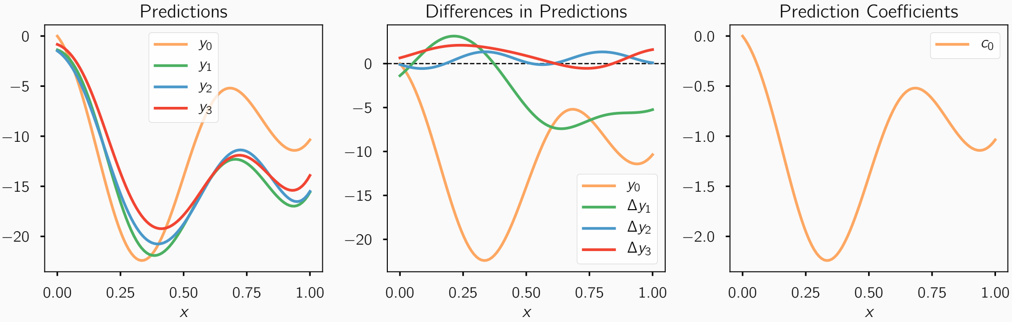
$$y_3 = y_0 + \Delta y_1 + \Delta y_2 + \Delta y_3$$



Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.
- $\Delta y_n = y_{\text{ref}} c_n Q^n$

$$y_0 = y_{\text{ref}} \left[c_0 Q^0 \right]$$



Toy Predictions

- Theoretical predictions could look like the following
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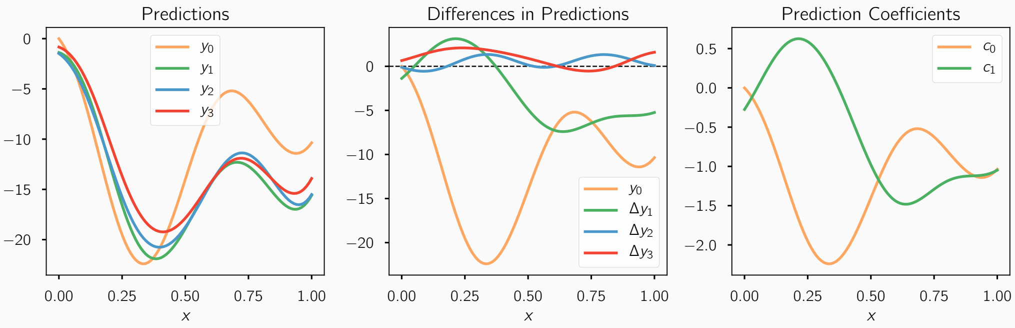
- $\Delta y_n = y_{\text{ref}} c_n Q^n$

$$y_1 = y_{\text{ref}} \left[c_0 Q^0 + c_1 Q^1 \right]$$



Toy Predictions

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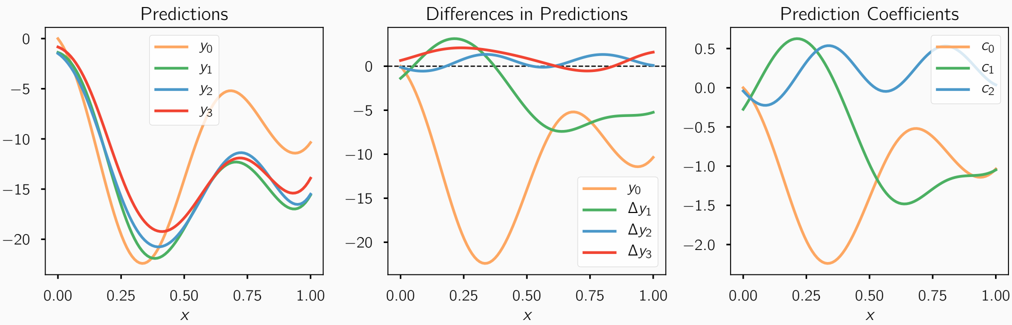
$$y_2 = y_{\text{ref}} \left[c_0 Q^0 + c_1 Q^1 + c_2 Q^2 \right]$$



Toy Predictions

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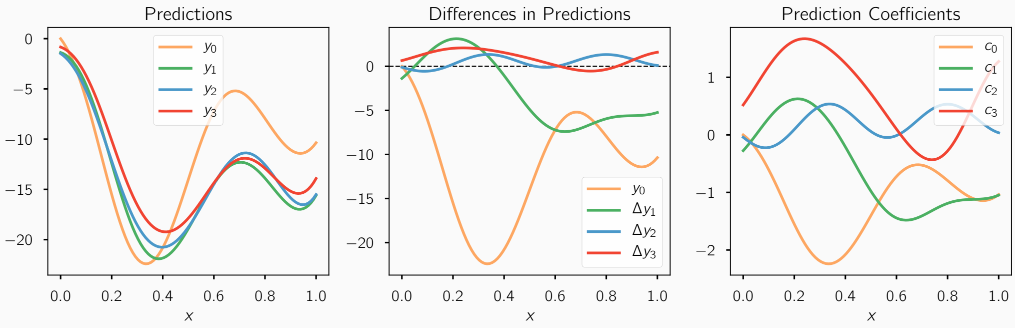
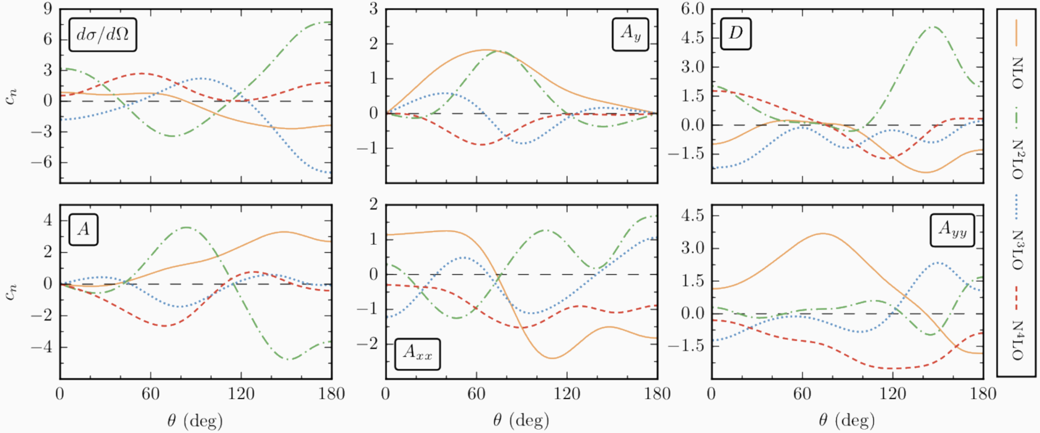
$$y_3 = y_{\mathrm{ref}} \left[c_0 Q^0 + c_1 Q^1 + c_2 Q^2 + c_3 Q^3 \right]$$



Real Life

Coefficients from NN scattering look like our toy model!

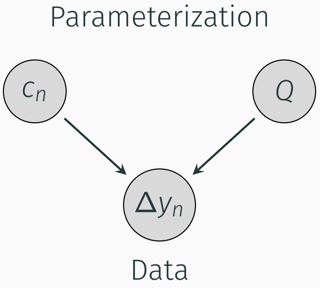


Statistical Model

- Decompose prediction

$$y_k = y_0 + \sum_{n=1}^{k} \Delta y_n$$

The Hierarchical Model

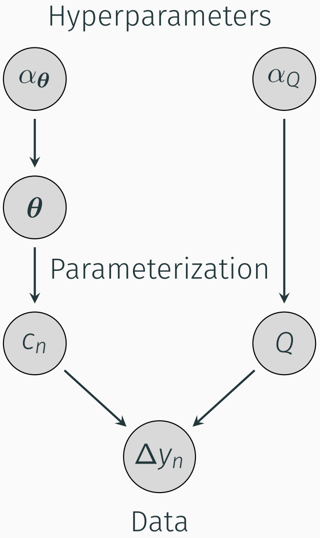


- Decompose prediction

$$y_k = y_0 + \sum_{n=1}^{k} \Delta y_n$$

$$= y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

The Hierarchical Model



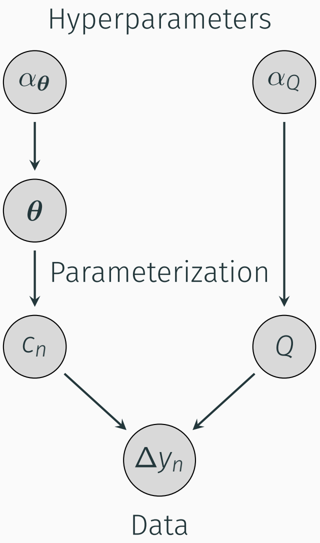
- Decompose prediction

$$
\begin{aligned}
y_k &= y_0 + \sum_{n=1}^{k} \Delta y_n \\
&= y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n
\end{aligned}
$$

- Put priors on c_n (and Q)

$$\text{pr}(c_n \mid \boldsymbol{\theta}) \overset{\text{iid}}{\sim} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$

The Hierarchical Model



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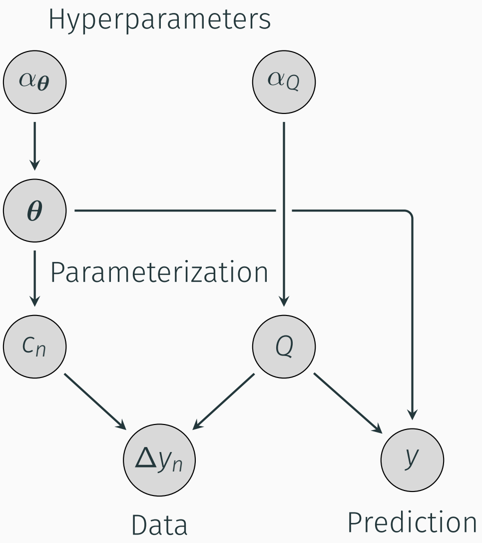
- Learn $\boldsymbol{\theta}$ and Q

The Hierarchical Model



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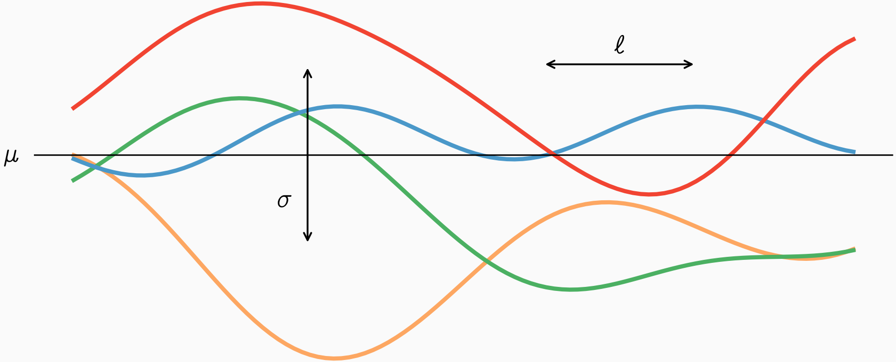
- Put priors on c_n (and Q)

$$\text{pr}(c_n \mid \boldsymbol{\theta}) \overset{\text{iid}}{\sim} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$

- Learn $\boldsymbol{\theta}$ and Q
- Predict $\text{pr}(y \mid \mathcal{D})$

Gaussian Process Priors on Observable Coefficients

$$c_n \mid \boldsymbol{\theta} \overset{\text{iid}}{\sim} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$



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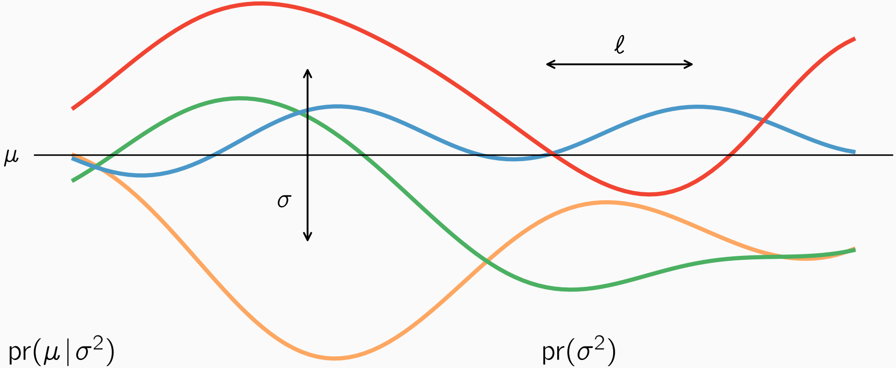
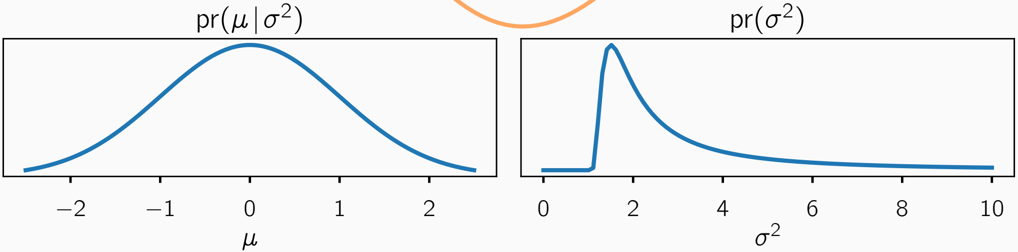


Conjugate priors:

$$\mu \mid \sigma^2 \sim \mathcal{N}(m, \sigma^2 V)$$
$$\sigma^2 \sim \mathrm{IG}(a, b)$$

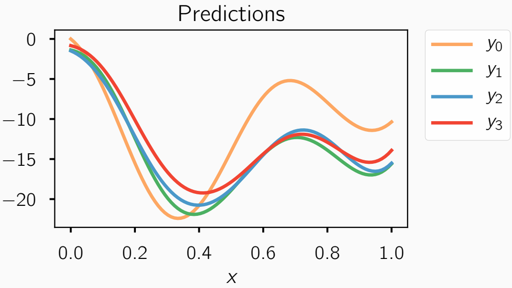


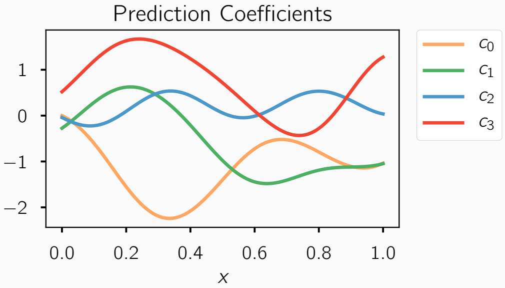
Model Building

Main equation

$$y_k = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

$$c_n \equiv \frac{y_n - y_{n-1}}{y_{\text{ref}} Q^n}$$





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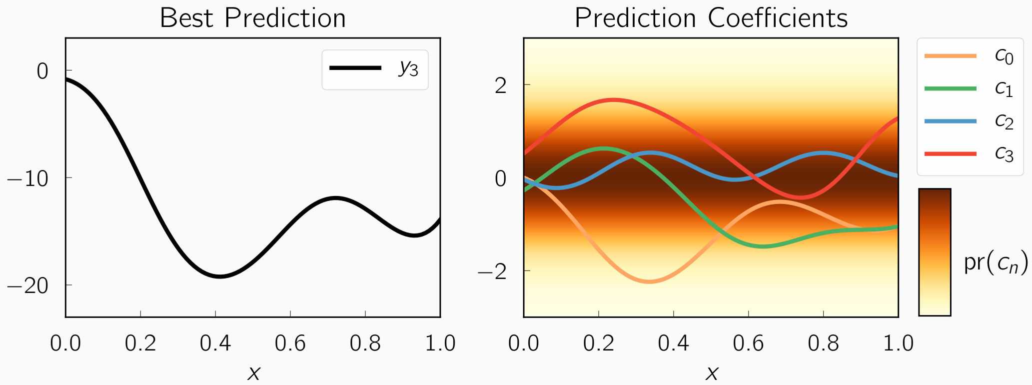
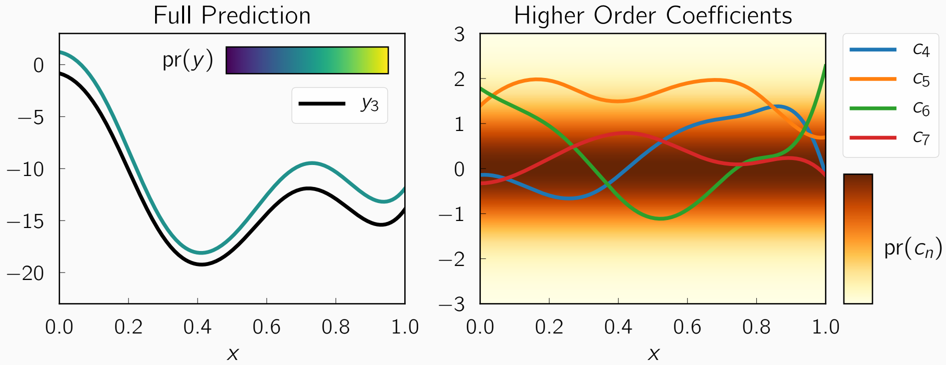


Model Building

Main equation

$$y = y_{\text{ref}} \sum_{n=0}^{\infty} c_n Q^n$$

$$c_n \equiv \frac{y_n - y_{n-1}}{y_{\text{ref}} Q^n}$$



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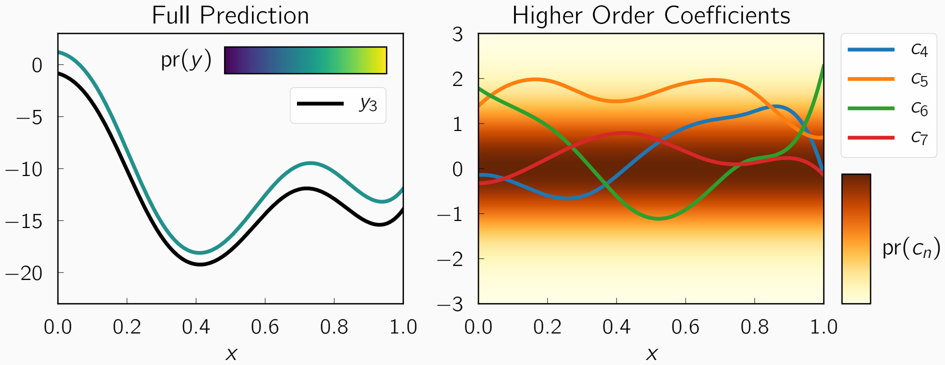

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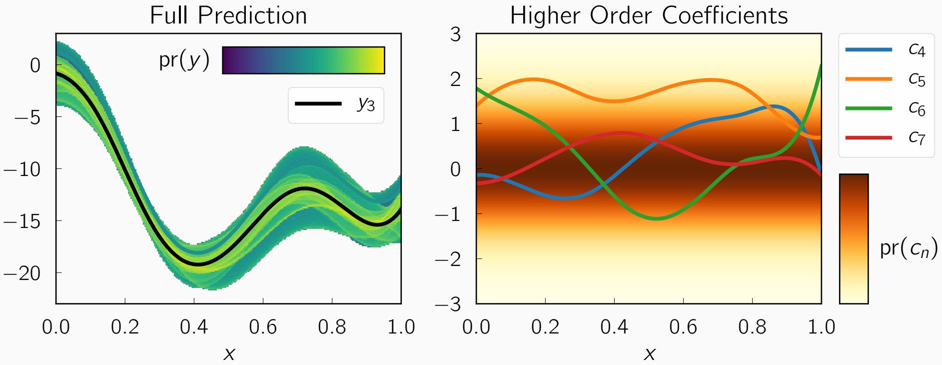

Discrepancy Distribution

Remember the goal:

$$y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}}$$

Discrepancy Distribution

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$$y_{\mathrm{exp}}(x) = y_{\mathrm{th}}(x, \vec{a}) + \delta y_{\mathrm{th}}(x) + \delta y_{\mathrm{exp}}$$

Our convergence assumptions

$$\mathrm{pr}(c_n \mid \boldsymbol{\theta}) \overset{\text{iid}}{=} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$

$$\delta y_{\mathrm{th}}(x) = y_{\mathrm{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

Discrepancy Distribution

Remember the goal:

$$y_{\mathrm{exp}}(x) = y_{\mathrm{th}}(x, \vec{a}) + \delta y_{\mathrm{th}}(x) + \delta y_{\mathrm{exp}}$$

Our convergence assumptions

$$\mathrm{pr}(c_n \mid \boldsymbol{\theta}) \overset{\text{iid}}{=} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$

$$\delta y_{\mathrm{th}}(x) = y_{\mathrm{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

Gaussian sum rules

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Discrepancy Distribution

Remember the goal:

$$y_{\mathrm{exp}}(x) = y_{\mathrm{th}}(x, \vec{a}) + \delta y_{\mathrm{th}}(x) + \delta y_{\mathrm{exp}}$$

Our convergence assumptions

$$\mathrm{pr}(c_n \mid \boldsymbol{\theta}) \overset{\mathrm{iid}}{=} \mathcal{GP}(\mu, \sigma^2 R_\ell)$$

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$$\mathrm{pr}(\delta y_{\mathrm{th}} \mid \boldsymbol{\theta}) = \mathcal{GP}\left(\mu_{\mathrm{th}}, \Sigma_{\mathrm{th}}\right) = \mathcal{GP}\left(\frac{\mu Q^{k+1}}{1-Q}, y_{\mathrm{ref}}^2 \frac{\sigma^2 Q^{2(k+1)}}{1-Q^2} R_\ell\right)$$

Implications for EFT Fitters

Standard χ^2

$$\sum_i \frac{[y_{\mathrm{exp},i} - y_{\mathrm{th},i}(\vec{a})]^2}{\sigma^2_{\mathrm{exp}}} = \sum_i \frac{r(x_i, \vec{a})^2}{\sigma^2_{\mathrm{exp}}}$$

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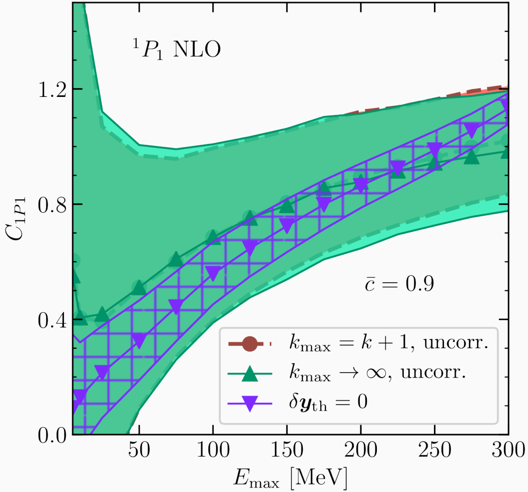
- Different correlation assumptions $\rightarrow$ different results!

What You Get for Free: Max Energy Insensitivity

- *y* axis: posterior median $\pm 1\sigma$
- *x* axis: max energy of data in fit



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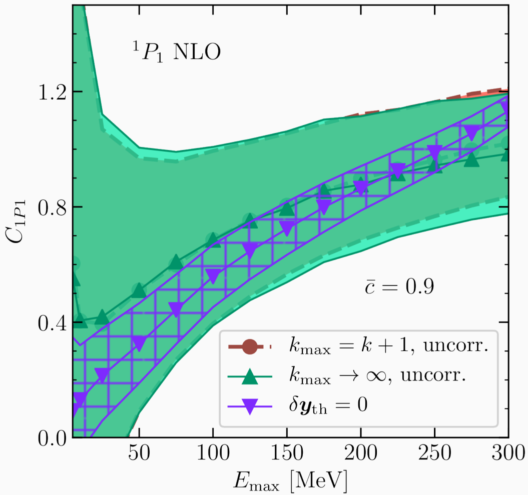
- This weights high energy data less!
- Stabilizes LEC fit as a function of E



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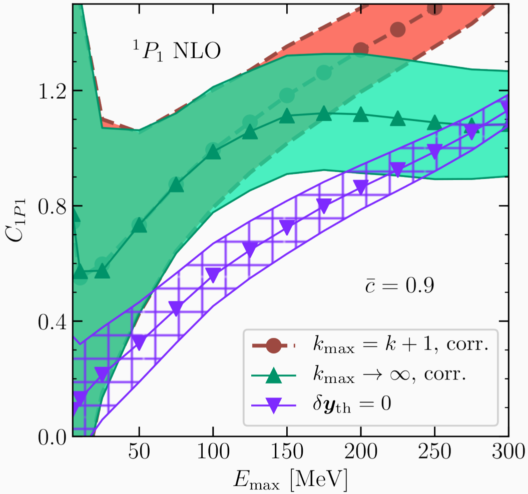
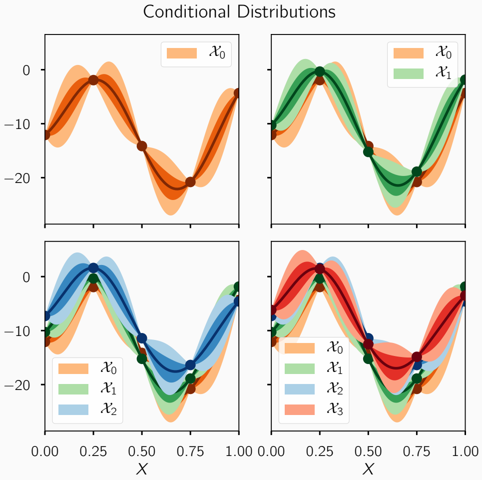


Quantifying Truncation Uncertainty



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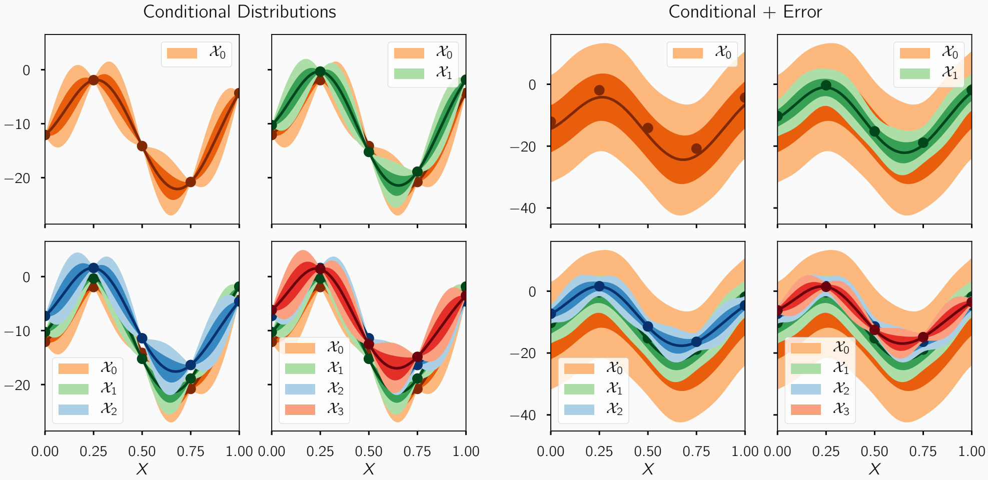

What You Get for (Almost) Free: Evidence, Length Scale, & Breakdown Scale

This model permits mostly analytic calculation of evidence

$$\mathrm{pr}(\mathcal{D} \mid \ell, Q) = \frac{\Gamma(a^\star)}{\Gamma(a)} \frac{b^a}{(b^\star)^{a^\star}} \sqrt{\frac{|V^\star|}{|V|}} \frac{|2\pi R_\ell|^{-(k+1)/2}}{|Q|^{k(k+1)/2}}$$

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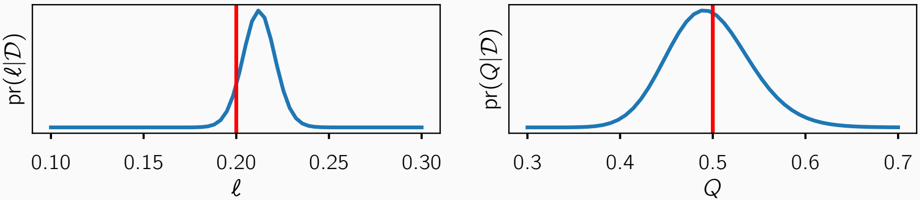

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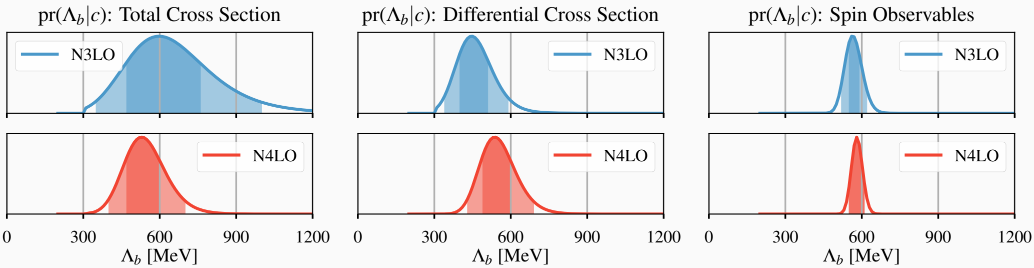
Here, $Q \propto \dfrac{1}{\Lambda_b}$



Model Checking

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Takeaway Points

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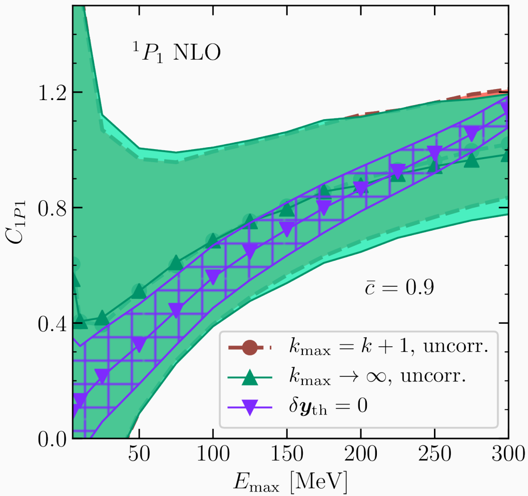

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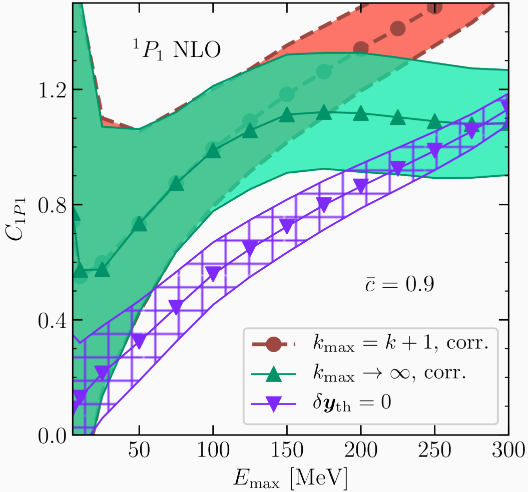

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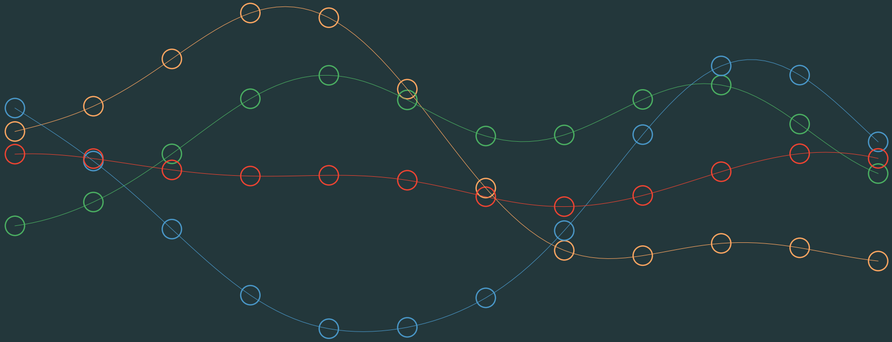
Suggestions welcome!

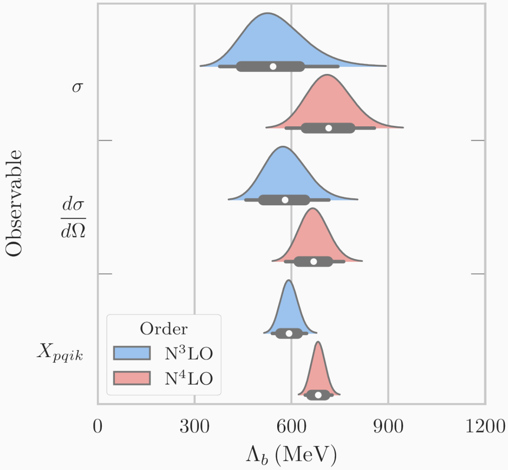
Thank you!

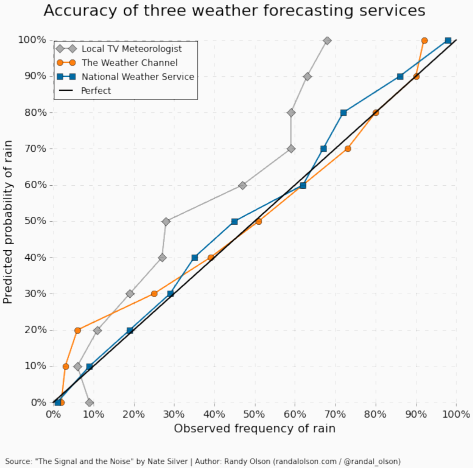
arXiv:1808.08211 arXiv:1704.03308



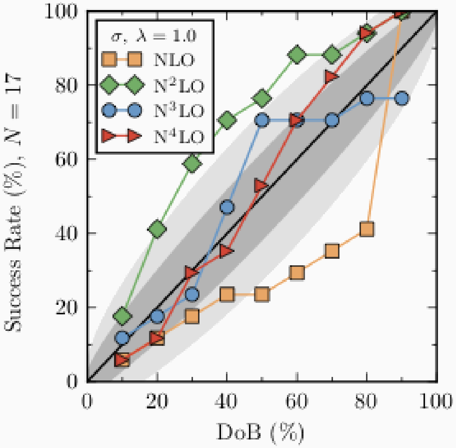
Uncorrelated Posteriors

Assumes that the variance of the c_n is independent at each point





Accuracy of three weather forecasting services
Local TV Meteorologist
The Weather Channel
National Weather Service
Perfect
Predicted probability of rain
Observed frequency of rain
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Source: "The Signal and the Noise" by Nate Silver | Author: Randy Olson (randalolson.com / @randal_olson)



σ, λ = 1.0
NLO
N²LO
N³LO
N⁴LO
Success Rate (%), N = 17
DoB (%)
0
20
40
60
80
100