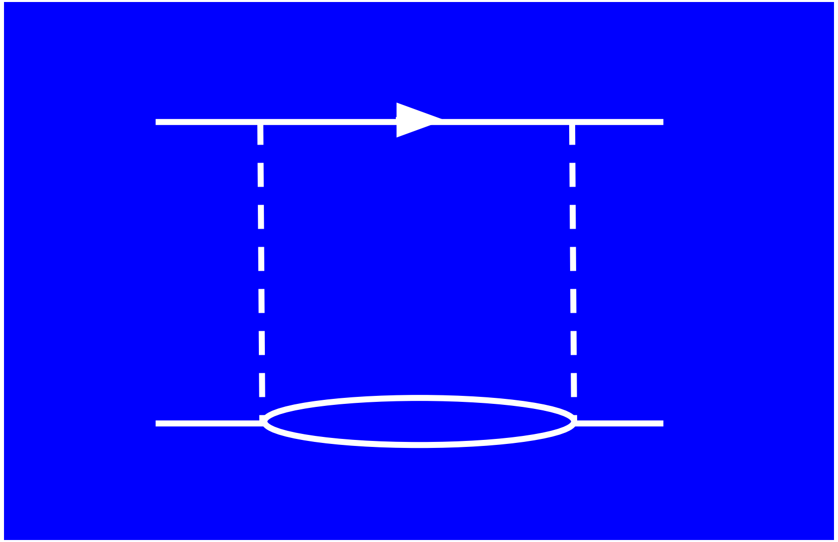
Presented By: Oscar Javier Hernandez

Nuclear structure corrections in muonic atoms: Quantifying theoretical uncertainties



Phys. Lett. B 778, 377-383, (2018)

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Sonia Bacca
Nir Barnea

THE UNIVERSITY OF BRITISH COLUMBIA

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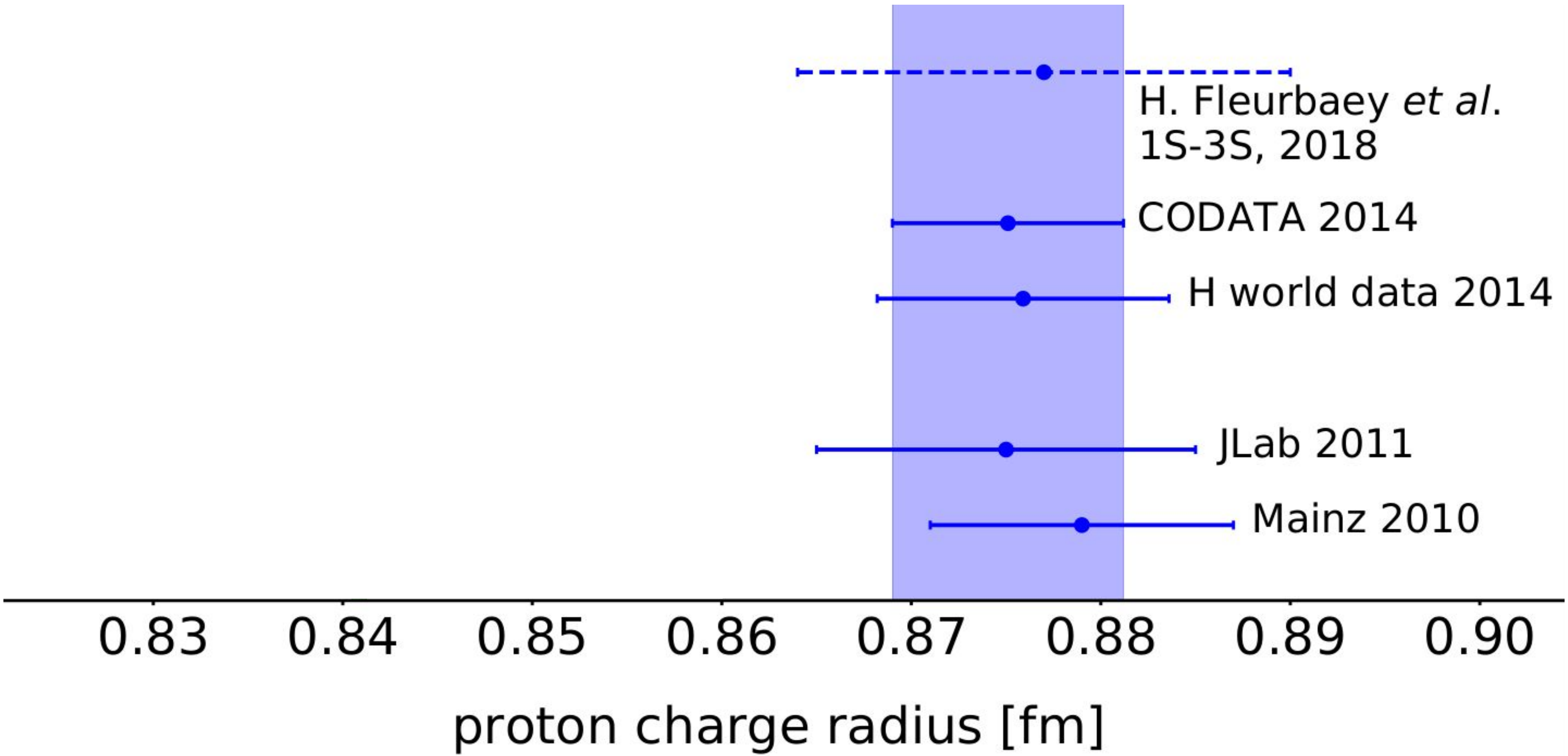
TRIUMF

The growing proton radius puzzle



[Chen et al, JPG 45, 093002, 2018.]

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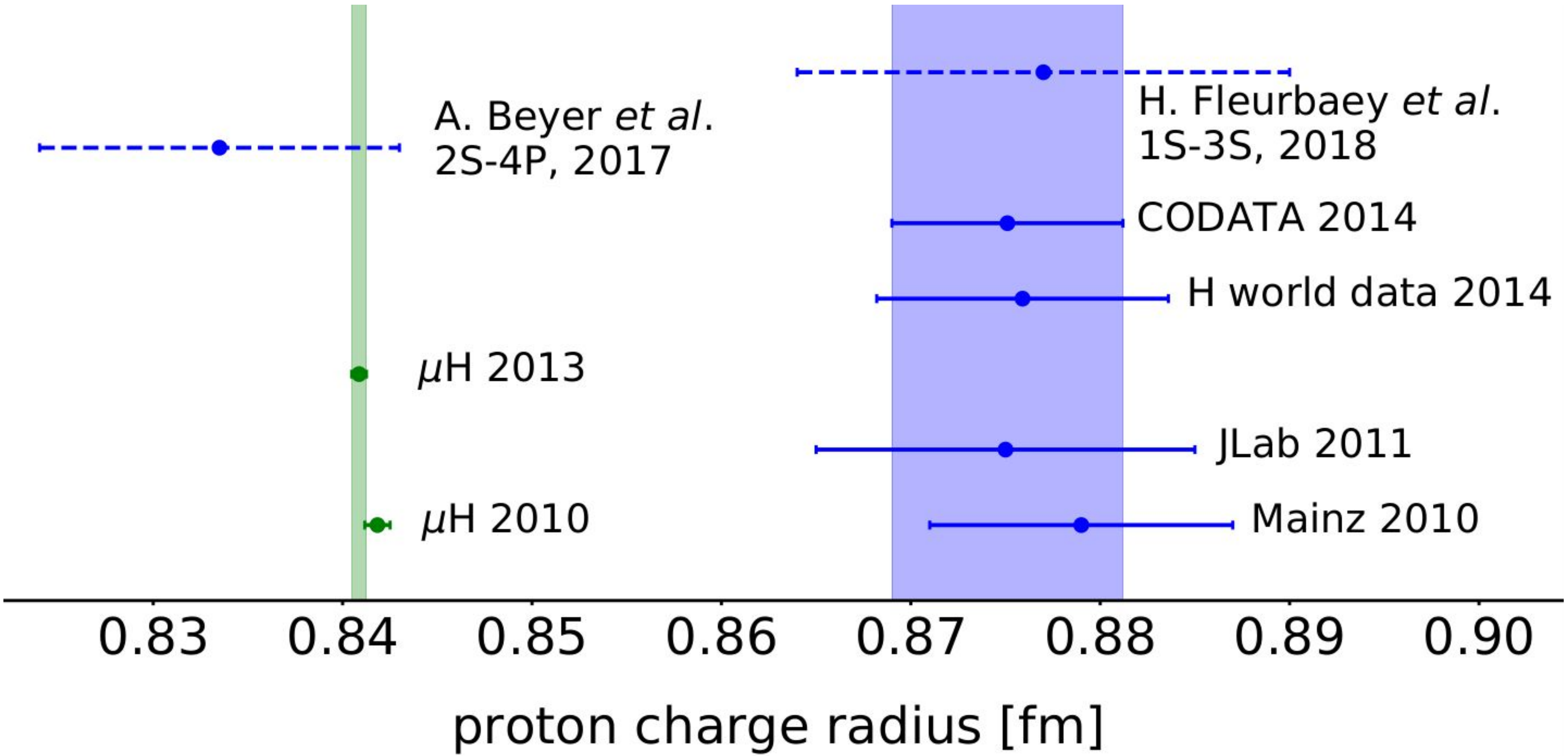

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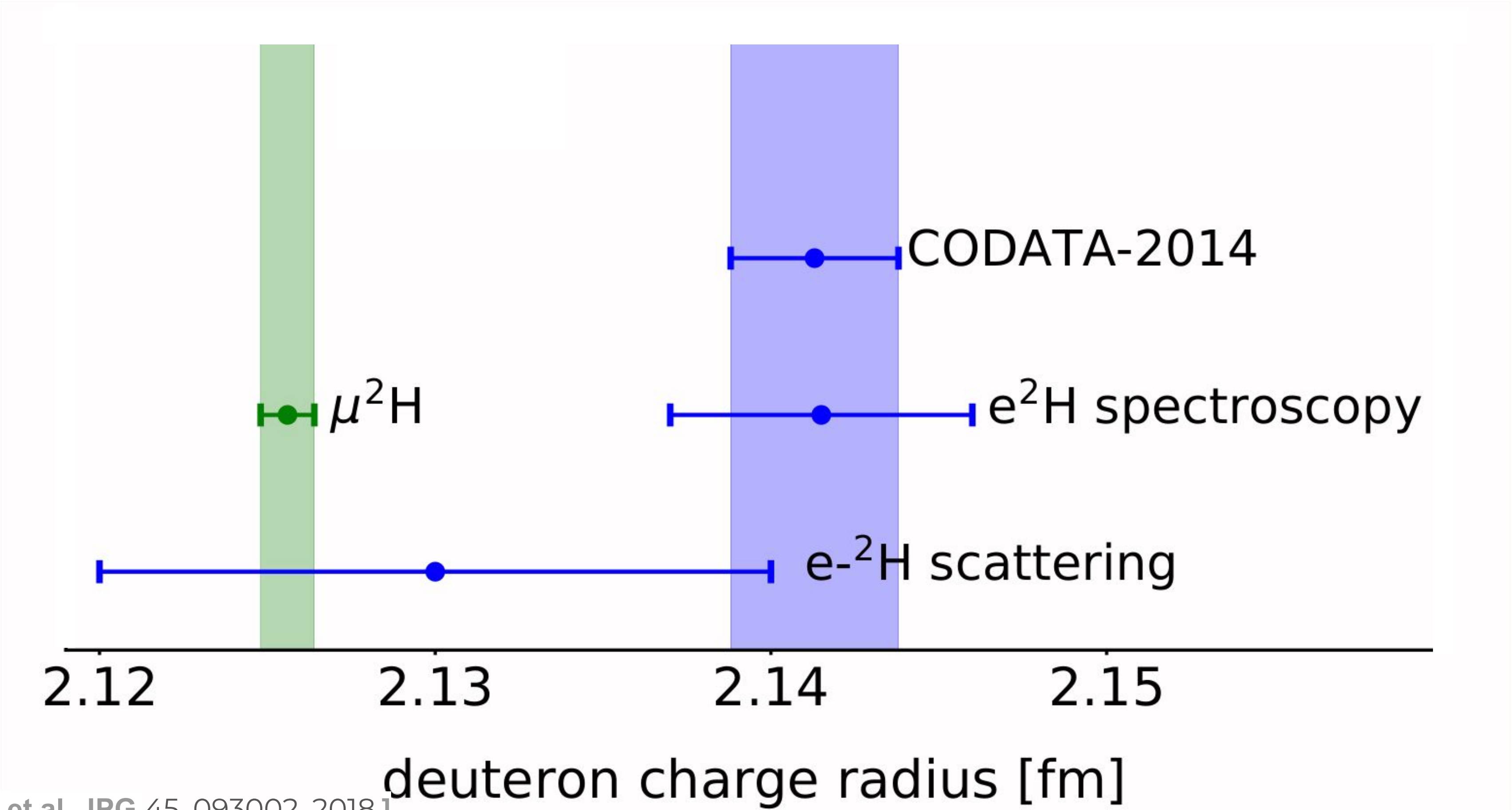
There is a discrepancy between eD and μD data



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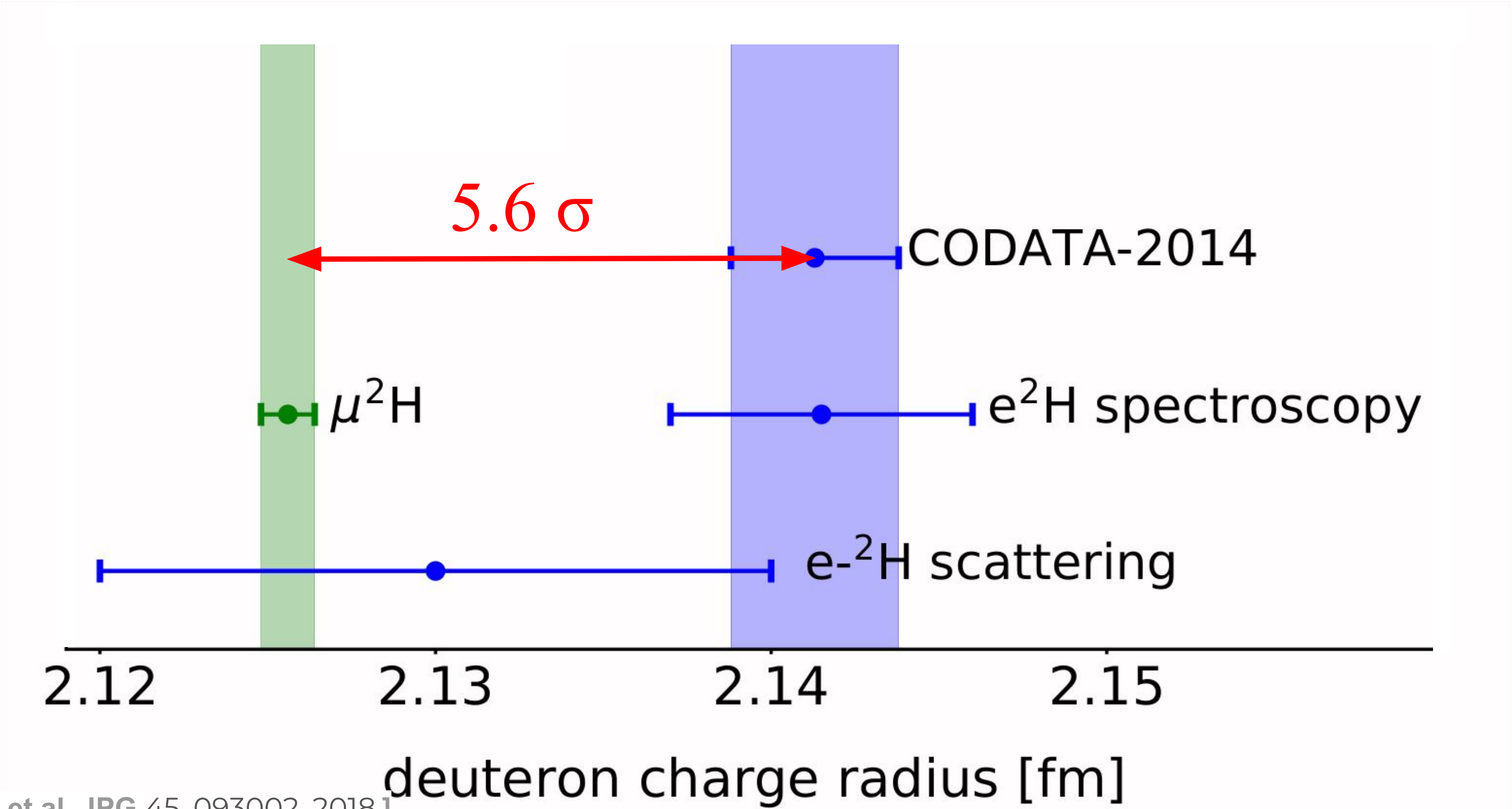

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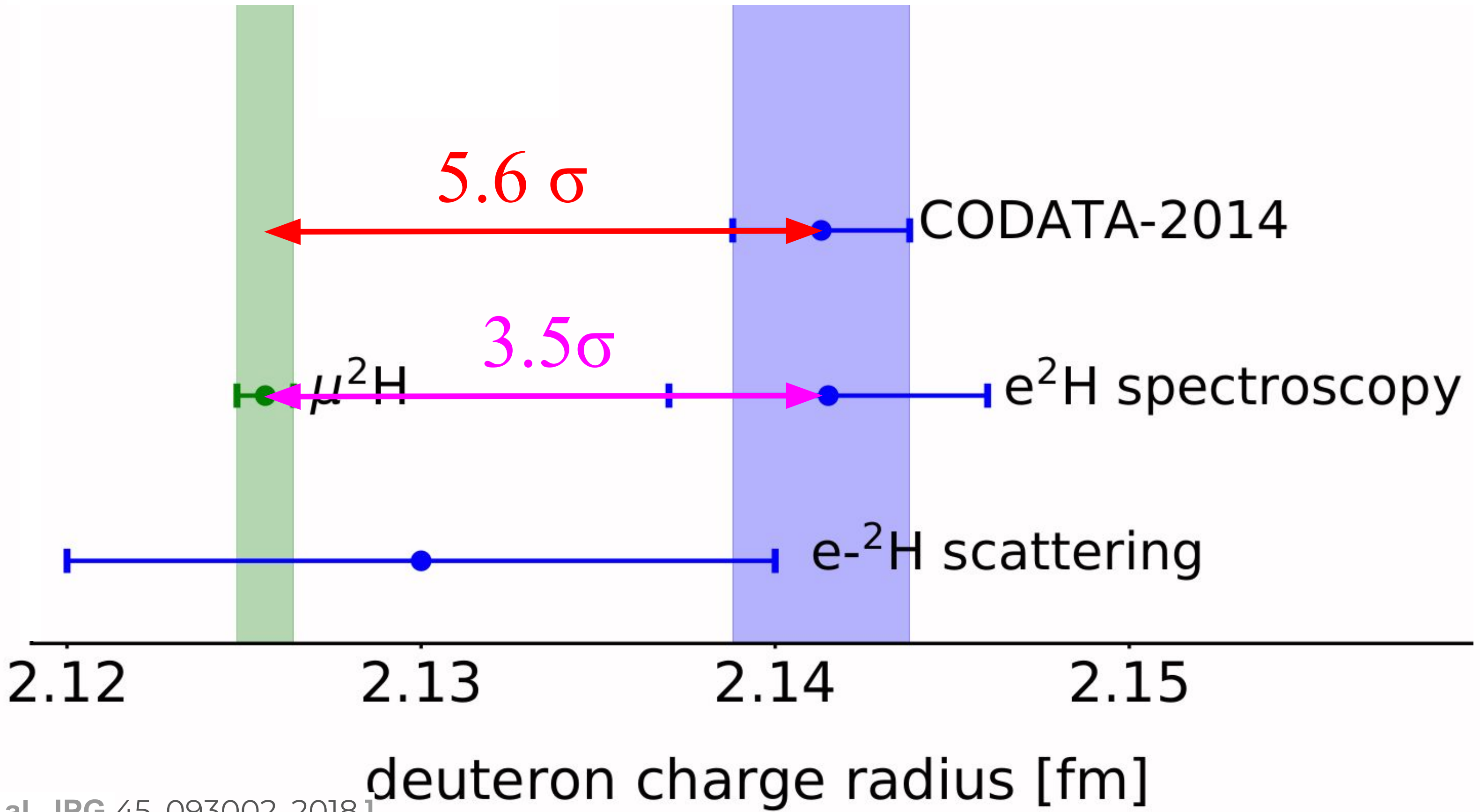

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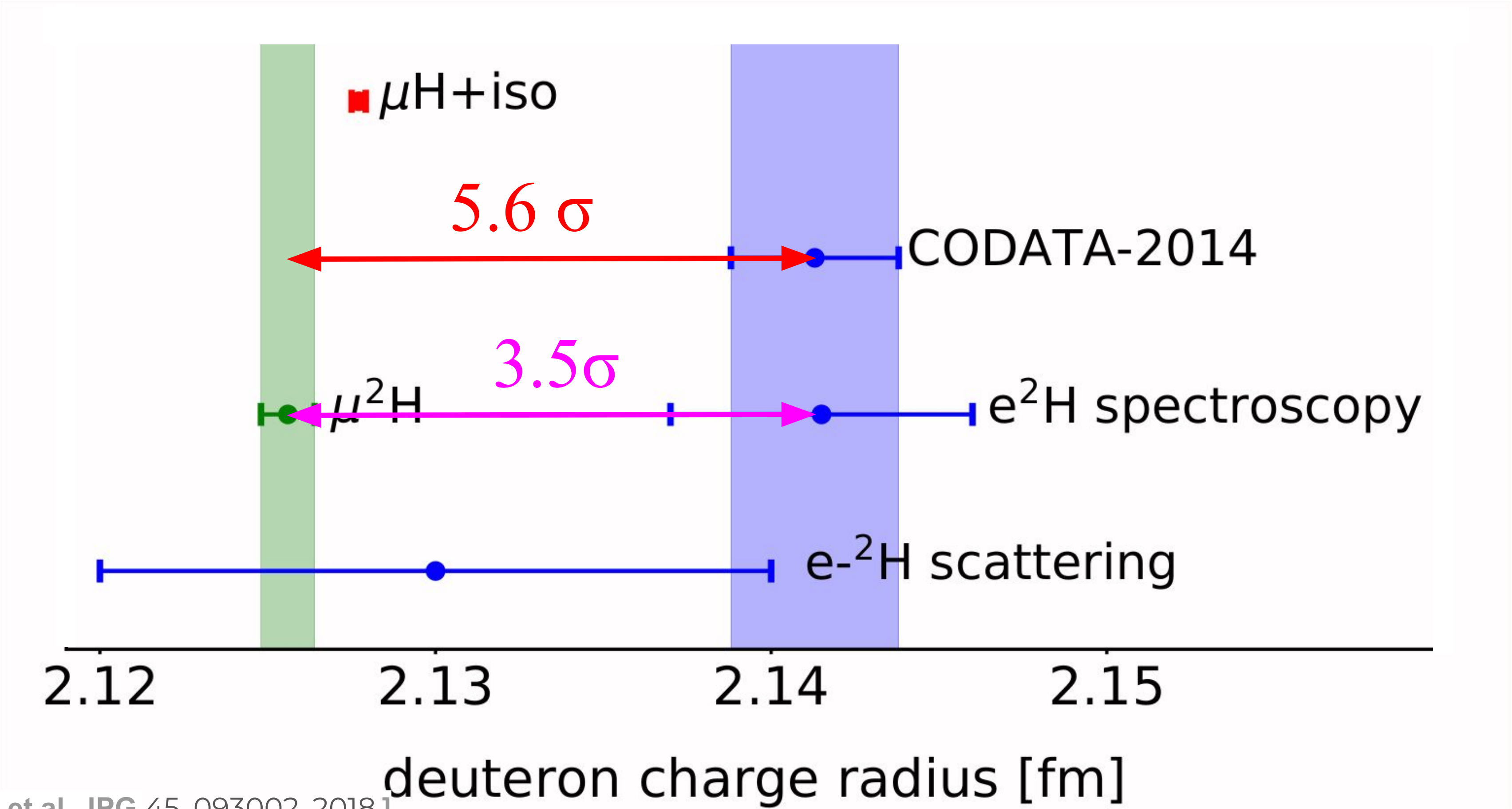

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The Lamb shift

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \delta_{\mathrm{FS}}(r_C^2) + \delta_{\mathrm{TPE}}$$

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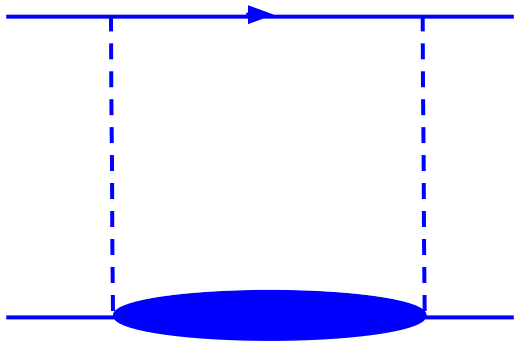
The dominant nuclear structure corrections are given by the two-photon exchange



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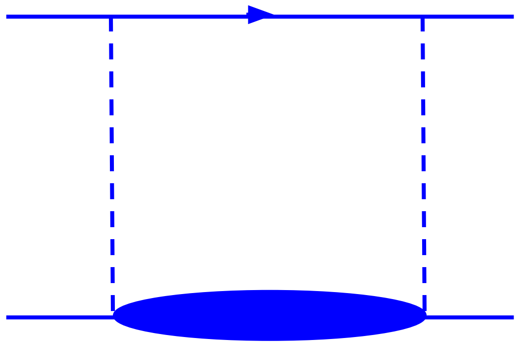
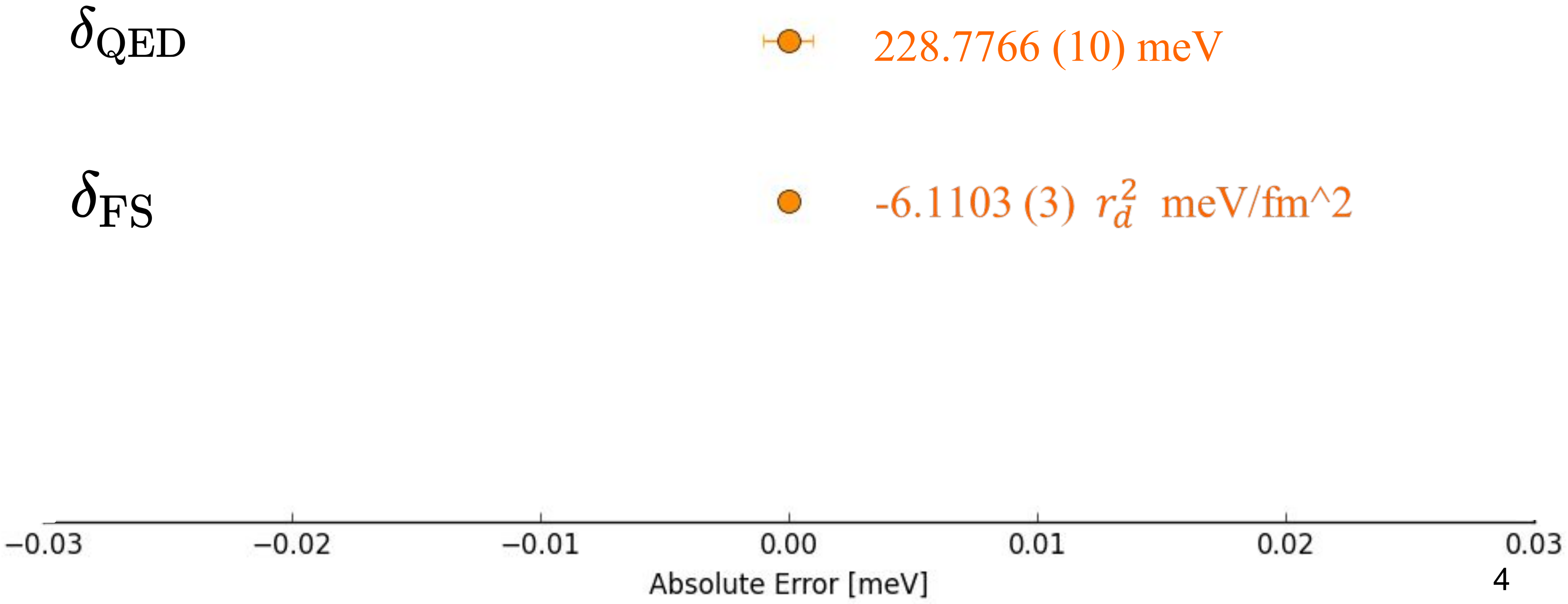


$$\delta_{\mathrm{TPE}} = \underbrace{\delta^{A}_{\mathrm{TPE}}}_{\text{Nuclear}} + \underbrace{\delta^{N}_{\mathrm{TPE}}}_{\text{Nucleonic}}$$

The physics problem

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \delta_{\mathrm{FS}}(r_C^2) + \delta_{\mathrm{TPE}}$$



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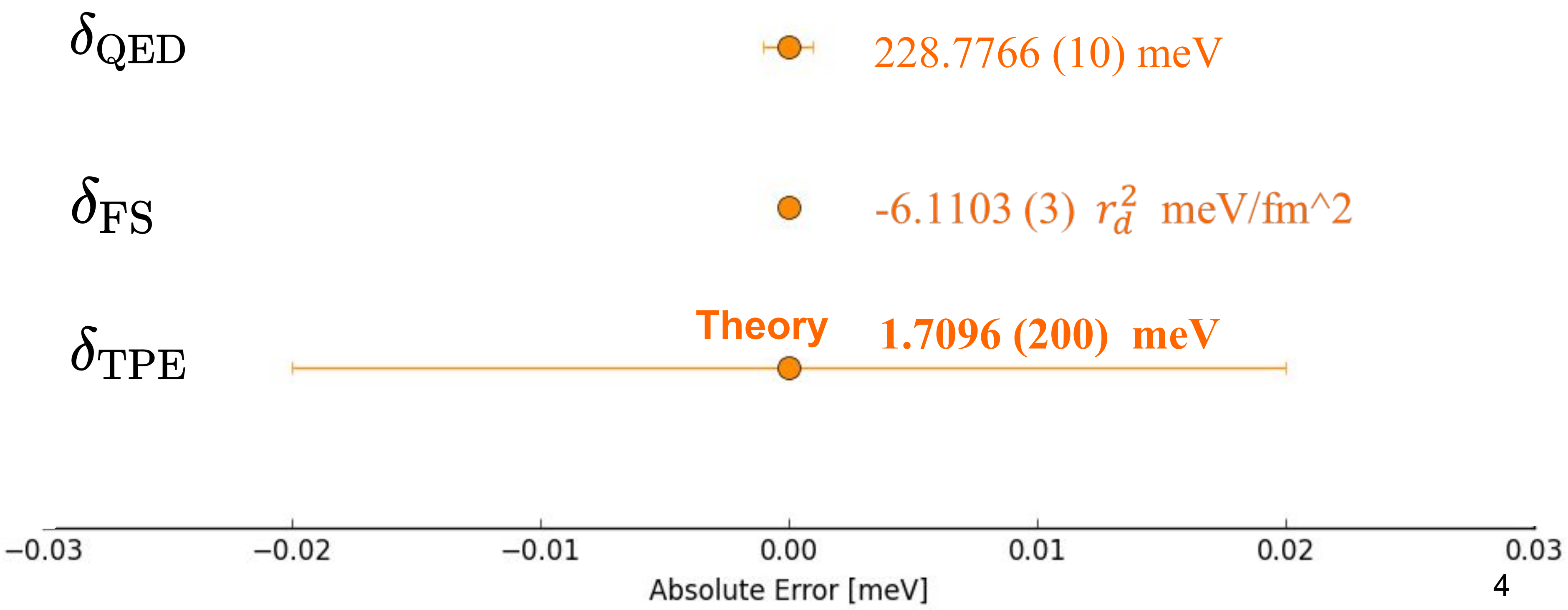

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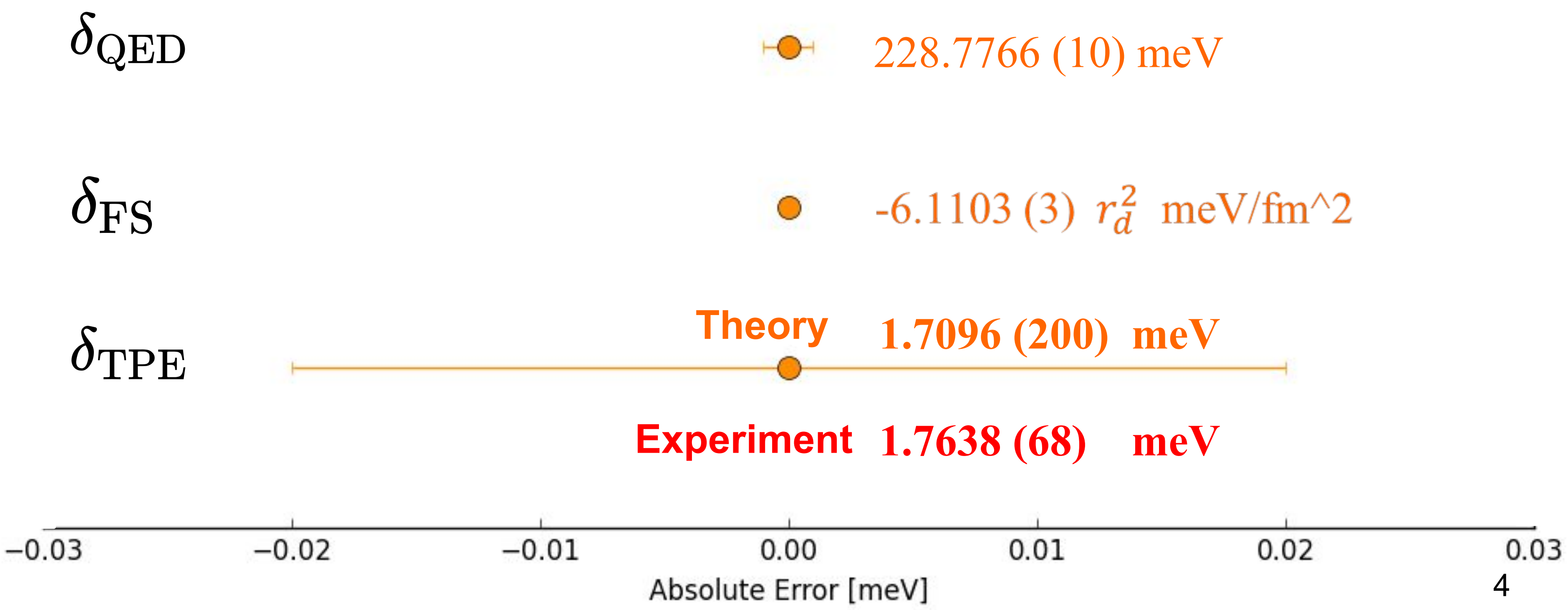
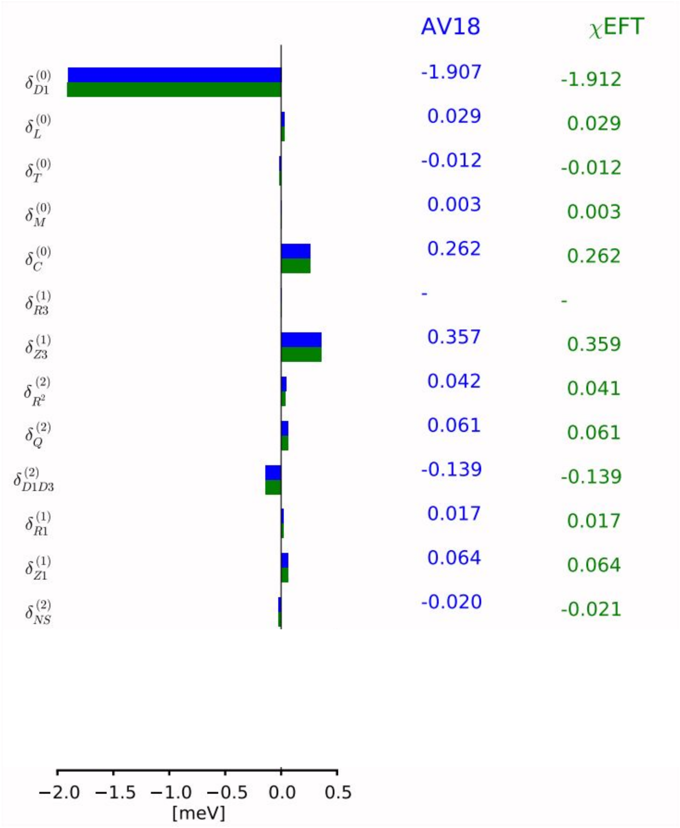


The nuclear polarizability is a sum of many terms



$$\delta_{\mathrm{TPE}} = \sum_k \delta^{(k)}$$

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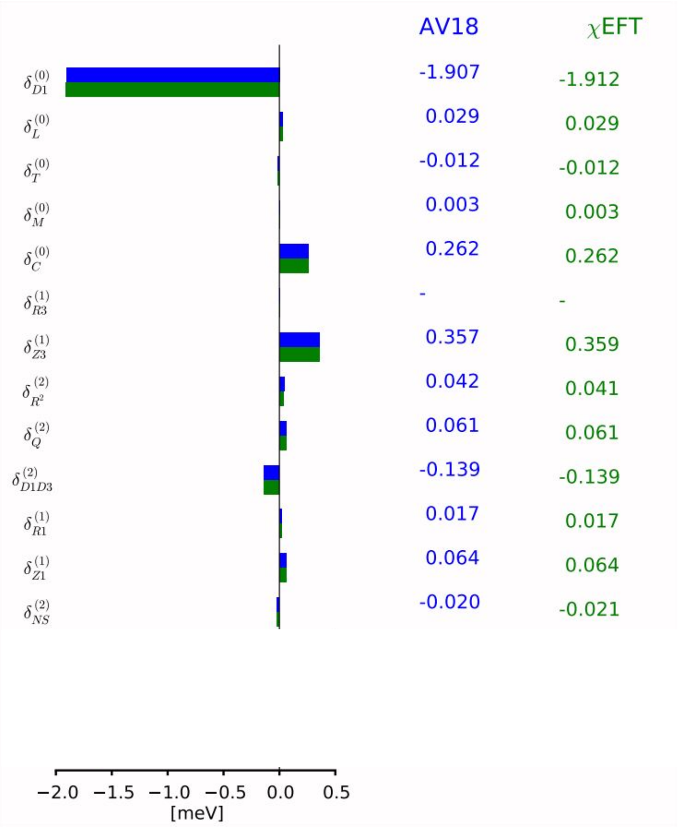


$$\delta_{\mathrm{TPE}} = \sum_k \delta^{(k)}$$

- Corrections are ordered according to the power of the expansion parameter η

$$\eta \approx \sqrt{\frac{m_r}{M}} \ll 1$$

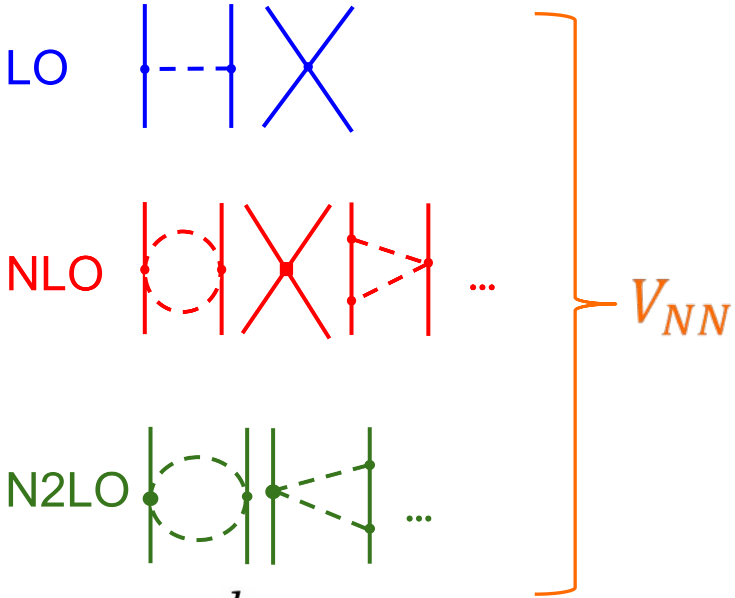
- Each correction is an integral over the response

$$\delta \propto \int d\omega\, g(\omega) S_{\hat{O}}(\omega)$$

[JPG 45, 093002, 2018.] μD

Improving the uncertainty estimates

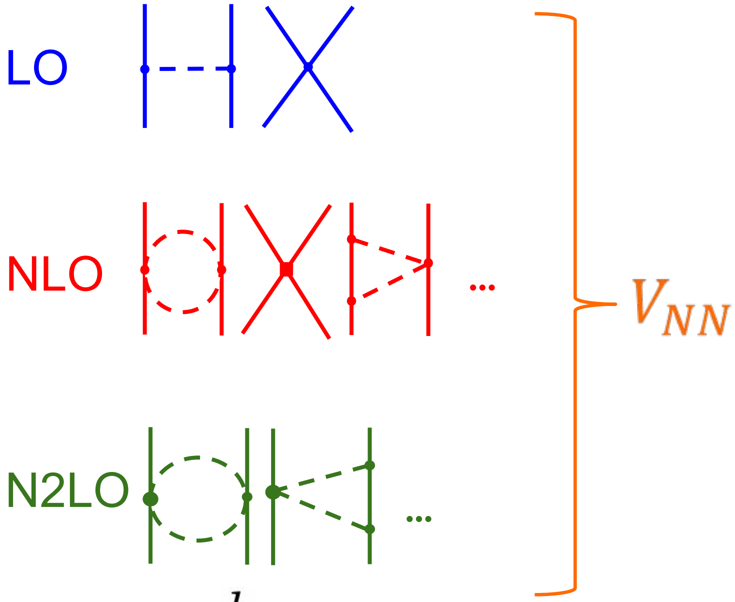
$$\delta^{A}_{TPE}\left(c_\mu, \Lambda, T^{Max}_{Lab}, k\right)$$



$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_\nu \left(\frac{Q}{\Lambda_\chi}\right)^{\nu}$$

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Improving the uncertainty estimates



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- Use N2LO potentials fit simulatenously to NN and πN data

Statistical uncertainties: c_μ

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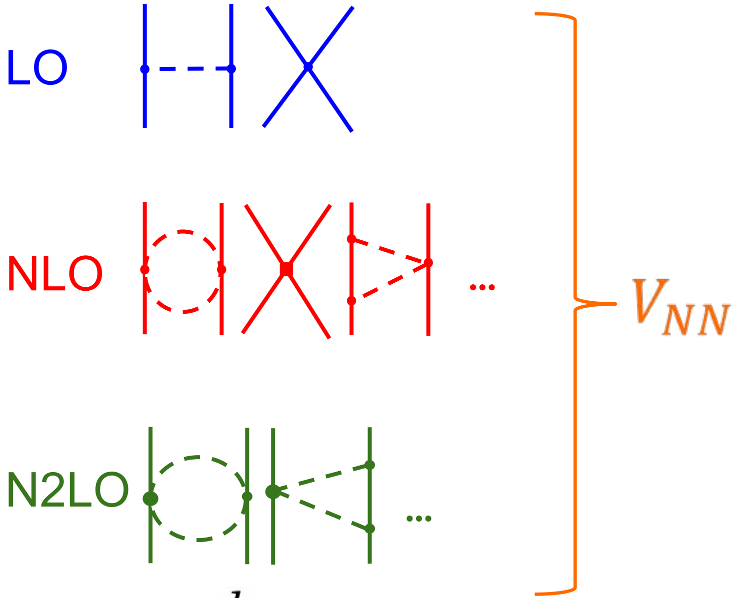

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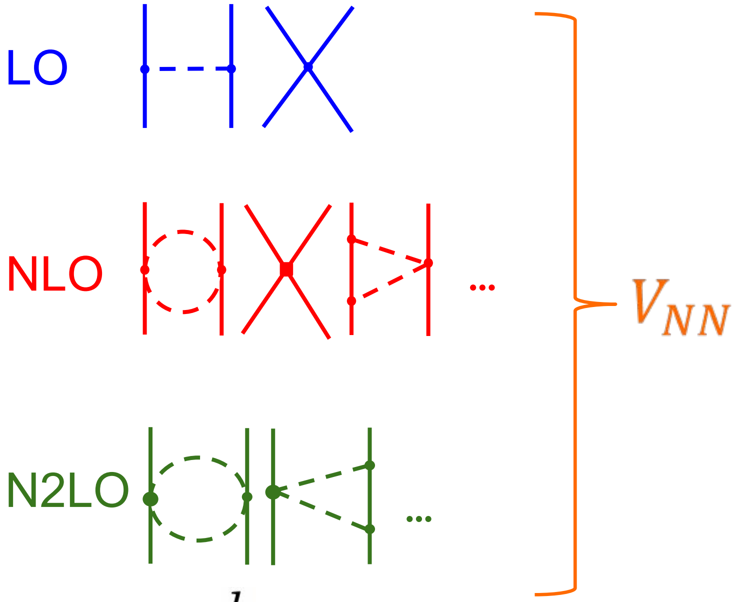

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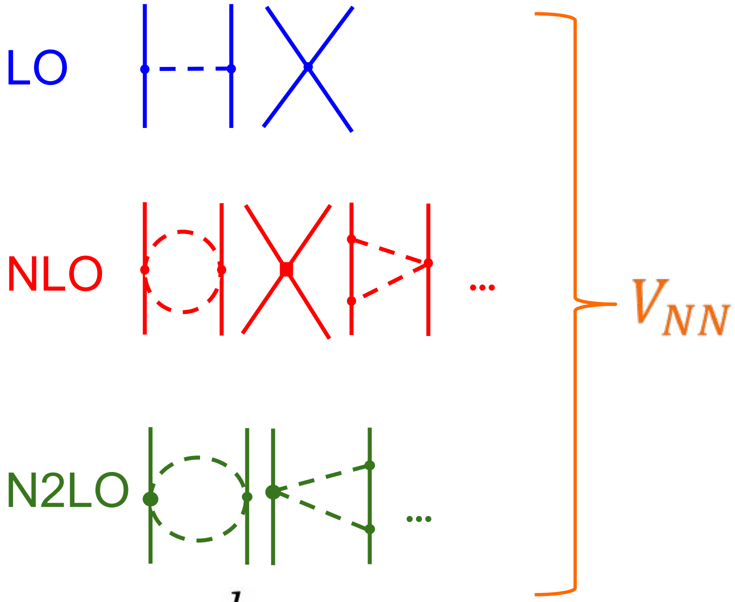

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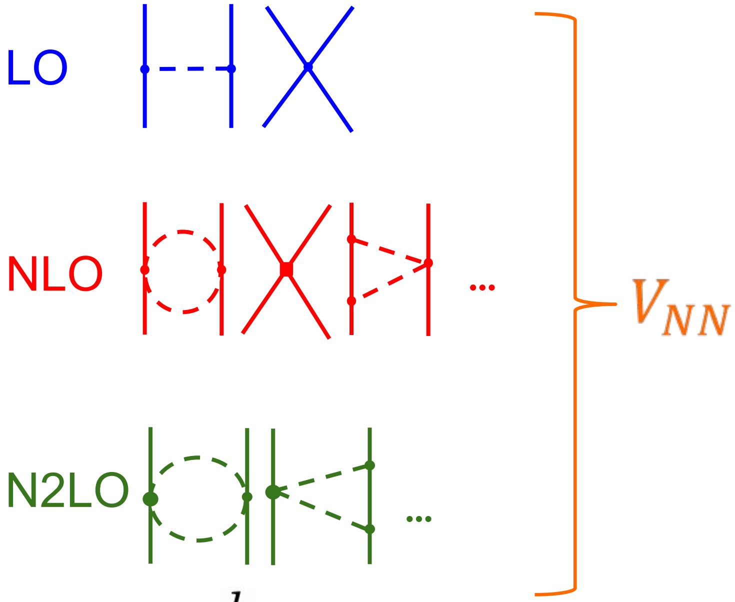

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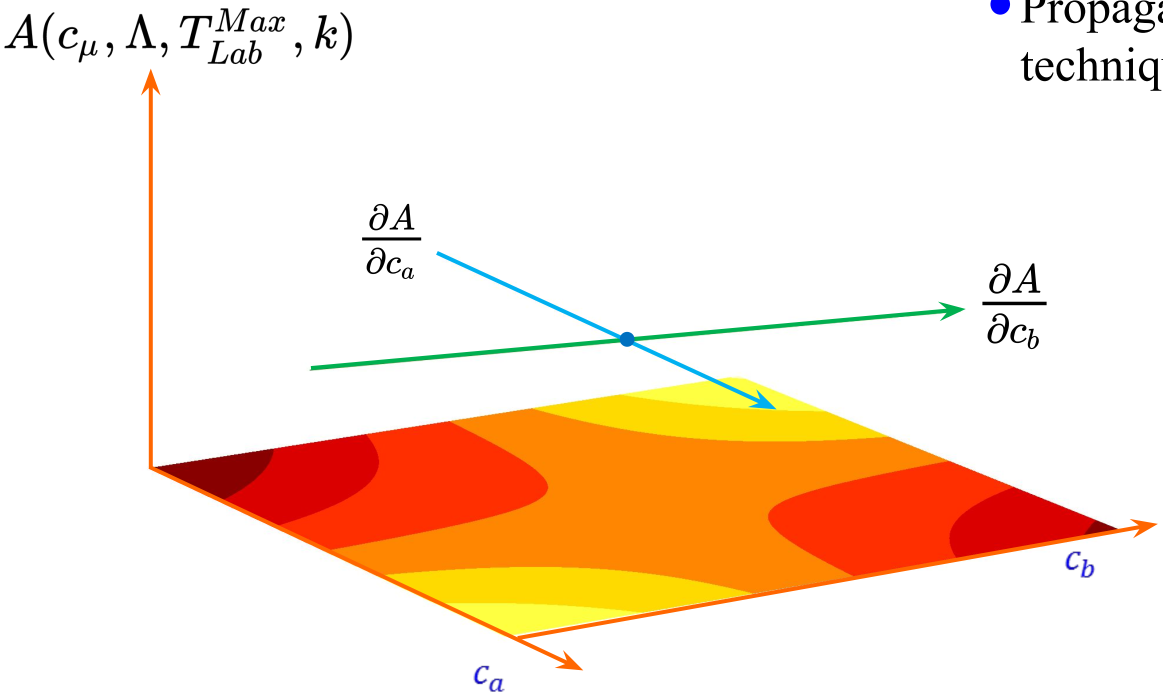
Single Nucleon: δ^{N}_{TPE}

Higher Order Corrections: $O(\alpha^6)$

Statistical uncertainties



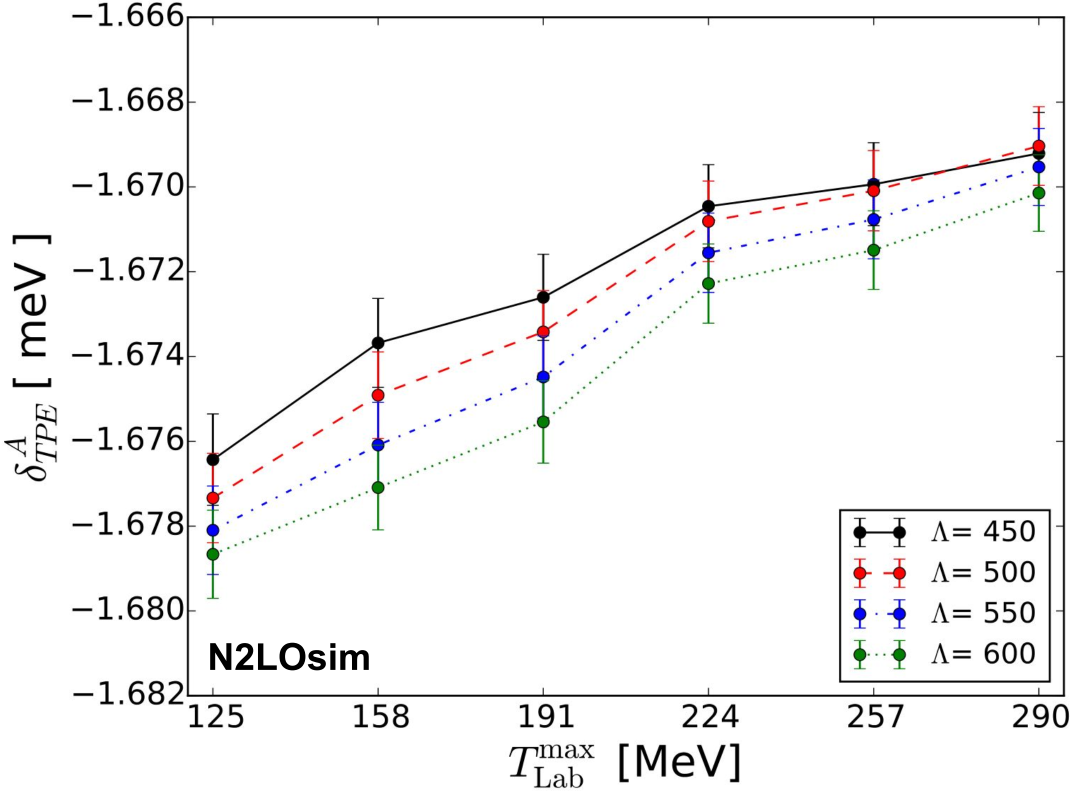
- Propagate uncertainty using standard techniques

$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

$$\mathrm{Cov}(A, B) = \boldsymbol{J}_A \mathrm{Cov}(c_\mu) \boldsymbol{J}_B^T$$

$$\sigma_{A,stat} = \sqrt{\mathrm{Cov}(A, A)}$$

Statistical uncertainties



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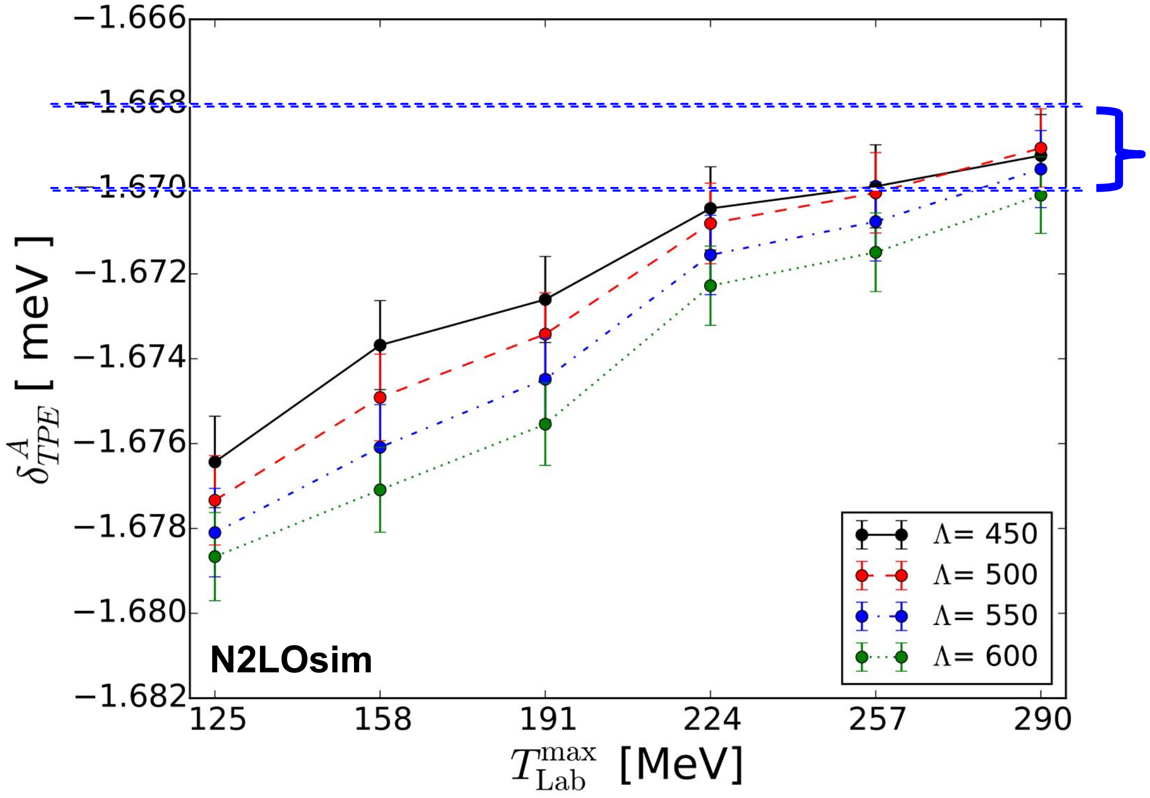
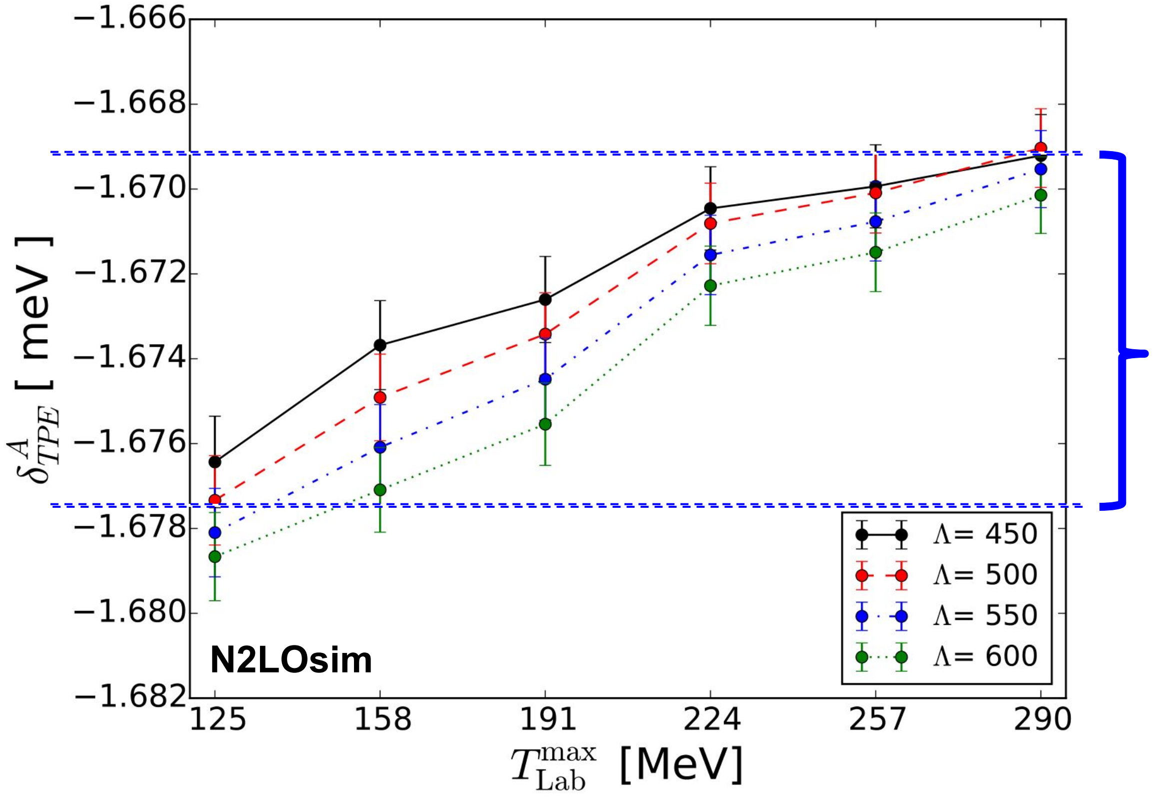


Statistical uncert. c_μ

Correction	% Uncert.
Statistical	0.06

Sytematic Tlab uncertainties

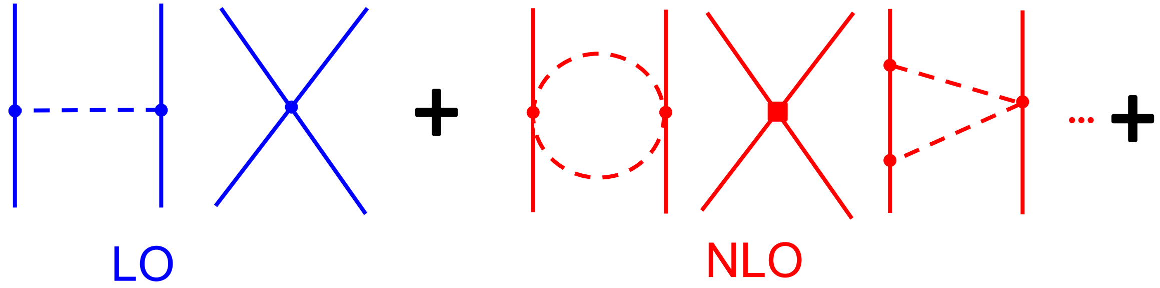


Systematic Tlab uncert.

Correction	% Uncert.
Statistical	0.06
Tlab Sys.	0.2

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} =$$



- Expand observable in the same Chiral EFT pattern,

$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_{\nu}(p) Q^{\nu} \qquad Q = max\left\{ \frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b} \right\}$$

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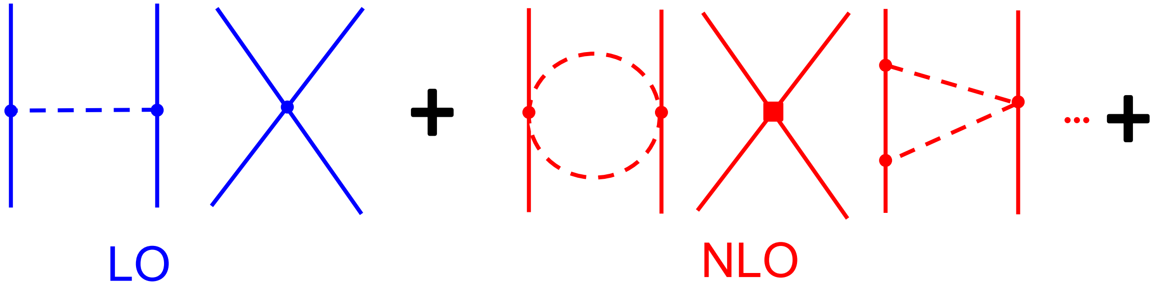

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- Truncation uncertainty can then be calculated according to

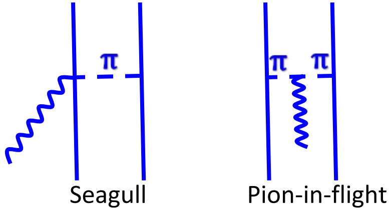
$$\sigma_{A,sys}^{N^k LO}(p) \approx Q \cdot |A_0 Q^{k+1} \beta_{k+1}|$$

$$\sigma_{A,sys}^{N^k LO}(p) = A_0 Q^{k+2} \; max\{|\beta_0|, \ldots, |\beta_{k+1}|\}$$

Correction	% Uncert.
Chiral Trunc.	0.4

Additional uncertainties

Two body currents + relativistic corr.



$$\rho, \vec{j}$$

Correction	% Uncert.
MEC	0.15
Rel. Corr.	0.05

Additional uncertainties

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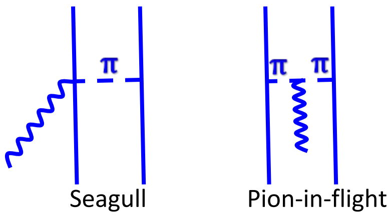
J. J. Krauth, et al. Ann. of Phy. 366, 168 (2016).

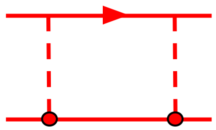
Two body currents + relativistic corr.



$\rho, \vec{j}$

Correction	% Uncert.
MEC	0.15
Rel. Corr.	0.05
Nucleon*	0.6

Single Nucleon Physics



δ^{N}_{TPE}

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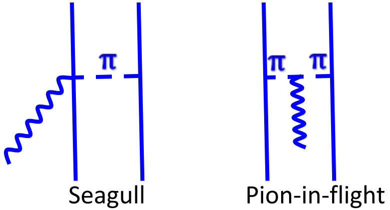
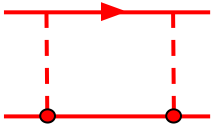


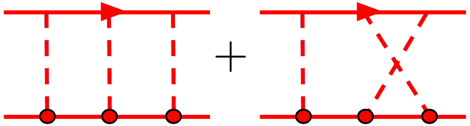
$\rho, \vec{j}$

Single Nucleon Physics



δ^{N}_{TPE}

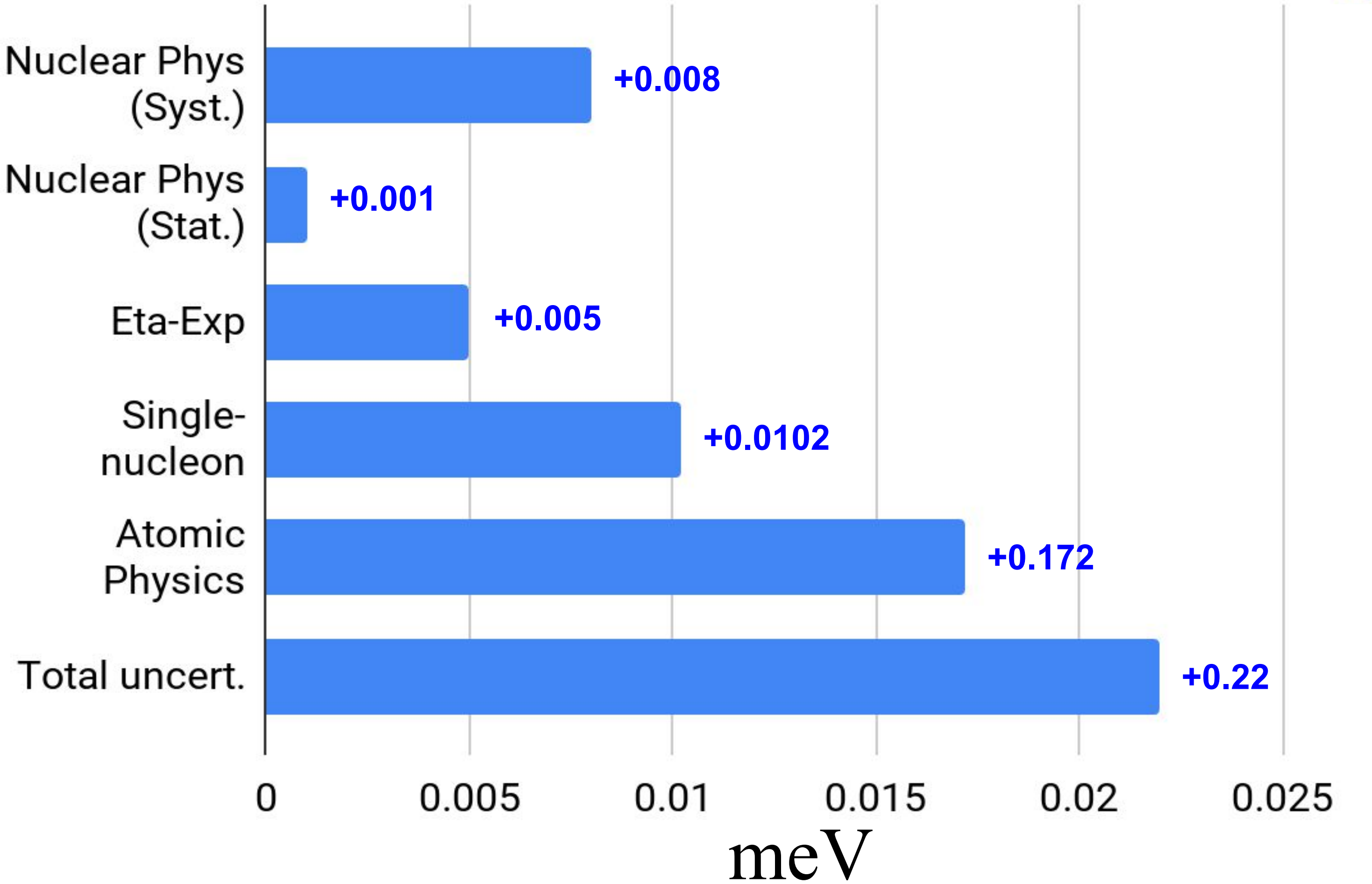
Atomic Physics uncert.



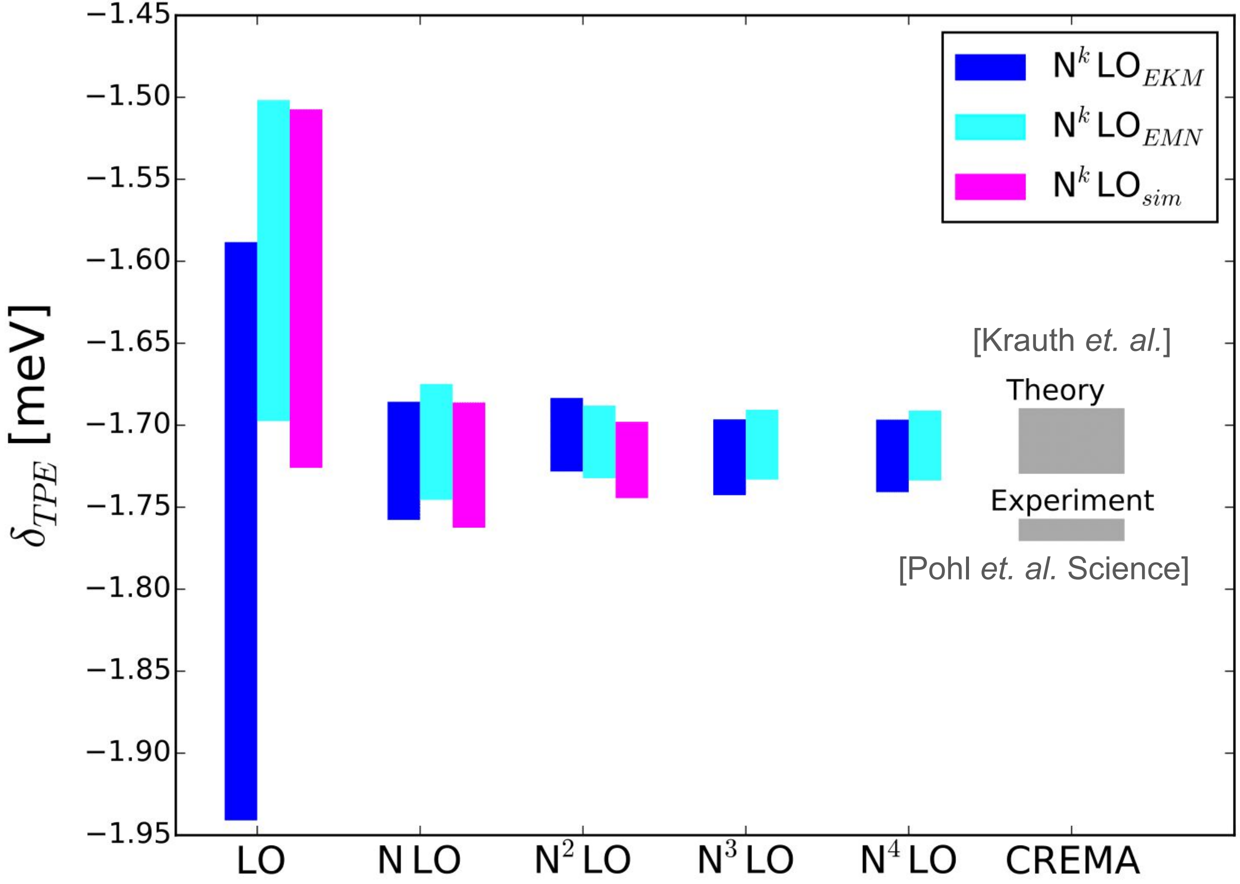
$+ \cdots \Big\} \; O(\alpha^6)$

Correction	% Uncert.
MEC	0.15
Rel. Corr.	0.05
Nucleon*	0.6
Atomic Phys.	1.0

Final uncertainty budget



Uncertainty comparisons



Uncertainty in other muonic atoms

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	-	0.0	0.1	-	0.1	0.4	-	0.1	0.1	-	0.0
Coulomb	0.4	-	0.3	0.5	-	0.3	3.0	-	0.9	0.4	-	0.1
η-expansion	0.4	-	0.3	1.3	-	0.9	1.1	-	0.3	0.8	-	0.2
Higher $Z\alpha$	0.7	-	0.5	0.7	-	0.5	1.5	-	0.4	1.5	-	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

[**Chen et al, JPG** 45, 093002, 2018.]

Uncertainty in other muonic atoms

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
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Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

$$\delta^A_{\rm TPE} = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \ldots$$

- η-expansion uncertainty estimates are quite large in A=3 systems.
- Can we confirm this estimate using Bayesian methods?

[Chen et al, JPG 45, 093002, 2018.]

Estimating the natural scale parameter

- Can we identify the scale parameter of our expansion (η) and unknown coefficients ?

$$\delta^A_{\text{TPE}} = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \dots$$

Estimating the natural scale parameter

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$$\begin{aligned}\delta^{A}_{\mathrm{TPE}} &= \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \ldots \\ &= c_0 + c_1\eta + c_2\eta^2 + \ldots\end{aligned}$$

Estimating the natural scale parameter

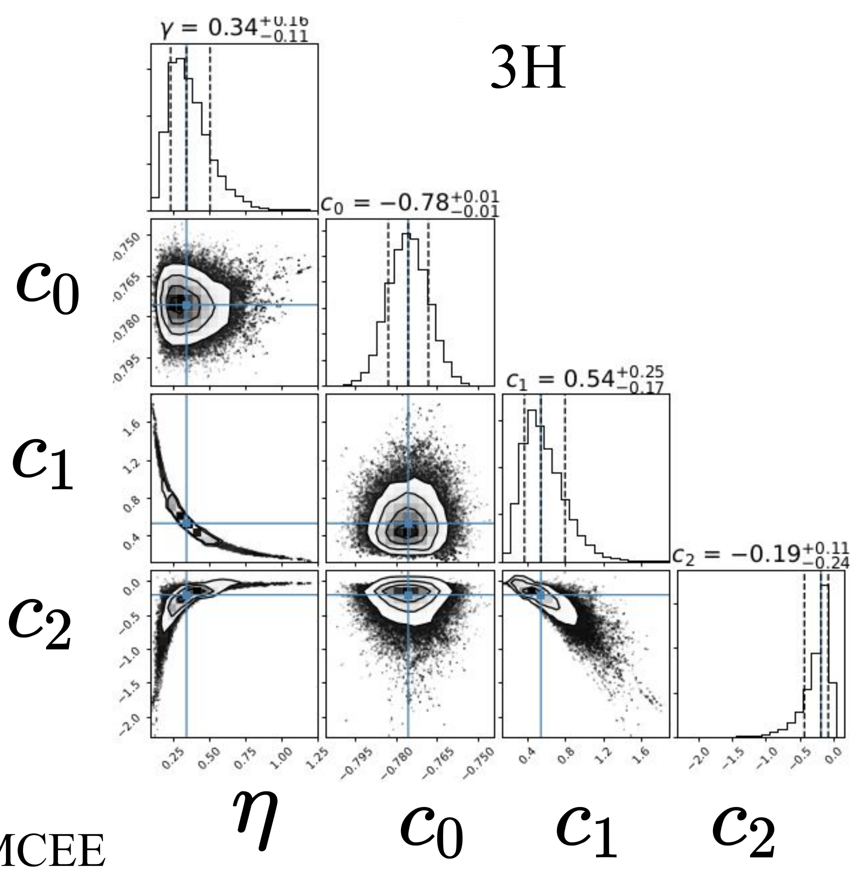
- Can we identify the scale parameter of our expansion (η) and unknown coefficients ?

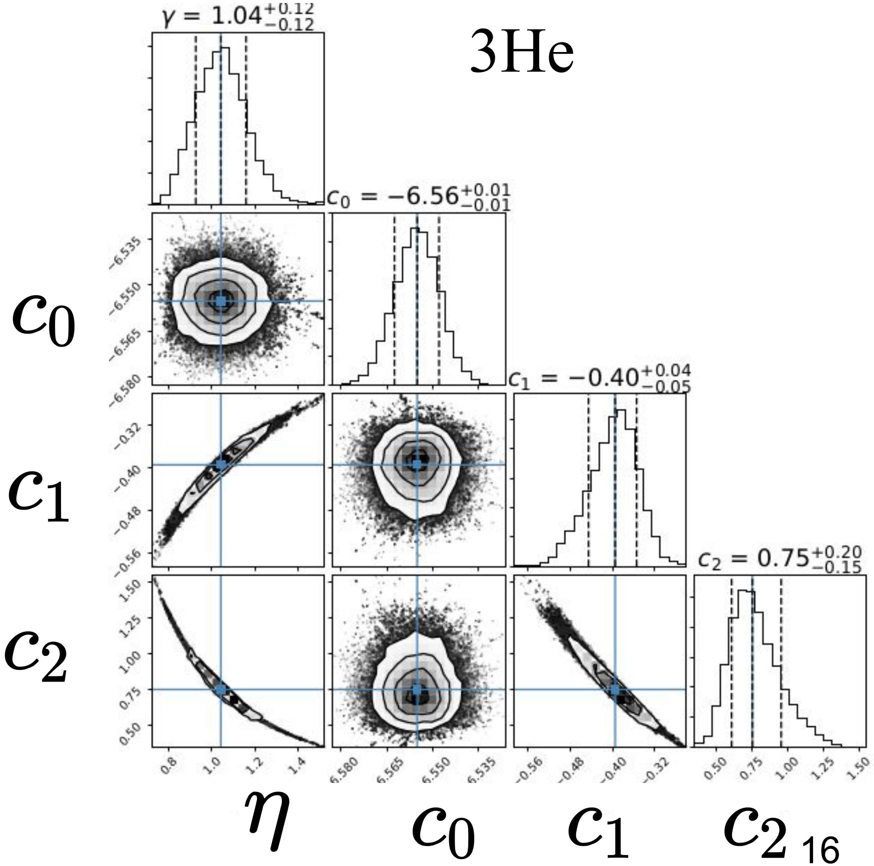
$$\begin{aligned}\delta^A_{\mathrm{TPE}} &= \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \dots \\ &= c_0 + c_1\eta + c_2\eta^2 + \dots\end{aligned}$$

- Bayesian parameter estimation problem

$$P(\eta|D,I) \propto \int d\vec{c}\, \underbrace{P(D|\vec{c},\eta,I)}_{e^{-\chi^2}} \underbrace{P(c_n|c_{n-1},\dots,c_0,\eta,I)\dots P(c_0|\eta,I)}_{\text{independent}} P(\eta|I).$$

MCMC Sampling





EMCEE

Preliminary estimates

	^{2}H	^{3}H	^{3}He	^{4}He
η	$0.51^{+0.17}_{-0.14}$	$0.34^{+0.16}_{-0.11}$	$1.04^{+0.12}_{-0.12}$	$0.84^{+0.18}_{-0.15}$
c_0	$-1.91^{+0.01}_{-0.01}$	$-0.78^{+0.01}_{-0.01}$	$-6.56^{+0.01}_{-0.01}$	$-4.56^{+0.01}_{-0.01}$
c_1	$0.70^{+0.26}_{-0.18}$	$0.54^{+0.25}_{-0.17}$	$-0.40^{+0.04}_{-0.05}$	$0.92^{+0.19}_{-0.16}$
c_2	$-0.14^{+0.06}_{-0.12}$	$-0.19^{+0.11}_{-0.24}$	$0.75^{+0.20}_{-0.15}$	$0.13^{+0.06}_{-0.04}$

Physics based estimates

η	0.107	0.109	0.109	0.110

Outlook

Results:

- Experimental vs theory difference improved by thorough analysis of nuclear TPE uncertainty.
- Uncertainty in TPE cannot solve the 5.6 σ discrepancy.

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Uncertainty Analysis:

- Use bayesian methods to combine statistical and chiral EFT truncation uncertainty
- Complete Bayesian η-expansion uncertainty analysis
- Reduce atomic physics uncert. $O(\alpha^6)$

Thank you!

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