A statistical study of the transition from isolated resonances to the continuum

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A Remark on Systematic Uncertainties

off topic!

- I am a firm believer in quantifying systematic uncertainties, and as collaborator, referee, and mentor of students I strive to make the world a better place, but ...

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- In most cases, if experimentalists have a firm understanding of systematic effects, they can be eliminated or correct for. So ...
- Systematic are seldom fully understood by experimentalists.

Outline

- Background and Motivations
- Astrophysics : $^{34}\mathrm{Ar}(\alpha, p)^{37}\mathrm{K}$
- Implementation
- Results

In what situation is this transition important?

- "Lighter" nuclei
- Near closed shells and/or driplines
- Low energies (or temperatures)
  - Gamow peak: $E_0 \propto T^{2/3}$
  - peak width: $\Delta \propto T^{5/6}$

Two Ways of Thinking about Cross Sections

- Isolated resonances:

$$\begin{aligned}\sigma_{cc'} &= \frac{\pi}{k^2}\omega_J \frac{\Gamma_c\Gamma_{c'}}{(E-E_R)^2+\Gamma^2/4} \\ \Gamma &= \sum_c \Gamma_c\end{aligned}$$

- Hauser-Feshbach (energy-averaged) result:

$$\sigma_{cc'} = \frac{\pi}{k^2}\omega_J \frac{T_c T_{c'}}{\sum_c T_c}$$

- How high must the level density be for the H-F formula to apply?

  Rule of thumb in astrophysics:
  10 levels in Gamow Window (?)

Astrophysical Motivations
Hauser-Feshbach with $20 \leq A \leq 50$

- x-ray bursts
  - (α, p) reactions
  - $T = 0.5$-2 GK
- type-II supernovae
  - (α, γ), (α, n), (α, p), p-induced reactions
  - $T = 1.5$-5 GK

Artist's conception of an x-ray burst:

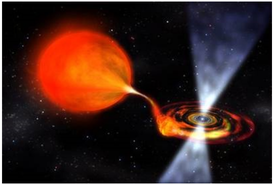


image courtesy of JINA

Reaction Rate Formalism

- General Result:

$$\langle \sigma v \rangle = \left(\frac{8\pi}{\mu} \right)^{1/2} (kT)^{-3/2} \int_0^\infty E\, \sigma(E)\, \exp(-\frac{E}{kT})\, dE$$

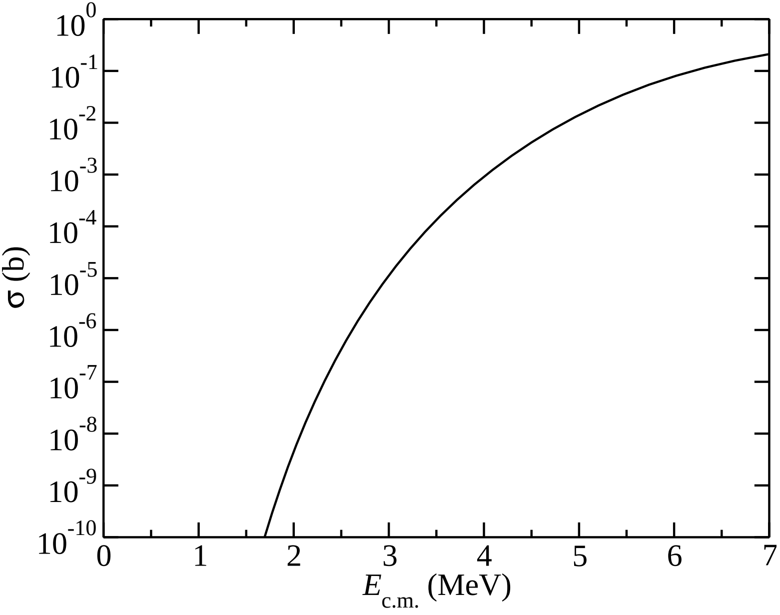
- Narrow resonance:

$$\langle \sigma v \rangle = c(\hbar c)^2 \left(\frac{2\pi}{\mu c^2 kT} \right)^{3/2} \omega_J \frac{\Gamma_c \Gamma_{c'}}{\Gamma} \exp(-\frac{E_R}{kT})$$

$^{34}Ar(\alpha, p)^{37}K$

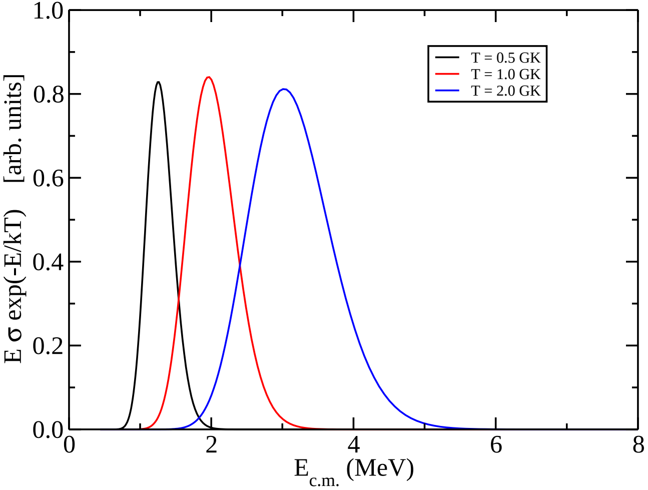
- ^{34}Ar is a waiting point in the RP process [(p, γ) Q value is very small]
- α threshold in ^{38}Ca: 6.1 MeV
- (α, p) Q value: 1.6 MeV
- Focus of considerable recent work:
  - C. Deibel *et al.* $^{37}K(p, \alpha)^{34}Ar$ at ATLAS/ANL
  - A. Lauer *et al.* $^{37}K(p, p)$ at ReA3/NSCL
  - A. Long *et al.* $^{40}Ca(p, t)^{38}Ca$ at iThemba
  - K. Schmidt *et al.* $^{34}Ar(\alpha, p)^{37}K$ at ReA3/NSCL
- There have been no direct measurements in the relevant energy range.

What does the average cross section look like?

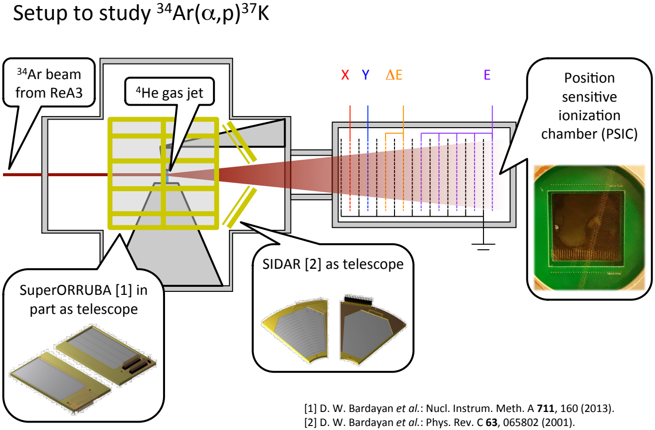


$^{34}Ar(\alpha, p)^{37}K$ Reaction Rate Integrand

assuming smoothed (HF) cross section



$^{34}Ar(\alpha, p)^{37}K$ at ReA3/NSCL



[1] D. W. Bardayan *et al.*: Nucl. Instrum. Meth. A **711**, 160 (2013).
[2] D. W. Bardayan *et al.*: Phys. Rev. C **63**, 065802 (2001).

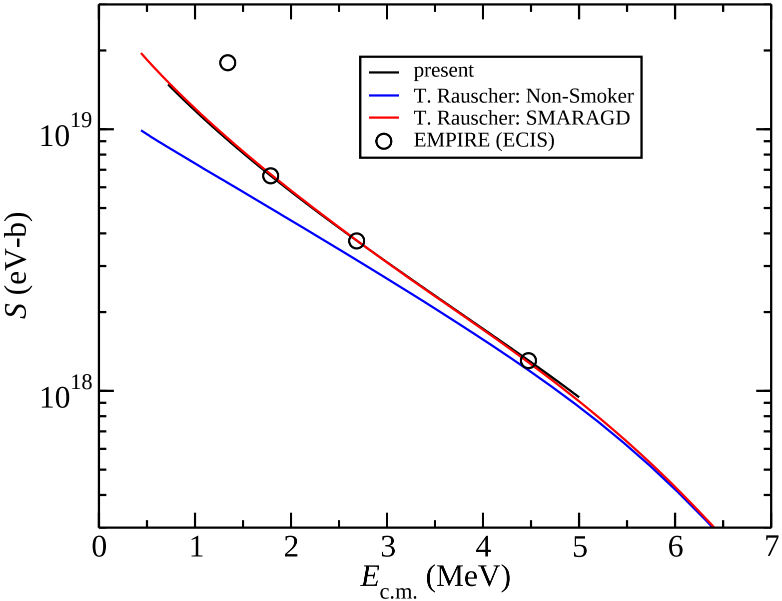
Measurements for $E = 5.7$ and 6.1 MeV (slide courtesy of Konrad Schmidt)
Important detail: $\Delta E_{\mathrm{c.m.}} = 180$ keV

Approaches used to estimate ^{34}Ar$(\alpha, p)^{37}$K so far

- Hauser-Feshbach calculations
  - $\Gamma_\alpha \ll \Gamma_p$
  - on the α transmission functions matter
  - $T_{\alpha l}$ determined from the α optical potential
  - everybody uses McFadden and Satchler (1966)
- Measured levels with estimated α widths (Long *et al.*, 2017)
  - Significantly lower than Hauser-Feshbach

$^{34}\mathrm{Ar}(\alpha,p)^{37}\mathrm{K}$ Results

glossing over computational methods



Monte Carlo Sampling of Hauser Feshbach

- Transmission coefficients, average widths, and level density are connected:

$$
\begin{aligned}
T_c &= 1 - \exp(-2\pi\langle\Gamma_c\rangle\rho) \\
&\rightarrow 2\pi\langle\Gamma_c\rangle\rho
\end{aligned}
$$

  Moldauer, Phys. Rev. **177**, 1841 (1969),
  Simonius, Phys. Lett. B **52**, 279 (1974).

- Approach:
  - Adopt model for level density
  - Sample levels, widths consistent with T_c and ρ
  - Assume Wigner distribution for the level spacing
  - Assume Porter-Thomas distribution for widths
  - Use narrow resonance formula for reaction rate
  - Similar scheme used by Kawano, Talou, Weidenmüller, Phys. Rev. C **92**, 044617 (2015), in a different regime, to study width-fluctuation corrections

Notes on Statistical Distributions

- Wigner Distribution of level spacings, for a given J^π:

$$P_W(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi s^2}{4}\right)$$

$0 \leq s < \infty$, $s =$ actual spacing / average spacing

- Porter-Thomas Distribution of level widths:

$$P(t) = \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)$$

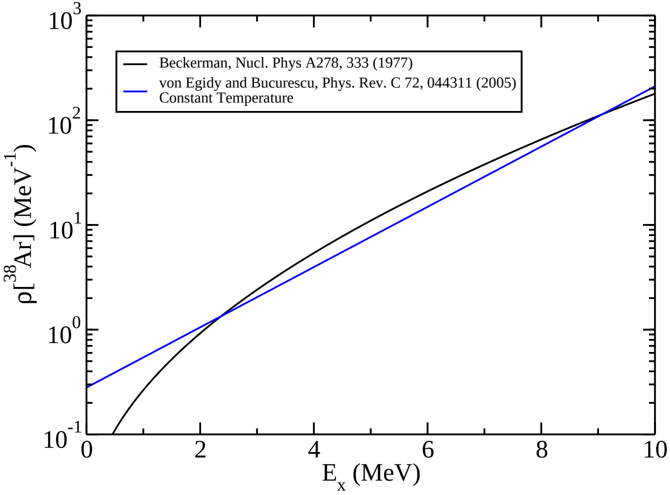
$-\infty < t < \infty \; , \quad \Gamma_c = t^2 \langle \Gamma_c \rangle$

- No correlations

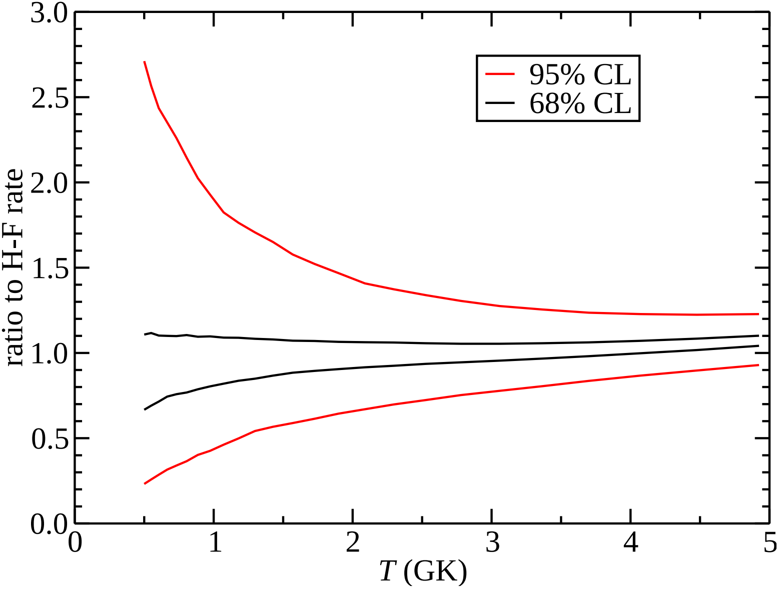
Level Density

- Use results from Mirror nucleus ^{38}Ar.
- von Egidy and Bucurescu, Phys. Rev. C **72**, 044311 (2005):
  - $\rho(U, J, \pi) = \frac{1}{2}\,\rho(U)\,f(J)$
  - $f(J) = \exp(-J^2/2\sigma^2) - \exp(-(J+1)^2/2\sigma^2)$
  - $\rho(U) = \frac{1}{T}\exp((U - E_0)/T)$
  - E_0, T, σ are given parameters

Level Density in ^{38}Ar

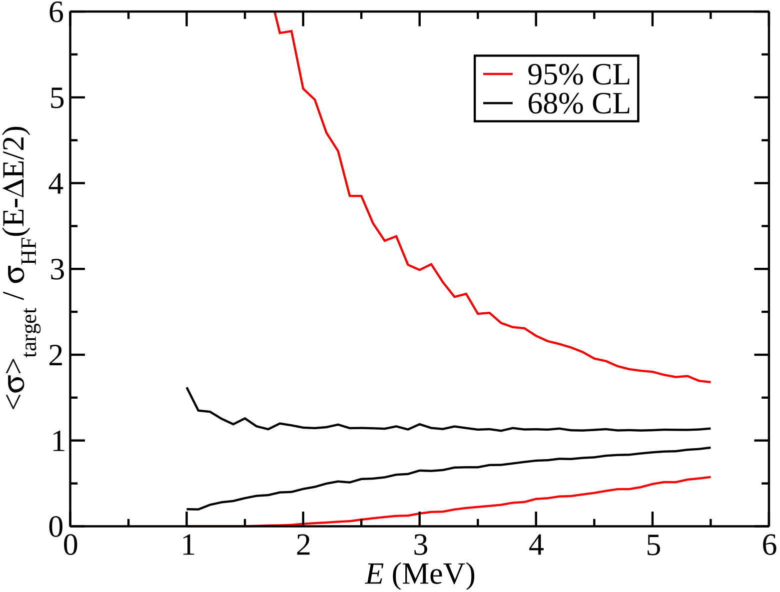


Monte Carlo Reaction Rates



Monte Carlo of Experimental Cross Section

Remember $\Delta E_{\rm exp} = 180$ keV?



Conclusions

- The variance in the reaction rate from statistical effects has been calculated.
- Variance is significant in the astrophysically-interesting regime.
- **To do**:
  - full R-matrix calculation
  - include interference effects
  - include known levels
  - include cross section measurements when available
- **Open Questions:**
  - How to model for α strength?
  - Near-threshold α-cluster enhancement?

Thanks to A. Voinov, S. Grimes, K. Chipps, and K. Schmidt.