Quantifying errors in effective theories for heavy nuclei

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Outline

Deformed systems

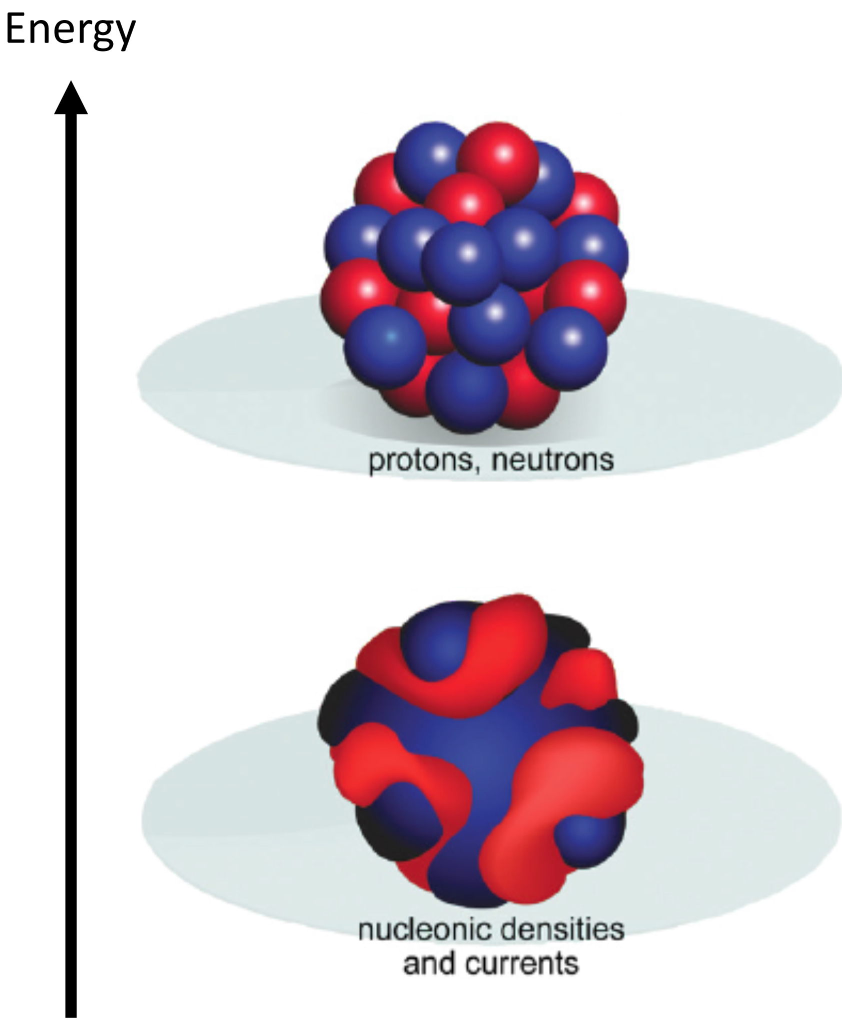
- Energy spectra
- Characterization of rotational bands

Spherical systems

- Energy spectra
- Electric quadrupole properties

Weak decays

- β decays to excited states
- $2\nu\beta\beta$ decays in the SSD approximation



Chiral EFT

- Nucleon and pion fields

BREAKDOWN SCALE Λ

ET

- Orientation angles $\xi \sim 100\mathrm{keV}$
- Phonons $\omega \sim 1000\mathrm{keV}$

Bertsch, Dean, Nazarewicz, SciDAC Rev. **6**, 42 (2007)

Rotations and vibrations in deformed nuclei

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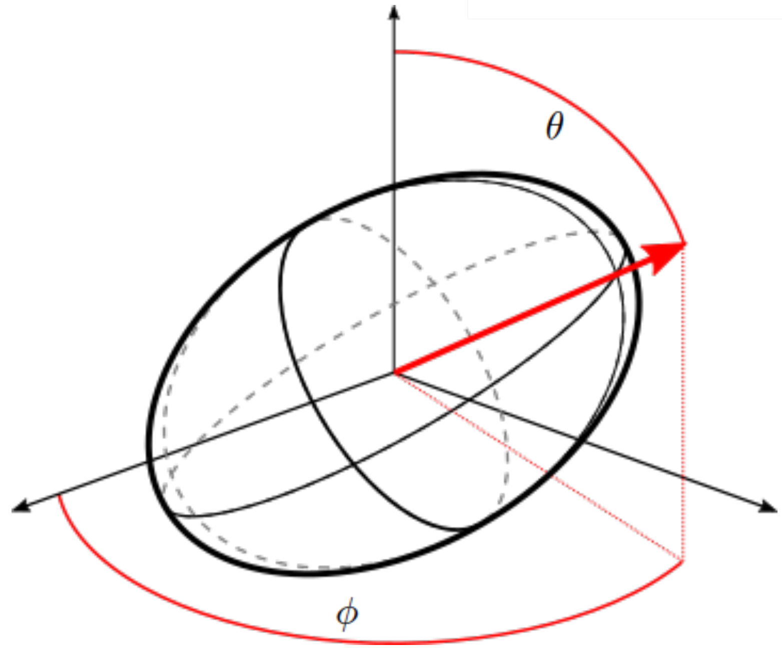
Degrees of freedom

$$v_{\pm 1} \equiv \mp \sqrt{\frac{1}{2}} \left(\dot\theta \pm i \dot\phi \sin\theta \right)$$

$$\Psi_0 = \zeta + \psi_0 \qquad \Psi_{\pm 2} = \psi_2 e^{\pm i 2\gamma}$$



The Hamiltonian

$$H \approx H^{(0)} + H^{(1)} + H^{(2)} + H^{(3)} + H^{(4)}$$

$H^{(0)}$: Harmonic excitations

$H^{(1)}$: Anharmonic corrections

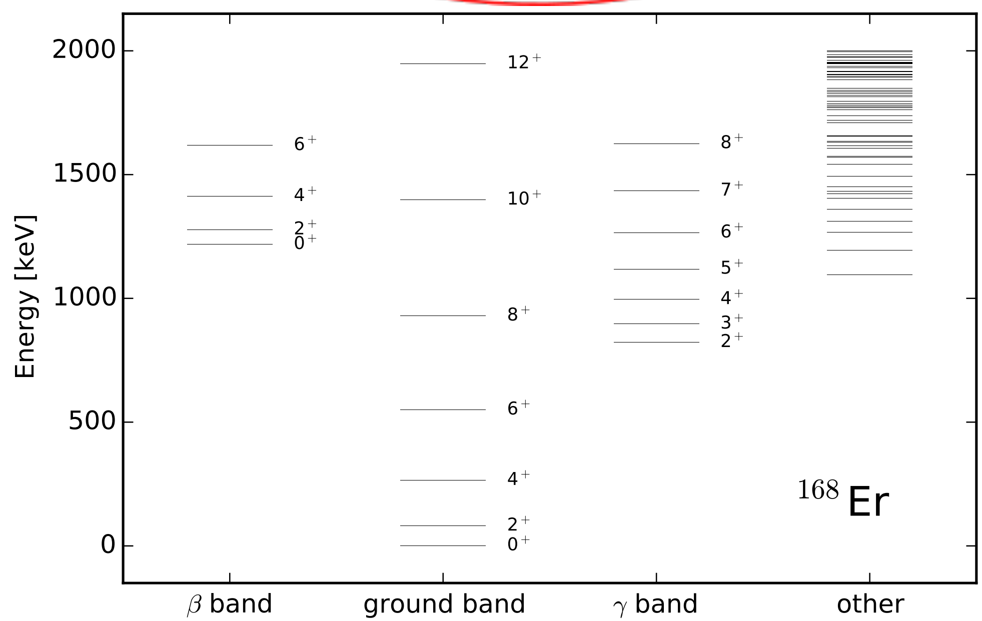
$H^{(2)}$: Rigid rotor

$H^{(3)}$: Off-diagonal corrections

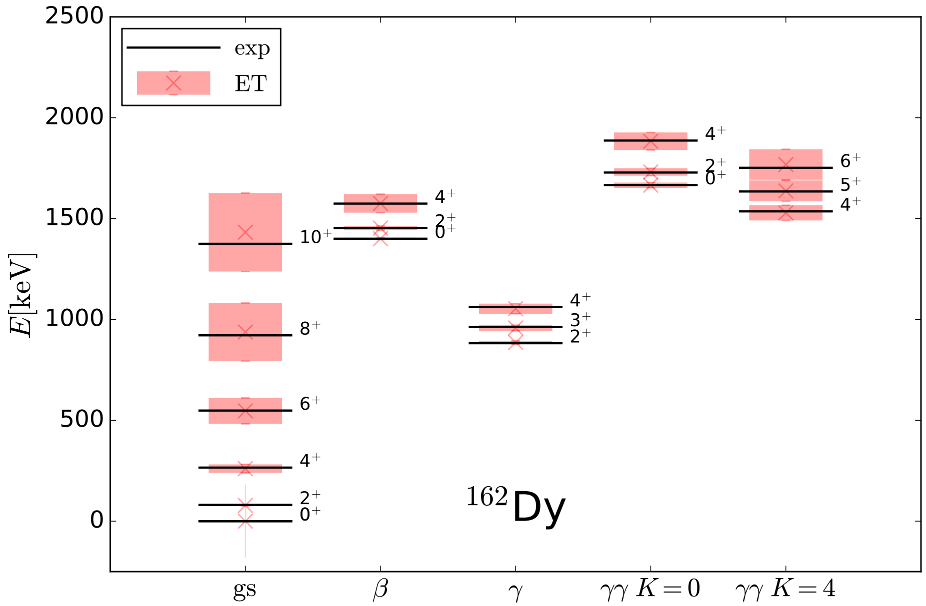
$H^{(4)}$: Corrections to moment of inertia

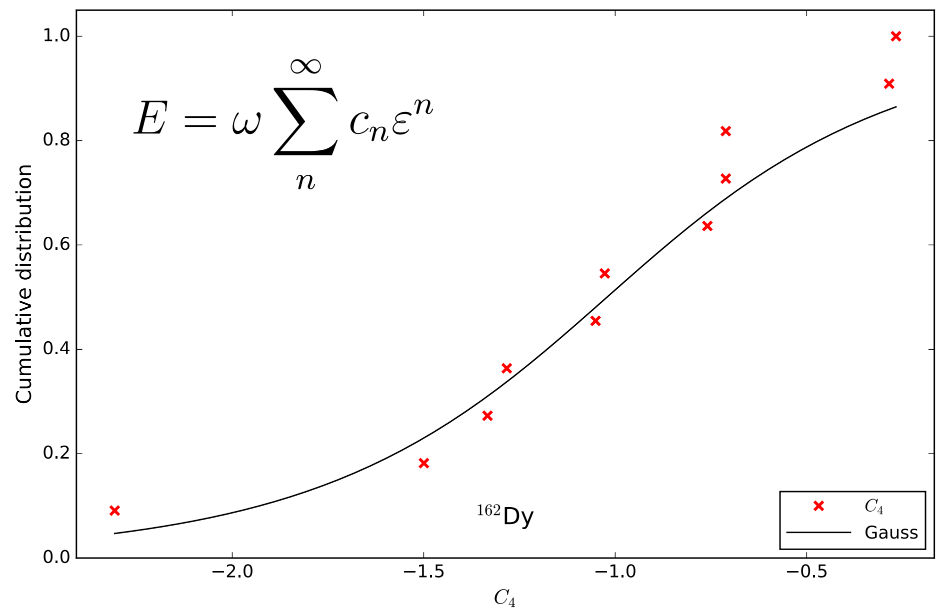
where the *n*-th term is of order

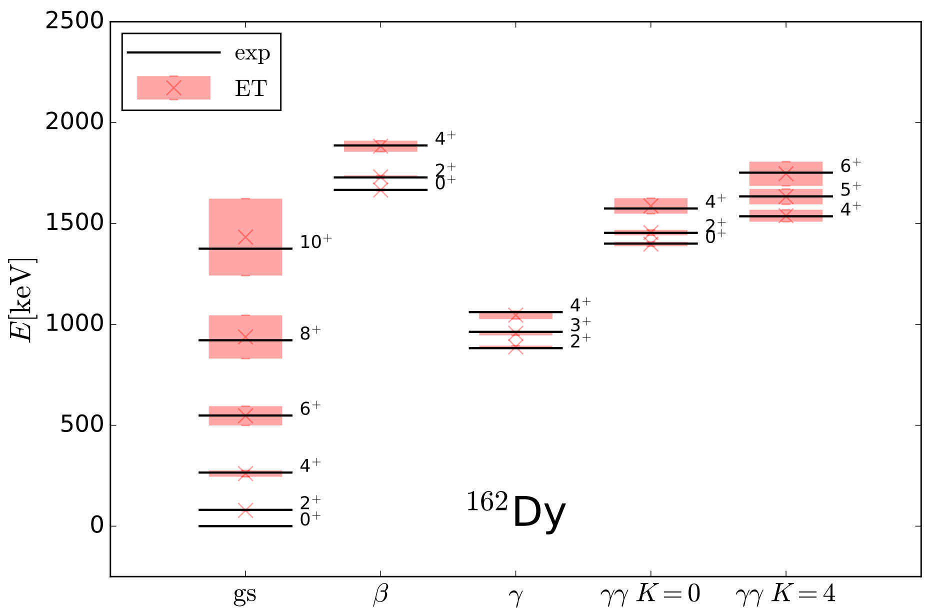
$$\mathcal{O}(\varepsilon^{n/2}) \qquad \varepsilon \equiv \xi/\omega$$



^{162}Dy case


exp
ET
E[keV]
gs
β
γ
$\gamma\gamma\ K=0$
$\gamma\gamma\ K=4$
^{162}Dy


$E = \omega \sum_{n}^{\infty} c_n \varepsilon^n$
Cumulative distribution
C_4
Gauss
^{162}Dy


exp
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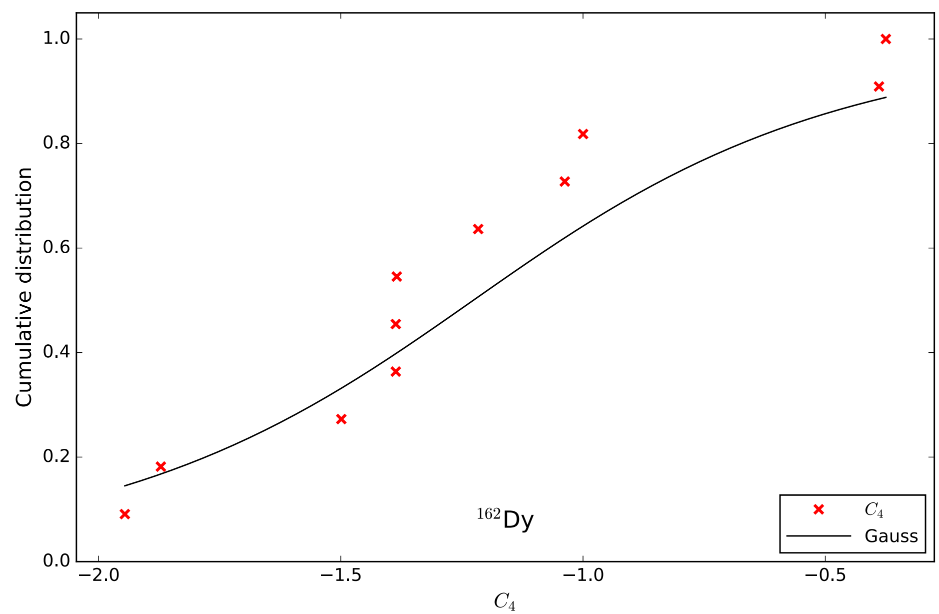
Cumulative distribution
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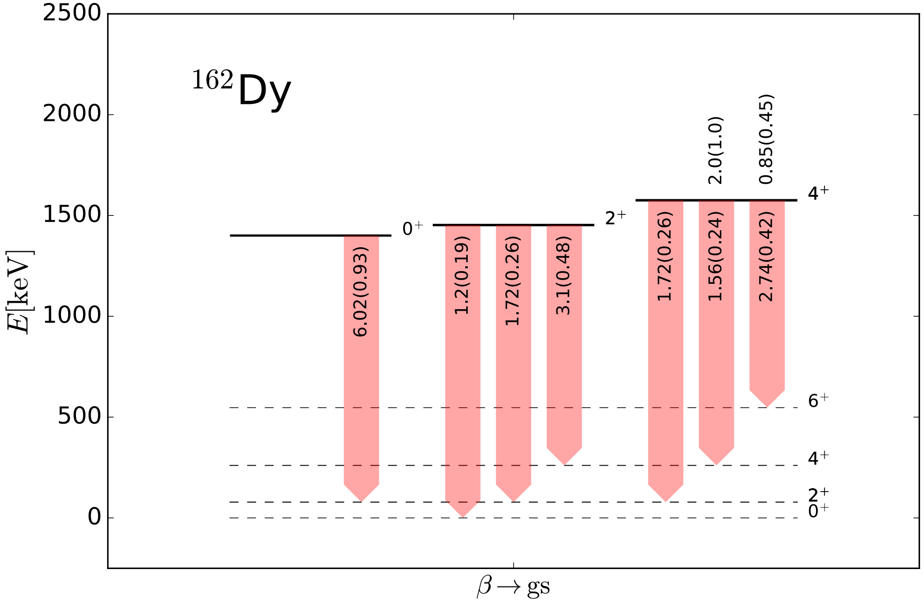
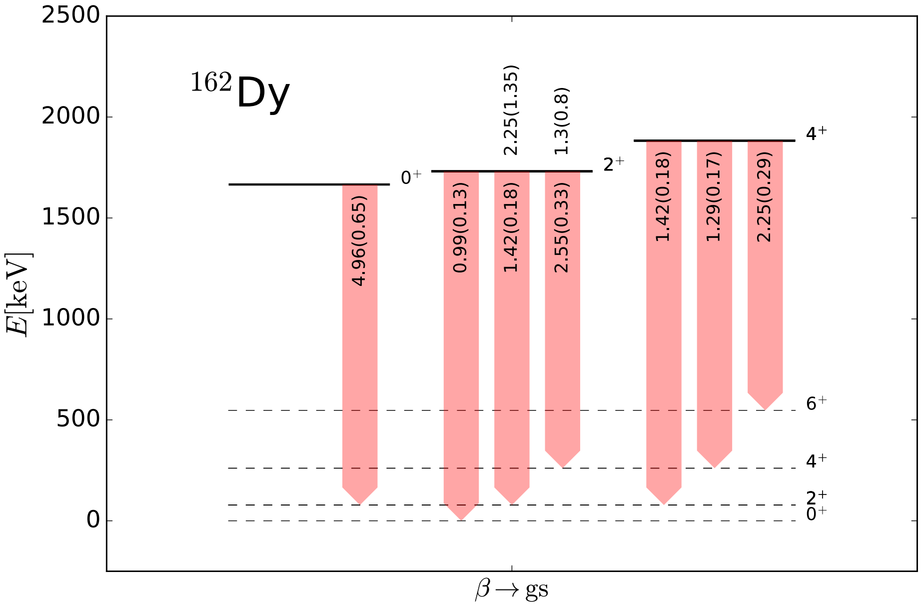




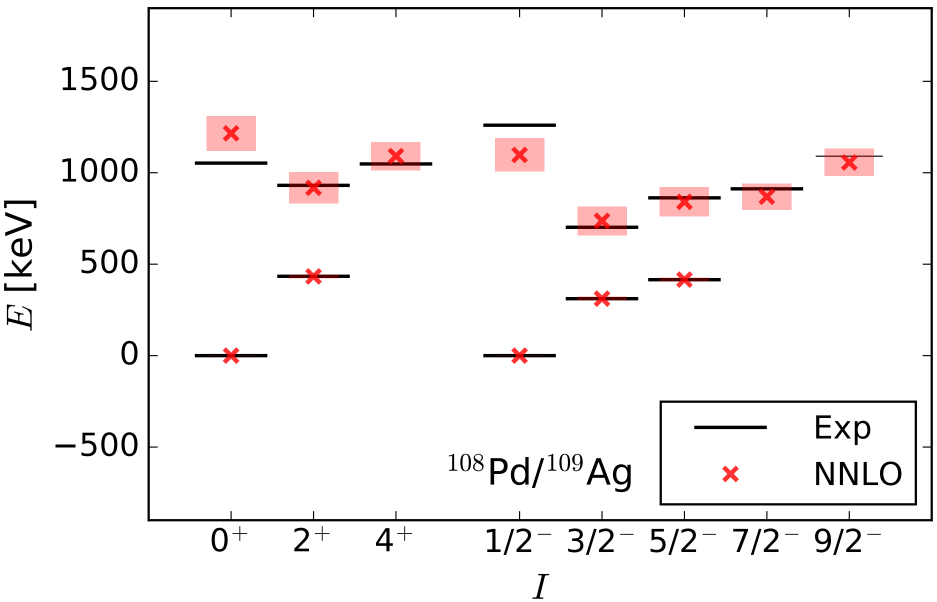
Data by courtesy of Ani Aprahamian

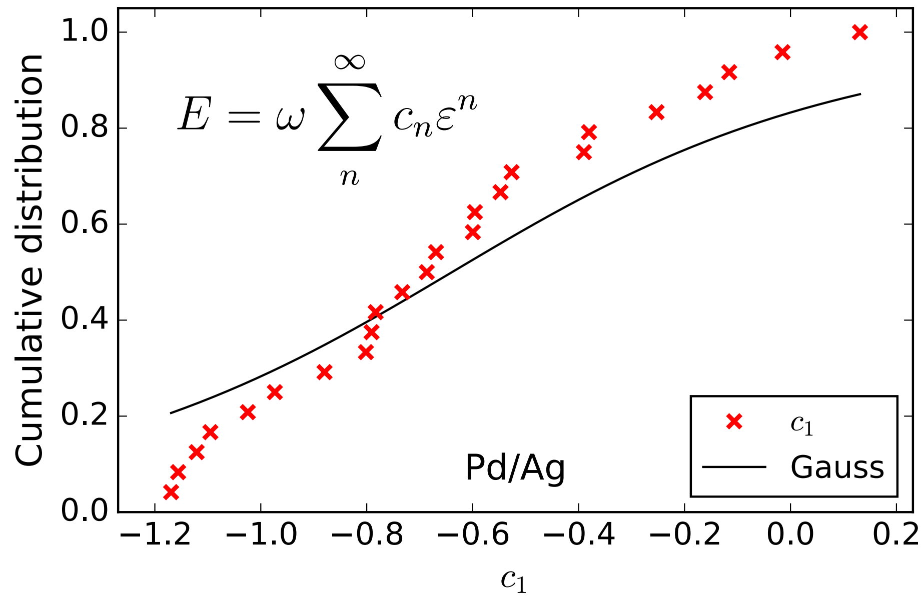
Electric quadrupole transition strengths depend on LECs that appear in the Hamiltonian

PDFs for these LECs allow us to estimate transition strengths between states in different bands

$$B(E2; i_\beta \xrightarrow{\beta} f_{gs}) = q^2 \frac{C_\beta^2}{4C_0^2\omega_0} \left(C_{I_i 020}^{I_f 0}\right)^2$$

$$C_\beta \sim C_\gamma \sim \xi^{-1/2}$$





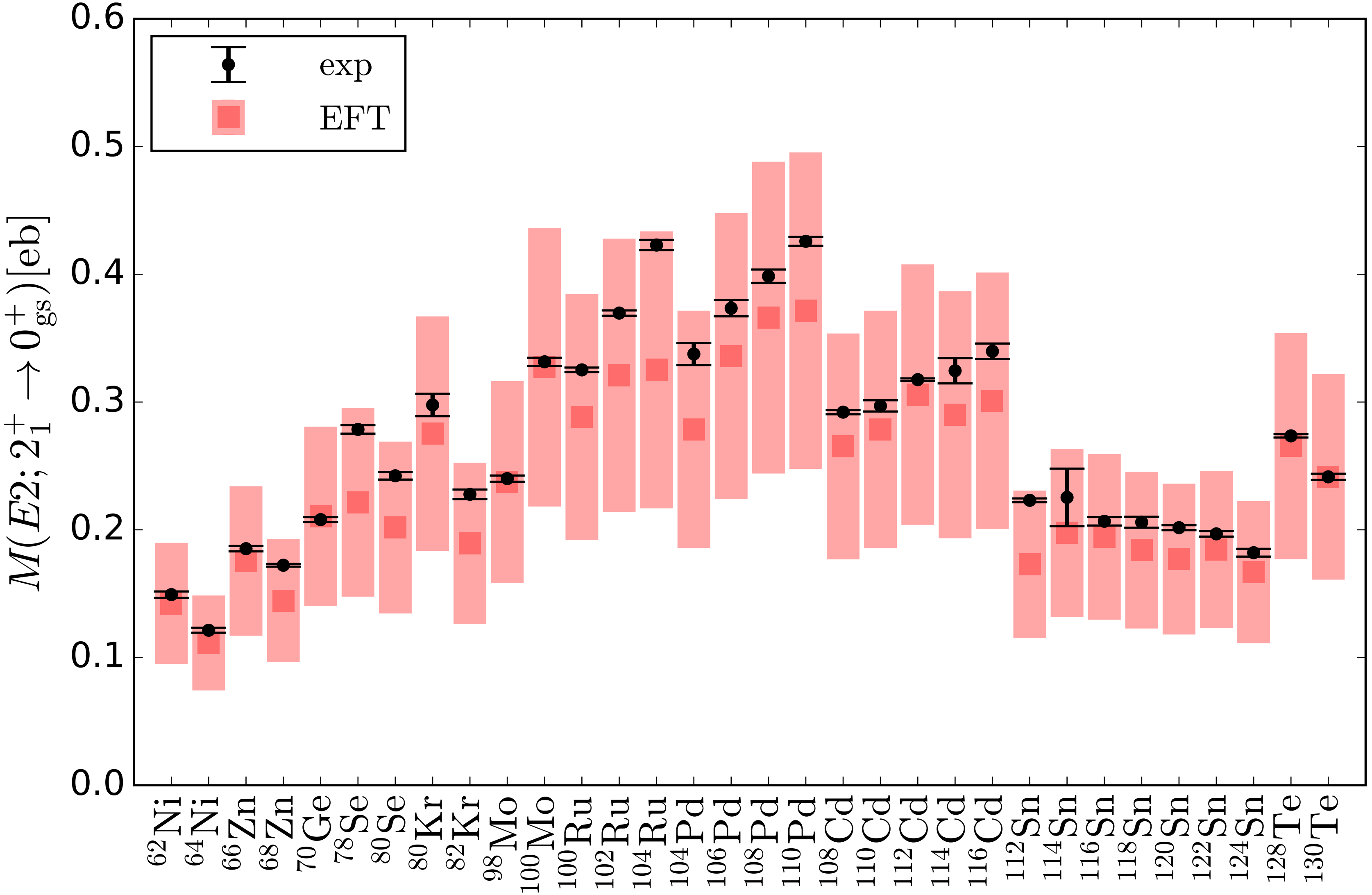
Hamiltonian written in terms of creation and annihilation operators

$$[d_\mu, d_\nu^\dagger] = \delta_{\mu\nu} \qquad \{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu}$$

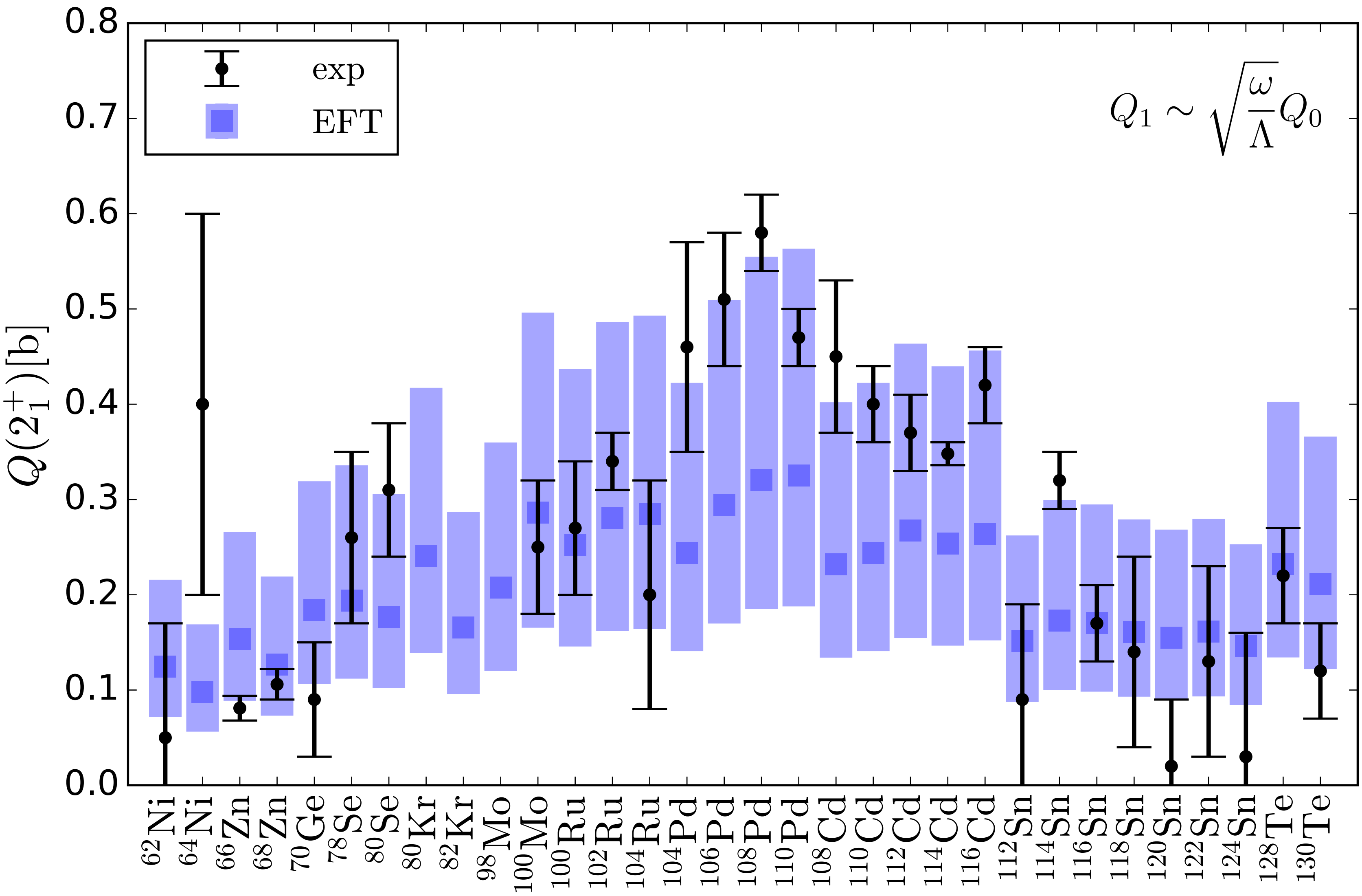
LO: Bohr and Mottelson model
NLO: Core-fermion interactions
NNLO: Anharmonicities

E2 transition matrix elements

$$\hat{Q} = Q_0 \left(d^\dagger + \tilde{d}\right) + Q_1 \left(d^\dagger \otimes \tilde{d}\right)^{(2)}$$

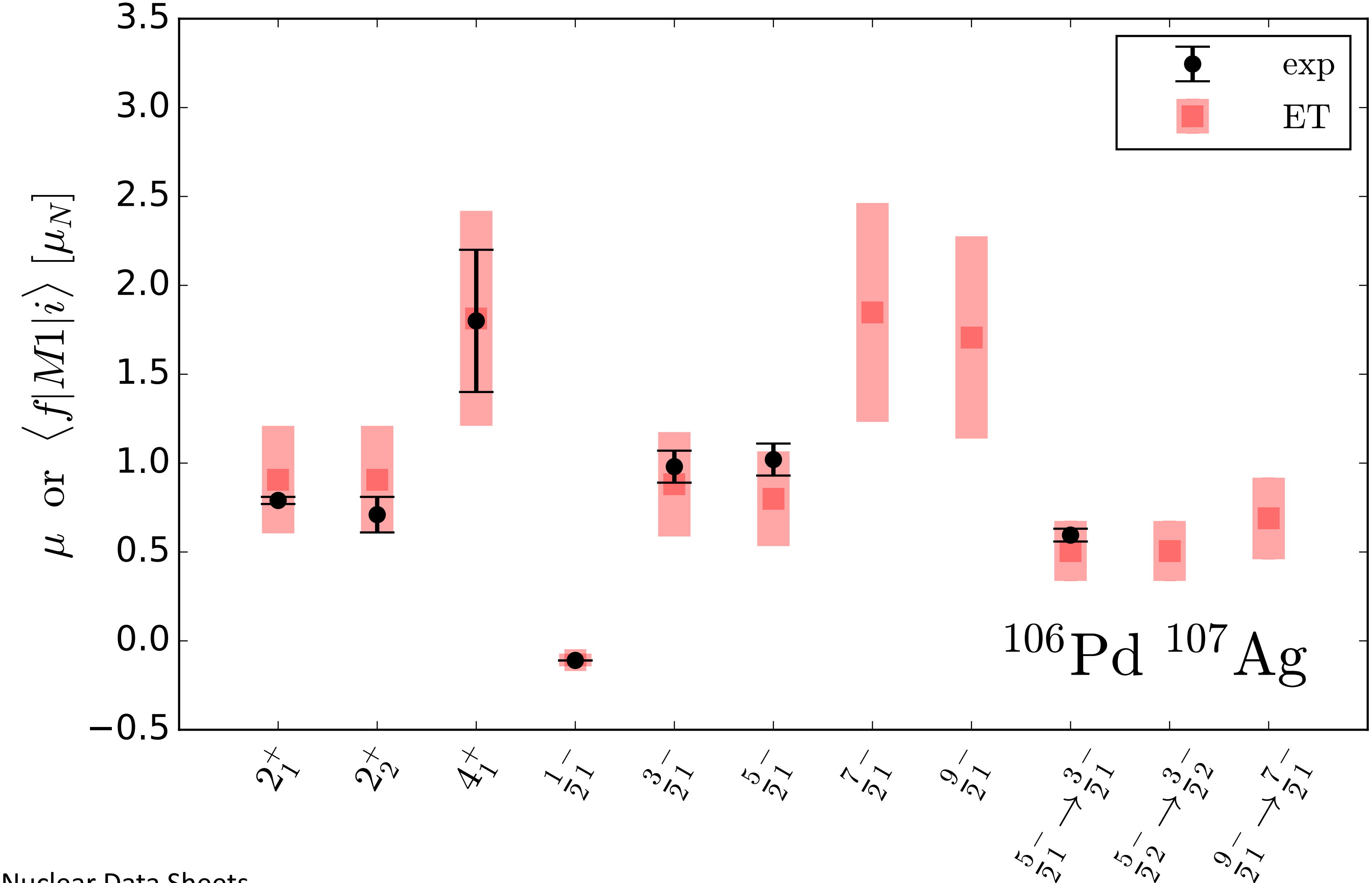


Static E2 moments



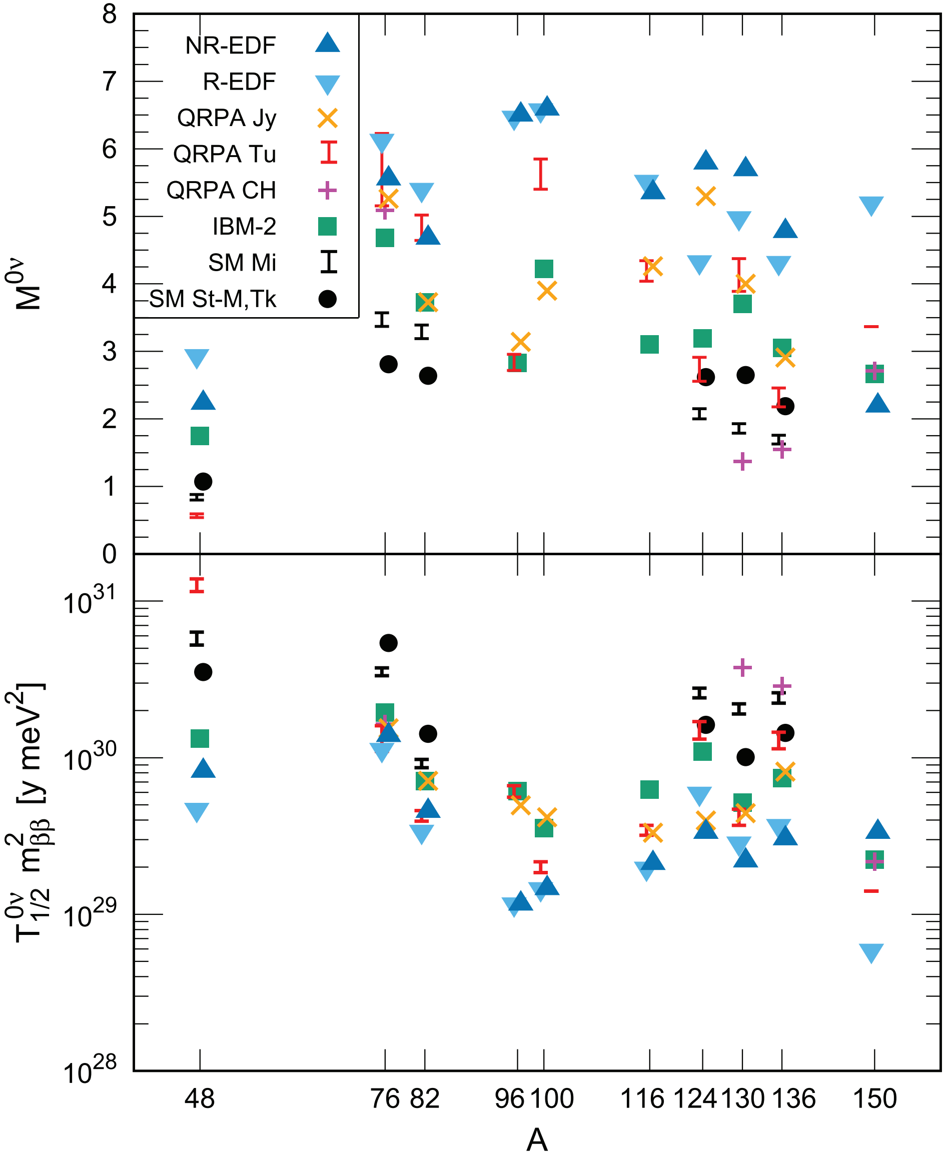
Static M1 static moments and M1 transition matrix elements

$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} \qquad \hat{\mathbf{J}} = \sqrt{10}\left(d^\dagger \otimes \tilde{d}\right)^{(1)} \qquad \hat{\mathbf{j}} = \frac{1}{\sqrt{2}}\left(a^\dagger \otimes \tilde{a}\right)^{(1)}$$



Data: Nuclear Data Sheets

Study of β and $2\nu\beta\beta$ decays



Matrix elements for $0\nu\beta\beta$ decays exhibit large disagreement from model to model

Provide $0\nu\beta\beta$ matrix elements with associated theoretical uncertainties

We start studying β and $2\nu\beta\beta$ for which experimental data is available

Engel, Menéndez; Rep. Prog. Phys. **80**, 046301 (2017)

Low-lying odd-odd states

$$|IM; j_p; j_n\rangle = \sum_{\mu\nu} C^{IM}_{j_n \mu j_p \nu} n^\dagger_\mu p^\dagger_\nu |0\rangle$$

From the power counting

$$\frac{C_{\beta\ell}}{C_\beta} \sim 0.58 \quad \text{and} \quad \frac{C_{\beta L\ell}}{C_\beta} \sim 0.33$$

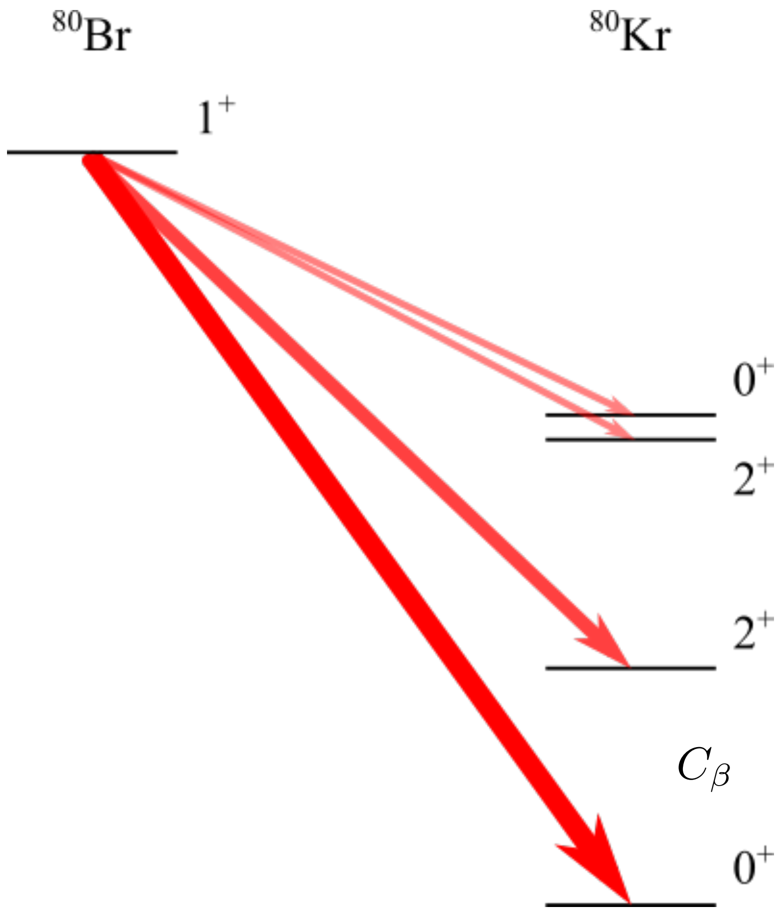
Effective Gamow-Teller operator

$$\begin{aligned}\hat{O}_\beta =& C_\beta \left(\tilde{p} \otimes \tilde{n}\right)^{(1)} \\ &+ \sum_\ell C_{\beta\ell} \left[\left(d^\dagger + \tilde{d}\right) \otimes \left(\tilde{p} \otimes \tilde{n}\right)^{(\ell)}\right]^{(1)} \\ &+ \sum_{L\ell} C_{\beta L\ell} \left[\left(d^\dagger \otimes d^\dagger + \tilde{d} \otimes \tilde{d}\right)^{(L)} \otimes \left(\tilde{p} \otimes \tilde{n}\right)^{(\ell)}\right]^{(1)}\end{aligned}$$

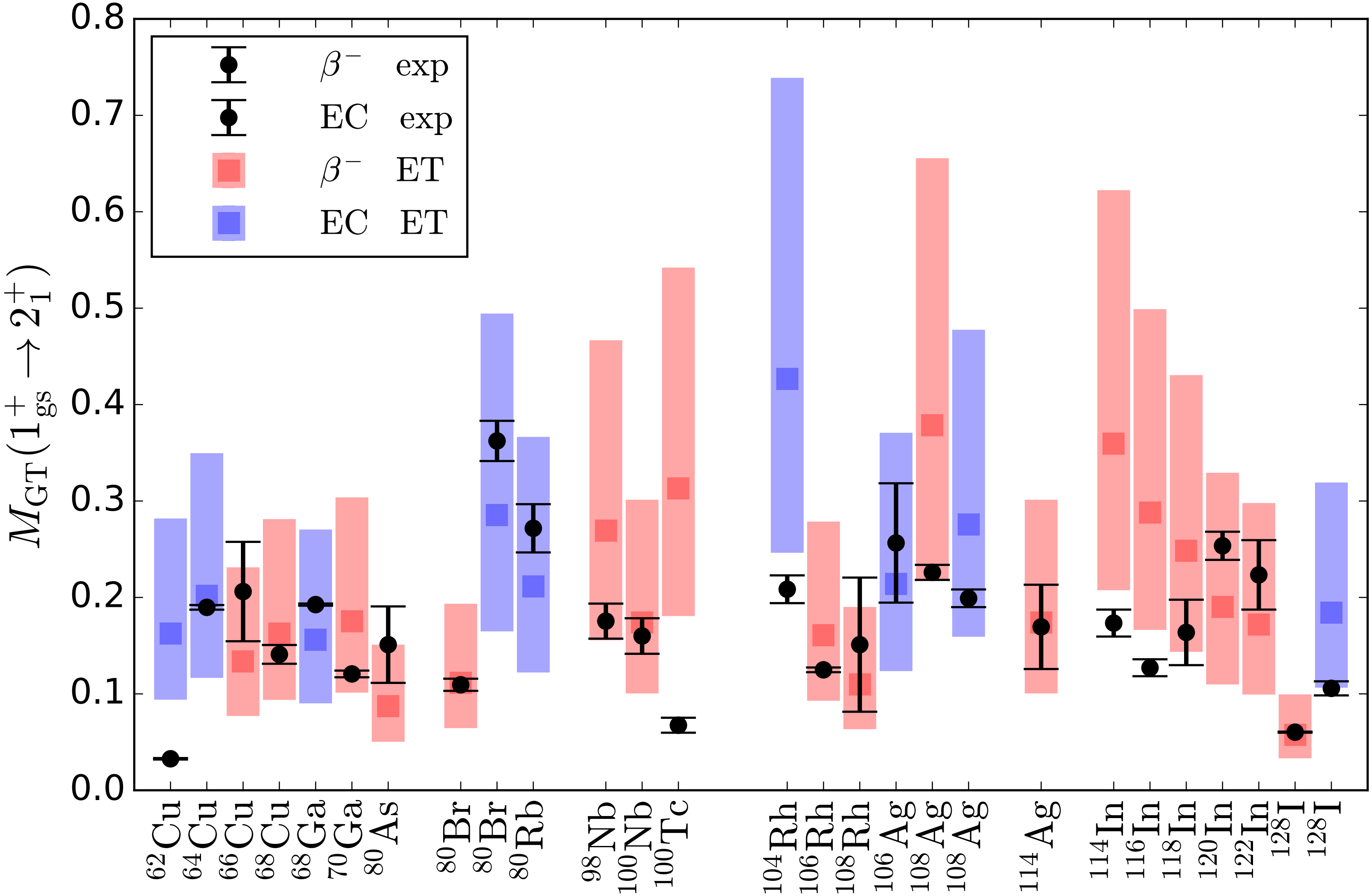
LO term:
- Couples states with $\Delta\mathcal{N} = 0$

NLO term:
- Couples states with $\Delta\mathcal{N} = 1$

NNLO term:
- Couples states with $\Delta\mathcal{N} = 2$

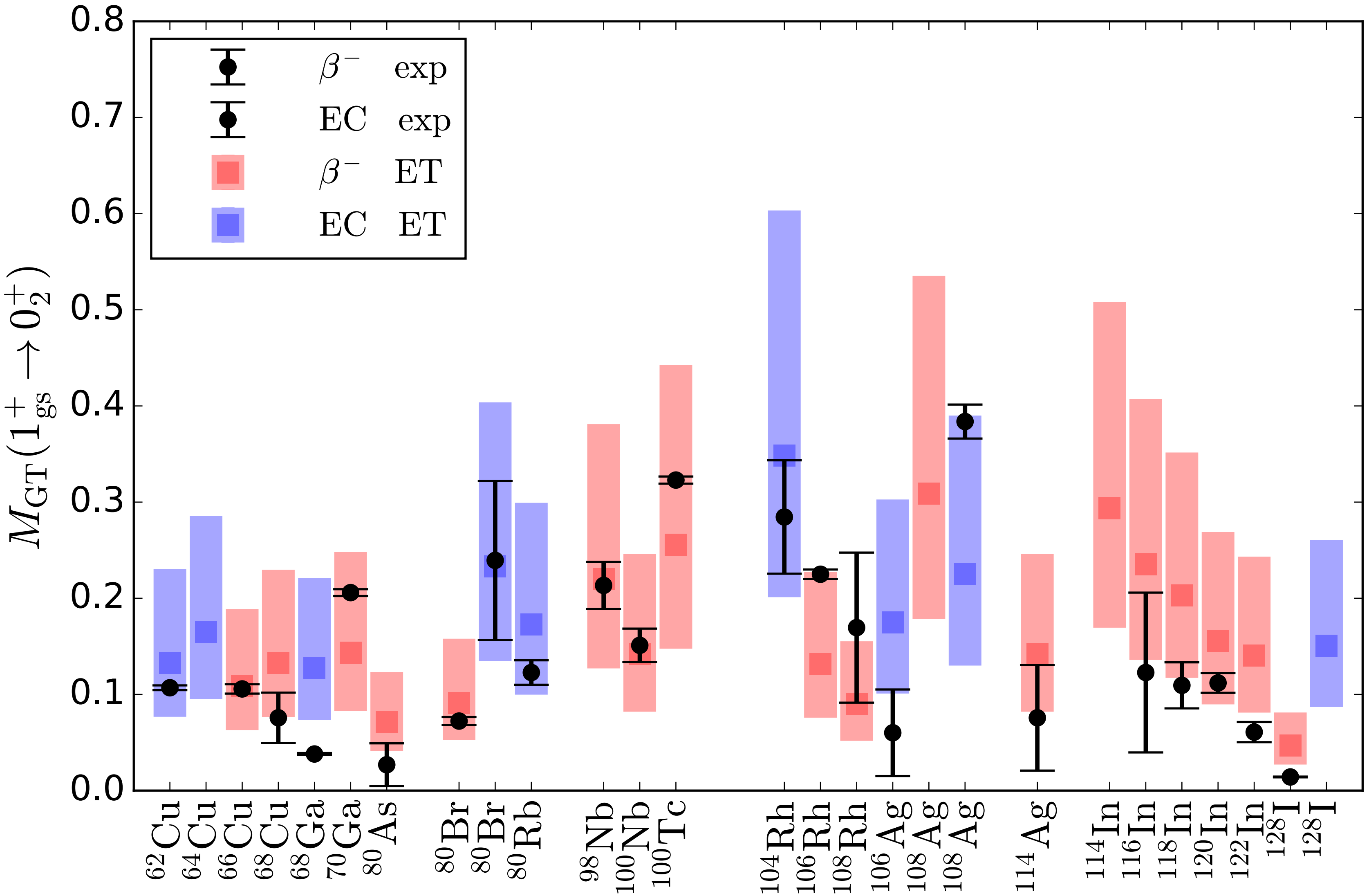


β decays to excited states



Coello Pérez, Menéndez, Schwenk; Phys. Rev. C **98**, 045501 (2018) Data: Nuclear Data Sheets

β decays to excited states



Coello Pérez, Menéndez, Schwenk; Phys. Rev. C **98**, 045501 (2018)   Data: Nuclear Data Sheets

GT matrix elements for $2\nu\beta\beta$ decay

$$M_{\rm GT}^{2\nu} = \sum_n \frac{\langle f || \sum_a \boldsymbol{\sigma}_a \tau_a^+ || 1_n^+ \rangle \langle 1_n^+ || \sum_b \boldsymbol{\sigma}_b \tau_b^+ || i \rangle}{D_{nf}/m_e}$$

SSD approximation

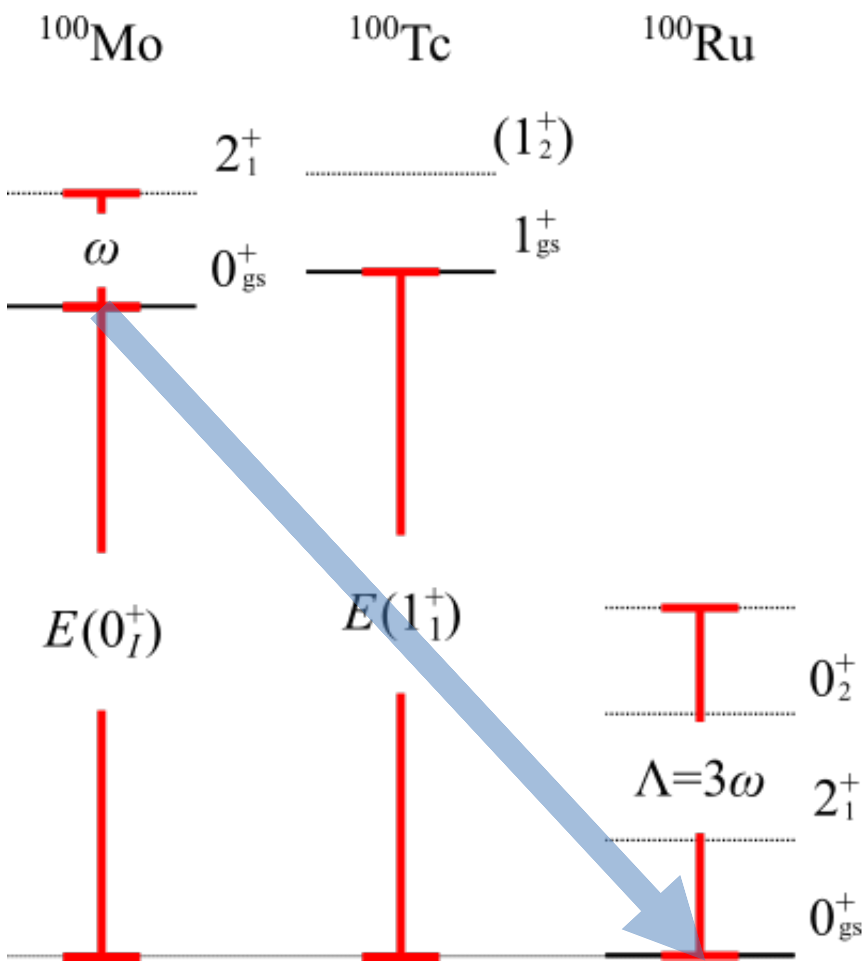
$$M_{\rm GT}^{2\nu}(i \to f) \approx \frac{M_{\rm GT}(1_1^+ \to 0_f^+) M_{\rm GT}(0_i^+ \to 1_1^+)}{D_{1f}/m_e c^2}$$

Percentual uncertainty estimate

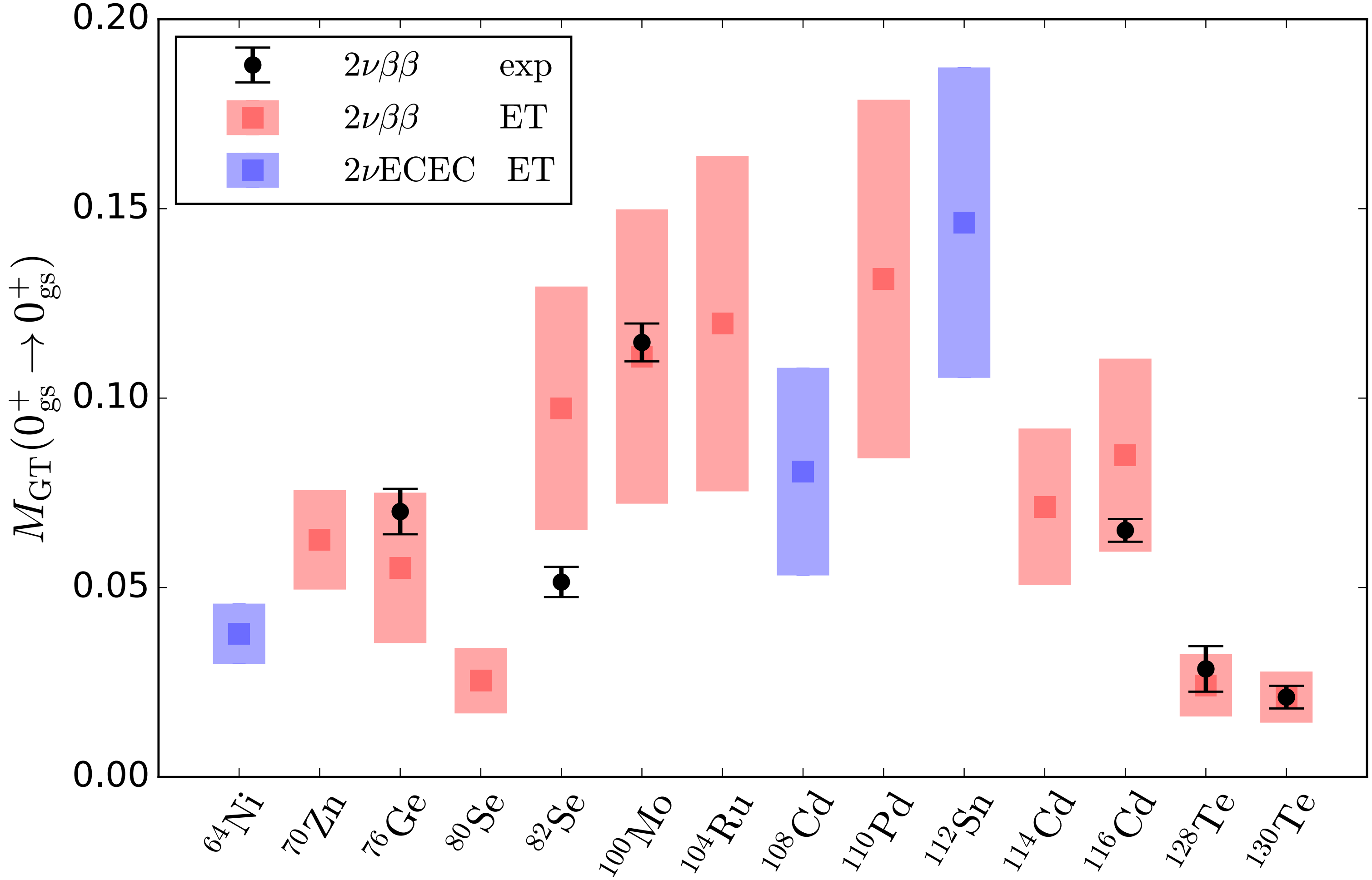
$$\delta(\text{gs} \to \text{gs}) = \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11}+\omega}{\omega}\right)$$

where

$$\Phi(z,s,a) \equiv \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s}$$



Good agreement with experiment where data exist

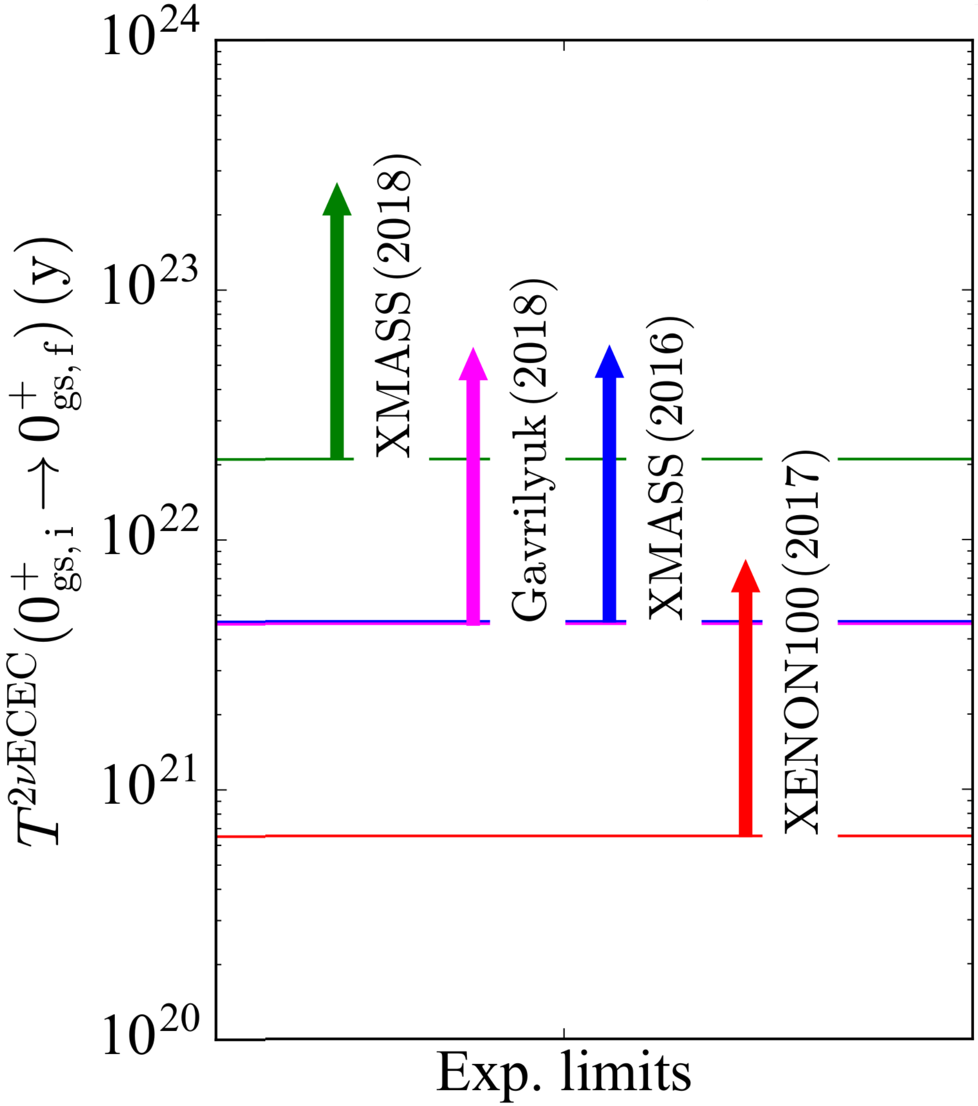


Coello Pérez, Menéndez, Schwenk; Phys. Rev. C **98**, 045501 (2018) Data: Barabash, Nucl. Phys. A **935**, 52 (2015)

Half-life for the 2νECEC on ^{124}Xe

Large xenon detectors for dark matter experiments have enough sensitivity to observe the double-EC on ^{124}Xe

The most recent experimental lower limit for its half-life excludes theoretical calculations (most of them performed with the QRPA)



Aprile et al. (XENON100), Phys. Rev. C **95**, 024605 (2017)

Abe et al. (XMASS), Progr. Theor. Exp. Phys. **2018**, 053D03 (2018)

Gavrilyuk et al., Phys. Part. Nucl. **49**, 36 (2018)

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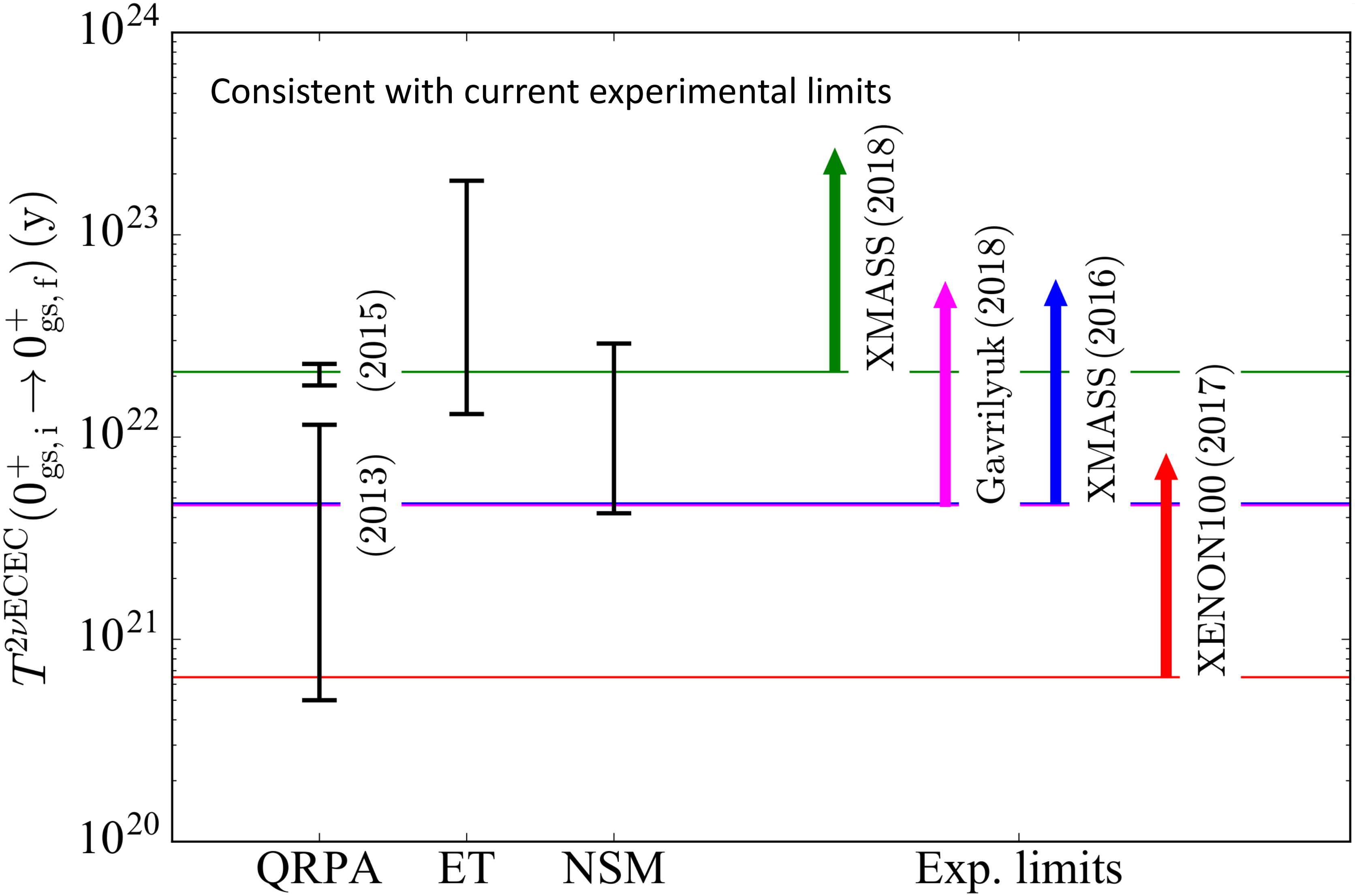

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Coello Pérez, Menéndez, Schwenk; arXiv:1809.04443

Summary

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We constructed effective theories to describe the low-energy properties of heavy nuclei

The systematic construction of the operators allows us to employ Bayesian methods to quantify the theoretical uncertainty associated to their matrix elements

The low-energy spectra and electromagnetic properties of heavy nuclei is consistently described once the theoretical uncertainty is taken into account

In spherical systems, the ET consistently describes observed $2\nu\beta\beta$ decays once the SSD approximation error is taken into account

A correlation between the double GT and the $0\nu\beta\beta$ matrix elements might allow us to provide an uncertainty for the latter

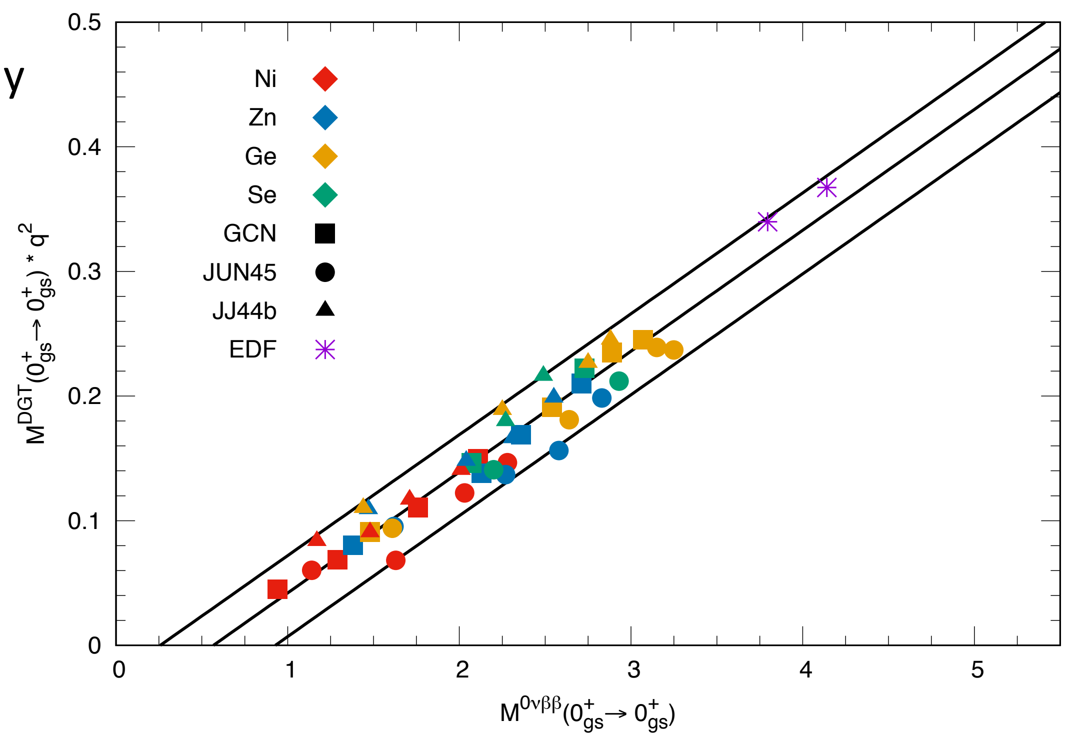

Figure by courtesy of Javier Menéndez