Quantifying Errors from Chiral Effective Field Theory

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Physical Motivation
Scales in Physics

Grav. force (short distances):

\[ F = -mg \]
Grav. force (short distances):

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Grav. force (large distances):

\[ F = -\frac{GMm}{r^2} \]

The laws look quite different!
Scales in Physics

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Connected via series expansion about radius of Earth \( R \):

\[ F \approx -mg + 2mg \left( \frac{r - R}{R} \right) - 3mg \left( \frac{r - R}{R} \right)^2 + \mathcal{O} \left( \left( \frac{r - R}{R} \right)^3 \right) \]
Scales in Physics

Grav. force (short distances):

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Can fit unknown parameters to data $\Rightarrow$ inverse problem!

$$F \approx a_0 + a_1 \left( \frac{r - R}{R} \right) + a_2 \left( \frac{r - R}{R} \right)^2 + \mathcal{O} \left[ \left( \frac{r - R}{R} \right)^3 \right]$$
Scales in Physics

Grav. force (short distances): 
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The laws look quite different!

Use prior info from physics:

\[ F \approx mg \left\{ a'_0 + a'_1 \left( \frac{r - R}{R} \right) + a'_2 \left( \frac{r - R}{R} \right)^2 + O \left[ \left( \frac{r - R}{R} \right)^3 \right] \right\} \]
Scales in Physics

Grav. force (short distances):

\[ F = -mg \]

Grav. force (large distances):

\[ F = \frac{GMm}{r^2} \]

The laws look quite different!

Propagate full uncertainty

\[
F \approx mg \left\{ a'_0 + a'_1 \left( \frac{r - R}{R} \right) + a'_2 \left( \frac{r - R}{R} \right)^2 + \mathcal{O} \left( \left( \frac{r - R}{R} \right)^3 \right) \right\}
\]
• There is interesting physics at all scales
Predictions in Low-Energy Nuclear Physics

- There is interesting physics at all scales
- Nuclear physics spans lengths from $10^{-15} - 10^9$ m
• There is interesting physics at all scales
• Nuclear physics spans lengths from $10^{-15}$–$10^9$ m
• Fine details at one level of analysis do not affect the physics at a coarser level of analysis
• There is interesting physics at all scales
• Nuclear physics spans lengths from $10^{-15} - 10^9$ m
• Fine details at one level of analysis do not affect the physics at a coarser level of analysis
• Start simple → add corrections to reach desired precision.
Chiral EFT

<table>
<thead>
<tr>
<th></th>
<th>NN force</th>
<th>3N force</th>
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<tbody>
<tr>
<td>LO</td>
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<tr>
<td>NLO</td>
<td><img src="NLO.png" alt="Diagram" /></td>
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<tr>
<td>N^2LO</td>
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- An expansion in the nuclear force
- Ordered by increasing factors of small parameter $Q$
- Truncation $\rightarrow$ main source of uncertainty
- Force convergence $\neq$ prediction convergence
- The debate on the “best” expansion is ongoing

We want to:

- Fit unknown parameters $\vec{a}$, or low-energy constants, with discrepancy $\delta$
- Quantify uncertainty in predictions (aka observables)
- Test existing EFTs, uncover physics
Chiral EFT

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<tr>
<td>NLO</td>
<td><img src="image3.png" alt="Diagram" /></td>
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<tr>
<td>$N^2$LO</td>
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Chiral EFT

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We want to

- Fit unknown parameters $\vec{a}$, or low-energy constants, with discrepancy $\delta y_{\text{th}}$
- Quantify uncertainty in predictions (aka observables) $y_{\text{th}}$
- Test existing EFTs, uncover physics
$y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}}$
To theorists, magic

\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}} \]

Parameters

Discrepancy
To theorists, magic

Parameters

Discrepancy

Can we build this?

Can we use it?

\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{exp}} \]
To theorists, magic

Parameters

Discrepancy

Can we build this?

Can we use it?

5

rigorous fit

\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}} \]
\[ y_{\exp}(x) = y_{\text{th}}(x, \bar{a}) + \delta y_{\text{th}}(x) + \delta y_{\exp} \]
\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}} \]

Can we build this?
Can we use it?
• Theoretical predictions could look like the following

\[ \Delta y_n = y_{\text{ref}} c_n Q_n \{ y_0 \} \]
Toy Predictions

• Theoretical predictions could look like the following

\[
\Delta y_n = y_{\text{ref}} c_n Q_n \{ y_0, y_1 \}
\]

\[
\begin{align*}
0:00 & \quad 0:25 & \quad 0:50 & \quad 0:75 & \quad 1:00 \\
\end{align*}
\]

Predictions

\[ y_0 \rightarrow \text{LO} \]
\[ y_1 \rightarrow \text{NLO} \]
• Theoretical predictions could look like the following

\[ \Delta y_n = y_{\text{ref}} c_n Q_n \]

\[ \{y_0, y_1, y_2\} \]

\[ y_0 \rightarrow \text{LO} \]
\[ y_1 \rightarrow \text{NLO} \]
\[ y_2 \rightarrow \text{N}^2\text{LO} \]
Theoretical predictions could look like the following

\[ \begin{align*}
\Delta y_n &= y_{\text{ref}} c_n Q_n \\
\end{align*} \]

\{y_0, y_1, y_2, y_3\}

\[ y_0 \rightarrow \text{LO} \]
\[ y_1 \rightarrow \text{NLO} \]
\[ y_2 \rightarrow \text{N}^2\text{LO} \]
\[ \vdots \]
\[ y_k \rightarrow \text{N}^k\text{LO} \]
• Theoretical predictions could look like the following
• One can change variables for convenience/insight.

\[ y_0 = y_0 \]
• Theoretical predictions could look like the following
• One can change variables for convenience/insight.

\[ y_1 = y_0 + \Delta y_1 \]
• Theoretical predictions could look like the following
• One can change variables for convenience/insight.

\[ y_2 = y_0 + \Delta y_1 + \Delta y_2 \]
Theoretical predictions could look like the following.

One can change variables for convenience/insight.

\[ \Delta y_n = y_{ref} c_n Q^n \]

\[ y_3 = y_0 + \Delta y_1 + \Delta y_2 + \Delta y_3 \]
Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.
- \[ \Delta y_n = y_{\text{ref}} c_n Q^n \]

\[ y_0 = y_{\text{ref}} \left[ c_0 Q^0 \right] \]
Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.
- \( \Delta y_n = y_{ref} c_n Q^n \)

\[
y_1 = y_{ref} \left[ c_0 Q^0 + c_1 Q^1 \right]
\]

![Graphs showing predictions, differences in predictions, and prediction coefficients.](image)
• Theoretical predictions could look like the following
• One can change variables for convenience/insight.
• $\Delta y_n = y_{\text{ref}} c_n Q^n$

$$y_2 = y_{\text{ref}} \left[ c_0 Q^0 + c_1 Q^1 + c_2 Q^2 \right]$$
Toy Predictions

- Theoretical predictions could look like the following
- One can change variables for convenience/insight.
- \[ \Delta y_n = y_{ref} c_n Q^n \]
  
  \[ y_3 = y_{ref} \left[ c_0 Q^0 + c_1 Q^1 + c_2 Q^2 + c_3 Q^3 \right] \]
Coefficients from NN scattering look like our toy model!
Statistical Model
The Hierarchical Model

• Decompose prediction

\[ y_k = y_0 + \sum_{n=1}^{k} \Delta y_n \]
The Hierarchical Model

- Decompose prediction

\[ y_k = y_0 + \sum_{n=1}^{k} \Delta y_n \]
\[ = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n \]
The Hierarchical Model

\[ y_k = y_0 + \sum_{n=1}^{k} \Delta y_n = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n \]

- Decompose prediction
- Put priors on \( c_n \) (and \( Q \))

\[ \text{pr}(c_n | \theta)^{\text{iid}} \sim \mathcal{GP}(\mu, \sigma^2 R_{\ell}) \]
The Hierarchical Model

- Decompose prediction
  \[ y_k = y_0 + \sum_{n=1}^{k} \Delta y_n = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n \]

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- Learn \( \theta \) and \( Q \)
The Hierarchical Model

Hyperparameters

• Decompose prediction

\[ y_k = y_0 + \sum_{n=1}^{k} \Delta y_n \]
\[ = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n \]

• Put priors on \( c_n \) (and \( Q \))

\[ \text{pr}(c_n | \theta) \overset{\text{iid}}{\sim} \mathcal{GP}(\mu, \sigma^2 R_{\ell}) \]

• Learn \( \theta \) and \( Q \)

• Predict \( \text{pr}(y | \mathcal{D}) \)
Gaussian Process Priors on Observable Coefficients

\[ c_n | \theta \sim \mathcal{GP}(\mu, \sigma^2 R_{\ell}) \]
Gaussian Process Priors on Observable Coefficients

\[ c_n \mid \theta \sim \mathcal{GP}(\mu, \sigma^2 R_{\ell}) \]

Conjugate priors:

\[ \mu \mid \sigma^2 \sim \mathcal{N}(m, \sigma^2 V) \]
\[ \sigma^2 \sim \text{IG}(a, b) \]
Model Building

**Main equation**

\[ y_k = y_{ref} \sum_{n=0}^{k} c_n Q^n \]

\[ c_n \equiv \frac{y_n - y_{n-1}}{y_{ref} Q^n} \]

**Predictions**

Prediction Coefficients

\[ x \]

\[ y \]

\[ c \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

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\[ y_k = y_{ref} \sum_{n=0}^{k} c_n Q^n \]

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Best Prediction

\[ y_3 \]

Prediction Coefficients

\[ c_0 \]
\[ c_1 \]
\[ c_2 \]
\[ c_3 \]

\[ pr(c_n) \]
Model Building

Main equation

\[ y = y_{\text{ref}} \sum_{n=0}^{\infty} c_n Q^n \]

\[ c_n \equiv \frac{y_n - y_{n-1}}{y_{\text{ref}} Q^n} \]

Full Prediction

Higher Order Coefficients
Model Building

Main equation

\[ y = y_{ref} \sum_{n=0}^{\infty} c_n Q^n \]

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Full Prediction

Higher Order Coefficients

\( pr(y) \)

\( pr(c_n) \)

0.0 0.2 0.4 0.6 0.8 1.0

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pr(y)

pr(c_n)
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Full Prediction

Higher Order Coefficients
Discrepancy Distribution

Remember the goal:

\[ y_{\exp}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\exp} \]
Discrepancy Distribution

Remember the goal:

\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \vec{a}) + \delta y_{\text{th}}(x) + \delta y_{\text{exp}} \]

Our convergence assumptions

\[ \text{pr}(c_n | \theta) \overset{\text{iid}}{=} \mathcal{G}\mathcal{P}(\mu, \sigma^2 R) \]

\[ \delta y_{\text{th}}(x) = y_{\text{ref}} \sum_{n=k+1}^{\infty} c_n Q^n \]
Discrepancy Distribution

Remember the goal:

\[ y_{\text{exp}}(x) = y_{\text{th}}(x, \tilde{a}) + \delta y_{\text{th}}(x) + \delta y_{\exp} \]

Our convergence assumptions

\[ \text{pr}(c_n | \theta) \overset{\text{iid}}{=} \mathcal{GP}(\mu, \sigma^2 R_\ell) \]

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Gaussian sum rules

\[ a\mathcal{N}(\mu_1, \Sigma_1) + b\mathcal{N}(\mu_2, \Sigma_2) = \mathcal{N}(a\mu_1 + b\mu_2, a^2\Sigma_1 + b^2\Sigma_2) \]
Discrepancy Distribution

Remember the goal:

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Discrepancy Distribution

\[ \text{pr}(\delta y_{\text{th}} | \theta) = \mathcal{GP}(\mu_{\text{th}}, \Sigma_{\text{th}}) = \mathcal{GP} \left( \frac{\mu Q^{k+1}}{1 - Q}, y_{\text{ref}}^2 \frac{\sigma^2 Q^{2(k+1)}}{1 - Q^2} R_\ell \right) \]
Implications for EFT Fitters

**Standard $\chi^2$**

$$\sum_i \frac{[y_{\text{exp},i} - y_{\text{th},i}(\vec{a})]^2}{\sigma_{\text{exp}}^2} = \sum_i \frac{r(x_i, \vec{a})^2}{\sigma_{\text{exp}}^2}$$

- Gaussian process correlations propagate via $\Sigma_{\text{th}}$ matrix (computed once!)
- Different correlation assumptions $\rightarrow$ different results!
Implications for EFT Fitters

\[ \chi^2_{\text{mod}}(\bar{a}) = \bar{r}^T(\bar{a})(\Sigma_{\text{th}} + \Sigma_{\text{exp}})^{-1}\bar{r}(\bar{a}) \]
Implications for EFT Fitters

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Prediction

$$y_{\exp}(x) \approx y_{\th}(x, \vec{a})$$

$\chi^2$ + Theory Error

$$\chi^2_{\text{mod}}(\vec{a}) = \vec{r}^\top(\vec{a})(\Sigma_{\th} + \Sigma_{\exp})^{-1}\vec{r}(\vec{a})$$
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**Prediction**

$$y_{\text{exp}}(x) \approx y_{\text{th}}(x, \vec{a})$$

**Prediction + Theory Error**

$$p_r(y_{\text{exp}}) = \mathcal{N}[y_{\text{th}}(x, \vec{a}) + \mu_{\text{th}}, \Sigma_{\text{th}}]$$
Implications for EFT Fitters

Standard $\chi^2$

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- Different correlation assumptions $\rightarrow$ different results!
What You Get for Free: Max Energy Insensitivity

- $y$ axis: posterior median ± 1σ
- $x$ axis: max energy of data in fit

$Q$, and hence $\delta y_{th}$, grows with energy

$$\delta y_{th} = y_{ref} - \sum_{n=k+1} Q_n$$

This weights high energy data less!

Stabilizes LEC fit as a function of $E$

Correlation assumptions can lead to different results
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Quantifying Truncation Uncertainty

Conditional Distributions

0.00 0.25 0.50 0.75 1.00
X

0.00 0.25 0.50 0.75 1.00
X

0.00 0.25 0.50 0.75 1.00
X

0.00 0.25 0.50 0.75 1.00
X
Quantifying Truncation Uncertainty

Conditional Distributions

Conditional + Error
This model permits mostly analytic calculation of evidence

\[ p_r(D \mid \ell, Q) = \frac{\Gamma(a^*)}{\Gamma(a)} \frac{b^a}{(b^*)^{a^*}} \sqrt{\frac{|V^*|}{|V|}} \frac{|2\pi R_\ell|^{-(k+1)/2}}{|Q|^{k(k+1)/2}} \]
This model permits mostly analytic calculation of evidence

\[
\text{pr}(\mathcal{D} \mid \ell, Q) = \frac{\Gamma(a^*)}{\Gamma(a)} \frac{b^a}{(b^*)^a^*} \sqrt{\frac{|V^*|}{|V|}} \frac{|2\pi R\ell|^{-(k+1)/2}}{|Q|^{k(k+1)/2}}
\]

Important for model comparison and for posteriors:

\[
\text{pr}(\ell \mid \mathcal{D}, Q) \propto \text{pr}(\mathcal{D} \mid \ell, Q) \text{pr}(\ell)
\]

\[
\text{pr}(Q \mid \mathcal{D}, \ell) \propto \text{pr}(\mathcal{D} \mid \ell, Q) \text{pr}(Q)
\]
This model permits mostly analytic calculation of evidence

\[ \text{pr}(D \mid \ell, Q) = \frac{\Gamma(a^*)}{\Gamma(a)} \frac{b^a}{(b^*)^{a^*}} \sqrt{|V^*|} \frac{|2\pi R_{\ell}|^{-(k+1)/2}}{|V| |Q|^{k(k+1)/2}} \]

Important for model comparison and for posteriors:

\[ \text{pr}(\ell \mid D, Q) \propto \text{pr}(D \mid \ell, Q) \text{pr}(\ell) \]
\[ \text{pr}(Q \mid D, \ell) \propto \text{pr}(D \mid \ell, Q) \text{pr}(Q) \]
This model permits mostly analytic calculation of evidence

\[
\text{pr}(\mathcal{D} | \ell, Q) = \frac{\Gamma(a^*)}{\Gamma(a)} \frac{b^a}{(b^*)^a^*} \sqrt{|V^*|} \frac{2\pi R_\ell}{|Q|^{(k+1)/2}} \frac{|V|}{|Q|^{k(k+1)/2}}
\]

Important for model comparison and for posteriors:

\[
\text{pr}(\ell | \mathcal{D}, Q) \propto \text{pr}(\mathcal{D} | \ell, Q) \text{pr}(\ell)
\]

\[
\text{pr}(Q | \mathcal{D}, \ell) \propto \text{pr}(\mathcal{D} | \ell, Q) \text{pr}(Q)
\]

Here, \( Q \propto \frac{1}{\Lambda_b} \)
Model Checking
As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.

— Albert Einstein
Model Checking

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Does our model refer to reality? How can we check?
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Does our model refer to reality? How can we check?

### Assumptions
1. \( c_n \) are iid stationary GPs

### Tests
1. Compare posteriors from individual curves & domains
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Does our model refer to reality? How can we check?

**Assumptions**

1. $c_n$ are iid stationary GPs
2. Error bands have statistical meaning

**Tests**

1. Compare posteriors from individual curves & domains
2. Credible interval diagnostic
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— Albert Einstein

Does our model refer to reality? How can we check?

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1. $c_n$ are iid stationary GPs
2. Error bands have statistical meaning
3. Squared exp. kernel $\rightarrow R_\ell$

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Truncation Model

• Replaces $\chi^2$ with its matrix analog, very easy!
• Full error can be propagated
• Reduces $E_{\text{max}}$ sensitivity and bias of LEC posterior, but need realistic correlations!
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$$\chi^2_{\text{mod}}(\vec{a}) = \vec{r}^T(\vec{a})(\Sigma_{\text{th}} + \Sigma_{\text{exp}})^{-1}\vec{r}(\vec{a})$$
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$$\text{pr}(y_{\text{exp}}) = \mathcal{N}[y_{\text{th}}(x, \vec{a}) + \mu_{\text{th}}, \Sigma_{\text{th}}]$$
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Suggestions welcome!
Thank you!

Uncorrelated Posteriors

Assumes that the variance of the $c_n$ is independent at each point