FCC RF beam dynamics and HOM power challenges

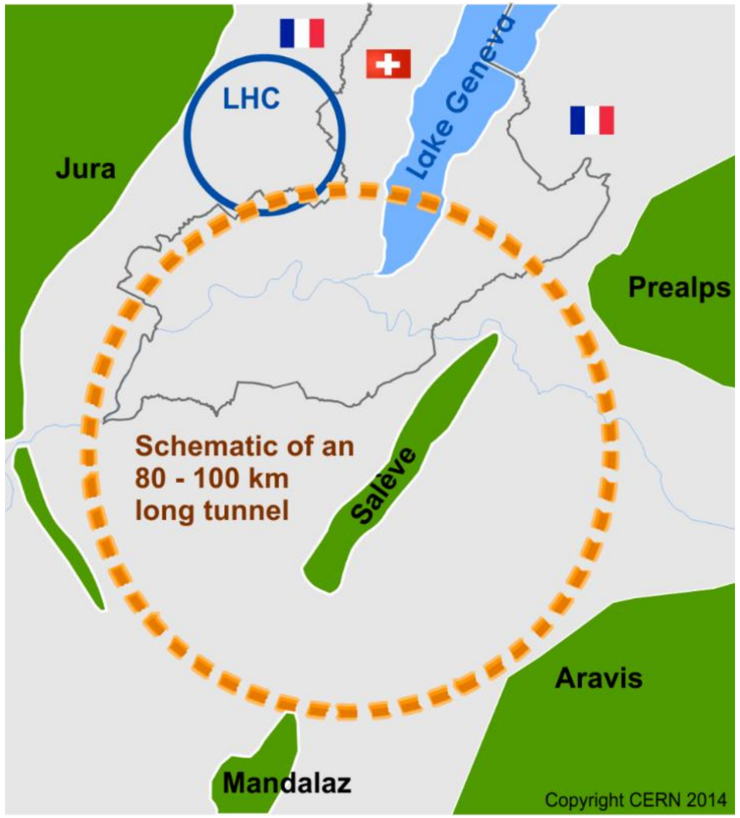
Ivan Karpov, CERN

Acknowledgments: P. Baudrenghien, O. Brunner, A. Butterworth, R. Calaga, E. Shaposhnikova, D. Teytelman

Future circular electron-positron collider FCC-ee

- Tunnel circumference, $C = 97.75$ km
- Experimental program has 4 energy stages
- Maximum acceptable power loss per beam due to synchrotron radiation is 50 MW

	Z	W	H	$t\bar{t}$
Energy/beam E [GeV]	45.6	80	120	182.5
Energy loss/turn U_0 [GeV]	0.036	0.34	1.72	9.21
RF voltage/beam V_{RF} [GV]	0.1	0.75	2.0	10.93
Beam current $I_{\mathrm{b,DC}}$ [mA]	1390	147	29	5.4



Beam-cavity interaction challenges in FCC-ee Z

The Z machine has the highest beam current and the lowest total RF voltage

- **Power losses** due to high order modes (HOM) must be extracted by HOM couplers (maximum 1 kW/coupler in SC RF)

- **Transient beam loading** leads to bunch length spread or/and to shift of collision point, therefore to possible reduction of luminosity

HOM power loss

Simulated cavity impedance

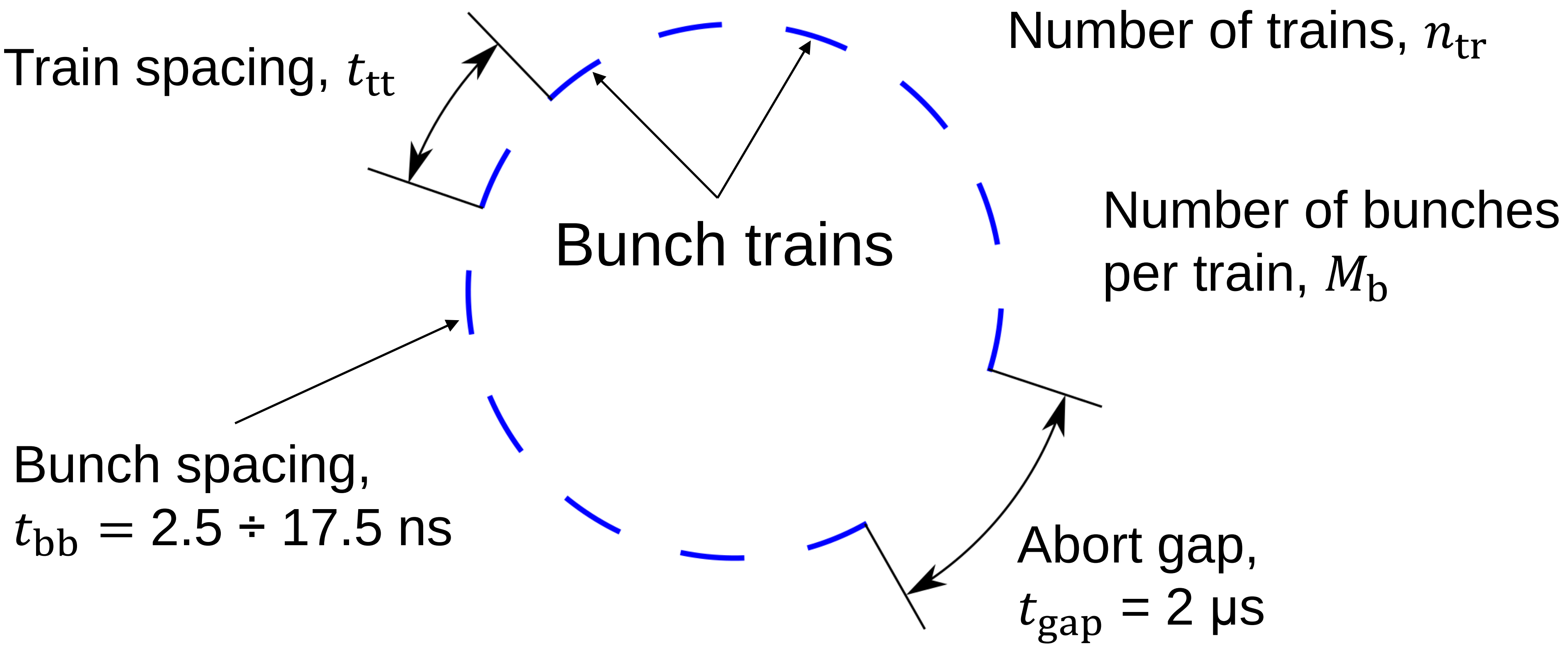
Normalized Fourier harmonics of beam current

$$P = I_{\mathrm{b,DC}}^2 \sum_{k=-\infty}^{\infty} \mathrm{Re}\left[Z_{||}(k f_{\mathrm{rev}})\right] |I_k|^2$$

$I_{\mathrm{b,DC}}$ – average beam current
f_{rev} – revolution frequency
k – revolution harmonic number

→ Power losses depend on HOM impedance and beam spectrum (ring filling schemes)

Possible beam filling schemes

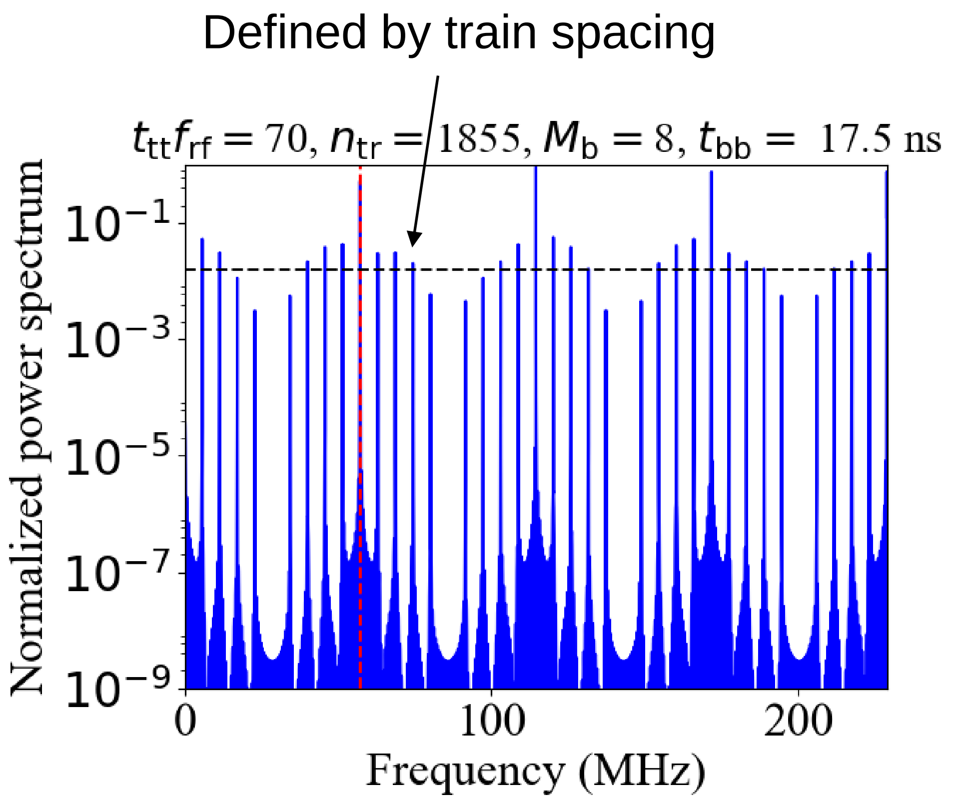


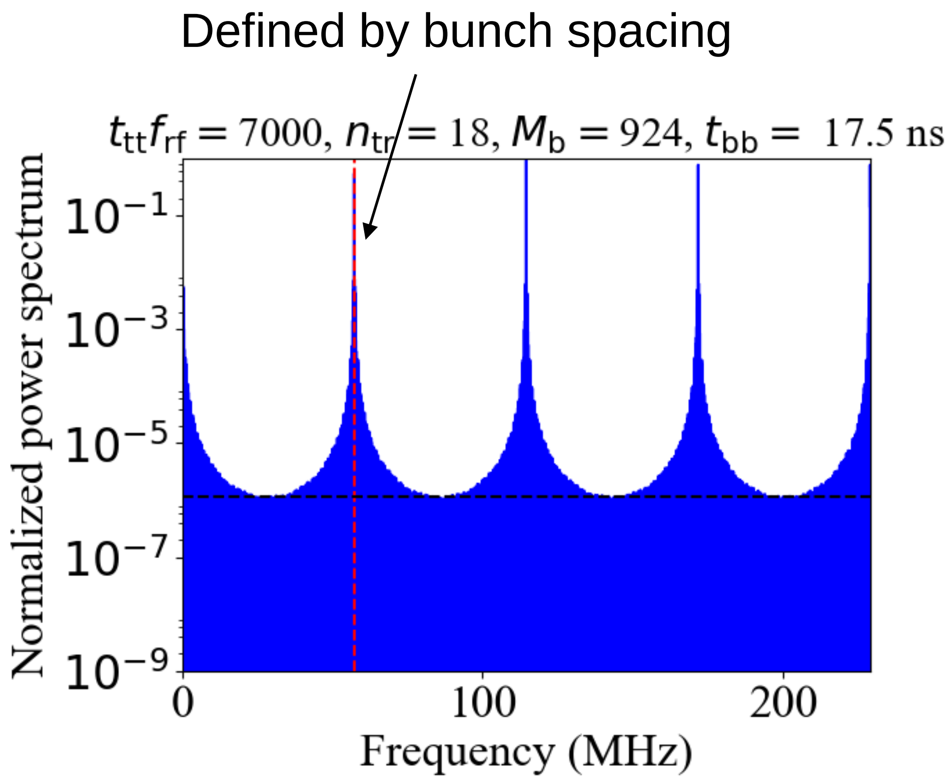
Beam spectrum is dominated by:

$1/t_{bb}$ lines (always present)

$1/t_{tt}$ lines (depending on number of trains)

Possible beam filling schemes



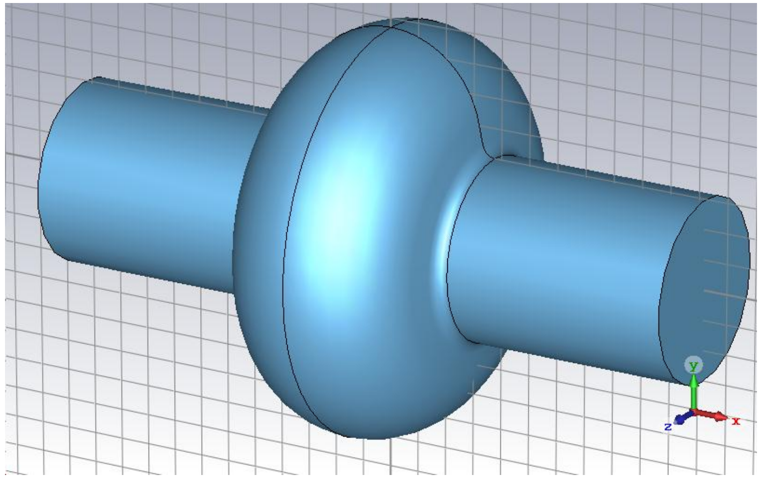


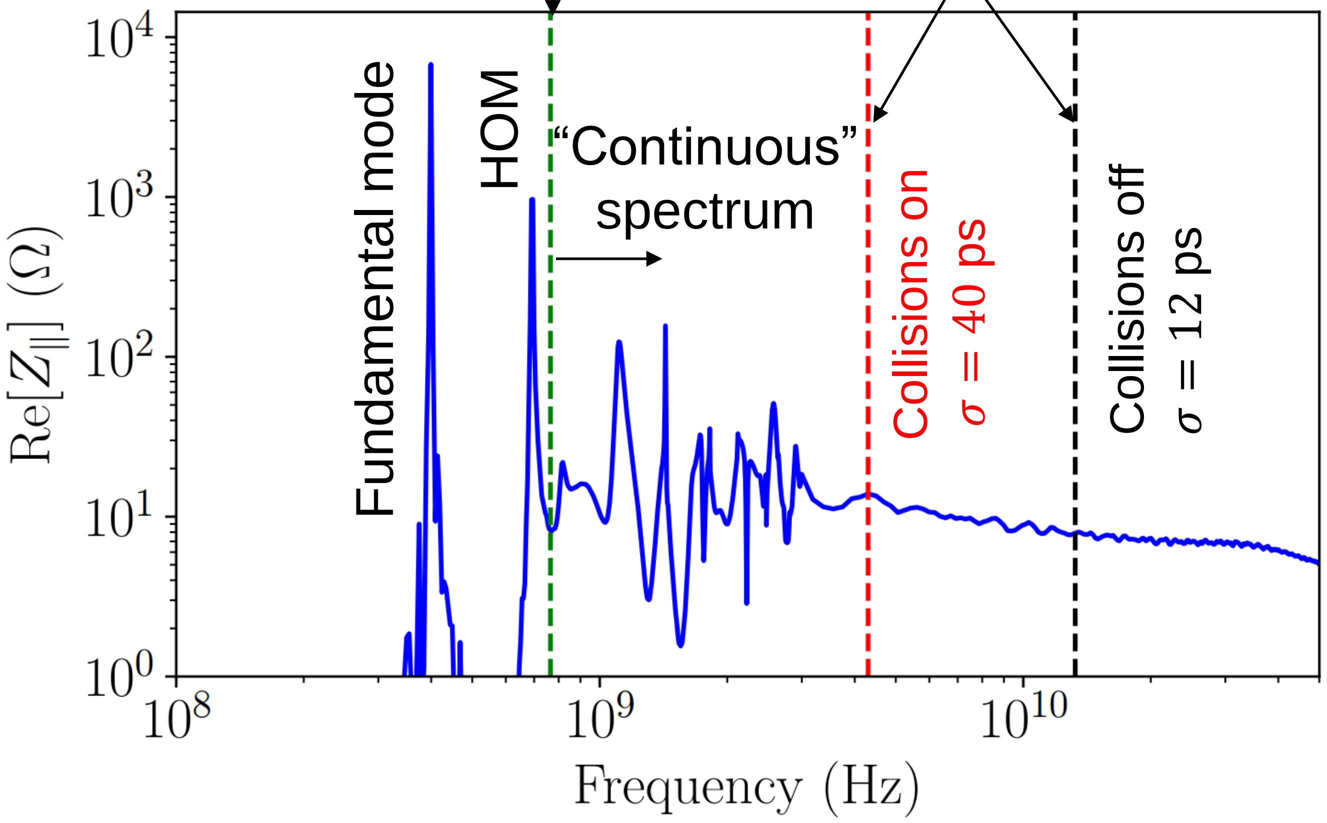
Beam spectrum is dominated by:

$1/t_{bb}$ lines (always present)

$1/t_{tt}$ lines (depending on number of trains)

LHC-like superconducting single-cell cavity





ABCI* and CST** results:

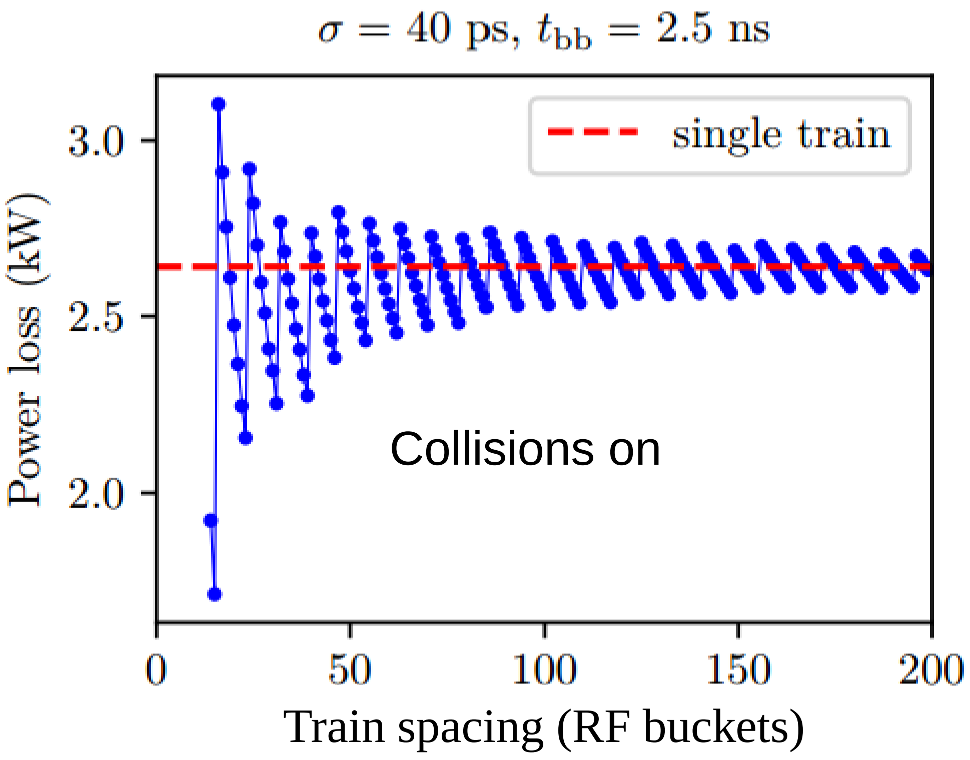
- Fundamental mode at 400.79 MHz with (R/Q) = 42.3 Ω
- HOM below cut-off frequency at 694 MHz, $(R/Q)_{\mathrm{HOM}} \approx 12\ \Omega$

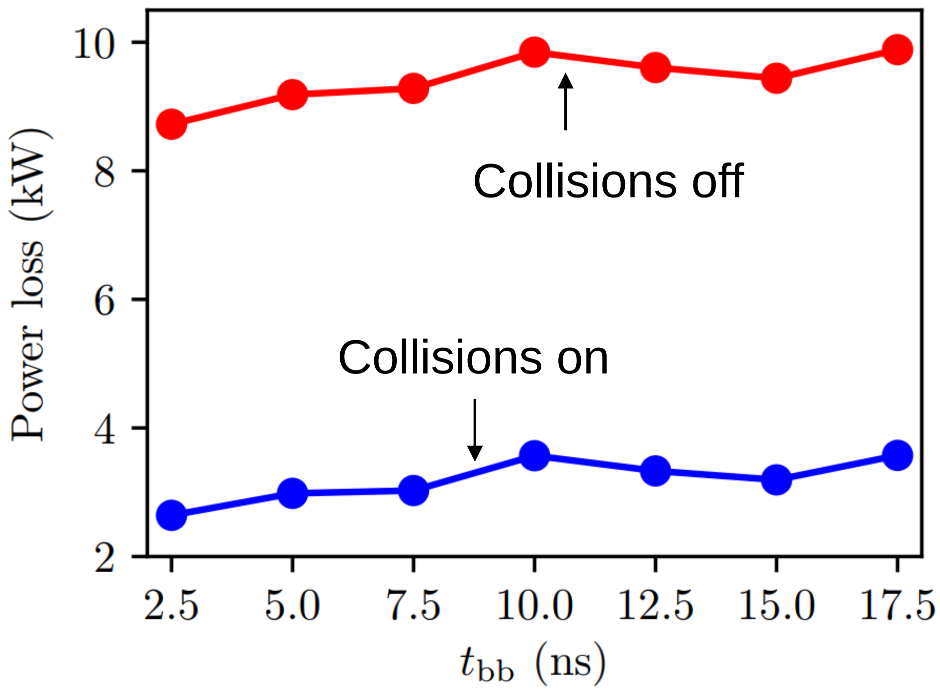
→ Contributions of impedance below and above cut-off frequency are important for power loss calculations

*Azimuthal Beam Cavity Interaction code, http://abci.kek.jp/abci.htm
**CST EM STUDIO, https://www.cst.com/products/cstems

Power loss above cut-off frequency

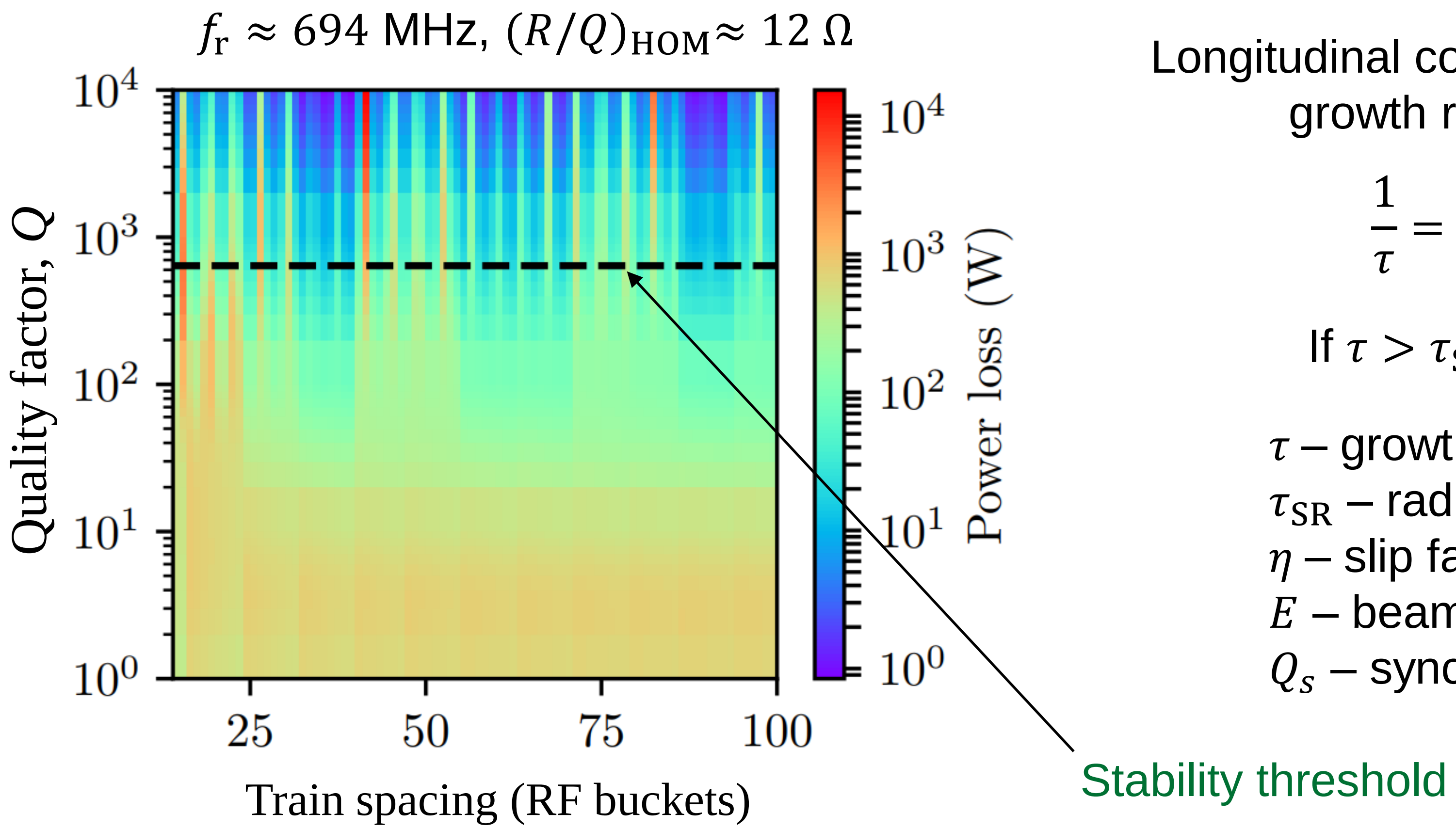
Assumptions: total current ≤ 1.4 A, number of bunches ≤ 16640, abort gap ≥ 2 μs, bunch population 1.7e11
Variable parameters: number of bunches in the train, number of trains, train spacing





→ Power loss is moderate for the present cavity design for bunches in collisions (≈ 3 kW)
→ There is a weak dependence on train spacing and bunch spacing

Power loss for HOM below cut-off frequency



Longitudinal coupled-bunch instability growth rate due to HOM

$$\frac{1}{\tau} = \frac{e|\eta|I_{b,DC}}{2EQ_s} f_r (R/Q)_{HOM} Q$$

If $\tau > \tau_{SR} \rightarrow$ stability

τ – growth time
τ_{SR} – radiation damping time
η – slip factor
E – beam energy
Q_s – synchrotron tune

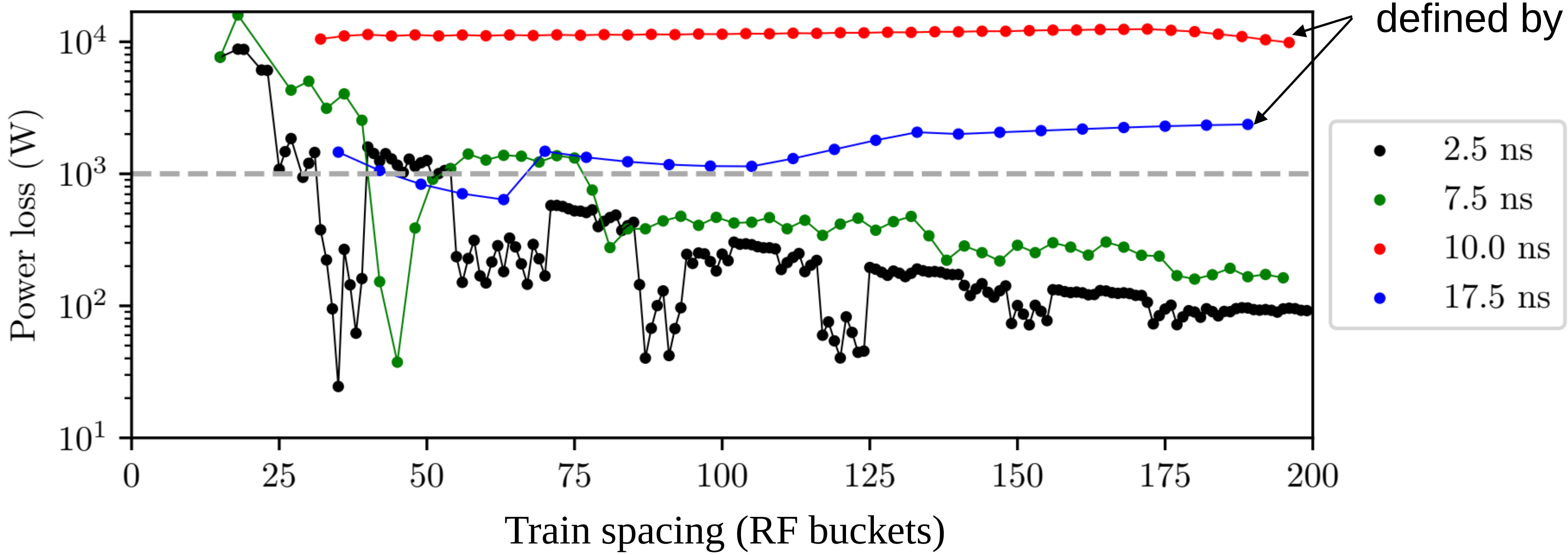
Power losses of about 1 kW are for small Q + "resonant" cases with high Q
→ Damping of the mode for longitudinal stability should be moderate
→ Resonant cases should be identified

Power losses for different filling schemes

Rounded off value

Resonant case when the beam spectral line overlaps with HOM* $\left|1 - \frac{[f_r t_{tt}]}{f_r t_{tt}}\right| < \frac{1}{Q}$

$f_r = 694 \pm 5$ MHz, $(R/Q)_{HOM} = 12\ \Omega, Q = 640$

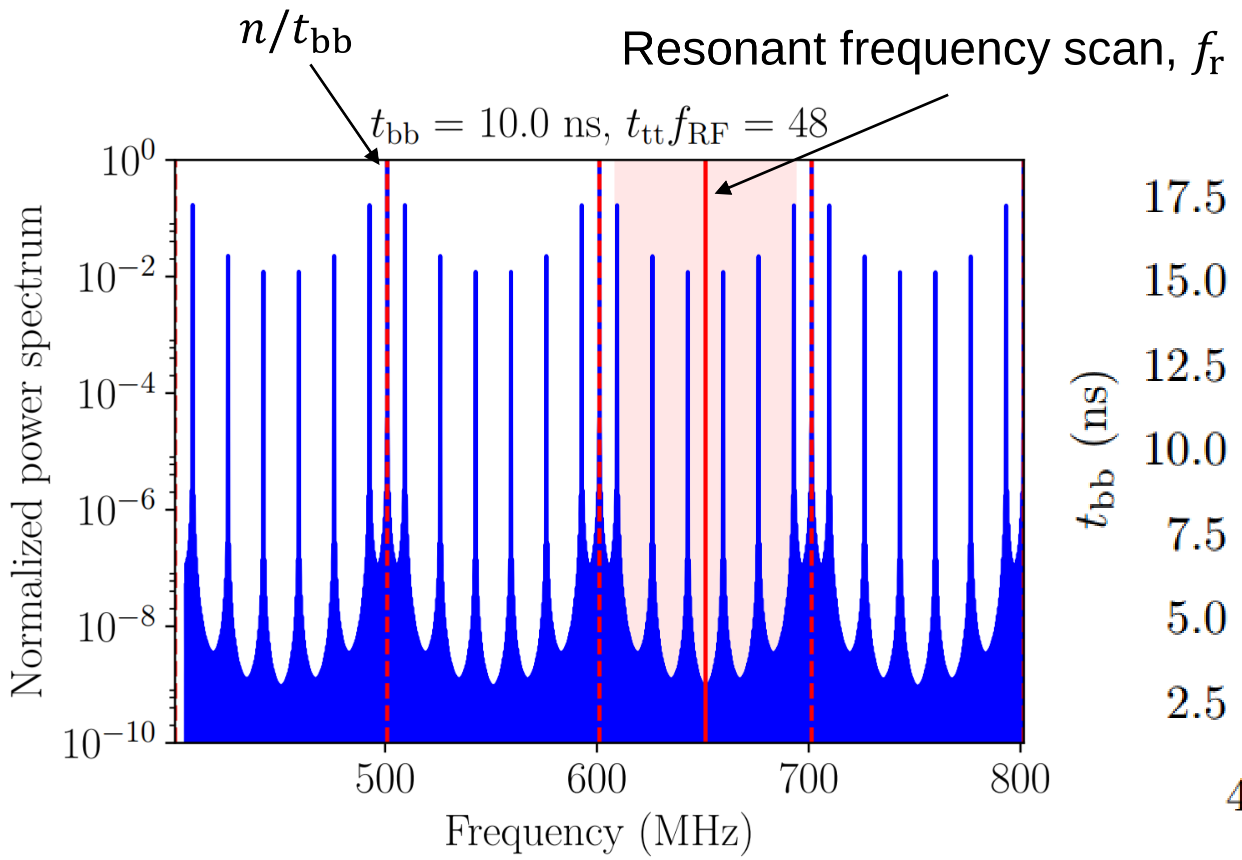


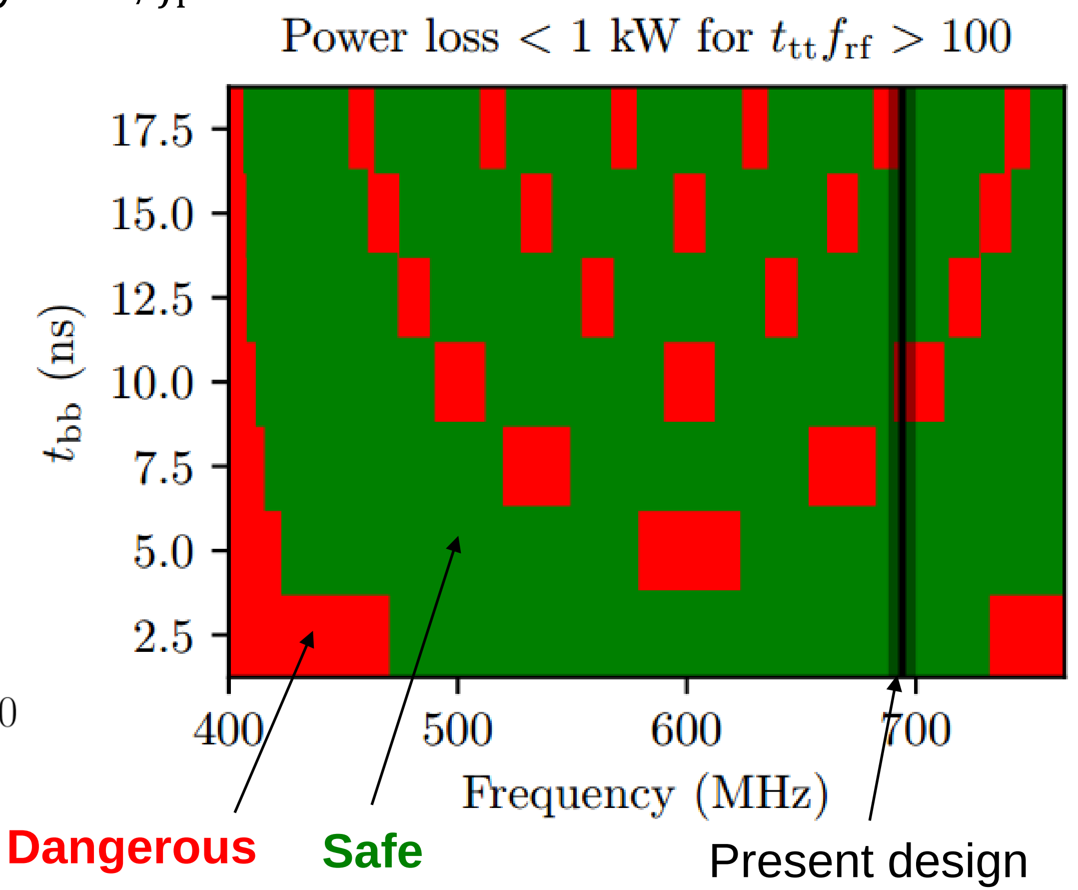
Overlap with spectral lines defined by bunch spacing

→ Some filling schemes should be avoided in machine operation (restrictions for train and bunch spacings)

*I.Karpov, R. Calaga, E. Shaposhnikova, Phys. Rev. Accel. Beams 21, 071001 (2018)

More “general” case

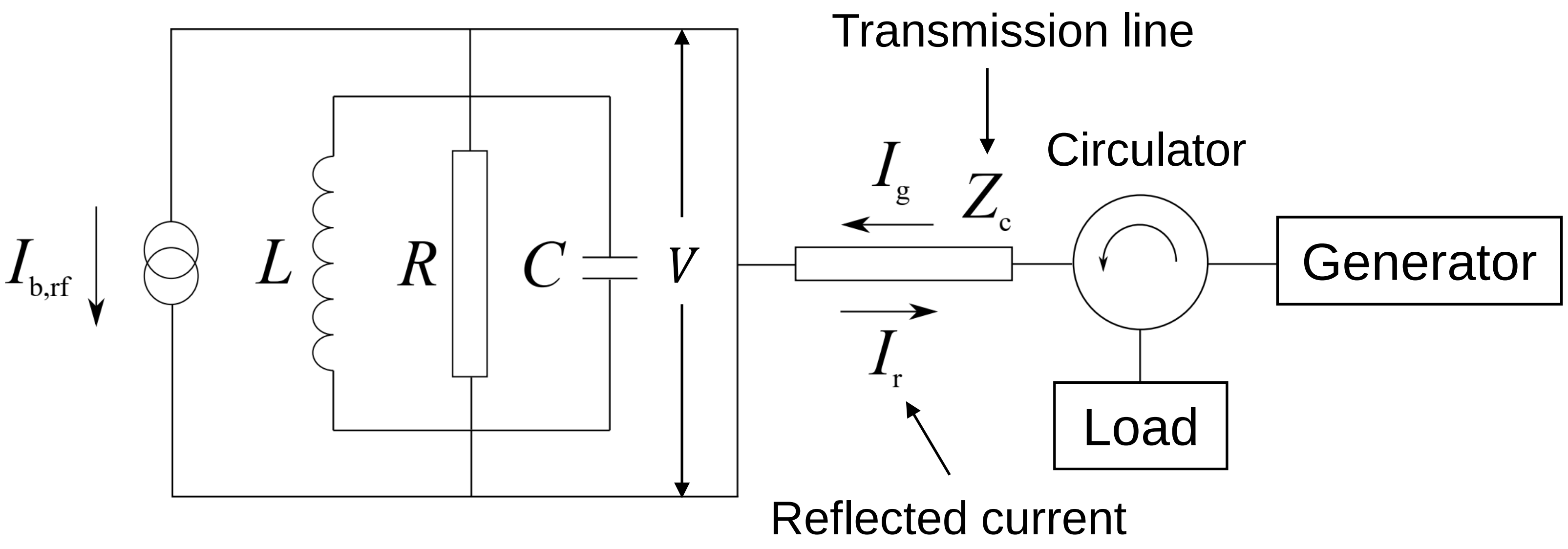




→ Operation settings define recommendations for the cavity geometry

Transient beam loading

Model of superconducting RF cavity linked to generator via circulator*



Generator current**

$$I_g = \frac{V(t)}{2(R/Q)}\left(\frac{1}{Q_0} + \frac{1}{Q_{ext}} - \frac{2i\Delta\omega}{\omega_{RF}}\right) + \frac{dV(t)}{dt}\frac{1}{\omega_{RF}(R/Q)} + \frac{I_{b,RF}(t)}{2}$$

Cavity detuning is $\Delta\omega = \omega_0 - \omega_{rf}$, $R = (R/Q)Q_0$, $Z_c = (R/Q)Q_{ext}$

*D. Boussard, CERN-SPS-ARF-DB-GW-NOTE-84-9, 1984
**J. Tückmantel, CERN-ATS-Note-2011-002, 2011

Beam induced transients

amplitude and phase modulations

Cavity voltage $V(t) = A(t)e^{i\phi(t)}$

amplitude and phase modulations

Beam current $I_{\mathrm{b,rf}}(t) = A_{\mathrm{b}}(t)e^{-i\phi_s + i\phi_{\mathrm{b}}(t)}$

Average synchronous phase
$eN_{\mathrm{cav}}V_{\mathrm{cav}}\cos\phi_s = U_0$

Generator current

$$I_{\mathrm{g}} = \frac{V(t)}{2(R/Q)}\left(\frac{1}{Q_0} + \frac{1}{Q_{\mathrm{ext}}} - \frac{2i\Delta\omega}{\omega_{\mathrm{RF}}}\right) + \frac{dV(t)}{dt}\frac{1}{\omega_{\mathrm{RF}}(R/Q)} + \frac{I_{\mathrm{b,RF}}(t)}{2}$$

Synchronous phase

$$eN_{\mathrm{cav}}A(t)\cos[\phi_s - \phi_{\mathrm{b}}(t) + \phi(t)] = U_0$$

→ System of equations for $A(t), \phi(t)$, and $\phi_b(t)$ can be obtained
→ Solution depends on the choice of $\Delta\omega$ and Q_{ext}

Optimal parameters

Generator power* $P_\mathrm{g} = \frac{1}{2} Z_\mathrm{c} \left|I_\mathrm{g}\right|^2 = \frac{1}{2} (R/Q) Q_\mathrm{ext} \left|I_\mathrm{g}\right|^2$

It can be minimized for given:

- average beam current $\langle I_\mathrm{b,rf} \rangle = F_\mathrm{b} I_\mathrm{b,dc} e^{-i\phi_s}$, with $F_\mathrm{b} = 2\exp[-(\omega_\mathrm{rf}\sigma)^2/2]$ - bunch form factor
- average cavity voltage $\langle V \rangle = V_\mathrm{cav}$

$Q_0 \gg Q_\mathrm{ext}$ for superconducting cavities

Optimal loaded quality factor $$\frac{1}{Q_\mathrm{L,opt}} = \frac{1}{Q_\mathrm{ext,opt}} + \frac{1}{Q_0} \approx \frac{1}{Q_\mathrm{ext,opt}} \approx \frac{V_\mathrm{cav}}{F_\mathrm{b} I_\mathrm{b,dc} (R/Q) \cos\phi_s}$$

Optimal detuning $$\Delta\omega_\mathrm{opt} = -\omega_\mathrm{rf} \frac{F_\mathrm{b} I_\mathrm{b,dc} (R/Q) \sin\phi_s}{2 V_\mathrm{cav}}$$

→ Beam is unstable for optimal parameters because the second Robinson limit** is reached if there are no loops around cavity

*J. Tückmantel, CERN-ATS-Note-2011-002, 2011

**K. W. Robinson, Stability of beam in radiofrequency system, Rep. CEAL-1010 (1964)

Small-signal Pedersen model

Small modulations: $A(t) = V_{\mathrm{cav}}[1 + a_V(t)]$, $A_{\mathrm{b}}(t) = F_{\mathrm{b}} I_{\mathrm{b,dc}}[1 + a_{\mathrm{b}}(t)]$

Transfer functions* from beam current amplitude $\tilde{a}_{\mathrm{b}}$

cavity voltage amplitude $\tilde{a}_V$

$$\frac{\tilde{a}_V(s)}{\tilde{a}_{\mathrm{b}}(s)} = -\frac{\Delta\omega_{\mathrm{opt}}\tau \cot\phi_s}{1 + \tau s}$$

cavity voltage phase $\tilde{\phi}$

$$\frac{\tilde{\phi}(s)}{\tilde{a}_{\mathrm{b}}(s)} = -\frac{\Delta\omega_{\mathrm{opt}}\tau}{1 + \tau s}$$

beam phase $\tilde{\phi}_{\mathrm{b}}$

$$\frac{\tilde{\phi}_{\mathrm{b}}(s)}{\tilde{a}_{\mathrm{b}}(s)} = -\frac{\Delta\omega_{\mathrm{m}}\tau}{1 + \tau s}$$

For “magic” detuning**

$$\Delta\omega_{\mathrm{m}} = \frac{\Delta\omega_{\mathrm{opt}}}{\sin^2\phi_s}$$

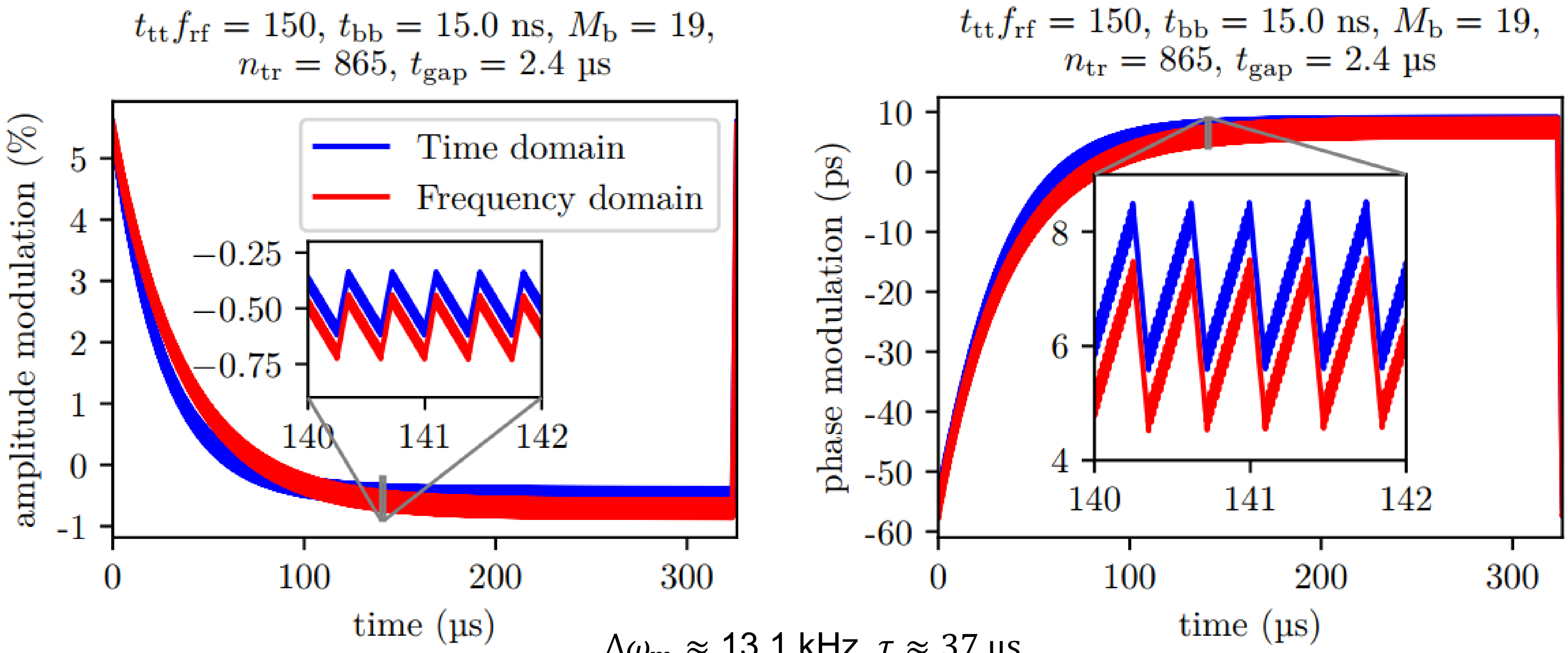
$\tau = 2Q_{\mathrm{L}}/\omega_{\mathrm{rf}}$ - cavity filling time, s – complex variable

→ Magic detuning can be used for transient beam loading calculations

*F. Pedersen, Beam loading effects in the CERN PS Booster, IEEE Trans. Nucl. Sci. **22**, 1906 (1975)

**F. Pedersen, RF cavity feedback, CERN-PS-92-59-RF, 17 (1992)

Transient beam loading in FCC-ee Z



$\Delta\omega_m \approx 13.1$ kHz, $\tau \approx 37$ µs

→ There is a strong modulation due to the abort gap and a fine structure due to the gaps between trains

→ Reasonable agreement between time-domain and frequency-domain calculations

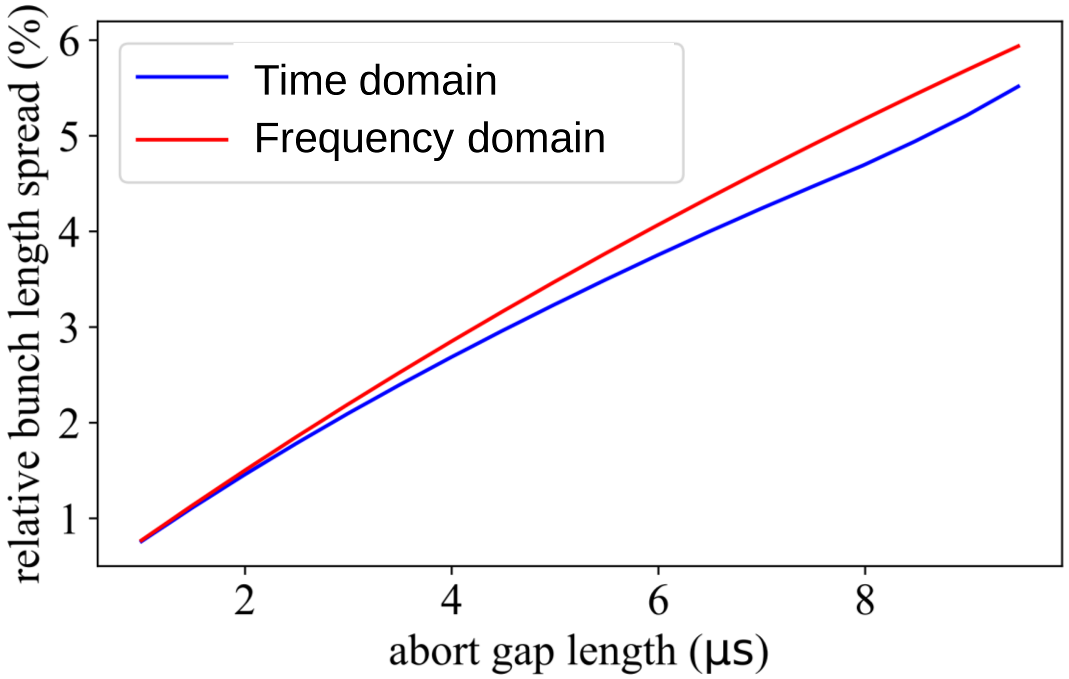
Results for FCC-ee Z

- Phase modulations can result in collision point shift, but It can be compensated by proper matching of abort gap transients (for example in the LHC*)
- Bunch length modulations can still affect luminosity

Bunch length $\sigma(t) = \frac{\eta \Delta E C}{2\pi Q_s(t)}$

Synchrotron tune $Q_s(t) = \sqrt{\frac{h\eta A(t) N_{\mathrm{cav}}}{2\pi E} \sin[\phi_s - \phi_b(t) + \phi(t)]}$

h – harmonic number
ΔE – energy spread



→ For baseline parameters, the abort gap length < 2 μs and bunch length spread can be neglected

*T. Mastoridis, P. Baudrenghien, and J. Molendijk, Phys. Rev. Accel. Beams 20, 101003 (2017)

Conclusions

- HOM power loss contributions:
    - From impedance above cutoff frequency is about 3 kW,
    - From overlap of HOM below cutoff frequency with beam spectral line is below 1 kW for train spacing larger than 100 RF buckets, if 10 ns and 17.5 ns bunch spacing are excluded from operation.
- HOM frequency ranges for new cavity designs which are “safe” for given bunch spacings were identified.
- Beam induced transient are dominated by abort gap:
    - collision point shift still can be eliminated by matching abort gap transients,
    - for $t_{\mathrm{gap}} < 2$ μs bunch length spread < 2%.

Thank you for your attention!

Second Robinson limit

Second Robinson limit above transition (no loops around cavity)*

$$Y < -\frac{2\sin\phi_s}{\sin(2\phi_z)}$$

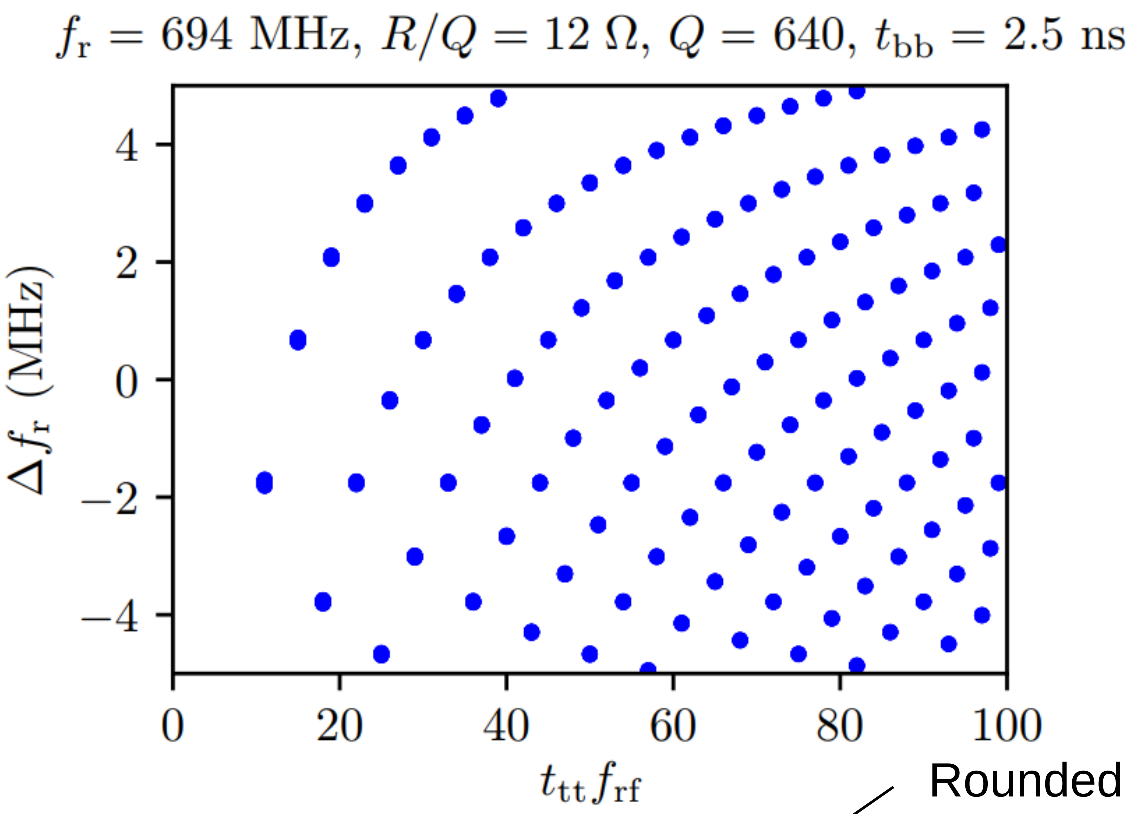
$$Y = F_{\mathrm{b}} I_{\mathrm{b,dc}} (R/Q) Q_{\mathrm{L}} / V_{\mathrm{cav}}$$

$$\phi_z = \arctan[2Q_{\mathrm{L}}\Delta\omega/\omega_{\mathrm{rf}}]$$

for optimal parameters ($Y = 1/\cos\phi_s$ and $\phi_z = -\phi_s$)

*K. W. Robinson, Stability of beam in radiofrequency system, Rep. CEAL-1010 (1964)

Shift of the resonant frequency



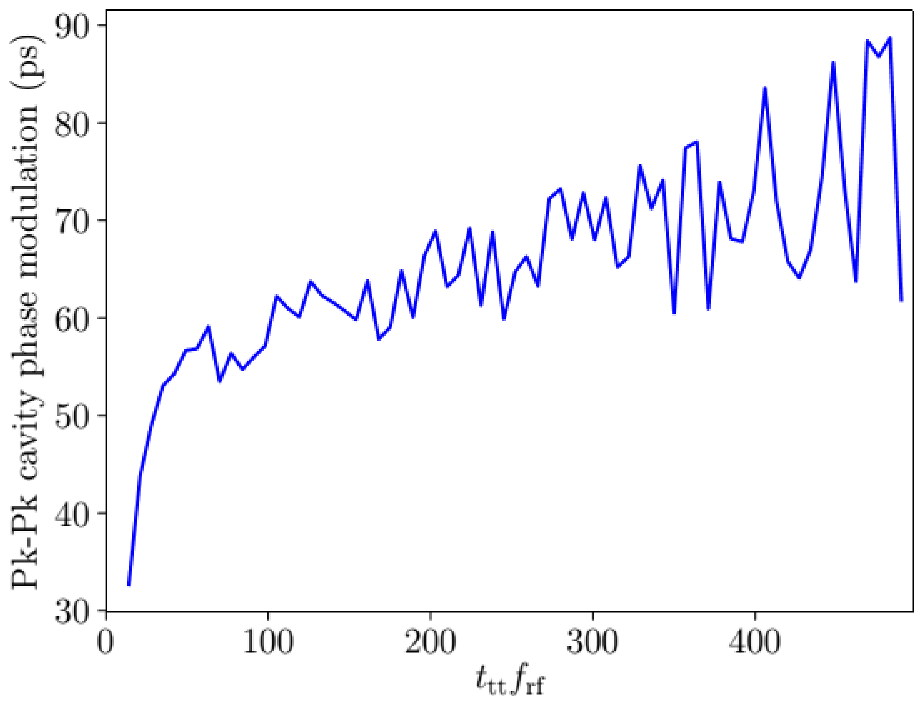
Rounded off value

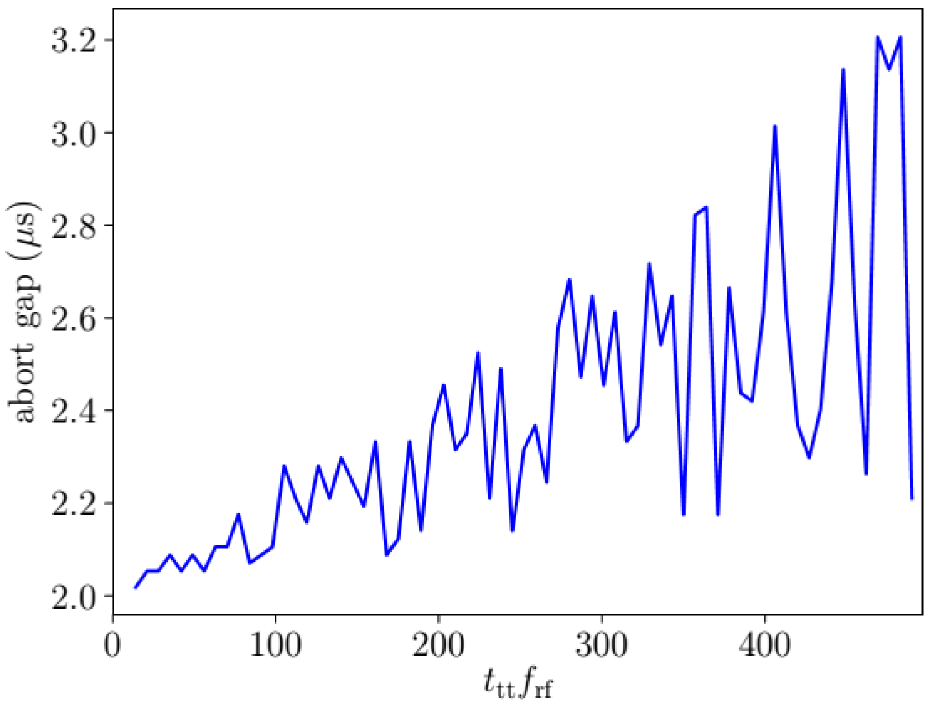
"Resonant" condition* $\left|1 - \frac{[f_r t_{tt}]}{f_r t_{tt}}\right| < \frac{1}{Q}$

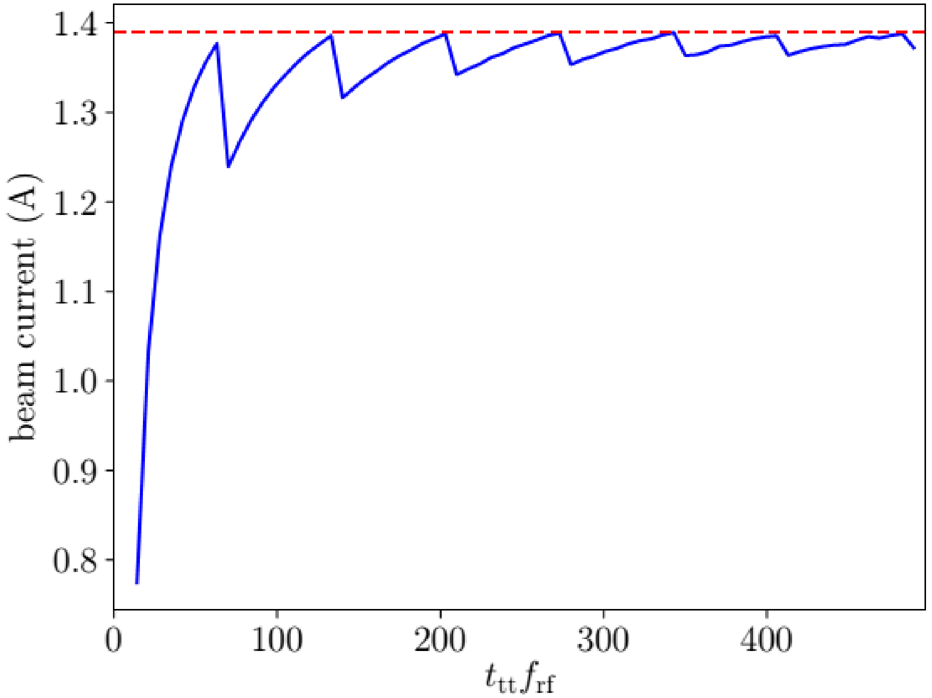
→ There are many cases when the spectrum line hits the resonant line

→ Not all of them are dangerous

*I.Karpov et al., CERN-ACC-NOTE-2018-0005 (2018)

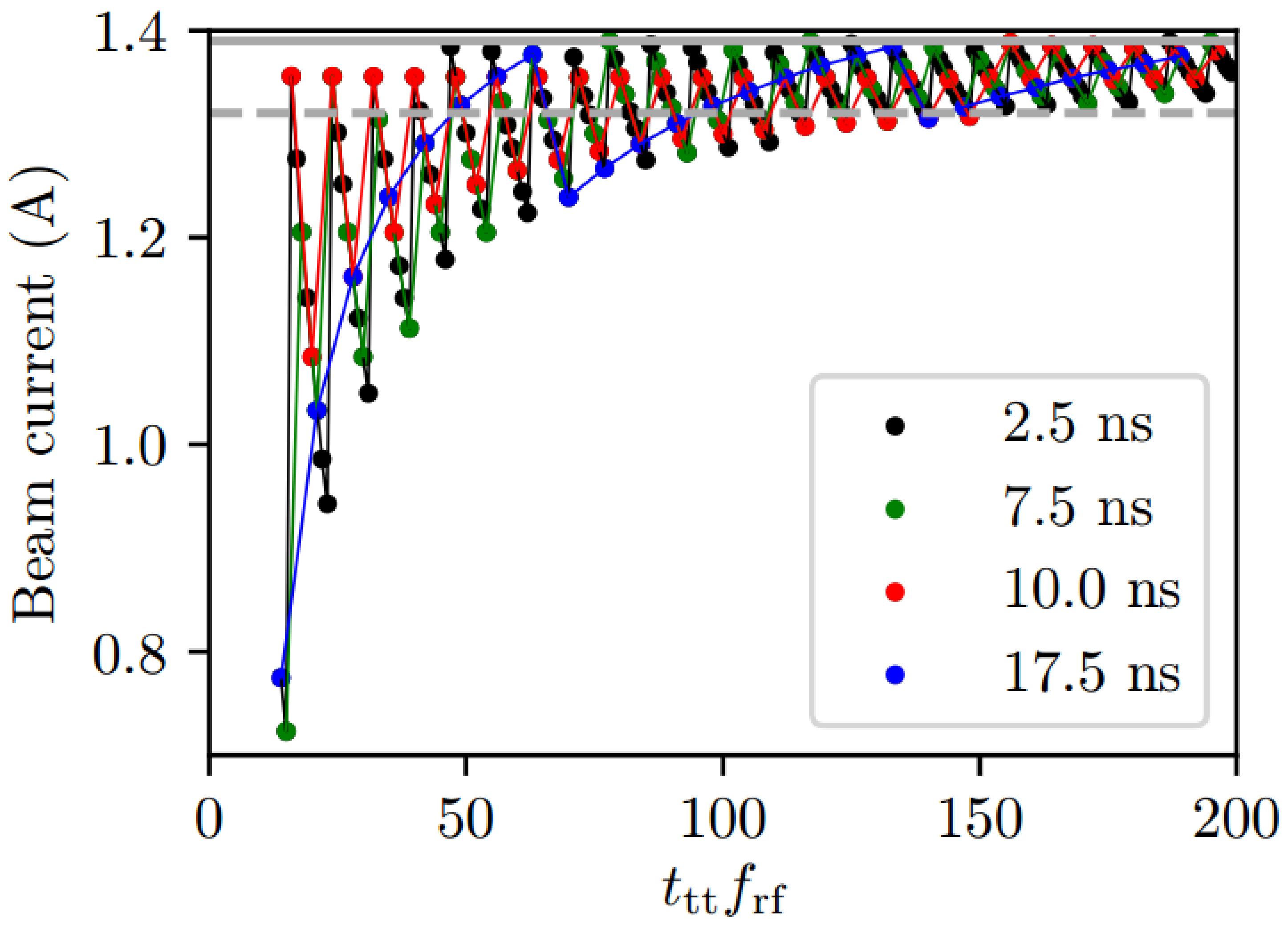






Dependence on the train spacing for bunch spacing of 17.5 ns

Beam current for different filling schemes



Beam-cavity interaction challenges in FCC-ee Z

Parameter	Value
Beam current, $I_{\mathrm{b,DC}}$	1.39 A
Number of bunches, M	16640
Minimum abort gap, τ_{gap}	2 μs
RF frequency, f_{RF}	400.79 MHz
R/Q of fundamental mode	42.3 Ω
Cavity voltage, V_{cav}	1.91 MV
Number of cavities, N_{cav}	52
Harmonic number, h	130680
Radiation damping time, τ_{SR}	414 ms

→ Power losses due to high order modes (HOM)

→ Transient beam loading

→ Longitudinal coupled-bunch instability

Impedance of LHC-like single-cell cavity

Impedance calculation using ABCI

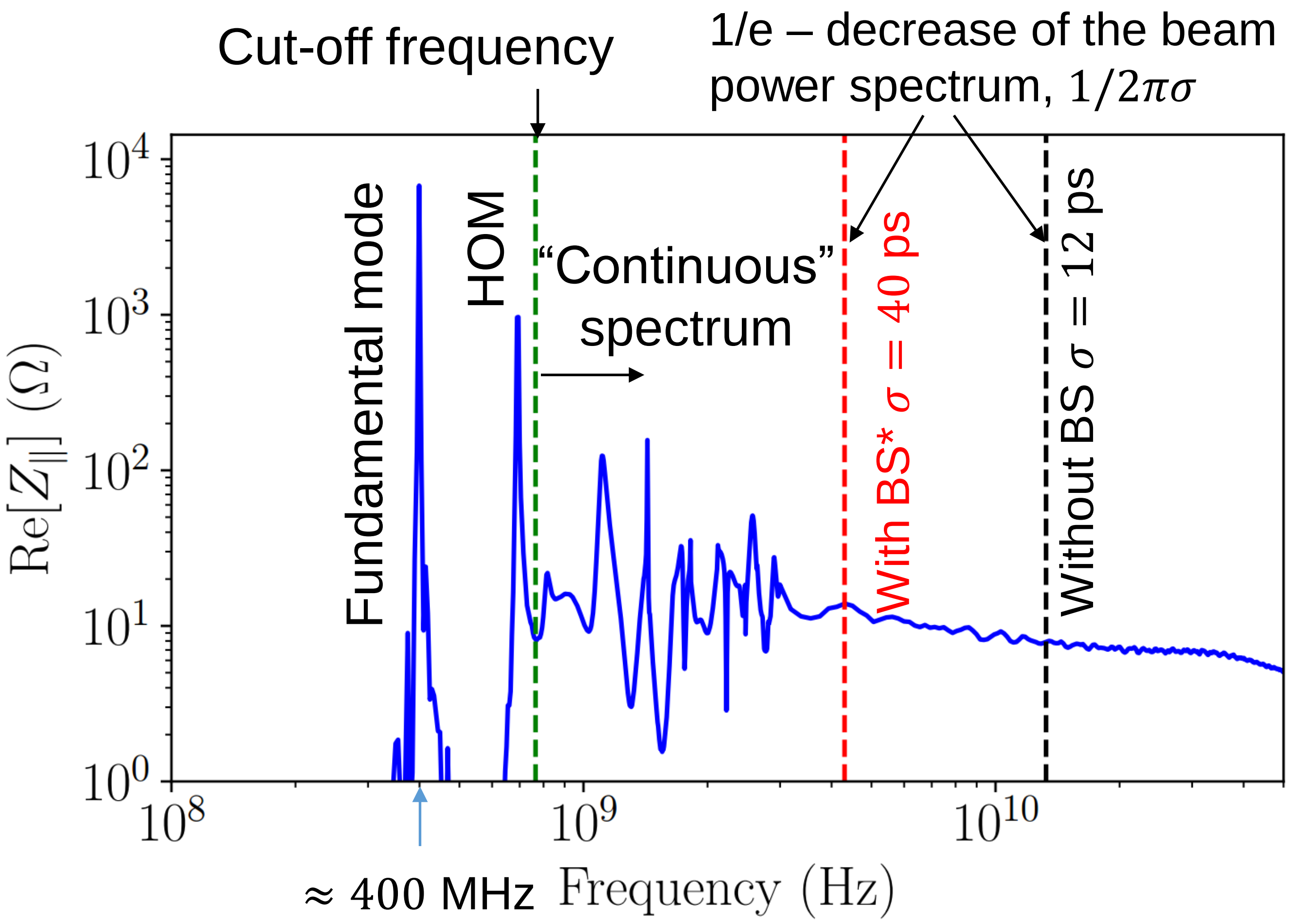
Axisymmetric structure
+
Gaussian bunch

↓

Wake potential

↓

Impedance

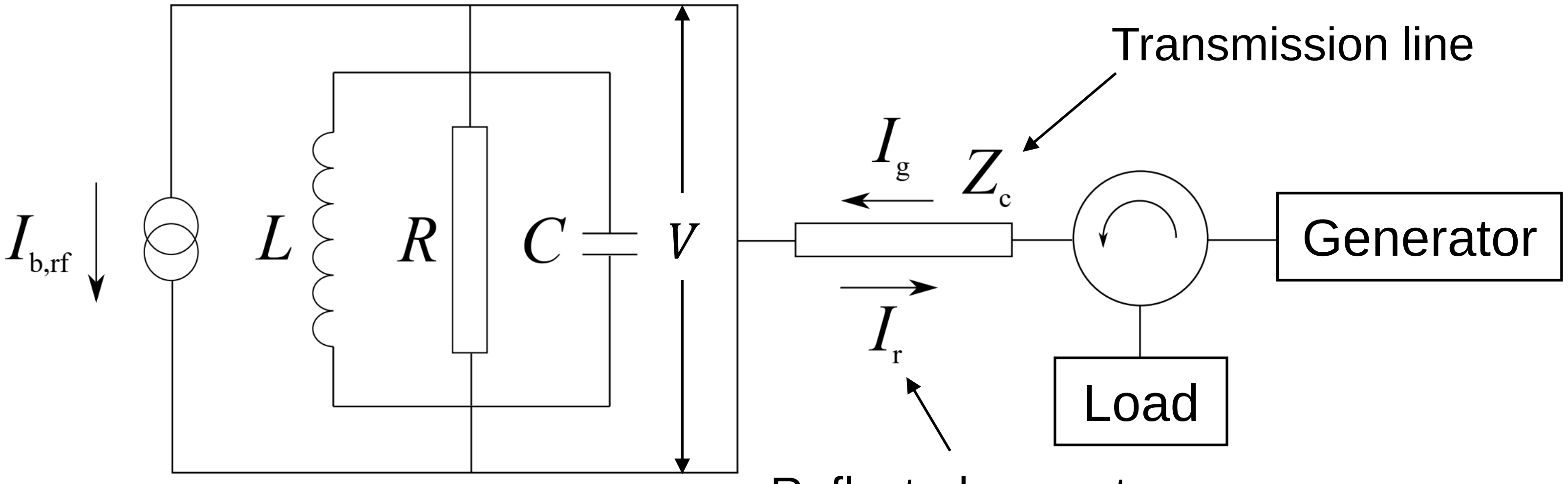


*Beamstrahlung effect

→ Only one mode below cut-off frequency with parameters:
$f_\mathrm{r} \approx 694$ MHz, $R/Q \approx 12\ \Omega$ (CST EMS simulations), quality factor Q = ?

Transient beam loading

Model of superconducting RF cavity linked to generator via circulator*



From circuit equations:

$$V = Z_c(I_g + I_r)$$

$$I_{LRC} = I_g - I_r - I_{b,RF}$$

Assumptions:

- all dynamic variables are proportional to $\exp(i\omega_{RF}t)$ with varying amplitude
- Slow change of the complex voltage amplitude, $|dV^2(t)/dt^2| \ll \left|\omega_{RF}^2 V(t)\right|$

Generator current**

$$I_g = \frac{V(t)}{2(R/Q)}\left(\frac{1}{Q_0} + \frac{1}{Q_{ext}} - \frac{2i\Delta\omega}{\omega_{RF}}\right) + \frac{dV(t)}{dt}\frac{1}{\omega_{RF}(R/Q)} + \frac{I_{b,RF}(t)}{2}$$

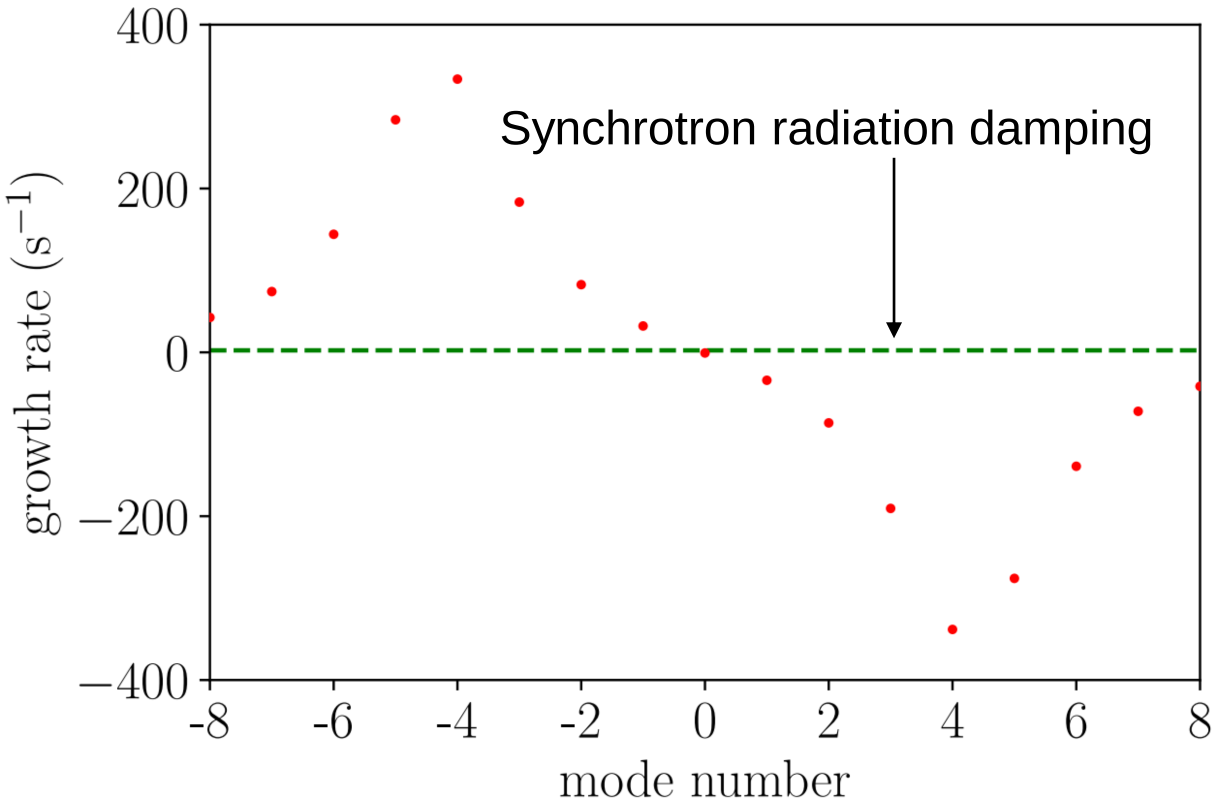
Cavity detuning is $\Delta\omega = \omega_0 - \omega_{rf}$, $R = (R/Q)Q_0$, $Z_c = (R/Q)Q_{ext}$

*D. Boussard, CERN-SPS-ARF-DB-GW-NOTE-84-9, 1984
**J. Tückmantel, CERN-ATS-Note-2011-002, 2011

Fundamental impedance driven instability

The growth rate of the longitudinal coupled-bunch mode m*

$$\frac{1}{\tau_\mathrm{m}} \approx \frac{e\eta\omega_\mathrm{RF}}{4\pi E Q_s} I_\mathrm{b,DC} N_\mathrm{cav} (\mathrm{Re}\{Z_\parallel[\omega_\mathrm{RF} + (m+Q_s)\omega_\mathrm{rev}]\} - \mathrm{Re}\{Z_\parallel[\omega_\mathrm{RF} - (m+Q_s)\omega_\mathrm{rev}]\})$$



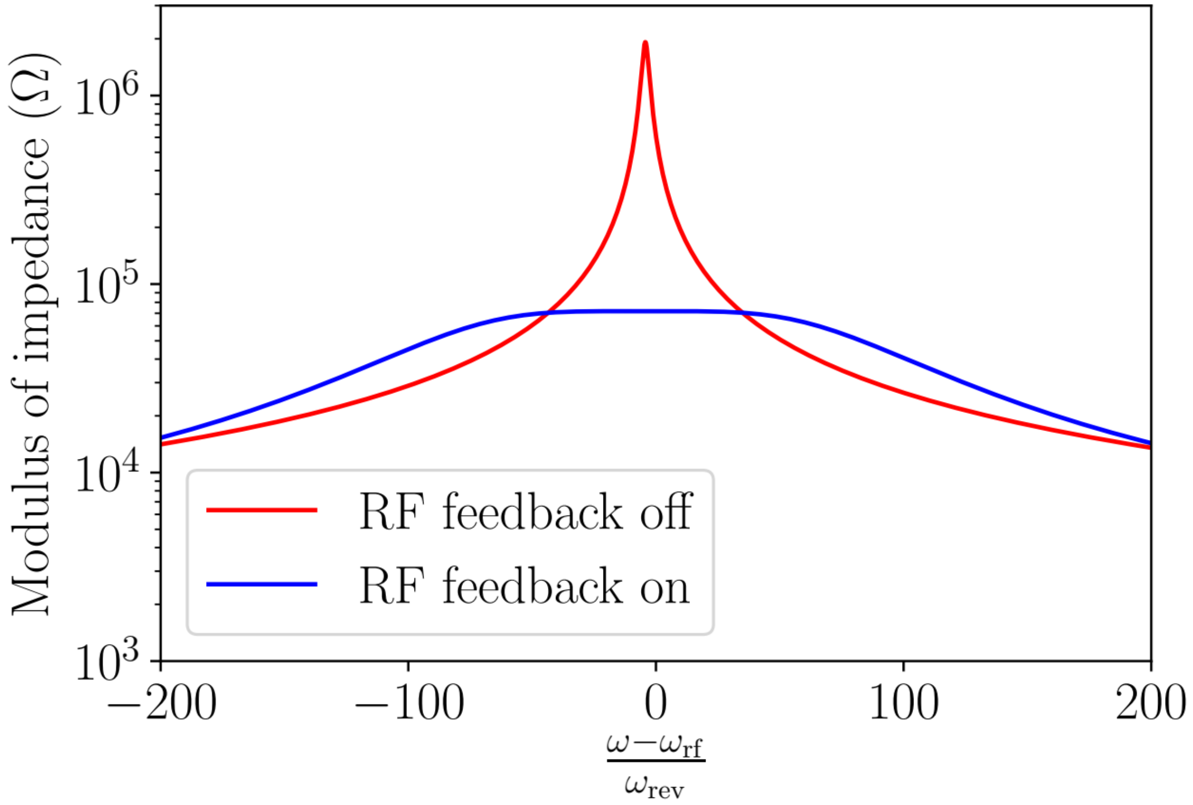
Fundamental cavity impedance

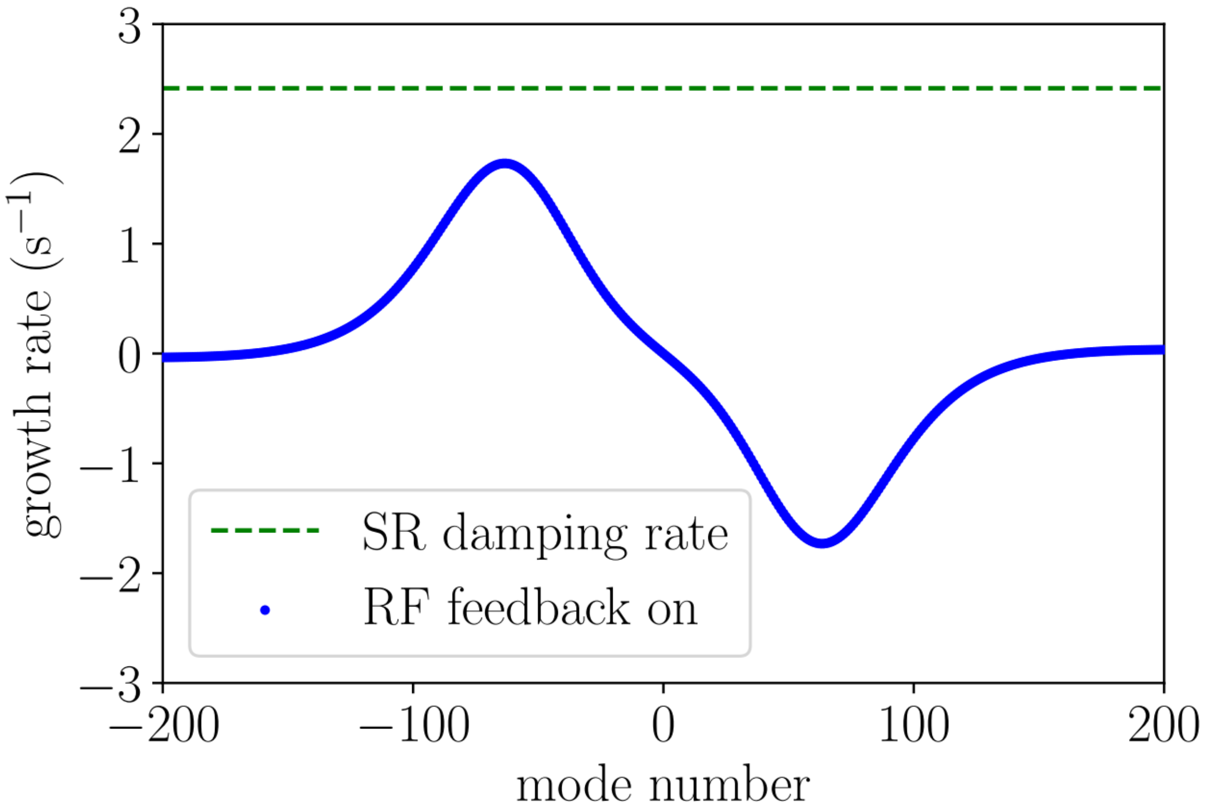
$$Z_\parallel(\omega) = \frac{(R/Q)Q_\mathrm{L}}{1 + iQ_L\left[\frac{\omega_0}{\omega_\mathrm{RF}} - \frac{\omega_\mathrm{RF}}{\omega_0}\right]}$$

→ For magic detuning (about $4 \times f_\mathrm{rev}$)
the most unstable mode is $m =$ -4

*For example in A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993

Mitigation by direct RF feedback*





The flat response is achieved for $1/G = R/Q\omega_{\mathrm{RF}}\tau_{\mathrm{d}}$, loop delay $\tau_{\mathrm{d}} \approx 700$ ns (similar to LHC)

→ Growth rates of all unstable modes are smaller than synchrotron radiation damping rate
→ To increase stability margins one-turn delay feedback (similar to SPS, LHC) or more sophisticated double peaked comb filter (PEP II) can be used

*D. Boussard, Control of cavities with high beam loading, IEEE NS-32 (1985)