Study of Two-Photon Corrections in the $p\bar{p} \rightarrow e^+e^-$ Process

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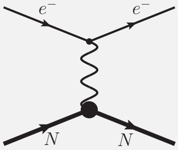
Electromagnetic Form Factors of the Nucleon

Space-like Region ($q^2 < 0$)

Elastic eN-scattering:



Electromagnetic current:

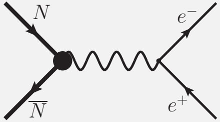
$$\langle N(p')|J^{\mu}_{em}|N(p)\rangle =$$

$$\bar{u}(p')\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m}\right]u(p)$$

- Form factors are real functions of $Q^2 = -q^2$

Time-like Region ($q^2 > 0$)

$p\bar{p}$-Annihilation



Crossing symmetry:

$$\langle N(p')|J^{\mu}_{em}|N(p)\rangle \rightarrow \langle 0|J^{\mu}_{em}|N(p)\overline{N}(p')\rangle$$

$$\langle 0|J^{\mu}_{em}|N(p)\overline{N}(p')\rangle =$$

$$\bar{u}(p')\left[F_1(q^2)\gamma^{\mu} - F_2(q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m}\right]u(p)$$

- Form factors are complex functions of q^2

Space-like Electromagnetic Form Factors

e-N-Scattering in the Born Approximation ...

There are two experimental methods to extract the ratio G_E/G_M:

Unpolarized Rosenbluth measurement:

$$d\sigma = \mathcal{C}(Q^2,\epsilon)\left[G_M^2(Q^2) + \frac{\epsilon}{\tau}G_E^2(Q^2)\right]$$

Polarisation transfer technique:

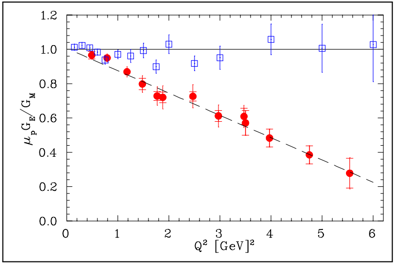
$$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\,\frac{G_E(Q^2)}{G_M(Q^2)}$$

with $\tau = \frac{Q^2}{4m^2}$ and $\frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$

- The two methods give different results:

  Rosenbluth exp: $\frac{\mu_p G_E}{G_M} \approx 1$

  Polarization exp.: linear decrease of $\frac{G_E}{G_M}$ with Q^2



(Arrington, PRC (2003))

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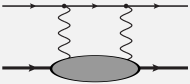
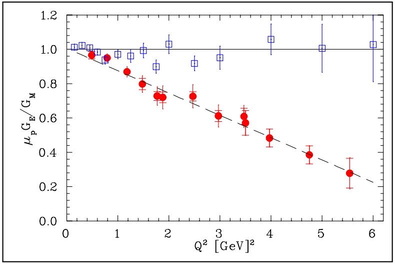
A possible explanation for the discrepancy:

Two-photon exchange

(Guichon, Vanderhaeghen (2003), Blunden et al. (2003), ...)



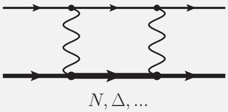


(Arrington, PRC (2003))

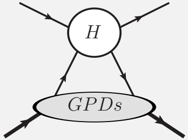
Calculation of the 2γ-Exchange Contribution

Models

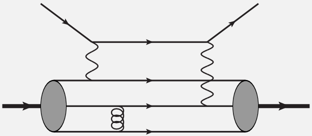
- Hadronic Calculation
  (Blunden et al. (2003,...))



- Partonic GPD-Calculation
  (Afanasev et al. (2005))

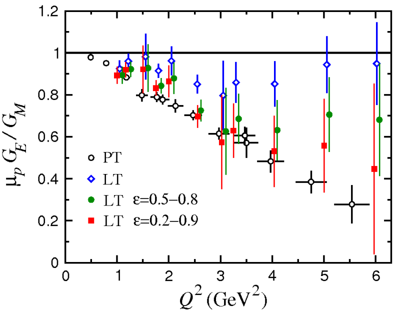


- pQCD factorization approach
  (Kivel, Vanderhaeghen (2009))



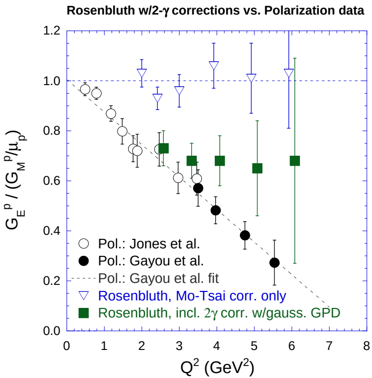
Calculation of the 2γ-Exchange Contribution

Hadronic Calculation



Blunden et al. PRC 72,034612

Partonic GPD-Calculation



Afanasev et al. PRD 72,013008

Time-like Electromagnetic Form Factors I

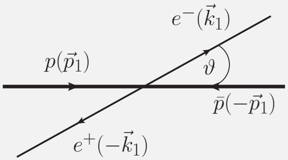
Born Approximation of $p + \overline{p} \rightarrow e^- + e^+$ ....

Matrix element of the reaction:

$$\mathcal{M} = \frac{-ie^2}{q^2} \overline{u}(k_1)\gamma_\mu v(k_2) \times \overline{v}(p_2) \left[G_M(q^2)\gamma^\mu - F_2(q^2)\frac{P^\mu}{m} \right] u(p_1)$$

G_M and F_2 are complex functions of q^2.

Kinematic: Choose c.m.-System



$$P = \frac{1}{2}(p_1 - p_2), \qquad K = \frac{1}{2}(k_1 - k_2)$$

Kinematical invariants:

$$q^2 = (p_1 + p_2)^2, \quad \nu = P \cdot K \propto \cos\vartheta$$

(unpolarized) **differential Cross Section**:

$$d\sigma_{cms,1\gamma} = \mathcal{C}(q^2) \left[|G_M|^2(1 + \cos^2\vartheta) + \frac{1}{\tau}|G_E|^2 \sin^2\vartheta \right]$$

Time-like Electromagnetic Form Factors II

...and beyond One-Photon Exchange

The matrix element can be expressed through 3 generalized form factors:

$$\mathcal{M} = \frac{-ie^2}{q^2}\bar{u}(k_1)\gamma_\mu v(k_2) \times \bar{v}(p_2)\left(\widetilde{G_M}\gamma^\mu - \widetilde{F_2}\frac{P^\mu}{m} + \widetilde{F_3}\frac{P^\mu}{m^2}\not{K}\right)u(p_1)$$

$\widetilde{G_M}$, $\widetilde{F_2}$, $\widetilde{F_3}$ are complex functions of q^2 and ν

$$\widetilde{G_M}(q^2,\nu) = G_M(q^2) + \delta\widetilde{G_M}(q^2,\nu), \quad \widetilde{F_3}(q^2,\nu) = \delta\widetilde{F_3}(q^2,\nu)$$

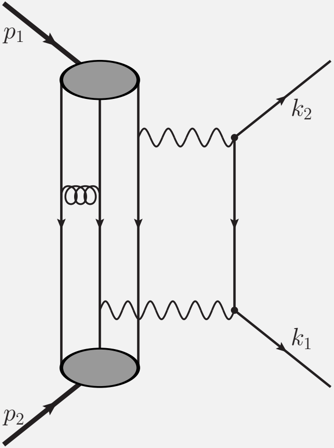
$$\widetilde{F_2}(q^2,\nu) = F_2(q^2) + \delta\widetilde{F_2}(q^2,\nu)$$

Cross section up to order α:

$$\begin{aligned} d\sigma_{cms} = \; & \mathcal{C}(q^2)\Big[|G_M|^2(1+\cos^2\vartheta) + \frac{1}{\tau}|G_E|^2\sin^2\vartheta \\ & + 2\mathrm{Re}[G_M\delta\widetilde{G_M}](1+\cos^2\vartheta) + 2\frac{1}{\tau}\mathrm{Re}[G_E\delta\widetilde{G_E}]\sin^2\vartheta \\ & + 2\left(\mathrm{Re}[G_M\widetilde{F_3}^*] - \frac{1}{\tau}\mathrm{Re}[G_E\widetilde{F_3}^*]\right)\sqrt{\tau(\tau-1)}\cos\vartheta\sin^2\vartheta\Big] \end{aligned}$$

(Gakh, Tomasi-Gustafsson NPA 761 (2005))

pQCD Factorization Approach



At large momentum transfer the leading twist contribution, where all valence quarks participate in the hard process, dominates

The leading pQCD contribution to the 2γ-exchange correction is given by a convolution of:

- Perturbative calculable hard kernel
- Nonpertubative contribution represented by proton **D**istribution **A**mplitudes

Nucleon Distribution Amplitudes I

Kinematic

$$q^2 \gg m^2 \quad \Longrightarrow \quad \begin{array}{rcl} p_1 & \simeq & \sqrt{s}\,\frac{\bar{n}}{2} \quad \text{with} \quad \bar{n} = (1,0,0,\ 1) \\ p_2 & \simeq & \sqrt{s}\,\frac{n}{2} \quad \text{with} \quad n = (1,0,0,-1) \end{array}$$

Three-quark Matrix Element

The proton-to-vacuum matrix element at leading twist is described by 3 DAs:

$$\begin{aligned} 4\langle 0|\varepsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)|P\rangle &= \boldsymbol{V}\, P^+ \left[\frac{1}{2}(\bar{n}\cdot\gamma)C\right]_{\alpha\beta} [\gamma_5 N^+]_\gamma \\ &+ \boldsymbol{A}\, P^+ \left[\frac{1}{2}(\bar{n}\cdot\gamma)\gamma_5 C\right]_{\alpha\beta} [N^+]_\gamma \\ &+ \boldsymbol{T}\, P^+ \left[\frac{1}{2} i\sigma_{\perp\bar{n}} C\right]_{\alpha\beta} [\gamma^\perp \gamma_5 N^+]_\gamma \end{aligned}$$

$$X(a_i, \lambda p^+) = \int d[x_i] exp(-i\lambda p^+ \sum x_i a_a) X(x_i), \qquad X = \{V, A, T\}$$

$$\text{with} \quad d[x_i] = dx_1 dx_2 dx_3\, \delta(1 - \sum x_i)$$

(Braun et al., NPB 589, (2000))

Nucleon Distribution Amplitudes II

Parametrization of the Distribution Amplitudes

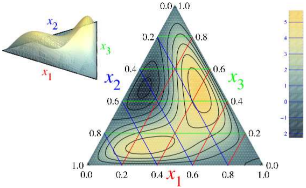
Asymptotic behavior of the DAs and the first corrections:

$$
\begin{aligned}
V(x_i) &\simeq 120 x_1 x_2 x_3\, f_N [1 + r_+(1 - 3x_3)] \\
A(x_i) &\simeq 120 x_1 x_2 x_3\, f_N r_- (x_2 - x_1) \\
T(x_i) &\simeq 120 x_1 x_2 x_3\, f_N \left[1 + \frac{1}{2}(r_- - r_+)(1 - 3x_3)\right]
\end{aligned}
$$

(Braun et al., NPB 589 (2000))

	f_N (10^{-3} GeV^2)	r_-	r_+
COZ	5.0 ± 0.5	4.0 ± 1.5	1.1 ± 0.3
BLW	5.0 ± 0.5	1.37	0.35
QCDSF	3.23	1.06	0.33

(Chernyak, Ogloblin, Zhitnitsky (1989);
Braun, Lenz, Wittmann (2006); Gockeler et al., (2008))



Nucleon DA from Lattice QCD
(PRL 101 (2008))

pQCD Factorization Approach II

Results

$$\begin{aligned}\delta\widetilde{G_M} &= -\left(\frac{4\pi}{3!}\right)^2 \frac{\alpha_{\mathrm{em}}\alpha_s}{q^4}(2\zeta-1)\int \frac{d[x_i]d[y_i]}{D}\frac{4x_2y_2}{x_1x_2(1-x_2)y_1y_2(1-y2)} \\ &\quad \times\Big[Q_u^2\left[(V'+A')(V+A)+4T'T\right](x_3,x_2,x_1) \\ &\quad + Q_uQ_d\left[(V'+A')(V+A)+4T'T\right](x_1,x_2,x_3) \\ &\quad + 2Q_uQ_d(V'V+A'A)(x_1,x_3,x_2)\Big]\end{aligned}$$

$$\begin{aligned}\frac{\nu}{m^2}\widetilde{F_3} &= -\left(\frac{4\pi}{3!}\right)^2 \frac{\alpha_{em}\alpha_s}{q^4}(2\zeta-1)\int \frac{d[x_i]d[y_i]}{D}\frac{x_2+y_2-2x_2y_2}{x_1x_2(1-x_2)y_1y_2(1-y2)} \\ &\quad \times\Big[Q_u^2\left[(V'+A')(V+A)+4T'T\right](x_3,x_2,x_1) \\ &\quad + Q_uQ_d\left[(V'+A')(V+A)+4T'T\right](x_1,x_2,x_3) \\ &\quad + 2Q_uQ_d(V'V+A'A)(x_1,x_3,x_2)\Big]\end{aligned}$$

$$\zeta = -\frac{t}{s}, \quad D = [x_2(1-\zeta)+y_2\zeta - x_2y_2 - i\varepsilon][x_2\zeta + y_2(1-\zeta) - x_2y_2 - i\varepsilon]$$

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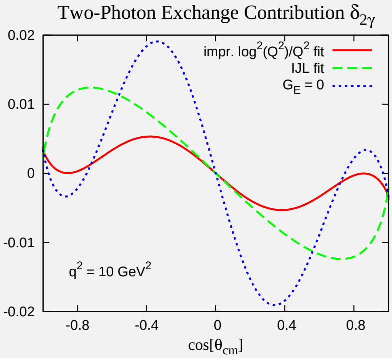
Results

Estimate of the Two-Photon Exchange Contribution



Cross Section:

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{2\gamma} = \left(\frac{d\sigma}{d\cos\theta}\right)_{1\gamma}\left(1+\delta_{2\gamma}\right)$$

Using:

- Form factor from QCD-fit:

$$G_M(q^2) = \frac{C}{Q^4 \log^2\left(\frac{Q^2}{\Lambda^2}\right)}$$

- DA: Model from Braun, Lenz, Wittmann (BLW) (PRD 73 (2006))

- Symmetry of 2γ contribution: $\delta_{2\gamma}(q^2, \cos\theta) = -\delta_{2\gamma}(q^2, -\cos\theta)$ (Gakh, Tomasi-Gustafsson NPA 771 (2006))

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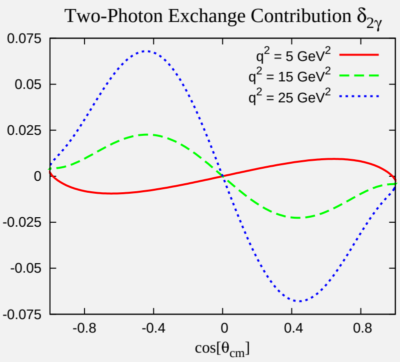

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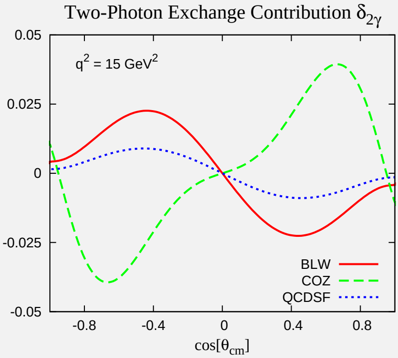

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Conclusion and Outlook

- **First calculation** and **estimate** of two-photon-exchange contribution for the reaction $p + \overline{p} \rightarrow e^- + e^+$ using a **pQCD factorization approach**
- Contributions from higher order corrections need to be studied
- Experiments at high q^2 are a good opportunity to check pQCD predictions
- Possible **measurement** of the process at the **PANDA@FAIR** project in the range of $q^2 = 5 - 28\ \mathrm{GeV}^2$