Central Depression of Nucleonic Densities

Trend analysis in the nuclear DFT approach

Bastian Schuetrumpf

Institut für Kerphysik, Technische Universität Darmstadt
GSI Helmholzzentrum für Schwerionenforschung

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Physical motivation

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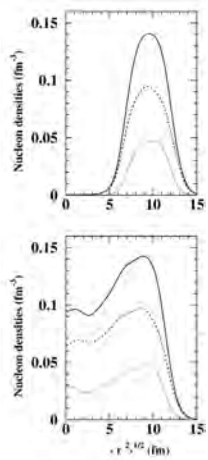
- Superheavy nuclei with vanishing/reduced central density (bubble/semi-bubble) expected already in the 1940s.
- Effect attributed to the electrostatic repulsion of protons.
- Central depression of light nuclei also expected in ^{34}Si, ^{46}Ar and ^{206}Hg. Here depression is due to low occupation of s-orbitals.
- Proton density distributions probed through electron scattering. PREX and CREX can probe neutron densities of ^{208}Pb and ^{48}Ca.

Questions:

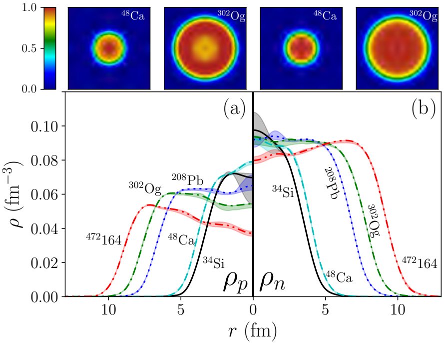
- What is the mechanism behind central depression in both light and heavy nuclei?
- Can we gain information about nuclear matter properties from density distributions?



Decharge et al., Nucl. Phys. A 716 (2003) 55

Nucleonic Densities

- Proton central depression for ^{34}Si, ^{302}Og and ^{472}164. Neutron central depression only for ^{472}164.
- ^{34}Si is very sensitive to EDF. (EDFs share about the same bulk properties).
- Central depression in ^{34}Si is large for SLy6 parametrization predicting large shell gap between $0d_{5/2}$ and $1s_{1/2}$.
- In general, all models agree in heavy and superheavy nuclei.



Radial Skyrme DFT densities of ^{34}Si, ^{48}Ca, ^{208}Pb, ^{302}Og, and ^{472}164. Shaded areas mark spread of results obtained with SV-min, SLy6, and UNEDF1.

Quantification of central depression

Modified Helm model:

- Parametrize density as:

$$\rho_H(r;\, R_d, \sigma) = \rho_h(r;\, R_d) * \rho_G(r;\, \sigma),$$

$$\rho_h(r;\, R_d) = (1 + wr^2)\rho_0\, \Theta(R_d - r)$$

  $\rho_G(r;\, \sigma)$: Gaussian with width σ.
  R_d: Diffraction radius.

- Quantification with

$$\bar{w} = w \cdot R_d^2$$

  which can be obtained from shift of first and second zero in form factor.

- Robust with respect to shell fluctuations.

Differences of densities:

- Quantification with

$$\bar{\rho}_{t,\mathrm{c}} = (\rho_{t,\mathrm{av}} - \rho_{t,\mathrm{c}})/\rho_{t,\mathrm{av}}$$

  $t = (n, p)$
  $\rho_{t,\mathrm{av}} = N_t/(4/3\pi R_d^3)$: Average density up to diffraction radius.
  $\rho_{t,\mathrm{c}}$: Central density.

- Straightforward way to quantify central depression.

- Sensitive to oscillations due to shell effects.

Coefficient of Determination (CoD)

Question: How well determined is an observable by one model parameter?

- Assessment through statistical analysis around the χ^2 minimum of EDF parametrization.
- Coefficient of determination:

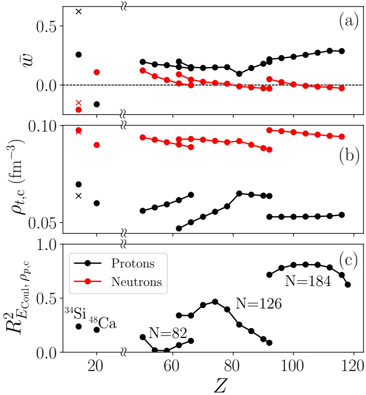
$$R^2_{x,y} = \left(\frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y} \right)^2$$

  where $\mathrm{cov}(x,y)$ is the covariance of x and y and σ_x is the standard deviation of x.

- Gives values between 0 and 1.
- Can be evaluated for correlations between two model parameters, two observables or one observable and one model parameter.
- CoD does not give any information about the rate of change (slope).

Central Depression

- ^{34}Si proton central depression increases dramatically without pairing (large Z=14 subshell closure).
- $\bar{w}_p$ larger than $\bar{w}_n$, flat up to Z=90. Increases in the N=184 isotonic chain.
- $\rho_{p,c}$ increasing for N=82 and 126 isotonic chains, but flat for N=184 chain.
- Dip at ^{208}Pb due to full occupation of 2s orbit.
- Correlation of central density with Coulomb energy only significant for N=184 chain.

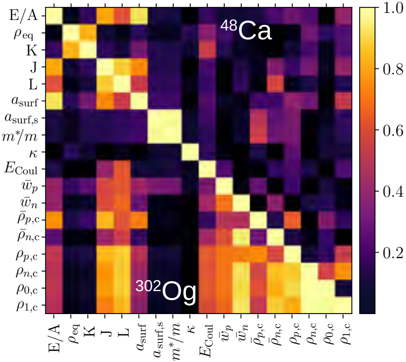


Proton central depression $\bar{w}_p$ (a), central density $\rho_{t,c}$ (b), and CoD between E_{Coul} and $\rho_{p,c}$ (c) predicted by SV-min. $\times$ marks ^{34}Si values obtained without pairing.

Correlation Analysis

- ^{48}Ca typical for nuclei lighter than ^{208}Pb, ^{302}Og for heavier nuclei.
- CoD between measures for central depression is low for ^{48}Ca because of shell effects, but high for ^{302}Og.
- CoD between observable and model parameters insignificant for ^{48}Ca, but large in ^{302}Og especially with E/A, J, L, a_{surf}.

$\rightarrow$ Central depression in nuclei below ^{208}Pb is governed by shell effects. For superheavy nuclei CoDs reveal strong correlation with LDM parameters and Coulomb.

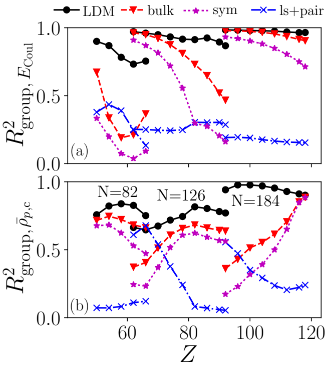


Matrices of CoD for SV-min parameters and selected observables in ^{48}Ca (upper triangle) and ^{302}Og (lower triangle).

Multiple Correlations

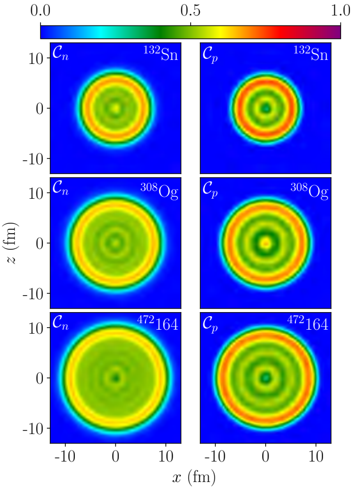
- *LDM*: (E/A, $\rho_{\rm eq}$, K, J, L, $a_{\rm surf}$, $a_{\rm surf,s}$)
- *bulk*: (E/A, $\rho_{\rm eq}$, K, J, L)
- *sym*: (J, L)
- *ls+pair*: ($C_t^{\rho\nabla J}$, $V_{{\rm pair},t}$, $\rho_{\rm pair}$), t=(n,p)

- $E_{\rm Coul}$ almost fully determined by LDM.
- Surface effects important for N=82 and for large Z due to competition between Coulomb and surface tension.
- $\bar{\rho}_{\rm p,c}$ largely determined by LDM, especially for superheavy nuclei.
- Symmetry energy very important for heavy and superheavy nuclei, where Coulomb determines the central depression.



Multiple correlation coefficients with $E_{\rm Coul}$ (a) and $\bar{\rho}_{p,c}$ (b) in heavy nuclei.

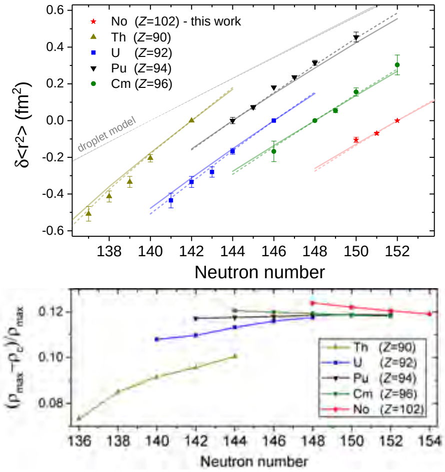
Localization for superheavy nuclei



Jerabek, B.S., Schwerdtfeger, Nazarewicz PRL 120, 053001 (2018)

- Localization:
  High value: One shell dominates.
  Low value: shells overlapping.
  0.5: Fermi gas limit.
- Neutron and proton localization function for ^{132}Sn, ^{302}Og, and ^{472}164.
- Going heavier, shell structure fades and extrema become fainter in NLF. Transition to Fermi gas.
- Effect is stronger for Neutrons, because of more particles occupying the same space (higher level density).

Charge radius isotopic shifts



Raeder et al. PRL 120, 232503 (2018)

- Measurement of nobelium charge radius isotopic shifts at GSI.
- DFT can describe isotopic shifts of charge radii for heavy nuclei.
- Simultaneously, DFT predicts appreciable semi-bubbles.
- Simple droplet models fail to predict isotopic shifts.

Conclusions

Light and medium mass nuclei:

- Central depression in ^{34}Si highly sensitive to the input used (interactions, EDFs).
- No strong correlation of density distribution of ^{48}Ca with specific parameter.

$\rightarrow$ Density distributions of light nuclei below ^{208}Pb especially ^{34}Si are governed by shell effects and carry little information about nuclear matter parameters.

Heavy nuclei:

- Central depression in heavy nuclei above lead is mainly driven by electrostatic repulsion.
- Since the Coulomb term has a large isovector component, the symmetry energy is strongly correlated to central depression.
- Heavy nuclei are closer to the leptodermous limit (determined by LDM parameters) and density distribution can be related better to nuclear matter parameters.
- Predicted densities are consistent with newest charge radii measurements.

Thanks!

Thanks to my collaborators: **Witek Nazarewicz** and **Paul-Gerhard Reinhard**.

Thank you for your attention!

Density Functional Theory with Skyrme Energy Density Functionals

- Nucleonic part of the energy density functional (EDF):

$$\mathcal{E}_{\rm Sk} = \sum_{t=0,1} \left(C_t^{\rho}(\rho_0)\rho_t^2 + C_t^{\rho\Delta\rho}\rho_t\Delta\rho_t + C_t^{\tau}\rho_t\tau_t + C_t^{\rho\nabla J}\rho_t\nabla \boldsymbol{J}_t \right)$$ (t=0: isoscalar; t=1: iso

$$\mathcal{E}_{\rm pair} = \sum_{q=p,n} \frac{V_{\rm pair,q}}{2}\left[1 - \frac{\rho_0}{\rho_{\rm pair}}\right]\tilde{\rho}^2$$

- Some constants can be expressed through nuclear matter properties:

$\rho_{\rm eq}$ Equilibrium density.

E/A Energy-per-nucleon at equilibrium.

K Incompressibility

m^*/m Effective mass characterizing the dynamical isoscalar response.

J Symmetry energy.

L Slope of symmetry energy.

κ Thomas-Reiche-Kuhn sum-rule enhancement characterizing the dynamical isovector response.

$a_{\rm surf}$ Surface energy coefficient

$a_{\rm surf,s}$ Surface-symmetry energy coefficient.

$C_t^{\rho\nabla J}$ Spin-orbit parameters.

$V_{\rm pair,q}$, $\rho_{\rm pair}$ Pairing parameters.

Multiple correlation coefficient (MMC)

Question: How well determined is an observable by a group of model parameters?

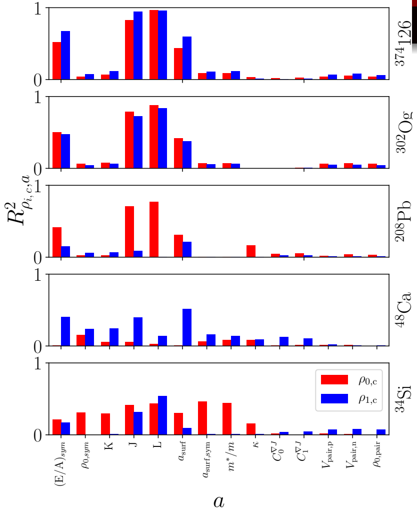
- CoDs are not an additive quantity due to correlations between model parameters.
- Multiple correlation coefficient:

$$R^2_{\boldsymbol{a},x} = \boldsymbol{c}^T (R_{\boldsymbol{a},\boldsymbol{a}})^{-1} \boldsymbol{c}$$

  where $R_{\boldsymbol{a},\boldsymbol{a}}$ contains correlations between the model parameters of group $\boldsymbol{a}$, $\boldsymbol{c} = (R_{a_{\mathbf{1}},x}, R_{a_{\mathbf{2}},x}, ...)$ contains correlations between the observable and single group members.
- Value of 0.3 means that 30% of the variance of x is predictable from group $\boldsymbol{a}$.
- If $\boldsymbol{a}$ contains all model parameters, $R^2_{\boldsymbol{a},x} = 1$ always.

Correlation Analysis

- Isoscalar and isovector densities correlate weakly with nuclear matter properties and other model parameters for nuclei below ^{208}Pb.
- Central densities in nuclei above lead carry information on nuclear matter properties, especially the symmetry energy.



Correlations of central densities and model parameters.