Thermodynamic and hydrodynamic description of relativistic heavy-ion collisions

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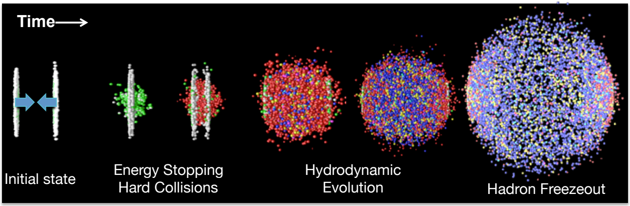
Jagiellonian University, Sept. 18, 2018, FAIR WORKSHOP

Outline

1.1 Standard model of heavy-ion collisions



T. K. Nayak, Lepton-Photon 2011 Conference

FIRST STAGE — HIGHLY OUT-OF EQUILIBRIUM ($0 < \tau_0 \lesssim 1\ \mathrm{fm}$)

- **initial conditions**, including fluctuations, reflect to large extent the distribution of matter in the colliding nuclei — Glauber model, works by A. Białas and W. Czyż
- **emission of hard probes**: heavy quarks, photons, jets
- **hydrodynamization stage** – the system becomes well described by equations of viscous hydrodynamics — crucial contributions from R. Janik and his collaborators

SECOND STAGE — HYDRODYNAMIC EXPANSION ($1\,\mathrm{fm} \lesssim \tau \lesssim 10\,\mathrm{fm}$)

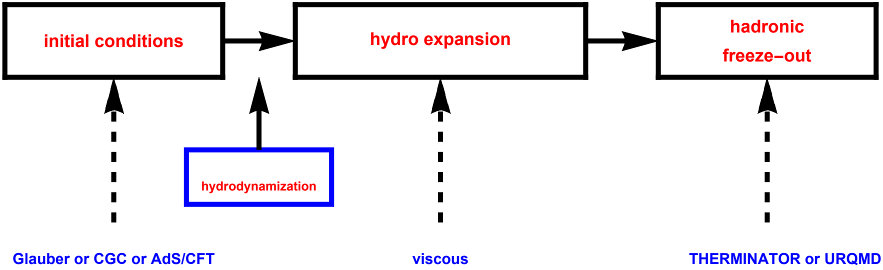
- expansion controlled by viscous hydrodynamics (effective description)
- **thermalization stage**
- **phase transition** from QGP to hadron gas takes place (encoded in the equation of state)
- **equilibrated hadron gas**

THIRD STAGE — FREEZE-OUT

- **freeze-out and free streaming of hadrons** ($10\,\mathrm{fm} \lesssim \tau$)

IN THIS TALK (except for the last part) EFFECTS OF FINITE BARYON NUMBER DENSITY ARE NEGLECTED

STANDARD MODEL (MODULES) of HEAVY–ION COLLISIONS



FLUCTUATIONS IN THE INITIAL STATE / EVENT–BY–EVENT HYDRO / FINAL–STATE FLUCTUATIONS

EQUATION OF STATE = lattice QCD

1 < VISCOSITY < 3 times the lower bound

Danielewicz and Gyulassy (quantum mechanics), Kovtun+Son+Starinets (AdS/CFT)

lower bound on the ratio of shear viscosity to entropy density $\eta/S = 1/(4\pi)$

1.2 From perfect-fluid to viscous hydrodynamics

$T(x)$ and $u^\mu(x)$ are fundamental fluid variables

the relativistic perfect-fluid energy-momentum tensor is the most general symmetric tensor which can be expressed in terms of these variables without using derivatives

dynamics of the perfect fluid theory is provided by the conservation equations of the energy-momentum tensor, four equations for the four independent hydrodynamic fields – a self-consistent (hydrodynamic) theory

$$\partial_\mu T^{\mu\nu}_{\rm eq} = 0, \qquad T^{\mu\nu}_{\rm eq} = \mathcal{E} u^\mu u^\nu - \mathcal{P}_{\rm eq}(\mathcal{E}) \Delta^{\mu\nu}, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \tag{1}$$

$$\mathcal{E}_{\rm eq}(T(x)) = \mathcal{E}(x), \qquad T^{\mu\nu}_{\rm eq}(x) u_\nu(x) = \mathcal{E}(x) u^\mu(x). \tag{2}$$

local rest frame: $\quad u^\mu = (1,0,0,0) \quad \rightarrow \quad T^{\mu\nu}_{\rm eq} = \begin{bmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P}{\rm eq} & 0 & 0 \\ 0 & 0 & \mathcal{P}{\rm eq} & 0 \\ 0 & 0 & 0 & \mathcal{P}{\rm eq} \end{bmatrix} \quad (3)$

DISSIPATION DOES NOT APPEAR! $u_\nu \partial_\mu T^{\mu\nu}_{\rm eq} = 0 \rightarrow \partial_\mu (\mathcal{S} u^\mu) = 0$

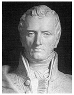
entropy conservation follows from the energy-momentum conservation and the form of the energy-momentum tensor

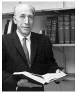
Navier-Stokes hydrodynamics

Claude-Louis Navier, 1785–1836, French engineer and physicist
Sir George Gabriel Stokes, 1819–1903, Irish physicist and mathematician

C. Eckart, Phys. Rev. 58 (1940) 919

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1959






complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu} \tag{4}$$

where $\Pi^{\mu\nu} u_\nu = 0$, which corresponds to the Landau definition of the hydrodynamic flow u^μ

$$T^\mu_{\ \nu} u^\nu = \mathcal{E}\, u^\mu. \tag{5}$$

It proves useful to further decompose $\Pi^{\mu\nu}$ into two components,

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Pi \Delta^{\mu\nu}, \tag{6}$$

which introduces the **bulk viscous pressure** Π (the trace part of $\Pi^{\mu\nu}$) and the **shear stress tensor** $\pi^{\mu\nu}$ which is symmetric, $\pi^{\mu\nu} = \pi^{\nu\mu}$, traceless, $\pi^{\mu}_{\ \mu} = 0$, and orthogonal to u^μ, $\pi^{\mu\nu} u_\nu = 0$.

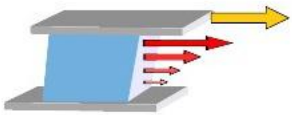
in the Navier-Stokes theory, the **bulk pressure** and **shear stress tensor** are given by the gradients of the flow vector

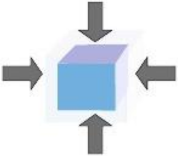
$$\Pi = -\zeta\, \partial_\mu u^\mu, \quad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}. \tag{7}$$

Here ζ and η are the bulk and shear viscosity coefficients, respectively, and $\sigma^{\mu\nu}$ is the shear flow tensor

shear viscosity η
$\Downarrow$
reaction to a change of shape

bulk viscosity ζ
$\Downarrow$
reaction to a change of volume





$$\pi^{\mu\nu}{}_{\rm Navier-Stokes} = 2\eta\, \sigma^{\mu\nu}$$

$$\Pi_{\rm Navier-Stokes} = -\zeta\theta$$

Navier-Stokes hydrodynamics

complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} = T^{\mu\nu}_{\rm eq} + 2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu} \qquad (8)$$

again four equations for four unknowns

$$\partial_\mu T^{\mu\nu} = 0 \qquad (9)$$

1) THIS SCHEME DOES NOT WORK IN PRACTICE!

ACAUSAL BEHAVIOR + INSTABILITIES!

2) NEVERTHELESS, THE GRADIENT FORM (8) IS A GOOD APPROXIMATION

FOR SYSTEMS APPROACHING LOCAL EQUILIBRIUM

Great progress has been made in the last years to understand the hydrodynamic gradient expansion by

R. Janik, M. Spaliński , M. P. Heller, P. Witaszczyk and their collaborators

Israel-Stewart equations

$\Pi, \pi^{\mu\nu}$ promoted to new hydrodynamic variables!

W. Israel and J.M. Stewart, *Transient relativistic thermodynamics and kinetic theory*, Annals of Physics 118 (1979) 341

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta, \qquad \tau_\Pi \beta_\Pi = \zeta \tag{10}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu}, \qquad \tau_\pi \beta_\pi = 2\eta \tag{11}$$

1) HYDRODYNAMIC EQUATIONS DESCRIBE BOTH HYDRODYNAMIC AND NON-HYDRODYNAMIC MODES

perturbations $\sim \exp(-\omega_k t)$, hydro modes $\omega_k \to 0$ for $k \to 0$, nonhydro modes $\omega_k \to \mathrm{const} \neq 0$ for $k \to 0$

2) HYDRODYNAMIC MODES CORRESPOND TO GENUINE HYDRODYNAMIC BEHAVIOR

3) NON-HYDRODYNAMIC MODES (TERMS) SHOULD BE TREATED AS REGULATORS OF THE THEORY

4) NON-HYDRODYNAMIC MODES GENERATE ENTROPY

1.3 Equation of state

Equation of state

in ultrarelativistic collisions (top RHIC and the LHC energies) we may neglect the baryon number

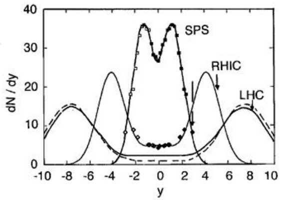
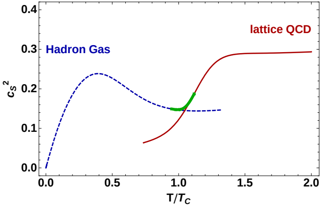


FIG. 1: Net-proton rapidity spectra in the Relativistic Diffusion Model (RDM), solid curves: Transition from the double-humped shape at SPS energies of $\sqrt{s_{NN}}$ = 17.3 GeV to a broad midrapidity valley in the three-sources model at RHIC (200 GeV) and LHC (5.52 TeV). See [11] for details.

R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)



M. Chojnacki, WF, Acta Phys.Pol. B38 (2007) 3249

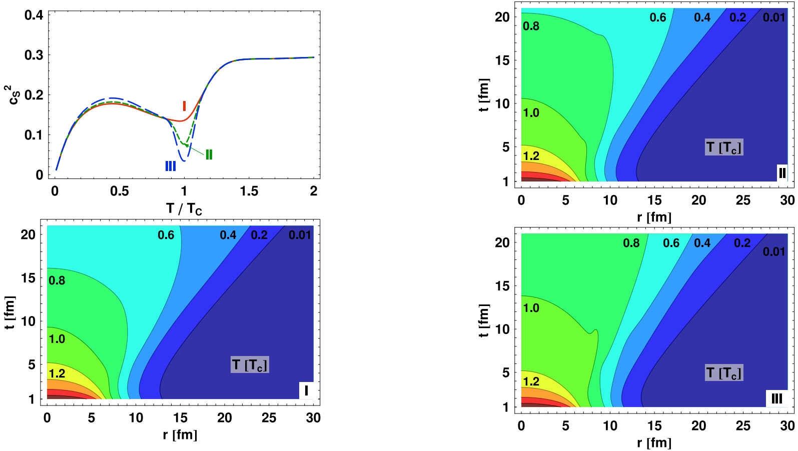
$$c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}}$$

$c_s^2 = \frac{1}{3}$ for conformal systems

$c_s^2 \to 0$ if $T \to T_{\rm critical}$ for the 1st order phase transition

Equation of state

EOS can be checked experimentally by looking at the HBT correlations that give information about the space-time extensions of the system



Equation of state

further evidence for semi-hard EOS (crossover) from complete perfect-fluid simulations

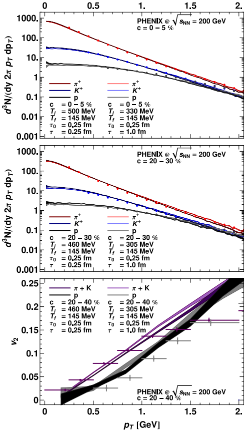
solution of the so-called HBT puzzle
$C(\mathbf{k},\mathbf{q}) \to C(k_\perp,\mathbf{q}) \to C(k_\perp,\mathbf{R})$
$R_{\rm out}/R_{\rm side} \sim 1$

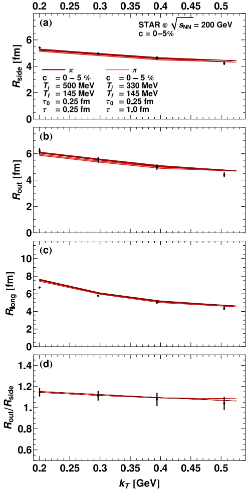
early start of hydro: 0.6 fm/c → early-thermalization puzzle

fast freeze-out process

overall short timescales due to fast expansion

W. Broniowski, M. Chojnacki, WF, A. Kisiel, Phys.Rev.Lett. 101 (2008) 022301





1.4 Shear and bulk viscosities

Harmonic flows

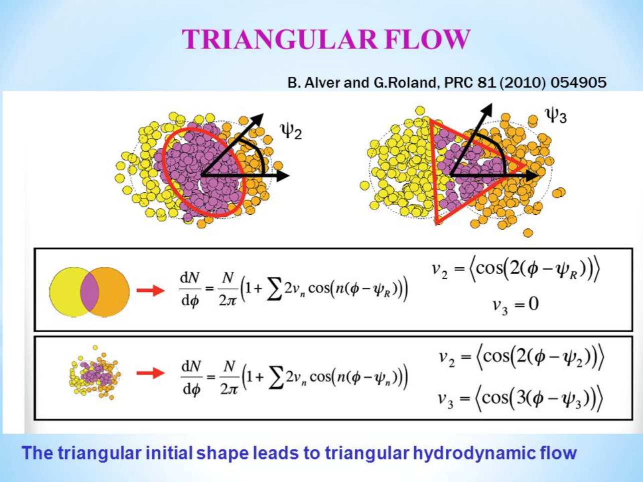
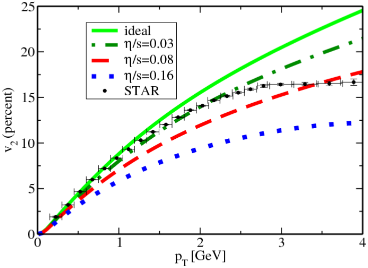


figure from L. Bravina's presentation at Quark Confinement and the Hadron Spectrum XI

Elliptic flow

shear viscosity affects elliptic flow

first principles tell us that one should use relativistic dissipative hydrodynamics, but better description of the data is also achieved with finite but small $\eta/\mathcal{S}$



P. Romatschke and U. Romatschke, PRL 99 (2007) 172301

E. Shuryak: small $\eta/\mathcal{S}$ means that QGP is strongly interacting, previous concepts of QGP hold at really asymptotic energies

QGP shear viscosity: large or small?

John Mainstone (Wikipedia)

Pitch

Start 1927

1^{st} drop 1938

8^{th} drop 2000

$\eta \sim 2\,10^8$ Pa s

$\sim 10^{11} \eta_{H_2O}$

Wikipedia: The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

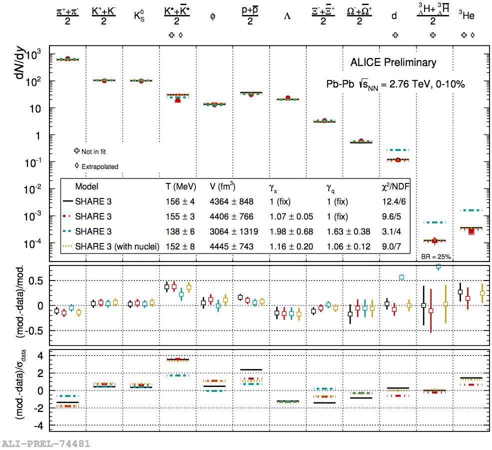
$$\eta_{\rm qgp} > \eta_{\rm pitch}$$

$$\eta_{\rm qgp} \sim 10^{11}\ \text{Pa s}, \qquad (\eta/s)_{\rm qgp} < 3/(4\pi)\hbar \qquad \text{(from experiment)}$$

2 Freeze-out models

2.1 Thermal models for the ratios of hadronic abundances

Thermal fit to hadron multiplicity ratios



M. Floris, Nucl. Phys. A931 (2014) c103

P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel

Hadron production in Au+Au collisions at RHIC, Phys.Lett. B518 (2001) 41

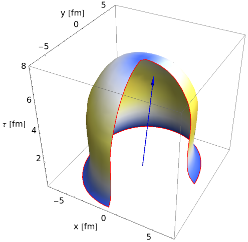
2.2 Single-freeze-out model/scenario

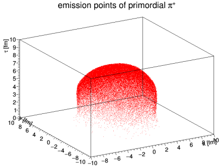
Cooper-Frye formula describing spectra of emitted particles (hadrons) on the freeze-out hyper surface $\Sigma_\mu(x)$

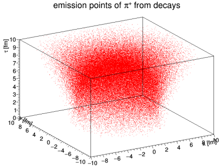
$$E_p \frac{dN}{d^3p} = \int d\Sigma_\mu(x) p^\mu f_{\rm eq}(x,p) \tag{12}$$

basis for the thermal models — expansion "cancels" in the ratios

$$\frac{N_i}{N_j} = \frac{n_i^{\rm eq}(T,\mu) \int d\Sigma_\mu u^\mu}{n_j^{\rm eq}(T,\mu) \int d\Sigma_\mu u^\mu} = \frac{n_i^{\rm eq}(T,\mu)}{n_j^{\rm eq}(T,\mu)} \tag{13}$$







"Monte-Carlo statistical hadronization in relativistic heavy-ion collisions" by R. Ryblewski, arXiv:1712.05213

W. Broniowski, WF, "Explanation of the RHIC p(T) spectra in a thermal model with expansion", Phys. Rev. Lett. 87 (2001) 272302

chemical freeze-out (fixed ratios of abundances) = kinetic freeze-out (fixed spectra)

- SHARE: Statistical hadronization with resonances G. Torrieri, S. Steinke (Arizona U.), W. Broniowski (Cracow, INP), WF, J. Letessier, J. Rafelski (Arizona U.) Comput. Phys. Commun. 167 (2005) 229
- THERMINATOR: THERMal heavy-IoN generATOR A. Kisiel, T. Taluc (Warsaw U. of Tech.), W. Broniowski (Cracow, INP), WF, Comput. Phys. Commun. 174 (2006) 669
- THERMINATOR 2: THERMal heavy IoN generATOR 2 M. Chojnacki (Cracow, INP), Adam Kisiel (CERN & Warsaw U. of Tech.), WF, Wojciech Broniowski (Cracow, INP & Jan Kochanowski U.), Comput. Phys. Commun. 183 (2012) 746.

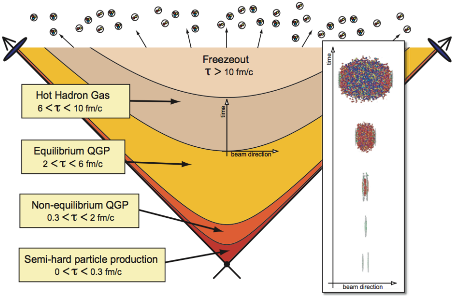
resonances important not only for the ratios but also for the spectra
theoretical basis -> virial expansion
open-source codes in heavy-ion physics

3 Anisotropic hydrodynamics

3.1 Problems of standard (IS) viscous hydrodynamics

Simplified space-time diagram

space-time diagram for a simplified, one dimensional and boost-invariant expansion



M. Strickland, Acta Phys.Polon. B45 (2014) 2355

evolution governed by the proper time $\tau = \sqrt{t^2 - z^2}$

Pressure anisotropy

space-time gradients in boost-invariant expansion increase the transverse pressure and decrease the longitudinal pressure

$$\mathcal{P}_T = \mathcal{P} + \frac{\pi}{2}, \quad \mathcal{P}_L = \mathcal{P} - \pi, \quad \pi = \frac{4\eta}{3\tau} \tag{14}$$

$$\left(\frac{\mathcal{P}_L}{\mathcal{P}_T}\right)_{\rm NS} = \frac{3\tau T - 16\bar{\eta}}{3\tau T + 8\bar{\eta}}, \quad \bar{\eta} = \frac{\eta}{\mathcal{S}}$$

using the AdS/CFT lower bound for viscosity, $\bar{\eta} = \frac{1}{4\pi}$

RHIC-like initial conditions, $T_0 = 400$ MeV at $\tau_0 = 0.5$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.50$
LHC-like initial conditions, $T_0 = 600$ MeV at $\tau_0 = 0.2$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.35$

3.2 Concept of aHydro

Thermodynamic & kinetic-theory formulations

Thermodynamic formulation

WF, R. Ryblewski
PRC 83, 034907 (2011), JPG 38 (2011) 015104

1. energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$
2. ansatz for the entropy source, e.g., $\partial(\sigma U^\mu) \propto (\lambda_\perp - \lambda_\parallel)^2/(\lambda_\perp \lambda_\parallel)$

Kinetic-theory formulation

M. Martinez, M. Strickland
NPA 848, 183 (2010), NPA 856, 68 (2011)

1. first moment of the Boltzmann equation = energy-momentum conservation
2. zeroth moment of the Boltzmann equation = specific form of the entropy source

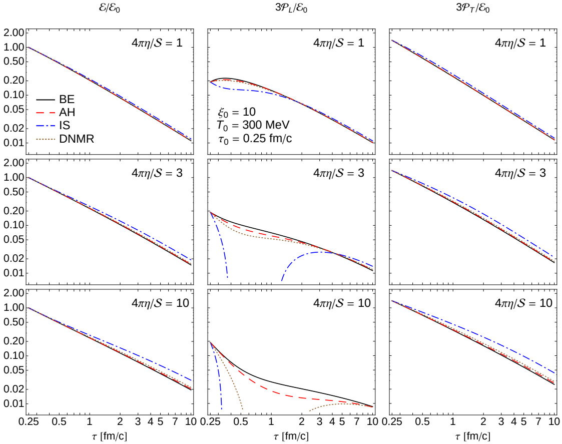
3. Generalized form of the equation of state based on the Romatschke-Strickland (RS) form

generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution in the local rest frame)

$$f_{RS} = \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right) = \exp\left(-\frac{1}{\Lambda}\sqrt{p_\perp^2 + (1+\xi)\, p_\parallel^2}\right)$$

anisotropy parameter $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ and transverse-momentum scale $\lambda_\perp = \Lambda$

WF, R. Ryblewski, M. Strickland, Phys.Rev. C88 (2013) 024903, $m = 0$, boost-invariant, transversally homogeneous system, (0+1) case



aHydro being used and developed now by Heinz (Columbus, Ohio), Strickland (Kent, Ohio), Schaeffer (North Carolina), Rischke (Frankfurt), ... ; applied in other branches of physics, cold atoms, ...

4 Hydrodynamics with spin

4.1 Is QGP the most vortical fluid?

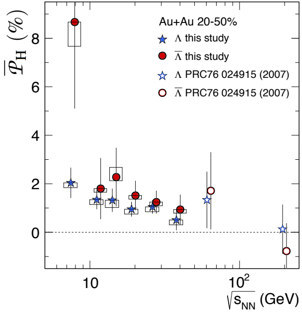
First positive measurements of Λ spin polarization

Non-central heavy-ion collisions create fireballs with large global angular momenta which may generate a spin polarization of the hot and dense matter in a way similar to the Einstein-de Haas and Barnett effects

Much effort has recently been invested in studies of polarization and spin dynamics of particles produced in high-energy nuclear collisions, both from the experimental and theoretical point of view

L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**, arXiv:1701.06657 (nucl-ex)

Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid
www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever



4.2 Perfect fluid with spin

WF, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (2018) 041901 new hydrodynamic variables connected with the conservation of angular momentum — spin polarisation (antisymmetric) tensor $\omega_{\mu\nu}$

$$\zeta = \frac{\Omega}{T} = \frac{1}{2}\sqrt{\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu}}, \qquad \xi = \frac{\mu}{T} \tag{15}$$

pressure P becomes a function of temperature, T, chemical potential, μ, and spin chemical potential, Ω, with

$$s = \left.\frac{\partial P}{\partial T}\right|_{\mu,\Omega}, \quad n = \left.\frac{\partial P}{\partial \mu}\right|_{T,\Omega}, \quad w = \left.\frac{\partial P}{\partial \Omega}\right|_{T,\mu}.$$

s - entropy density, n - charge density, w - spin density

The conservation of energy and momentum requires that $\partial_\mu T^{\mu\nu} = 0$
This equation can be split into two parts, one longitudinal and the other transverse with respect to u^μ:

$$\begin{aligned} \partial_\mu[(\mathcal{E}+P)u^\mu] &= u^\mu\partial_\mu P \equiv \frac{dP}{d\tau} \quad \text{entropy conservation} \\ (\mathcal{E}+P)\frac{du^\mu}{d\tau} &= (g^{\mu\alpha} - u^\mu u^\alpha)\partial_\alpha P \quad \text{relativistic Euler equation} \end{aligned}$$

Evaluating the derivative on the left-hand side of the first equation we find

$$T\,\partial_\mu(su^\mu) + \mu\,\partial_\mu(nu^\mu) + \Omega\,\partial_\mu(wu^\mu) = 0.$$

The middle term vanishes due to charge conservation,

$$\partial_\mu(nu^\mu) = 0.$$

Thus, in order to have entropy conserved in our system (for the perfect-fluid description we are aiming at), we demand that

$$\partial_\mu(wu^\mu) = 0.$$

Consequently, we self-consistently arrive at the conservation of entropy, $\partial_\mu(su^\mu) \;=\; 0$
Equations above form dynamic background for the spin dynamics.

Spin dynamics

Using the conservation law for the spin tensor and introducing the rescaled spin polarisation tensor $\bar{\omega}^{\mu\nu} = \omega^{\mu\nu}/(2\zeta)$, with, $\zeta = \Omega/T$, we obtain

$$u^\lambda \partial_\lambda \, \bar{\omega}^{\mu\nu} = \frac{d\bar{\omega}^{\mu\nu}}{d\tau} = 0,$$

with the normalization condition $\bar{\omega}_{\mu\nu} \, \bar{\omega}^{\mu\nu} = 2$.

TRANSPORT OF THE SPIN POLARIZATION DIRECTION ALONG THE FLUID STREAM LINES

CHANGE OF THE POLARIZATION MAGNITUDE DESCRIBED BY THE BACKGROUND EQUATIONS

Quasi-realistic model for low-energy collisions

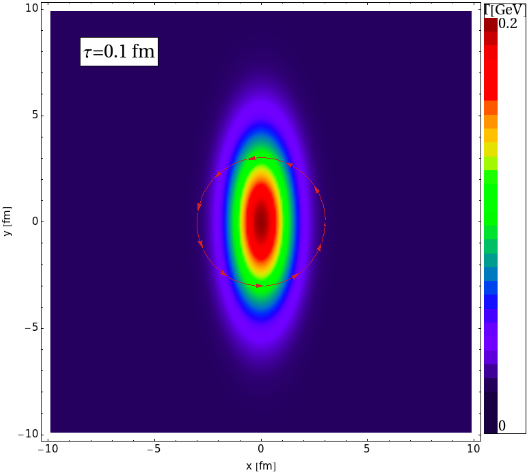
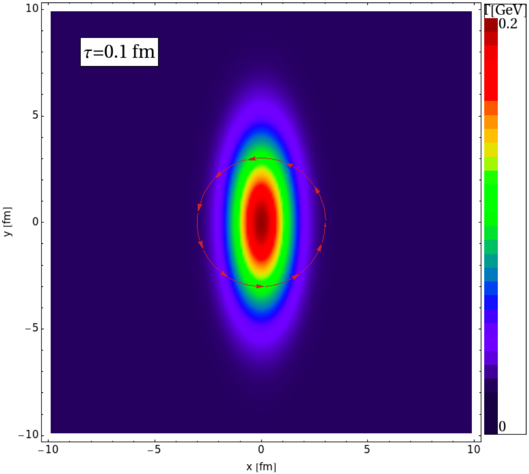
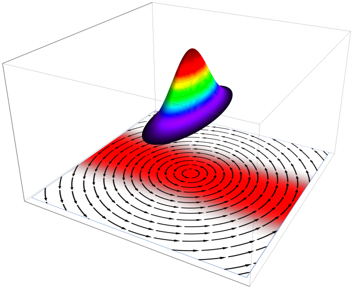


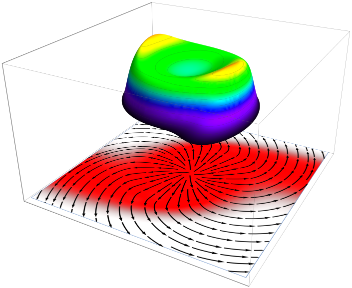
Figure: Initial conditions for the quasi-realistic model

Quasi-realistic model for low-energy collisions



Quasi-realistic model for low-energy collisions

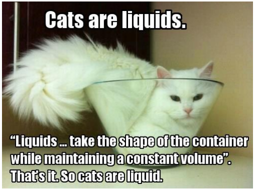




2-D temperature profiles over stream lines denoted by arrows, the region with polarised particles is marked in red, it evolves in time following the flow lines

5 Summary

- golden era of heavy-ion collisions
- enormous progress in both experiment and theory
- great success of statistical methods



a few more advertisments:

- WF "Phenomenology of ultra-relativistic heavy-ion collisions" World Scientific 2010
- WF, M. P. Heller, M. Spaliński, "New theories of relativistic hydrodynamics in the LHC era", Rept. Prog. Phys. 81 (2018) 046001
- Quark Matter 2021 in Kraków, Oct. 4-9, 2021, ICE Congress Center