The QCD equation of state at finite density, from the known to the unknown

Jan Steinheimer
with contributions from
V. Vovchenko, A. Motornenko, A. Mukherjee, S. Schramm, M. Hanauske, L. Rezzolla and H. Stöcker

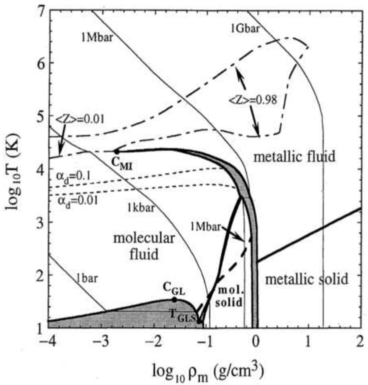
05/15/2018

FIAS Frankfurt Institute for Advanced Studies

Motivation

The legacy of high energy nuclear physics?

Hydrogen

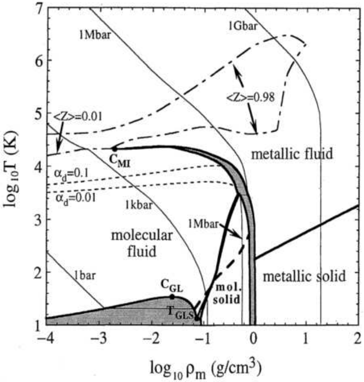


Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

Motivation

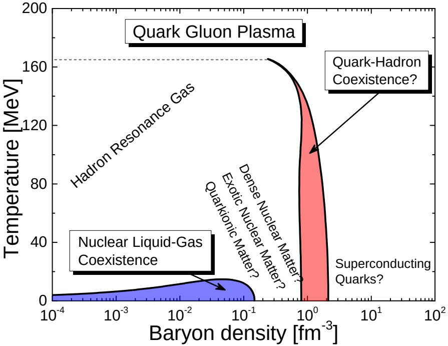
The legacy of high energy nuclear physics?

Hydrogen



Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

QCD



V. A. Dexheimer and S. Schramm, Phys. Rev. C **81**, 045201 (2010)

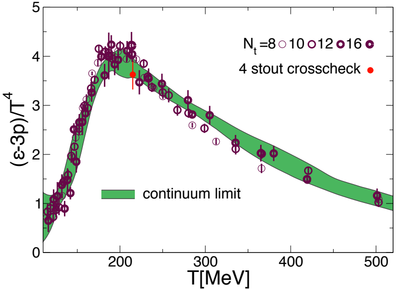
used in: **JS** , V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm and H. Stoecker, Phys. Rev. C **81**, 044913 (2010)

Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV and $\mu_B/T \leq \pi$.

Constraints from lQCD:

- The Interaction measure, thermodynamics at $\mu_B = 0$
- Derivatives of the pressure wrt μ_B. Expansion into finite μ_B.
- Calculations at imaginary μ.



S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B **730**, 99 (2014)

Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV and $\mu_B/T \leq \pi$.
- Neutron stars, for $T = 0$ and $\rho_B \leq 6\rho_0$.

Constraints from neutron stars:

- The mass-radius diagram.
- with the TOV equation: a unique mapping

Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV and $\mu_B/T \leq \pi$.
- Neutron stars, for $T = 0$ and $\rho_B \leq 6\rho_0$.
- Nuclear matter, for $T = 0$ and $\rho_B \leq 1\rho_0$.

Constraints from nuclear matter:

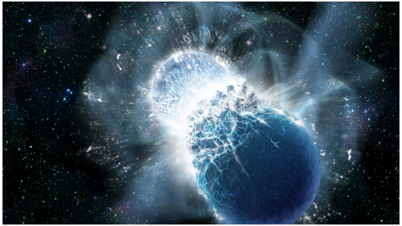
- Saturation density.
- Binding energies, also of hyperons
- Vacuum masses

Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV and $\mu_B/T \leq \pi$.
- Neutron stars, for $T = 0$ and $\rho_B \leq 6\rho_0$.
- Nuclear matter, for $T = 0$ and $\rho_B \leq 1\rho_0$.
- Neutron star mergers, Part of this talk.

Constraints from neutron star mergers:

- **The Gravitationally Driven Super Massive Ion Collider.**
- Compression and heating at the same time.
- How does the EoS play in here?
- A new playground for physicists.



Credit: NASA/Swift/Dana Berry

Getting the most out of lattice QCD $\rightarrow$ the CEM model

Using only the Fourier coefficients b_k from imaginary μ_B simulations as input:

Getting the most out of lattice QCD $\rightarrow$ the CEM model

Using only the Fourier coefficients b_k from imaginary μ_B simulations as input:

- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$

Getting the most out of lattice QCD $\rightarrow$ the CEM model

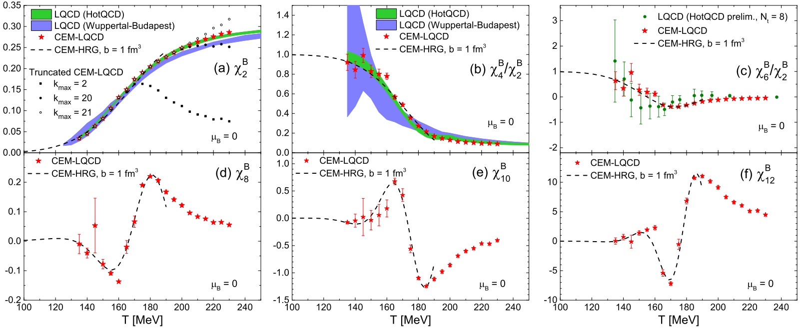
Using only the Fourier coefficients b_k from imaginary μ_B simulations as input:

- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$
- Assuming the proper SB limit and using only the first two coefficients on can exactly predict finite μ_B thermodynamics
- $b_k(T) = \alpha_k \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}$.

Getting the most out of lattice QCD $\rightarrow$ the CEM model

Using only the Fourier coefficients b_k from imaginary μ_B simulations as input:

- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$
- Assuming the proper SB limit and using only the first two coefficients on can exactly predict finite μ_B thermodynamics
- $b_k(T) = \alpha_k \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}.$



Results on the possibility of a critical point

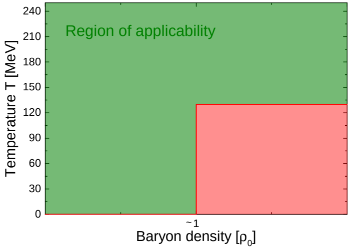
Consistent with: No critical endpoint for $\mu_B/T < \pi$

V. Vovchenko, **JS** , O. Philipsen and H. Stoecker, arXiv:1711.01261 [hep-ph], accepted by PRD.

Getting the most out of lattice QCD → the CEM model

Using only the Fourier coefficients b_k from imaginary μ_B simulations as input:

- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$
- Assuming the proper SB limit and using only the first two coefficients on can exactly predict finite μ_B thermodynamics
- $b_k(T) = \alpha_k \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}$.



Current limitations on lattice data.
Below $T = 130$ MeV no guidance → GSI and NSM?

Results on the possibility of a critical point

Consistent with: No critical endpoint for $\mu_B/T < \pi$

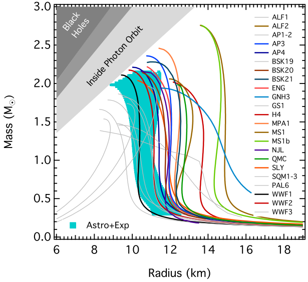
V. Vovchenko, **JS** , O. Philipsen and H. Stoecker, arXiv:1711.01261 [hep-ph], accepted by PRD.

Constraints at $T = 0$

- Here we have guidance from measured neutron star masses

Constraints at $T = 0$

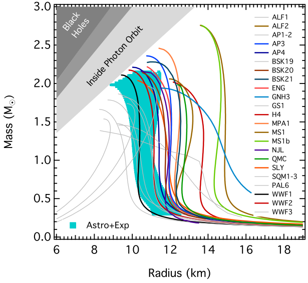
- Here we have guidance from measured neutron star masses
- Without Radii no real constraints!



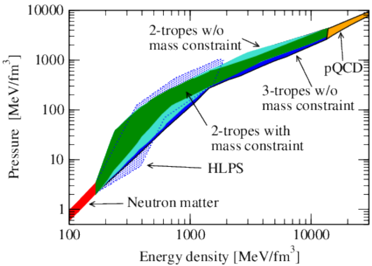
F. Özel and P. Freire, Ann. Rev. Astron. Astrophys.

Constraints at $T = 0$

- Here we have guidance from measured neutron star masses
- Without Radii no real constraints!
- Add constraints from PQCD.



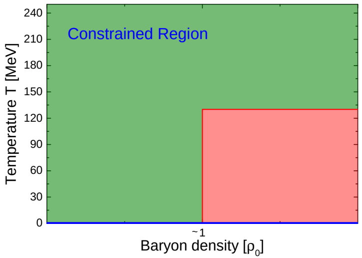
F. Özel and P. Freire, Ann. Rev. Astron. Astrophys.



A. Kurkela, E. S. Fraga, J. Schaffner-Bielich and A. Vuorinen, Astrophys. J. **789**, 127 (2014)

Constraints at $T = 0$

- Here we have guidance from measured neutron star masses
- Without Radii no real constraints!
- Add constraints from PQCD.
- Still missing the important region. Extension to finite temperature $\rightarrow$ New degrees of freedom.

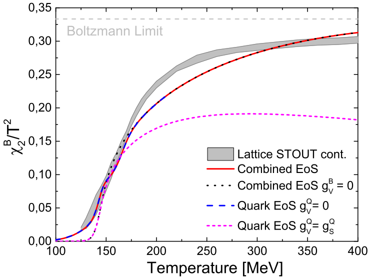


Effective models

- Quark based models: (P)NJL, (P)QM, etc.
- Usually use lattice constraints

Effective models

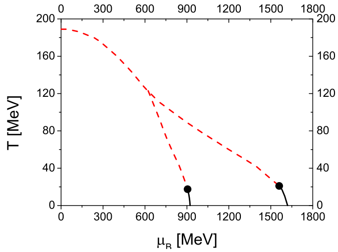
- Use thermodynamics or order parameters
- But also susceptibilities give valuable insights
- No vector repulsion for quarks in the deconfined phase!



JS and S. Schramm, Phys. Lett. B **736**, 241 (2014)

Effective models

- Quark based models: (P)NJL, (P)QM, etc.
- Usually use lattice constraints
- Nucleon based models: Walecka type, σ-ω-models, parity-doublet models
- Usually use nuclear matter and neutron star constraints.



JS , S. Schramm and H. Stöcker, Phys. Rev. C **84**, 045208 (2011)

Effective models

- Quark based models: (P)NJL, (P)QM, etc.
- Usually use lattice constraints
- Nucleon based models: Walecka type, σ-ω-models, parity-doublet models
- Usually use nuclear matter and neutron star constraints.
- Combined models: Phase matching, Mott-dissociation or additional couplings.
- Tend to be complex/challenging

However necessary for:

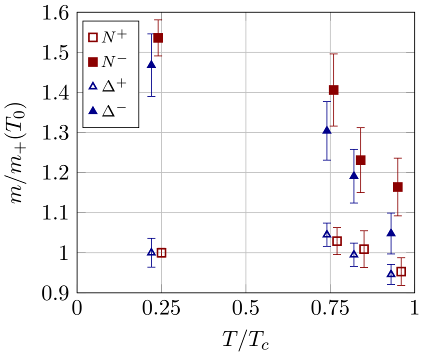
- Low energy HIC (GSI, FAIR, RHIC-BES, NICA, J-PARC)
- Neutron Star Mergers!

Needs a good description of hadronic matter + the correct asymptotic degrees of freedom.

Our approach: A combined Chiral Mean field Model (CMF)

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!



G. Aarts, C. Allton, D. De Boni, S. Hands, B. Jäger, C. Praki and J. I. Skullerud, JHEP **1706**, 034 (2017)

Our approach: A combined Chiral Mean field Model (CMF)

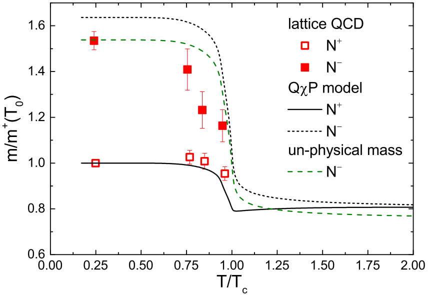
Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!
- $\mathrm{SU}(3)_f$ + mesons, including a scalar σ,ζ and vector ω,ϕ interactions.
- Scalar Interaction Potential:
  $V = V_0 + \frac{1}{2}k_0 I_2 - k_1 I_2^2 - k_2 I_4 + k_6 I_6$

Our approach: A combined Chiral Mean field Model (CMF)

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!
- $SU(3)_f$ + mesons, including a scalar σ,ζ and vector ω,ϕ interactions.
- Scalar Interaction Potential:
  $V = V_0 + \frac{1}{2}k_0 I_2 - k_1 I_2^2 - k_2 I_4 + k_6 I_6$
- Parameters fixed by vacuum and nuclear matter properties.



A. Mukherjee, S. Schramm, **JS** and V. Dexheimer, Astron. Astrophys. **608**, A110 (2017)

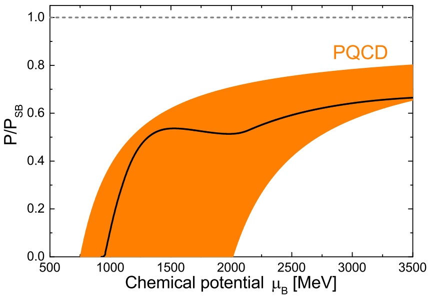
Our approach: A combined Chiral Mean field Model (CMF)

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!
- $SU(3)_f$ + mesons, including a scalar σ,ζ and vector ω,ϕ interactions.
- Scalar Interaction Potential:
  $V = V_0 + \frac{1}{2}k_0 I_2 - k_1 I_2^2 - k_2 I_4 + k_6 I_6$
- Parameters fixed by vacuum and nuclear matter properties.

- Quarks and gluons are included in a Polyakov Loop inspired approach
- $U = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\ln[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2]$,
- Only single quarks. Transition appears naturally through excluded volume of hadrons.

Pressure at $T = 0$



A. Mukherjee, S. Schramm, **JS** and V. Dexheimer, Astron. Astrophys. **608**, A110 (2017)

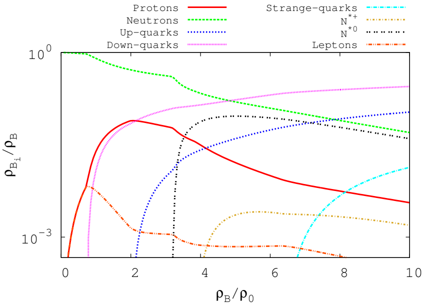
Our approach: A combined Chiral Mean field Model (CMF)

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!
- $SU(3)_f$ + mesons, including a scalar σ,ζ and vector ω,ϕ interactions.
- Scalar Interaction Potential:
  $V = V_0 + \frac{1}{2}k_0 I_2 - k_1 I_2^2 - k_2 I_4 + k_6 I_6$
- Parameters fixed by vacuum and nuclear matter properties.

- Quarks and gluons are included in a Polyakov Loop inspired approach
- $U = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\ln[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2]$,
- Only single quarks. Transition appears naturally through excluded volume of hadrons.

Chemical composition of neutron star matter.
A smooth transition from hadrons to quarks.

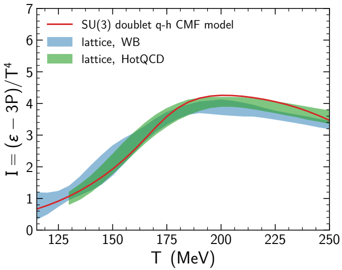


A. Mukherjee, S. Schramm, **JS** and V. Dexheimer, Astron. Astrophys. **608**, A110 (2017)

The CMF at finite temperature

Work by Anton Motornenko.

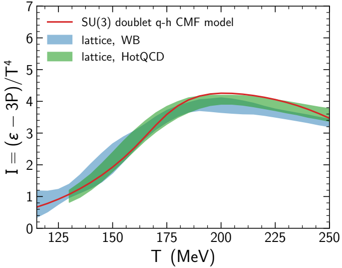
- Fit parameters of Polyakov potential and quark couplings to lattice thermodynamics at $\mu_B = 0$.

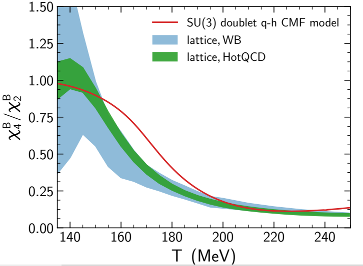


The CMF at finite temperature

Work by Anton Motornenko.

- Fit parameters of Polyakov potential and quark couplings to lattice thermodynamics at $\mu_B = 0$.
- Works well for the susceptibility ratios.

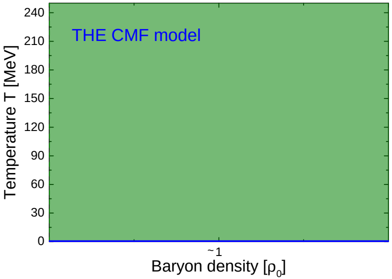


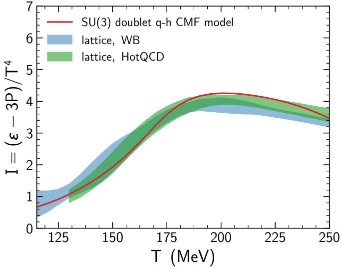


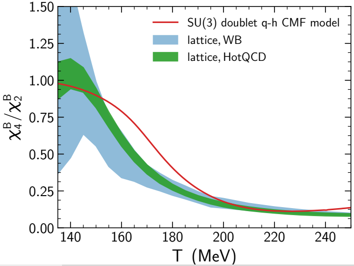
The CMF at finite temperature

Work by Anton Motornenko.

- Fit parameters of Polyakov potential and quark couplings to lattice thermodynamics at $\mu_B = 0$.
- Works well for the susceptibility ratios.
- Transition is always a smooth crossover.



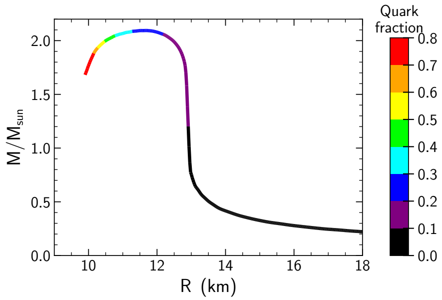




The CMF at finite temperature

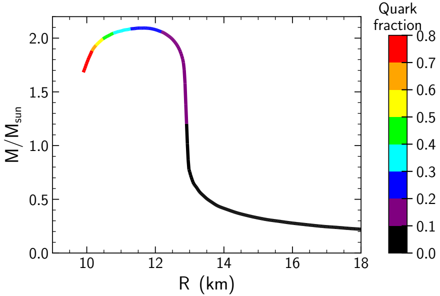
Applications

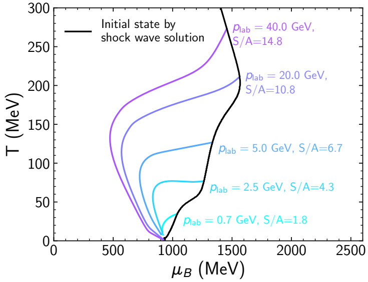
- This approach describes well the mass radius relation.



Applications

- This approach describes well the mass radius relation.
- All relevant degrees of freedom are included $\rightarrow$ finite T.
- Isentropic trajectories for compression by one dimensional shock wave.





The CMF at finite temperature

Applications

- Can be used in fluid dynamical simulations of:
- Heavy ion collisions.

The CMF at finite temperature

Applications

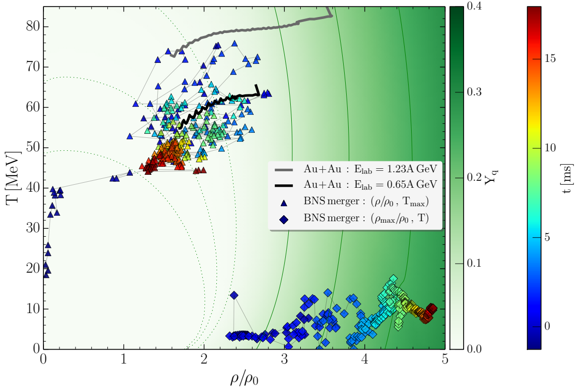
- Can be used in fluid dynamical simulations of:
- Heavy ion collisions.
- Neutron Star mergers?

The CMF and neutron star mergers

- This EoS enables us to treat heavy ion collisions and NS mergers on the same footing
- What area of the phase diagram are tested and what is the overlap?

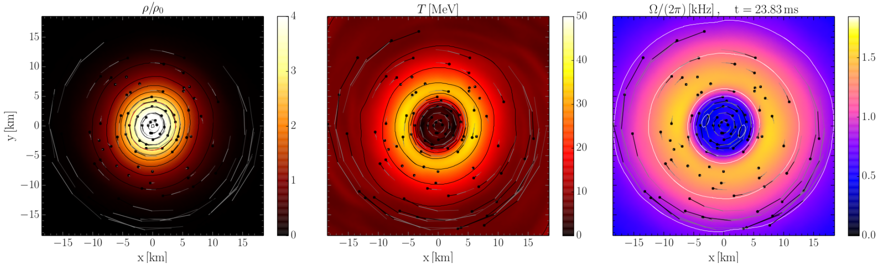
The CMF and neutron star mergers

- This EoS enables us to treat heavy ion collisions and NS mergers on the same footing
- What area of the phase diagram are tested and what is the overlap?
- Low beam energy HIC compared to NS merger simulations.
- Disclaimer: Not the same EoS used yet.



The CMF and neutron star mergers

- This EoS enables us to treat heavy ion collisions and NS mergers on the same footing
- What area of the phase diagram are tested and what is the overlap?
- Low beam energy HIC compared to NS merger simulations.
- Disclaimer: Not the same EoS used yet.
- A dense and cold core with a hot hadronic corona.



M. Hanauske, **JS** *et al.*, J. Phys. Conf. Ser. **878**, no. 1, 012031 (2017).

Summary

- The time of niche models for the EoS are over. (We are talking about p(e,n) here)
- Combined/Complex models are necessary to describe the matter in low energy HIC and neutron star mergers.
- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe the same region in the phase diagram.

Summary

- The time of niche models for the EoS are over. (We are talking about p(e,n) here)
- Combined/Complex models are necessary to describe the matter in low energy HIC and neutron star mergers.
- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe the same region in the phase diagram.
- Treat both on the same footing $\rightarrow$ Combining QCD thermodynamics, relativistic fluid dynamics and GR.

Summary

- The time of niche models for the EoS are over. (We are talking about p(e,n) here)
- Combined/Complex models are necessary to describe the matter in low energy HIC and neutron star mergers.
- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe the same region in the phase diagram.
- Treat both on the same footing $\rightarrow$ Combining QCD thermodynamics, relativistic fluid dynamics and GR.

Future

- Extend the parity doublet model to include also the decuplett baryons $\rightarrow$ Better description of the chiral condensate.
- Can we apply statistical/machine learning methods to determine all parameters in the model(s) consistently?
- How to interpret these complex/combined models in terms of physics?