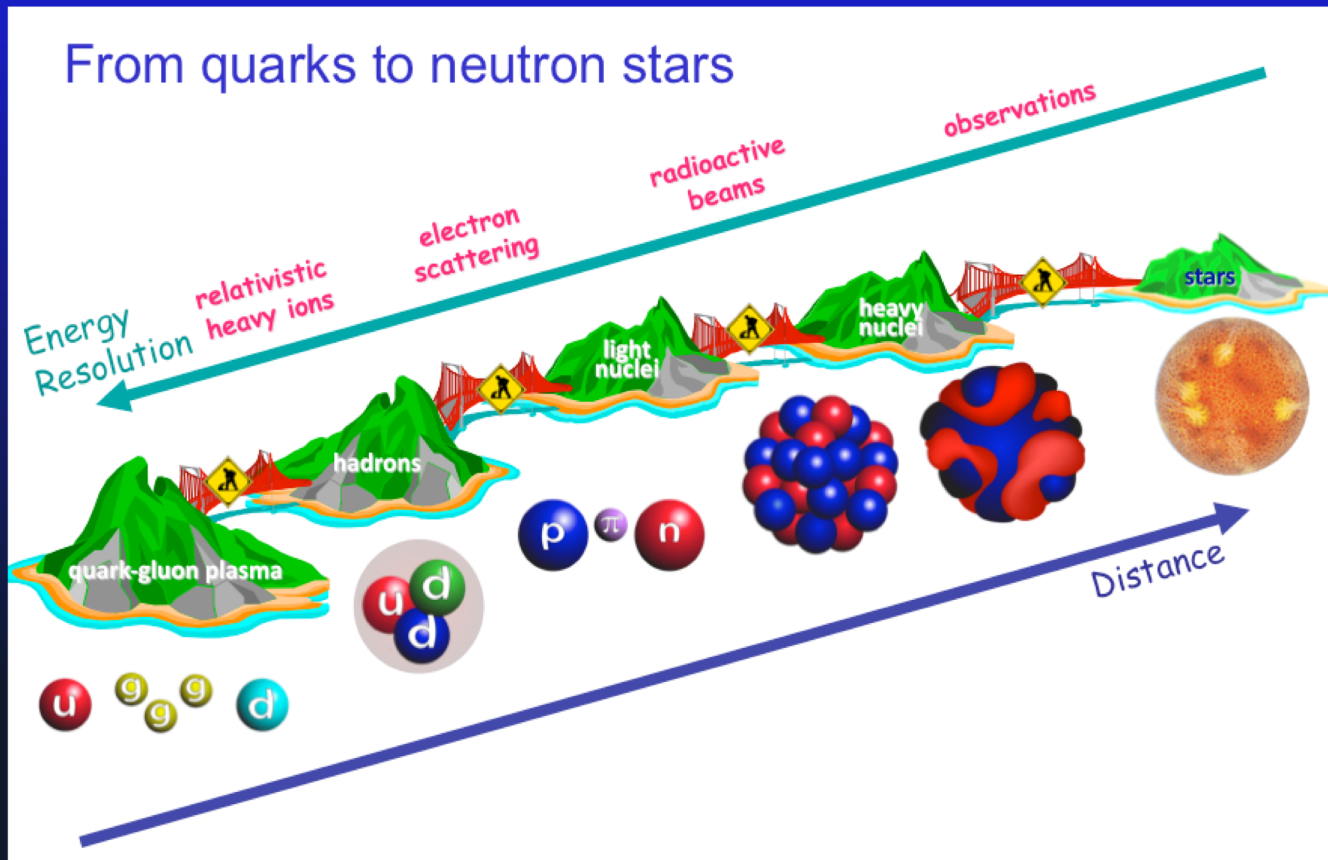
The neutron star equation of state



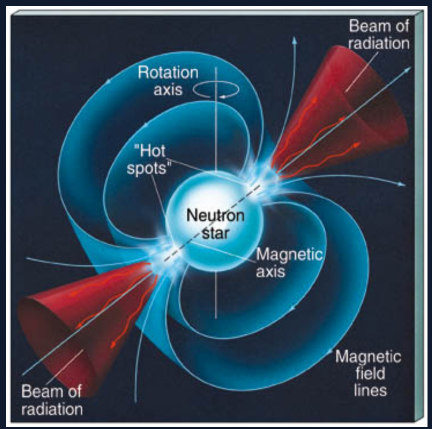
EMMI 05 Jun 2018

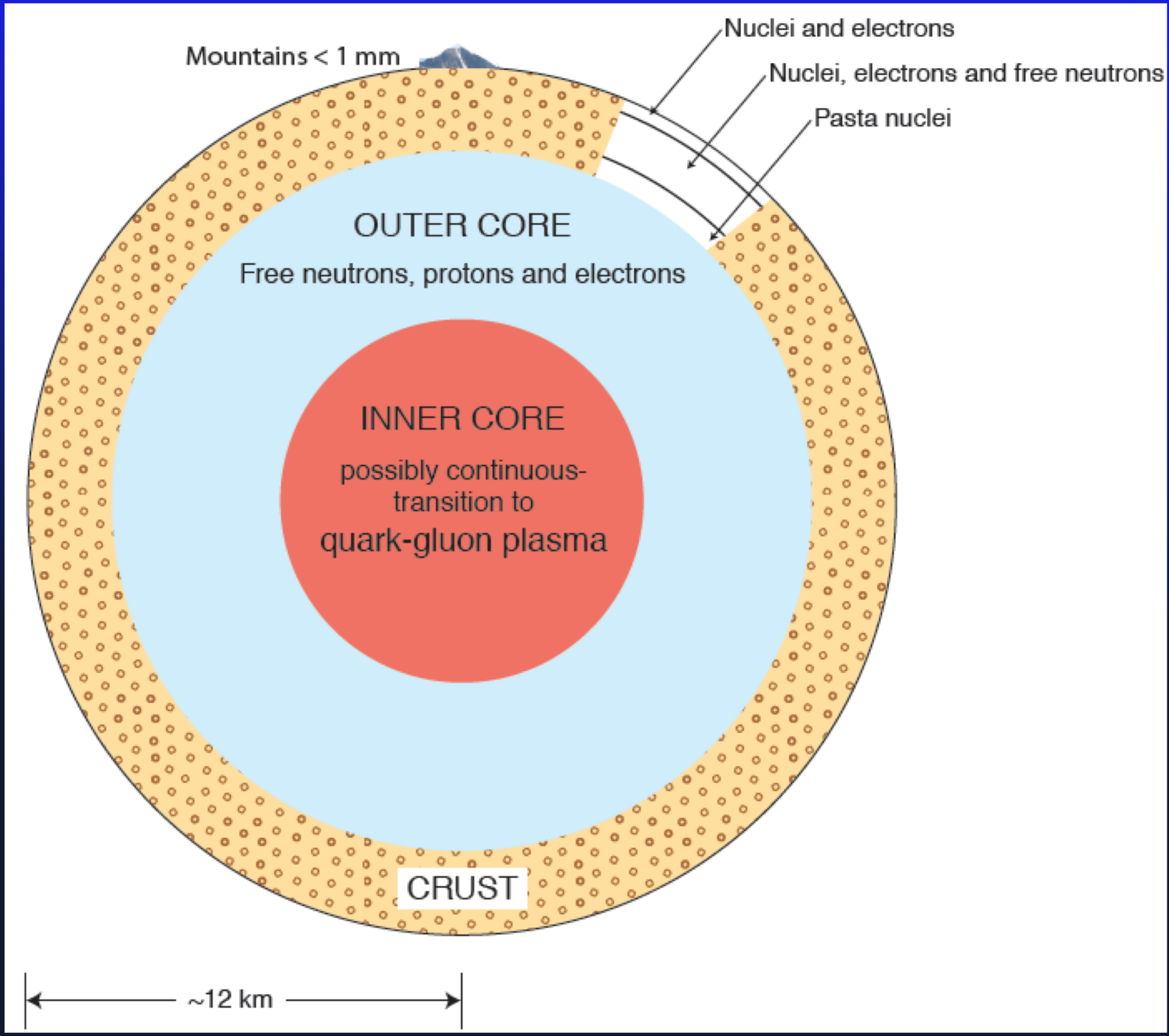
GSI

Neutron star interior

Mass ~ 1.4-2 M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

Surface gravity
~10^{14} that of Earth
Surface binding
~ 1/10 mc^2



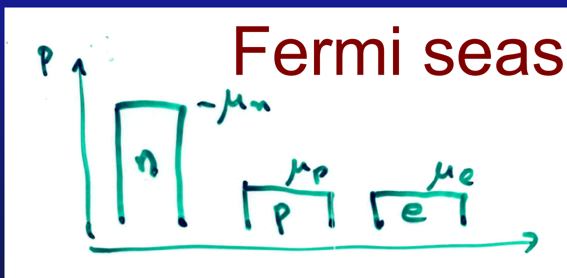


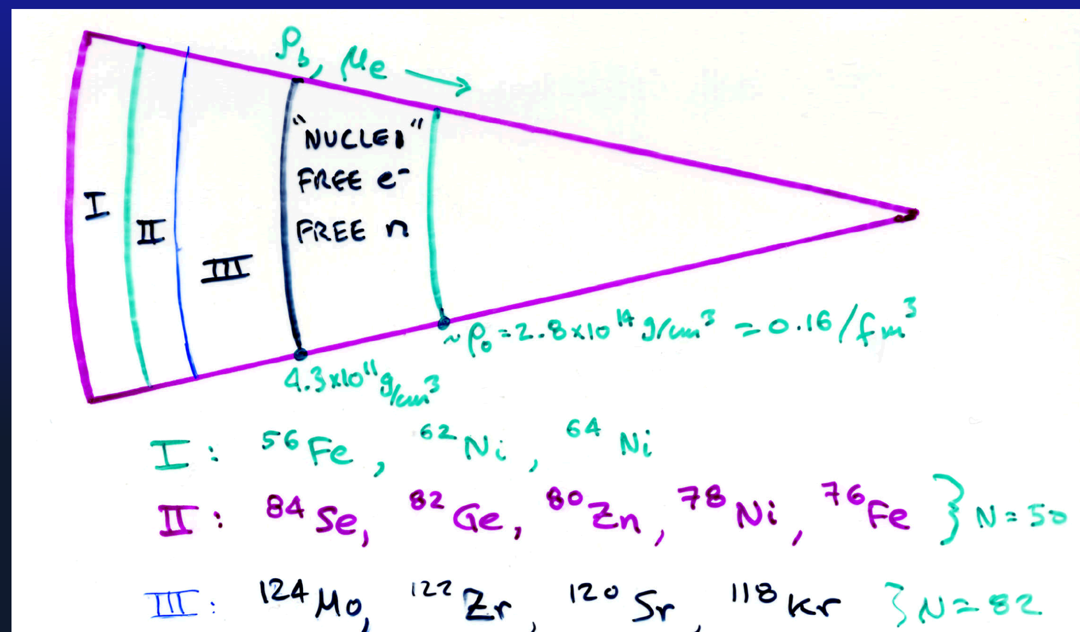
Nuclei before neutron drip

$e^- + p \longrightarrow n + \nu$: makes nuclei neutron rich as electron Fermi energy increases with depth

$n \longrightarrow p + e^- + \bar{\nu}$: not allowed if e^- state already occupied

Beta equilibrium: $\mu_n = \mu_p + \mu_e$



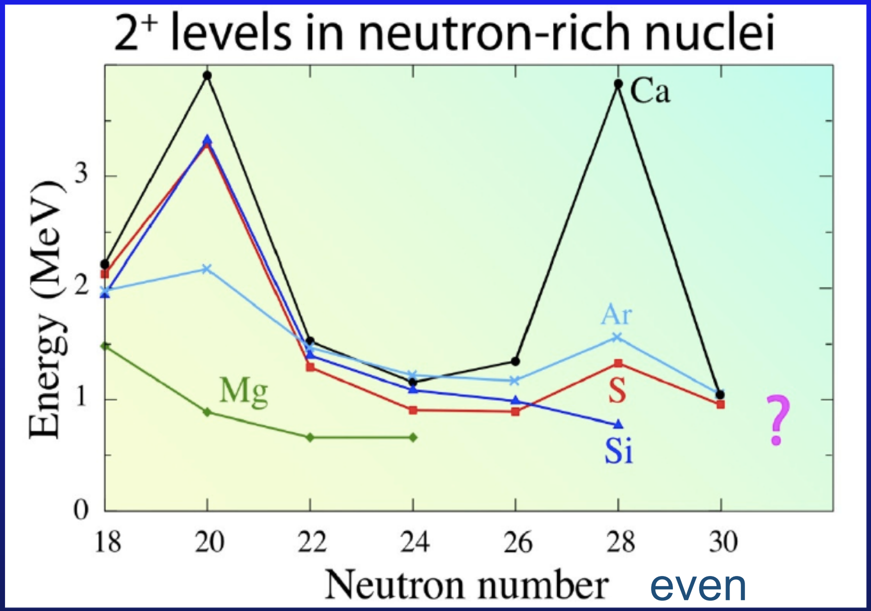


Shell structure (spin-orbit forces) for very neutron rich nuclei?

Do N=50, 82 remain neutron magic numbers? Proton shell structure?

Being explored at rare isotope accelerators: RIKEN Rare Ion Beam Facility, and later GSI (MINOS), FRIB, RAON (KoRIA)

Modification of shell structure for N >> Z

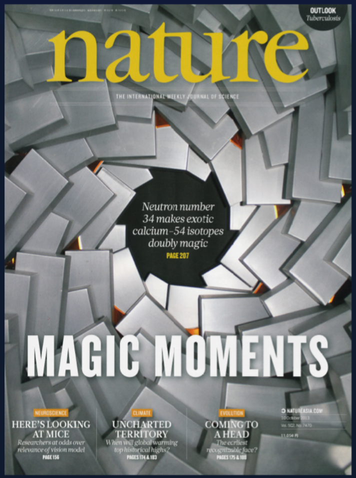


Usual shell closings
(N ~ Z) at 20, 28, 50, 82, 126

No shell effect for Mg(Z=12), Si(14), S(16), Ar(18) at N=20 and 28
Bastin. et al. PRL (2007, ...

Oxygen has new shell closure at N=16
Otsuka et al PRL (2005)

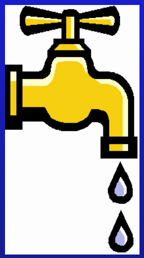
Calcium has new shell closure at N=34
D. Steppenbeck et al. Nature (2013)

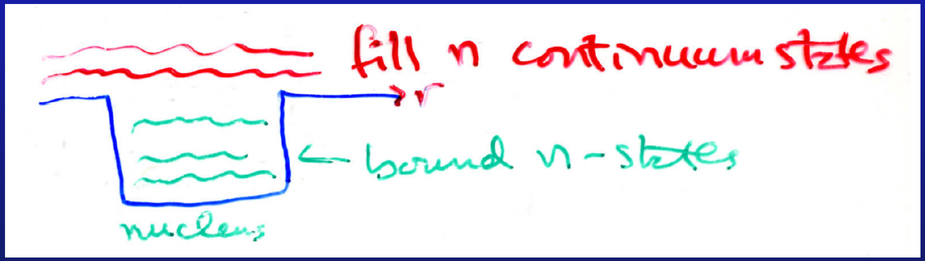


Spin-orbit forces and hence shell structure modified
by tensor and 3-body forces in neutron rich nuclei

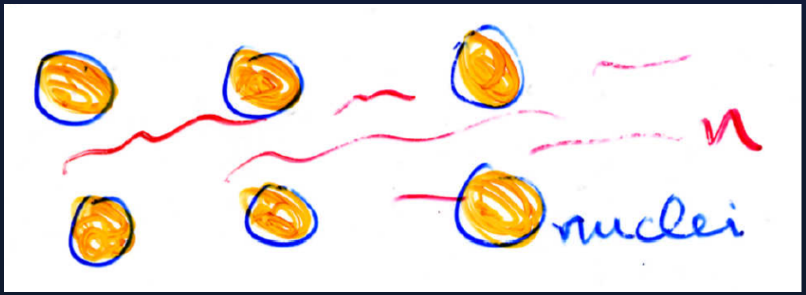
Neutron drip

Beyond density ρ_{drip} ~ 4.3 X 10^{11} g/cm^3 neutron bound states in nuclei become filled. Further neutrons must go into continuum states. Form degenerate neutron Fermi sea.





Neutrons in neutron sea are in equilibrium with those inside nucleus (common μ_n)

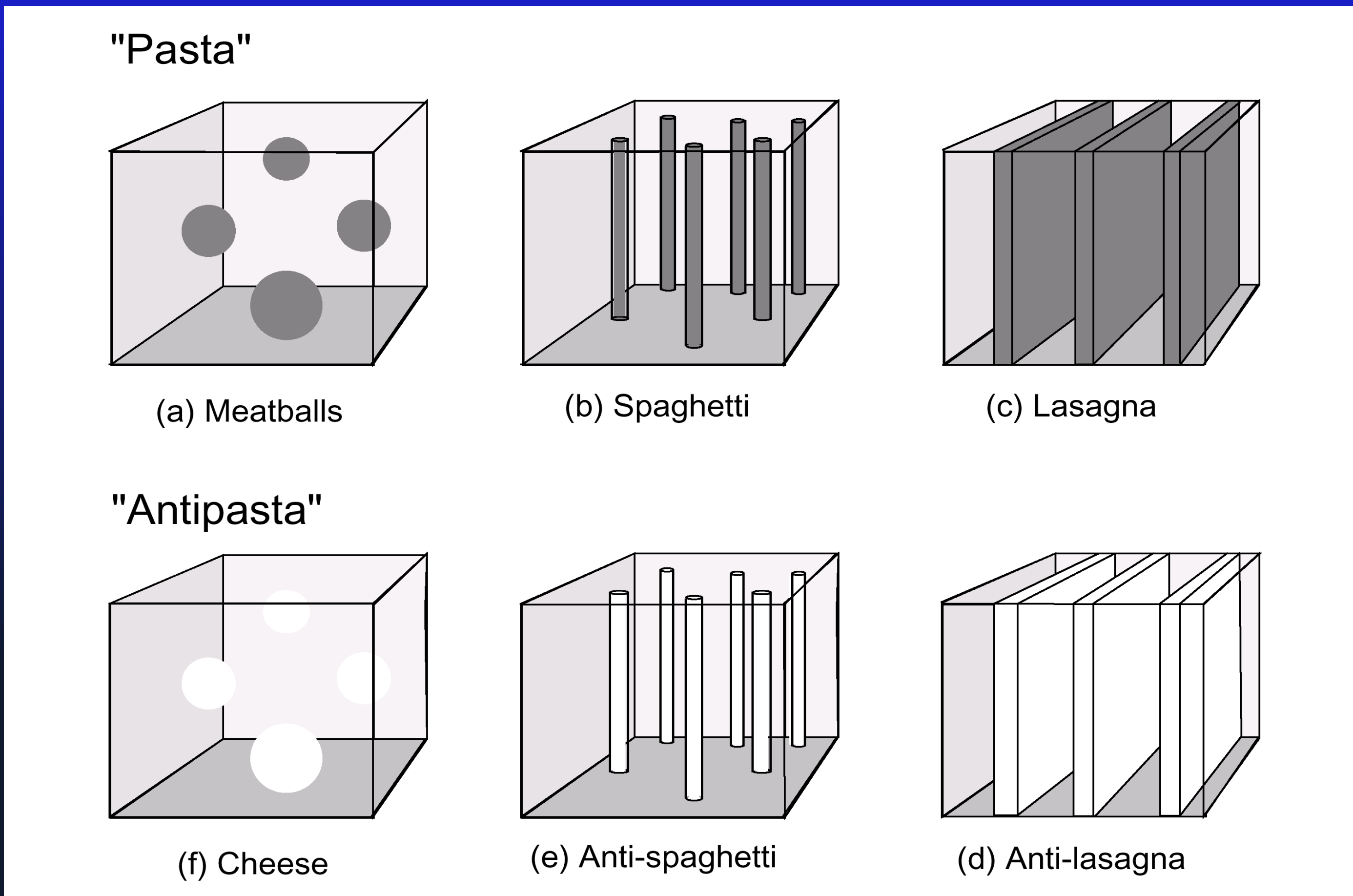


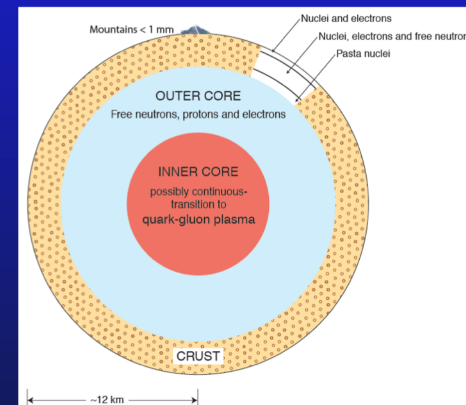
Protons appear not drip, but remain in bound states until nuclei merge in interior liquid.

Pasta Nuclei in inner crust

Lorentz, Pethick, and Ravenhall PRL (1993)

When Coulomb wins over surface energies





Involves over half the mass of the crust !! Effects on crust bremsstrahlung of neutrinos, pinning of n vortices, modes of crust, ...

The liquid interior

Neutrons (likely superfluid) ~ 95% Non-relativistic
Protons (likely superconducting) ~ 5% Non-relativistic
Electrons (normal, $T_c \sim T_f e^{-137}$) ~ 5% Fully relativistic

Eventually muons, hyperons??, quark matter and possible exotica:

- pion condensation
- kaon condensation
- quark droplets

Phase transition from crust to liquid at $n_b \simeq 0.7\ n_0 \simeq 0.09\ \text{fm}^{-3}$ (mass density ~ $2 \times 10^{14}\ \text{g/cm}^3$). 10% uncertainty!

$n_0 \simeq 0.16\ \text{fm}^{-3}$

Uncertainities in nuclear matter liquid: interpolations between pure neutron matter and symmetric nuclear matter.

Properties of liquid interior near nuclear matter density

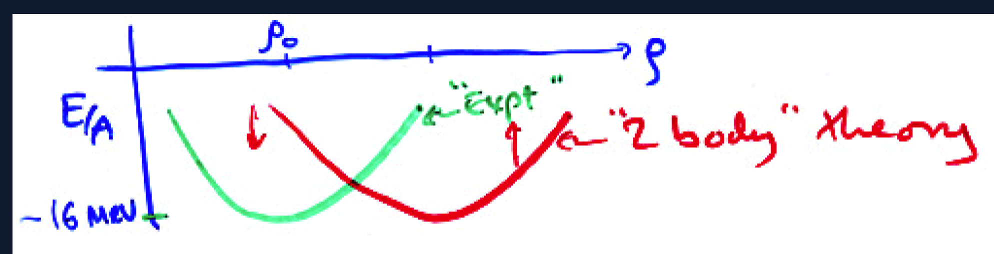
Determine N-N potentials from
- scattering experiments E<300 MeV
- deuteron, 3 body nuclei (^3He, ^3H)

ex., Paris, Argonne, Urbana 2 body potentials

Solve Schrödinger equation by variational techniques

Large theoretical extrapolation from low energy laboratory nuclear physics at near nuclear matter density

Two body potential alone:



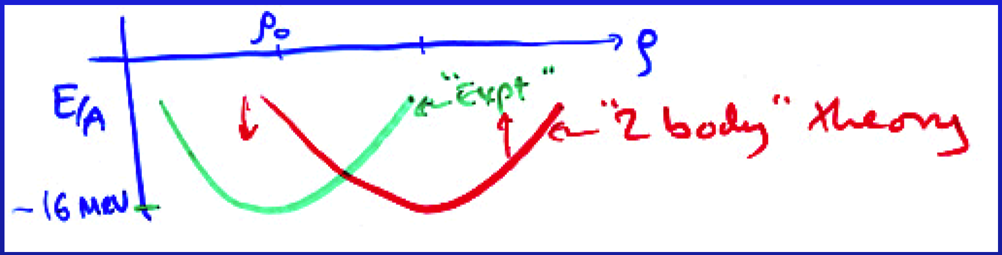
Underbind ^3H: Exp = -8.48 MeV, Theory = -7.5 MeV

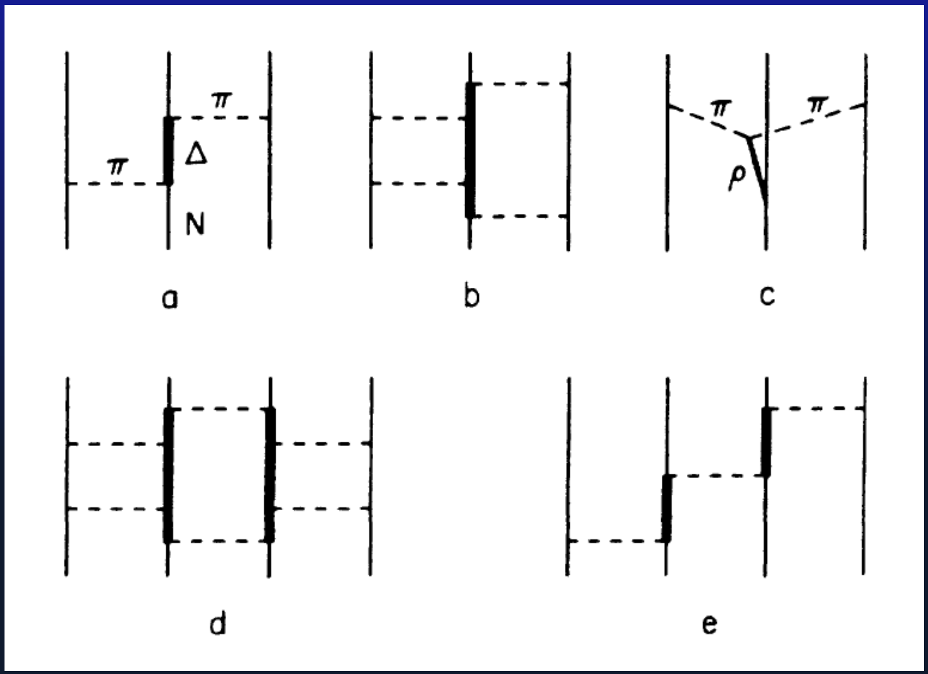
^4He: Exp = -28.3 MeV, Theory = -24.5 MeV

Importance of 3 body interactions

Attractive at low density

Repulsive at high density





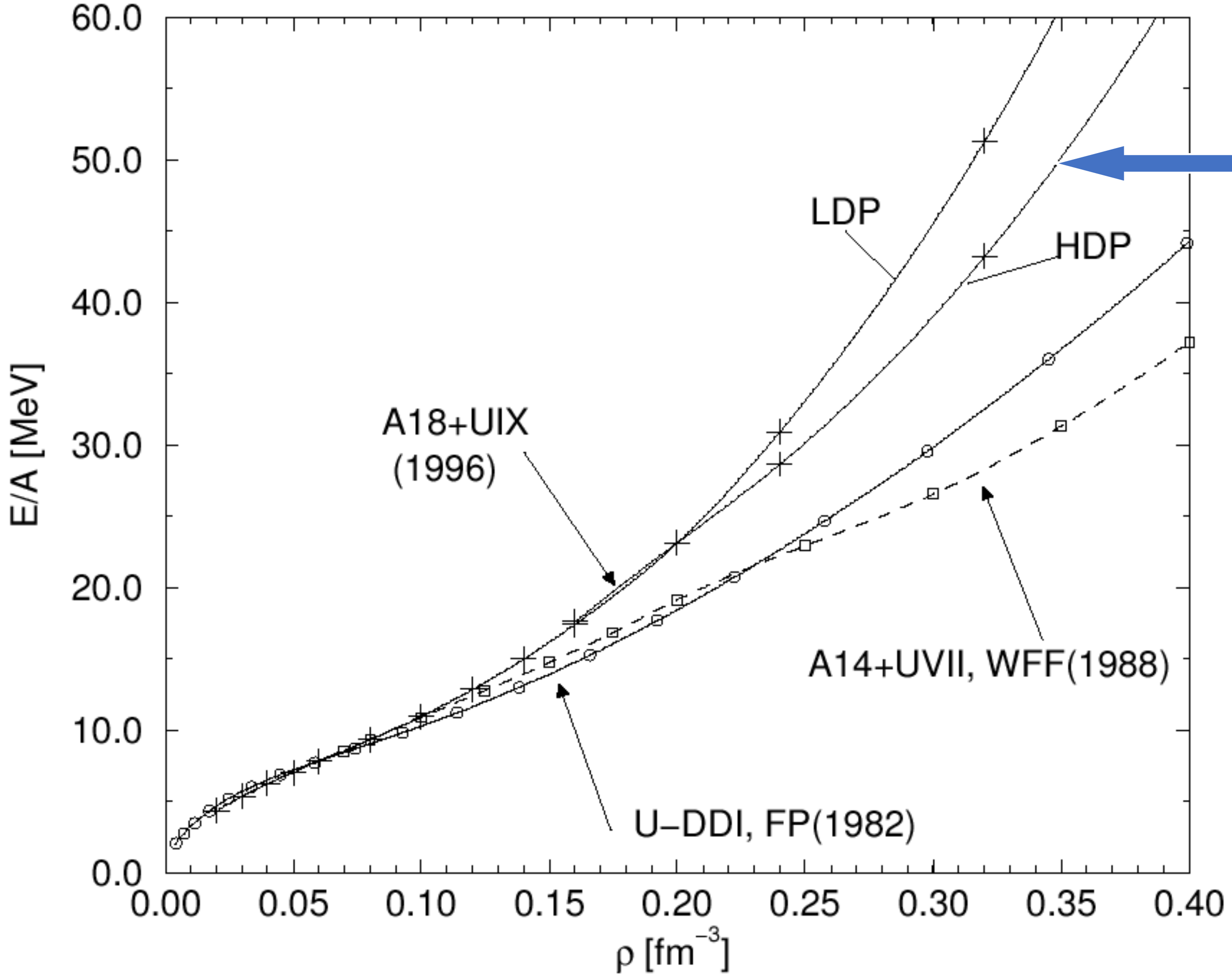
Various processes that lead to three and higher body intrinsic interactions (not described by iterated nucleon-nucleon interactions).

Stiffens equation of state at high density
Large uncertainties!

Standard construction of neutron star models

1) Compute energy per nucleon in neutron matter (pure or in beta equilibrium: $\mu_n = \mu_p + \mu_e$). Include 2 and 3 body forces between nucleons

Akmal, Pandharipande & Ravenhall, Phys. Rev. C58 (1998) 1804



π^0 condensate

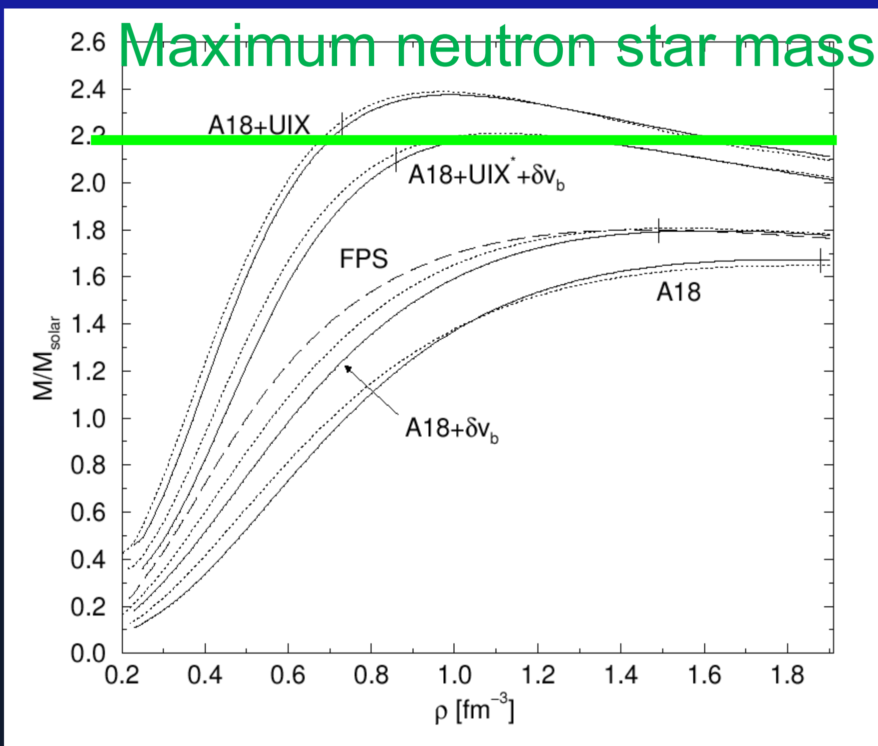
Neutron star models using *static interactions* between *nucleons*

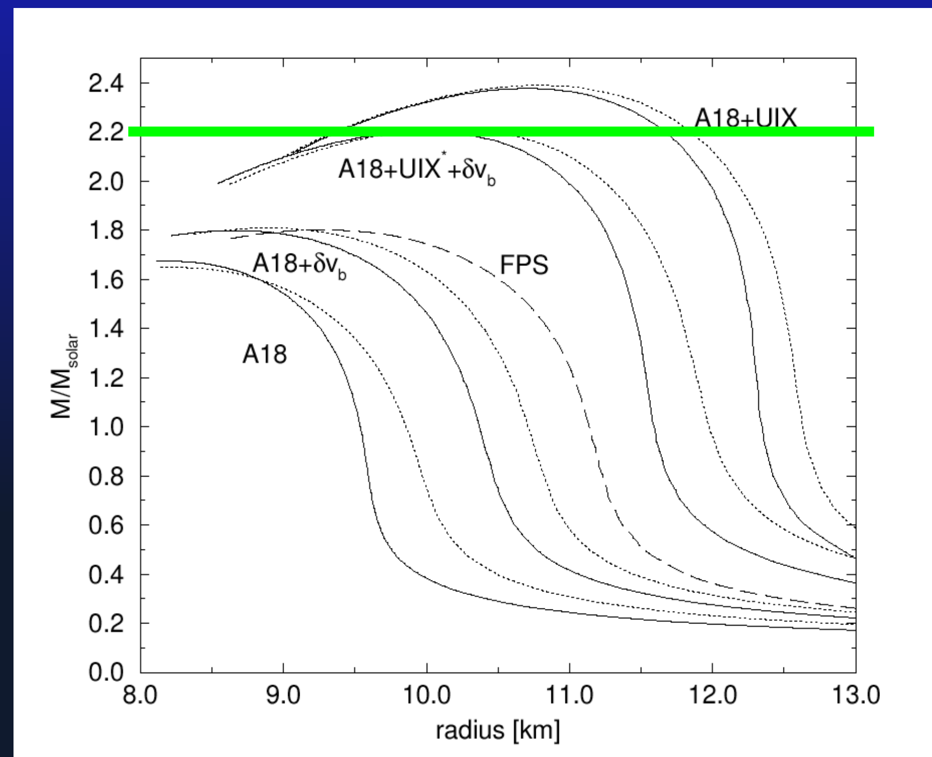
E = energy density = ρc^2
n_b = baryon density
P(r) = pressure = $n_b^2\, d(E/n_b)/dn_b$

$$\frac{\partial P(r)}{\partial r} = -\frac{G}{r^2}\frac{\left(\rho(r)+P(r)/c^2\right)}{1-2m(r)G/rc^2}\left(m(r)+4\pi P(r)r^3/c^2\right)$$

TOV equation

$$M = \int_0^R \rho(r) 4\pi r^2 dr$$





Mass vs. central density

Mass vs. radius

Akmal, Pandharipande and Ravenhall, 1998

APR equation of state

Neutron stars: cold quark matter

Fundamental limitations of eq. of state based on NN interactions alone

Accurate for $n \sim n_0$.   But for $n >> n_0$:

-can forces be described with static few-body potentials?

Given all information on Nb+Nb atomic scattering could one predict that Nb is a superconductor?

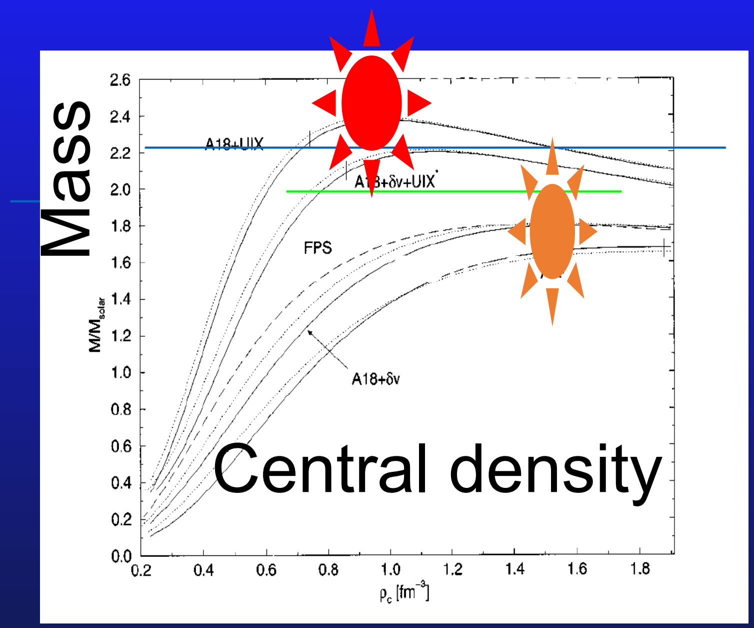
-Force range ~ $1/2m_\pi$ => relative importance of 3 (and higher) body forces ~ $n/(2m_\pi)^3 \sim 0.4n_{\text{fm}^{-3}}$.

-No well defined expansion in terms of 2,3,4,...body forces.

-Can one even describe system in terms of well-defined ``asymptotic'' laboratory particles?   Early percolation of nucleonic volumes!   Wrong degrees of freedom!!

Squeeze:  => 

The equation of state is very stiff



Softer equation of state => lower maximum mass and higher central density

Binary neutron stars ~ 1.4 $M_\odot$: consistent with soft eq. of state

PSR J1614-2230 : $M_{\text{neutron star}} = 1.93 \pm 0.02 M_\odot$

PSR J0348+0432: $M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot$

require very stiff equation of state! How possible?

Neutron star masses

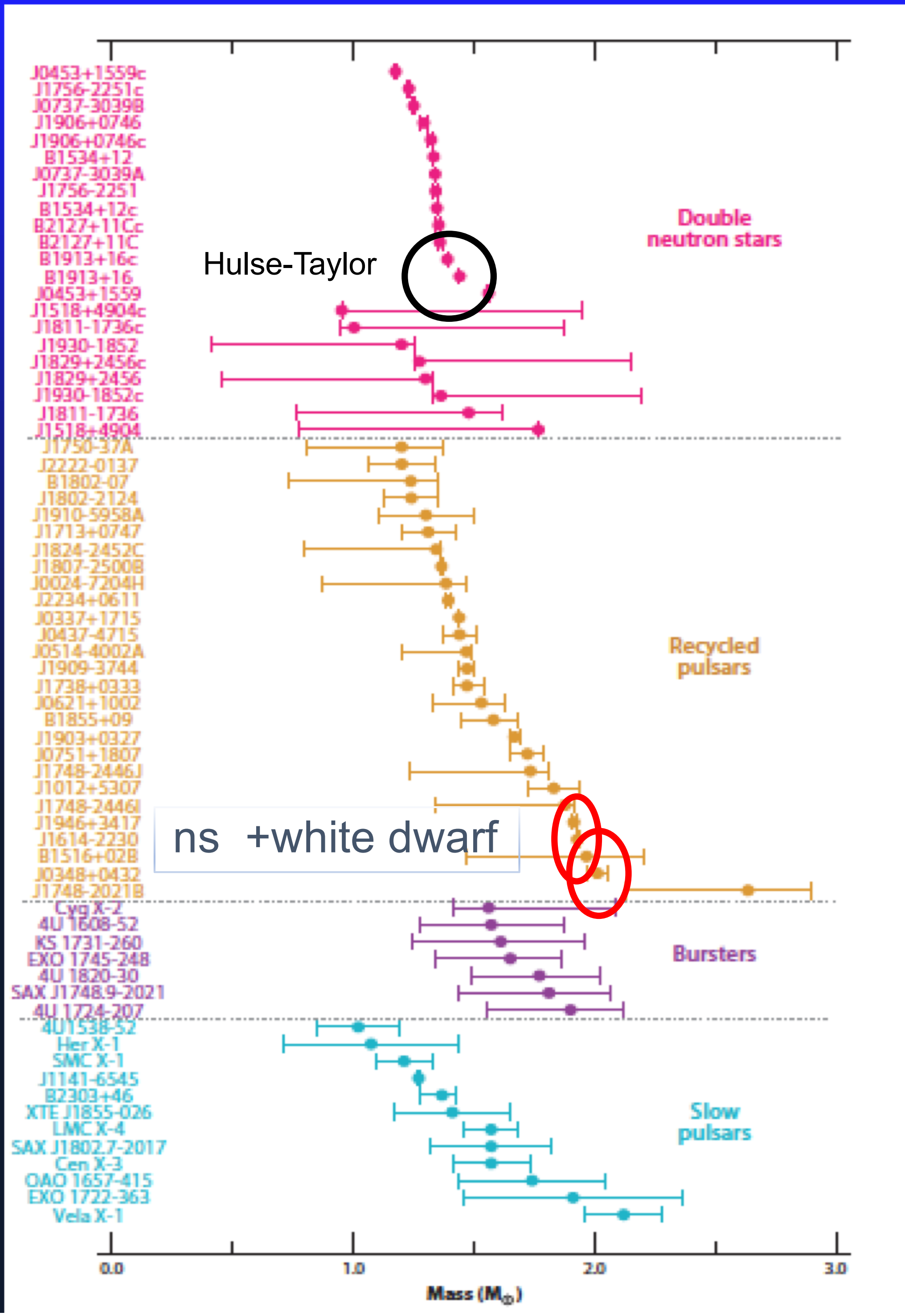
Õzel & Freire, Ann Rev AA (2016)

PSR J1614-2230 :

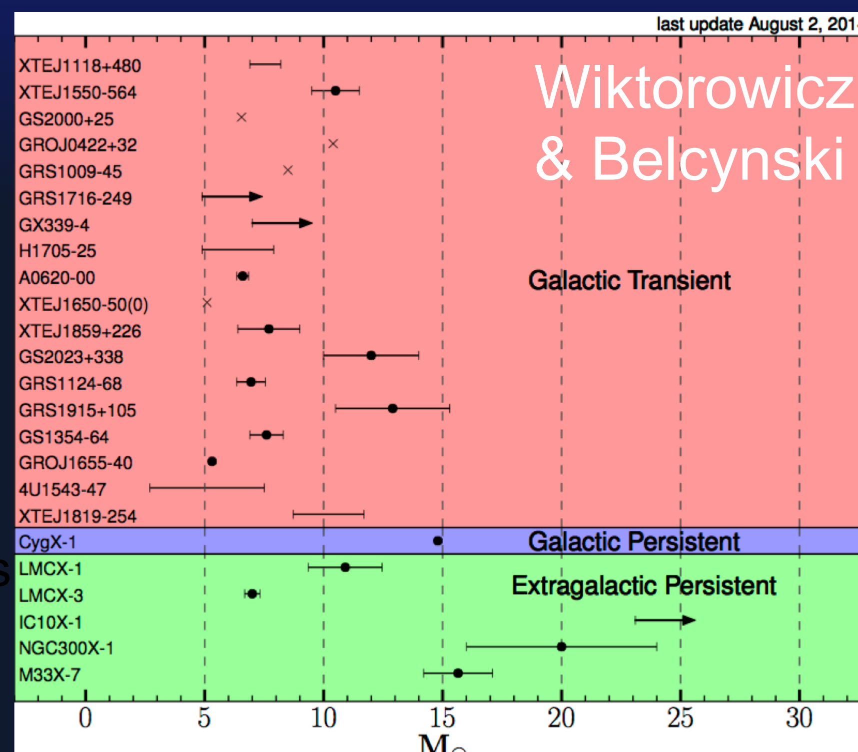
$M_{nstar} = 1.928 \pm 0.017 M_{\odot}$

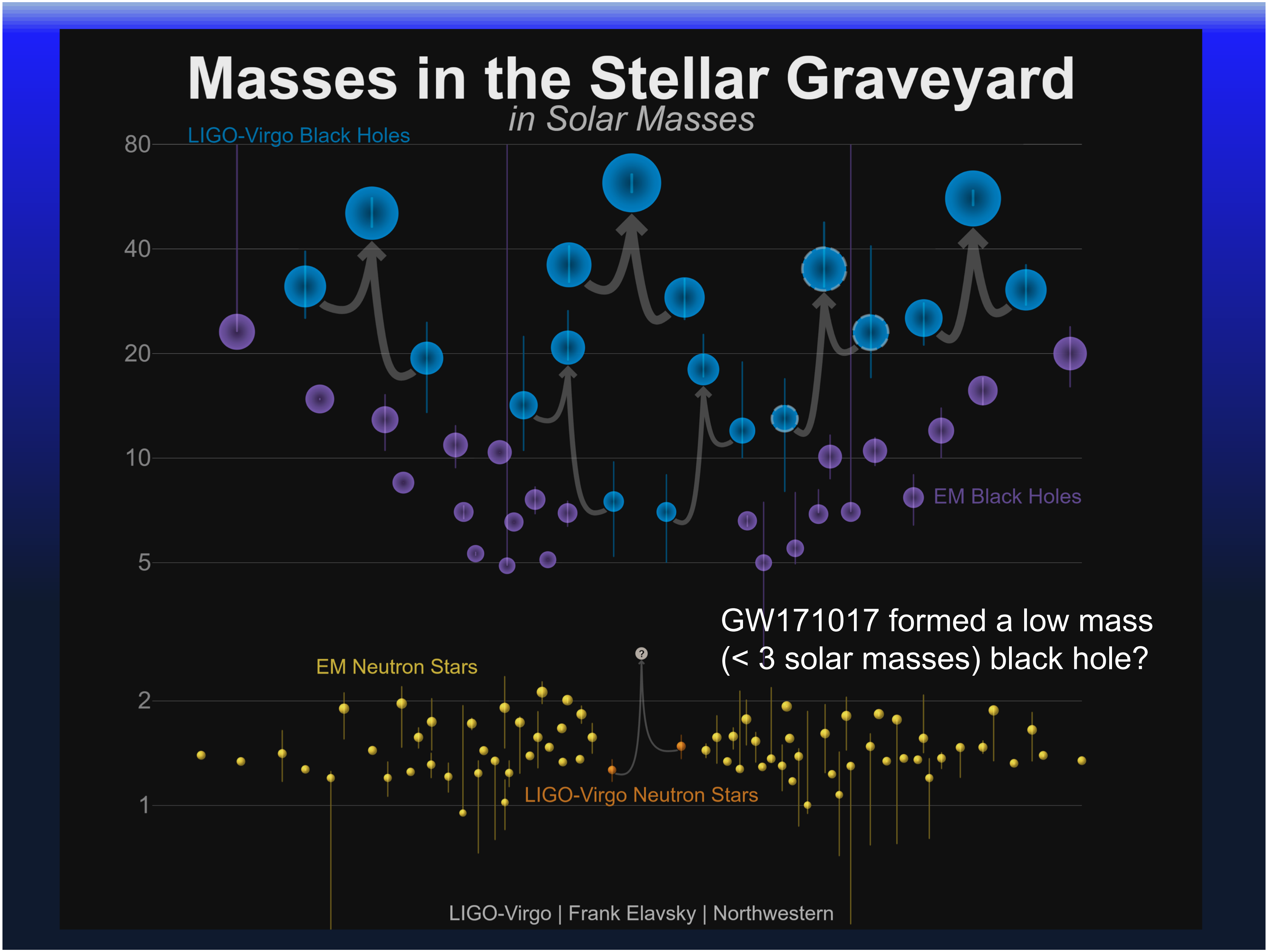
PSR J0348+0432:

$M_{nstar} = 2.01 \pm 0.04 M_{\odot}$



Galactic black hole masses



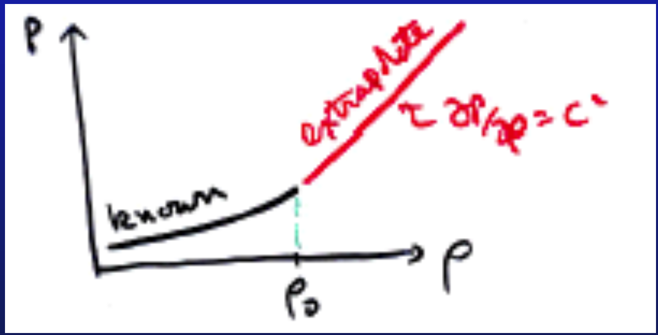

Masses in the Stellar Graveyard
in Solar Masses
LIGO-Virgo Black Holes
80
40
20
10
5
2
1
EM Black Holes
GW171017 formed a low mass
(< 3 solar masses) black hole?
EM Neutron Stars
?
LIGO-Virgo Neutron Stars
LIGO-Virgo | Frank Elavsky | Northwestern

Upper bound to neutron star mass:

require speed of sound, c_s, in matter in core not to exceed speed of light: $c_s^2 = \partial P/\partial\rho \leq c^2$

Maximum core mass when $c_s = c$

Rhodes and Ruffini (PRL 1974)



$\rho_0 = 4\rho_{nm}$ => $M_{max} = 2.2\ M_\odot$

$2\rho_{nm}$ => $2.9\ M_\odot$

V. Kalogera and G.B., Ap. J. 469 (1996) L61

Earliest phase diagram of dense qcd matter

N. Cabbibo and G. Parisi, Phys. Lett. B58, 67 (1975)

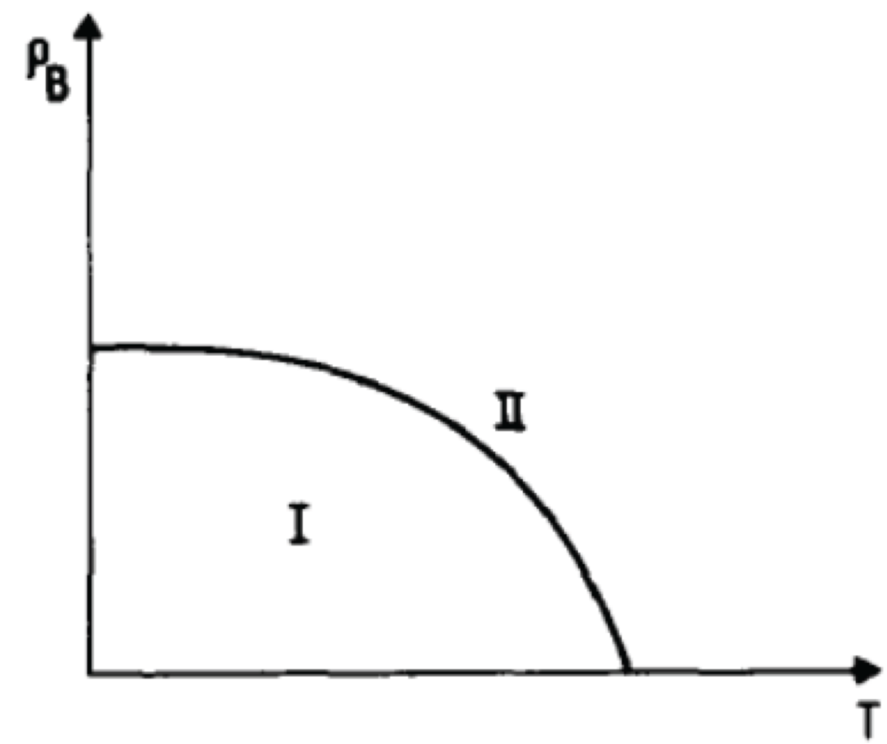
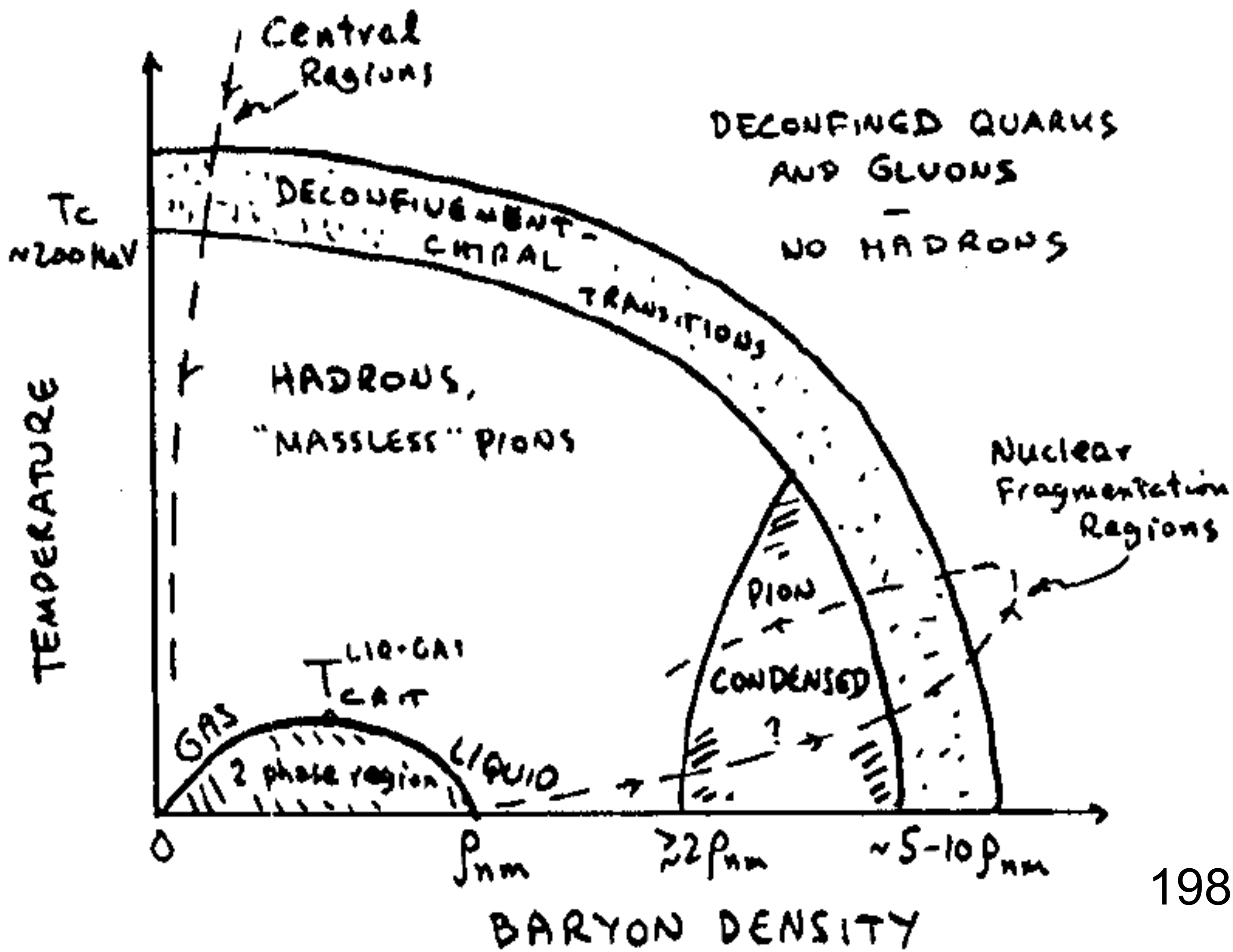


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

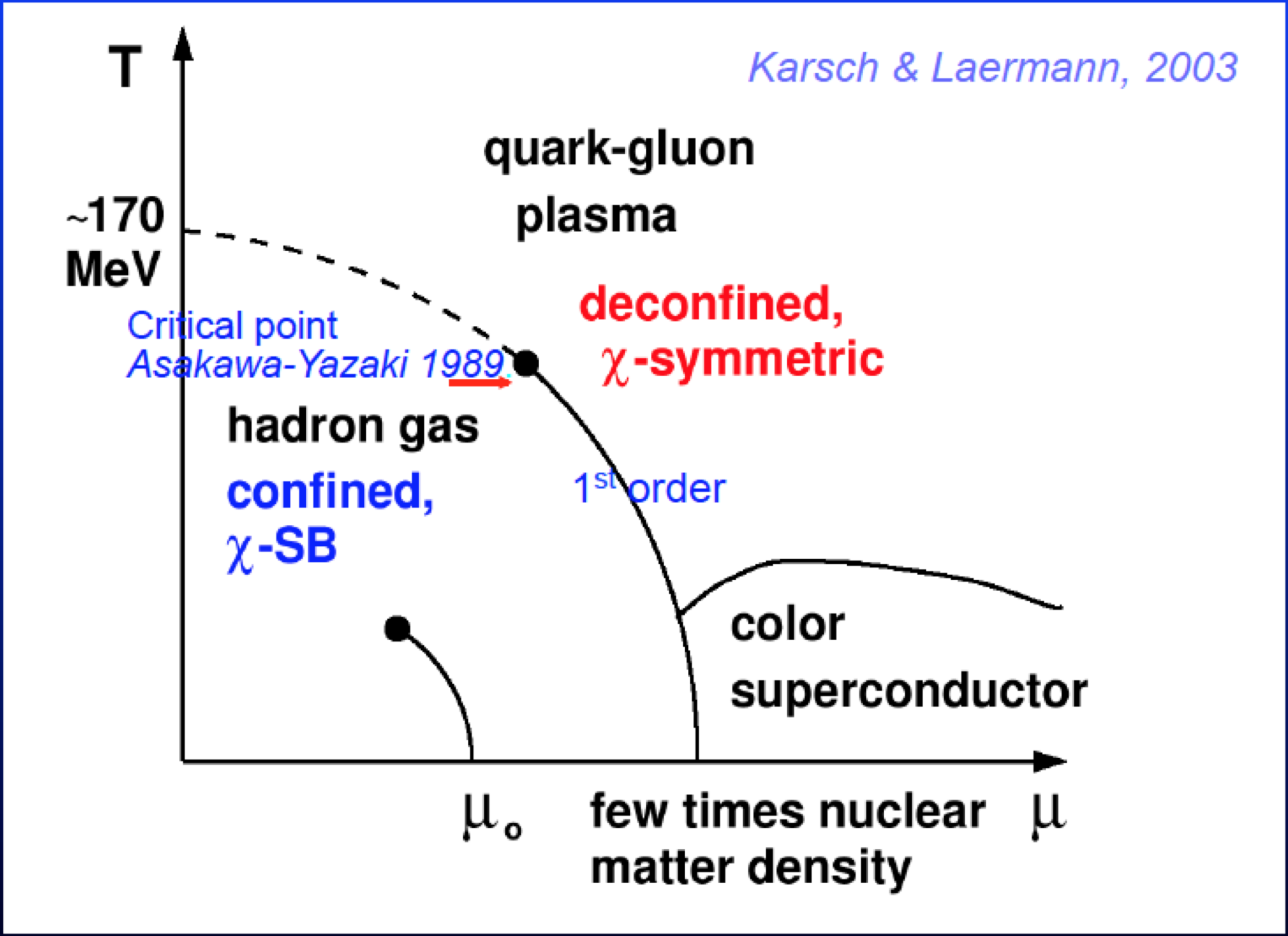
"Quark liberation"

PHASE DIAGRAM OF NUCLEAR MATTER



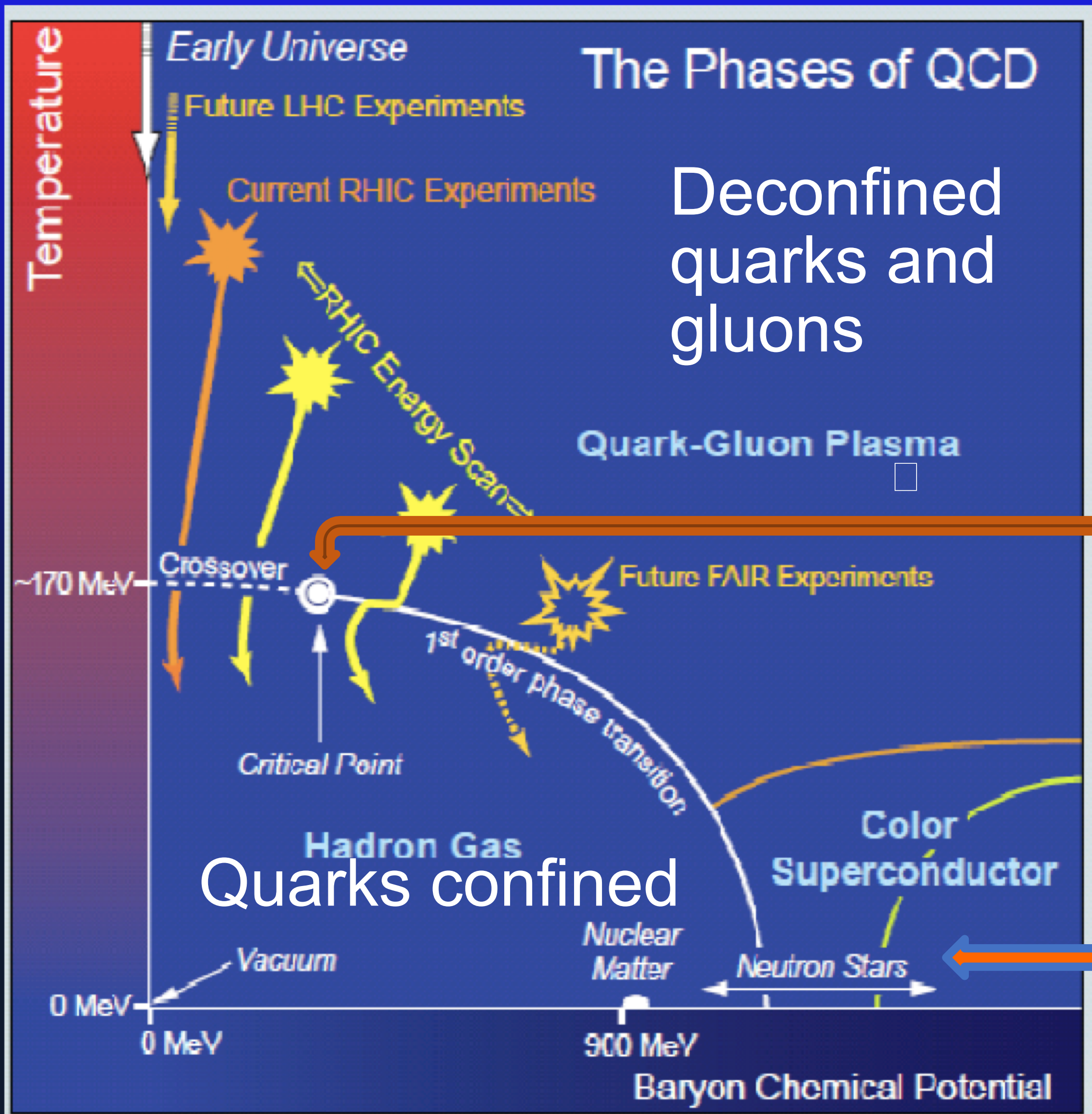
1983

Arrival of the Asakawa-Yazaki critical point



Note that the baryon chemical potential repaces the baryon density

Modern phase diagram



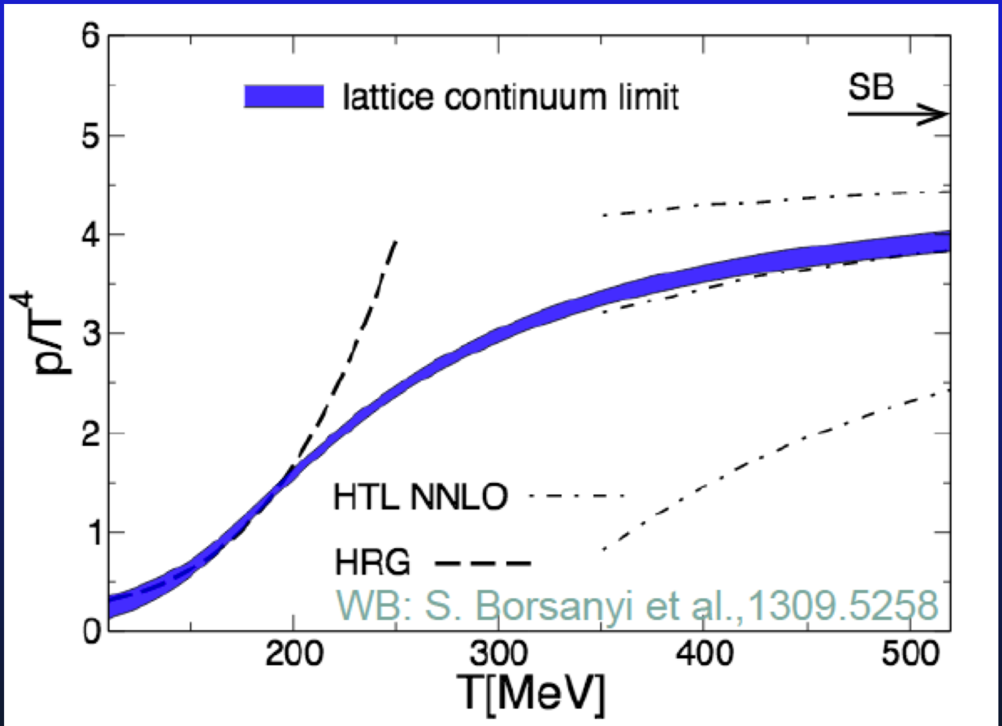
Asakawa-Yazaki critical point (1989)

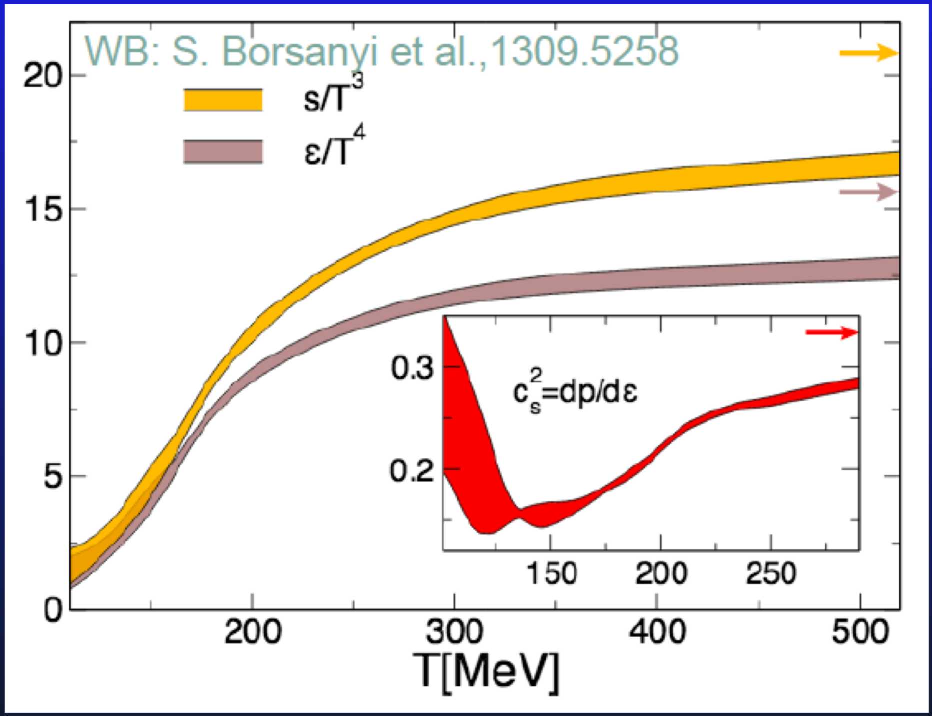
Search in RHIC & SPS energy scans.

States of color superconductivity – diquark BCS pairing

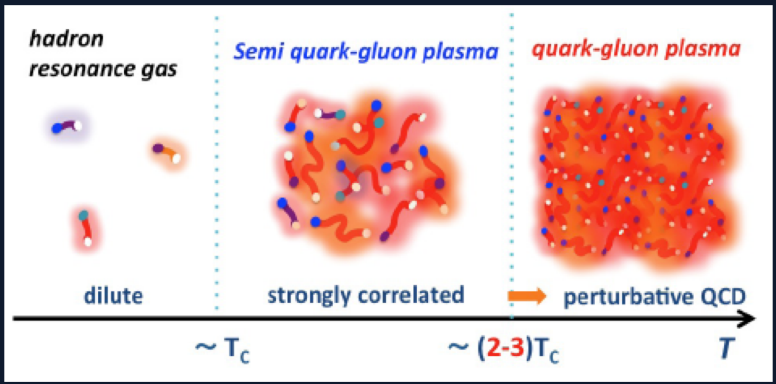
2SC / Color flavor locked
(Alford, Rajagopal, Wilczek, ...)

Crossover at zero net density: see no evidence of phase transition in pressure, entropy, or energy density.



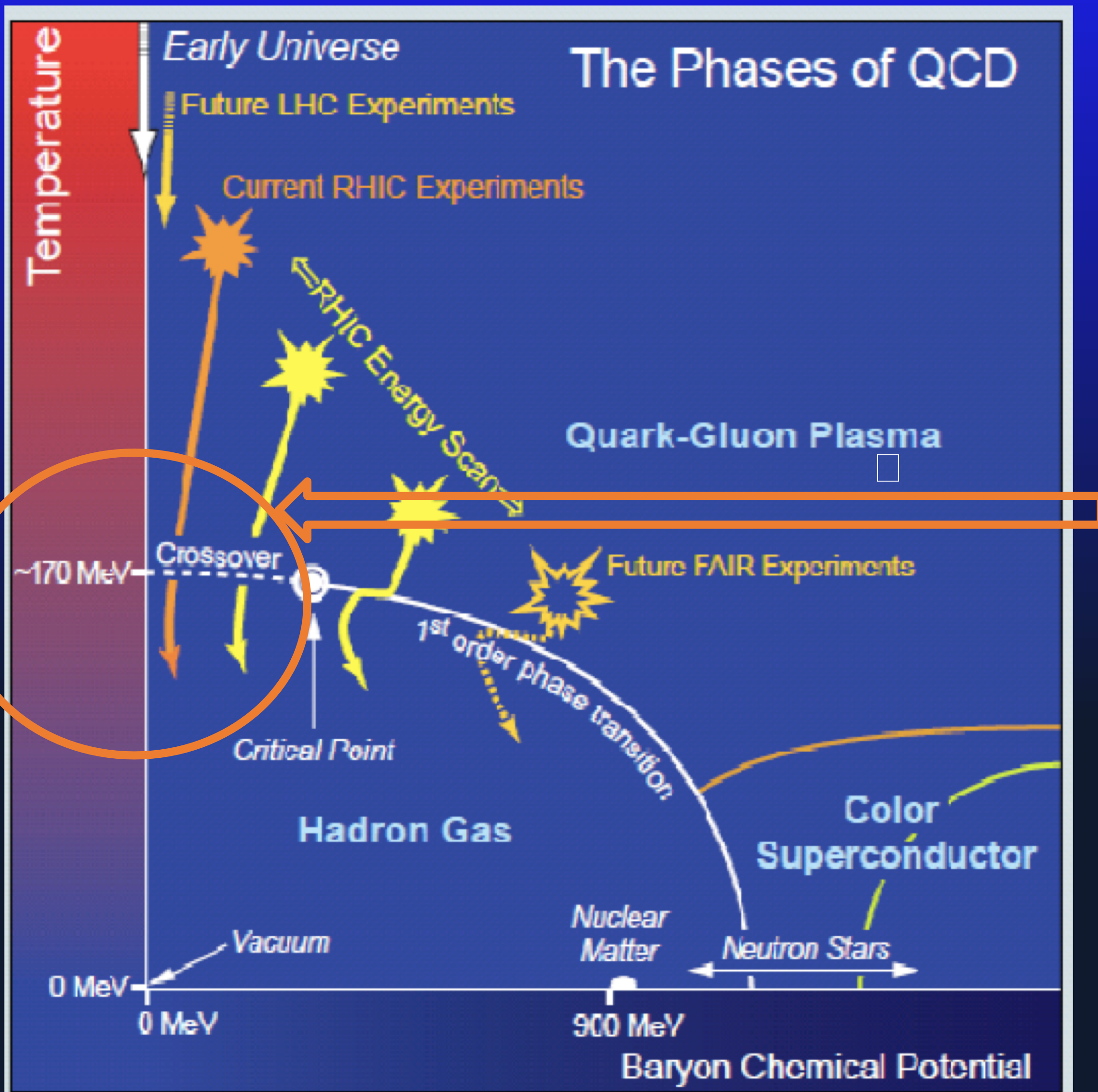


Wuppertal-Budapest lattice collaboration
WB: S. Borsanyi et al., PLB (2014)
HotQCD: A. Bazavov et al., PRD (2014)



Lattice gauge theory not yet well implemented for finite baryon density!!
Fermion sign problem

Crossover at zero net baryon density

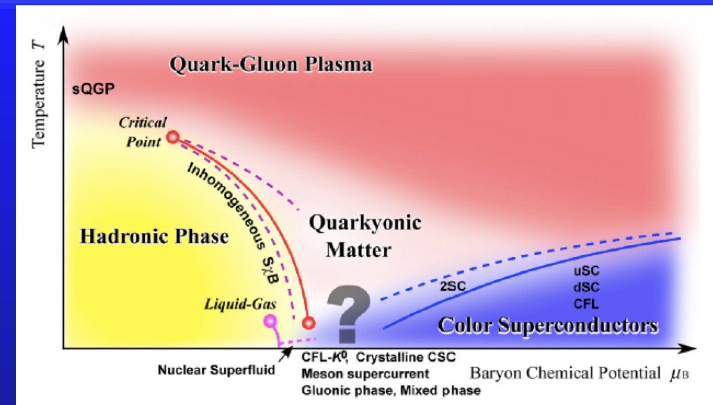


QCD lattice gauge theory -- for finite light quark masses -- predicts crossover from confined phase at lower T to deconfined phase at higher T.

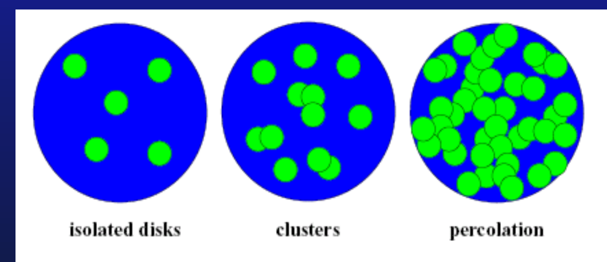
Do quarks roam freely in the deconfined phase? If so, they must also roam freely at lower T.

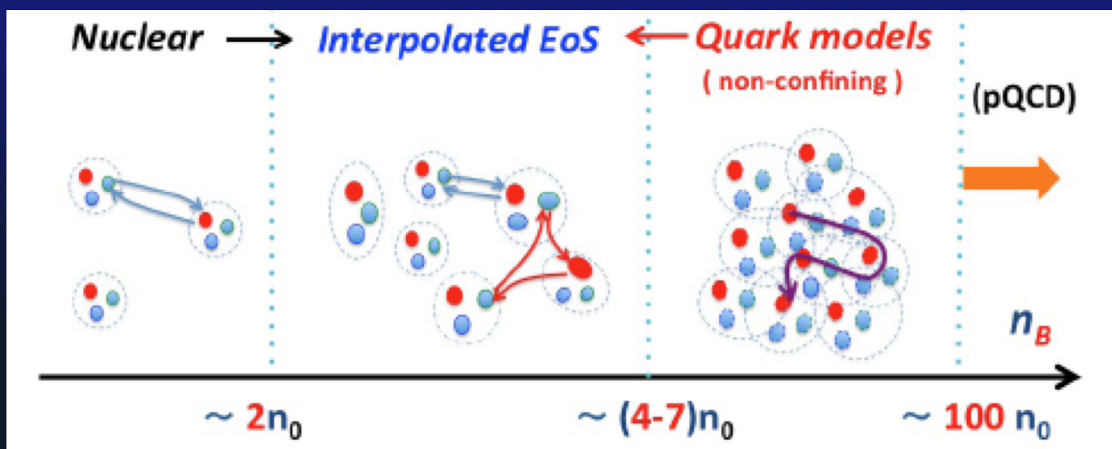
Are there really quarks running about freely in this room?

No free quarks even above the crossover!



In confined region quarks are inside hadrons. Also have quarks and antiquarks in the QCD forces between hadrons. With higher density or temperature, form larger clusters, which percolate at the crossover. In deconfined regime clusters extending across all of space.





Percolation of clusters along the density axis, at zero temperature.

Quarks can still be bound even if deconfined.

$n_{perc} \sim 0.34\ (3/4\pi\ r_n^3)\ \ fm^{-3}$

r_n = nucleon radius

n_0 = density of matter inside large nucleus.

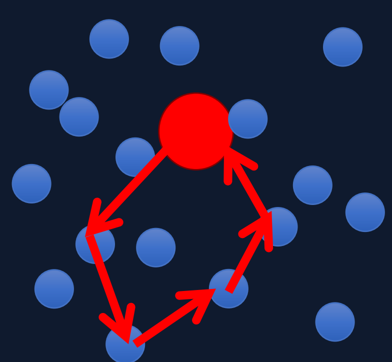
Classical vs. quantum percolation

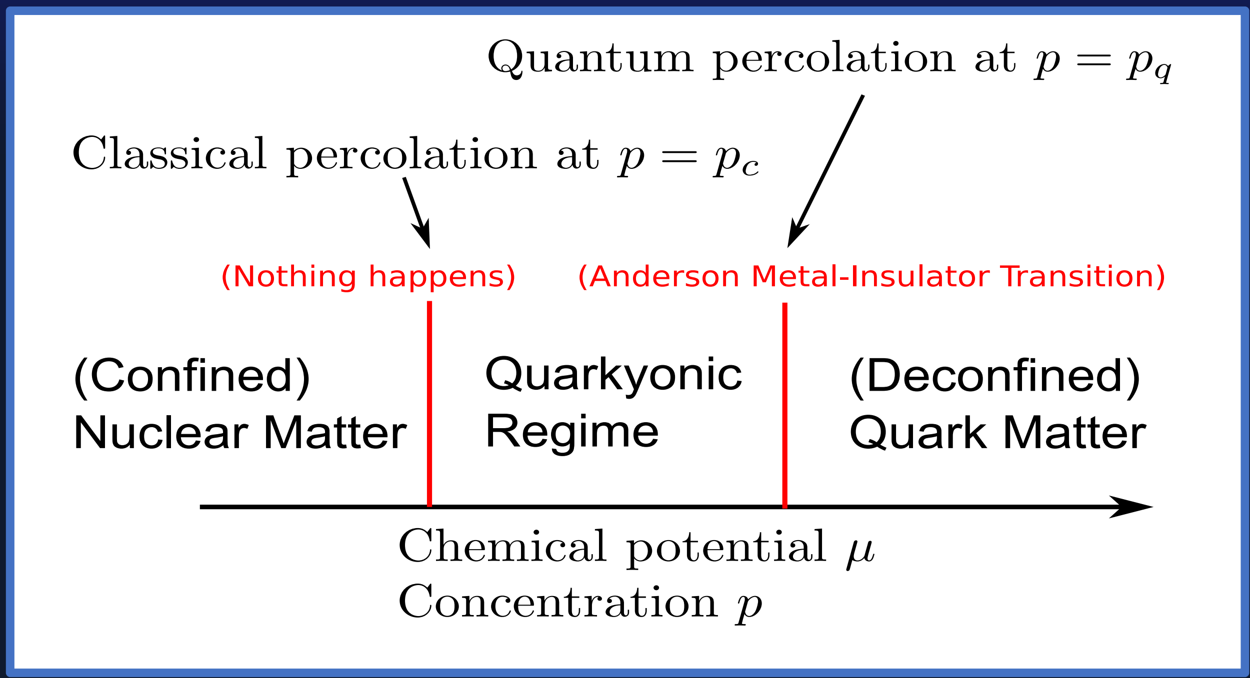
But aren't nucleons, with long distance cloud of mesons always overlapping?

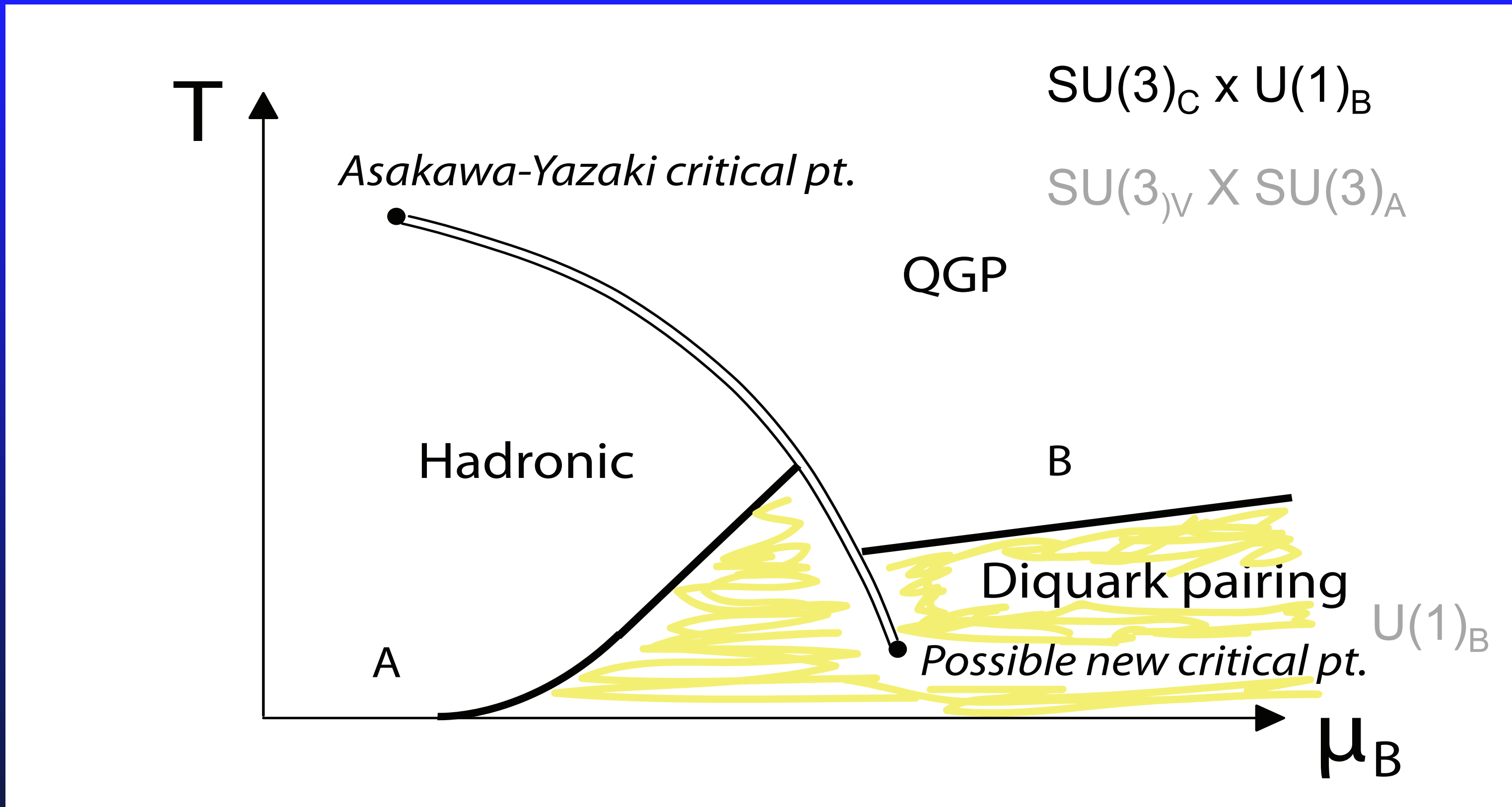
Does anything actually happen at classical percolation transition? No obvious lattice calculation to do!

Distinguish classical (geometric) percolation
  from quantum percolation in terms of wave functions

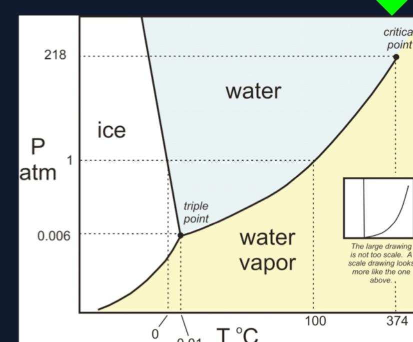
Deconfinement as (inverse) Anderson localization
 (K. Fukushima):







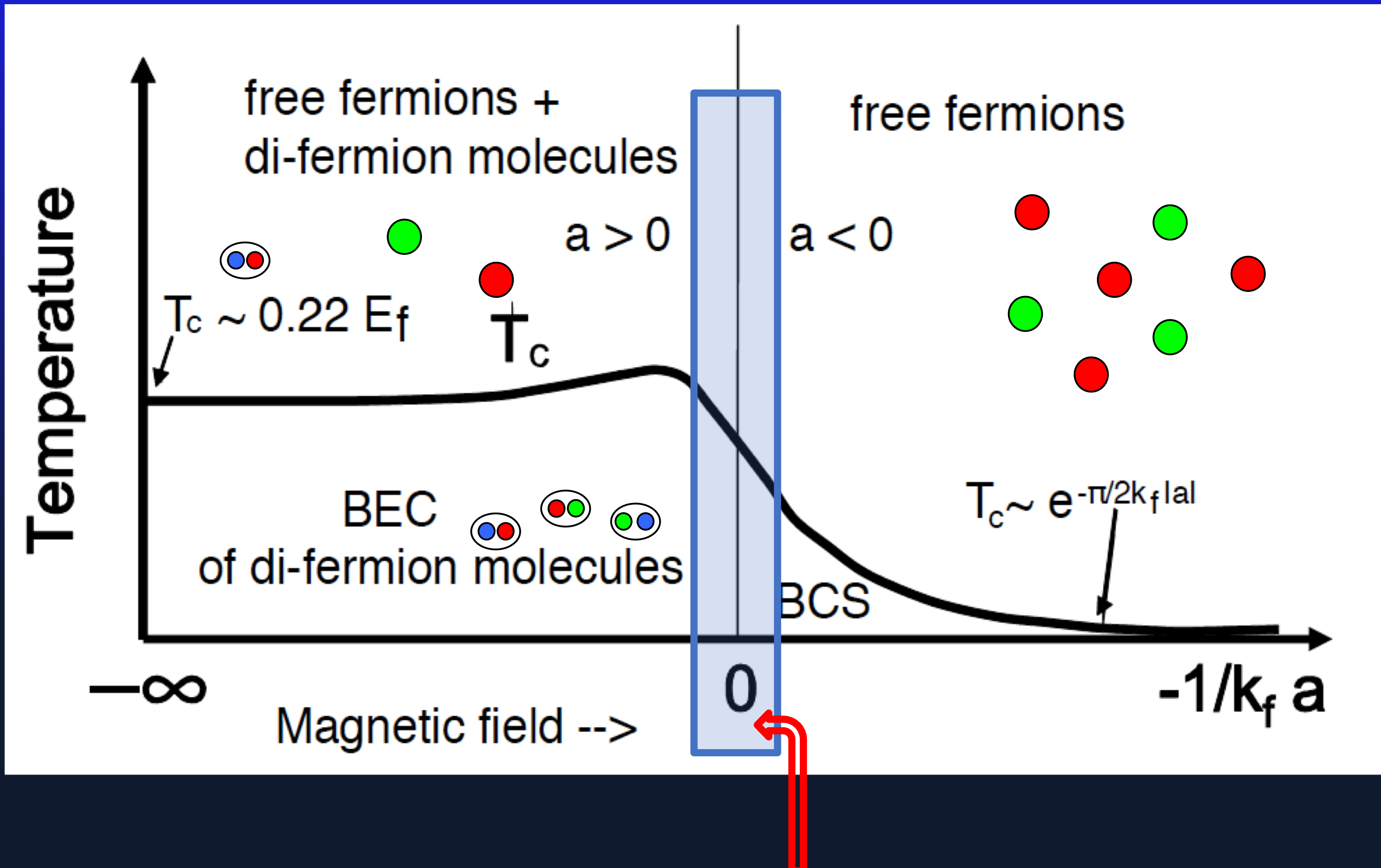
Critical points similar to those in liquid-gas phase diagram (H_2O). Neither critical point necessary!!



Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

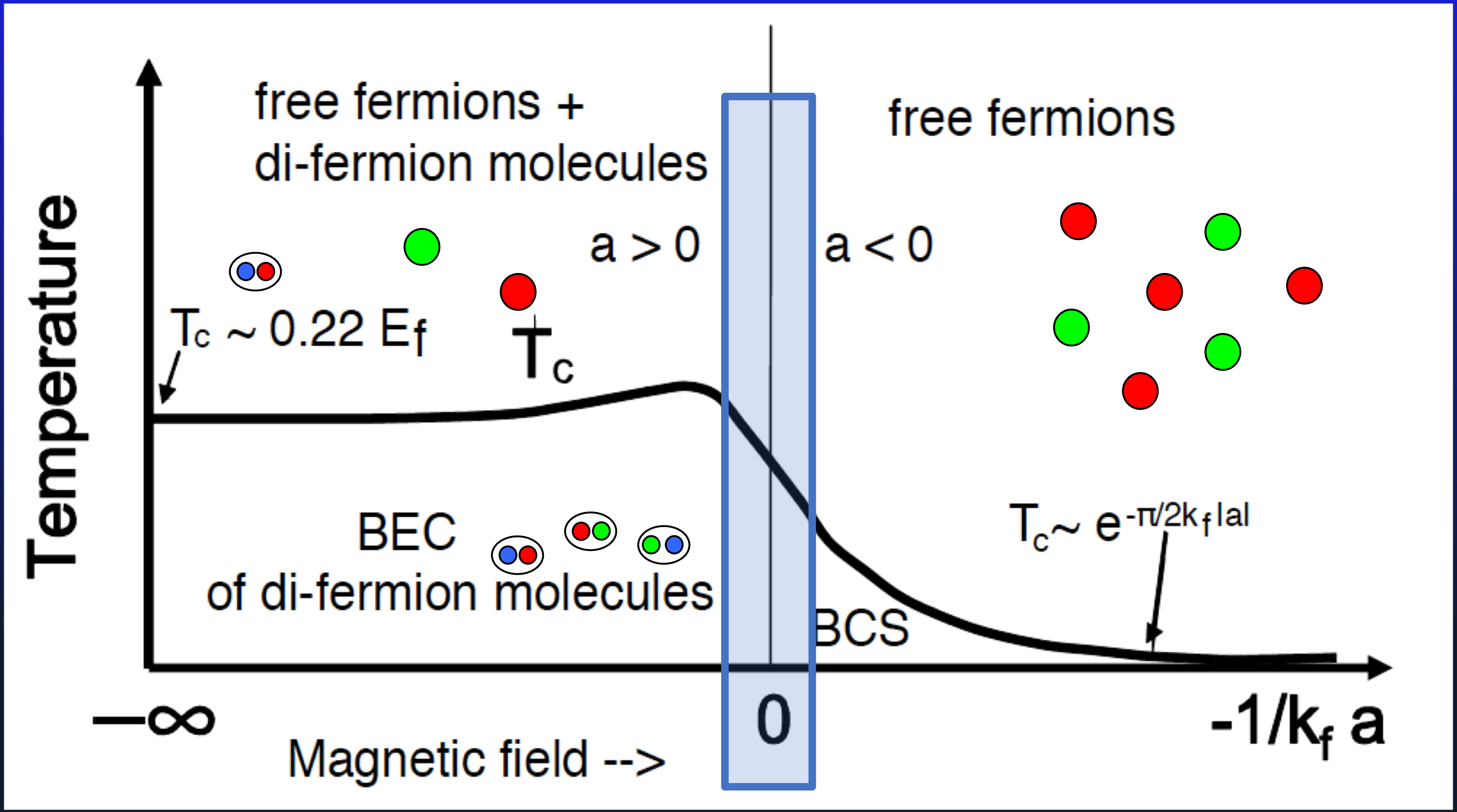
In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates). Different symmetry structure than at higher T.

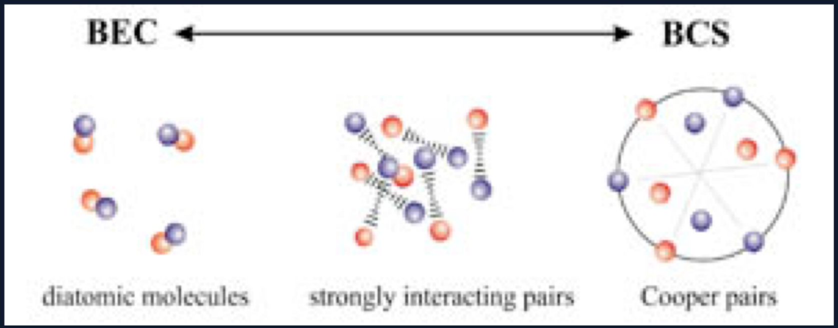
Phase diagram of ultracold atomic fermion gases: in T and strength of the particle interactions



Unitary regime (Feshbach resonance) – BEC-BCS crossover.   No phase transition through crossover

Phase diagram of ultracold atomic fermion gases: in T and strength of the particle interactions

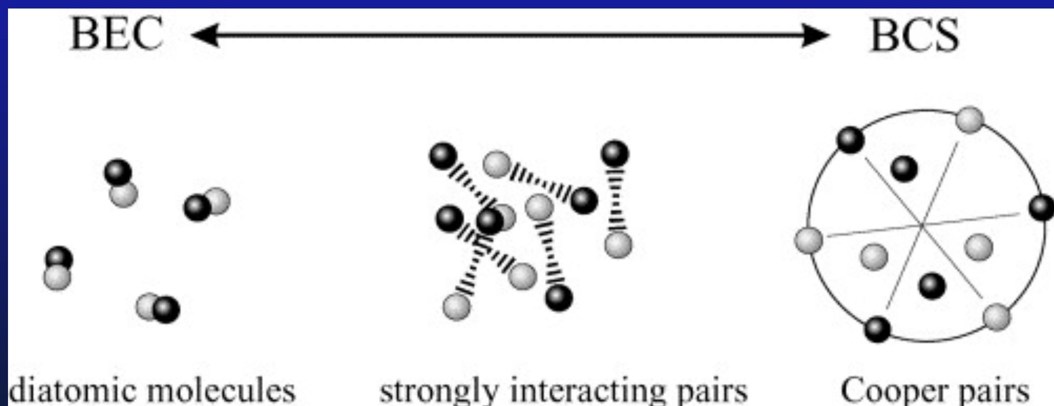




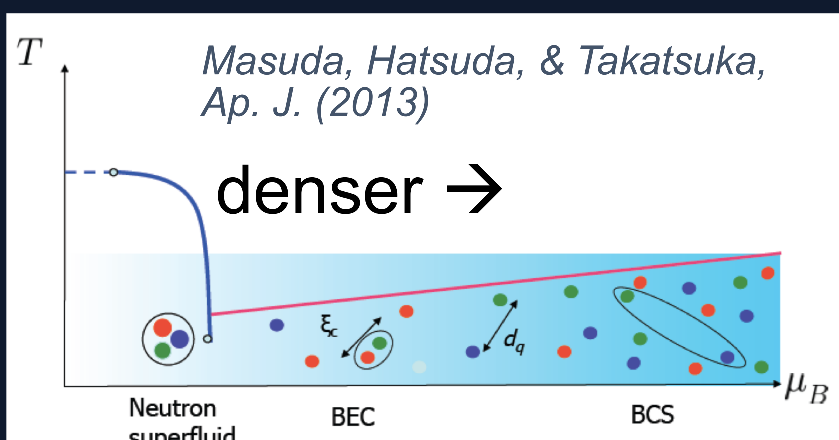
Smooth evolution of states in atomic clouds -- and nuclear matter (?)

GB, T.Hatsuda, M.Tachibana, & N.Yamamoto. J. Phys. G: Nucl. Part. 35, 10402 (2008)

Evolution of Fermi atoms with weakening attraction between atoms:

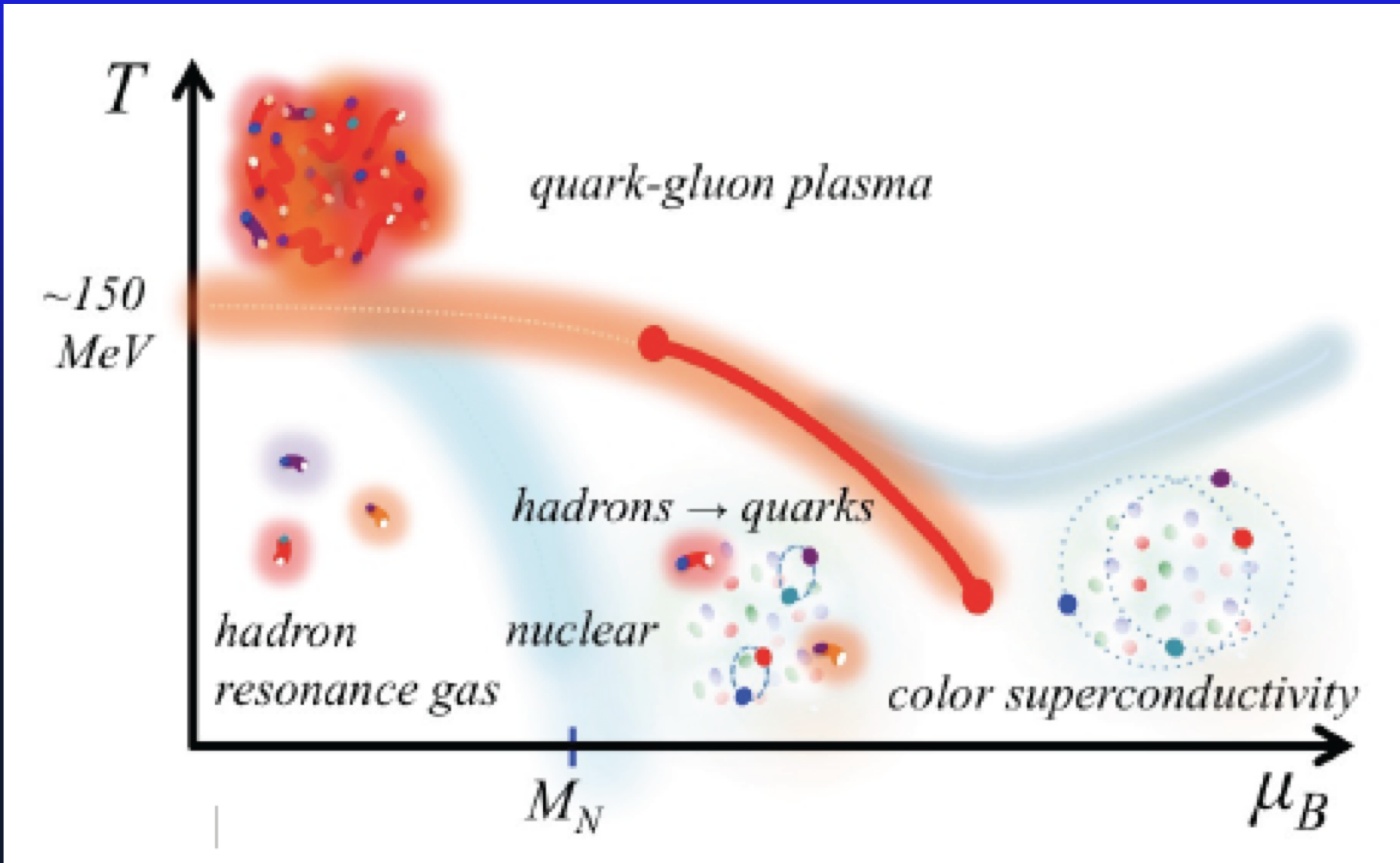


Similarly, as nuclear matter becomes denser can one expect "continuous" evolution from hadrons (nucleons) to quark pairs (diquarks)?

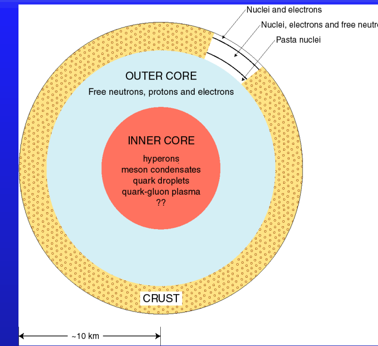


Quark hadron continuity

(Schäfer-Wilczek 1999)


T
~150
MeV
quark-gluon plasma
hadrons → quarks
hadron
resonance gas
nuclear
color superconductivity
M_N
μ_B

Quark matter cores in neutron stars

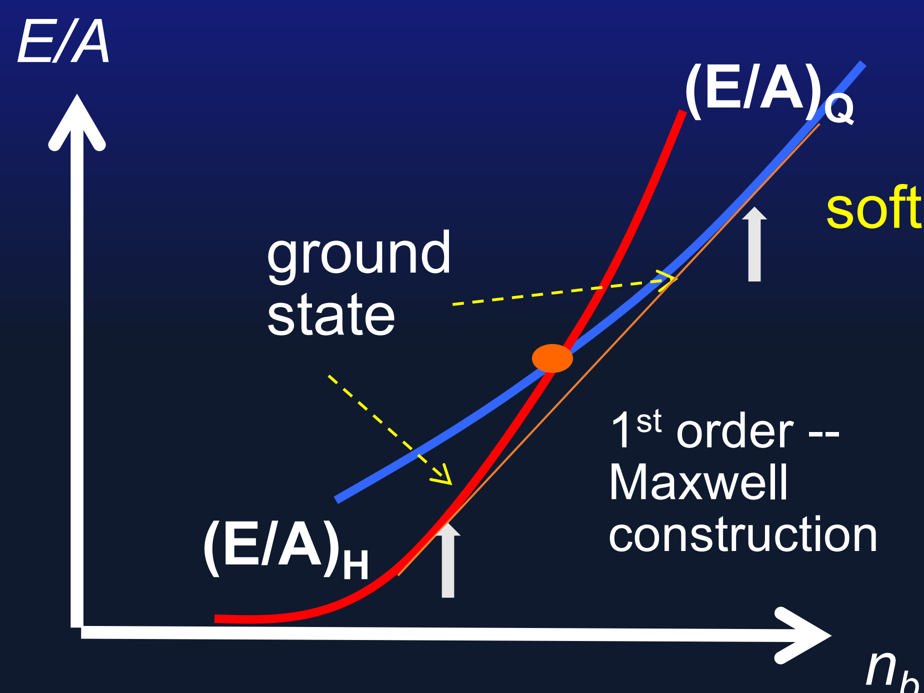


Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter.

GB & S.A. Chin (1976)

Crossing of thermodynamic potentials => first order phase transition.

ex. nuclear matter using 2 & 3 body interactions, vs. perturbative expansion or bag models.

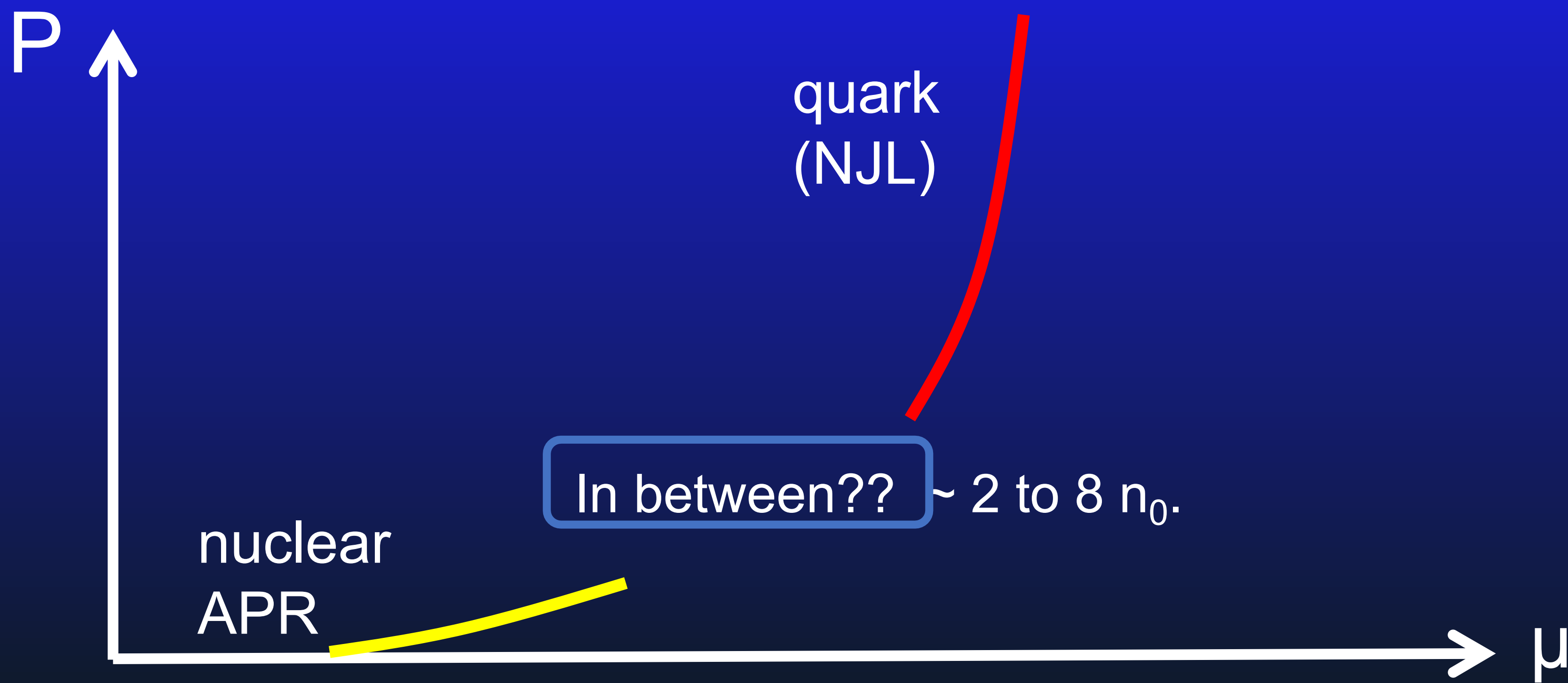


Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Allows only quark equations of state lying under hadronic at high density. Soft only and therefore can't support two solar mass stars.

Typically conclude transition at $n \sim 10 n_{nm}$ -- would not be reached even in high mass neutron stars => at most small quark matter cores

Have good idea of equation of state at nuclear densities and at high densities. Look at pressure vs. baryon chemical potential



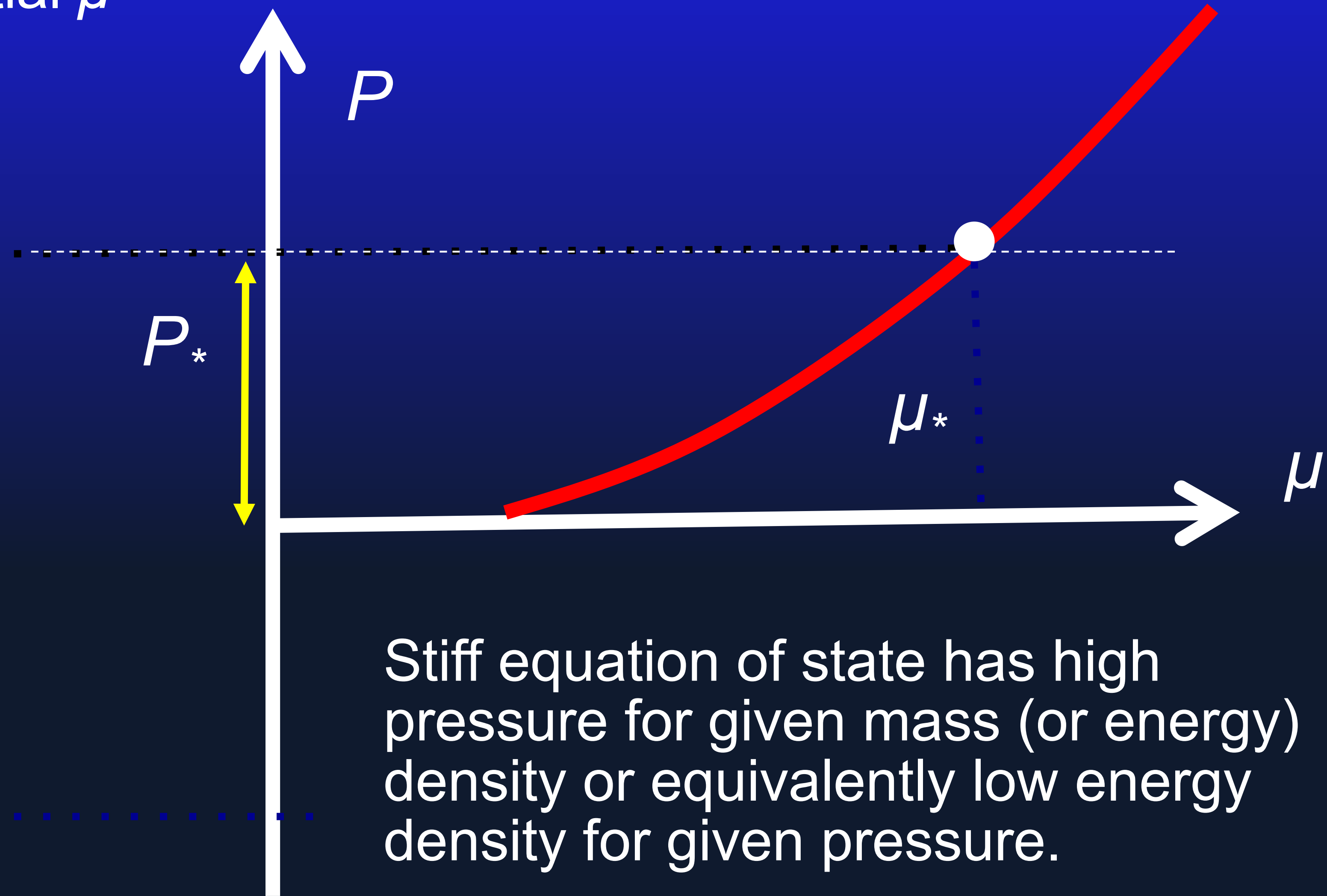
Quarks in Nambu-Jona-Lasinio (NJL) model with universal repulsive short-range qq coupling (*Kunihiro*)

$$\mathcal{L}_V^{(4)} = -g_V \left(\overline{q}\gamma^\mu q\right)^2$$

APR = Akmal, Pandharipande, Ravenhall nucleonic equation of state with nucleonic potentials (2 and 3 body) fit to NN scattering and light nuclei

How can QCD give large mass neutron stars?

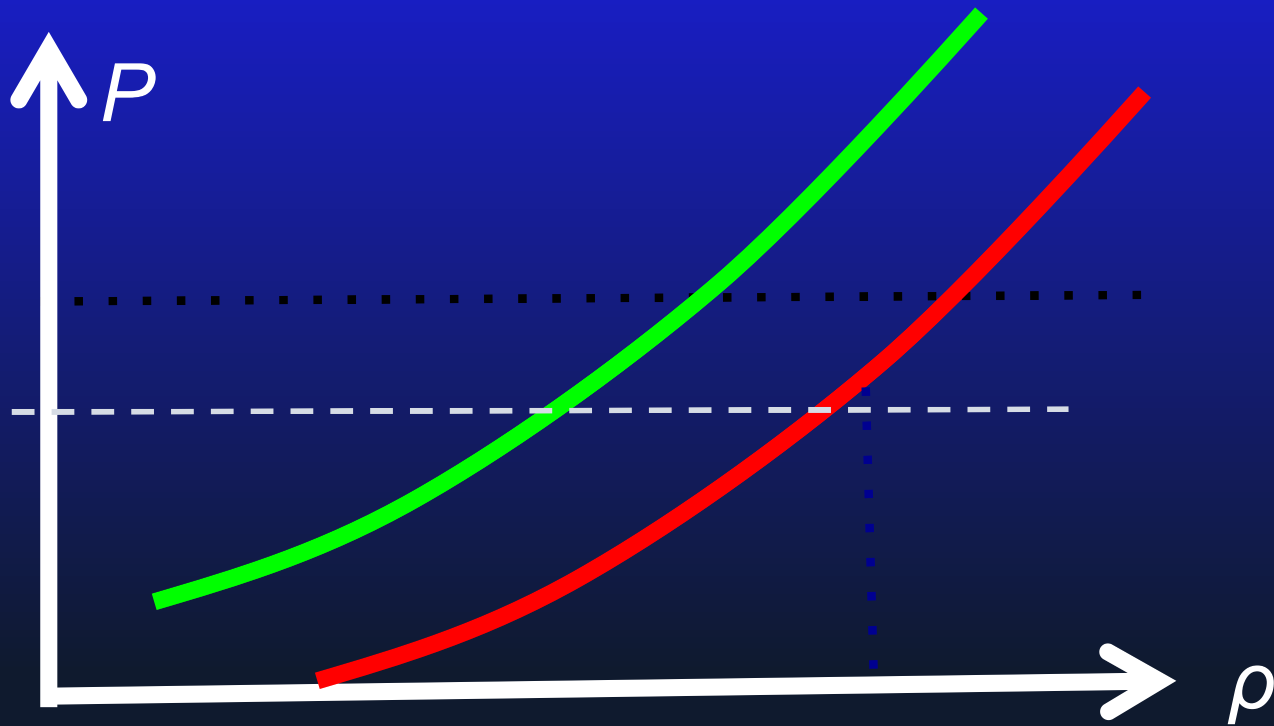
Pressure P is a continuous function of baryon chemical potential μ



Stiff equation of state has high pressure for given mass (or energy) density or equivalently low energy density for given pressure.

T. Kojo, P. D. Powell, Y. Song, & GB, PR D 91, 045003 (2015)

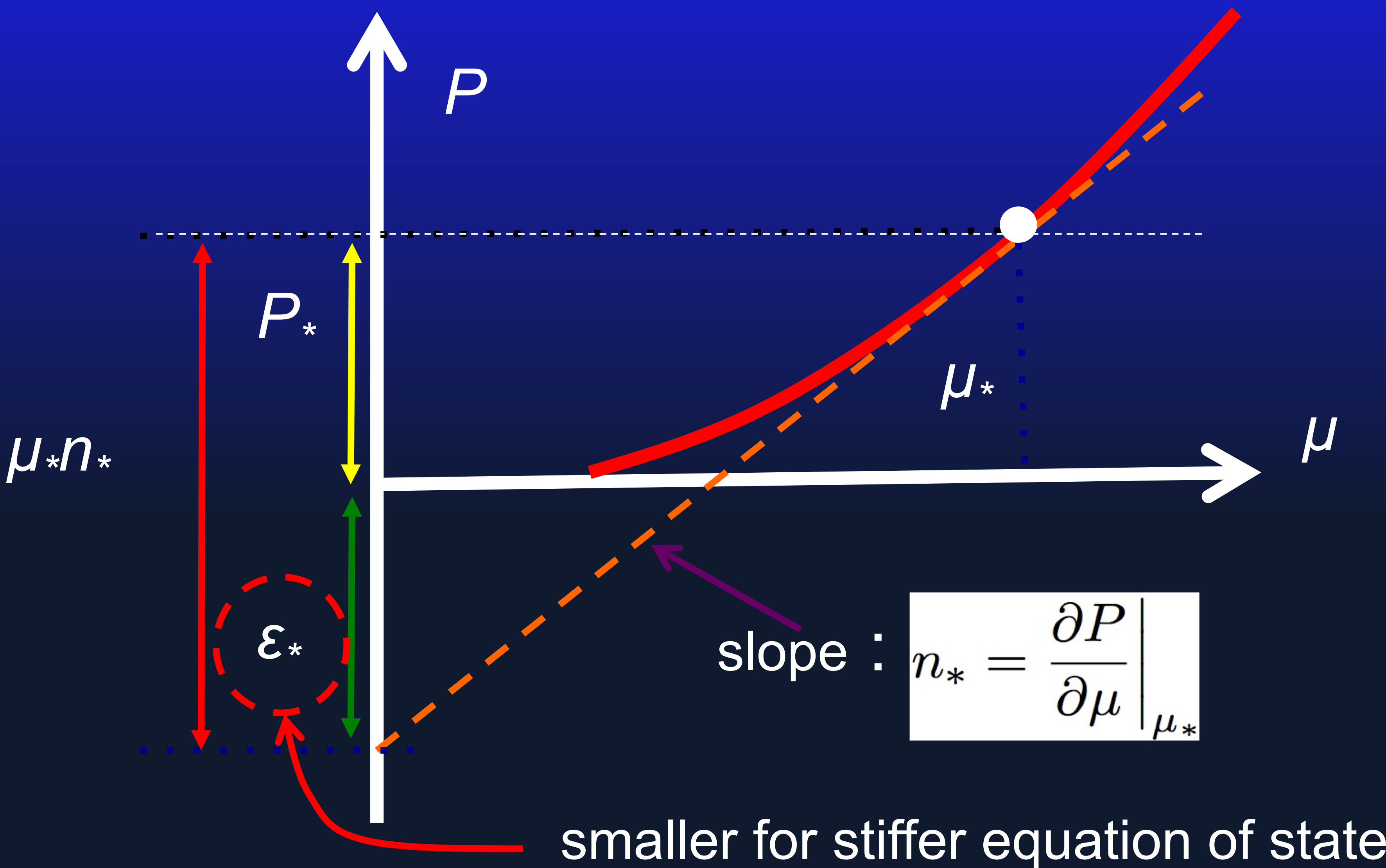
Stiffer equations of state given more massive neutron stars, with lower central densities



Green equation of state is stiffer than red.
Has larger pressure for given mass density *ρ*,
and has smaller *ρ* for given pressure P

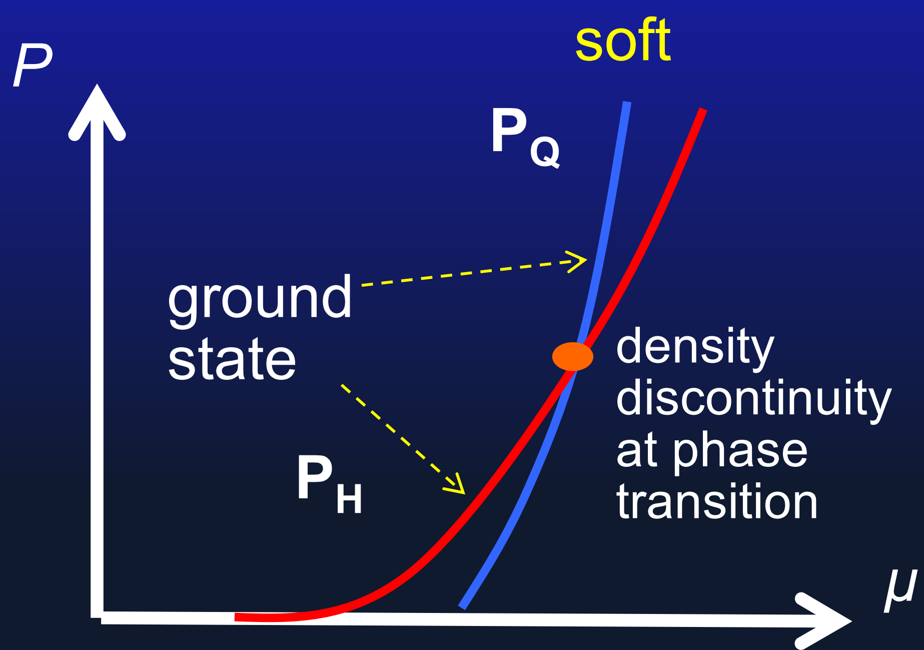
How can QCD give large mass neutron stars?

Energy or mass density $\varepsilon = \rho c^2 = \mu n - P$



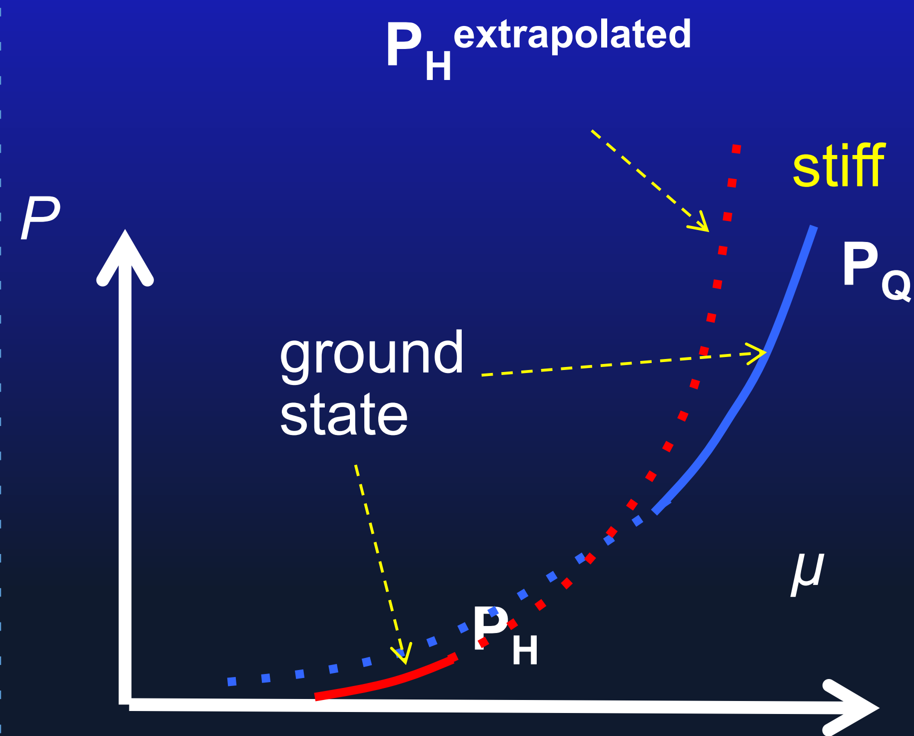
Hybrid eqs. of state are intrinsically softer

Phase with larger P at given μ thermodynamically preferred



Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Continuous eqs. of state can be much stiffer



Hadrons only at low density and quark matter at high density. In between???

Model calculations of neutron star matter within NJL model

NJL Lagrangian $\mathcal{L} = \bar{q}(i\gamma_\mu \partial^\mu - m_q + \mu\gamma_0)q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}$

$$\mathcal{L}^{(4)}_\chi = G \sum_{a=0}^{8} [(\bar{q}\tau_a q)^2 + (\bar{q} i\gamma_5 \tau_a q)^2]$$

chiral interactions

$$\mathcal{L}^{(4)}_d = H \sum_{A,A'=2,5,7} [(\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i\gamma_5 \tau_A \lambda_{A'} q)$$

BCS pairing interactions

$\mathcal{L}^{(6)}$ = Kobayashi-Maskawa-‘t Hooft six quark axial anomaly

plus universal repulsive quark-quark vector coupling

$$\mathcal{L}^{(4)}_V = -g_V \left(\bar{q}\gamma^\mu q\right)^2$$

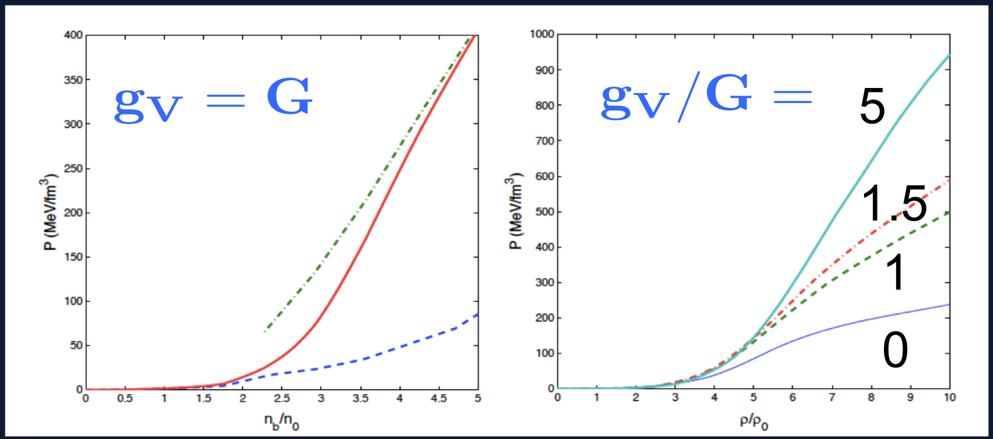
T. Kunihiro

Include u,d, and s quarks

K. Masuda, T. Hatsuda, & T. Takatsuka, Ap. J.764, 12 (2013)

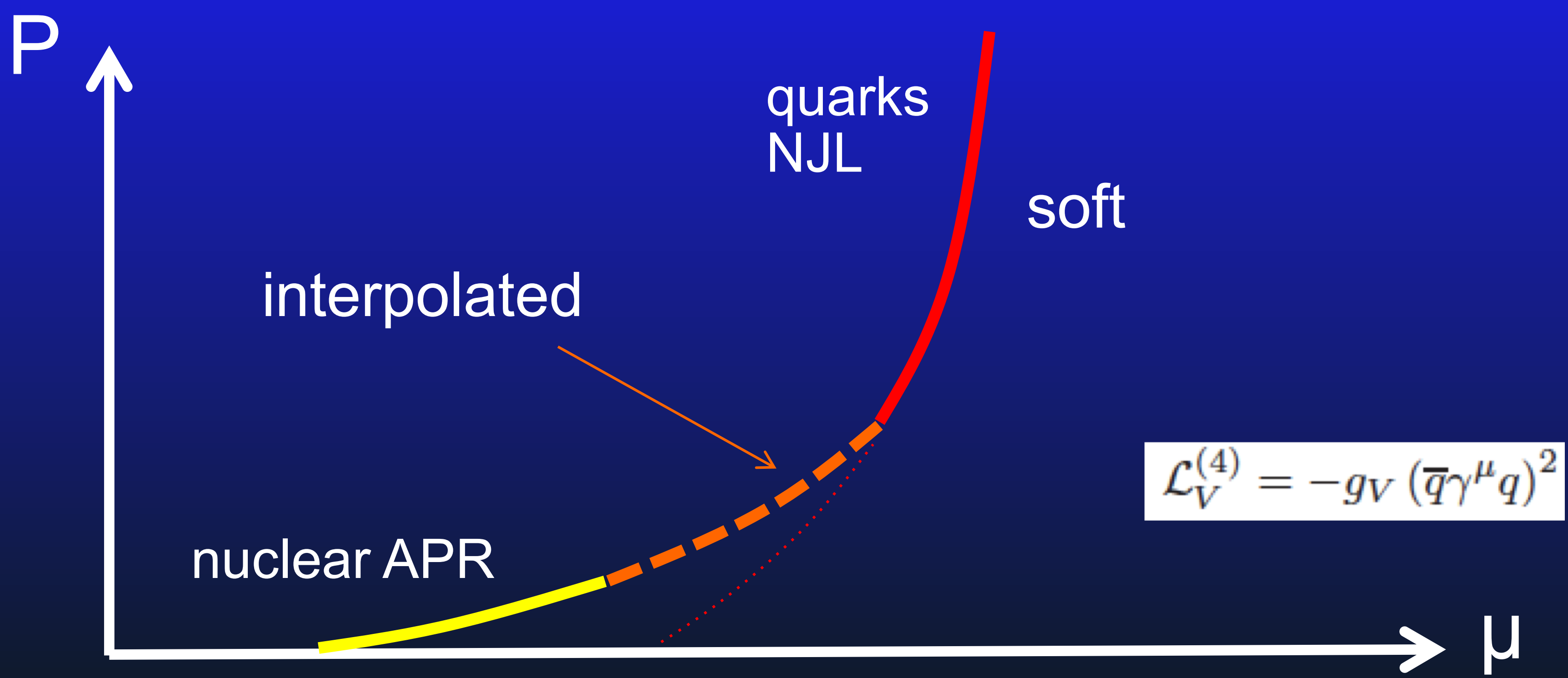
GB, T. Kojo, T. Hatsuda, T. Takatsuka, & Y. Song ROPP 81 (2018) 056902

pressure



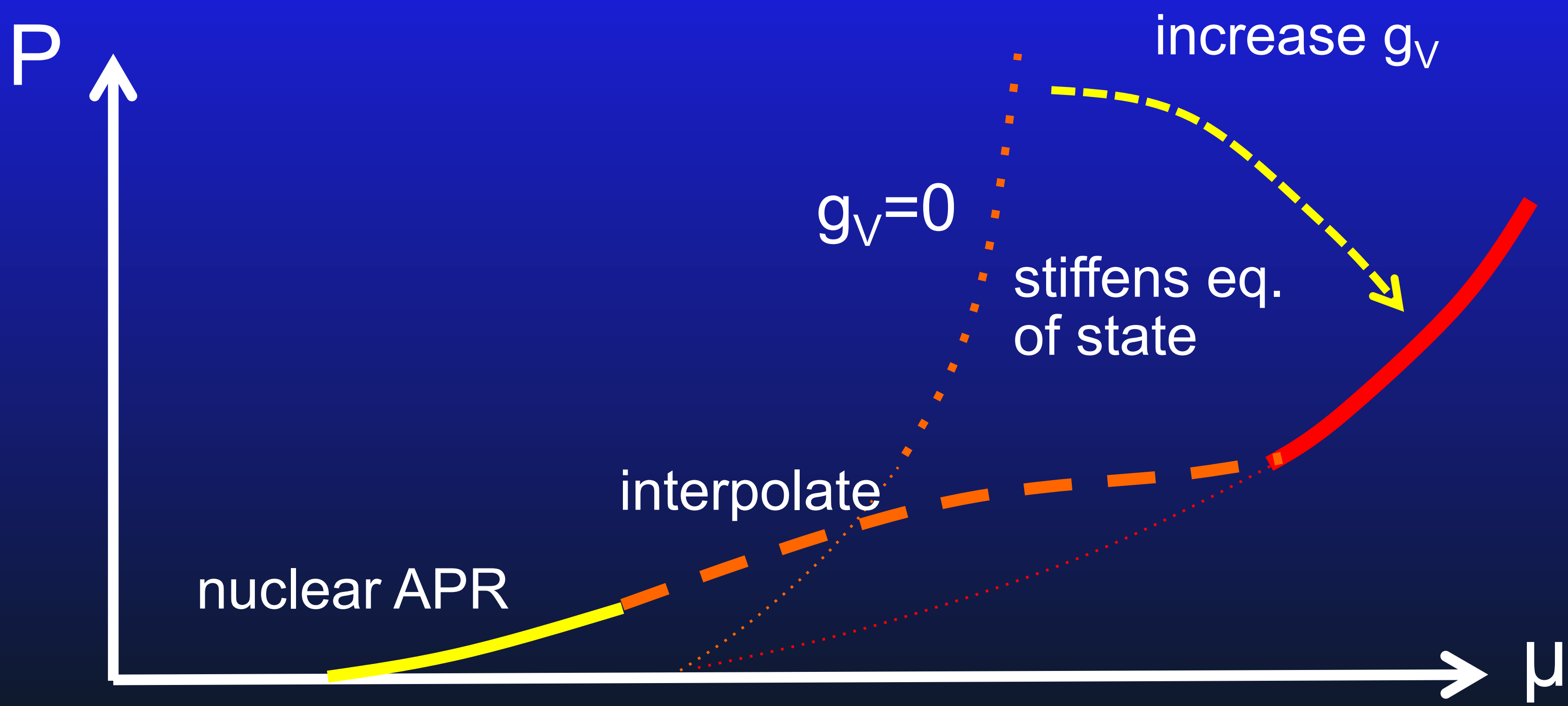
baryon density mass density

Minimal model: $g_V = 0$



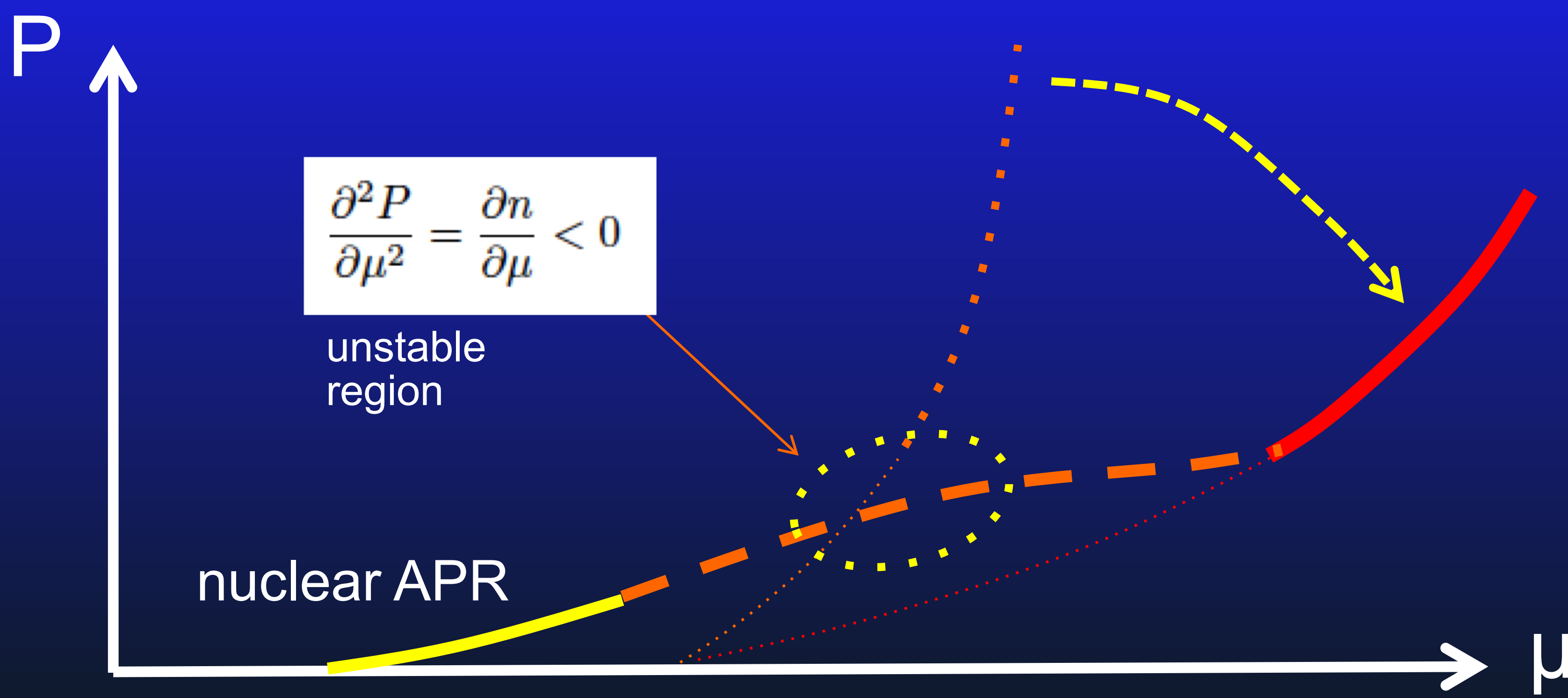
Soft quark equation of state does not allow high mass neutron stars

Vector interaction stiffens eq. of state



Shift of pressure in quark phase towards higher μ

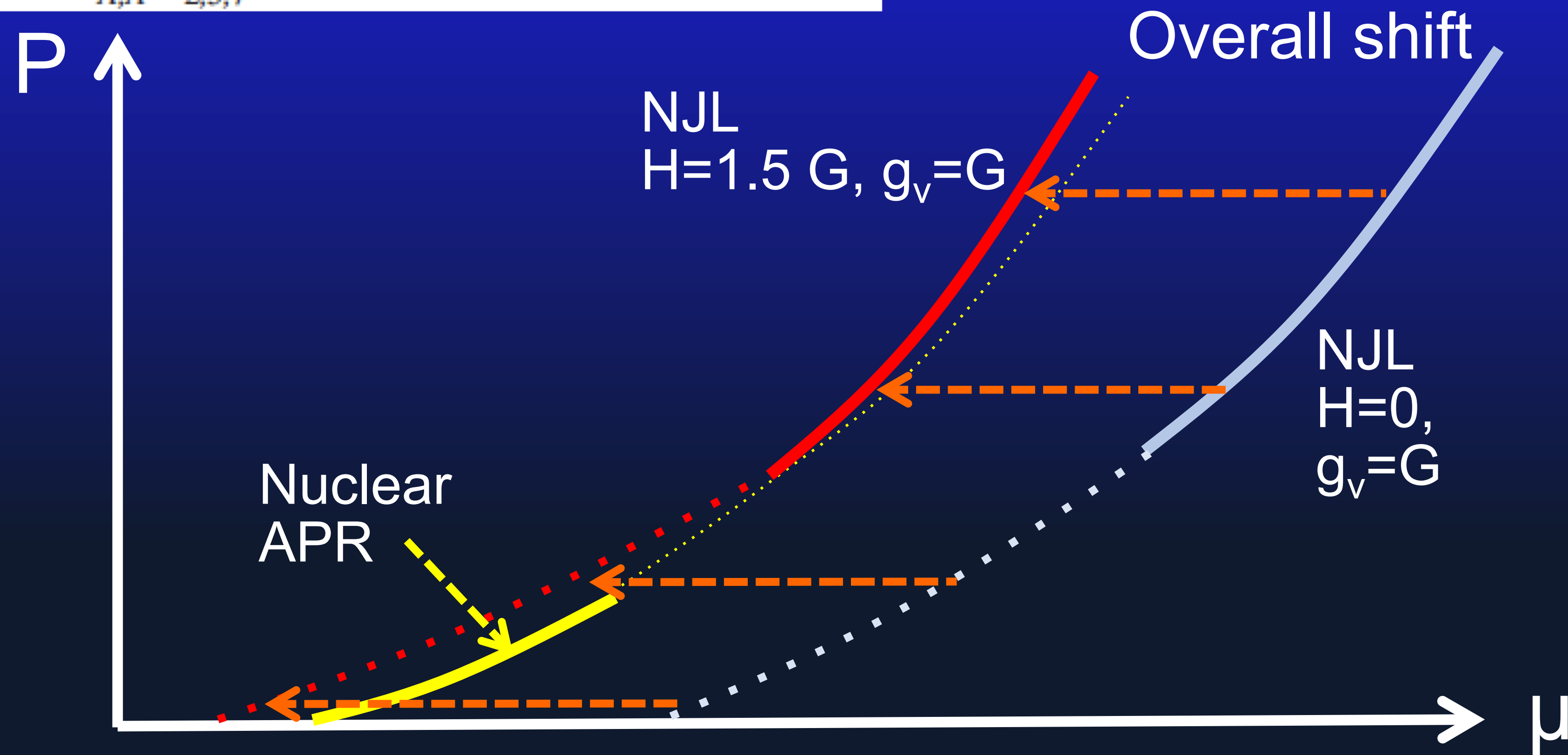
Vector interaction stiffens eq. of state



Larger g_V leads to unphysical thermodynamic instability

Restore stability with increased BCS (diquark) pairing interaction, H

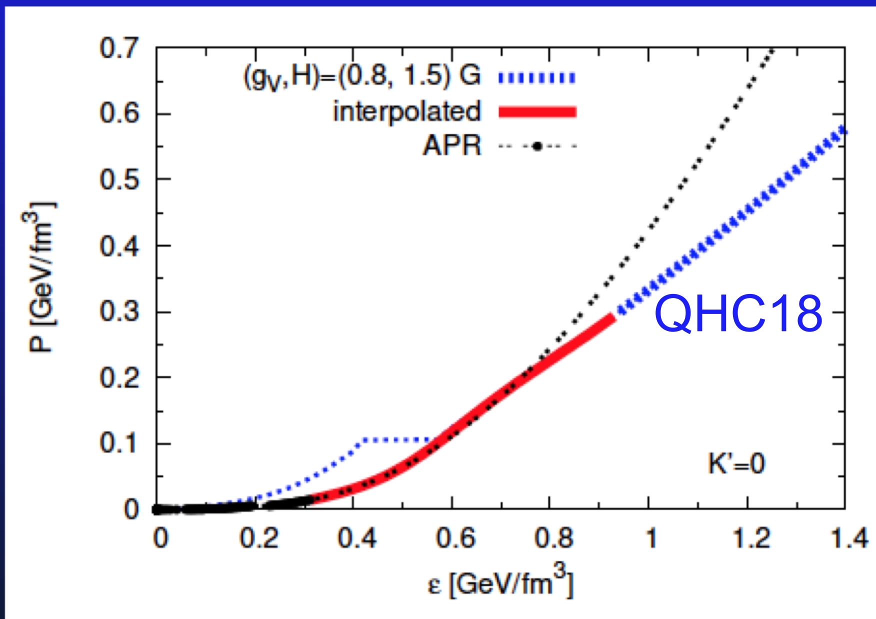
$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q} i \gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i \gamma_5 \tau_A \lambda_{A'} q)$$

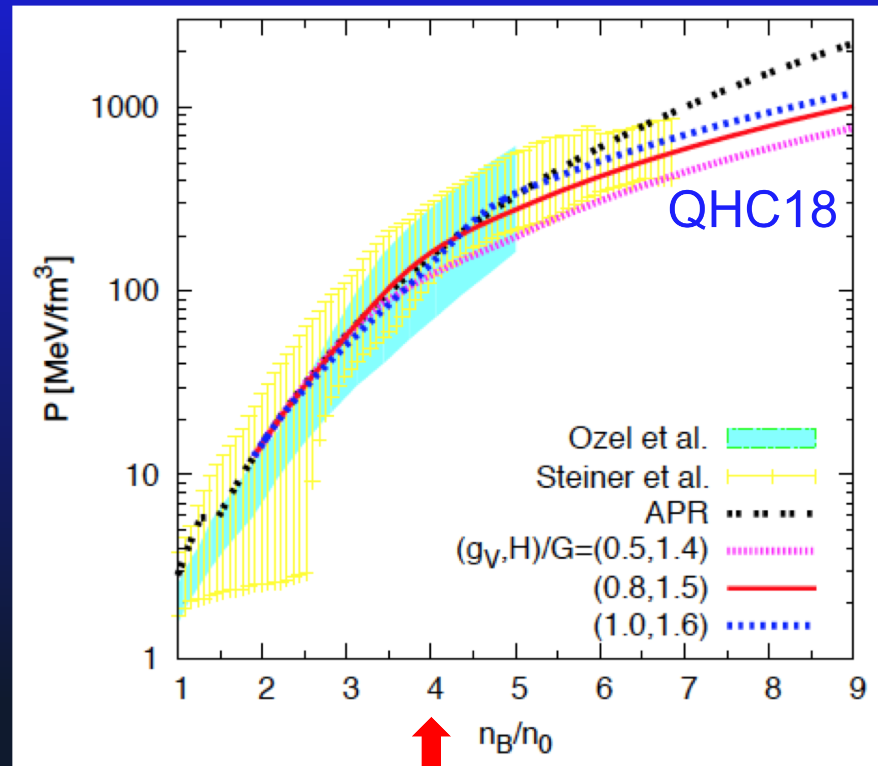


Increased BCS pairing (onset of stronger 2-body correlations) as quark matter comes nearer to becoming confined

Sample "unified" equation of state: QHC18 (quark-hadron crossover)

T. Kojo, T. Hatsuda, GB, et al.





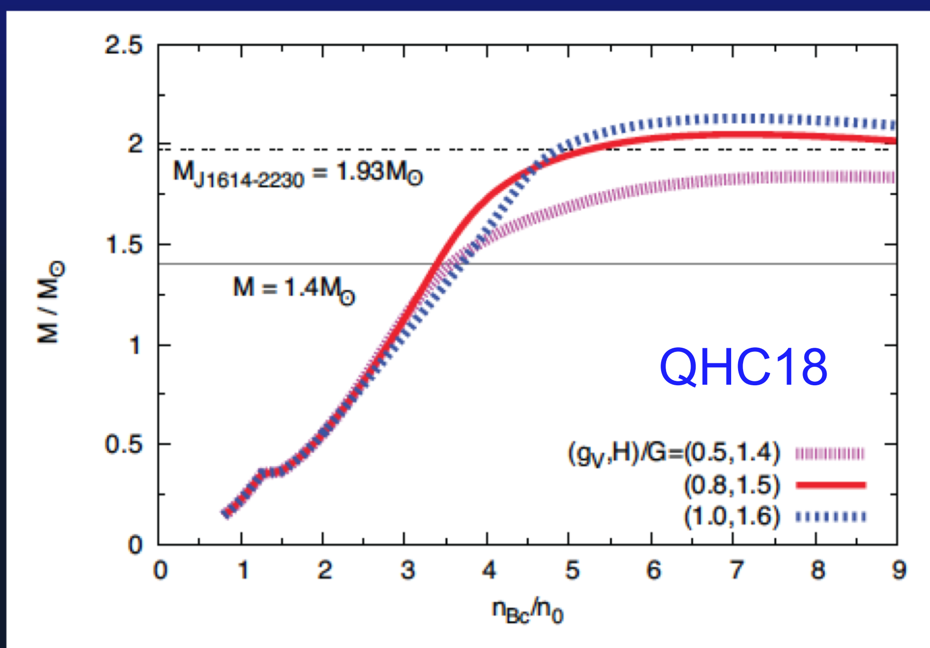
Consistent with eq. of state inferred from M vs. R observations

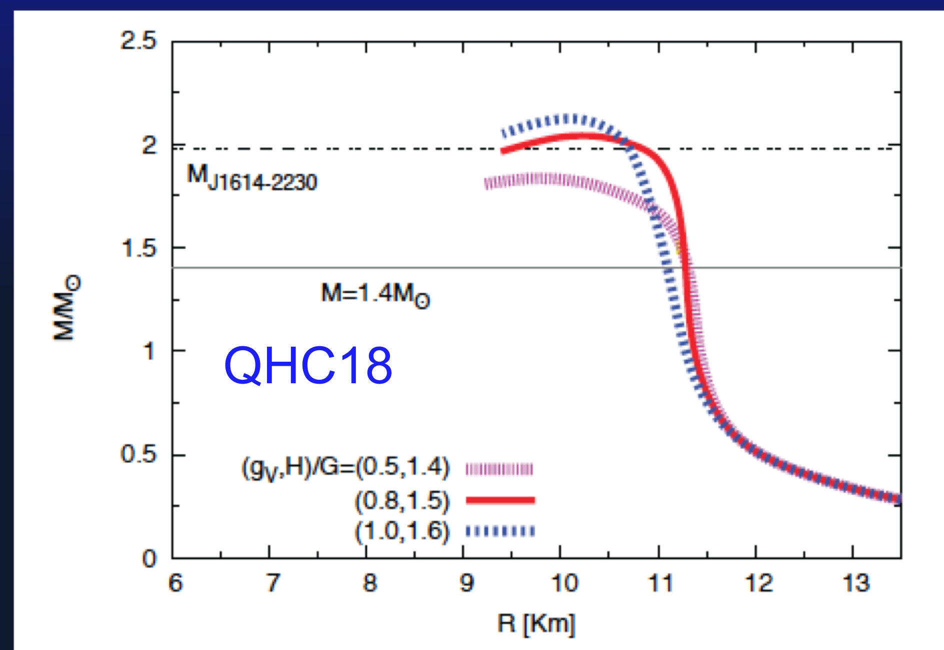
Quark eqs. of state can be stiffer than previously thought: allow for n.s. masses > 2 $M_{\odot}$, and with substantial quark cores in neutron stars!!!

Masses and radii of neutron stars vs. central mass density from integrating TOV equation with QHC18

GB et al. Rep. Prog. Phys. 81 (2018) 056902.

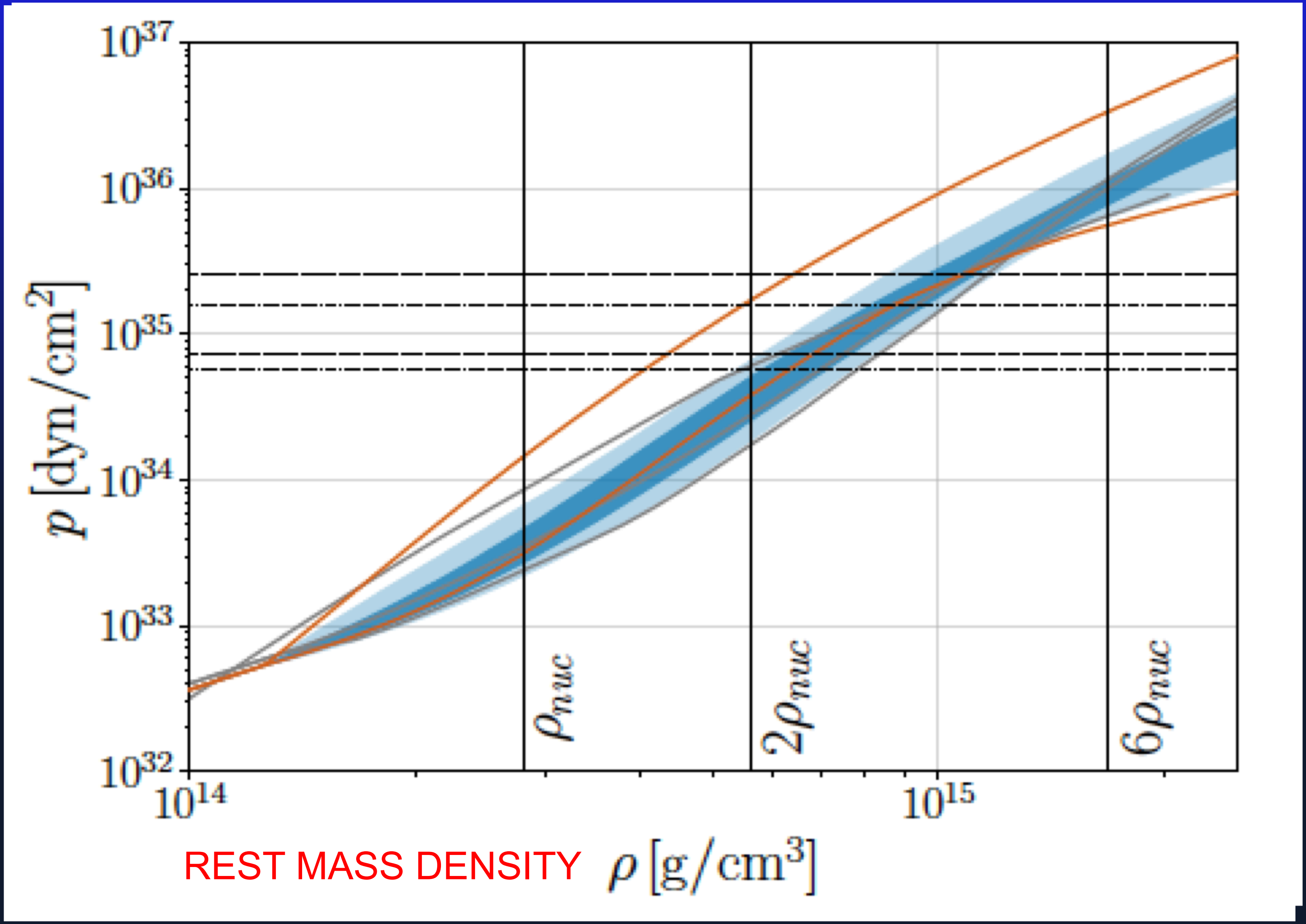
Include stronger correlations between quarks by increasing the effective pairing interaction H between quarks beyond standard NJL
H ~ 1.5 G
Increased vector repulsion between quarks: g_V ~ 0.5-1.0 G





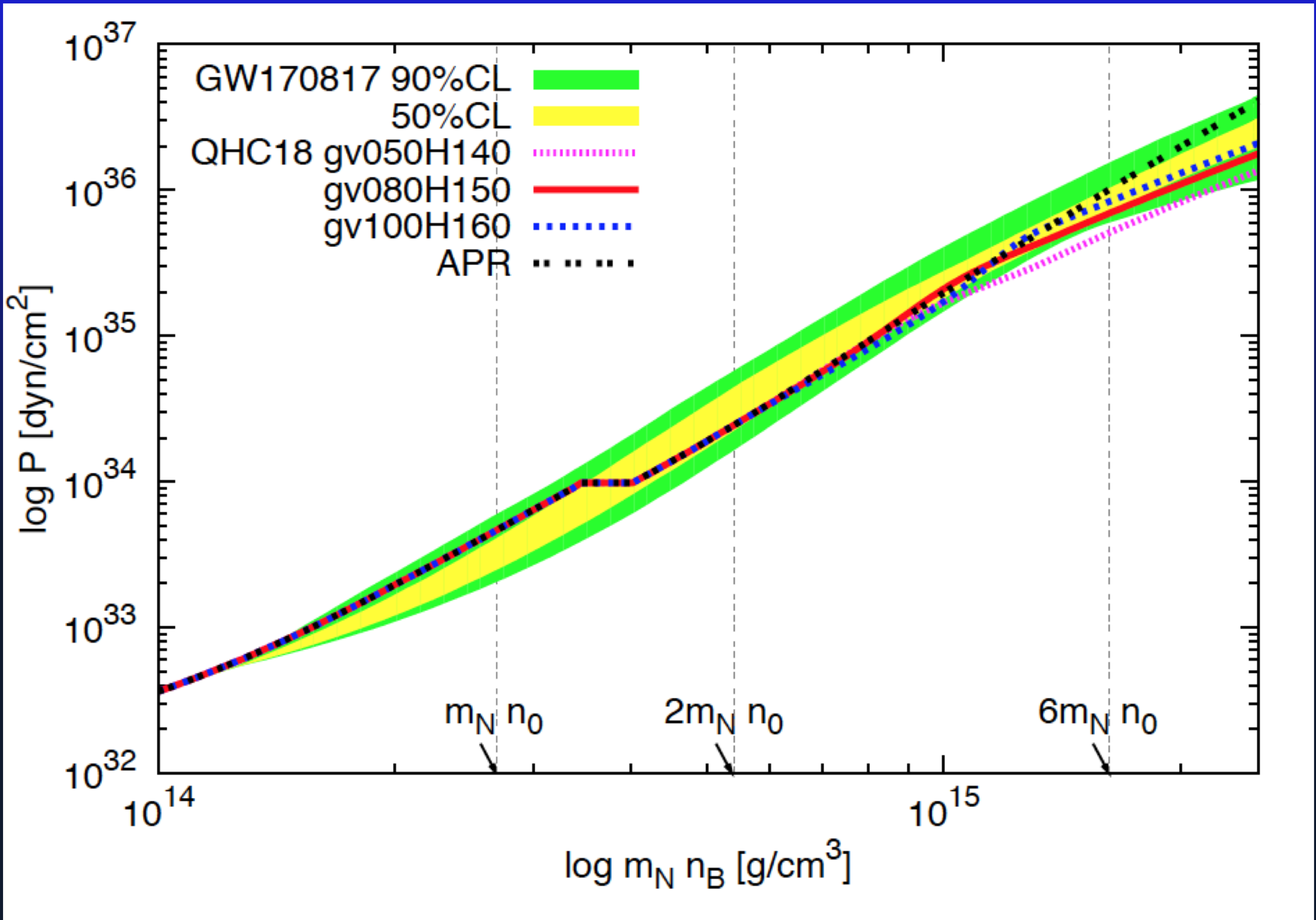
Mass vs. central baryon density: Mass vs. radius:

LIGO plot of pressure vs. rest mass density, arXiv:1805.11581



LIGO plot of pressure vs. rest mass density

arXiv:1805.11581



Calculating neutron star tidal deformability with QHC18

Metric tensor with neutron star of mass M and quadrupole moment Q_{ij}

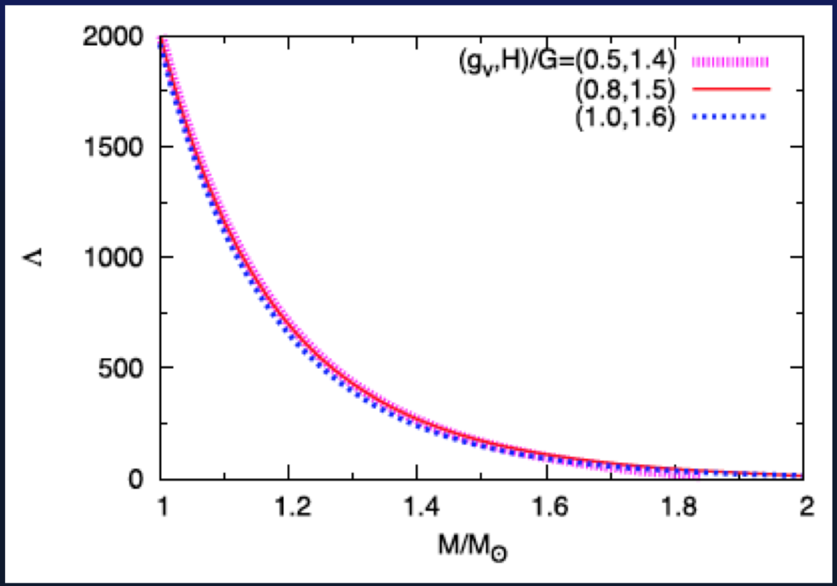
$$g_{00} \sim 1 - G\left(\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\hat{n}_i\hat{n}_j \cdots\right) + \frac{E_{ij}}{2}\hat{n}_i\hat{n}_j$$

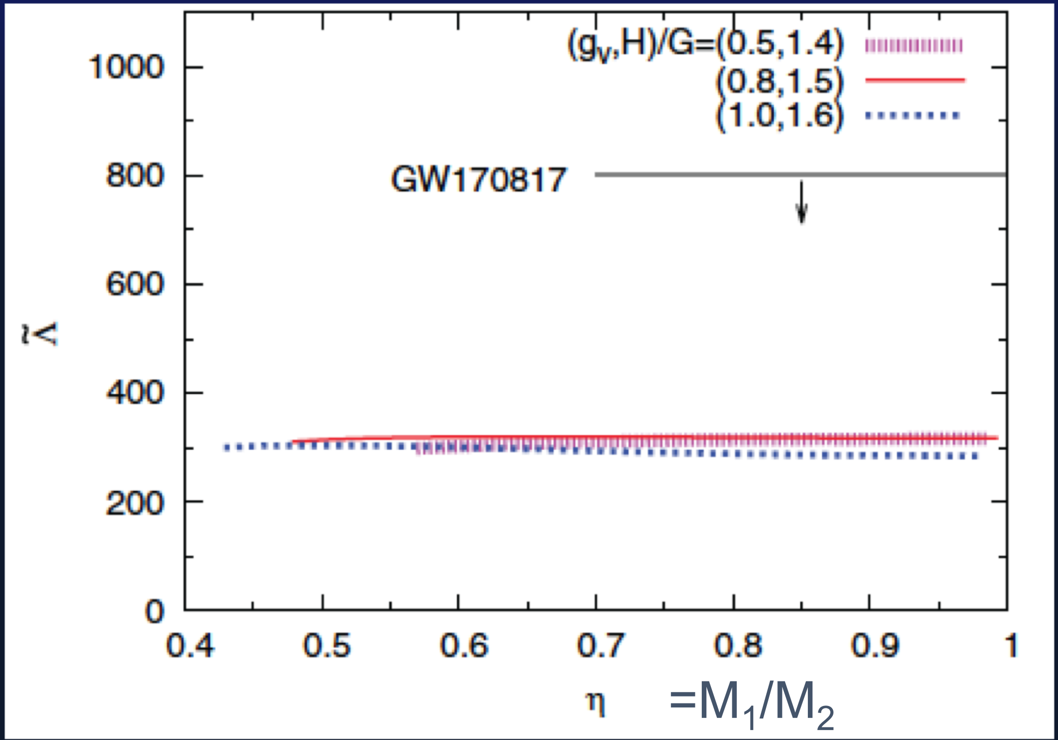
E_{ij} = external tidal force

Tidal deformability λ = response of Q to E: $Q_{ij} = -\lambda E_{ij}$

Dimensionless deformability Λ: $\lambda \equiv \frac{32G}{r_s^5}\Lambda$

$r_s = \frac{2MG}{c^2}$ = neutron star Schwarzschild radius





Quark Hadron continuity

Hadronic Matter	Quark Matter
Baryons 8+1 (low-lying)	Quarks 3color × 3flavor = 9

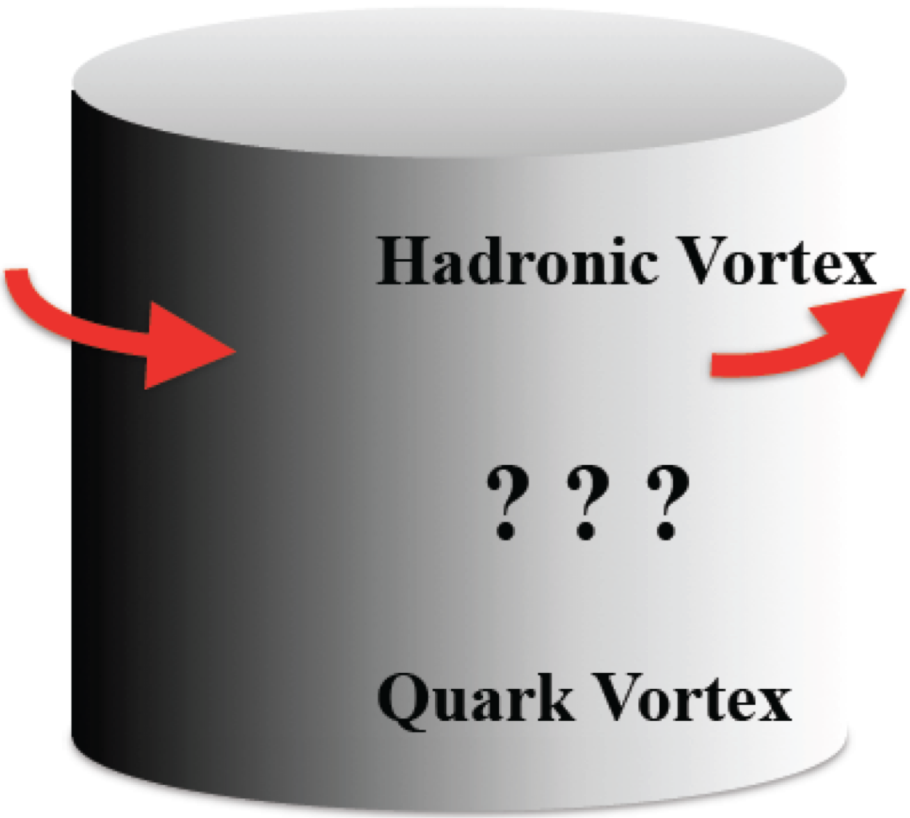
All the condensates and excitations have correspondence

8 vector mesons	8 gluons
8 pseudo-scalar mesons	8 tetra-quark mesons

What about topological configurations?

K. Fukushima

Vortices in neutron stars: quark-hadron continuity??



Rotate the bucket filled with quarks

Upper part : Hadronic Vortex

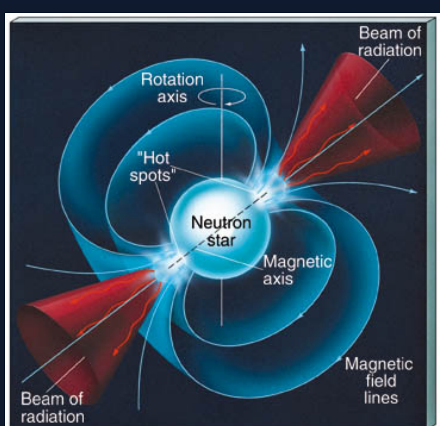
Lower part : Quark Vortex

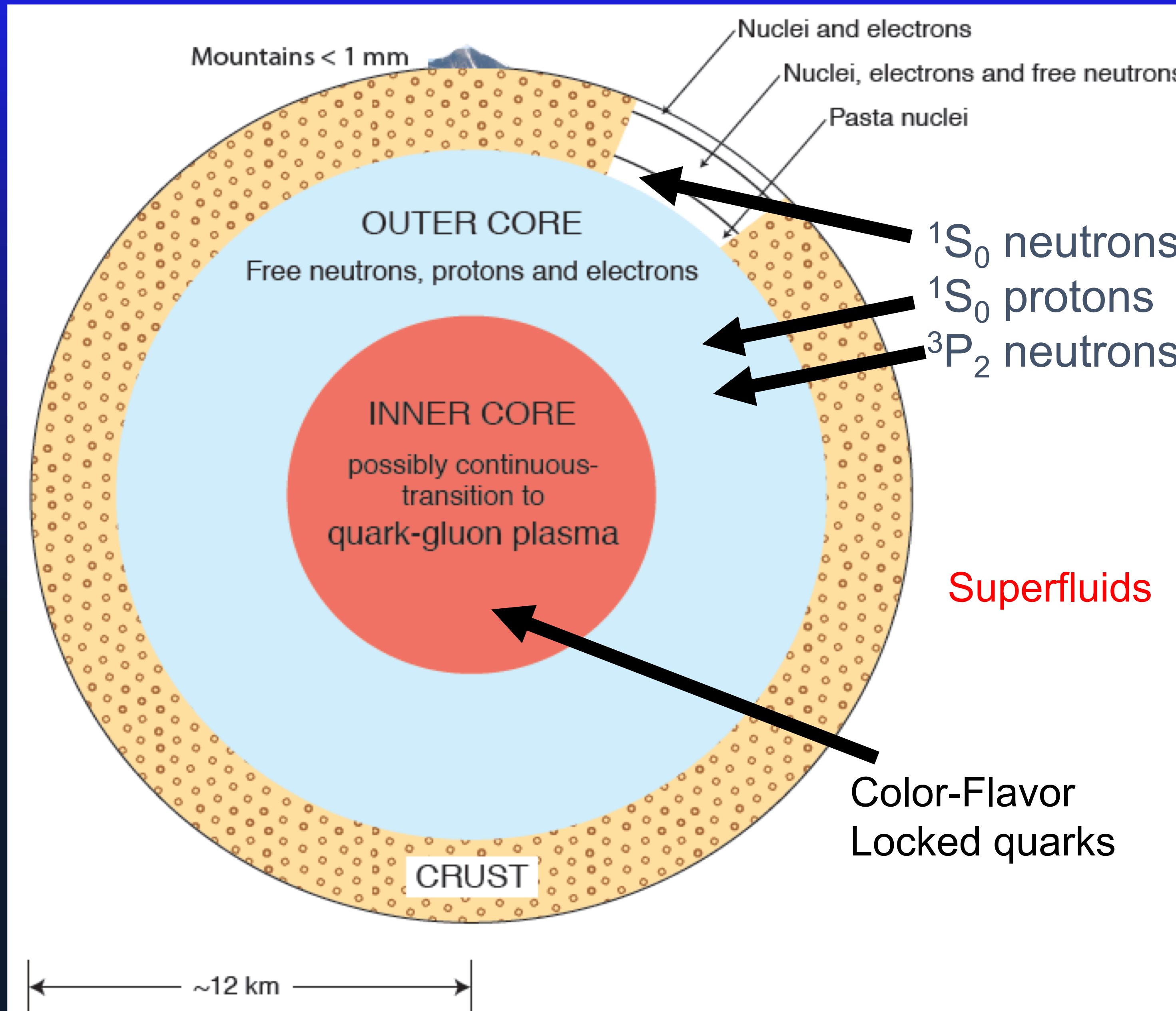
How can they be connected?

Neutron star interior

Mass ~ 1.4-2 M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

Surface gravity
~10^{14} that of Earth
Surface binding
~ 1/10 mc^2





Difficult to have continuity from BCS paired neutrons to CFL paired quarks

Neutron Pair $\langle nn \rangle \sim \langle udd\, udd \rangle$

In quark matter $\langle ud \rangle \langle ud \rangle \langle dd \rangle$

"Good" Diquark (→ $\langle ud \rangle$)

"Bad" Diquark (→ $\langle dd \rangle$)

"Bad" Diquark (favored in hadrons)

Color Anti-sym. (attractive) Flavor Sym.
Spin Sym. → p-wave pairing

$^3P_0$ or $^3P_2$

Even worse diquark (favored in matter)

Color Sym. (repulsive) Flavor Sym.
Spin Anti-sym. → s-wave pairing

orbital?

K. Fukushima

Summary

For 2 $n_0 < n_B <$ 7-8 n_0 mattter is intermediate between purely hadronic and purely quark

Quark model eqs. of state can be stiffer than previously thought, allowing for neutron star masses > 2 $M_\odot$

Much more to do:

Uncertainties in nuclear matter equation of state (APR, etc.)

Uncertainties in interpolating from nuclear matter to quark matter lead to errors in maximum neutron star masses and radii

Uncertainties in the vector coupling and pairing forces;

Going beyond the NJL model -- running g_v

Determine maximum mass neutron star! Black hole in GW171017?

Need to produce finite temperature equation of state ($\leq$ 100 MeV) for modelling neutron star -- neutron star (or black hole) mergers as sources of gravitational radiation.

Cooling and transport properties???