FAST Action WG3 meeting

Multi-Photon Time Resolution and Applications

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13th June 2018
Schwetzingen, Germany

ICASIPM– the International Conference on the Advancement of Silicon Photomultipliers

Motivation

There are many applications demanding for a photon-number-resolving detection of light pulses, some of them also require an extreme timing resolution at the <u>multi-photon</u> level (TOF PET, LIDAR, 4D calorimetry)

Why we are interested in SPTR?

We expect that good SPTR provides good timing resolution
One group of people wants to select the best detectors for their application
Another group of people wants to develop SiPMs most suitable for these applications

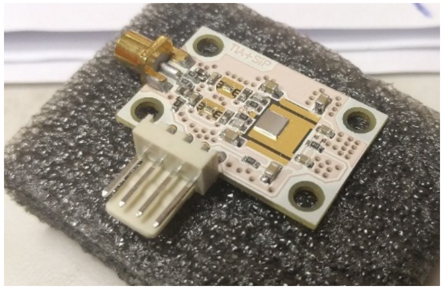
Goals of presentation:

1. **How to extract SPTR if it hardly measurable due significant electronic noise contribution**
2. **What is influence of SPTR and another parameters of SiPM and light pulse shape on multi-photon time resolution TR**

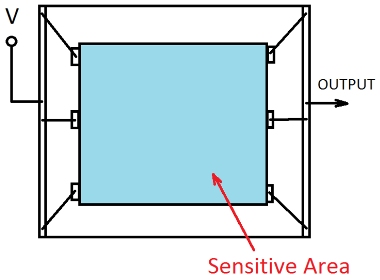
Timing measurements with KETEK SiPM+amplifiers assembly

Experimental setup:

- picosecond laser (405 nm, FWHM ≈ 40 ps)
- advanced timing optimized 3x3 mm^2 KETEK SiPM chip and specially designed (by S. Ageev) and produced monolithic trans-impedance amplifier(s) (BW 1.5GHz) on PCB assembly
- External KETEK evaluation kit amplifier
- thermal chamber with light protection T=-30° C
- digital oscilloscope LeCroy WaveRunner 620Zi (2GHz, 20GS/s)
- PMT-monitor for calibration light intensity into Npe

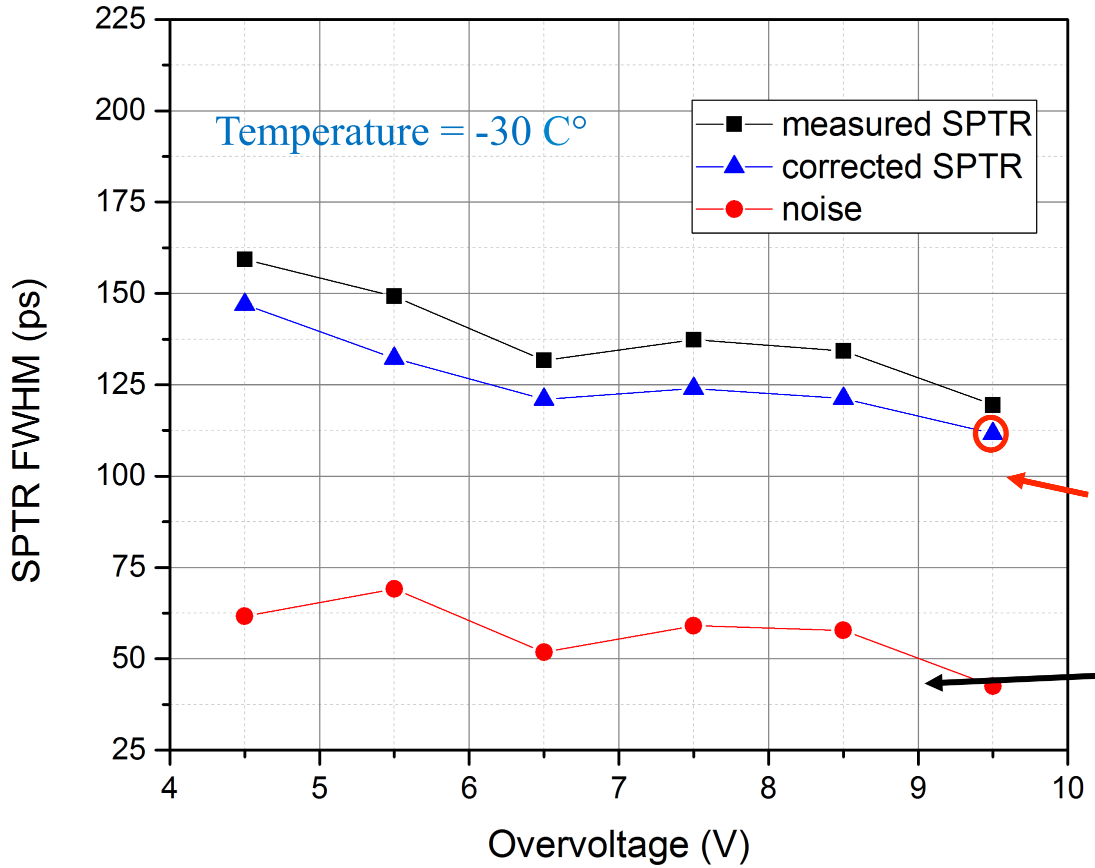


SiPM + Amplifiers PCB

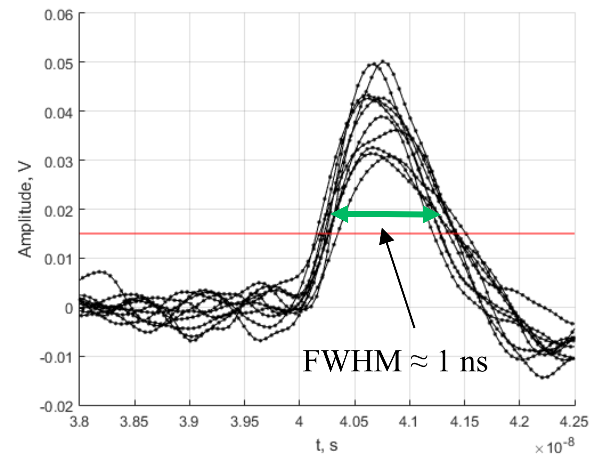


New timing optimized SiPM

SPTR measurements

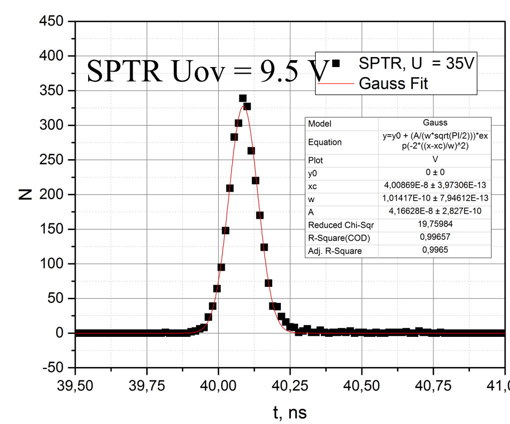


3x3 mm² SiPM,
SPTR = 112 ps



1_phe pulse shape, Uov = 4.5 V

$$\bar{\sigma}_{t,noise} = \frac{\bar{\sigma}_{A,noise}}{\frac{\overline{dU}}{dt}}$$

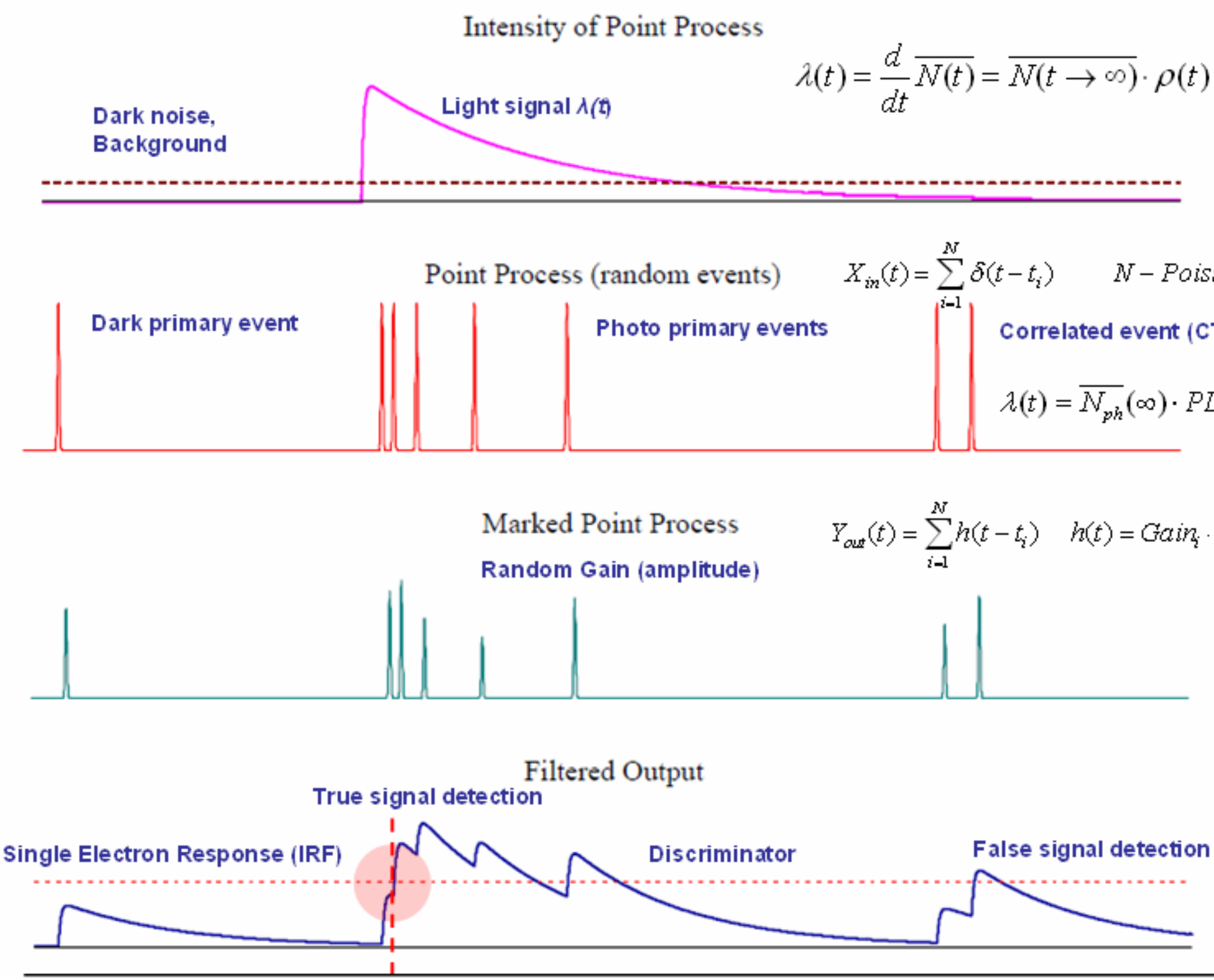


Multi-photon time resolution

Analytical model “Amplitude noise” for timing resolution (S.Vinogradov)

S.Vinogradov. Evaluation of performance of silicon photomultipliers in LIDAR application. Proceedings of the SPIE, Volume 10229, id. 102290L 10 pp. (2017)
S.Vinogradov. Approximations of coincidence time resolution models of scintillator detectors with leading edge discriminator. NIM A https://doi.org/10.1016/j.nima.2017.11.009

Filtered Marked Point Process, Campbell theorem



$$\lambda(t) = \frac{d}{dt}\overline{N(t)} = \overline{N(t \to \infty)} \cdot \rho(t)$$

$$X_{in}(t) = \sum_{i=1}^{N} \delta(t - t_i) \qquad N - Poissonian \qquad t_i - iidrv(i = 1 \ldots N)$$

$$\lambda(t) = \overline{N_{ph}}(\infty) \cdot PDE \cdot \left[\rho_{ph}(t) * \rho_{sptr}(t) * \rho_{sec}(t)\right]$$

$$Y_{out}(t) = \sum_{i=1}^{N} h(t - t_i) \qquad h(t) = Gain_i \cdot h_{ser}(t) \qquad Gain_i - iidrv(i = 1 \ldots N)$$

Campbell theorem :

$$E\left[A(t)\right] = \lambda(t) * h(t)$$

$$Var\left[A(t)\right] = \lambda(t) * h^2(t)$$

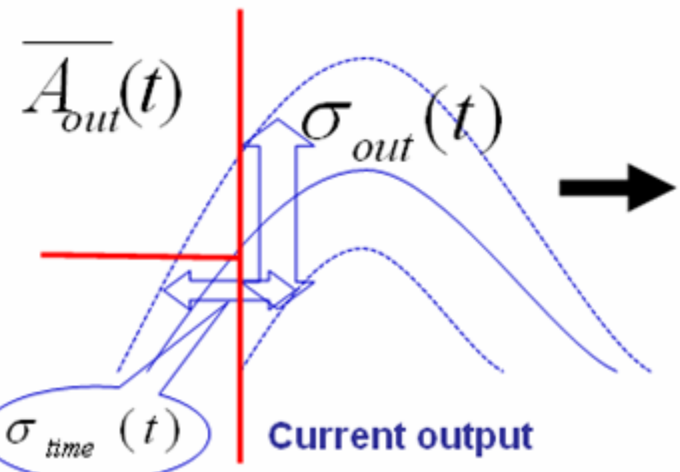
Time Resolution and Photon Number Resolution

Time Resolution combines

Photon number resolution and ***filtered point process convolutions***

- distribution of photon arrival times $\rho_{ph}(t)$
- distribution of single photon detection times $\rho_{sptr}(t)$
- distribution of CT & AP event times $\rho_{sec}(t)$
- IRF = SER pulse shape $h_{ser}(t)$

$$TR = \sigma_{time} = \left. \frac{\sigma_{out}(A(t))}{\frac{d}{dt}\overline{A(t)}} \right|_{\overline{A(t = T_{discr})} = Discr}$$



$$\sigma_{time} = \left. \frac{\sqrt{ENF_{tot} \cdot \left[\rho_{\det} * h_{ser}^2\right](t) + \frac{V_{noise}^2}{V_{ser}^2}}}{\sqrt{N_{ph}} \cdot \frac{d\left[\rho_{\det} * h_{ser}\right](t)}{dt}} \right|_{t = t_{Discr}}$$

$$\rho_{\det}(t) = [\rho_{ph} * \rho_{sptr} * \rho_{sec}](t)$$

Convolution: slower the slowest function $\rho_i(t)$ @ t_{Discr}

The narrower *i-th* process distribution $\rho_i(t)$ – the better

Analytical model (short laser light & no noise)

Gaussian shape of laser pulse and SPTR allows to get TR dependence on SPTR:

- in case if SER is a Heaviside step response it has an analytical form:

$$\sigma_t(N_{pe}) = \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot \sqrt{\pi \cdot e \cdot \left[1 - erf\left(\frac{1}{\sqrt{2}}\right)\right]} \approx \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot 1.646$$

- in case if SER is a bi-exponential with rise Tr and fall Tf times it has an analytical form:

For typical SiPM pulses (Tr = 0.5..1 ns, Tf = 1…100 ns) dependence of CTR on Tr and Tf is rather weak, so it can be approximated as:

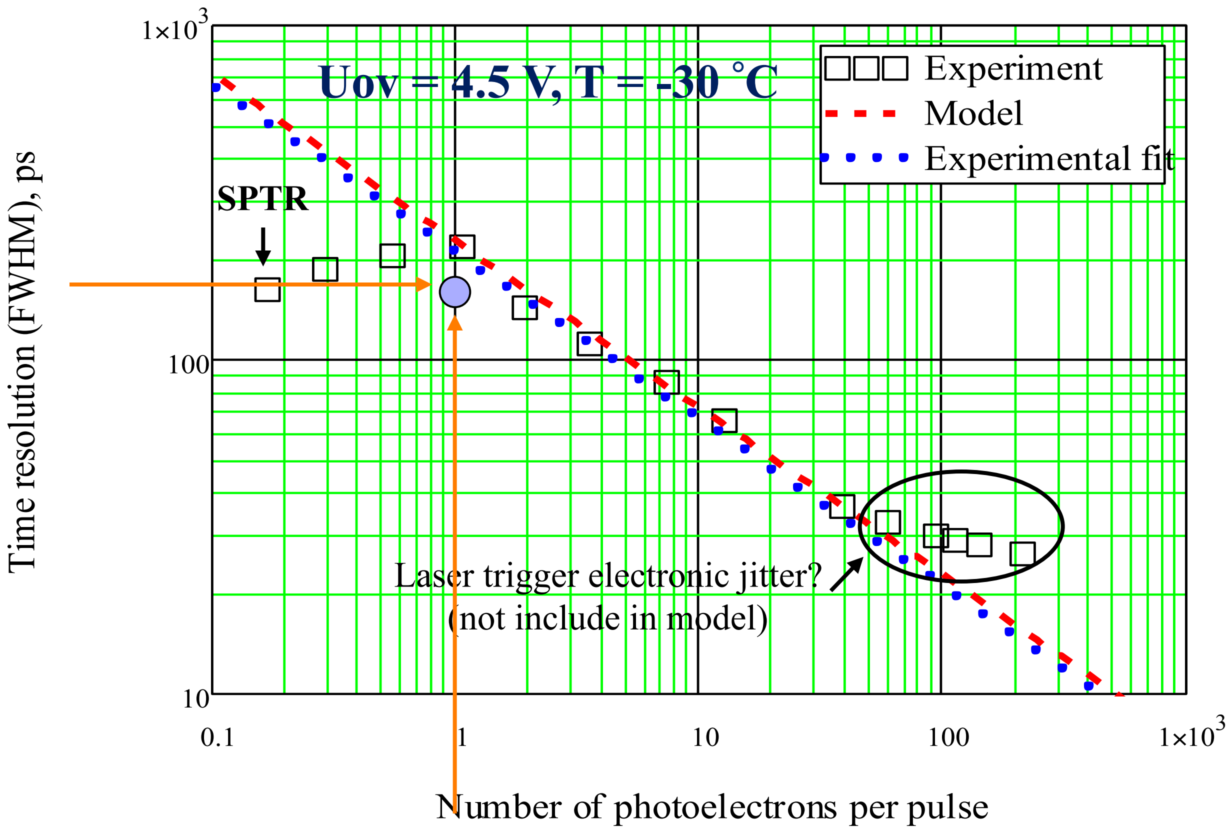
$$\sigma_t(N_{pe}) \approx \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot (1.4 \div 1.6)$$

TR vs Light intensity for short laser pulse

(T = -30°C, Uov = 4.5V, SPTR (true SPTR without noise contribution)= 147 ps

Pct=0.13, ENFct=1.16, no Dark rate)

Light source – laser, FWHM = 40 ps



$$FWHM_t(N_{pe}) \approx \frac{SPTR}{\sqrt{N_{pe}}} \cdot 1.5$$

Analytical model:
Tr = 0.5 ns, Tf = 1 ns

$$FWHM_t(N_{pe}) \approx \frac{210\,ps}{\sqrt{N_{pe}}}$$

Experimental Fit

$$\frac{210}{1{,}5} = 140 \approx SPTR_{corr}$$

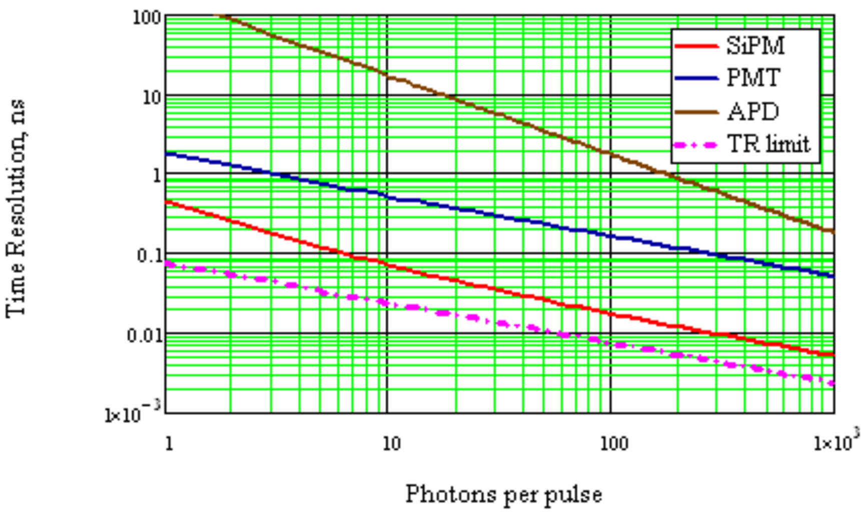
Extracted SPTR

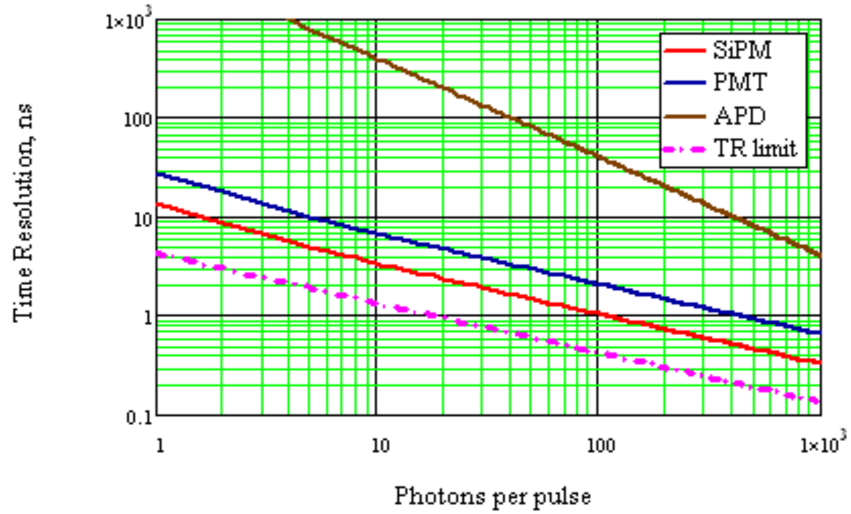
TR of SiPM, PMT, APD vs photons/pulse

LIDAR

Gaussian laser pulse shape

Laser pulse 40 ps FWHM   Nph=100   Laser pulse 10 ns FWHM





Analytical approximation of time resolution model based on filtered marked point process model for Gaussian laser pulse shape, Gaussian SPTR and bi-exponential single electron response shape SER with τ_{rise} and τ_{dec}

$$TR = \frac{\sigma_{out}(A(t))}{\frac{d}{dt}\overline{A(t)}} \approx \sqrt{\frac{2\sigma_{tts}^2 \cdot ENF_{tot}}{N_{ph} \cdot PDE} + \frac{4\tau_{rise}^2}{(N_{ph} \cdot PDE)^2}\left(\frac{DCR \cdot \tau_{dec} \cdot ENF_{tot}}{2} + \frac{V_{noise}^2}{V_{ser}^2}\right)}$$

$$\sigma_{tts}^2 = \sigma_{laser}^2 + \sigma_{sptr}^2 \qquad ENF_{tot} = ENF_{gain} \cdot ENF_{dcr} \cdot ENF_{corr} \cdot ENF_{nl}$$

TOF PET, scintillator readout

we are interested to estimate a coincidence time resolution CTR on the basis of known photodetector and scintillator parameters.

- Choosing of the best photodetector
- Choosing of the best scintillator
- Choosing of the best photodetector and scintillator

* *Photodetector – analogue SiPM*

SiPM

- single photon time resolution SPTR
- pulse shape SER, t_{rise}, t_{dec}
- PDE
- crosstalk
- Dark rate
- Electronic noise

LIGHT

- T_r rise time
- T_d decay time
- photon numbers

Common understanding of the CTR dependence for scintillator light

$$\sigma_t \sim \frac{1}{\sqrt{N_{pe}}}$$

$$\sigma_t \Big|_{\substack{\tau_d \gg \tau_r \\ \tau_d \gg \sigma_{sptr}}} \sim \sqrt{\tau_d}$$

CTR depends on

Number of photons

slightly on τr and σ_{sptr}

Too small for analysis

Monte-Carlo simulations of the Time Resolution

the Time Resolution (TR) of SiPMs is extensively studied in experiments and Monte-Carlo simulations,

Analytical extraction of parametric dependences from Monte-Carlo simulations

$$TR = \sqrt{\frac{\tau_d}{N_{pe}}} \cdot B\left(\tau_r, \tau_{otts}, \sigma_{sptr}, \frac{\tau_d}{N_{pe}}\right)$$

$$B = \sqrt{\begin{array}{l} 5.545\frac{\tau_d}{N_{pe}} + 2.424 \cdot (\tau_r + \tau_{otts}) + 2.291 \cdot \sigma_{sptr} + 4.938 \cdot \tau_r \cdot \tau_{otts} + 3.332 \cdot \sigma_{sptr}^2 + \\ 8.969 \cdot \sigma_{sptr}^2 \cdot \sqrt{\frac{\tau_d}{N_{pe}}} + 9.821 \cdot (\tau_r^2 + \tau_{otts}^2) \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - 0.6637 \cdot (\tau_r + \tau_{otts}) \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - \\ 3.305 \cdot (\tau_r + \tau_{otts}) \cdot \sigma_{sptr} - 6.149 \cdot \sigma_{sptr} \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - 0.3232 \cdot (\tau_r^2 + \tau_{otts}^2) - 3.530 \cdot \sigma_{sptr}^3 \\ -5.361 \cdot \tau_r \cdot \tau_{otts} \cdot \sigma_{sptr} - 9.287 \cdot \tau_r \cdot \tau_{otts} \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - 5.814 \cdot (\tau_r^2 + \tau_{otts}^2) \cdot \sigma_{sptr} \end{array}}$$

But after obtaining of MC-simulation results is quite difficult to analyse them…

S.E. Derenzo, W.-S. Choong, W.W. Moses, Fundamental limits of timing resolution for scintillation detectors, Phys. Med. Biol. 59 (2014) 3261–3286. http://dx.doi.org/10.1088/0031-9155/59/13/3261

TOF PET bi-expanentional light pulse
Analytical Approximation of model for CTR :)

signal:

$$\sigma_{t\_sig}(\sigma_s, \tau_d, \tau_r) = \sqrt{\frac{ENF_{tot}}{N_{pe}} \cdot \left[\frac{\pi}{2}\tau_d\tau_r + \frac{\sqrt{2\pi}(3\pi-4)}{12}(\tau_d+\tau_r)\cdot\sigma_s\right]}.$$

where $\sigma_s = \sqrt{\sigma_{otts}^2 + \sigma_{sptr}^2}.$

tr – rise time, td – decay time for scint

If Tr<<Td

$$\sigma_{t\_sig}(\sigma_s, \tau_d, \tau_r) = \sqrt{\frac{ENF_{tot}}{N_{pe}} \cdot \tau_d\left[\frac{\pi}{2}\tau_r + \frac{\sqrt{2\pi}(3\pi-4)}{12}\sigma_s\right]}$$

Scint rise time 1.57 **1.13 SPTR&OTTS**

Almost equal contributions!!!

noise:

$$\sigma_{t\_noise}(\sigma_s, \tau_d, \tau_r, \tau_{ser}) = \sqrt{2\pi \cdot \left(ENF_{tot}\cdot DCR\cdot\tau_{ser} + \frac{\sigma_n^2}{V_{ser}^2}\right)\frac{\tau_d^2}{N_{pe}^2}\left(\sqrt{\frac{\pi}{2}\frac{\tau_r}{\sigma_s}} + \frac{\tau_r^2}{\sigma_s^2}\right)}$$

full (combined):

$$CTR = \sqrt{2}\cdot 2\sqrt{2\ln(2)}\cdot\sigma_t \approx 3.33\sqrt{\sigma_{t\_sig}^2 + \sigma_{t\_noise}^2}.$$

MEPHI MPTR measurements

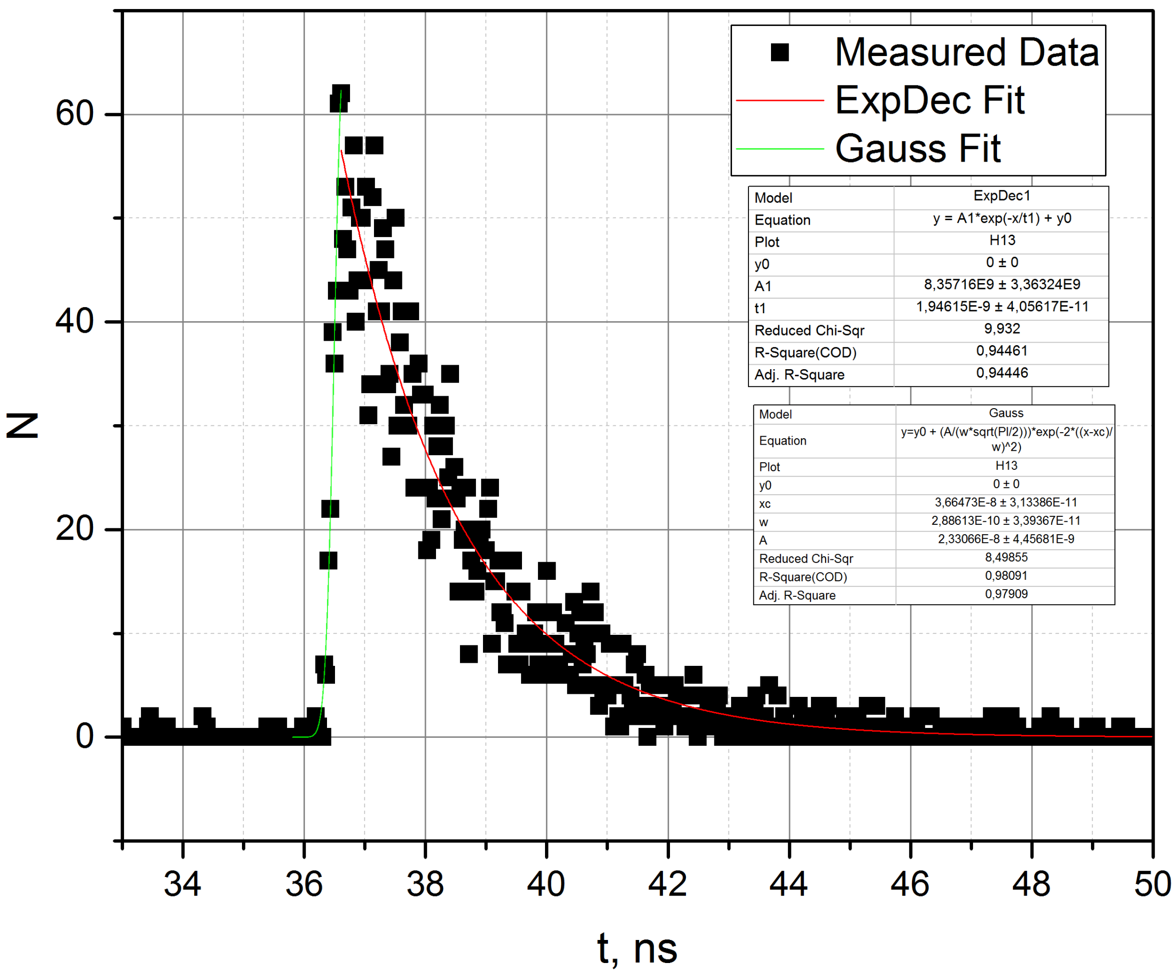
(T = -30°C, Uov = 4.5V, SPTR = 147 ps, ENFct=1.16):

Light source –

laser + WLS-fiber,

Tr ≈ 80 ps, Td ≈ 1.8ns,

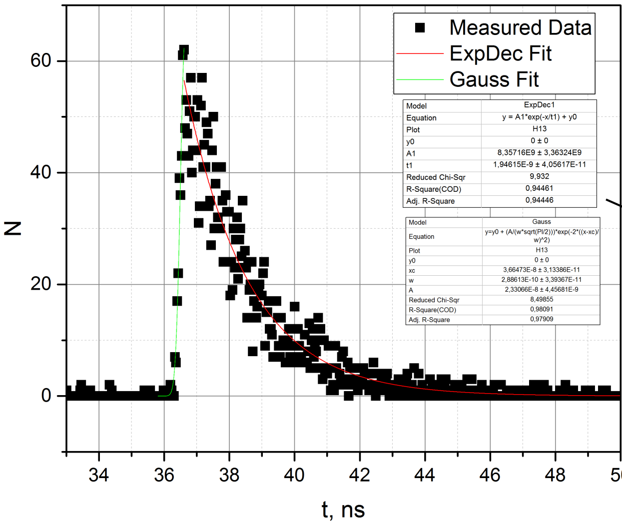
scintillator-simulated experiment

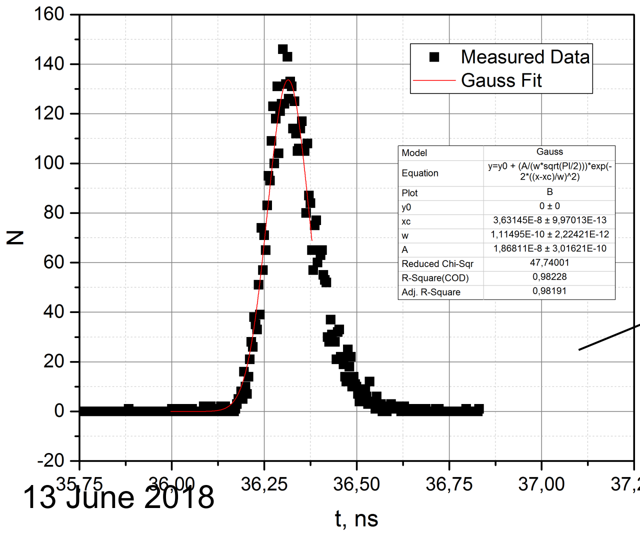


Experiment MPTR with laser+WLS-fiber

MPTR histograms (Tr ≈ 80ps, Td ≈ 1.9ns) :

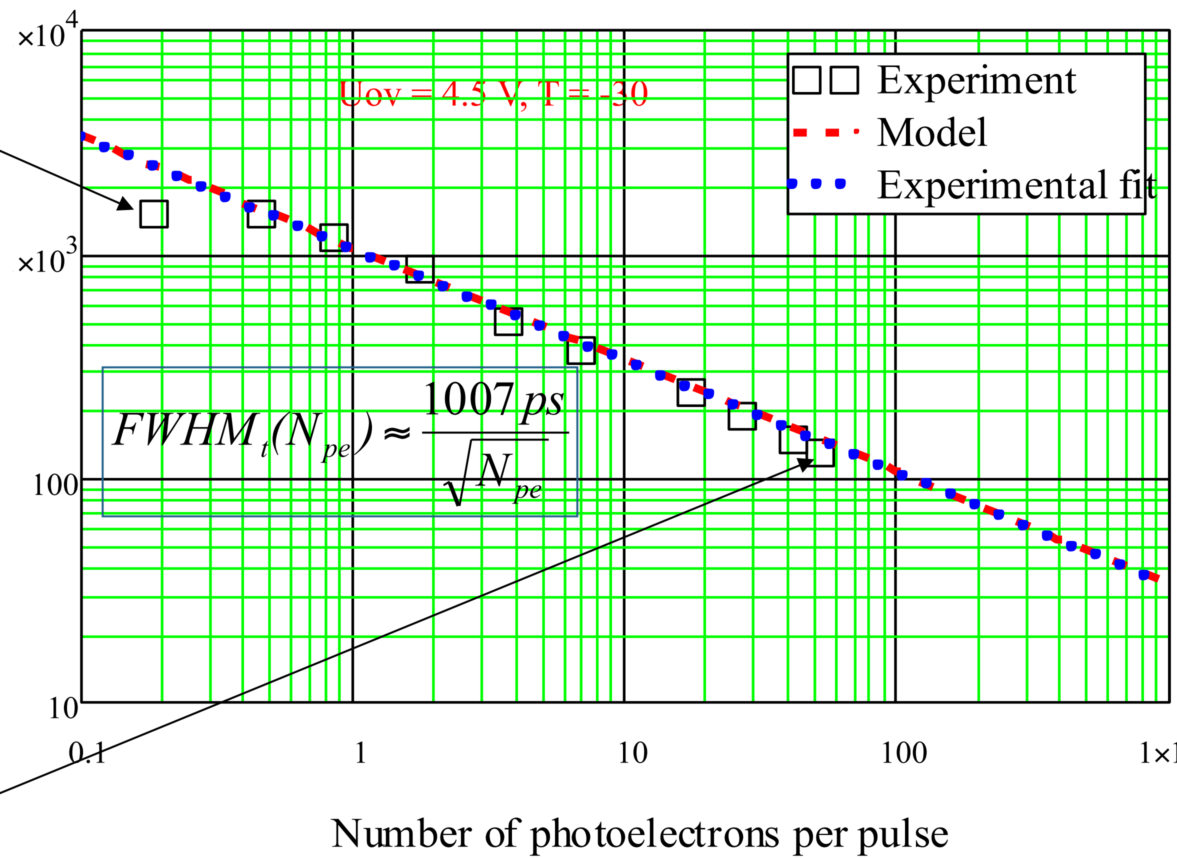
top – Npe ≈ 0.2 bottom – Npe ≈ 52.3



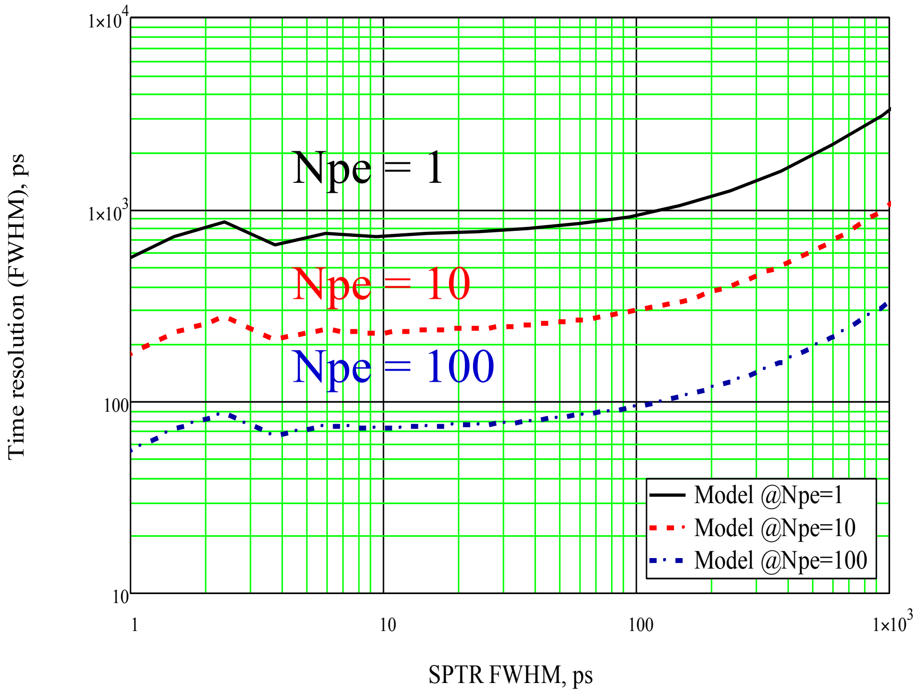


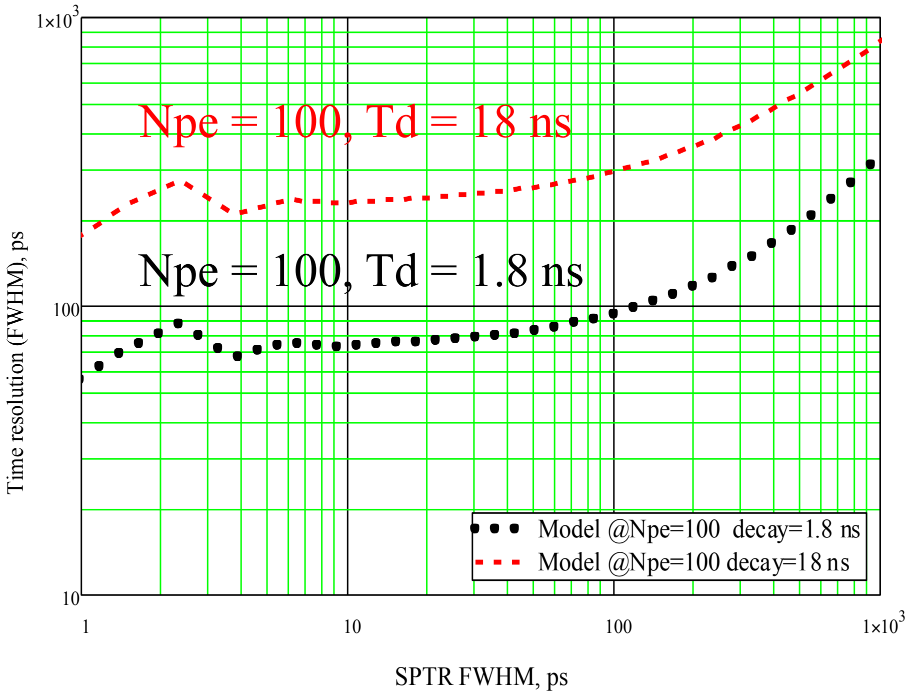
Experimental Fit

MPTR FWHM, ps (CTR with scintillator simulation) vs Light intensity



Analytical model calculations: MPTR as function of SPTR for scintillator-simulated pulse





MPTR has regions with different dependence on SPTR

Kind of plateau for smaller SPTR value is connected with WLS rise time (80 ps)

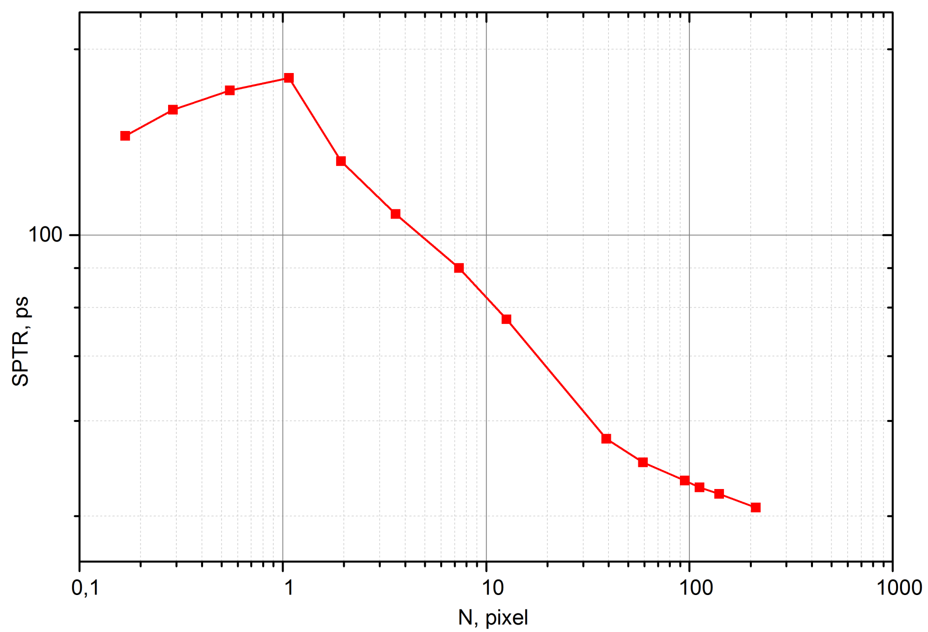
Summary

- The multi-photon timing measurements with different pulse shapes were carried out to show how coincidence timing resolution depends on SPTR.
- Analytical model of "Amplitude noise" has a good agreement with experiment results for light intensity Npe > 1.
- MPTR for short light pulse may allow to extract true SPTRdetector (not affected by noise) – should be checked
- Analytical model shows how MPTR depends on SPTR for long scintillator-like pulses, but it should be checked with more experimental data.

Supported by Russian grants #3.2989.2017/4.6 and 3.8484.2017/9.10
And FAST COST (European Cooperation in Science and Technology) action TD1401.

BACKUP

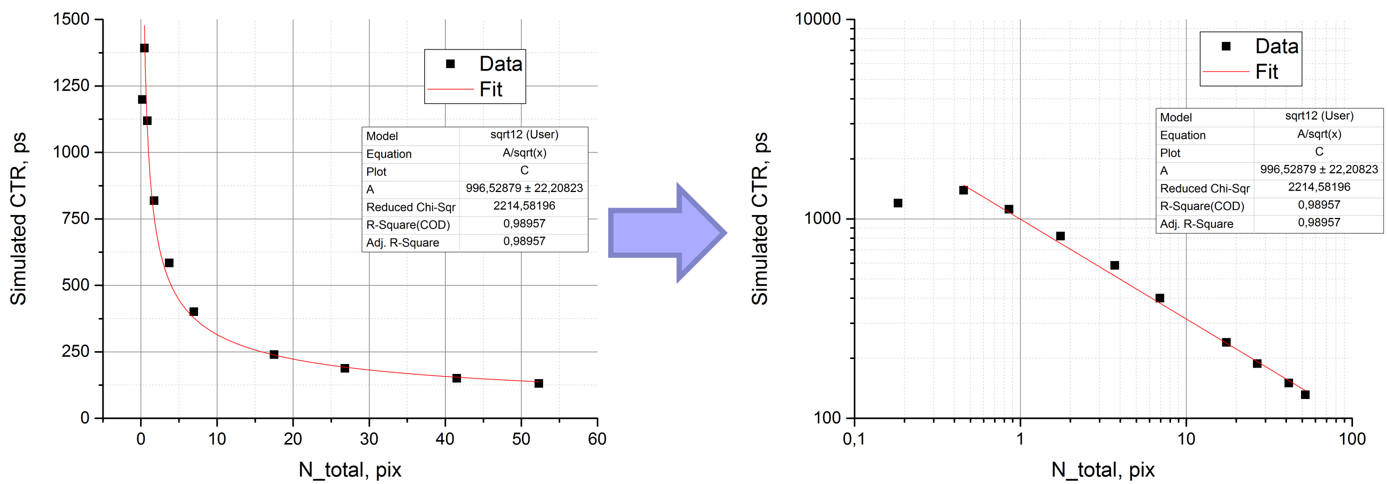
Timing measurements with new PCB – multi-photon TR results



Timing resolution vs Light intensity (in fired pixels), Uov = 4.5 V

N, pixel	SPTR, ps
0.168	163
0.288	185.5
0.549	204
1.072	217
1.932	144
3.585	111
7.353	85
12.6	66
39	36.6
59	32.6
95	29,8
112	28,8
140	27,9
212	26,1

Timing measurements with new PCB – CTR simulation experiment – results



N, pixel	CTR, ps
0.18	1200
0.46	1501
0.86	1202
1.75	850
3.71	499
6.94	376
17.5	240
26.8	188
41.5	151
52.3	131

Simulated coincidence timing resolution vs Light intensity (in fired pixels), Uov = 4.5 V

Analytical model: CTR as function of SPTR and other parameters

<u>Modern analytical approaches:</u>

- Monte Carlo simulations,
- Detection event statistics,
- Order statistics of photoelectron detection time,
- Cramer-Rao lower bound estimation.

Light, N photons	SPTR, σ	SER, ideal	CTR, min	Remark
←T→	$\delta(t)$		$\frac{T}{N}$	= Erlang distribution, No SPTR
$\delta(t)$	σ		$\frac{\sigma}{\sqrt{N}}$	Light distribution = SPTR
←T→	σ		$\sqrt{\frac{T\sigma}{N}}$	Mixed roles of light T & SPTR
Tr Td	σ		$\sqrt{\frac{Td}{N}\cdot\begin{Bmatrix}\sim Tr \\ \sim \sigma\end{Bmatrix}}$	Not clear analytical function

Timing resolution - analytical model (S.Vinogradov)

$$\sigma_t(N_{pe}) = \left.\frac{\sqrt{Var[V_{out}(t)]}}{\frac{d\overline{V_{out}}(t)}{dt}}\right|_{V_{out}(t_{Discrim}) = Discrim} = \frac{\sqrt{N_{pe} \cdot ENF_{SiPM} \cdot \left[\rho_{ph} * \rho_{sptr} * h_{ser}^2\right](t) + \frac{V_{noise}^2}{\overline{V}_{ser}^2}}}{N_{pe} \cdot \frac{d\left[\rho_{ph} * \rho_{sptr} * h_{ser}\right](t)}{dt}}$$

Filtered marked point process

Analytical model “Amplitude noise” for timing resolution

N_{pe} - Number of photoelectrons

ENF_{SiPM} - Excess noise factor of SiPM (include DCR, XT, AP)

ρ_{ph} - Probability density function of light

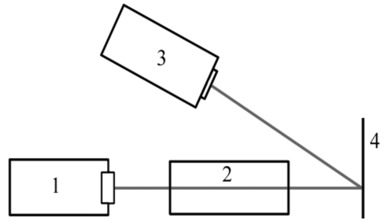
ρ_{sptr} - Probability density function of SiPM SPTR

h_{ser} - Single-electron response function (SER)

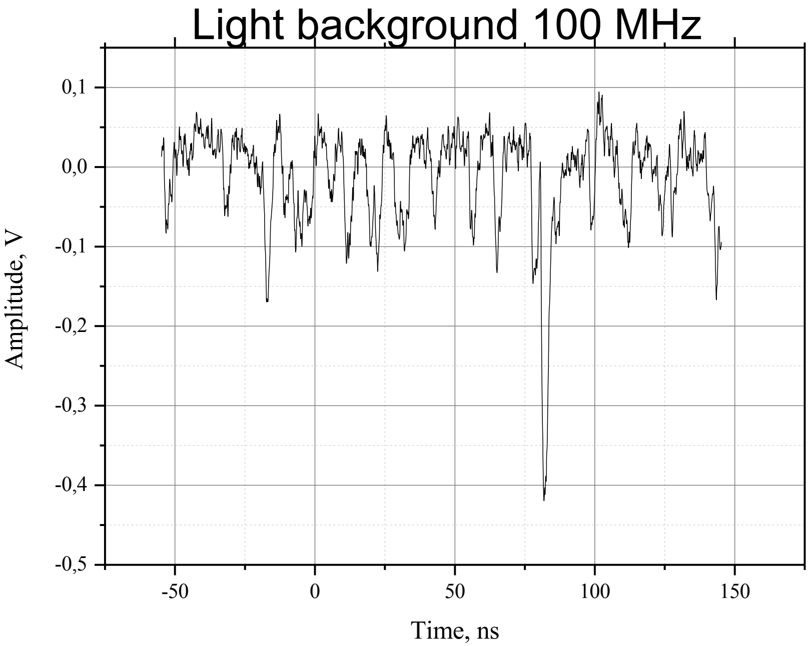
Constant threshold at the first photon- no CT, no AP, no dark rate

ENF≈1

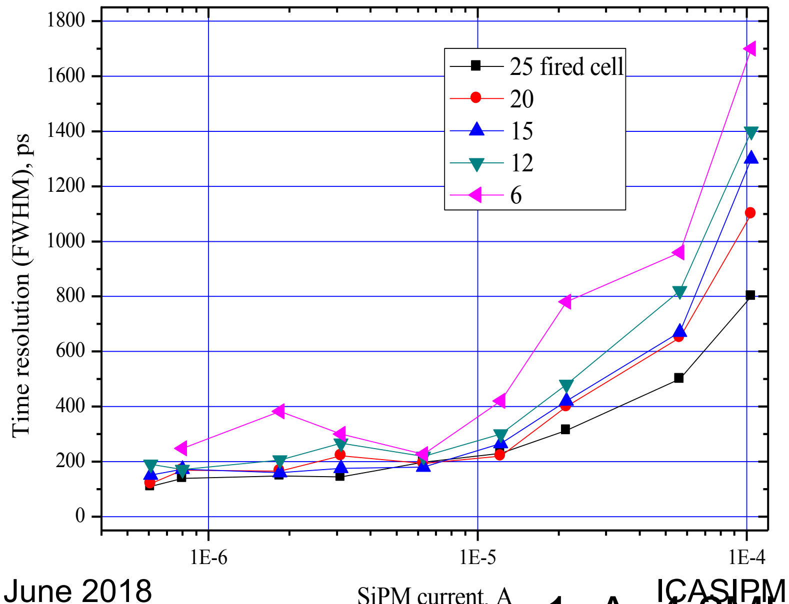
Experimental data with lidar prototype laser 40 ps FWHM 405nm



Scanning lidar



A M Antonova and V A Kaplin 2018 J. Phys.: Conf. Ser. 945 012012
SiPM timing characteristics under conditions of a large background for lidars



$1\mu A = 4.2 MHz$

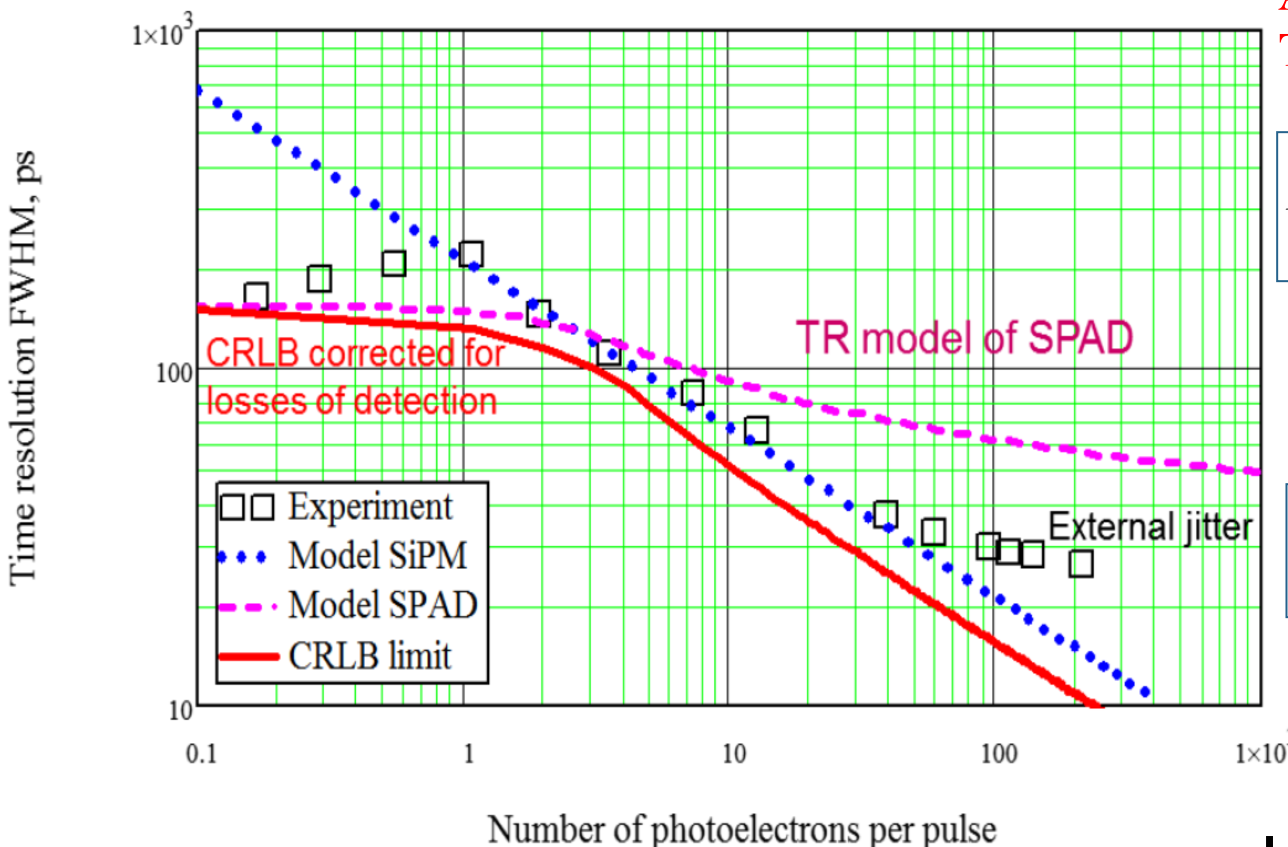
Experiment

TR dependence (T = -30°C, Uov = 4.5V, SPTR (true SPTR without noise contribution)= 147 ps, ENFct= $\frac{1}{1+\ln(1-Pct)}$

Pct=0.13, ENFct=1.16, no Dark rate)
Light source – laser, FWHM = 40 ps

$$FWHM_t(N_{pe}) \approx \frac{SPTR}{\sqrt{N_{pe}}} \cdot 1.5$$

Analytical model:
Tr = 0.5 ns, Tf = 1 ns



$$FWHM_t(N_{pe}) \approx \frac{210\,ps}{\sqrt{N_{pe}}}$$

Experimental Fit

$$\frac{210}{1{,}5} = 140 \approx SPTR_{corr}$$

LED threshold