Multi-Photon Time Resolution and Applications

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Motivation

There are many applications demanding for a photon-number-resolving detection of light pulses, some of them also require an extreme timing resolution at the multi-photon level (TOF PET, LIDAR, 4D calorimetry)

Why we are interested in SPTR?

We expect that good SPTR provides good timing resolution
One group of people wants to select the best detectors for their application
Another group of people wants to develop SiPMs most suitable for these applications

Goals of presentation:
1. How to extract SPTR if it hardly measurable due significant electronic noise contribution
2. What is influence of SPTR and another parameters of SiPM and light pulse shape on multi-photon time resolution TR
Timing measurements with KETEK SiPM+amplifiers assembly

Experimental setup:

- picosecond laser (405 nm, FWHM ≈ 40 ps)
- advanced timing optimized 3x3 mm² KETEK SiPM chip and specially designed (by S. Ageev) and produced monolithic trans-impedance amplifier(s) (BW 1.5GHz) on PCB assembly
- External KETEK evaluation kit amplifier
- thermal chamber with light protection T=-30° C
- digital oscilloscope LeCroy WaveRunner 620Zi (2GHz, 20GS/s)
- PMT-monitor for calibration light intensity into Npe
SPTR measurements

Temperature = -30 °C

3x3 mm² SiPM, SPTR = 112 ps

1_phe pulse shape, Uov = 4.5 V

FWHM ≈ 1 ns

SPTR Uov = 9.5 V, U = 35 V

\[ \bar{\sigma}_{t,\text{noise}} = \frac{\bar{\sigma}_{A,\text{noise}}}{\frac{dU}{dt}} \]

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Multi-photon time resolution

Analytical model “Amplitude noise” for timing resolution (S.Vinogradov)

S.Vinogradov. Approximations of coincidence time resolution models of scintillator detectors with leading edge discriminator. NIM A https://doi.org/10.1016/j.nima.2017.11.009
Filtered Marked Point Process, Campbell theorem

Intensity of Point Process
\[ \lambda(t) = \frac{d}{dt} \overline{N(t)} = \overline{N(t \rightarrow \infty)} \cdot \rho(t) \]

Point Process (random events)
\[ X_m(t) = \sum_{i=1}^{N} \delta(t - t_i) \quad N - Poissonian \quad t_i - iid \nu(i = 1 \ldots N) \]

Correlated event (CT, AP)
\[ \lambda(t) = \overline{N_{ph}(\infty)} \cdot PDE \cdot [\rho_{ph}(t) \ast \rho_{sp}(t) \ast \rho_{sec}(t)] \]

Marked Point Process
\[ Y_{out}(t) = \sum_{i=1}^{N} h(t - t_i) \quad h(t) = Gain_i \cdot h_{ref}(t) \quad Gain_i - iid \nu(i = 1 \ldots N) \]

Campbell theorem:
\[ E[A(t)] = \lambda(t) \ast h(t) \quad Var[A(t)] = \lambda(t) \ast h^2(t) \]

Single Electron Response (IRF)
Time Resolution combines photon number resolution and filtered point process convolutions:
- distribution of photon arrival times $\rho_{ph}(t)$
- distribution of single photon detection times $\rho_{sptr}(t)$
- distribution of CT & AP event times $\rho_{sec}(t)$
- IRF = SER pulse shape $h_{ser}(t)$

$$\sigma_{time} = \left| \frac{d}{dt} \frac{A(t)}{dA(t) \bigg| A(t = T_{discr}) = Discr} \right|$$

$$\sigma_{time} = \sqrt{\frac{\frac{1}{ENF_{tot}} \left[ \rho_{det} * h_{ser}^2 \right] (t) + \frac{V_{noise}^2}{V_{ser}^2}} {\frac{\sqrt{N_{ph}}}{d\left[ \rho_{det} * h_{ser} \right] (t)}}}$$

$$\rho_{det}(t) = [\rho_{ph} * \rho_{sptr} * \rho_{sec}](t)$$

Convolution: slower the slowest function $\rho_i(t) \oplus t_{Discr}$
The narrower $i$-th process distribution $\rho_i(t)$ – the better
Analytical model
(Short laser light & no noise)

Gaussian shape of laser pulse and SPTR allows to get TR dependence on SPTR:

- In case if SER is a Heaviside step response it has an analytical form:

\[
\sigma_t(N_{pe}) = \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot \sqrt{\pi \cdot e \cdot \left[1 - \text{erf}\left(\frac{1}{\sqrt{2}}\right)\right]} \approx \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot 1.646
\]

- In case if SER is a bi-exponential with rise \(T_r\) and fall \(T_f\) times it has an analytical form:

For typical SiPM pulses (\(T_r = 0.5..1\) ns, \(T_f = 1…100\) ns) dependence of CTR on \(T_r\) and \(T_f\) is rather weak, so it can be approximated as:

\[
\sigma_t(N_{pe}) \approx \frac{\sigma_{sptr}}{\sqrt{N_{pe}}} \cdot (1.4 \div 1.6)
\]
TR vs Light intensity for short laser pulse

(T = -30°C, Uov = 4.5V, SPTR (true SPTR without noise contribution)= 147 ps
Pct=0.13, ENFct=1.16, no Dark rate)

Light source – laser, FWHM = 40 ps

Analytical model:
Tr = 0.5 ns, Tf = 1 ns

Experimental Fit

Extracted SPTR

\[
FWMH_t(N_{pe}) \approx \frac{SPTR}{\sqrt{N_{pe}}} \times 1.5
\]

\[
FWMH_t(N_{pe}) \approx \frac{210 \text{ ps}}{\sqrt{N_{pe}}}
\]

\[
\frac{210}{1.5} = 140 \approx SPTR_{corr}
\]
Analytical approximation of time resolution model based on filtered marked point process model for Gaussian laser pulse shape, Gaussian SPTR and bi-exponential single electron response shape SER with $\tau_{\text{rise}}$ and $\tau_{\text{dec}}$

\[ TR = \frac{\sigma_{\text{out}}(A(t))}{\frac{d}{dt}A(t)} \approx \sqrt{\frac{2\sigma_{\text{its}}^2 \cdot ENF_{\text{tot}}}{N_{ph} \cdot PDE} + \frac{4\tau_{\text{rise}}^2}{(N_{ph} \cdot PDE)^2} \left( \frac{DCR \cdot ENF_{\text{tot}} \cdot V_{\text{noise}}^2}{2} + \frac{V_{\text{ser}}^2}{ENF_{\text{tot}}} \right)} \]

\[ \sigma_{\text{its}}^2 = \sigma_{\text{laser}}^2 + \sigma_{\text{sptr}}^2 \]

\[ ENF_{\text{tot}} = ENF_{\text{gain}} \cdot ENF_{\text{dcr}} \cdot ENF_{\text{corr}} \cdot ENF_{\text{nl}} \]
we are interested to estimate a coincidence time resolution CTR on the basis of known photodetector and scintillator parameters.

• Choosing of the best photodetector
• Choosing of the best scintillator
• Choosing of the best photodetector and scintillator
  *Photodetector – analogue SiPM

SiPM
• single photon time resolution SPTR
• pulse shape SER, $t_{rise}$, $t_{dec}$
• PDE
• crosstalk
• Dark rate
• Electronic noise

LIGHT
• $T_r$ rise time
• $T_d$ decay time
• photon numbers
Common understanding of the CTR dependence for scintillator light

\[ \sigma_t \sim \frac{1}{\sqrt{N_{pe}}} \]

\[ \sigma_t \left| \begin{array}{c} \tau_d \gg \tau_r \\ \tau_d \gg \sigma_{sptr} \end{array} \right\} \sim \sqrt{\tau_d} \]

CTR depends on
Number of photons
slightly on \( \tau_r \) and \( \sigma_{sptr} \)

Too small for analysis
Monte-Carlo simulations of the Time Resolution

The Time Resolution (TR) of SiPMs is extensively studied in experiments and Monte-Carlo simulations,

Analytical extraction of parametric dependences from Monte-Carlo simulations

\[ TR = \sqrt{\frac{\tau_d}{N_{pe}}} \cdot B \left( \tau_r, \tau_{otts}, \sigma_{s.ptr}, \frac{\tau_d}{N_{pe}} \right) \]

\[ B = \sqrt{\frac{5.545 \tau_d}{N_{pe}}} + 2.424 \cdot (\tau_r + \tau_{otts}) + 2.291 \cdot \sigma_{s.ptr} + 4.938 \cdot \tau_r \cdot \tau_{otts} + 3.332 \cdot \sigma_{s.ptr}^2 + 8.969 \cdot \sigma_{s.ptr}^2 \cdot \sqrt{\frac{\tau_d}{N_{pe}}} + 9.821 \cdot (\tau_r^2 + \tau_{otts}^2) \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - 0.6637 \cdot (\tau_r + \tau_{otts}) \cdot \sqrt{\frac{\tau_d}{N_{pe}}} - 0.3232 \cdot (\tau_r^2 + \tau_{otts}^2) - 3.530 \cdot \sigma_{s.ptr}^3 \]

But after obtaining of MC-simulation results is quite difficult to analyse them...

TOF PET bi-exponential light pulse
Analytical Approximation of model for CTR : )

signal:

$$\sigma_{t_{\text{sig}}}(\sigma_s, \tau_d, \tau_r) = \sqrt{\frac{E NF_{\text{tot}}}{N_{pe}} \cdot \left[ \frac{\pi}{2} \tau_d \tau_r + \frac{\sqrt{2\pi(3\pi - 4)}}{12} (\tau_d + \tau_r) \cdot \sigma_s \right]}.$$  

If $Tr << Td$

$$\sigma_{t_{\text{sig}}}(\sigma_s, \tau_d, \tau_r) = \sqrt{\frac{E NF_{\text{tot}}}{N_{pe}} \cdot \tau_d \left[ \frac{\pi}{2} \tau_r + \frac{\sqrt{2\pi(3\pi - 4)}}{12} \sigma_s \right]}.$$  

where $\sigma_s = \sqrt{\sigma_{ott}^2 + \sigma_{sptr}^2}$.

tr – rise time, td – decay time for scint

noise:

$$\sigma_{t_{\text{noise}}}(\sigma_s, \tau_d, \tau_r, \tau_{ser}) = \sqrt{2\pi \cdot \left( E NF_{\text{tot}} \cdot DCR \cdot \tau_{ser} + \frac{\sigma_n^2}{V_{ser}^2} \right) \frac{\tau_d^2}{N_{pe}^2} \left( \sqrt{\frac{\pi \tau_r}{2 \sigma_s}} + \frac{\tau_r^2}{\sigma_s^2} \right)}.$$  

full (combined):

$$CTR = \sqrt{2.2 \sqrt{2 \ln(2)} \cdot \sigma_t} \approx 3.33 \sqrt{\sigma_{t_{\text{sig}}}^2 + \sigma_{t_{\text{noise}}}^2}.$$  

Scint rise time 1.57 1.13 SPTR&OTTS
Almost equal contributions!!!

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MEPHI MPTR measurements

(T = -30°C, Uov = 4.5V, SPTR = 147 ps, ENFct=1.16):

Light source – laser + WLS-fiber,
Tr ≈ 80 ps, Td ≈ 1.8ns, scintillator-simulated experiment

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Experiment MPTR with laser+WLS-fiber

MPTR histograms (Tr ≈ 80ps, Td ≈ 1.9ns):
- top – Npe ≈ 0.2
- bottom – Npe ≈ 52.3

Experimental Fit

MPTR FWHM, ps (CTR with scintillator simulation) vs Light intensity

\[
N_{FWHM}(t(N_{pe})) \approx \frac{1007 \text{ ps}}{\sqrt{N_{pe}}}
\]

Number of photoelectrons per pulse

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Analytical model calculations: MPTR as function of SPTR for scintillator-simulated pulse

MPTR has regions with different dependence on SPTR
Kind of plateau for smaller SPTR value is connected with WLS rise time (80 ps)
Summary

• The multi-photon timing measurements with different pulse shapes were carried out to show how coincidence timing resolution depends on SPTR.
• Analytical model of “Amplitude noise” has a good agreement with experiment results for light intensity Npe > 1.
• MPTR for short light pulse may allow to extract true SPTRdetector (not affected by noise) – should be checked
• Analytical model shows how MPTR depends on SPTR for long scintillator-like pulses, but it should be checked with more experimental data.

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BACKUP
Timing measurements with new PCB – multi-photon TR results

Timing resolution vs Light intensity (in fired pixels), $U_{ov} = 4.5$ V

<table>
<thead>
<tr>
<th>N, pixel</th>
<th>SPTR, ps</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.168</td>
<td>163</td>
</tr>
<tr>
<td>0.288</td>
<td>185.5</td>
</tr>
<tr>
<td>0.549</td>
<td>204</td>
</tr>
<tr>
<td>1.072</td>
<td>217</td>
</tr>
<tr>
<td>1.932</td>
<td>144</td>
</tr>
<tr>
<td>3.585</td>
<td>111</td>
</tr>
<tr>
<td>7.353</td>
<td>85</td>
</tr>
<tr>
<td>12.6</td>
<td>66</td>
</tr>
<tr>
<td>39</td>
<td>36.6</td>
</tr>
<tr>
<td>59</td>
<td>32.6</td>
</tr>
<tr>
<td>95</td>
<td>29.8</td>
</tr>
<tr>
<td>112</td>
<td>28.8</td>
</tr>
<tr>
<td>140</td>
<td>27.9</td>
</tr>
<tr>
<td>212</td>
<td>26.1</td>
</tr>
</tbody>
</table>
Timing measurements with new PCB – CTR simulation experiment – results

Simulated coincidence timing resolution vs Light intensity (in fired pixels), $U_{ov} = 4.5$ V

<table>
<thead>
<tr>
<th>N, pixel</th>
<th>CTR, ps</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>1200</td>
</tr>
<tr>
<td>0.46</td>
<td>1501</td>
</tr>
<tr>
<td>0.86</td>
<td>1202</td>
</tr>
<tr>
<td>1.75</td>
<td>850</td>
</tr>
<tr>
<td>3.71</td>
<td>499</td>
</tr>
<tr>
<td>6.94</td>
<td>376</td>
</tr>
<tr>
<td>17.5</td>
<td>240</td>
</tr>
<tr>
<td>26.8</td>
<td>188</td>
</tr>
<tr>
<td>41.5</td>
<td>151</td>
</tr>
<tr>
<td>52.3</td>
<td>131</td>
</tr>
</tbody>
</table>

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Analytical model: CTR as function of SPTR and other parameters

Modern analytical approaches:

- Monte Carlo simulations,
- Detection event statistics,
- Order statistics of photoelectron detection time,
- Cramer-Rao lower bound estimation.

<table>
<thead>
<tr>
<th>Light, N photons</th>
<th>SPTR, $\sigma$</th>
<th>SER, ideal</th>
<th>CTR, min</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Upward Arrow] ← ![Downward Arrow]</td>
<td>![Upward Arrow]</td>
<td>$\delta(t)$</td>
<td>![Upward Arrow]</td>
<td>$\frac{T}{N}$ = Erlang distribution, No SPTR</td>
</tr>
<tr>
<td>![Upward Arrow]</td>
<td>![Upward Arrow]</td>
<td>$\sigma$</td>
<td>![Upward Arrow]</td>
<td>$\frac{\sigma}{\sqrt{N}}$ Light distribution = SPTR</td>
</tr>
<tr>
<td>![Upward Arrow] ← ![Downward Arrow]</td>
<td>![Upward Arrow]</td>
<td>$\sigma$</td>
<td>![Upward Arrow]</td>
<td>$\sqrt{\frac{T\sigma}{N}}$ Mixed roles of light $T$ &amp; SPTR</td>
</tr>
<tr>
<td>![Upward Arrow] ![Downward Arrow]</td>
<td>![Upward Arrow]</td>
<td>$\sigma$</td>
<td>![Upward Arrow]</td>
<td>$\sqrt{\frac{Td}{N}} \ {\sim} \frac{Tr}{\sigma}$ Not clear analytical function</td>
</tr>
</tbody>
</table>
Timing resolution - analytical model (S. Vinogradov)

\[
\sigma_t(N_{pe}) = \frac{\sqrt{\text{Var}[V_{out}(t)]}}{dV_{out}(t)/dt} \bigg| V_{out}(t_{\text{Discrim}}) = \text{Discrim}
\]

\[
= \sqrt{N_{pe} \cdot \text{ENF}_{\text{SiPM}} \cdot \left[ \rho_{ph} \cdot \rho_{\text{sptr}} \cdot h_{\text{ser}}^2 \right](t) + \frac{V_{\text{noise}}^2}{V_{\text{ser}}^2}}
\]

Filtered marked point process

Analytical model “Amplitude noise” for timing resolution

- \(N_{pe}\) - Number of photoelectrons
- \(\text{ENF}_{\text{SiPM}}\) - Excess noise factor of SiPM (include DCR, XT, AP)
- \(\rho_{ph}\) - Probability density function of light
- \(\rho_{\text{sptr}}\) - Probability density function of SiPM SPTR
- \(h_{\text{ser}}\) - Single-electron response function (SER)

Constant threshold at the first photon- no CT, no AP, no dark rate
\(\text{ENF} \approx 1\)
Experimental data with lidar prototype laser 40 ps FWHM 405nm

SiPM timing characteristics under conditions of a large background for lidars

Scanning lidar

Light background 100 MHz

Amplitude, V

Time, ns

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SiPM current, A

1μA=4.2MHz
Experiment

TR dependence (T = -30°C, Uov = 4.5V. SPTR (true SPTR without noise contribution)= 147 ps, ENFct = \( \frac{1}{1 + \ln(1 - Pct)} \))

Pct = 0.13, ENFct = 1.16, no Dark rate

Light source – laser, FWHM = 40 ps

Analytical model:

\[
FWHM_t(N_{pe}) \approx \frac{SPTR}{\sqrt{N_{pe}}} \cdot 1.5
\]

Experimental Fit:

\[
FWHM_t(N_{pe}) \approx \frac{210\, ps}{\sqrt{N_{pe}}}
\]

LED threshold

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