Statistics and models of SiPM nonlinearity and saturation

Sergey Vinogradov

Lebedev Physical Institute of the Russian Academy of Sciences, Moscow, Russia
National Research Nuclear University «MEPhI», Moscow, Russia

Scope & outline

- Photon detection – a series of stochastic processes described by statistics
    - It could be described as a filtered marked correlated history-dependent point process
    - SiPM response – a result of the stochastic process – a random variable

- <u>Nonlinearity and saturation – the most sophisticated topic in SiPM statistics</u>

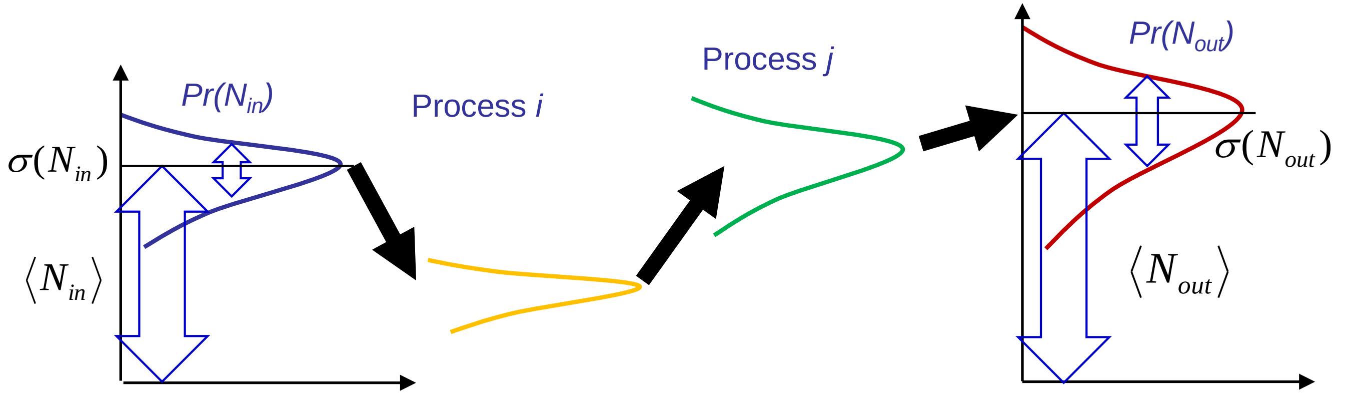
- Binomial nonlinearity – detection of short light pulses (Tpulse < Trecovery)
    - Conventional model: Poisson Npe in N pixels
    - Adjustments to CT to account for
        - Crosstalk
        - Recovery

- Recovery nonlinearity – detection of long light pulses (Tpulse > Trecovery)
    - Conventional model: non-paralizible counting with dead time
    - Advanced model of exponential recovery process
    - Advanced+ model of Markov reward-renewal process

Statistics of linear photon detection

- Full characterization of random variable *Nout* resulted from photon detection processes
  - Probability distribution Pr(*Nout*) conditional on Pr(*Nin)*
    - — Mean <*Nout*>
    - — Variance (*Nout*) or $\sigma(N_{out})$
- Partial characterization (the most demanded in practice):
  - Mean and Var of *Nout* conditional on Mean and Var of *Nin*
    - — Supported by Burgess variance theorem
    - — ENF approach for independent process chains allows to analyse specific noise contributions
- Linear detection: responsivity *R* = <*Nout*>/<*Nin*> = *const*
  - Resolution σ/μ is degraded by specific ENFs

$$Res_{in} < Res_{process\_i} < Res_{process\_j} < Res_{out} = Res_{in} \cdot \sqrt{ENF}$$



Input photon distribution **Photodetection processes** **Output charge distribution**

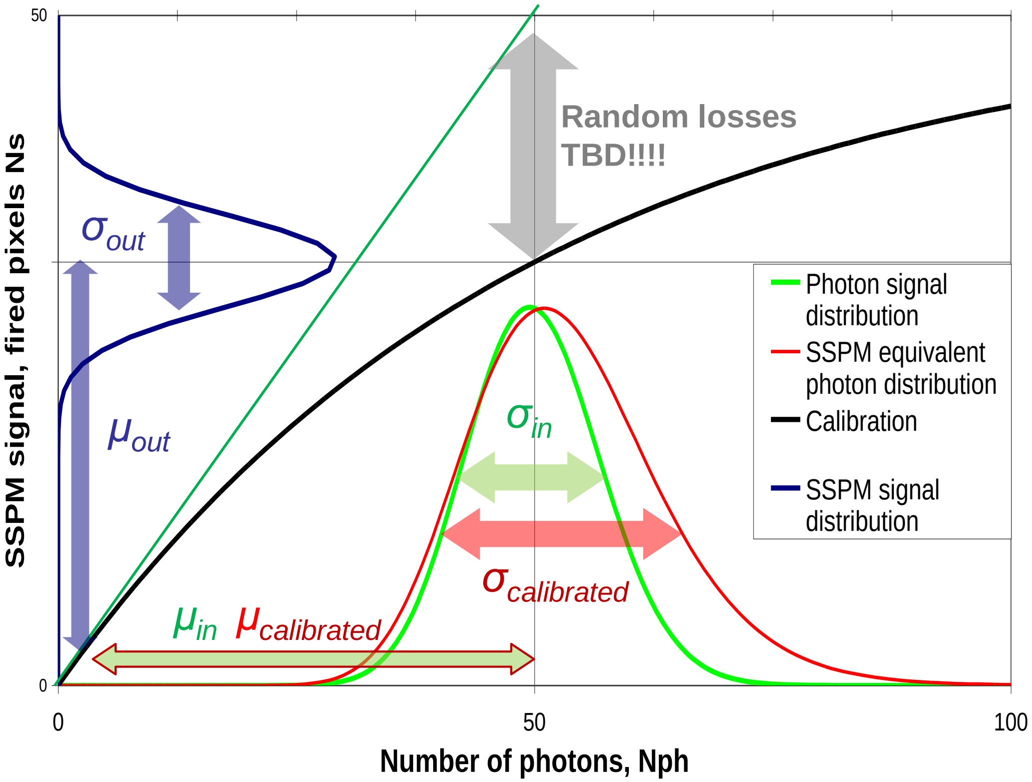
Statistics of nonlinear photon detection

- Nonlinear: $R = R(Nph)$
  - Nonlinearity = random losses
  - Losses depend on load *Nph*
  - Output resolution is “improved”
  - Calibrated resolution is degraded

    $\sigma_{out} < \sigma_{in} < \sigma_{calibrated}$

    even for ideal nonlinear detector
  - Excess noise of nonlinearity

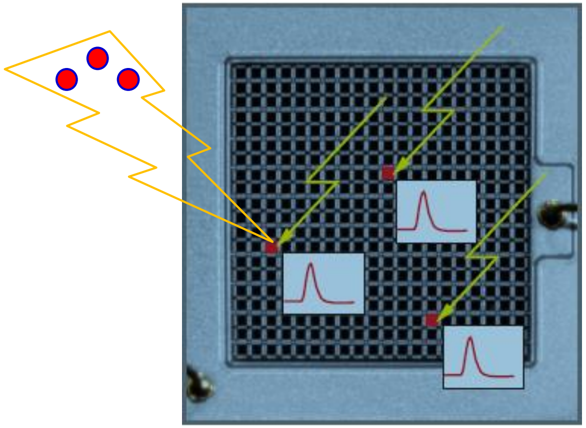
$$ENF = \frac{Res^2_{calib}}{Res^2_{in}} = \frac{\sigma^2_{calib}}{\sigma^2_{in}} = \frac{1}{\sigma^2_{in}} \cdot \frac{\sigma^2_{out}}{\left(\frac{d\mu_{out}}{d\mu_{in}}\right)^2}$$



- Nonlinear statistics:
  - Linear processes + Nonlinear processes =>
  - Severe complications in statistics (quantity/history/mutually-dependent processes)
    - New nonlinear distribution Pr(*Nout*), <*Nout*>, σ(*Nout*)
    - New nonlinear responsivity $R = R(Nph)$
    - New ENF of nonlinearity

Binomial distribution of SiPM response

- Binomial distribution – detection of short light pulses (Tpulse < Trecovery)

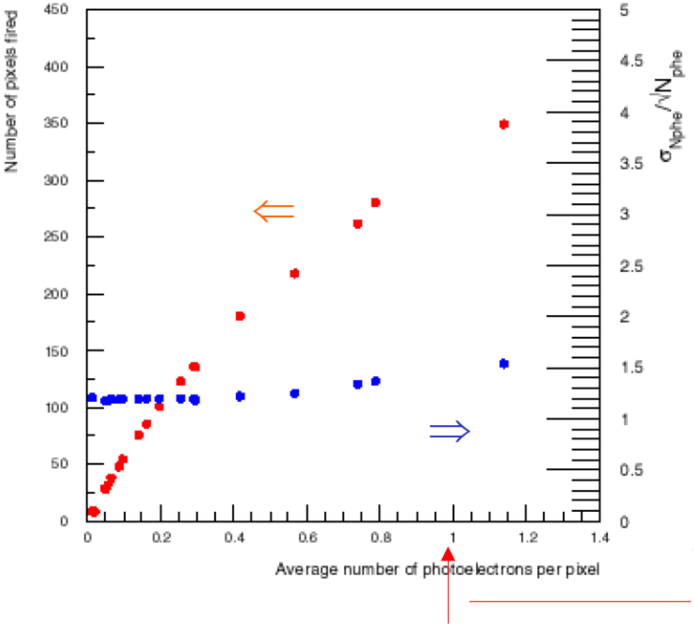


Binomial distribution is presumed

SiPM dynamic range

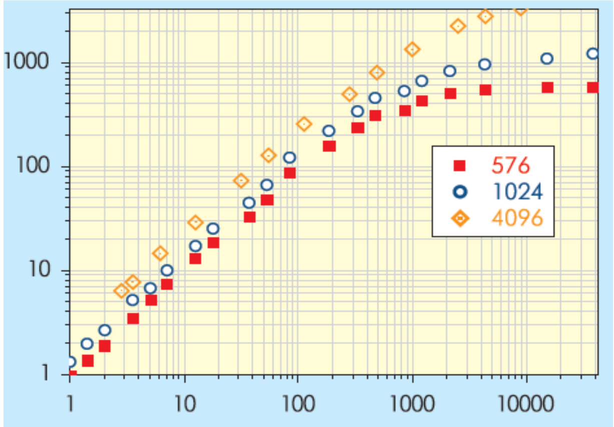
Dynamic range is limited due to finite total number of pixels *m*

Signal ~ $m(1-\exp(-N_{ph.e}/m))$



"Dispersion limit" of dynamic range: $N_{ph.e}/m < 1$

B. Dolgoshein et al., 1998 - 2003



SiPM signal

- The output SiPM signal is proportional to the number of pixels fired N

$$S \approx N_{pixel\_fired} = m\cdot(1-e^{-\frac{N_{ph}\cdot\varepsilon}{m}})$$

where m- total number of pixels

N_{ph}- number of photons

ε - photon detection efficiency

⇨ saturation of the signal

Urn model with non-random Npe is approximated to normal distribution with binomial μ and σ

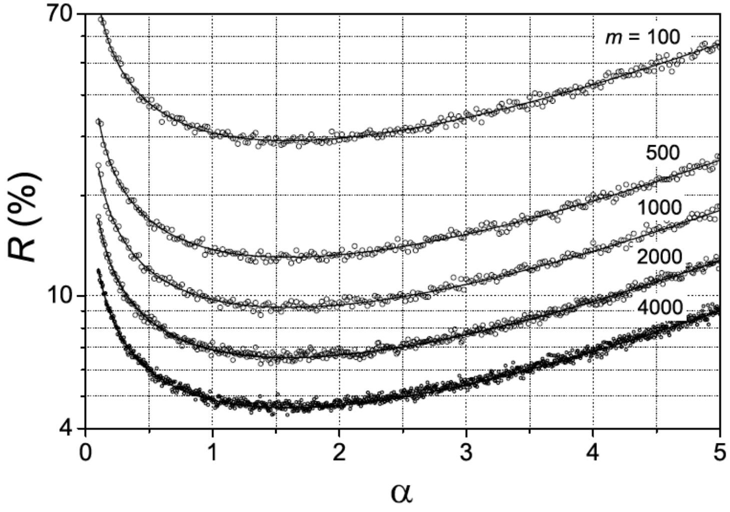
The considered problem is equivalent to a well-known problem in mathematical statistics of distributing (randomly) n balls (photoelectrons) into m urns (cells), see e.g. [4]. The number N of urns containing one or more balls is a random variable, its expected value and variance are:

$$\begin{aligned} \bar{N} &= m\left[1-\left(1-m^{-1}\right)^{n}\right] \\ \sigma_N^2 &= m\,(m-1)\left(1-2m^{-1}\right)^{n} + m\left(1-m^{-1}\right)^{n} - m^2\left(1-m^{-1}\right)^{2n} \end{aligned} \tag{1}$$

The distribution of N is approximately normal when $m, n \to \infty$ and the ratio $\alpha = n/m$ is bounded [4]:

$$\begin{aligned} \bar{N} &= m\left(1-e^{-\alpha}\right) \\ \sigma_N^2 &= m\,e^{-\alpha}\left[1-(1+\alpha)\,e^{-\alpha}\right]. \end{aligned} \tag{2}$$

In practice the number of cells in a G-APD is usually greater than ~ 100, which justifies using the asymptotic formulae (2) in the following analysis.



A. Stoykov et al., On the limited amplitude resolution of multipixel Geiger-mode APDs, JINST, 2007

SiPM binomial nonlinearity:
losses of photons firing the same pixel

- Binomial distribution of fired pixels N_{det} in SiPM (*Tpulse* < *Trecovery*)

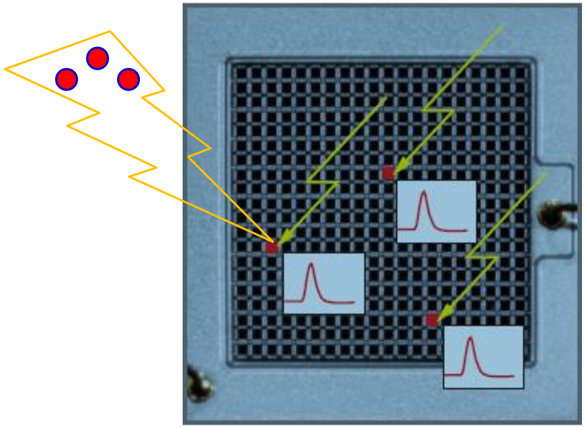
$$\Pr(N_{\det}, N_{pix}, p)) = \frac{N_{pix}!}{(N_{pix} - N_{\det})!\,N_{\det}!} p^{k} (1-p)^{N_{pix} - N_{\det}} \qquad p = 1 - \mathrm{e}^{-\frac{N_{pe}}{N_{pix}}}$$

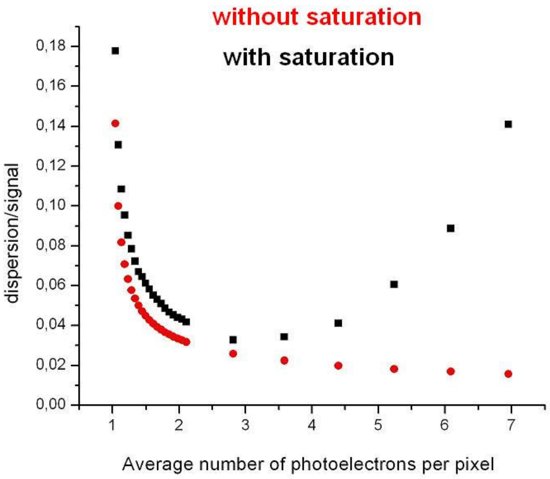
$$P(0) = \exp\left(-\frac{N_{pe}}{N_{pix}}\right) \qquad E[N_{\det}] = N_{pix} \cdot [1 - P(0)] \qquad Var[N_{\det}] = N_{pix} \cdot [1 - P(0)] \cdot P(0)$$

$$PNR = \frac{\sqrt{ENF_{nonlin}}}{\sqrt{N_{pe}}} \qquad ENF_{nonlin} = \frac{\exp\left(\frac{N_{pe}}{N_{pix}}\right) - 1}{\frac{N_{pe}}{N_{pix}}} \approx 1 + \frac{N_{pe}}{2N_{pix}} + \dots$$

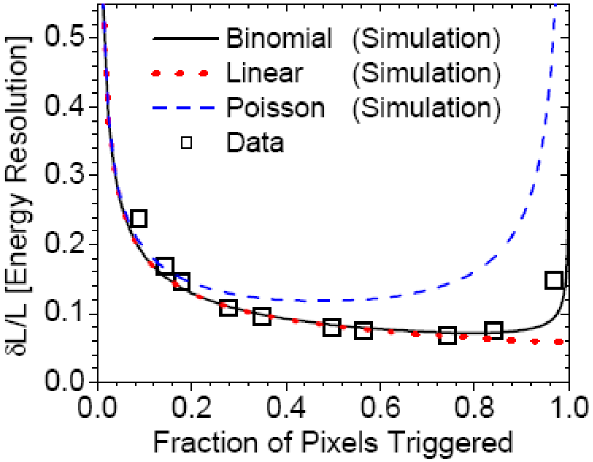
Excess noise factor - S. Vinogradov et al., IEEE NSS/MIC 2009

Losses of simultaneous photons in a pixel results in nonlinearity and excess noise



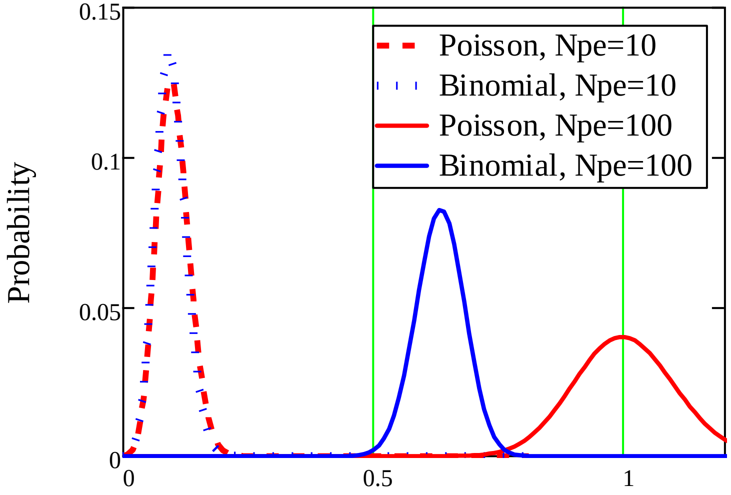


J. Barral, Study of SiPMs, MPI, 2004

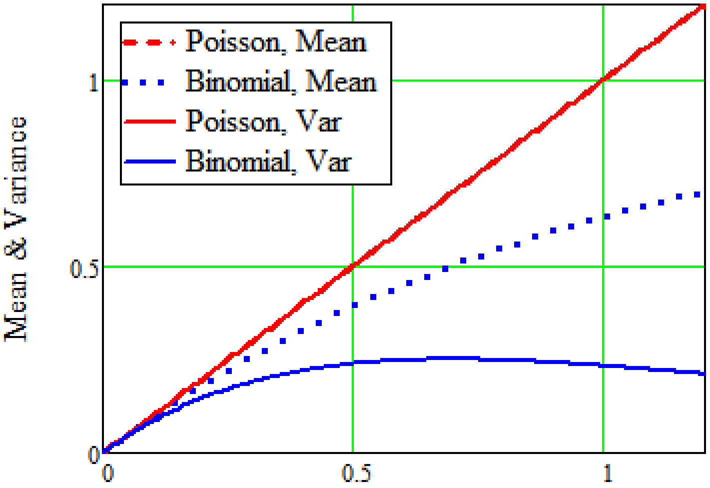


E.B. Johnson et al., IEEE NSS/MIC 2008

Binomial vs Poisson



Pixel load, Npe/Npix



Pixel load, Npe/Npix

Adjustments of binomial model

- Crosstalk
  - Typical approach: extension of mean Npe to include mean CT events μ_{CT}
    - Npe → Npe(1+μ_{CT})
  - So, Mean $N_{det} = N_{pix}\left(1 - e^{-\frac{N_{pe}}{N_{pix}}}\right) \rightarrow N_{pix}\left(1 - e^{-\frac{N_{pe}(1+\mu_{CT})}{N_{pix}}}\right)$
  - Reasonable from common sense, looks nice and simple but…
  - In general, incorrect because Poisson with CT is not Poisson
  - What about Variance - N_{det} ???
    - What about Resolution ???
      - And finally - probability distribution ???

Adjustments of binomial model

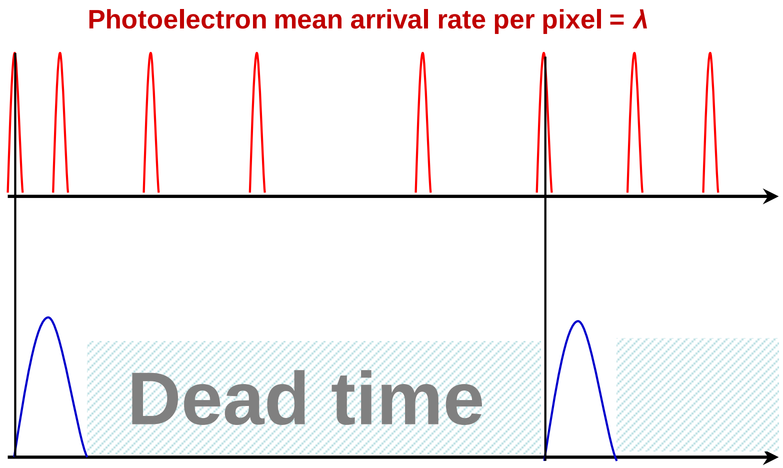
- Recovery
  - Typical approach: extension of mean Npix to include mean retriggering events μ_{RT}
    - — Npix → Neff = Npix(1+μ_{RT})=Npix(1+Tpulse/Trec)
  - So, Mean $$N_{det} = N_{pix}\left(1-e^{-\frac{N_{pe}}{N_{pix}}}\right) \to N_{pix}\left(1+\frac{T_{pulse}}{T_{rec}}\right)\left(1-e^{-\frac{N_{pe}}{N_{pix}\left(1+\frac{T_{pulse}}{T_{rec}}\right)}}\right)$$
  - Reasonable from common sense, looks nice and simple…
  - In general, possible because Poisson Npe over fixed Neff is still binomial
    - — More reasons in support ???
  - What about Variance - N_{det} ???
    - —What about Resolution ???
      - – And finally - probability distribution ???
- Main question: what about lower Gain due to incomplete recovery?

Adjustments of binomial model

- Crosstalk + Recovery
  - Typical approach: both corrections are applied as fitting parameters
  - So, Mean $N_{det} = N_{pix}\left(1 - e^{-\frac{N_{pe}}{N_{pix}}}\right) \rightarrow N_{eff\_1}\left(1 - e^{-\frac{N_{pe}}{N_{eff\_2}}}\right)$
  - So, the same concerns and questions

Recovery nonlinearity of SiPM response

- Recovery nonlinearity – detection of long light pulses (Tpulse > Trecovery)



SiPM recovery nonlinearity

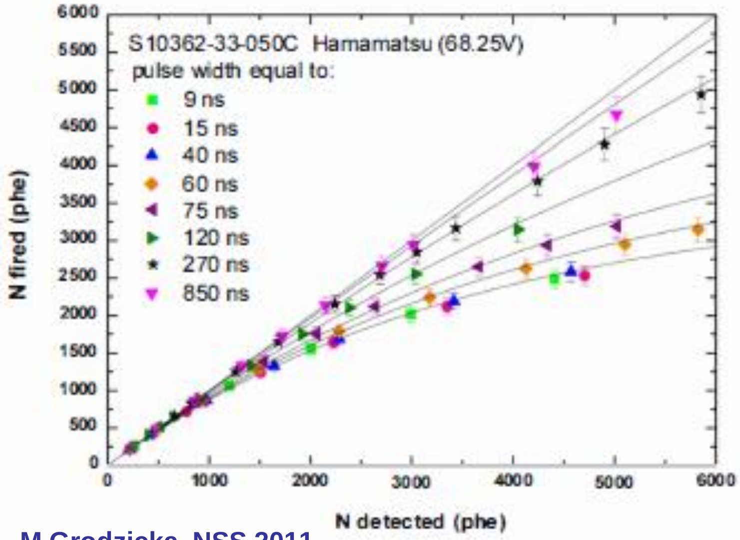
$$T_{pulse} >> T_{rec}$$

Nonparalizible dead time model
Probability distribution (~ Gaussian)

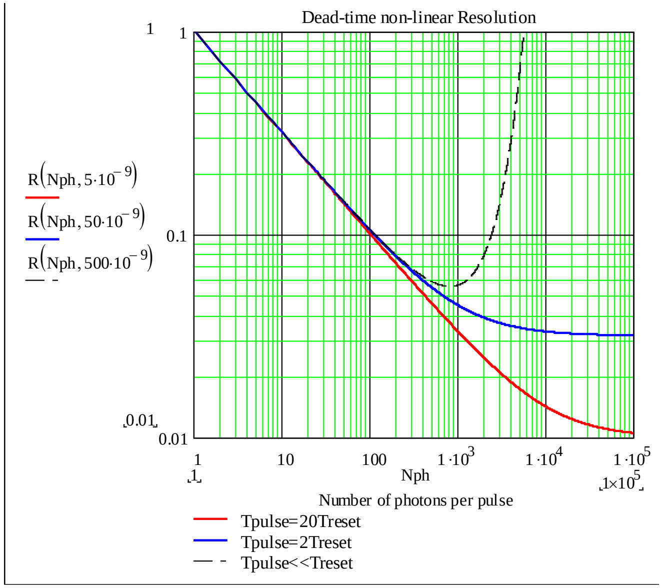
W. Feller, *An Introduction to Probability Theory and Its Applications,* Vol. 2, Ch. XI, John Willey & Sons, Inc., 1968

$$\mu_{Ns} = \frac{\lambda \cdot t}{1 + \lambda \cdot \tau_{dead}}$$

$$\sigma^2{}_{Ns} = \frac{\lambda \cdot t}{(1 + \lambda \cdot \tau_{dead})^3}$$



M.Grodzicka, NSS 2011



Recovery nonlinearity of SiPM → ENF

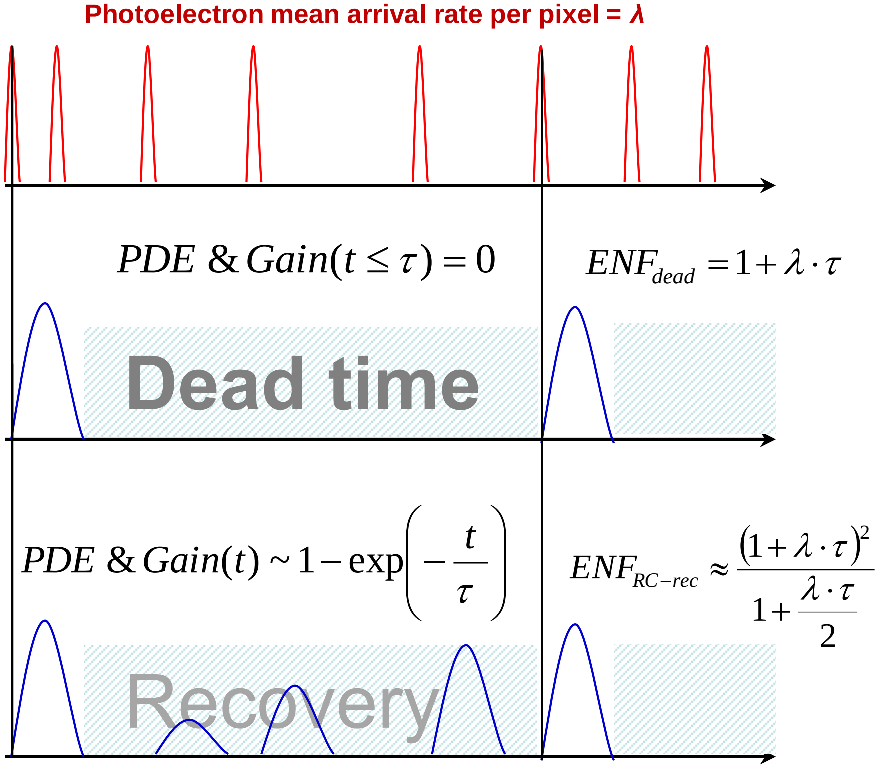
S. Vinogradov et al., IEEE NSS/MIC 2009

$$ENF_{eq(Nph)}(\lambda) = 1 + \lambda \cdot \tau_{dead}$$

SiPM recovery nonlinearity: advanced model of exponential recovery

- Non-paralizible dead time model for SiPM (*ENF*)
- Exponential RC recovery model (*Gain*, *PDE*, *ENF*)

Losses of sequential photons due to incomplete pixel recovery



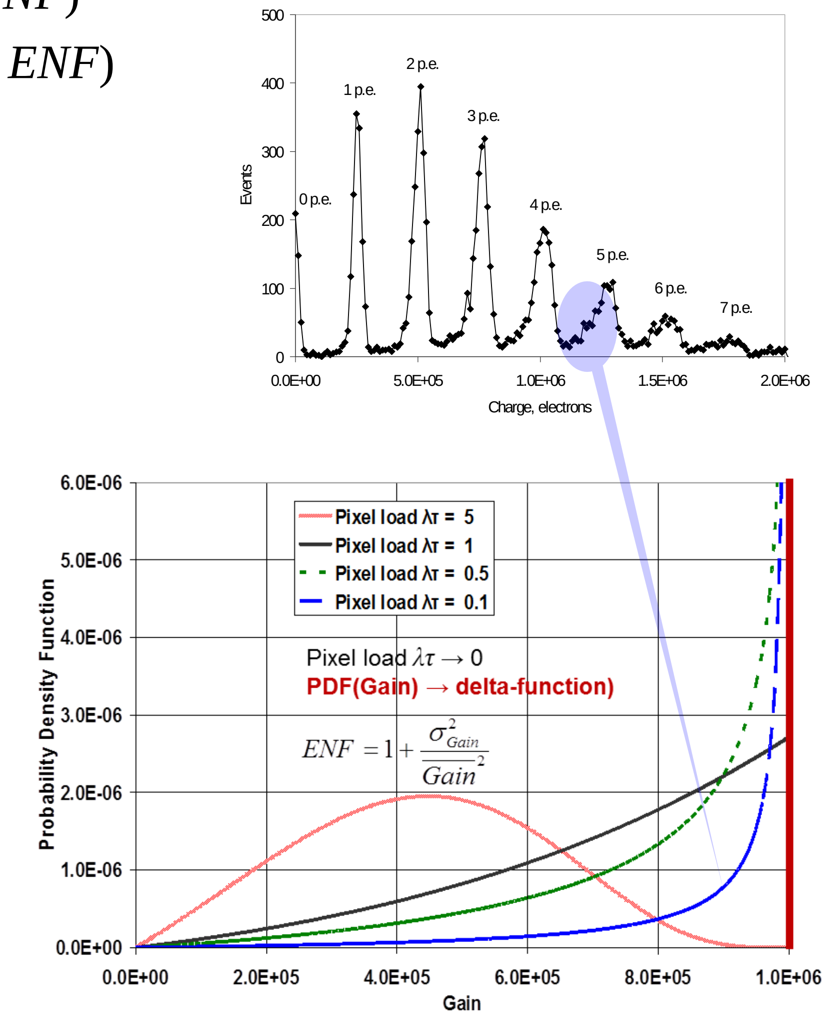
Photoelectron mean arrival rate per pixel = λ

$$PDE\ \&\ Gain(t \le \tau) = 0$$

$$ENF_{dead} = 1 + \lambda \cdot \tau$$

$$PDE\ \&\ Gain(t) \sim 1 - \exp\left(-\frac{t}{\tau}\right)$$

$$ENF_{RC-rec} \approx \frac{(1+\lambda\cdot\tau)^2}{1+\frac{\lambda\cdot\tau}{2}}$$



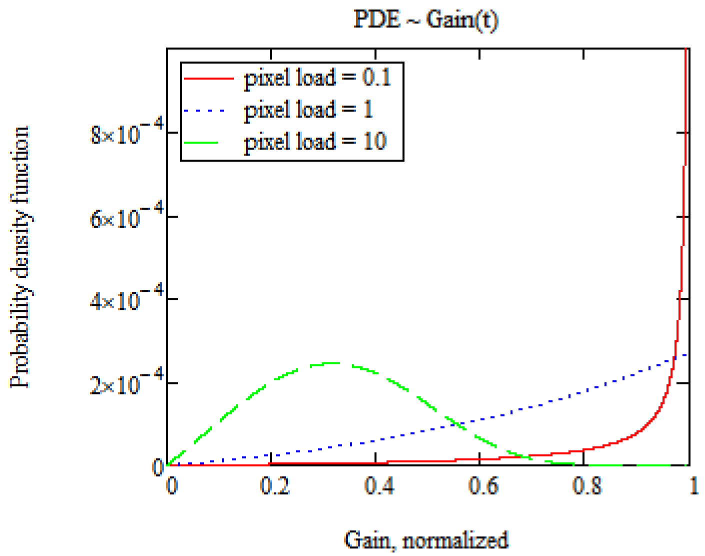
Pixel load $\lambda\tau \to 0$

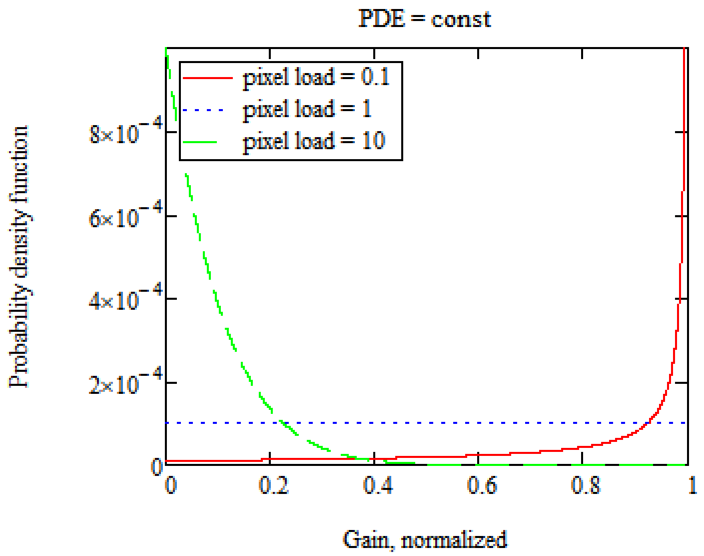
PDF(Gain) → delta-function)

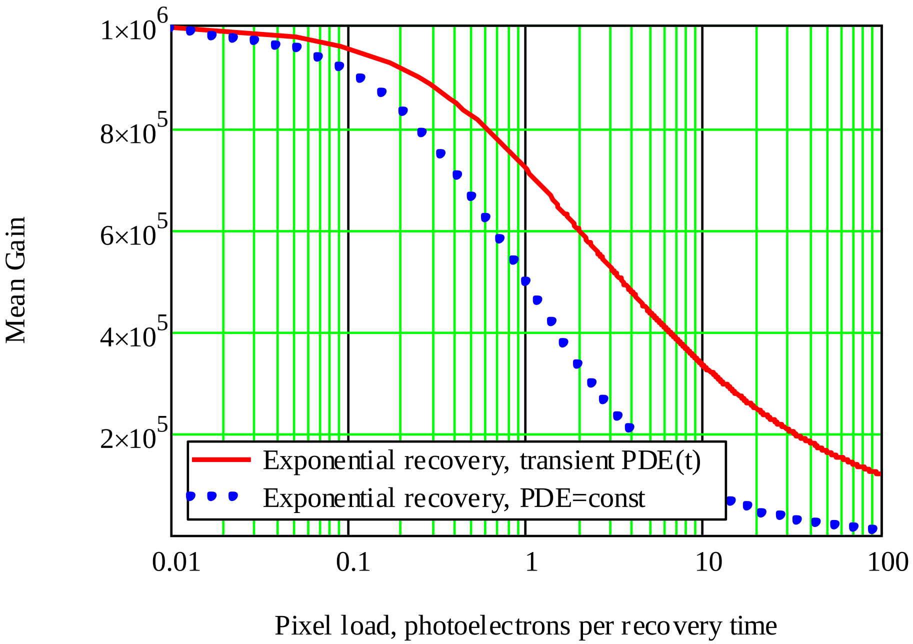
$$ENF = 1 + \frac{\sigma^2_{Gain}}{Gain^2}$$

S. Vinogradov, SPIE Adv. Photon Counting 2012

Advanced model of exponential recovery: accounting for a transient PDE is a must


PDE ~ Gain(t)


PDE = const



Comparison of models

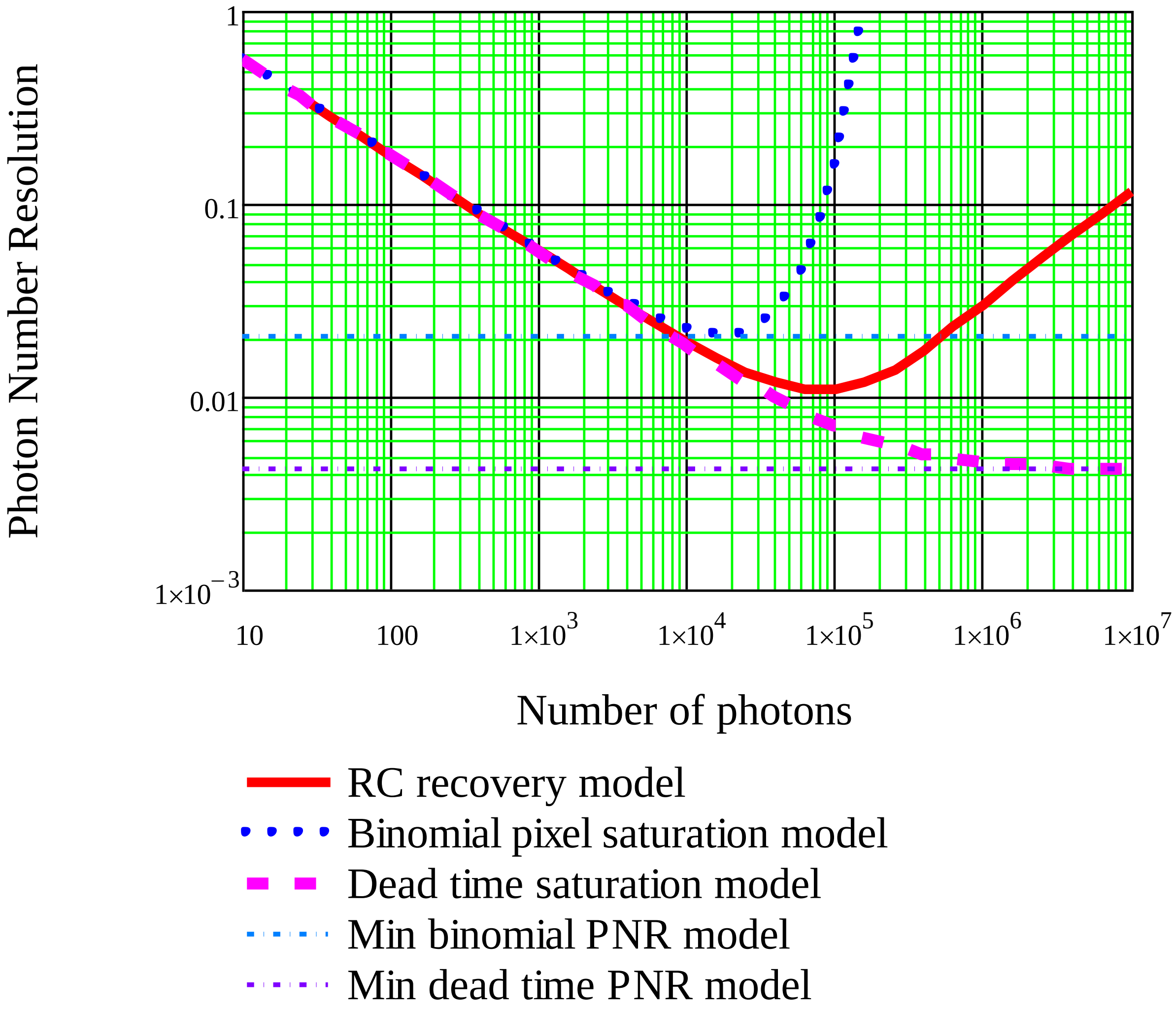


Figure 3. Photon Number Resolution for short (binomial model) and long (dead time and RC recovery models) light pulses calculated for MPPC KSX-I50015-E_S12573 Series 50 um pixel size, 3x3 sq. mm area.

Advanced+ model of SiPM recovery: reward-renewal Markov process:

- Renewal process: conditional probabilities of times between events (photon arrivals & avalanche triggers)
- Reward process: random gain dependent on time delay
- Exponential RC recovery model: *Gain(t), PDE(t)*

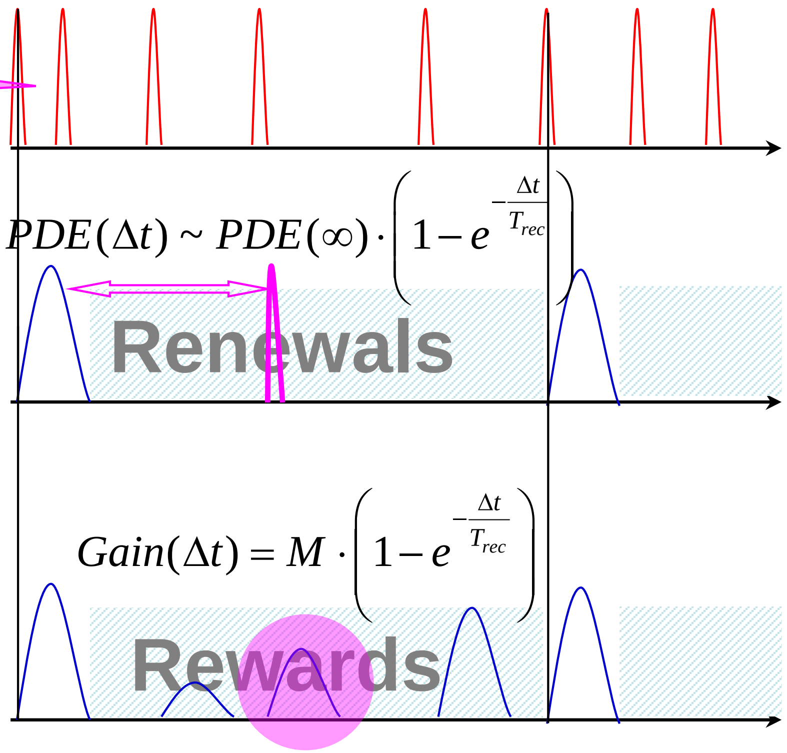
$$\rho_{ph}(t) = \lambda(t)\cdot\exp\left(-\int_0^t \lambda(t')dt'\right)$$ photon inter - arrival time PDF

$$\rho_{spdr}(t) = PDE(t)\cdot\rho_{sptr}(t)$$ single photon detection PDF

$$PDE(t) \sim Gain(t) = Gain(\infty)\cdot\left[1-\exp\left(-t/\tau_{rec}\right)\right]$$ exponential pixel recovery

$$\rho_{\det}(t) = \rho_{ph}(t) * \rho_{spdr}(t) \qquad P_{\det}(t) = \int_0^t \rho_{\det}(t')dt'$$ detected event PDF & CDF

$$\overline{\Delta t} \sim \frac{1}{\overline{I_{ph}(t)}}$$



$$PDE(\Delta t) \sim PDE(\infty)\cdot\left(1-e^{-\frac{\Delta t}{T_{rec}}}\right)$$

$$Gain(\Delta t) = M\cdot\left(1-e^{-\frac{\Delta t}{T_{rec}}}\right)$$

Reward-renewal Markov process model: qualitative correspondence for transient mean

Renewal equation and mean reward rate (SiPM response):

$$E[N_{\mathrm{det}}(t)] = P_{\mathrm{det}}(t) + \int_0^\infty E[N_{\mathrm{det}}(t'-t)]\rho_{\mathrm{det}}(t')dt'$$

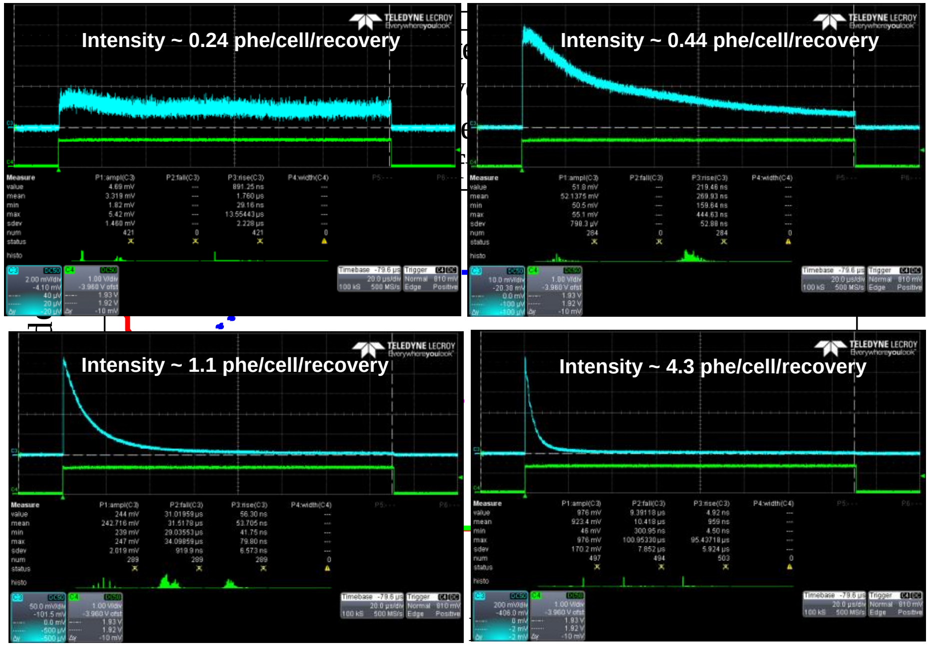
$E[N_{\mathrm{det}}(t)]$ - mean number of detected events

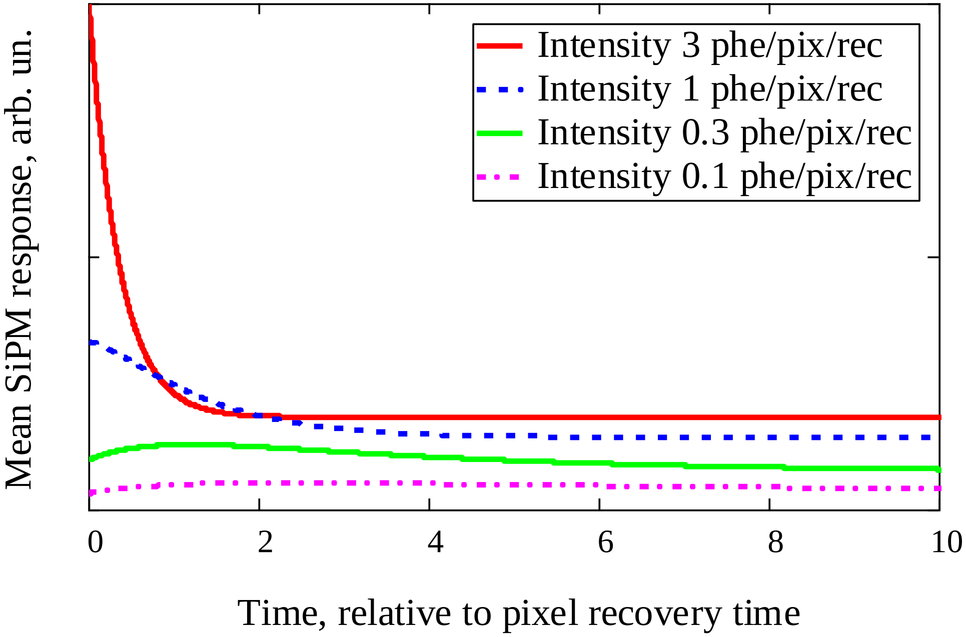
$$\tilde{L}\{E[N_{\mathrm{det}}(t)]\}(s) = \frac{\tilde{L}\{P_{\mathrm{det}}(t)\}(s)}{1-\tilde{L}\{P_{\mathrm{det}}(t)\}(s)}$$

solution of renewal equation (Laplace transform)

$$E[I_{SiPM}(t)] = E[\mathit{renewal\ rate}]\cdot E[\mathit{reward}] = \frac{dE[N_{\mathrm{det}}(t)]}{dt}\cdot E[Gain(t)]$$

$E[I_{SiPM}(t)]$ – SiPM response





SiPM nonlinearity and saturation papers

- [1] C. Adloff et al, “Construction and commissioning of the CALICE analog hadron calorimeter prototype,” *J. Instrum.*, vol. 5, no. 5, 2010.
- [2] M. L. Ahnen, “Over Saturation in SiPMs: The Difference Between Signal Charge and Signal Amplitude,” *Archiv.org*, p. 4, Jul. 2015.
- [3] A. Arodzero, S. Boucher, J. Hartzell, S. V. Kutsaev, R. C. Lanza, V. Palermo, S. Vinogradov, and V. Ziskin, “High speed, low dose, intelligent X-ray cargo inspection,” *2015 IEEE Nucl. Sci. Symp. Med. Imaging Conf. NSS/MIC 2015*, 2016.
- [4] A. Arodzero, S. Member, S. Boucher, S. V Kutsaev, V. Ziskin, A. Abstract, and M. I. X. Inspec-, “MIXI : Mobile Intelligent X-Ray Inspection System,” in *IEEE NSS/MIC 2015*, 2017, vol. 64, no. 7, pp. 1629–1634.
- [5] T. Bretz, T. Hebbeker, M. Lauscher, L. Middendorf, T. Niggemann, J. Schumacher, M. Stephan, A. Bueno, S. Navas, and A. G. Ruiz, “Dynamic range measurement and calibration of SiPMs,” *J. Instrum.*, vol. 11, no. 3, 2016.
- [6] P. BUZHAN et al, “THE ADVANCED STUDY OF SILICON PHOTOMULTIPLIER,” in *Advanced Technology & Particle Physics - Proceedings of the 7th International Conference on ICATPP-7*, 2002, pp. 717–728.
- [7] M. Danilov, “The Calice Analog Scintillator-Tile Hadronic Calorimeter Prototype,” in *SNIC Symposium*, 2006, no. April, pp. 1–6.
- [8] S. Dolinsky, “Novel approach for calibration breakdown voltage of large area SiPM Geiger Mode APD and Gain,” in *PhotoDet 2012*, 2012.
- [9] S. Dolinsky, “Novel approach for calibration breakdown voltage of large area SiPM,” *Proc. Sci.*, pp. 1–6, 2012.
- [10] P. Eckert, R. Stamen, and H. C. Schultz-Coulon, “Study of the response and photon-counting resolution of silicon photomultipliers using a generic simulation framework,” *J. Instrum.*, vol. 7, no. 8, 2012.
- [11] L. Gallego, J. Rosado, F. Blanco, and F. Arqueros, “Modeling crosstalk in silicon photomultipliers,” *J. Instrum.*, vol. 8, no. 5, 2013.
- [12] E. Garutti, “Silicon photomultipliers for high energy physics detectors,” *J. Instrum.*, vol. 6, no. 10, 2011.
- [13] M. Grodzicka, T. Szczęśniak, M. Moszyński, M. Szawłowski, and K. Grodzicki, “New method for evaluating effective recovery time and single photoelectron response in silicon photomultipliers,” *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.*, vol. 783, pp. 58–64, 2015.
- [14] L. Gruber, S. E. Brunner, J. Marton, and K. Suzuki, “Over saturation behavior of SiPMs at high photon exposure,” *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.*, vol. 737, pp. 11–18, 2014.
- [15] L. Gruber, S. E. Brunner, C. Curceanu, J. Marton, A. R. Vidal, and A. Scordo, “Recovery Time Measurements of Silicon Photomultipliers Using a Pulsed Laser,” *Proc. Sci.*, vol. 835, no. July 2015, pp. 0–7, 2012.
- [16] Z. Guoqing, L. Lina, and L. Hanchen, “Demonstration of the over dynamic range of MPPC by high intensity pulsed light illumination,” *Opt. - Int. J. Light Electron Opt.*, vol. 127, no. 5, pp. 2936–2938, Mar. 2016.
- [17] P. Hallen, “Determination of the Recovery Time of Silicon Photomultipliers,” RWTH Aachen University, 2011.
- [18] A. Heering, A. Karneyeu, I. Musienko, and M. Wayne, “SiPM linearization status update,” in *CERN CMS*, 2017, no. January.
- [19] D. Jeans, “Modeling the response of a recovering SiPM,” *Archiv.org*, no. 1, pp. 1–5, Nov. 2015.
- [20] J. Jiang, J. Jia, T. Zhao, K. Liang, R. Yang, and D. Han, “Recovery Time of Silicon Photomultiplier with Epitaxial Quenching Resistors,” *Instruments*, vol. 1, no. 1, p. 5, 2017.

- [21] K. Kotera, W. Choi, and T. Takeshita, “Describing the response of saturated SiPMs,” *Archiv.org*, pp. 1–9, 2015.
- [22] K. Kotera, W. Choi, and T. Takeshita, “Functions Represent SiPM Response Especially Linear Behavior After Saturation,” *Archiv.org*, p. 10, 2015.
- [23] T. Kraehenbuehl, “The First Semiconductor-Based Camera for Imaging Atmospheric Cherenkov Telescopes,” ETH Zurich, 2013.
- [24] E. Popova, “Charge and Recovery Time for Oversaturation Conditions,” in *PhotoDet 2015*, 2015, pp. 1–20.
- [25] E. Popova, P. Buzhan, A. Pleshko, S. Vinogradov, A. Stifutkin, A. Ilyin, D. Besson, and R. Mirzoyan, “Amplitude and timing properties of a Geiger discharge in a SiPM cell,” *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.*, vol. 787, pp. 270–274, 2015.
- [26] E. Popova and M. Chadeeva, “SiPM mini-workshop,” in *CERN CMS*, 2017, no. November.
- [27] J. Pulko, F. R. Schneider, a Velroyen, D. Renker, and S. I. Ziegler, “A Monte-Carlo model of a SiPM coupled to a scintillating crystal,” *J. Instrum.*, vol. 7, no. 2, pp. P02009–P02009, Feb. 2012.
- [28] J. Rosado, “Performance of SiPMs in the nonlinear region,” *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.*, no. October, 2017.
- [29] Shaojun Lu, “Correction for the SiPM non-linearity (new perspective on saturation curve) for AHCAL SiPM with scintillator tile,” in *CALICE ECAL/AHCAL 05/07/2010*.
- [30] A. Stoykov, Y. Musienko, A. Kuznetsov, S. Reucroft, and J. Swain, “On the limited amplitude resolution of multipixel Geiger-mode APDs,” *J. Instrum.*, vol. 2, no. 6, pp. P06005–P06005, Jun. 2007.
- [31] S. Uozumi, “Study and development of Multi Pixel Photon Counter for the GLD calorimeter readout,” *Proc. Sci.*, 2007.
- [32] H. T. Van Dam, S. Seifert, and D. R. Schaart, “The statistical distribution of the number of counted scintillation photons in digital silicon photomultipliers: Model and validation,” *Phys. Med. Biol.*, vol. 57, no. 15, pp. 4885–4903, 2012.
- [33] E. Van Der Kraaij, “SiPM Saturation Scans,” in *LCD ECAL meeting 20/02/2014*, 2014.
- [34] S. Vinogradov., “Challenges of arbitrary waveform signal detection by SiPM in beam loss monitoring systems with Cherenkov fibre readout,” in *Proceedings of Science*, 2015, vol. 6-9-NaN-2, no. July, pp. 4–8.
- [35] S. Vinogradov, “Performance of silicon photomultipliers in photon number and time resolution,” in *Proceedings of Science*, 2015, vol. 6-9-NaN-2.
- [36] S. Vinogradov, A. Arodzero, R. C. Lanza, and C. P. Welsch, “SiPM response to long and intense light pulses,” *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.*, vol. 787, pp. 148–152, 2015.
- [37] S. Vinogradov, “Probabilistic analysis of solid state photomultiplier performance,” in *Proceedings of SPIE - The International Society for Optical Engineering*, 2012, no. 1, p. 83750S–83750S–9.
- [38] S. Vinogradov, A. Arodzero, and R. C. Lanza, “Performance of X-ray detectors with SiPM readout in cargo accelerator-based inspection systems,” *Nucl. Sci. Symp. Conf. Rec. (NSS/MIC), 2013 IEEE*, vol. 58, no. 1, pp. 5–6, 2013.
- [39] S. Vinogradov, T. Vinogradova, V. Shubin, D. Shushakov, and C. Sitarsky, “Efficiency of Solid State Photomultipliers in Photon Number Resolution,” *IEEE Trans. Nucl. Sci.*, vol. 58, no. 1, pp. 9–16, Feb. 2011.
- [40] N. Wattimena, “Commissioning of an LED Calibration & Monitoring System for the Prototype of a Hadronic Calorimeter,” Hamburg, 2006.
- [41] P. A. Amaudruz et al, “The T2K fine-grained detectors,” Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip., vol. 696, pp. 1–31, 2012.
- [42] J. Repond et al, “Construction and response of a highly granular scintillator-based electromagnetic calorimeter,” Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip., vol. 887, no. July 2017, pp. 150–168, 2018.

The end

Thank you for your attention!

Questions?

Objections?

Opinions?

…

vin@lebedev.ru