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An Analytical Approach to Predict Fundamental Cryogenic Properties of Silicon Photomultipliers

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

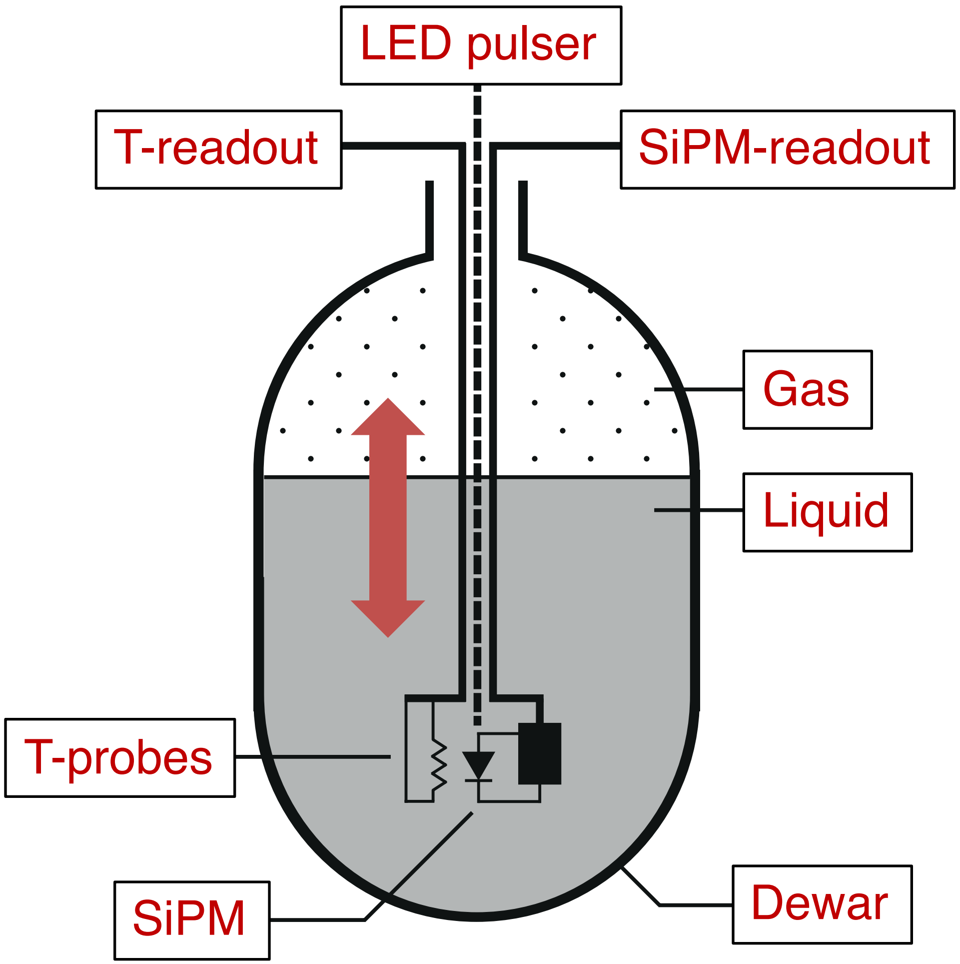
Investigated SiPM Properties

Goal of this work is to find physical and analytical descriptions for 3 property groups:

- ~~Single-pixel capacitance and breakdown voltage~~
- Photon detection probability and Crosstalk
- Optimum signal-to-noise ratio

All these calculations were proofed by data taken with a SensL C-series 30035 with an active area of (3x3) mm^2 and pixel size of (35x35) μm^2

SiPM Cryogenic Test Setup



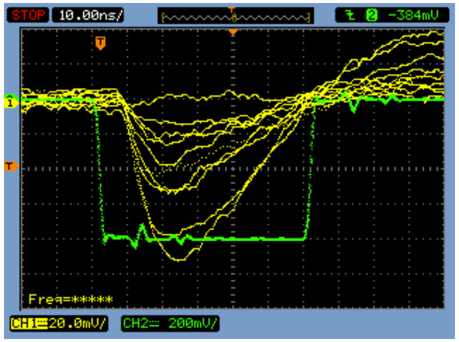
- Junction temperature adjustment by height over liquid Nitrogen / Helium
- 485nm LED pulser with adjustable single-photon intensity
- 4-wire readout of platinum and carbon temperature sensors
- Fully differential SiPM readout and charge integrating ADC (QDC)

M. Biroth, et al., A low-noise and fast pre-amplifier and readout system for SiPMs Nucl. Instr. Meth. A 787 (2015) 185-188

SiPM Low Intensity Pulse Response and Data Evaluation

Measuring pulse response under variation of reverse voltage and temperature at fixed intensity

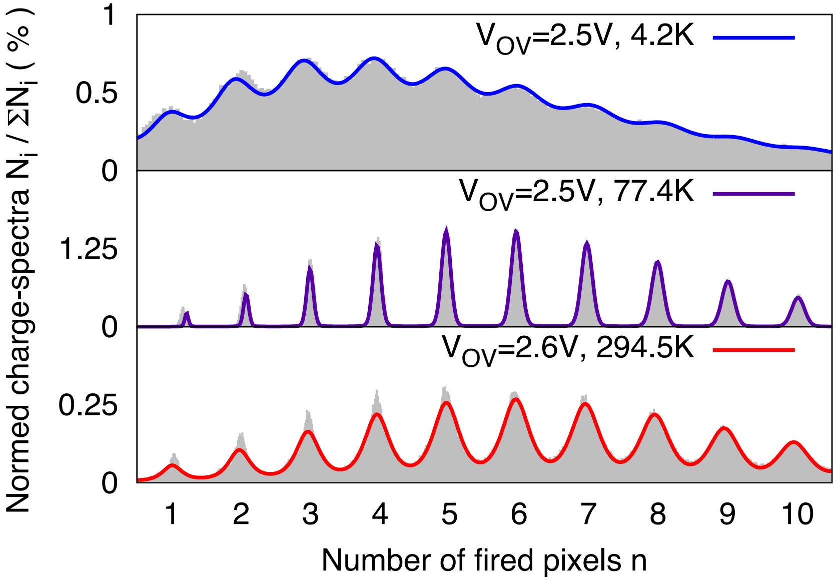
- 6 temperatures (4.2K, 46.3K, 77.2K, 126.6K, 216.0K, 294.5K)
- 6-8 different bias voltage settings per temperature
- Data Acquisition is gated by the LED pulser
- Charge is integrated within the gate width



SiPM response at LHe temperature (yellow)
Charge integration gate (green)

A curve-fit with charge spectrum function extracts the following parameters:

- Poisson Mean intensity ϕ
- Crosstalk probability P_{CT}
- Pedestal position Q_0 and width σ_0
- Single-pixel gain G_{SP} and variation σ_{SP}



M. Biroth, et al., Silicon photomultiplier properties at cryogenic temperatures, Nucl. Instr. Meth. A 787 (2015) 68-71

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PDE and Crosstalk

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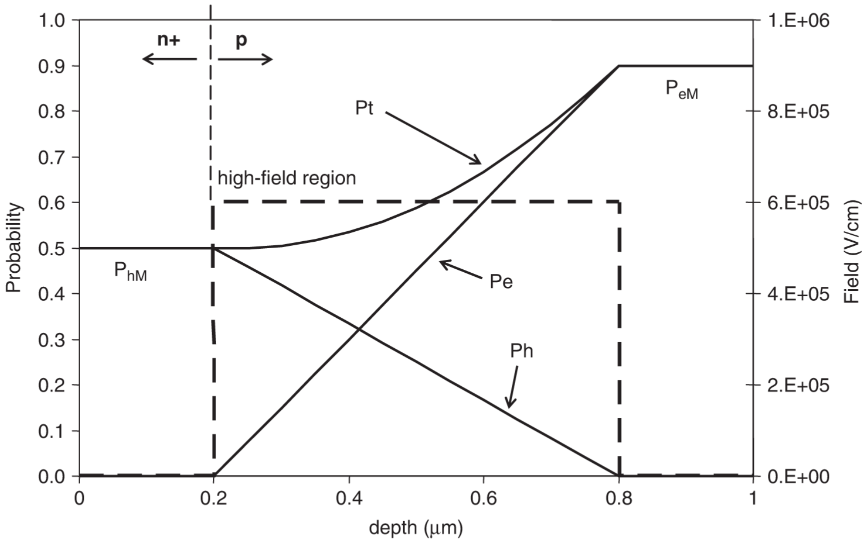
Influence of the Doping Profile to Avalanche Multiplication

- Electron-Hole pairs induced by absorbed light or particles will be accelerated in the high field region.
- The avalanche triggering probability (ATP) describes the turn-on probability of the charge build up.
- Depending on the penetration depth the avalanche can be purely electron or hole triggered.
- The used SensL SiPM has a p+n profile and optical light absorbed at the top of the high field region induces electron triggered avalanches.



C. Piemonte, A new Silicon Photomultiplier structure for blue light detection
Nucl. Instr. Meth A 568 (2006) 224-232

Doping	Top	Bottom
n+p	holes	electrons
p+n	electrons	holes

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Model for the Avalanche Triggering Probability

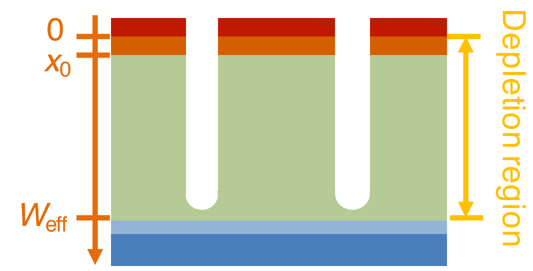
The ATP is a probability distribution of the number of ionizations γ. The ballot box model describes the turn-on probability for $\kappa = 2$ randomly distributed carriers (e-h-pair) in the depletion region of the junction.

$$P_{\mathrm{AT}}^{\mathrm{e}\|\mathrm{h}}[\gamma] = 1 - \gamma_{\mathrm{e}\|\mathrm{h}}^{-k} := 1 - \gamma_{\mathrm{e}\|\mathrm{h}}^{-2}$$

R.J. McIntyre, Theory of microplasma instability in silicon
J. Appl. Phys. 32 (1961) 983-995

In McIntyre's simple model the number of ionizations (γ - 1) $\propto$ V_{OV} is proportional to the overvoltage. But this is not valid for low temperature conditions. Therefore γ has to be calculated under sufficient assumptions.

$$\gamma_{\mathrm{e}\|\mathrm{h}} = \int_0^{x_0} \mathrm{d}x\ \alpha_{\mathrm{e}\|\mathrm{h}} + \int_{x_0}^{W_{\mathrm{eff}}} \mathrm{d}x\ \alpha_{\mathrm{h}\|\mathrm{e}}$$



The integral is defined over the junction of width W_{eff},
wherein α_{e} and α_{h} are the ionization coefficients of electrons and holes.

Simplify the Integral Expression of the Number of Ionizations

Reducing the integral by considering only the triggering carrier and assume a box like electrical field ε with height ε_{max}.

$$\gamma_{\mathrm{e}\,\|\mathrm{h}} \approx \int_0^{W_{\mathrm{eff}}} \mathrm{d}x \;\; \alpha_{\mathrm{e}\|\mathrm{h}} \approx W_{\mathrm{eff}} \cdot \alpha_{\mathrm{e}\|\mathrm{h}}[\mathcal{E}_{\mathrm{max}}]$$

R.J. McIntyre, Theory of microplasma instability in silicon J. Appl. Phys. 32 (1961) 983-995

The effective width of the depletion layer can be calculated in dependence of the total potential V_{tot}. Therein $\varphi_{\mathrm{BI}} = 2k_{\mathrm{B}}T \cdot \log\left[\sqrt{N_{\mathrm{A}}N_{\mathrm{D}}}/n_{\mathrm{i}}\right]$ is the build-in electrostatic potential, $w = \sqrt{2\varepsilon_0\varepsilon_{\mathrm{Si}}/e_0 N_{\mathrm{B}}}$ the width parameter depending on the background doping $N_{\mathrm{B}} \equiv N_{\mathrm{D}}$ what equals the donor concentration for p+n semiconductors.

$$W_{\mathrm{eff}} = w\, V_{\mathrm{tot}}^{1/2} \quad \text{with} \quad V_{\mathrm{tot}} = \underbrace{\frac{\varphi_{\mathrm{BI}} - 2k_{\mathrm{B}}T}{e_0} + V_{\mathrm{BD}}}_{=:V_{\mathrm{BD}}^{\mathrm{tot}}} + V_{\mathrm{OV}}$$

S.M. Sze, Physics of semiconductor devices John Wiley & Sons (1981)

The maximum field is proportional to average field ⟨ε⟩ and scaled by the doping profile.

$$\langle\mathcal{E}\rangle = \frac{V_{\mathrm{tot}}}{W_{\mathrm{eff}}} \qquad \mathcal{E}_{\mathrm{max}} = 2\,\langle\mathcal{E}\rangle = \frac{2}{w}\, V_{\mathrm{tot}}^{\frac{1}{2}}$$

A. Goetzberger, et al., Avalanche effects in silicon p-n junctions. II. structurally perfect junctions. J. Appl. Phys. 34 (1963) 1591-1600

Model for the Ionization Coefficient

There are many theoretical approaches how to approximate the Boltzmann transport equation. They differ in the definition of the threshold energies $\epsilon_{\mathrm{pair}}^{\mathrm{e}\|\mathrm{h}}, \hbar\omega_{\mathrm{opt}}$ for pair production and optical phonon scattering and the mean free paths $\Lambda_{\mathrm{pair}}^{\mathrm{e}\|\mathrm{h}}, \Lambda_{\mathrm{opt}}$.

- Medium field strength W. Shockley, Solid-State Electr. 2 (1961) 35-67
- High field strength P.A. Wolff, Phys. Rev. 128 (1962) 2507-2517
- Consider Angular distribution G.A. Baraff, Phys. Rev. 128 (1962) 2507-2517
- Introduced field transformation Y. Okuto, and C.R. Crowell, Phys. Rev. B 6 (1972) 3076-3081
- Universal approach K.K. Thornber, J. Appl. Phys. 52 (1981) 279-290
- Consider energy dependence of mean free paths B.K. Ridley, J. Phys. C 16 (1983) 4733-4751 / Y.Z. Chen, and T.W. Tang, J. Appl. Phys. 65 (1989) 4279-4286

In this work the model of Wolff for high field strengths is used, which is orientating at gas discharge. The parameter $r_{\mathrm{e}\|\mathrm{h}} = \Lambda_{\mathrm{pair}}^{\mathrm{e}\|\mathrm{h}} / \Lambda_{\mathrm{opt}}$ was adopted from Shockley's "Lucky electron" model.

$$\alpha_{e\|h}[\mathcal{E}] = \frac{e_0\,\mathcal{E}}{r_{\mathrm{e}\|\mathrm{h}}\,\hbar\omega_{\mathrm{opt}}} \cdot \exp\left[-\frac{3\,\epsilon_{\mathrm{pair}}^{\mathrm{e}\|\mathrm{h}}\,\hbar\omega_{\mathrm{opt}}}{\left(e_0\,\Lambda_{\mathrm{opt}}\,\mathcal{E}\right)^2}\right]$$

Calculation of the Avalanche Triggering Probability

Combining the previous results leads to an analytical expression for the ATP with 2+1 parameters. The Heaviside-theta function shrinks the range to positive values.

$$P_{\mathrm{AT}}^{e\|h}[V_{\mathrm{tot}}] = \Theta[V_{\mathrm{tot}} - V_{\mathrm{BD}}^{\mathrm{tot}}] \cdot \left\{ 1 - \left(\frac{r_{\mathrm{e\|h}}\,\hbar\omega_{\mathrm{opt}}}{2\,e_0\,V_{\mathrm{tot}}} \right)^2 \cdot \exp\left[\frac{3}{2} \left(\sqrt{\frac{\hbar\omega_{\mathrm{opt}}}{e_0}} \frac{w}{\Lambda_{\mathrm{opt}}} \right)^2 \cdot \frac{\epsilon_{\mathrm{pair}}^{\mathrm{e\|h}}}{e_0\,V_{\mathrm{tot}}} \right] \right\}$$

The threshold energies for pair production depend from the indirect band gap ΔE_{indi}. Their sum is $\epsilon_{\mathrm{pair}}^{\mathrm{e}} + \epsilon_{\mathrm{pair}}^{\mathrm{h}} = 3\,\Delta E_{\mathrm{indi}}$. The mixing parameter $\chi = m_{\mathrm{v}}^{*}/m_{\mathrm{c}}^{*}$ is given by the ratio from the masses of a hole at the top of the valence band and an electron at the bottom of the conduction band.

$$\epsilon_{\mathrm{pair}} = \begin{cases} \frac{2+\chi}{1+\chi} \cdot \Delta E_{\mathrm{indi}} = 1.37\,\Delta E_{\mathrm{indi}} & | \ \mathrm{e-triggered} \\ \frac{1+2\chi}{1+\chi} \cdot \Delta E_{\mathrm{indi}} = 1.63\,\Delta E_{\mathrm{indi}} & | \ \mathrm{h-triggered} \end{cases}$$

C.L. Anderson, and C.R. Crowell, Threshold energies for electron-hole pair production by impact ionization in semiconductors, Phys. Rev. B 5 (1972) 2267-2272

For the threshold for scattering of optical phonons is given as identical for electrons and holes. It can be identified in dispersion calculations as the transverse optical branch.

$$\hbar\omega_{\mathrm{opt}} = 63\ \mathrm{meV}$$

S.M. Sze, Physics of semiconductor devices (1981) John Wiley & Sons

Model for the Photon Detection Efficiency

The photon detection efficiency η (PDE) is proportional to the wavelength specific quantum efficiency ξ (QE) and the avalanche triggering probability P_{AT}.

$$\eta_{\mathrm{e}\|\mathrm{h}}[\lambda] \propto \xi[\lambda] \cdot P_{\mathrm{AT}}^{\mathrm{e}\|\mathrm{h}}$$

The number of incident photons creating an avalanche is reduced by the PDE. In case of light coupling to the surface the avalanche is electron triggered for p+n.

$$\phi = \eta_{\mathrm{e}}[\lambda] * \phi_{\mathrm{true}}[\lambda] \overset{\lambda\,=\,\mathrm{const}}{=} \eta_{\mathrm{e}}\ \phi_{\mathrm{true}} \propto \xi\ \phi_{\mathrm{true}}\ P_{\mathrm{AT}}^{\mathrm{e}}$$

The charge created by these initial photons is calculated by aid of the gain G_{SP}.

$$Q_{\mathrm{init}} = \phi\ G_{\mathrm{SP}} \propto \xi\ \phi_{\mathrm{true}}\ P_{\mathrm{AT}}^{\mathrm{e}}\ \frac{C_{\mathrm{eff}}\ V_{\mathrm{OV}}}{e_0}$$

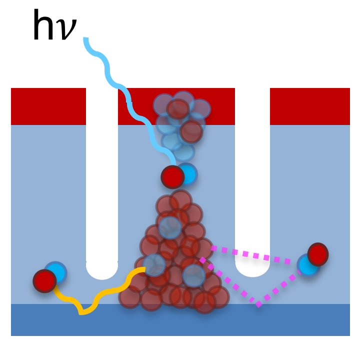
Optical Crosstalk and Hot Carrier Emission

Hot avalanche emits optical photons in the infrared-to-yellow range. The intensity is proportional to the initial charge.

R.H. Haitz, Studies on optical coupling between p-n junctions Solid-State Electr. 8 (1965) 417-425

Hot avalanche emits charge carriers with high kinetic energy whose can tunnel through the trenches.

C.T. Wang, Hot carrier design considerations for MOS devices and circuits Springer US (1992)



Number of photons which couple to neighboring pixels

$$\phi_{\mathrm{optCT}} = \kappa_{\mathrm{opt}}[\lambda_{\mathrm{optCT}}] \cdot \frac{Q_{\mathrm{init}}}{e_0}$$

Number of carriers which couple to neighboring pixels

$$N_{\overline{\mathrm{optCT}}} = \kappa_{\overline{\mathrm{opt}}} \frac{Q_{\mathrm{init}}}{e_0}$$

Model for the Crosstalk Probability

- Optical crosstalk depends not only from the coupling κ_{opt} but from the PDE at the bottom of the depletion layer to produce electron-hole-pairs
- Non-optical crosstalk depends from the ATP at the bottom of the depletion layer and the coupling $\kappa_{\overline{\text{opt}}}$ which is affected by the carrier mobility

$$Q_{\text{CT}} = \left(\eta_{\text{h}}[\lambda_{\text{optCT}}] * \phi_{\text{optCT}}[\lambda_{\text{optCT}}] + N_{\overline{\text{optCT}}} \cdot P_{\text{AT}}^{\text{h}}\right) \cdot G_{\text{SP}}$$

$$= \left(\xi[\lambda_{\text{optCT}}] * \phi_{\text{optCT}}[\lambda_{\text{optCT}}] + N_{\overline{\text{optCT}}}\right) \cdot P_{\text{AT}}^{\text{h}} G_{\text{SP}}$$

$$= Q_{\text{init}} \cdot \left(\xi[\lambda_{\text{optCT}}] * \kappa_{\text{opt}}[\lambda_{\text{optCT}}] + \kappa_{\overline{\text{opt}}}\right) \cdot P_{\text{AT}}^{\text{h}} \frac{G_{\text{SP}}}{e_0}$$

The crosstalk probability (CTP) P_{CT} can be calculated from the ratio of secondary and initial charge. It depends linearly from the overvoltage.

$$P_{\text{CT}} = \int_0^\infty \mathrm{d}\lambda_{\text{optCT}} \frac{Q_{\text{CT}}}{Q_{\text{init}}} = \left(\left\langle \xi * \kappa_{\text{opt}} \right\rangle_{\lambda_{\text{optCT}}} + \kappa_{\overline{\text{opt}}}\right) \cdot P_{\text{AT}}^{\text{h}} \frac{C_{\text{eff}} V_{\text{OV}}}{e_0}$$

Combined Curve-fit of PDE and CTP

As well the PDE as the CTP are proportional to the ATP.

$$\frac{\phi}{\phi_{\mathrm{true}}^{\infty}} = P_{\mathrm{AT}}^{\mathrm{e}}[V_{\mathrm{tot}}] \qquad \frac{P_{\mathrm{CT}}}{P_{\mathrm{CT}}^{\infty}} = P_{\mathrm{AT}}^{\mathrm{h}}[V_{\mathrm{tot}}] \cdot \frac{C_{\mathrm{eff}}}{e_0} \cdot (V_{\mathrm{tot}} - V_{\mathrm{BD}}^{\mathrm{tot}})$$

- Because of the shared parameter set a combined fit to the intensity and crosstalk data is sufficient.

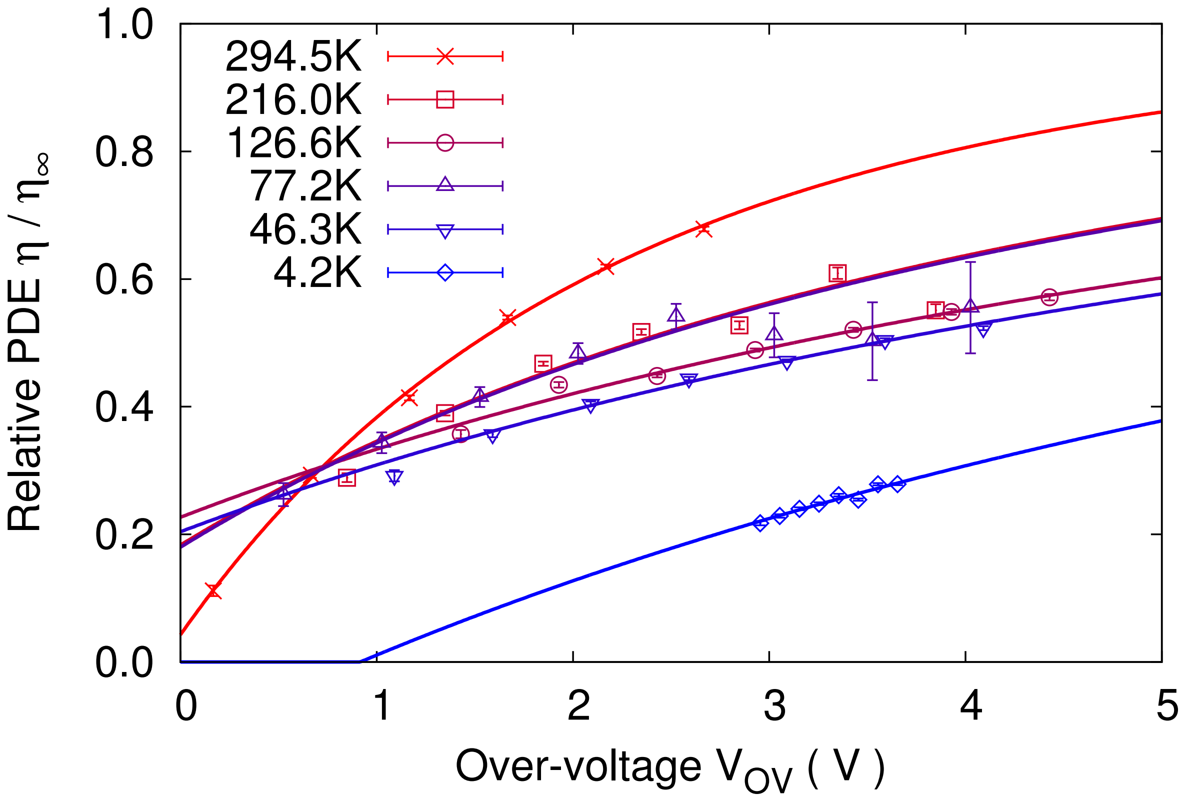
- The 2+1 parameters (not shown) of the ATP are correlated with optical phonon mean free path and have to follow Bose-Einstein statistics. The mean free path can be parameterized as $\Lambda_{\mathrm{opt}} = \Lambda_{\mathrm{opt,0}} \cdot \tanh\left[\frac{\hbar\omega_{\mathrm{opt}}}{2k_{\mathrm{B}}T}\right]$ with its value $\Lambda_{\mathrm{opt,0}}$ at 0 K.

S.M. Sze, Physics of semiconductor devices
John Wiley & Sons (1981)

- Charge carrier mobility effects are not included in the model. Therefore, the 2+1 fit parameters of the ATP do not converge correctly at 77 K.

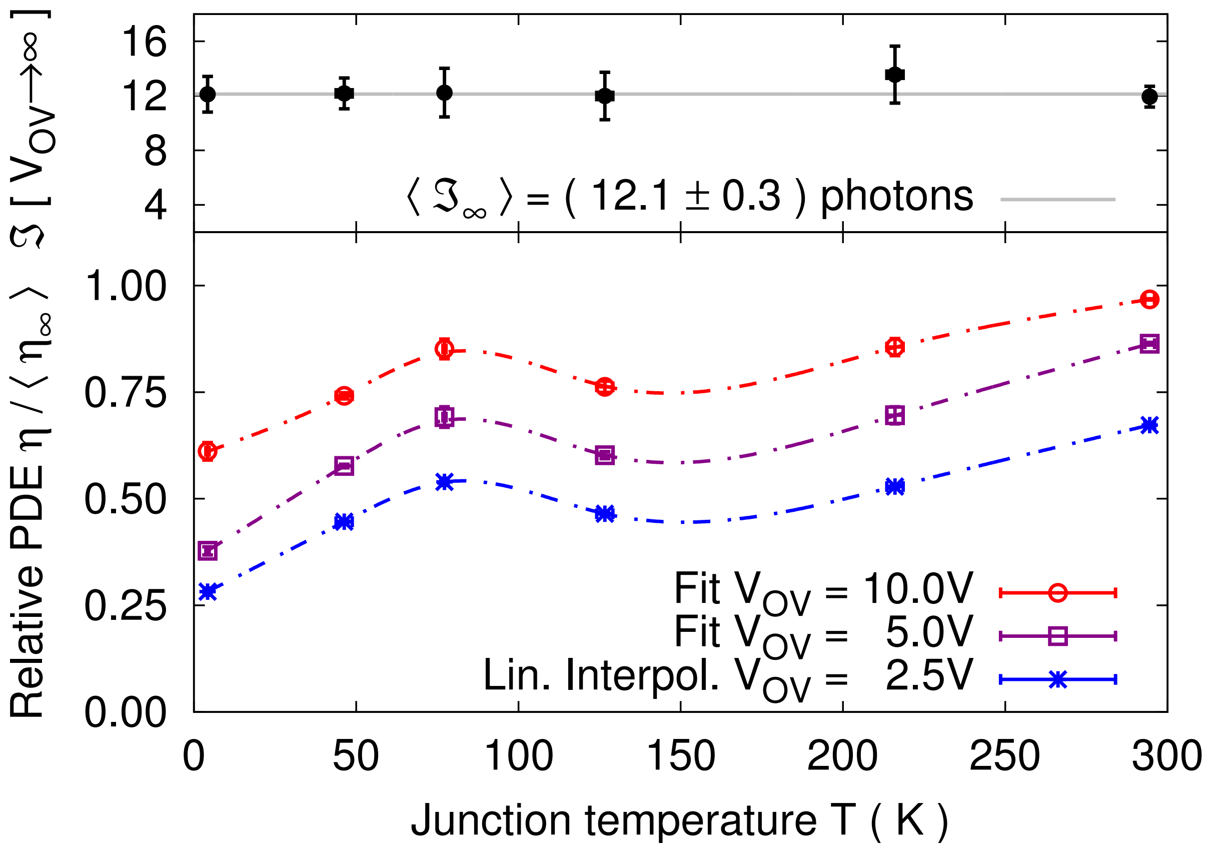
Results from the Intensity Curve-fit

Interesting are the different breakdown voltages for turn-on of the Geiger mode amplification and start of photon detection sensitivity, especially in case of LHe.



The Measured Photon Detection Efficiency

All independent intensity fits share the same saturation value for $V_{\mathrm{OV}} \rightarrow \infty$. This means the QE is not or slightly temperature dependent ${}^{\mathrm{d}\xi}/_{\mathrm{d}T} \approx 0$.



Discussion of Degenerative Effects on the PDE

- $T > 150$ K
  - PDE decreases because of optical phonon freeze-out
  - Direct band gap of silicon without phonon assistance $\Delta E_{\mathrm{di}} = 3.4\mathrm{eV}$ ($\Gamma_{25'} \longrightarrow \Gamma_{15}$) is large compared to $\Delta E_{\mathrm{indi}} = 1.1\mathrm{eV}$ ($\Gamma_{25'} \longrightarrow X_1$)
  - Global trend of PDE follows the photo-absorption coefficient α_{photo} as the emission dominated sum over all phonon branches

$$\alpha_{\mathrm{photo}} \approx \sum_{\text{all branches}} A^2_{\mathrm{phoNon}} \cdot \frac{\left(\hbar\omega_{\mathrm{phoTon}} - \Delta E_{\mathrm{indi}} - \hbar\omega_{\mathrm{phoNon}}\right)^2}{1 - \exp\left[-\frac{\hbar\omega_{\mathrm{phoNon}}}{k_{\mathrm{B}} T}\right]}$$

S.E. Aw, et al., Optical absorption measurements of band-gap shrinkage in moderately and heavily doped silicon
J. Phys.: Condens. Mat. 3 (1991) 8213-8223

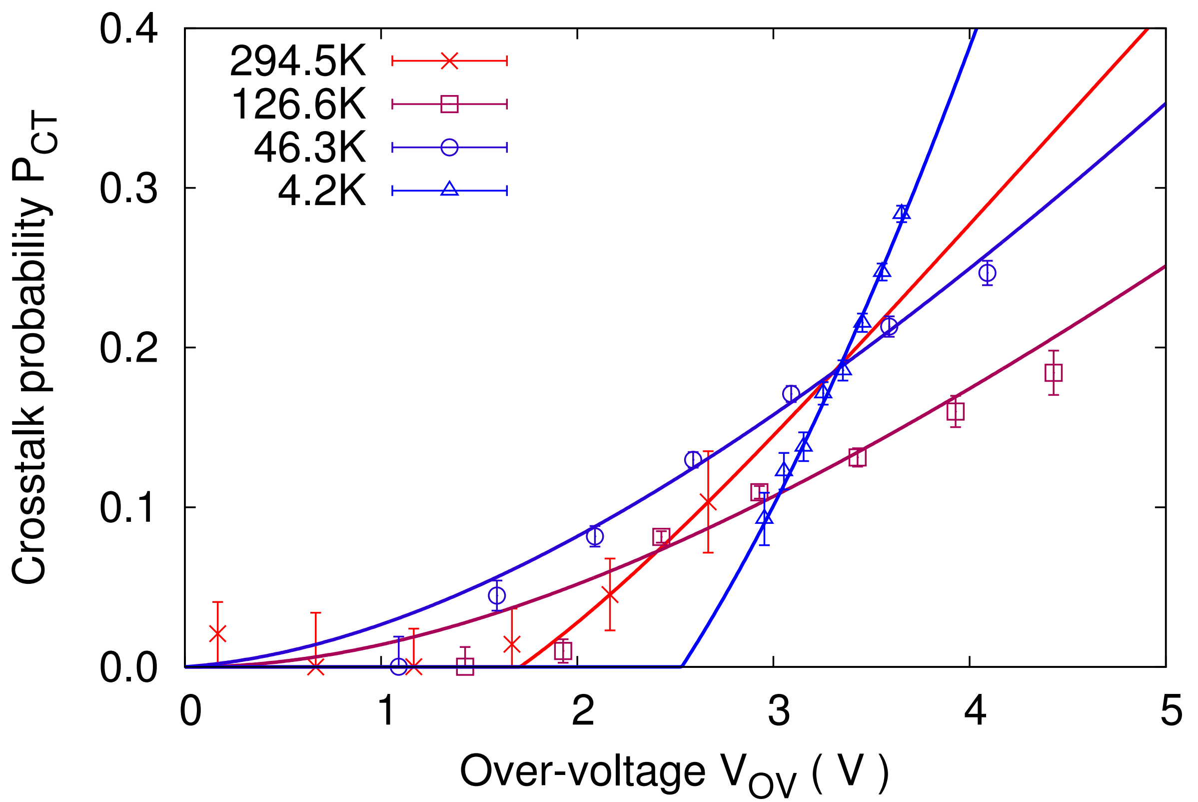
- $T > 77$ K
  - PDE increases because of increasing charge carrier mobility

- $T > 0$ K
  - PDE decreases because of charge carrier freeze-out compensates charge carrier mobility effects

G. Collazuol, Studies of silicon photomultipliers at cryogenic temperatures
Nucl. Instr. Meth. A 628 (2011) 389-392

Results from the Crosstalk Curve-fit

The measured value in this work $P_{\mathrm{CT}}[T = 294.5\ \mathrm{K}] = (9 \pm 3)\%$ matches the reference value from the datasheet $P_{\mathrm{CT}}^{\mathrm{ref}}[T = 294.2\ \mathrm{K}] = 7\%$, both at $V_{\mathrm{OV}} = 2.5\ \mathrm{V}$.



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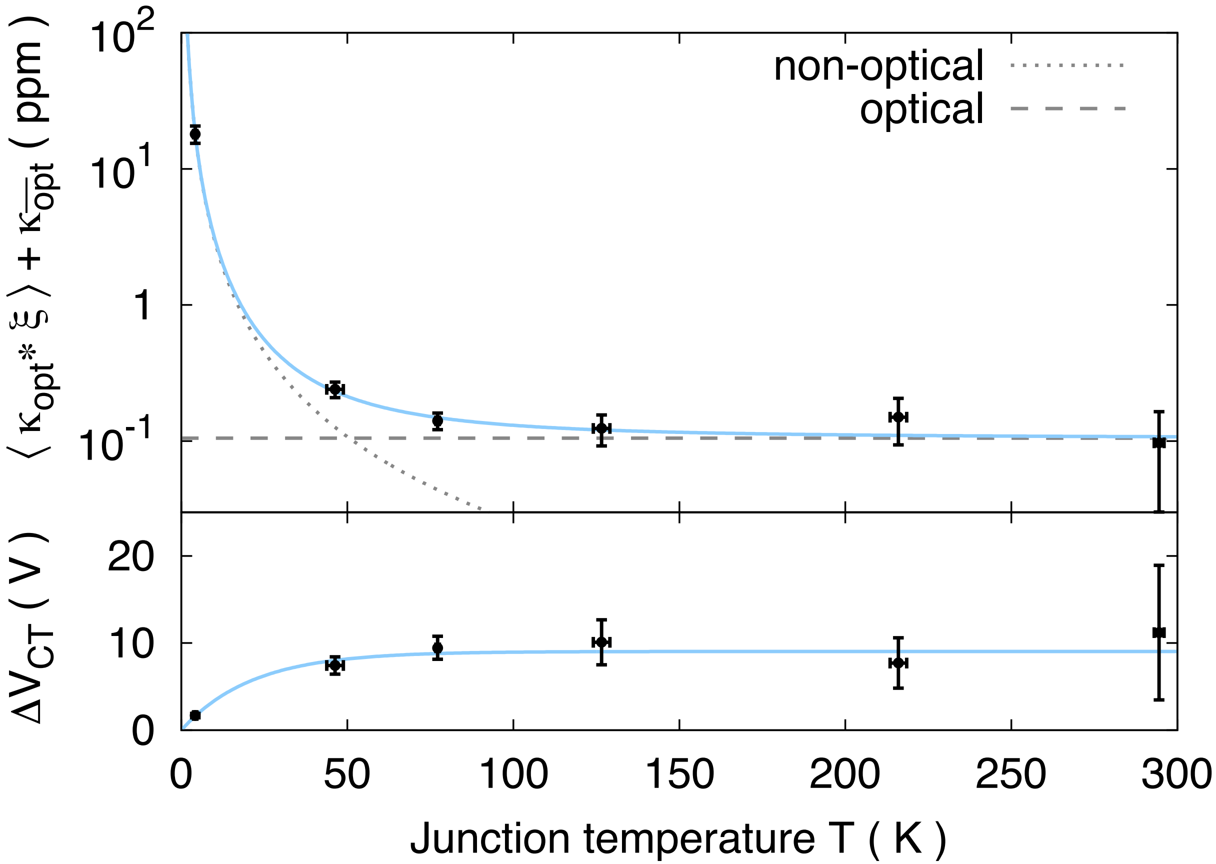
The Crosstalk Coupling Coefficients

The optical coupling $\langle \kappa_{\text{opt}} * \xi \rangle = (0.10 \pm 0.02)$ ppm is supposed to be constant over temperature like the QE. The non-optical coupling follows the charge carrier mobility with exponent $m = (-2.1 \pm 0.2)$ and matches therefore the reference value $m^{\text{h}}_{\text{ref}} = (-2.3 \pm 0.2)$. Saturation effects at high fields could explain the not-significant lower value.



S.S. Li, The dopant density and temperature dependence of hole mobility and resistivity in boron doped silicon
Solid-St. Electr. 21 (1978) 1109-1117

$$\kappa_{\overline{\text{opt}}} = 10^{-6} \cdot \left(\frac{k_{\text{B}}T}{\epsilon_\mu}\right)^m$$

The maximal usable overvoltage $\Delta V_{\text{CT}} = V_{\text{OV}}[P_{\text{CT}} := 1]$ has a constant value over a wide temperature range:

$$\Delta V^{\text{max}}_{\text{CT}} = (9.0 \pm 0.6)\ \text{V}$$

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Single-pixel Resolution

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Definition of the Single-pixel Resolution

The single-pixel resolution is a SiPM specific definition of the signal-to-noise ratio. It is based on the Gaussian distribution of the single-pixel capacity variation. Its FWHM value is given by Γ_n if *n* pixels are involved in the charge collection process.

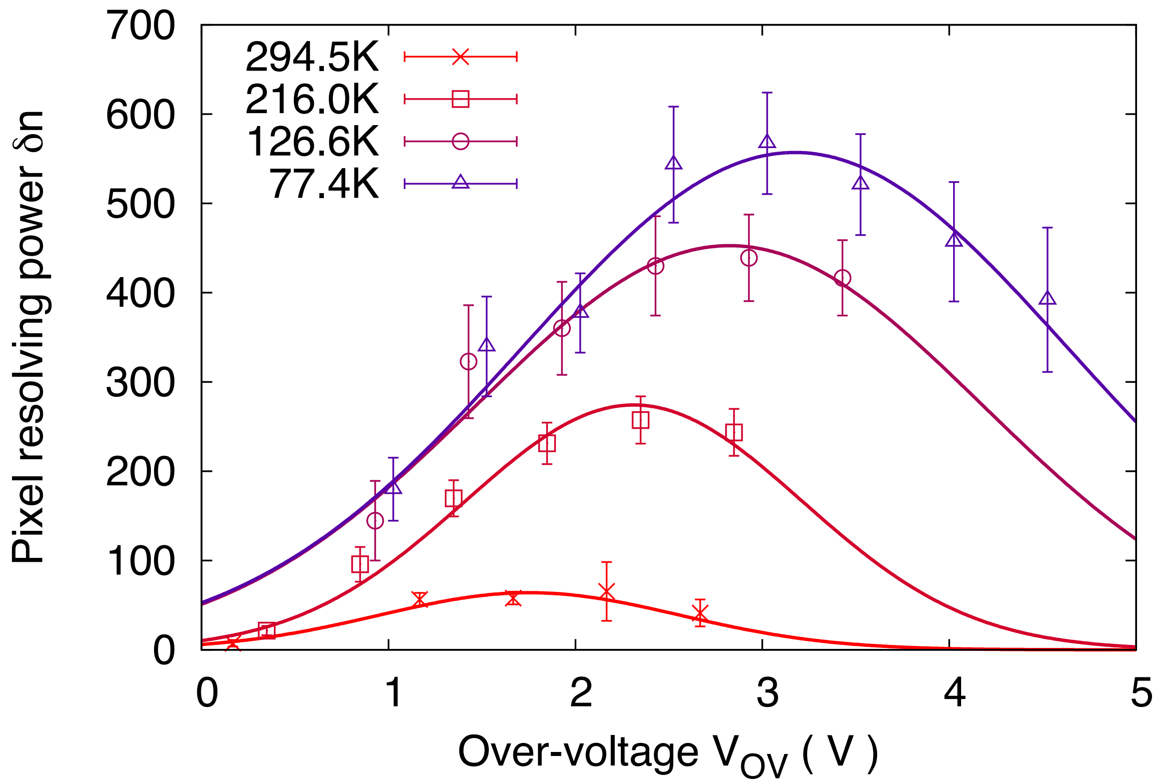
$$\Gamma_n = \sqrt{8 \log 2 \cdot \left(\sigma_0^2 + n\, \sigma_{SP}^2\right)}$$

The single-pixel resolving power δn can be defined as the order, from which two neighboring peaks in the charge spectrum are overlapping.

$$\frac{\Gamma_{\delta n} + \Gamma_{\delta n+1}}{2} \stackrel{\text{def}}{=} G_{SP} \qquad \delta n = \frac{G_{SP}^2 - 8 \log 2 \cdot \sigma_0^2}{8 \log 2 \cdot \sigma_{SP}^2} + \frac{1}{2}\left(\log 2 \cdot \left(\frac{\sigma_{SP}}{G_{SP}}\right)^2 - 1\right)$$

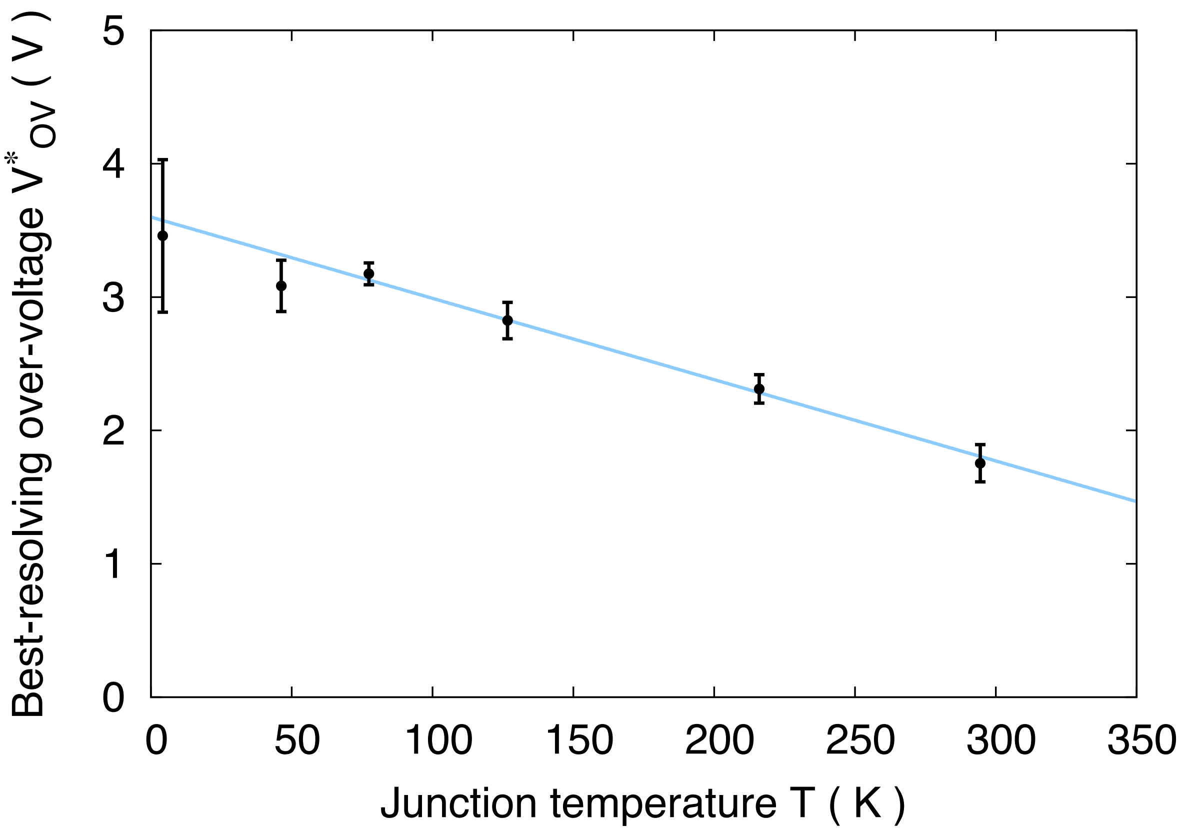
The Single-pixel Resolving Power

The resolving power follows a Gaussian distribution for each temperature setting. Therefore, it exists an optimum overvoltage for the best charge peak resolution.



The Optimum-resolving Overvoltage

The overvoltage has to be decreased linearly with increasing temperature because the noise rate rises.



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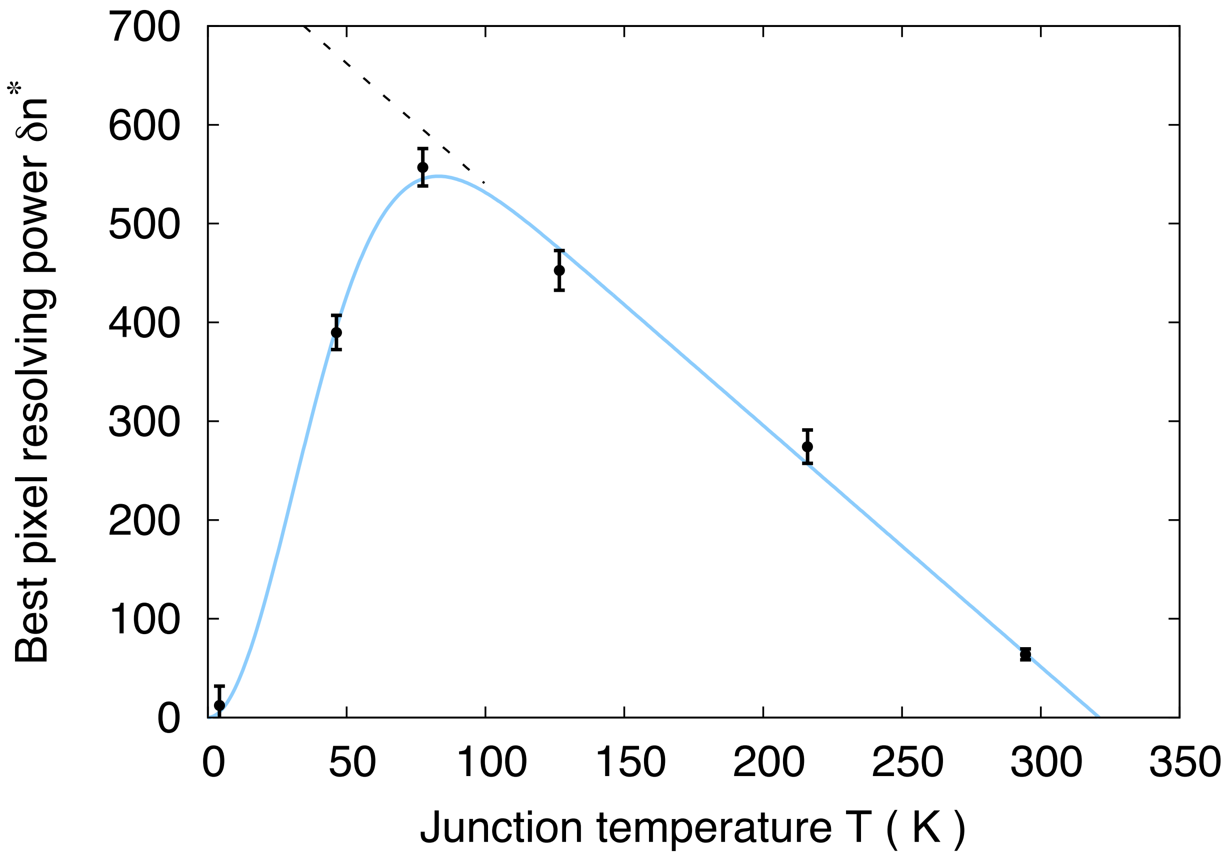
The Maximum Reachable Resolving Power

The maximum resolving power has an optimum at LN temperature and decreases for lower temperature because the increasing charge carrier mobility destroys the single-pixel charge uniformity (see non-optical crosstalk).



Thank you for your attention

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Gain and Breakdown Voltage

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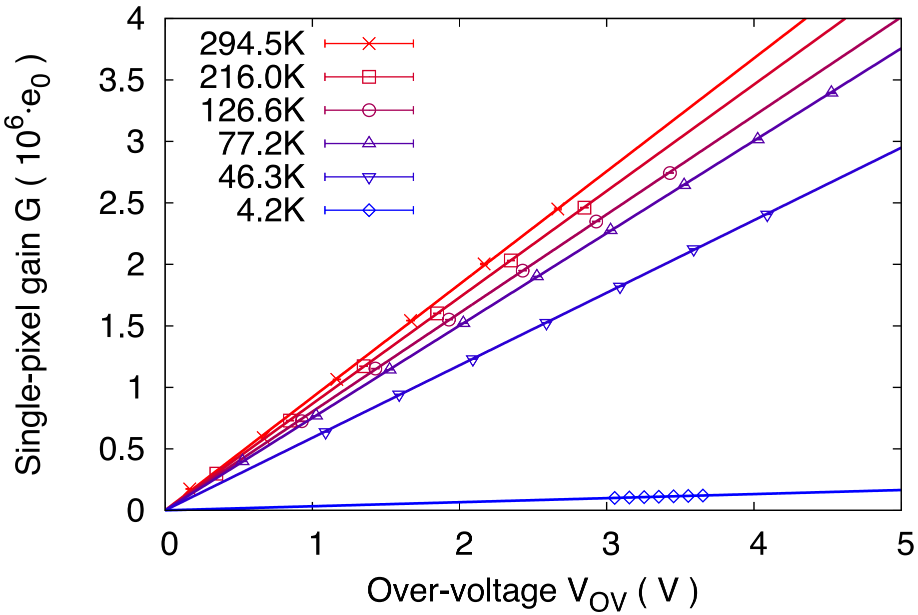
Extracted Single-pixel Gain from Peak Distance

The single-pixel gain G_{SP} is the product from the effective single-pixel capacity C_{eff} and the overvoltage V_{OV}

$$G_{SP}[T,V] = C_{eff}[T] \cdot (V - V_{BD}[T]) = C_{eff}[T] \cdot V_{OV}[T,V]$$



- The usable overvoltage range at LHe is limited
- The single-pixel capacity (slope) drops with temperature

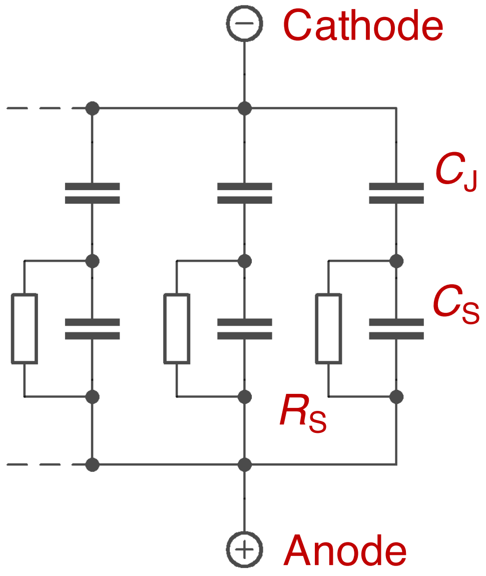
Single-pixel capacity should be constant over temperature (see Zecotek MAPD-3N at LHe temperature)

M. Biroth, et al., Silicon photomultiplier properties at cryogenic temperatures Nucl. Instr. Meth. A 787 (2015) 68-71

The Equivalent Circuit of the Effective Single-pixel Capacitance

The junction capacitance C_J is connected in series with series resistance R_S and its stray capacitance C_S. The size of the series network effects the measured effective single-pixel capacitance C_{eff}.

B.M. Vul, and É.I. Zavaritskaya, The capacitance of p-n junctions at low temperatures, J. Exp. Theo. Phys. 11 (1960) 6-11



$$C_{\mathrm{eff}}[\omega,T] = -\frac{1}{\omega}\cdot\Im\left[\frac{1}{j\omega\, C_J} + \frac{R_S[T]}{1 + j\omega\, C_S\, R_S[T]}\right]^{-1}$$

$$= C_J \cdot \frac{1 + \omega^2\, C_S^2\, R_S^2[T]}{1 + \omega^2\, C_S \cdot (C_J + C_S)\cdot R_S^2[T]}$$

The frequency dependency can be solved by integration over the full acquired bandwidth $\Delta\omega$.

$$C_{\mathrm{eff}}[T] = \frac{1}{\Delta\omega}\int_{\Delta\omega} \mathrm{d}\omega\, C_{\mathrm{eff}}[\omega,T] \approx \frac{C_J\, C_S}{C_J + C_S}\cdot\left(1 + \frac{C_J}{C_S}\cdot\frac{\tan^{-1}\left[\Delta\omega\sqrt{C_S\cdot(C_J + C_S)}\, R_S[T]\right]}{\Delta\omega\sqrt{C_S\cdot(C_J + C_S)}\, R_S[T]}\right)$$

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The Single-pixel Series Resistance

For a SiPM the series resistance is the sum from the background low-doped silicon and the quenching resistor.

$$R_S = R_{Base} + R_Q$$

For not-degenerated doped junctions the charge carrier freeze-out blows up the base resistance and a higher electrical field is necessary to ionize the impurities. In case of a p+n structure $\epsilon_{BG} := \epsilon_{nSi}$ is the ionization energy of the donor impurities.

$$R_{Base} \propto \exp[\epsilon_{BG}/k_B T] := \exp[\epsilon_{nSi}/k_B T]$$

B.M. Vul, and É.I. Zavaritskaya, The capacitance of p-n junctions at low temperatures, J. Exp. Theo. Phys. 11 (1960) 6-11

The Single-pixel Quenching Resistance

The quenching resistance can be extracted from forward voltage measurements.

M. Biroth, et al., Modeling and characterization of SiPM parameters at temperatures between 95 K and 300 K IEEE Trans. Nucl. Sci. 64 (2017) 1619-1624

A common parameterization for poly-silicon resistors is $R_\mathrm{Q} \propto \sqrt{T} \cdot \exp[e_0 V_\mathrm{B}/k_\mathrm{B} T]$.

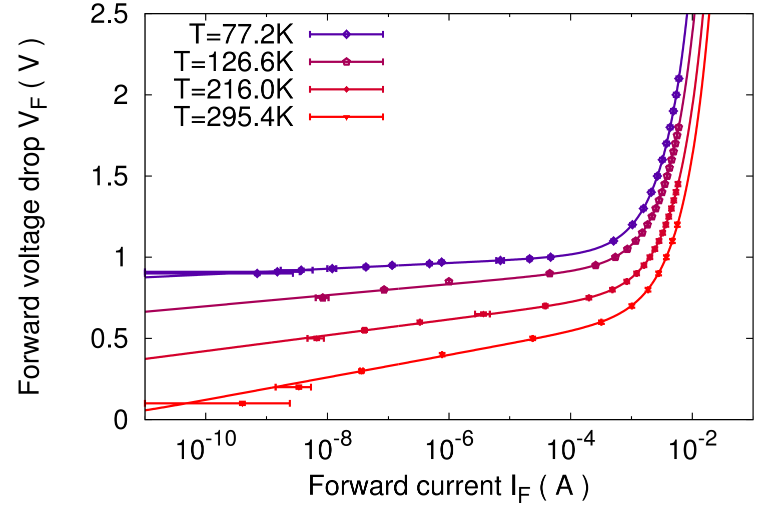
H. Muro, et al., Determination of electrical properties of n-type and p-type polycristalline silicon thin films as sensors Sens. Mat. 18 (2006) 433-444

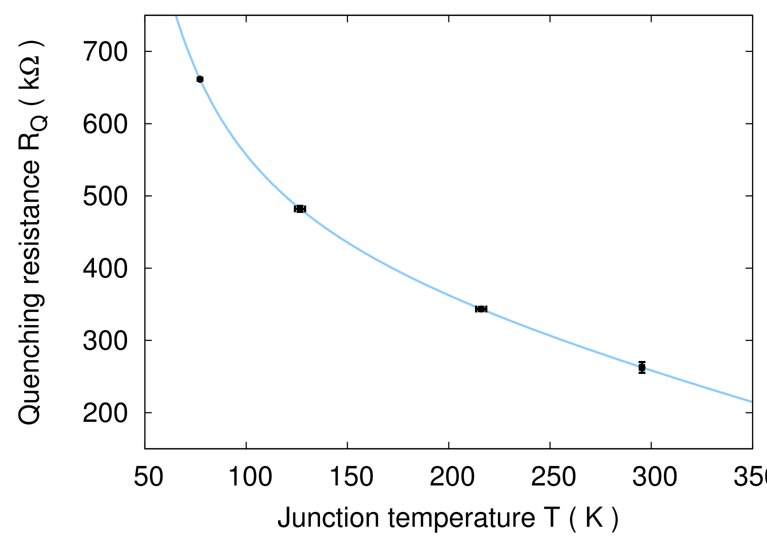
The SensL C-type shows a linear instead of the square-root behavior.

$$R_\mathrm{Q} = \left\{R_{\mathrm{Q},0} + \left(\frac{\mathrm{d}R_\mathrm{Q}}{\mathrm{d}T}\right)_\infty \cdot T\right\} \cdot \exp\left[\frac{e_0 V_\mathrm{B}}{k_\mathrm{B} T}\right]$$

The slope $(\mathrm{d}R_\mathrm{Q}/\mathrm{d}T)_\infty = (-677 \pm 4)\,\Omega/\mathrm{K}$ is not deviant from the reference value for the J-type $(\mathrm{d}R_\mathrm{Q}/\mathrm{d}T)_\infty^\mathrm{ref} = (-689 \pm 28)\,\Omega/\mathrm{K}$.

A.N. Otte, et al., Characterization of three high efficiency and blue sensitive silicon photomultipliers Nucl. Instr. Meth. A 846 (2017) 106-125





The Total Single-pixel Series Resistance

For a SiPM the series resistance is the sum from the background low-doped silicon and the quenching resistor.

$$R_S = R_{Base} + R_Q$$

For not-degenerated doped junctions the charge carrier freeze-out blows up the base resistance and a higher electrical field is necessary to ionize the impurities. In case of a p+n structure $\epsilon_{BG} := \epsilon_{nSi}$ is the ionization energy of the donor impurities.

$$R_{Base} \propto \exp[\epsilon_{BG}/k_B T] := \exp[\epsilon_{nSi}/k_B T]$$

B.M. Vul, and É.I. Zavaritskaya,
The capacitance of p-n junctions at
low temperatures, J. Exp. Theo.
Phys. 11 (1960) 6-11

Because $e_0 V_B = (3.79 \pm 0.02)\ \mathrm{meV} \ll \epsilon_{nSi}$ is true for all known ionization energies, the total series resistance is dominated at cryogenics by the freezing-out base silicon and at higher temperatures by the quenching resistor.

$$R_s \approx r_{Base} \cdot \exp\left[\frac{\epsilon_{nSi}}{k_B T}\right] + R_{Q,0} + \left(\frac{dR_Q}{dT}\right)_\infty \cdot T$$

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The Intrinsic Breakdown Voltage of the Junction

The voltage which is applied to the SiPM if a junction breakdown is observed should be called the extrinsic breakdown voltage V_{BD}. The voltage could be measured over the junction should be defined as the intrinsic breakdown voltage $V_{\mathrm{J}}^{\mathrm{BD}}$.

The intrinsic breakdown voltage has a constant value $V_{\mathrm{J},0}^{\mathrm{BD}}$ below 50 K, where all phonon branches are frozen out. At higher temperatures acoustical and optical branch, including longitudinal and transverse modes of lattice vibrations (LA, TA, LO, TO) are present. Scattering on longitudinal modes changes only momentum, while scattering on transverse phonon effects even the kinetic energy.

S. Mahadevan, et al., Electrical breakdown in semiconductors
Phys. Stat. Sol. 8 (1971) 335-374

This lost in energy with increasing temperature has to compensated by the electrical field $V_{\mathrm{J}}^{\mathrm{BD}} \propto 1/\Lambda_{\mathrm{phon}}$ and the intrinsic breakdown voltage increases, while Λ_{phon} is the mean free path for phonon scattering. The threshold energies of the transverse modes are $\hbar\omega_{\mathrm{TA}} = 18.6\ \mathrm{meV}$, $\hbar\omega_{\mathrm{TO}} = 63\ \mathrm{meV}$. They are mixing to the effective transverse value $\langle\hbar\omega\rangle_{\perp} = (27.5 \pm 0.8)\ \mathrm{meV}$ held from the curve-fit.

$$V_{\mathrm{J}}^{\mathrm{BD}} = V_{\mathrm{J},0}^{\mathrm{BD}} + \left(\frac{\mathrm{d}V_{\mathrm{J}}^{\mathrm{BD}}}{\mathrm{d}T}\right)_{\infty} \cdot \frac{\langle\hbar\omega\rangle_{\perp}}{2\ k_{\mathrm{B}}} \cdot \tanh\left[\frac{\langle\hbar\omega\rangle_{\perp}}{2\ k_{\mathrm{B}}T}\right]^{-1}$$

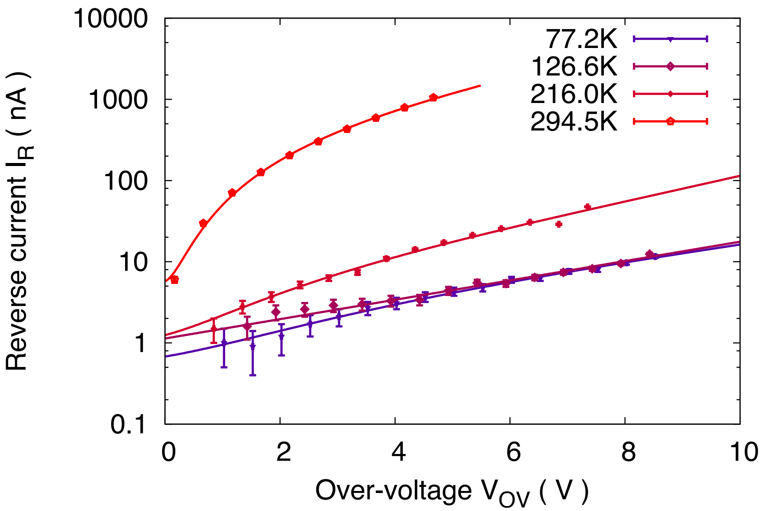
S.M. Sze, Physics of semiconductor devices
John Wiley & Sons (1981)

The Extrinsic Breakdown Voltage and Junction Resistance

At temperatures below 50 K the freezing-out series resistance reaches values in the order of the junction and cuts-off the electrical field. Therefore, extrinsic and intrinsic breakdown voltage are connected by a voltage divider formalism.

$$V_{\mathrm{BD}} = \left(1 + \frac{R_{\mathrm{S}}}{R_{\mathrm{J}}}\right) \cdot V_{\mathrm{J}}^{\mathrm{BD}}$$

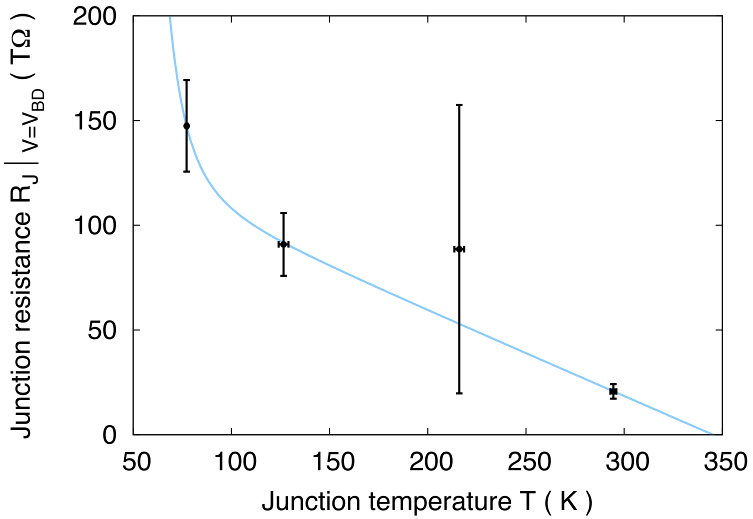
K.I. Nuttall, and M.W. Nield, Behavior of silicon pn junctions at temperatures between 4.2 and 300 K Int. J. Electr. 24 (1968) 69-78



To investigate the junction resistance, the dark current at $V_{\mathrm{OV}} = 0$ can be extrapolated from curve-fits to reverse current measurements $R_{\mathrm{J}} + R_{\mathrm{S}} = V_{\mathrm{BD}}/I_{\mathrm{R}}[V_{\mathrm{OV}} = 0]$.

This leads to a linear behavior with additional freeze-out term of the silicon in the depletion layer, analog to the series resistance.

$$R_{\mathrm{J}} = r_{\mathrm{DL}} \cdot \exp\left[\frac{\epsilon_{\mathrm{nSi}}}{k_{\mathrm{B}}T}\right] + R_{\mathrm{J},0} + \left(\frac{\mathrm{d}R_{\mathrm{J}}}{\mathrm{d}T}\right)_{\infty} \cdot T$$



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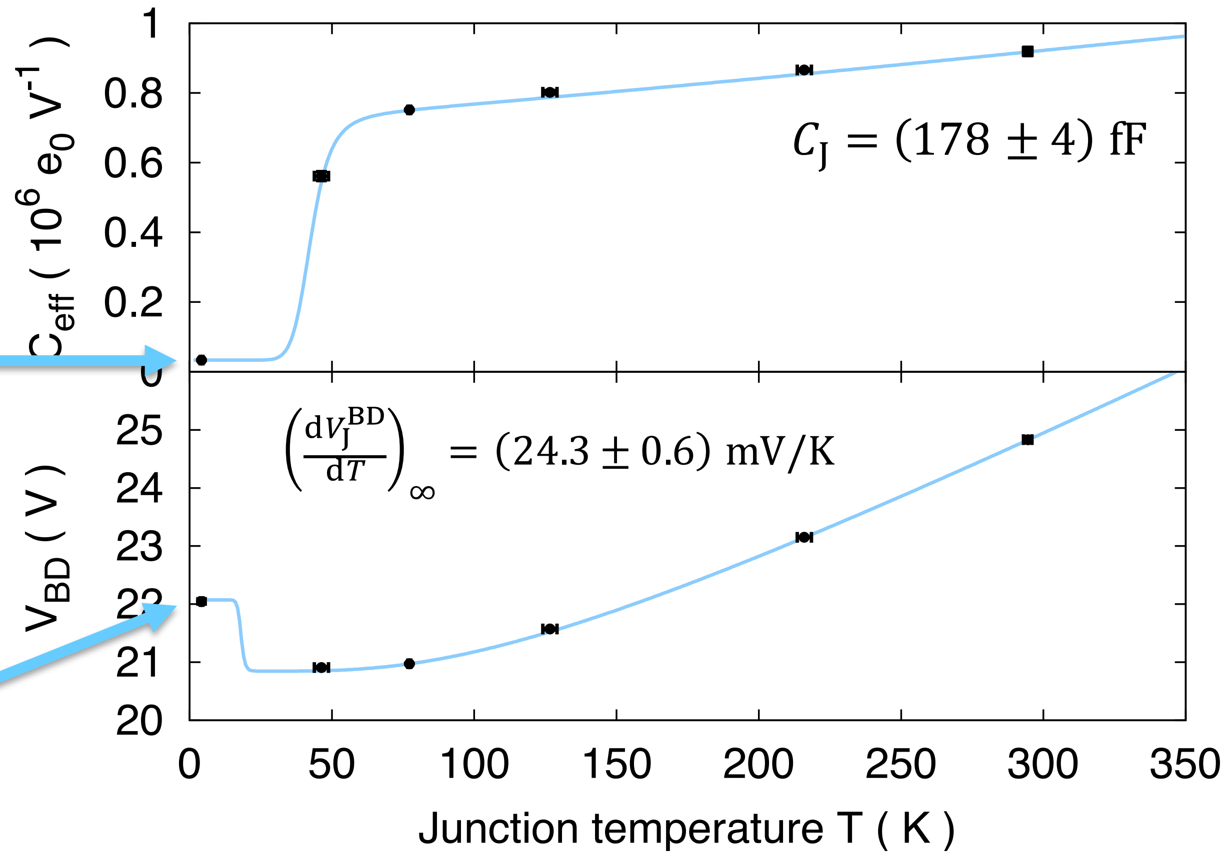
Results from the Combined Gain Curve-fit

Single-pixel capacitance and breakdown voltage can be described by a combined curve-fit ($\chi^2/\mathrm{dof} = 1.1$) using the knowledge about series and junction resistances.

Effective single-pixel capacitance saturates for $T \to 0$

$$C_\mathrm{eff} \to \frac{C_\mathrm{J}\, C_\mathrm{S}}{C_\mathrm{J} + C_\mathrm{S}}$$



Breakdown voltage saturates if junction is not-degenerated and freezes out

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The Charge Spectra Function

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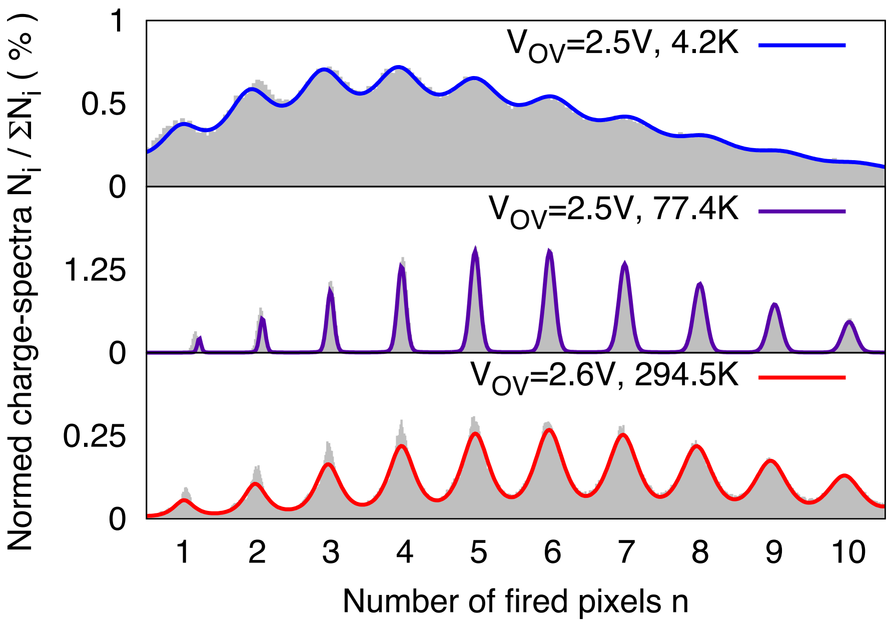
Mainz, Germany

Extracting SiPM Properties from Curve-fits to QDC Spectra



At low intensities ϕ of some photons per pulse the charge spectra are Poisson distributed

Properties can be extracted by a curve-fit

The spectrum S can be modeled as the product from a charge peak distribution Λ_n and a response probability function P_n

$$S[Q,\phi] = \mathrm{e}^{-\phi} \cdot \Lambda_0[Q] + \left(1 - \mathrm{e}^{-\phi}\right) \sum_{j=1}^{\infty} P_j[\phi]\, \Lambda_j[Q]$$

M. Biroth, et al., Silicon photomultiplier properties at cryogenic temperatures
Nucl. Instr. Meth. A 787 (2015) 68-71

The Analytically SiPM Spectrum Function

Response probability P_n for a n-pixel event in spectrum

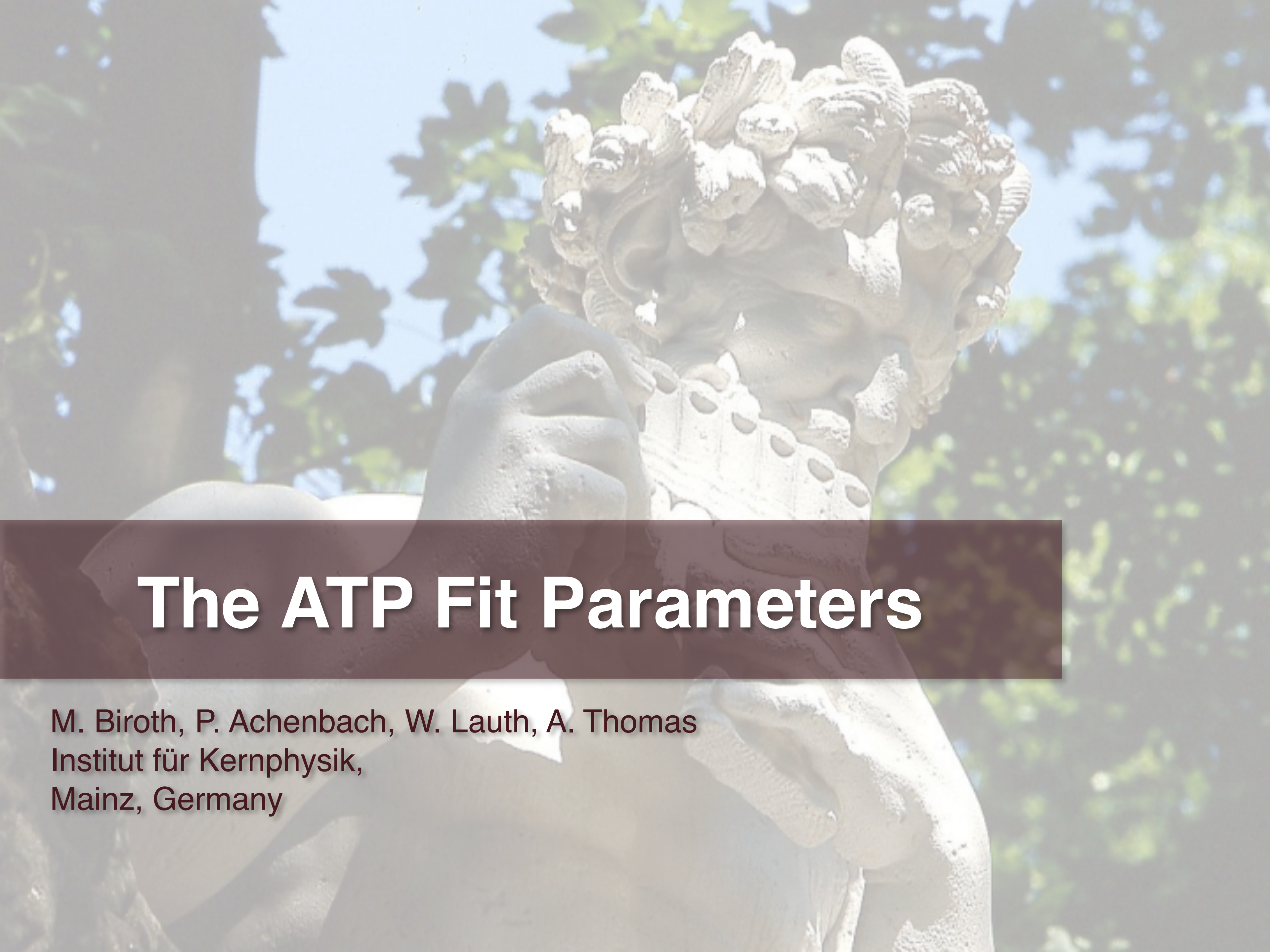
- Crosstalk is geometrical distributed with crosstalk probability P_{CT}
- Combinatorics of n-pixel events is given by the Binomial coefficient
- Intensity is Poisson distributed with mean number of photons ϕ
  Normalized term can be written with n-th Laguerre polynomial $\mathcal{L}_n$

$$P_n[\phi] = \frac{(1-P_{\mathrm{CT}}) \cdot P_{\mathrm{CT}}^n}{\mathrm{e}^{\phi} - 1} \cdot \left(\mathcal{L}_n \left[-\phi \, \frac{1-P_{\mathrm{CT}}}{P_{\mathrm{CT}}} \right] - 1 \right)$$

Charge peaks Λ_n of the n-th order are Gaussian distributed

- Pedestal position Q_0 and width σ_0
- Single-pixel gain G_{SP} and variation σ_{SP}

$$\Lambda_n[Q] \propto \exp\left[-\frac{1}{2} \frac{\{Q - (Q_0 + n\, G_{\mathrm{SP}})\}^2}{\sigma_0^2 + n\, \sigma_{\mathrm{SP}}^2} \right]$$



The ATP Fit Parameters

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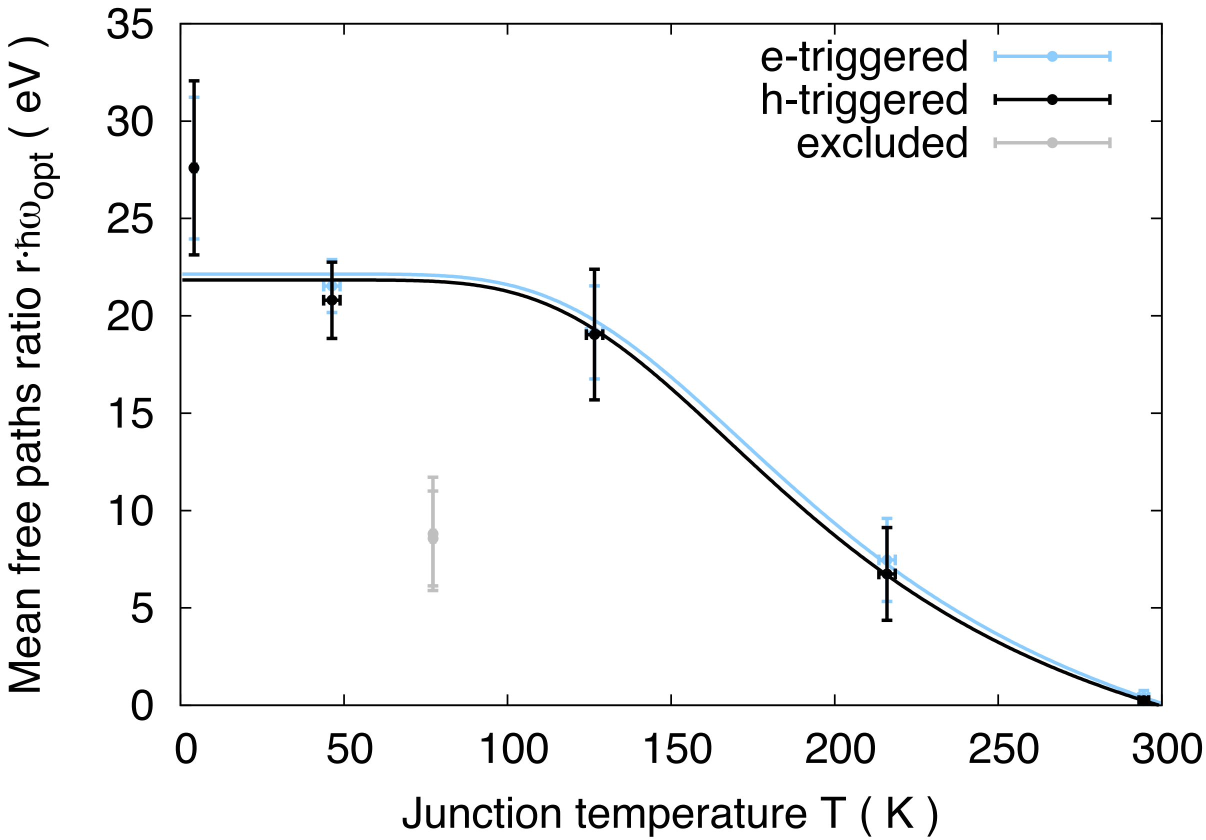
Mainz, Germany

The Ratio of Mean Free Paths

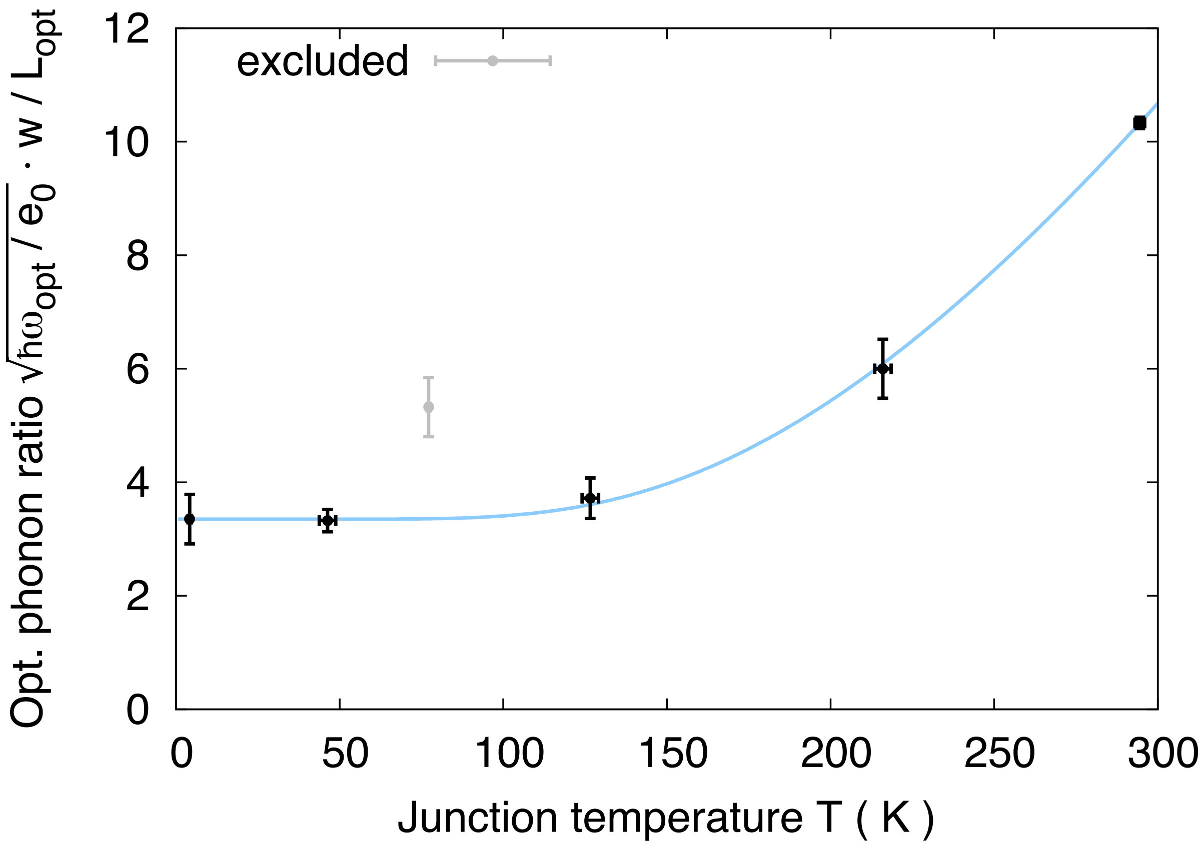
The ratio from the mean free paths $r \cdot \hbar\omega_{\mathrm{opt}} \propto \tanh\left[\frac{\hbar\omega_{\mathrm{opt}}}{2k_{\mathrm{B}}T}\right]$ follows Bose-Einstein statistics like expected.



The Fit Parameter around the Phonon Mean Free Path

Also the mean free path parameter follows Bose-Einstein statistics with the scaling factor *s*.

$$\sqrt{\frac{\hbar\omega_{\mathrm{opt}}}{e_0}} \cdot \frac{w}{\Lambda_{\mathrm{opt}}} \propto s \cdot \tanh\left[\frac{\hbar\omega_{\mathrm{opt}}}{2k_{\mathrm{B}}T}\right]^{-1}$$



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The Absolute Value of Phonon Energy and Mean Free Path

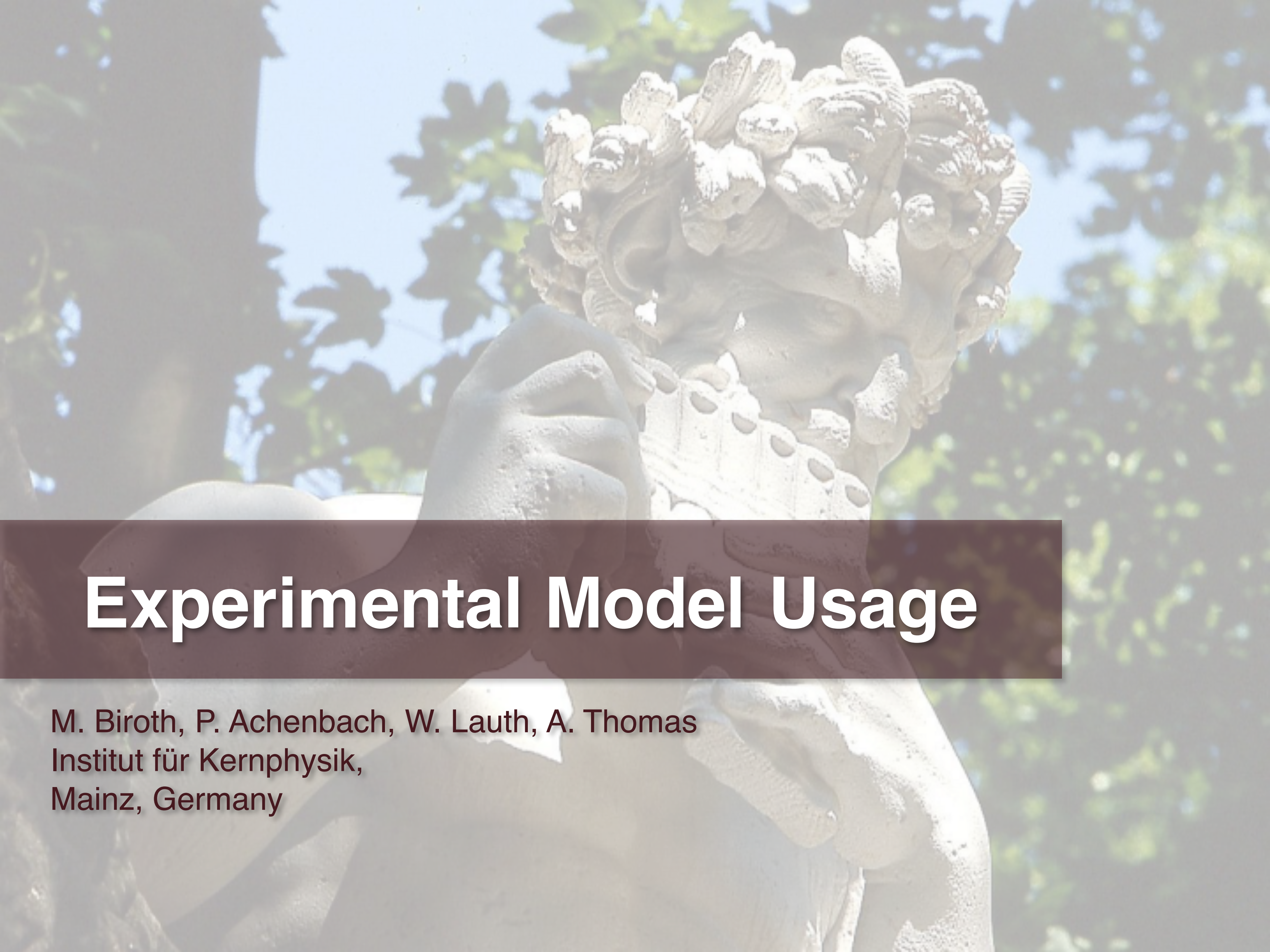
The extracted energy threshold $\hbar\omega_{\text{opt}} = (61 \pm 5)\ \text{meV}$ for scattering on optical phonons is statistically compatible with the reference $\hbar\omega_{\text{opt}}^{\text{ref}} = 63\ \text{meV}$.

The donator density was calculated by the phenomenological formula $N_{\text{D}} \approx 10^{16} \cdot (\Delta E_{\text{indi}}/1.1)^2 \cdot (60/V_{\text{BD}})^{4/3}$. The mean free path was calculated at room temperature, but without significance interval.

$$\Lambda_{\text{opt}}[T = 300\ \text{K}] = \underbrace{\sqrt{\frac{\hbar\omega_{\text{opt}}}{e_0}} \cdot \sqrt{\frac{2\varepsilon_0\varepsilon_{\text{Si}}}{e_0\ N_{\text{D}}}} \cdot \frac{1}{s}}_{\approx\Lambda_{\text{opt},0}} \cdot \tanh\left[\frac{\hbar\omega_{\text{opt}}}{2k_{\text{B}}T}\right]^{-1} = 75.9\ \text{Å}$$

As reference one find different values for electrons $\Lambda_{\text{opt}}^{\text{e}} = 76\ \text{Å}$ and holes $\Lambda_{\text{opt}}^{\text{h}} = 55\ \text{Å}$.

S.M. Sze, Physics of semiconductor devices John Wiley & Sons (1981)



Experimental Model Usage

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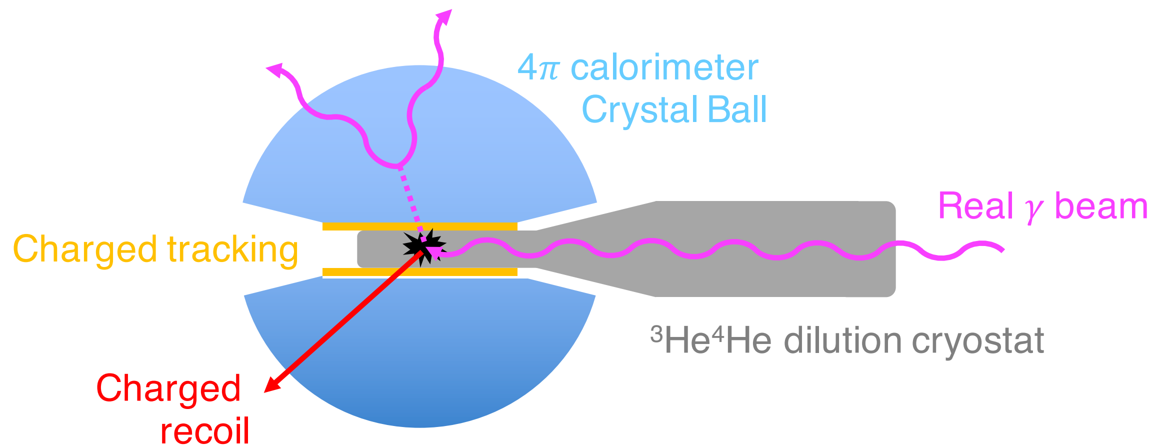
The Frozen-spin Target at the Real Photon Facility A2 in Mainz

Photoproduction experiments at the electron accelerator MAMI up to 1.6 GeV

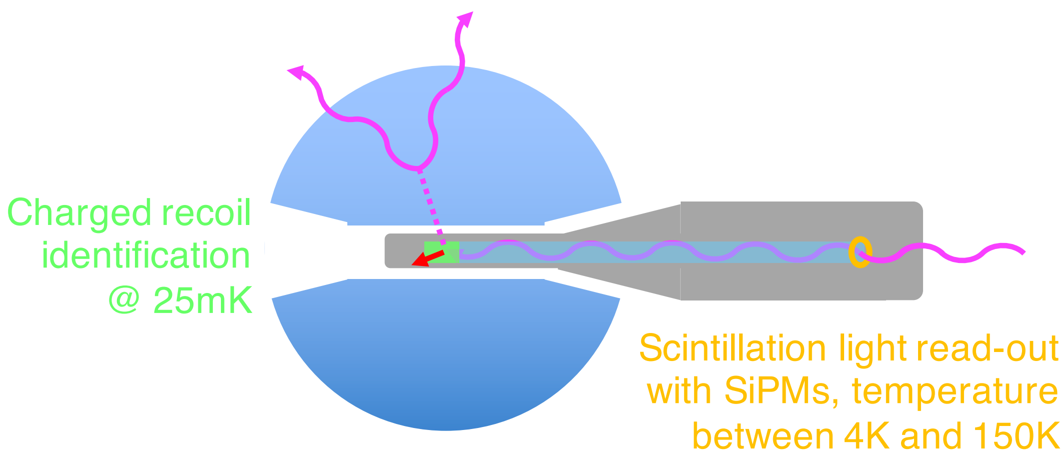
Spin polarized targets, normally doped Butanol



Low energy measurements where recoils can not escape:

Active Polarized Proton Target

- SiPMs temperature depends from placement
- Temperature not stable during operation

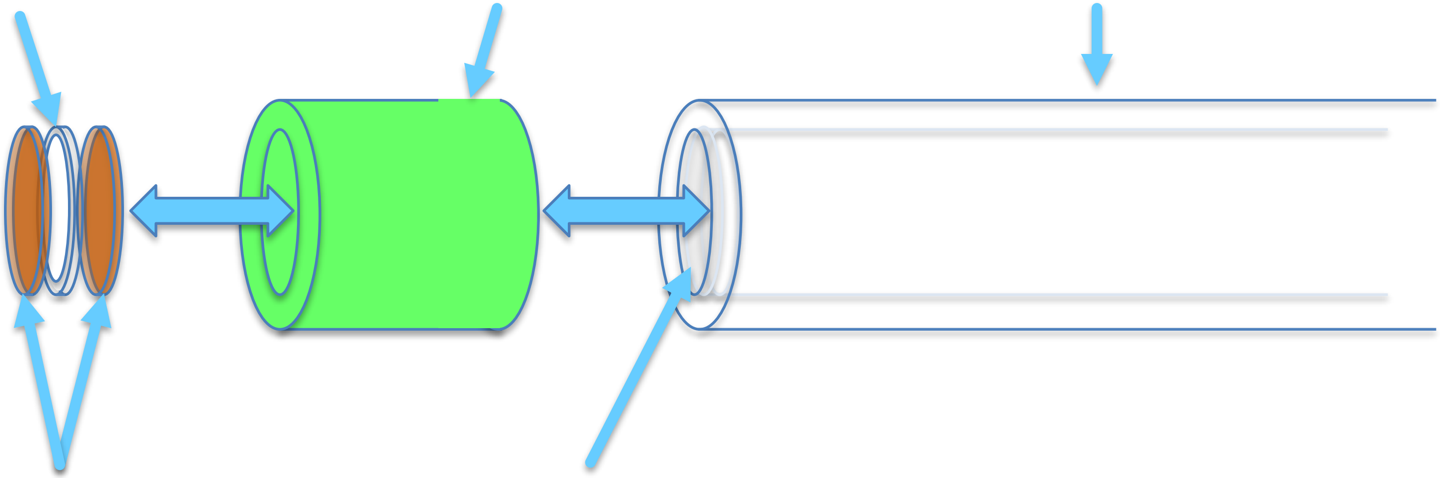


Optical Design of the Active Polarized Proton Target

Spacers / PMMA
9x 0.5mm thickness

Wavelength-shifting head
o ⌀26mm / i ⌀20mm / L 20mm

Light guide tube / PMMA
o ⌀26mm / i ⌀20mm / L 1.5m

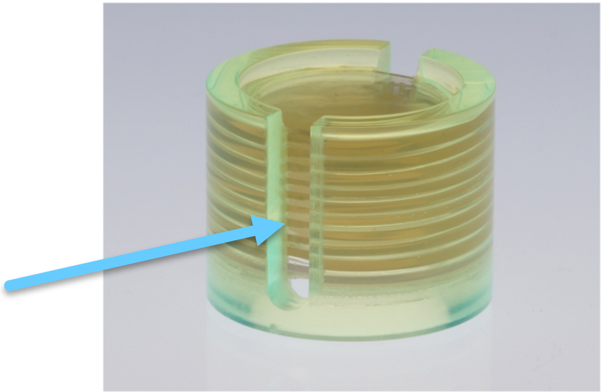


Spin polarizable scintillator
10x ⌀20mm / 1mm thickness
Doping: $1.5 \cdot 10^{-19} cm^{-3}$

Inner vacuum window
PMMA 1mm thickness

Scintillator produced at the UMass Amherst

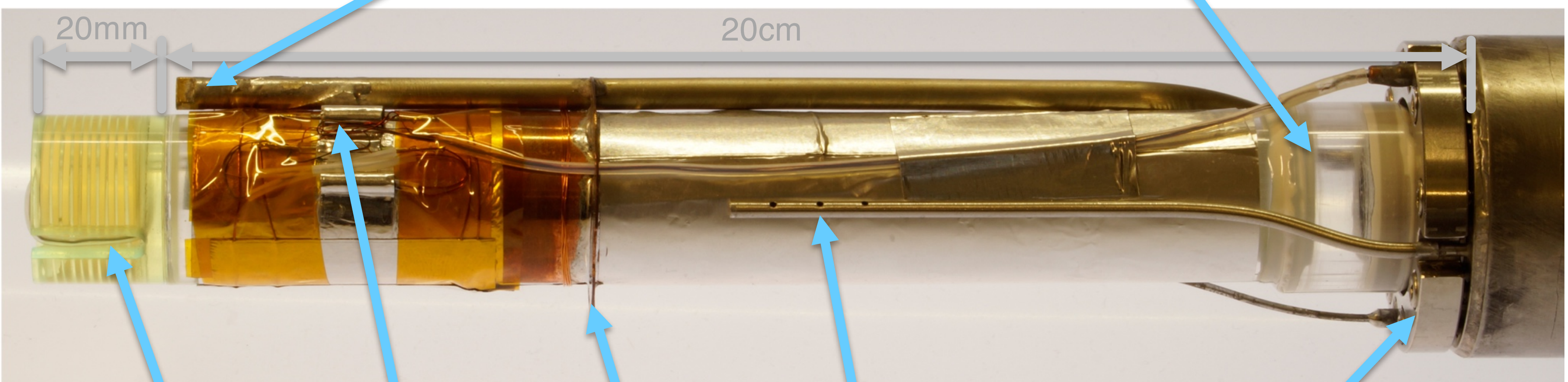
Slit for cooling and NMR coil



Target Mounted on the Cryostat Insert

μWave guide with UHF tuner
and outlet port for DNP

He-vacuum feed-through for
the light guide tube



Temperature probes

Outlet nozzle of the
^{4}He pre-cooling line

He-vacuum feed-through for
cabling, μWave and pre-cooling

Target head with cooling slit
and transverse NMR coil

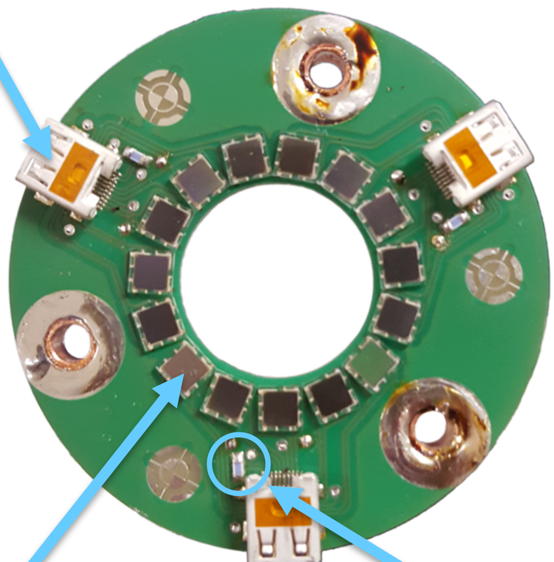
μWave radiation shield

Developed by KPH Mainz & JINR Dubna

Maik Biroth, Institut für Kernphysik, Mainz, Germany
mbiroth@uni-mainz.de

SiPM Based Read-out Board

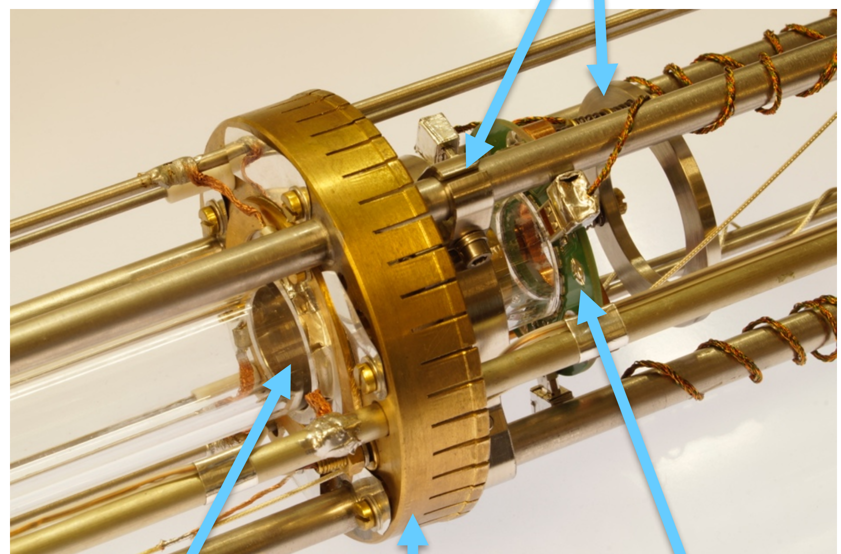
3x Micro-HDMI Connector
(5x Differential SiPM, 1x 4-wire Pt-1000)



15x SensL 3x3mm2
C-Type 35 μm

3x Temperature
Probe Pt-1000

Spring-mechanism to
work under thermal cycling



Light guide

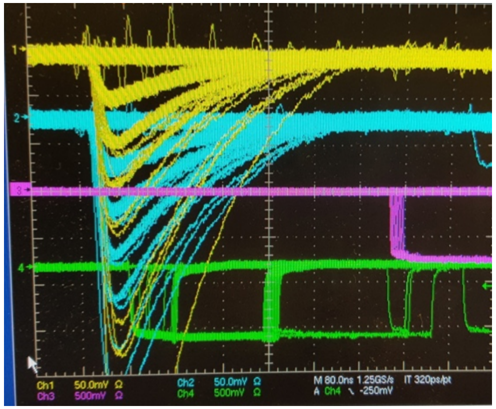
Thermal connection
for radiation shield

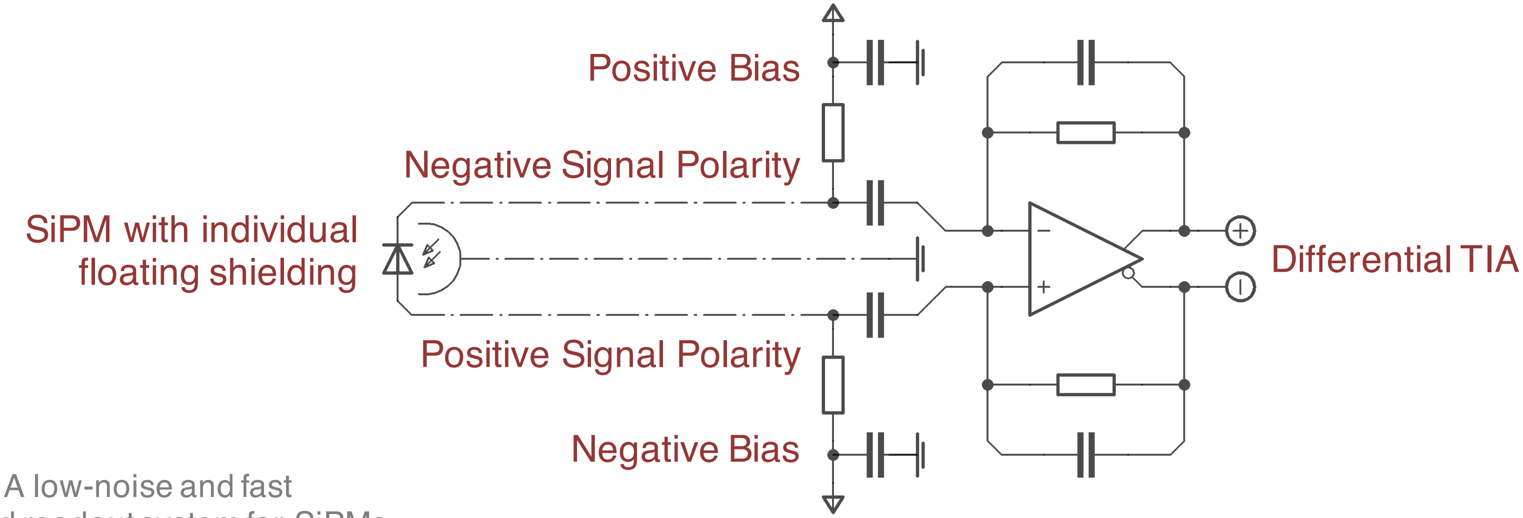
Detector board

Differential SiPM Read-out Concept

Challenges:

- Noisy environment
- Low dynamic range, single-photon scale
- Amplifier outside of the cryostat, distance 2.5 m





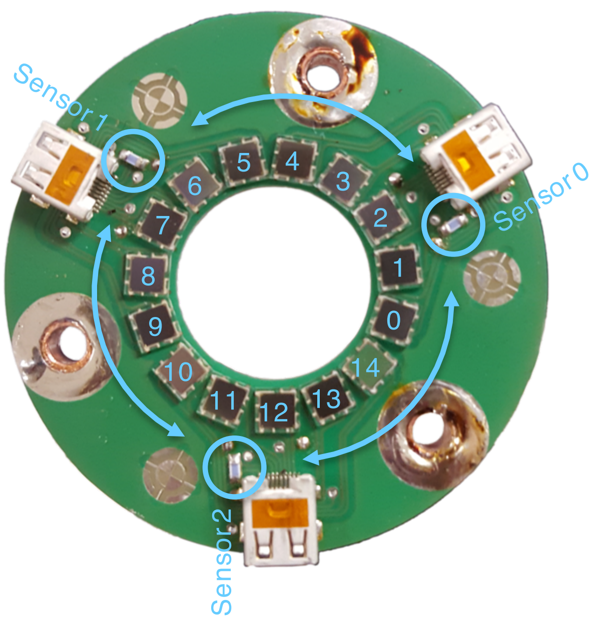
M. Biroth, et al., A low-noise and fast pre-amplifier and readout system for SiPMs Nucl. Instr. Meth. A 787 (2015) 185-188

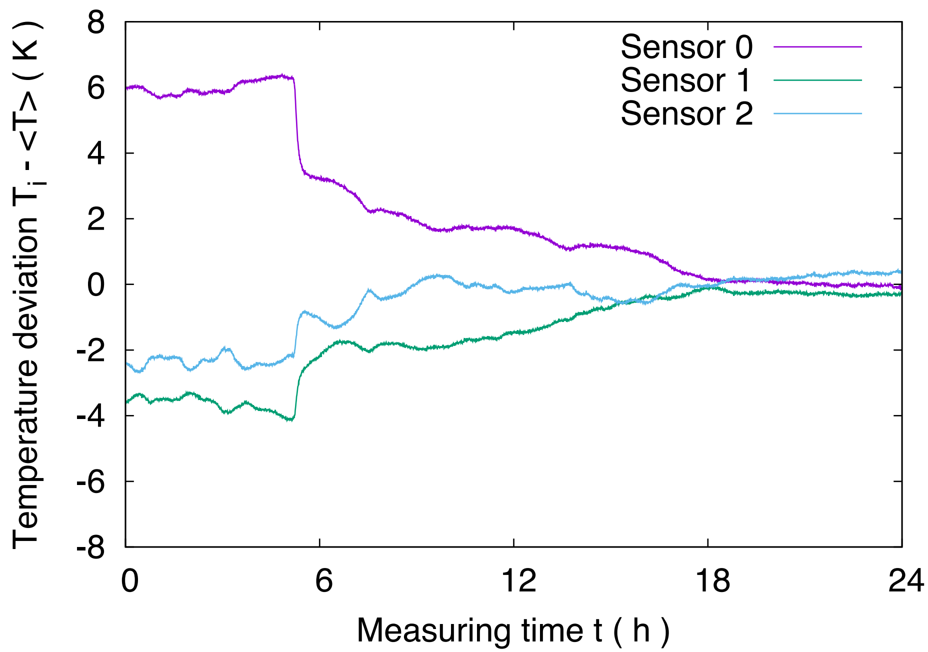
Temperature Measurement

SiPMs gain depends strongly from the temperature: Calculating the individual junction temperatures by circular interpolation between the 3 temperature probes to compensate temperature gradients.

Sketchy formula:

$$T_n = T_{Sensor(i)} + \frac{\measuredangle(SiPM, Sensor) + n \cdot \measuredangle(SiPM, SiPM)}{\measuredangle(Sensor, Sensor)} \cdot (T_{Sensor(i+1)} - T_{Sensor(i)})$$





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