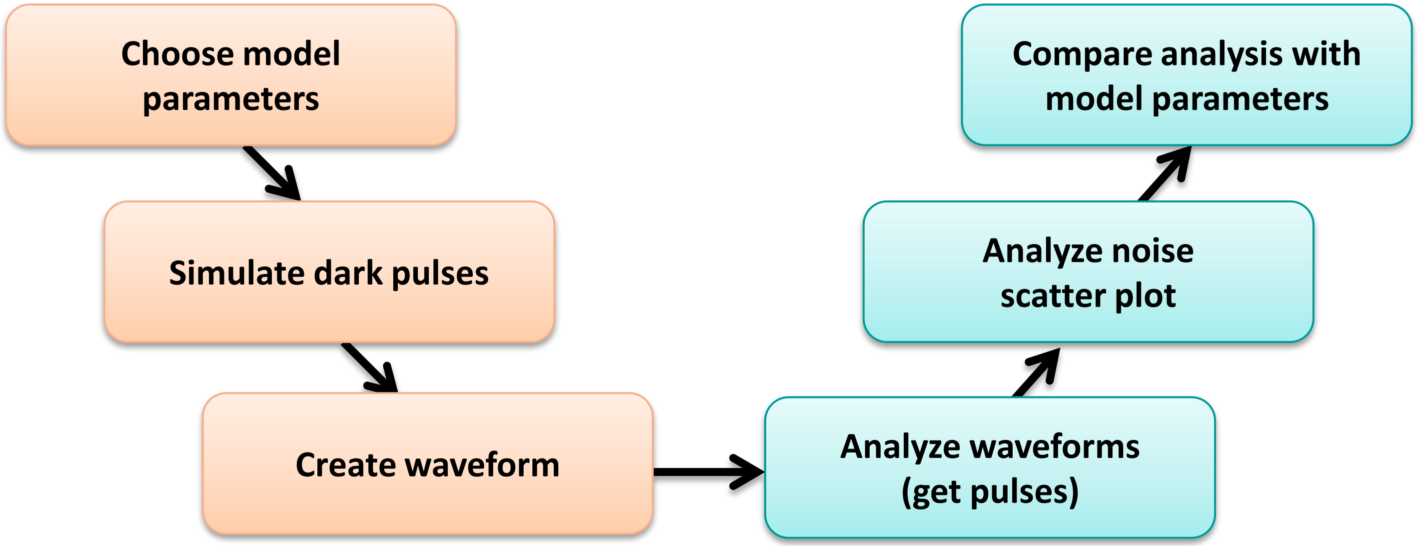
Simulation of SiPM Noise

Johannes Breuer on behalf of nuisance parameters topical group
ICASiPM, June 2018

Overview

- **Simple Monte Carlo model of dark noise**
  → Dark pulses (time / amplitude of pulses)
  → Waveforms

- **Simulated data can be fed into waveform analysis SW**
  → Does analysis identify SiPM model parameters correctly?
  → What are the statistical and systematic errors?



Monte Carlo model
Included effects

- **Creation of primary dark pulses**
  (Poisson-distributed time intervals)
- **Creation of secondary pulses**
  (prompt / delayed crosstalk, afterpulsing)
- **Recovery of micro cells**
- **Convolution with SPE pulse shape**
  → waveforms
- **Electronic noise (white noise)**
- **Band pass filter**

Monte Carlo model
Parameters

SiPM:

- Primary dark rate
- Afterpulsing
- Prompt crosstalk (*)
- Delayed crosstalk (*)
- Micro cell recovery time
- SPE amplitude
- Standard deviation of SPE amplitude
- Number of micro cells x/y

Afterpulsing:

- Probability
- Delay

Crosstalk:

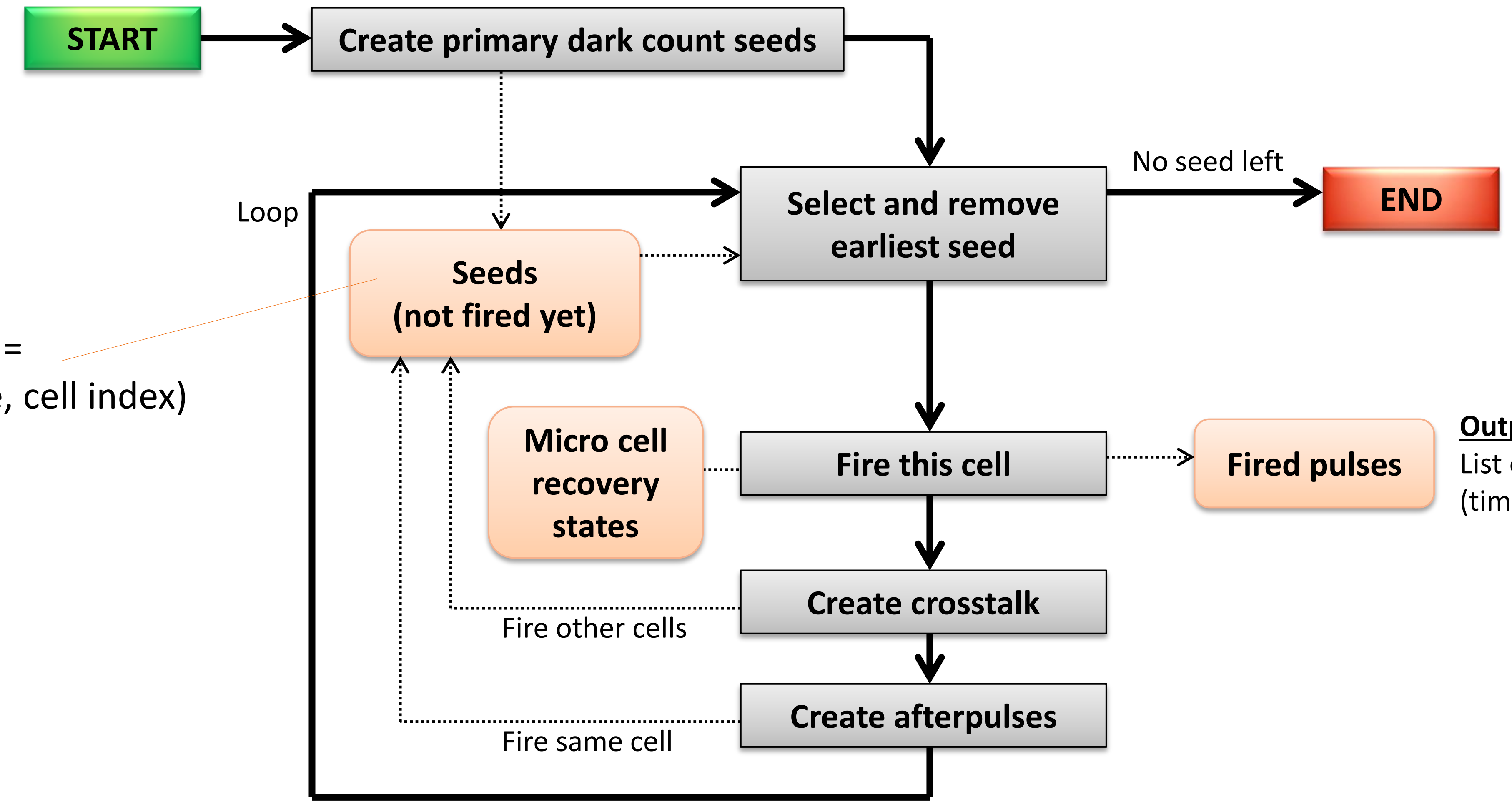
- Poisson lambda
- Delay
- Range

$$P_\lambda(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

Electronics:

- Pulse shape (bi-exp: rise/fall time)
- Noise amplitude
- Filter corner frequencies

Monte Carlo model
Creation of primary and secondary pulses



seed =
(time, cell index)

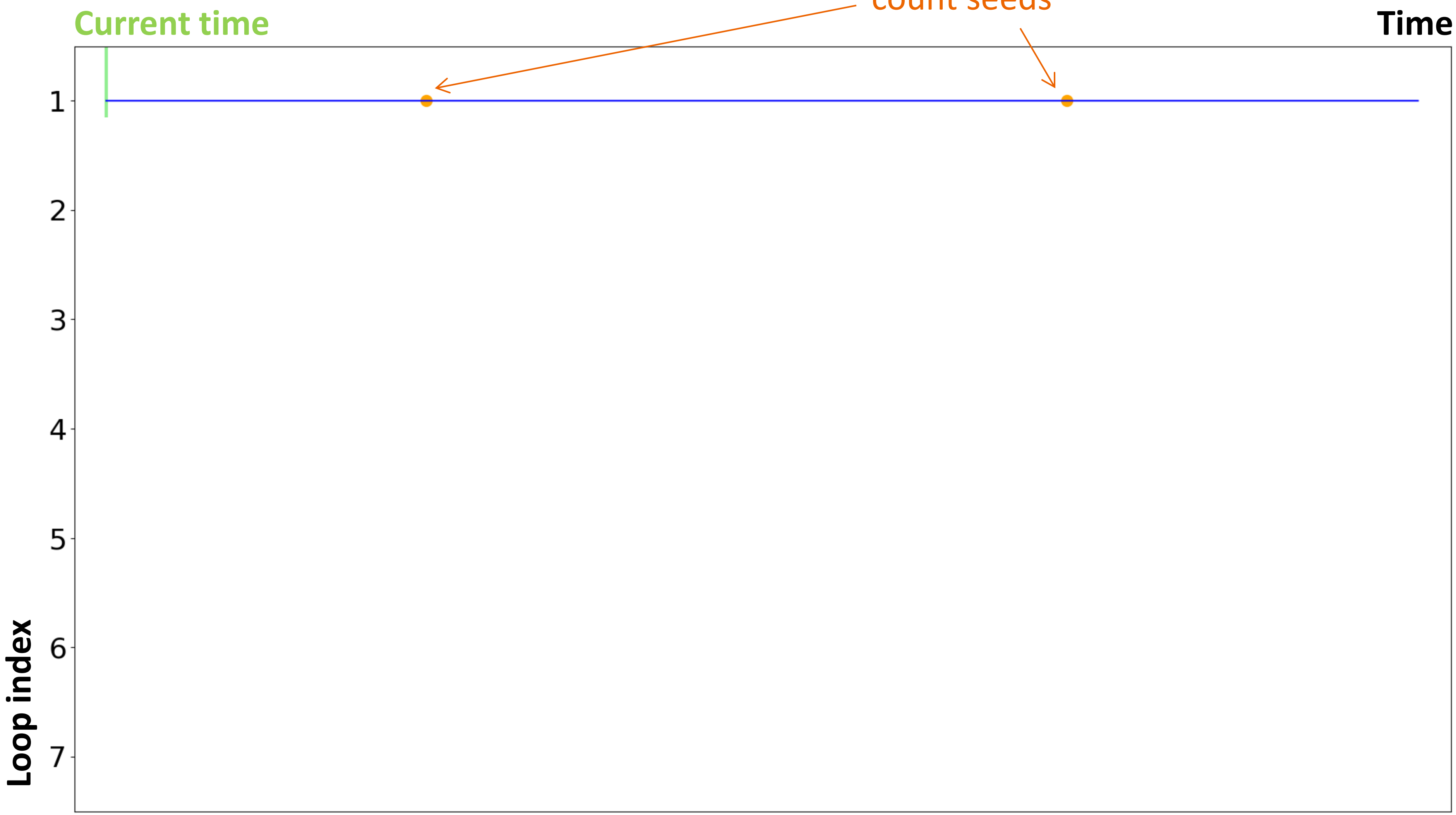
Output:
List of pulses
(time, cell index, amp)

Monte Carlo model
Creation of primary and secondary pulses

Waveform

Output:
Set of fired
micro cells

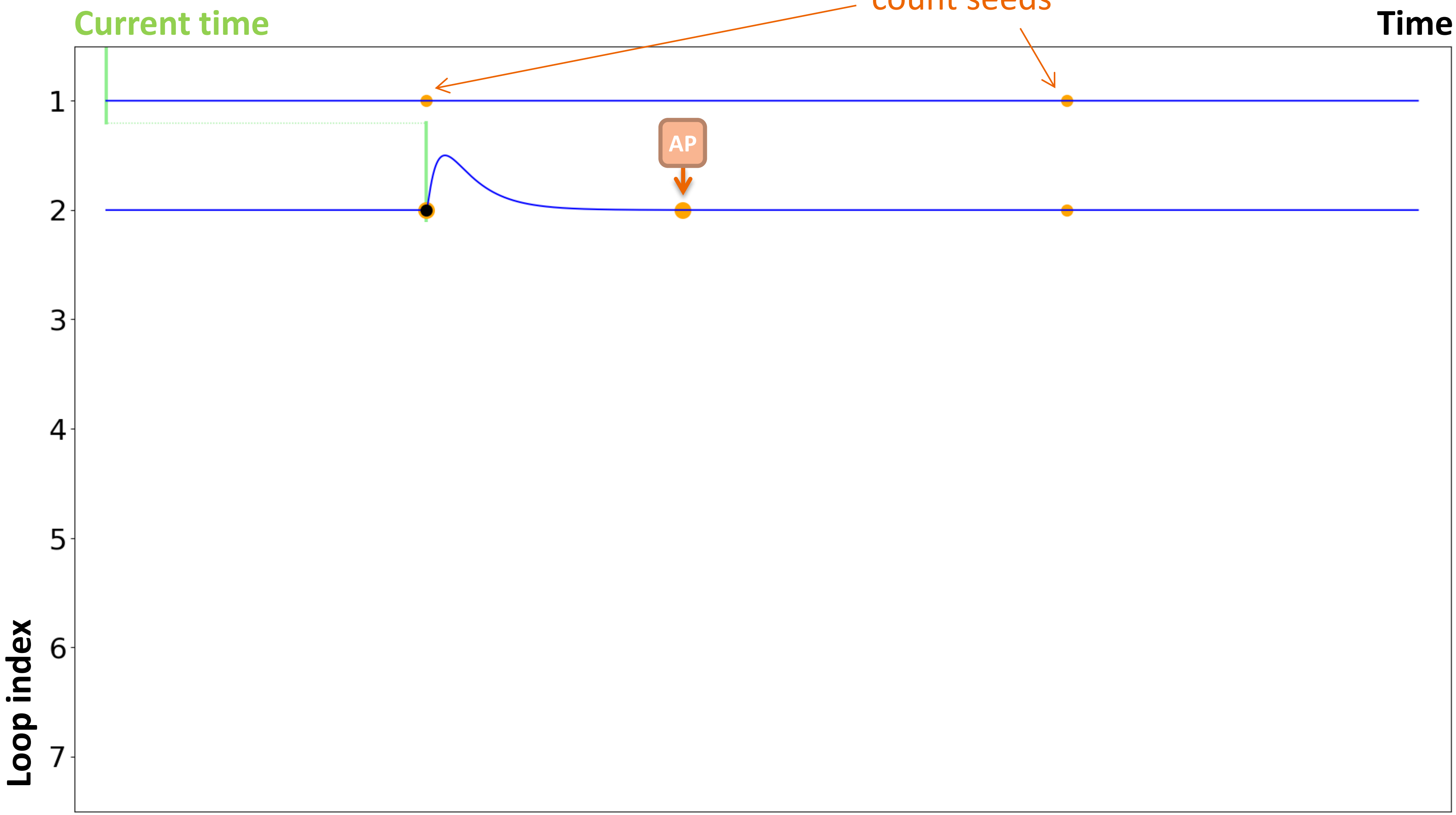
Input:
Set of primary dark
count seeds



Monte Carlo model
Creation of primary and secondary pulses

Waveform

Output:
Set of fired
micro cells

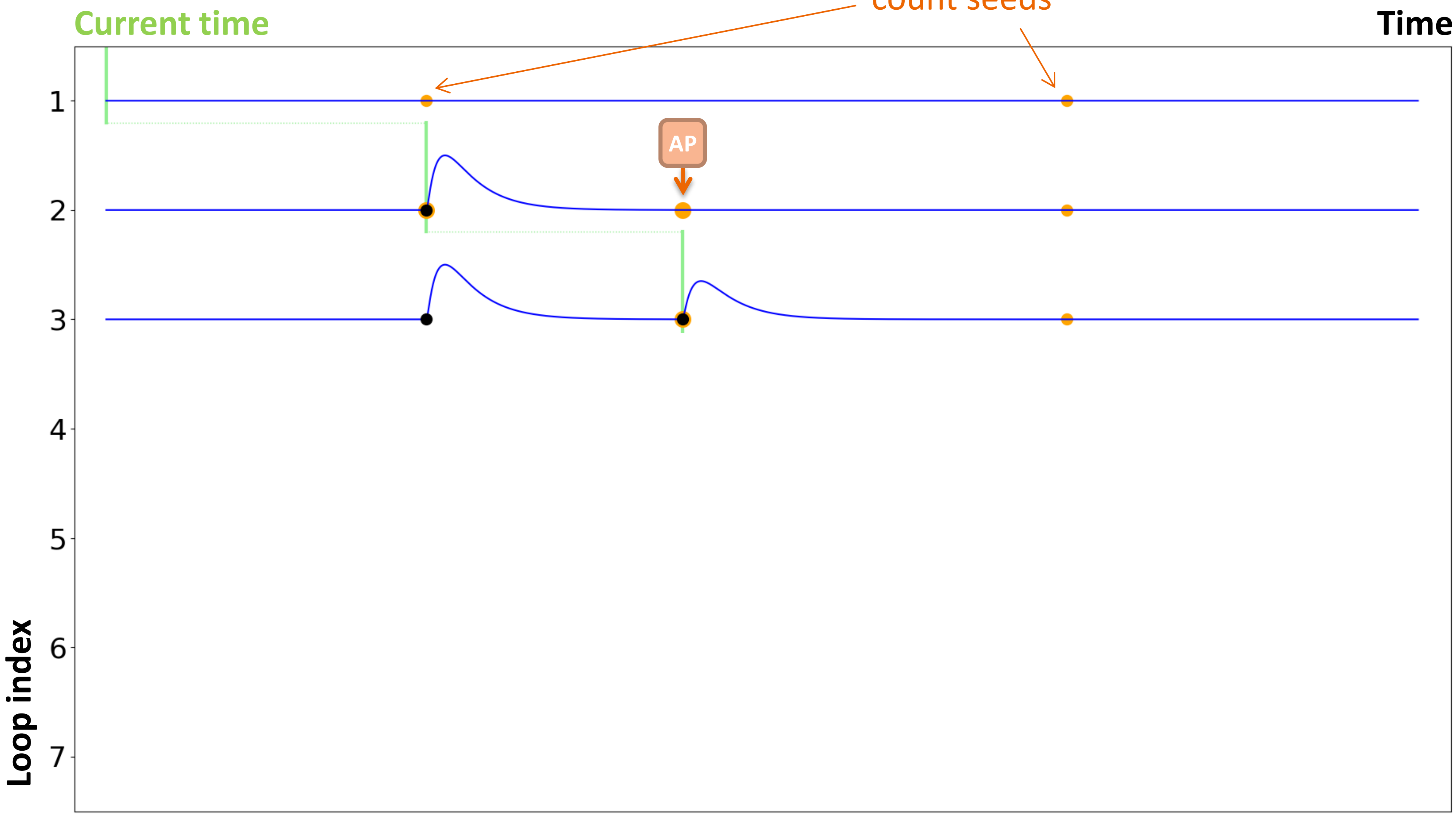


Monte Carlo model
Creation of primary and secondary pulses

Input:
Set of primary dark count seeds

Waveform

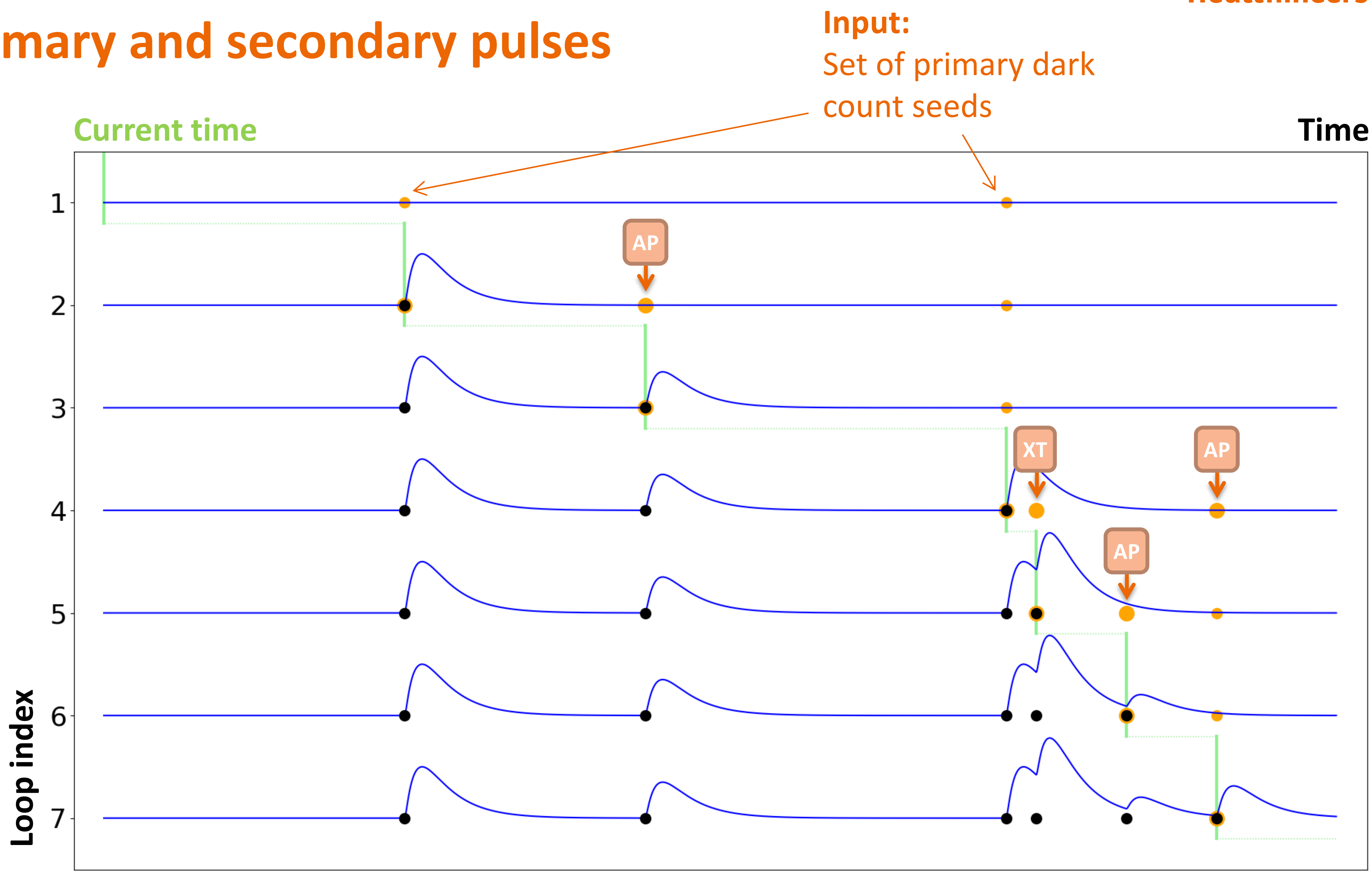
Output:
Set of fired micro cells



Monte Carlo model
Creation of primary and secondary pulses

Waveform

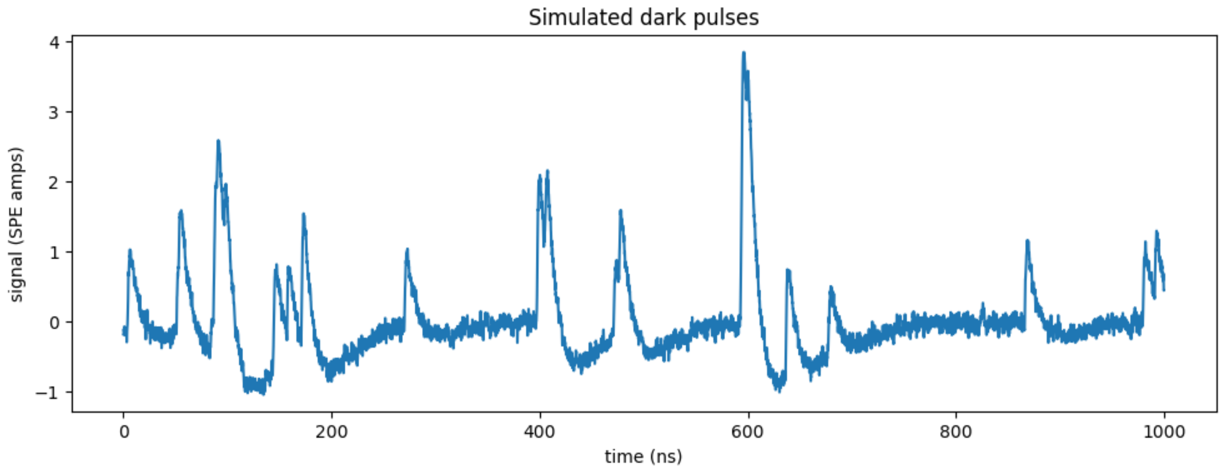
Output:
Set of fired
micro cells

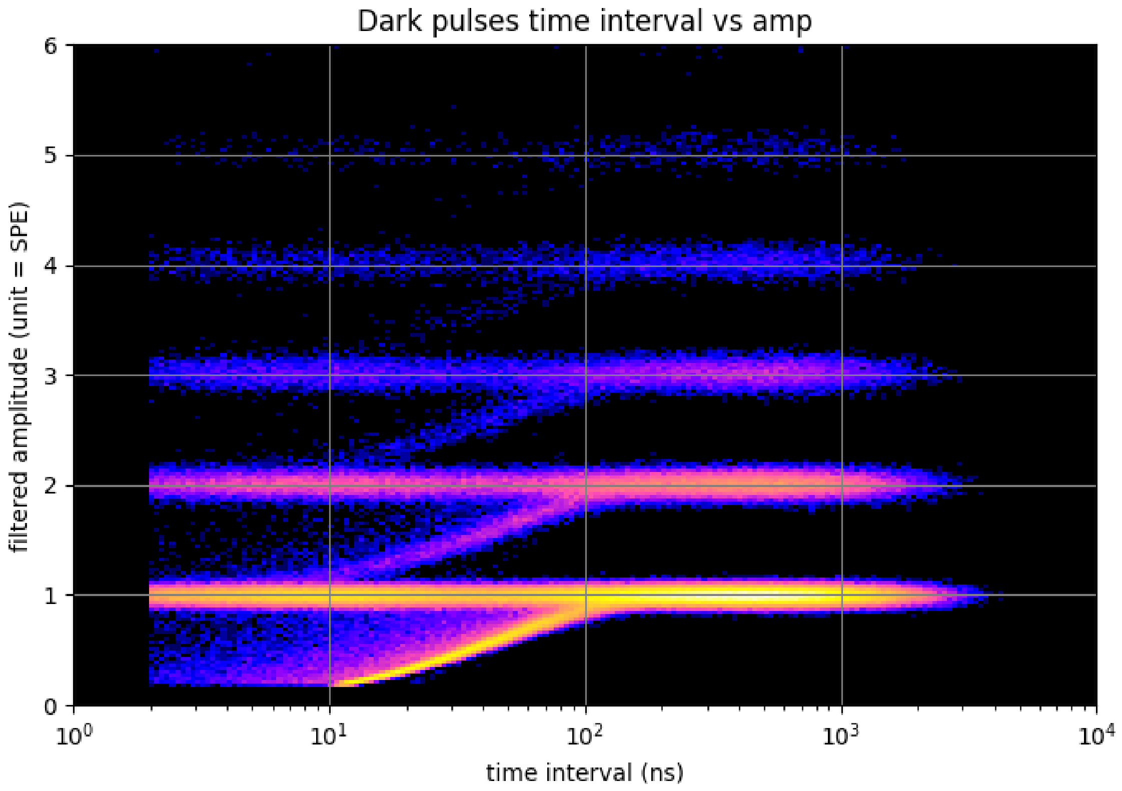


Monte Carlo model
Example waveform and noise scatter plot

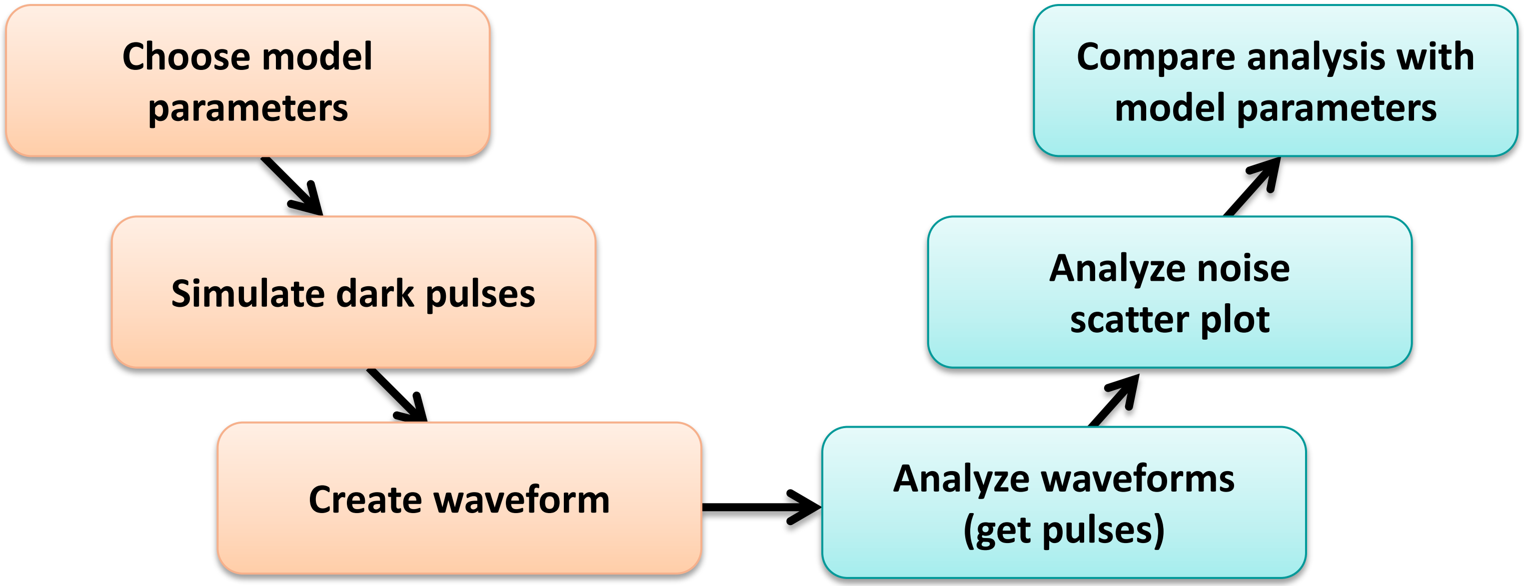
Waveform post-processing:

- Addition of Gaussian white noise
- Band pass filter

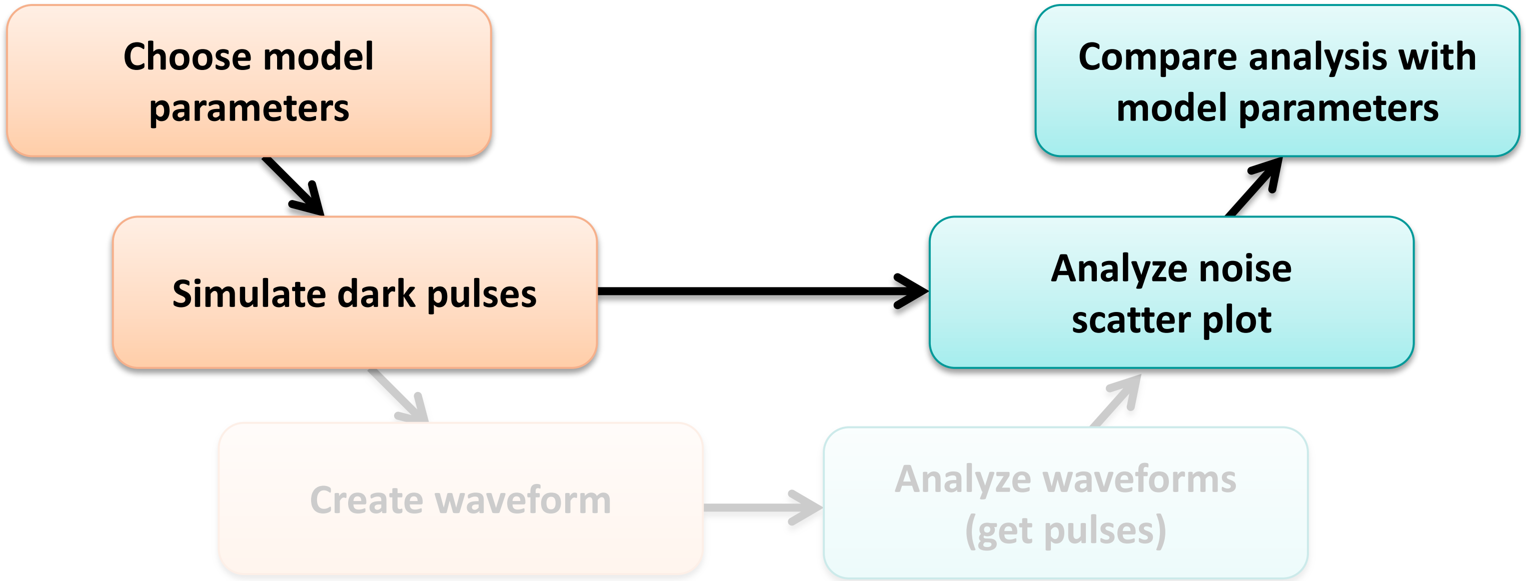




Testing waveform analysis

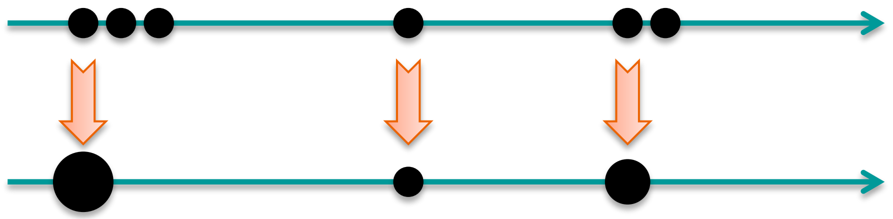


Testing waveform analysis

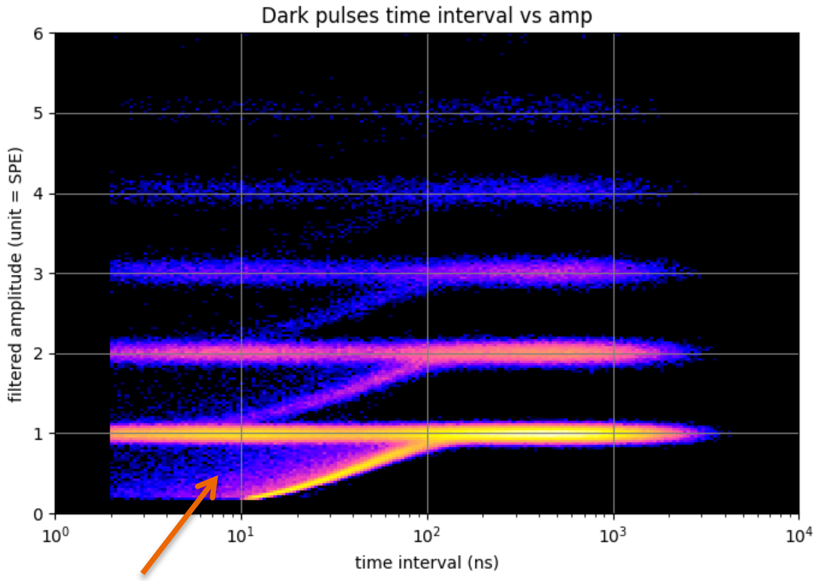


Omitting waveform step
→ allows faster processing

Note:
Nearby pulses must be consolidated into one

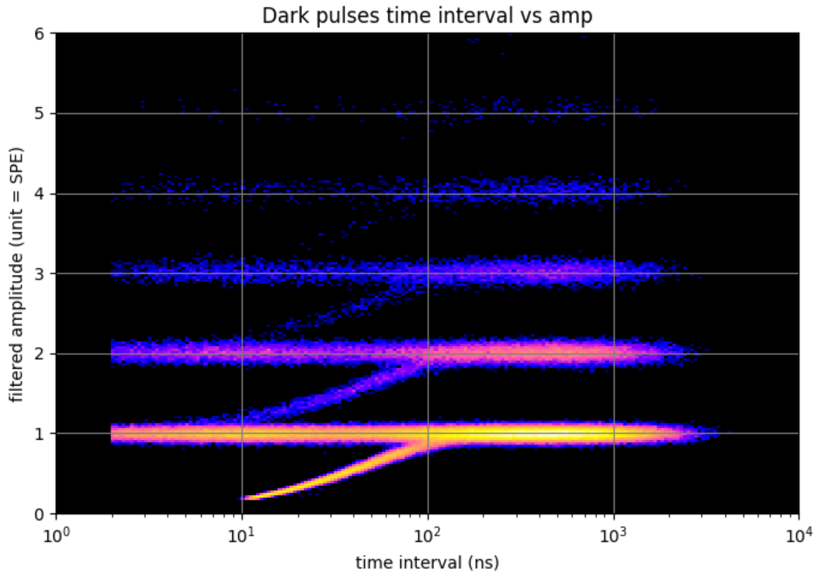


Noise scatter plot analysis:
Fit of distribution function → Get parameters back



Criterion for time intervals:

- First pulse must have 1 p.e. amp
- No pulse in last 250 ns before first pulse



Noise scatter plot analysis:
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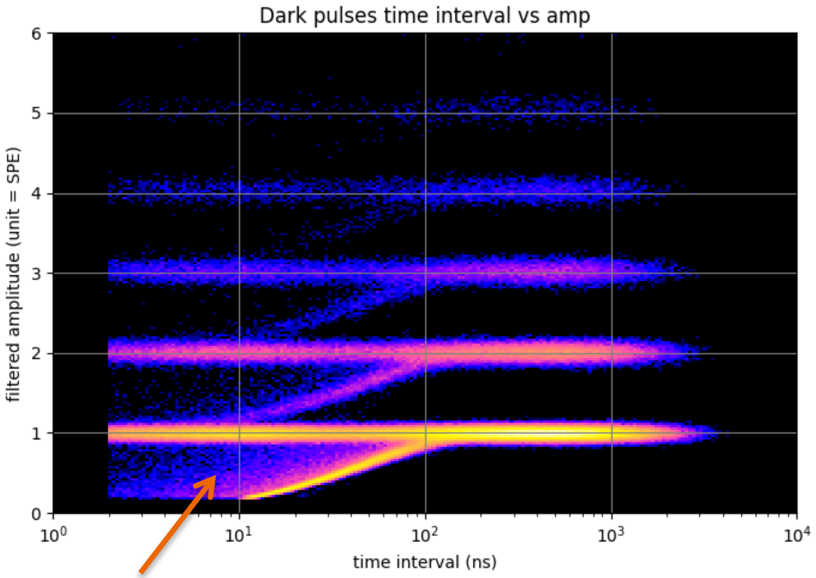

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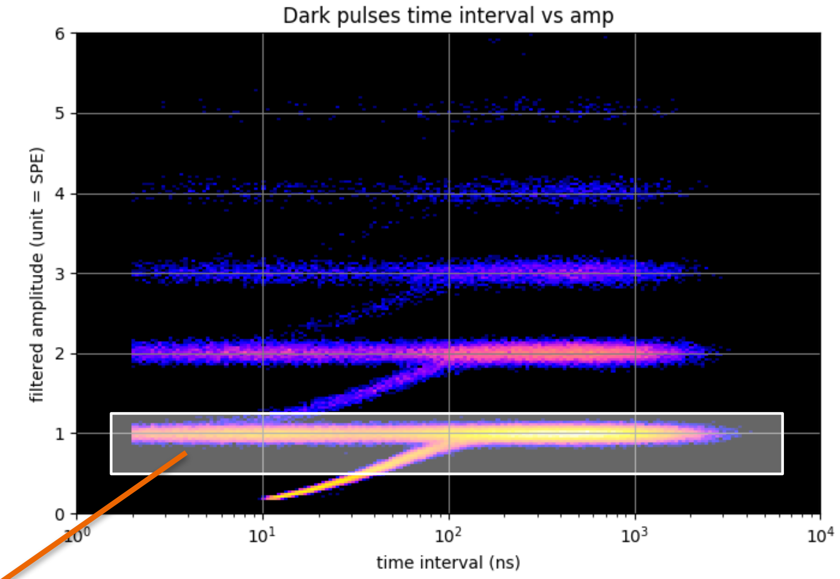
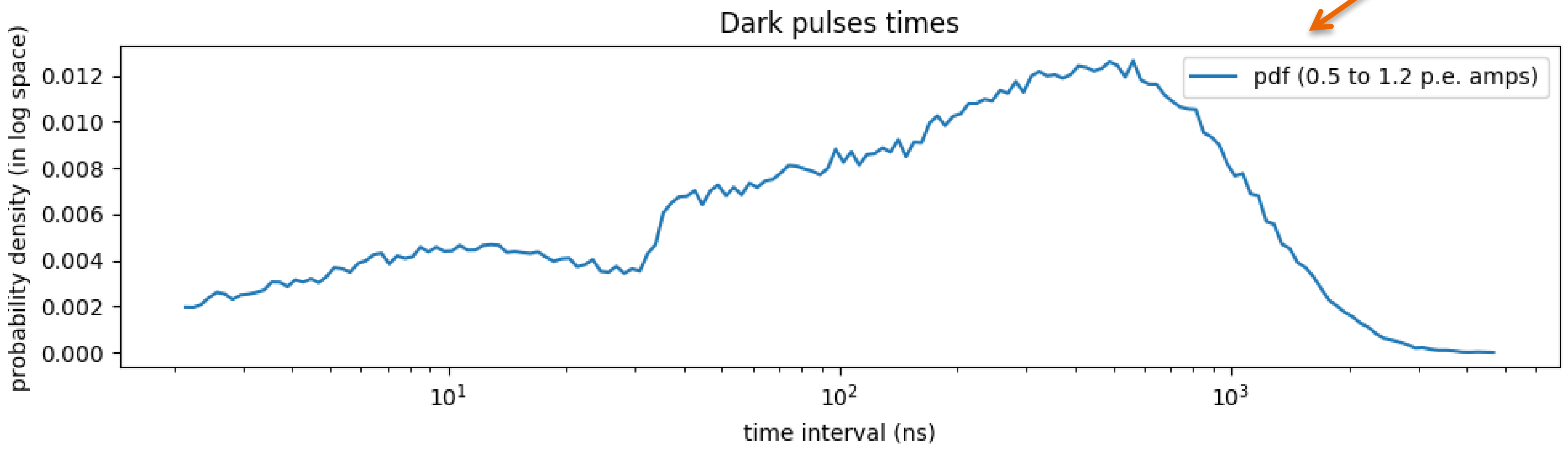




PDF of interval time only for these events

bins equidistant in log space

Design of a fit function for PDF of pulse time intervals

SIEMENS Healthineers

Dark pulses

$$p_{dp}(t) = \frac{1}{\tau_{dp}} \cdot e^{-t/\tau_{dp}}$$

Design of a fit function for PDF of pulse time intervals

Afterpulsing

$$p_{ap}(t) = P_{ap} \cdot \frac{1}{\tau_{ap}} \cdot e^{-t/\tau_{ap}} \cdot P_{trig}(t) \cdot \frac{1 + \mathrm{erf}(v \cdot (t - T_{1/2}))}{2}$$

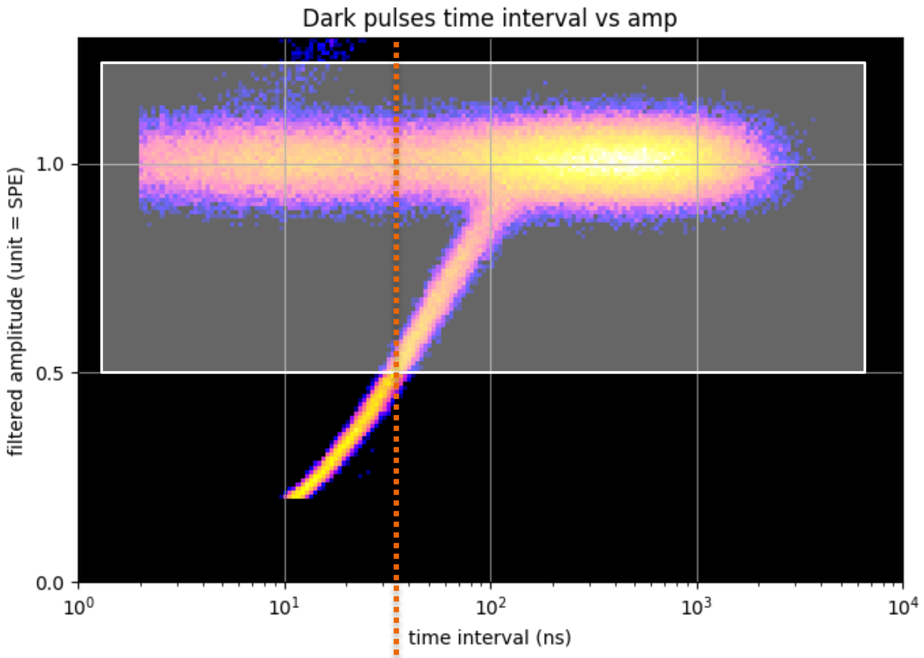
- P_{ap}: Probability of having trapped charge carrier
- $\frac{1}{\tau_{ap}} \cdot e^{-t/\tau_{ap}}$: Poisson process
- $P_{trig}(t)$: Probability of triggering avalanche
- $\frac{1 + \mathrm{erf}(v \cdot (t - T_{1/2}))}{2}$: Probability that amplitude of afterpulse is above 0.5 p.e. → smooth step function

$$P_{trig}(t) = 1 - e^{-k \cdot V_{OV}\left(1 - e^{-\frac{t}{\tau_{recov}}}\right)}$$

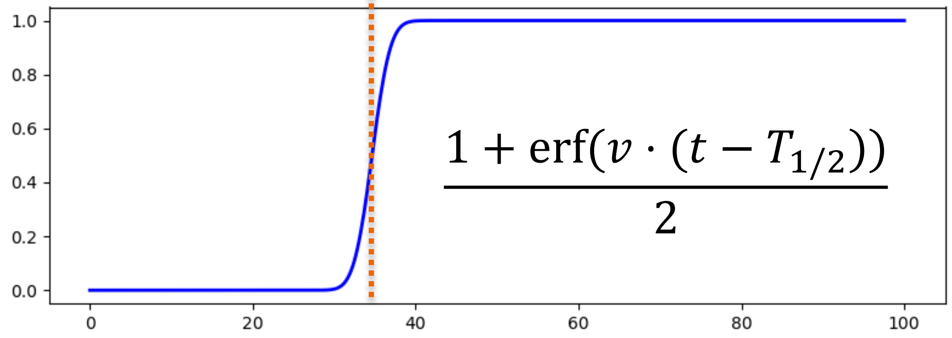
However:

In this Monte Carlo model, I used: $P_{trig}(t) = 1$

→ Needs to be corrected



$T_{1/2}$ = 50% recovery time



Design of a fit function for PDF of pulse time intervals

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Delayed crosstalk

$$p_{dct}(t) = \sum_{k=1} P_\lambda(k) \cdot \frac{k}{\tau_{dct}} \cdot e^{-\frac{k \cdot t}{\tau_{dct}}} = \frac{e^{-\lambda}}{\tau_{dct}} \cdot \sum_{k=1} \frac{\lambda^k}{(k-1)!} \cdot e^{-k \cdot t/\tau_{dct}}$$

Poisson distribution

$$P_\lambda(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

Distribution of first arrival time of k Poisson processes with same decay time τ_{dct}

First arrival time for multiple Poisson processes

$$p_{(k=2)}(t_{first}) = p_a(t) \cdot \int_t^\infty p_b(t')dt' + p_b(t) \cdot \int_t^\infty p_a(t')dt' = \frac{2}{\tau} \cdot e^{-\frac{2 \cdot t}{\tau}}$$

$$p_{(k)}(t_{first}) = \frac{k}{\tau} \cdot e^{-\frac{k \cdot t}{\tau}}$$

Design of a fit function for PDF of pulse time intervals

Dark pulses

$$p_{dp}(t) = \frac{1}{\tau_{dp}} \cdot e^{-t/\tau_{dp}}$$

Afterpulsing

$$p_{ap}(t) = P_{ap} \cdot \frac{1}{\tau_{ap}} \cdot e^{-t/\tau_{ap}} \cdot P_{trig}(t) \cdot \frac{1 + \mathrm{erf}(v \cdot (t - T_{1/2}))}{2}$$

Delayed crosstalk

$$p_{dct}(t) = \frac{e^{-\lambda}}{\tau_{dct}} \cdot \sum_{k=1} \frac{\lambda^k}{(k-1)!} \cdot e^{-k \cdot t/\tau_{dct}}$$

First arrival for all 3 processes

$$p(t_{first}) = p_{dct}(t) \cdot \left(1 - \int_0^t p_{ap}(t')dt'\right) \cdot \left(1 - \int_0^t p_{dp}(t')dt'\right)$$
$$+ \left(1 - \int_0^t p_{dct}(t')dt'\right) \cdot p_{ap}(t) \cdot \left(1 - \int_0^t p_{dp}(t')dt'\right)$$
$$+ \left(1 - \int_0^t p_{dct}(t')dt'\right) \cdot \left(1 - \int_0^t p_{ap}(t')dt'\right) \cdot p_{dp}(t)$$

→ **can be calculated numerically**

Overall 7 parameters:

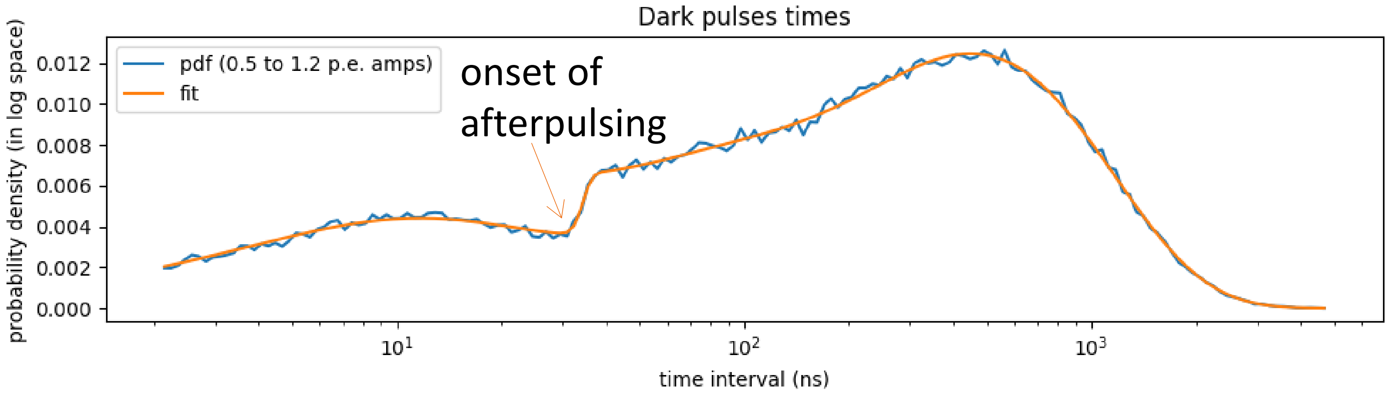
τ_{dp},

$P_{ap}, \tau_{ap}, v, T_{1/2}$,

λ, τ_{dct}

→ good initialization required

Example fit



PDF calculated with
162k selected pulses

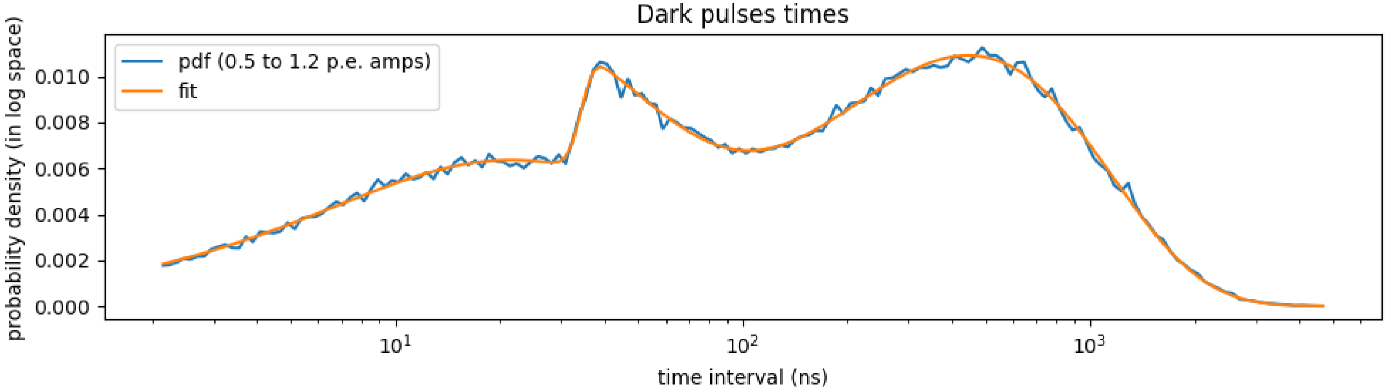
		Simulation	Curve fit
Dark pulses	tau (ns)	500	499.7
Afterpulsing	prob	0.2	0.22
	tau (ns)	50	51.1
	50% recovery time (ns)	34.7	34.2
	v		0.43
Delayed XT	lambda	0.2	0.21
	tau	10	9.98
Prompt XT	lambda	0.2	
Translated	dark rate (Mcps)	2	2.00
	prompt crosstalk (%)	18.1	
	delayed crosstalk (%)	18.1	19.1
	afterpulsing (%)	10.0	11.1

Good match

All 7 parameters can be identified well.

Delayed crosstalk and afterpulsing can be distinguished.

Example fit



PDF calculated with 127k selected pulses

		Simulation	Curve fit
Dark pulses	tau (ns)	500	494.8
Afterpulsing	prob	**0.3**	0.39
	tau (ns)	**30**	28.0
	50% recovery time (ns)	34.7	34.3
	v		0.31
Delayed XT	lambda	**0.3**	0.33
	tau	**20**	18.9
Prompt XT	lambda	**0.3**	
Translated	dark rate (Mcps)	2	2.02
	prompt crosstalk (%)	25.9	
	delayed crosstalk (%)	25.9	27.8
	afterpulsing (%)	9.5	11.5

Still a reasonable match

However, differences show that the method can still be improved.

Separation of pulses with very short time intervals
Pulse undershoot subtraction

Development of an automatic procedure for the characterization of silicon photomultipliers

Claudio Piemonte, Alessandro Ferri, Alberto Gola, Antonino Picciotto, Tiziana Pro, Nicola Serra, Alessandro Tarolli and Nicola Zorzi

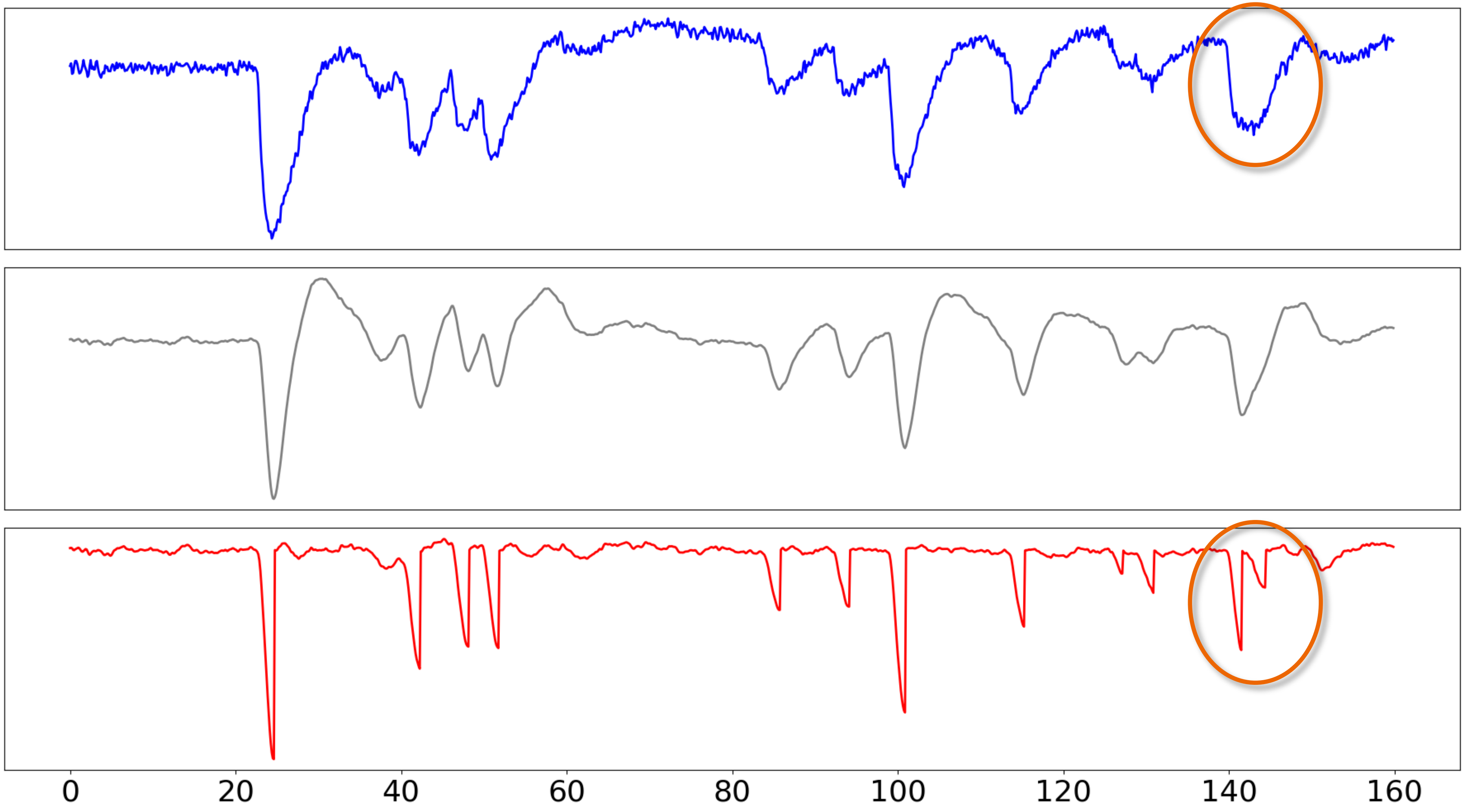
2012 IEEE NSS/MIC

Separation of pulses with very short time intervals
Pulse undershoot subtraction

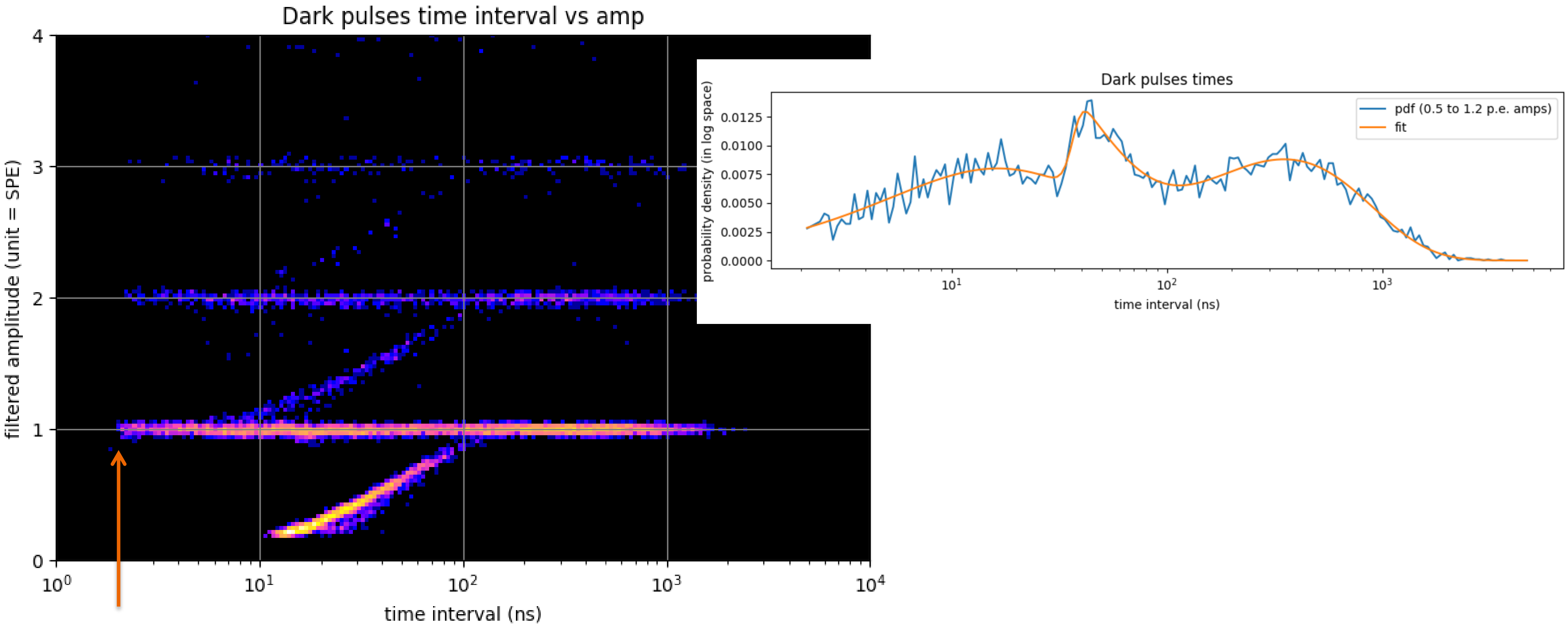
Original waveform (real measurement)

Filtered + DLED

Pulse tail eliminated



Separation of pulses with very short time intervals
Pulse undershoot subtraction



2ns intervals can be measured

Summary and outlook

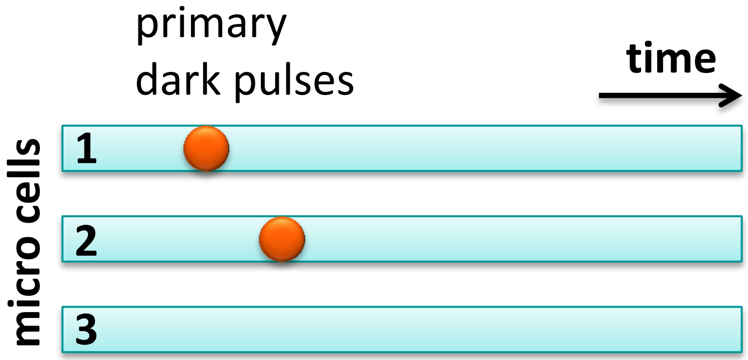
- Simple Monte Carlo model of dark noise
  (implemented in Python)
- Allows verification of waveform analysis and estimation of systematic errors

Next steps:

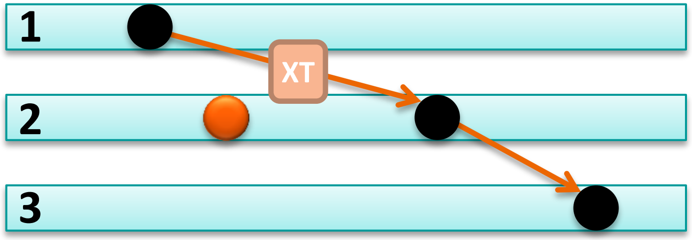
- Check analysis method with SiPMs of different designs / over-voltages
- Improvement of formulas
  (e.g. reduced triggering probability in afterpulses must be considered)

Thank you for your attention!

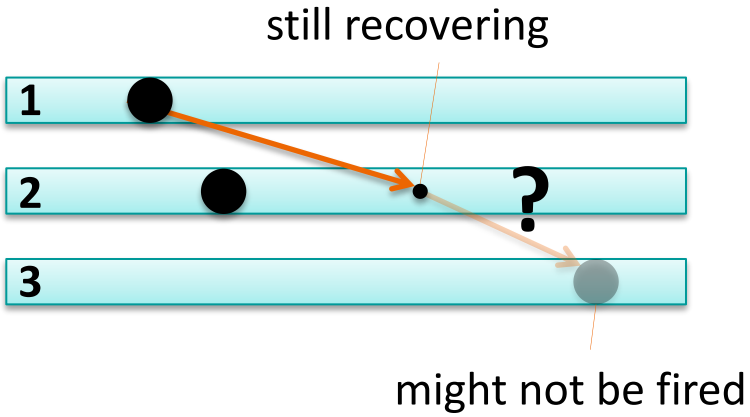
Monte Carlo model
Why all seeds must be processed in temporal order



A) Secondary pulses created immediately



B) Seeds processed in temporal order

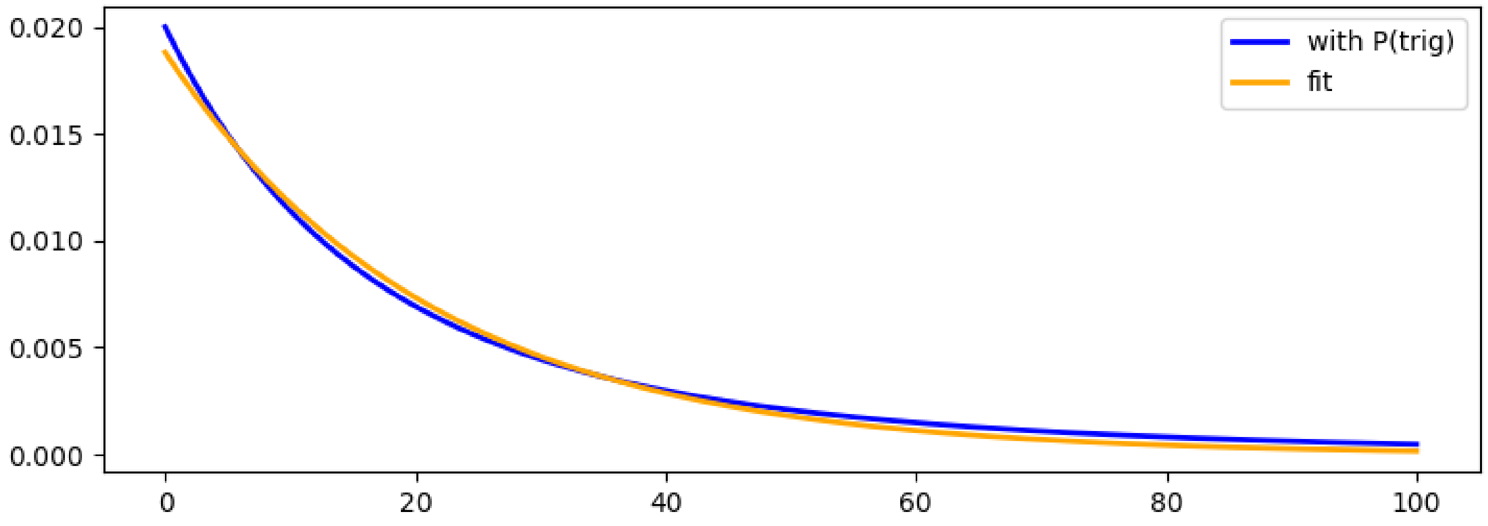


Effective afterpulsing decay function

$$p_{ap}(t) = P_{ap} \cdot \frac{1}{\tau_{ap}} \cdot e^{-t/\tau_{ap}} \cdot P_{trig}(t) \cdot \frac{1 + \mathrm{erf}(v \cdot (t - T_{1/2}))}{2}$$

$$P_{trig}(t) = 1 - e^{-k \cdot V_{OV}\left(1 - e^{-\frac{t}{\tau_{recov}}}\right)}$$

$$P_{trig}(t) = \quad P_{ap} \cdot \frac{1}{\tau_{ap}} \cdot e^{-t/\tau_{ap}} \cdot \left(1 - e^{-k \cdot V_{OV}\left(1 - e^{-\frac{t}{\tau_{recov}}}\right)}\right) \quad = \quad P'_{ap} \cdot \frac{1}{\tau'_{ap}} \cdot e^{-t/\tau'_{ap}}$$



$k = 0.4\ V^{-1}$
$\tau_{recov} = 50\ ns$
$\tau_{ap} = 50\ ns$
$V_{OV} = 5\ V$

↓

$\tau'_{ap} = 50\ ns$