# Calculations of Shear, Bulk viscosities in Polyakov-Quark-Meson model

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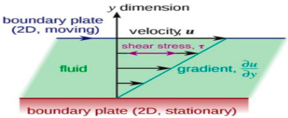
Based on the work done by P. Singha, A. Abhishek, G. Kadam, S. Ghosh and H. Mishra, arXiv:1705.03084 [nucl-th].

# Shear viscosity



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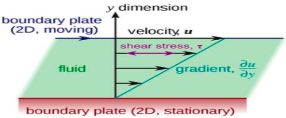
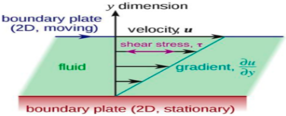


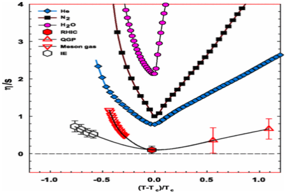

$$F = \eta \frac{\partial v}{\partial y}.$$

# Shear viscosity




$$F = \eta \frac{\partial v}{\partial y}.$$

- Temperature dependence of $\eta/s$:




-R. A. Lacey et al., Phys. Rev. Lett. 98 (2007) 092301

# Bulk viscosity

• The bulk viscosity or second viscosity appears in the processes which are accompanied by the change in volume of the fluid.

• Due to this compression or expansion, the fluid ceases to be in thermodynamic equilibrium and internal processes are set up in it which tend to restore the equilibrium.

• The processes which establish equilibrium are irreversible, they increase the entropy , therefore energy dissipation occurs.

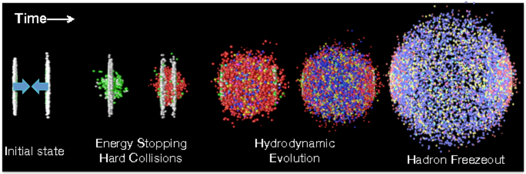
• This energy dissipation is determined by the second viscosity, $\zeta$.

• The value of $\zeta$ depends on the relation between the rate of compression and expansion and the relaxation time.

-Landau Lifshitz -Fluid Mechanics

# Brief introduction to Quark Gluon Plasma

• At sufficient high beam energy nuclei collision, a deconfined state of quarks and gluons may be created.That state is referred as Quark Gluon Plasma.



# Historical Overview

• In 1953 Landau proposed to exploit the law of ideal hydrodynamics to explore the strongly interacting matter that is formed in high energy heavy ion collision.

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An accurate hydrodynamic code with small but essential dissipative effects⇒ **Transport coefficients!!**

# Analytic expressions for viscosity

•Energy Momentum tensor for dissipative system:

$$T^{\mu\nu} = -Pg^{\mu\nu} + \omega U^{\mu} U^{\nu} + \Delta T^{\mu\nu}$$

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Where,

$$\Delta T^{\mu\nu} = \eta(D^{\mu} U^{\nu} + D^{\nu} U^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\rho} U^{\rho}) - \zeta\Delta^{\mu\nu}\partial_{\rho} U^{\rho} \ .$$

$$\Delta^{\mu\nu} = U^{\mu} U^{\nu} - g^{\mu\nu} \quad D_{\mu} = \partial_{\mu} - U_{\mu} U^{\beta}\partial_{\beta}$$

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# Analytic expressions for viscosity

$$\eta = \frac{2}{15}\Sigma_a \int \frac{d^3p}{(2\pi)^3} \frac{|p|^4}{E_a} f_a^{eq} C_a$$

$$\zeta = \frac{1}{3}\Sigma_a \int \frac{d^3p}{(2\pi)^3} \frac{|p|^2}{E_a} f_a^{eq} A_a$$

Where, $C^a_{\mu\nu} = C_a p_\mu p_\nu$.

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$\bullet A_a$, $C_a$ contain the informations of particle interactions.

# Analytic expressions for viscosity

- **Boltzmann Equation:**

$$\frac{\partial f_a}{\partial t} + \mathbf{v}.\nabla f_a = -\omega_a \delta f_a$$

Where $\omega_a$ is the frequency of interaction taking into account both gain and loss rates.

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- **Relaxation Time approximation** suggests in those interaction $f_i = f_i^{eq}$ unless $i = a$.

# Analytic expressions for viscosity

- Final expressions:

$$\eta = \frac{g\beta}{15}\int \frac{(d^3p)\tau}{(2\pi)^3}(\frac{p^2}{E_p})^2 f(1 \pm f)$$

$$\zeta = g\beta \int \frac{(d^3p)\tau}{(2\pi)^3}(\frac{(\frac{1}{3} - c_s^2)p^2 - c_s^2 \frac{d}{d\beta^2}(\beta^2 M^2)}{E_p})^2 \times (f(1 \pm f))$$

# Effective model

• In the non perturbative region extracting useful informations for most of the physical processes of interest from QCD lagrangian gets way too complicated.
Effective model:**Polyakov Quark Meson Model**

# Construction of model lagrangian

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- Sigma like state: $\bar{\psi}\psi$

$$(\pi^2+\sigma^2) \xrightarrow[\Lambda_V]{} (\pi^2+\sigma^2)$$

$$(\pi^2+\sigma^2) \xrightarrow[\Lambda_A]{} (\pi^2+\sigma^2)$$

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$$(\pi^2+\sigma^2)\xrightarrow[\Lambda_V]{}(\pi^2+\sigma^2)$$

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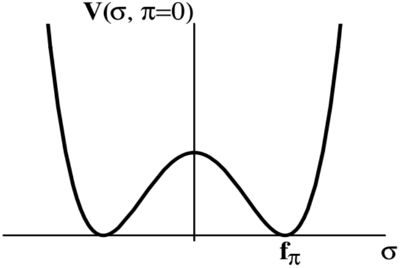
- interaction term:

$$g((i\bar{\psi}\gamma_5\vec{\tau}\psi)\vec{\pi}+(\bar{\psi}\psi)\sigma)$$

# Construction of the Lagrangian

- Pion Sigma potential: $V = V(\pi^2 + \sigma^2) = \frac{\lambda}{4}(\pi^2 + \sigma^2 - v^2)$




- $\sigma_0 = v = f_\pi;\ m_\sigma = \lambda f_\pi^2;\quad m_\pi = 0$

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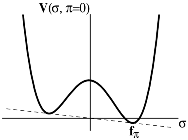


- $c = f_\pi m_\pi^2; \quad v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda}; \quad m_\sigma^2 = m_\pi^2 + 2\lambda f_\pi^2$

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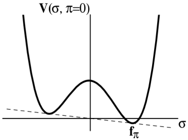


- $c = f_\pi m_\pi^2;\quad v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda};\quad m_\sigma^2 = m_\pi^2 + 2\lambda f_\pi^2$

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - g((i\bar{\psi}\gamma_5\vec{\tau}\psi)\vec{\pi} + (\bar{\psi}\psi)\sigma) + \frac{1}{2}(\partial^\mu\pi\partial_\mu\pi + \partial^\mu\sigma\partial_\mu\sigma)$$
$$-(\frac{\lambda}{4}(\pi^2+\sigma^2-v^2) - c\sigma) - U_P(\Phi,\bar{\Phi})$$

# Polyakov loop

- Polyakov loop:

$$L(x) = \mathcal{P}(\int_0^\beta dx_0 A_0(x_0, \vec{x}))$$

- The Polyakov loop variable $\Phi = \Phi(\vec{x}) = \frac{1}{N_c}\langle tr_c L(\vec{x})\rangle_\beta$

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- Polyakov Loop potential:

$$U_p(\Phi, \bar{\Phi}) = T^4\,[-\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{2}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2]$$

# Thermodynamic potential

$$\Omega(T,\mu) = \Omega_{\bar{Q}Q} + U_{\chi} + U_P(\Phi,\bar{\Phi}) \ .$$

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- $P = -\Omega$
- $s = -\frac{\partial\Omega}{\partial T}$
- $\epsilon = P + Ts$
- $c_s^2 = \frac{\partial P}{\partial \epsilon}|_s$
- $M_\sigma^2 = \frac{\partial^2\Omega}{\partial\sigma^2}$ and $M_{\pi_i}^2 = \frac{\partial^2\Omega}{\partial\pi_i^2}$.

# Calculation of thermal width

$$\eta = \frac{g\beta}{15}\int \frac{(d^3p)}{(2\pi)^3}\tau(\frac{p^2}{E_p})^2 f(1\pm f)$$

•$\omega$ = Total interaction frequency$= 1/\tau$ = Inverse of relaxation time.

# Calculation of thermal width

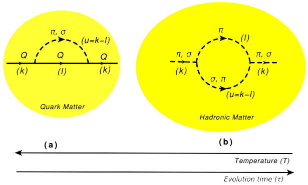


Figure : Interaction picture taken into consideration

- $\Gamma =$ Thermal width=Imaginary part of self energy.

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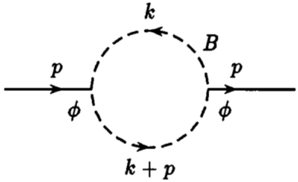


Figure : a bosonic self energy diagram

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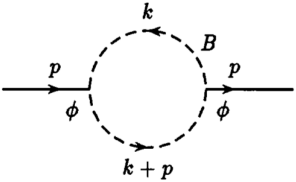



Figure : a bosonic self energy diagram

- There are real particles present in the medium.
- Available channels with appropriate statistical weight factors.

# calculation of thermal width

$$Im\Pi(\omega) = -\frac{g^2}{16}\int \frac{d^3k}{(2\pi)^2}\frac{1}{\omega_k \omega_{k+p}}$$
$$[(1+n(\omega_k))(1+n(\omega_{k+p})) - n(\omega_k)n(\omega_{k+p}))$$
$$\times(\delta(\omega+\omega_k+\omega_{k+p}) - \delta(\omega-\omega_k-\omega_{k+p})$$
$$+ n(\omega_k)(1+n(\omega_{k+p})) - n(\omega_{k+p})(1+n(\omega_k))$$
$$\times(\delta(\omega-\omega_k+\omega_{k+p}) - \delta(\omega+\omega_k-\omega_{k+p})]$$

# calculation of thermal width

$$
\begin{aligned}
Im\Pi(\omega) = -\frac{g^2}{16}\int \frac{d^3k}{(2\pi)^2}\frac{1}{\omega_k\omega_{k+p}} \\
[(1+n(\omega_k))(1+n(\omega_{k+p})) - n(\omega_k)n(\omega_{k+p})) \\
\times(\delta(\omega+\omega_k+\omega_{k+p}) - \delta(\omega-\omega_k-\omega_{k+p}) \\
+ n(\omega_k)(1+n(\omega_{k+p})) - n(\omega_{k+p})(1+n(\omega_k)) \\
\times(\delta(\omega-\omega_k+\omega_{k+p}) - \delta(\omega+\omega_k-\omega_{k+p})]
\end{aligned}
$$

- $\phi \to B + B$ and $B + B \to \phi$ Decay processes.
- $\phi + B \to B$ and $B \to \phi + B$ Scattering processes.

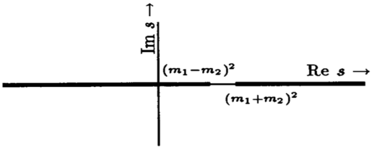
# Calculation of thermal width

- Branch Cuts:

$s = \omega^2 - \vec{p}^2$

For decay process:$\infty \geq s \geq (m_1 + m_2)^2$ Unitary cut.

for scattering process:$(m_1 - m_2)^2 \geq s \geq -\infty$ Landau cut.



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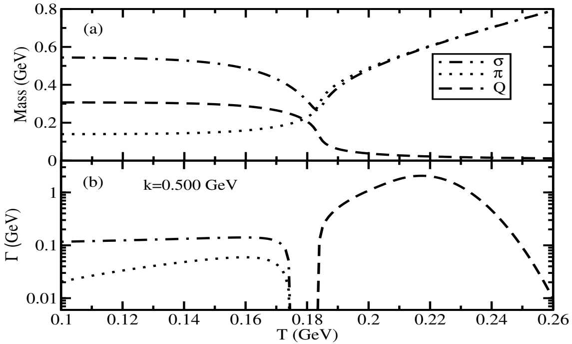


Figure : Temperature dependence of mass (a) and thermal width (b) for quark, pion and sigma meson( $|\vec{k}| = 0.500$ GeV for panel (b)).

# Results for constant thermal width

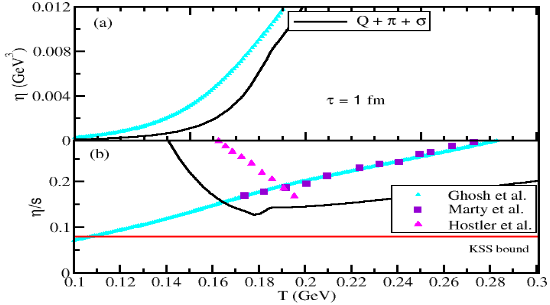


Figure : Temperature dependence of $\eta$ (a) and $\eta/s$ (b)

# Entropy Density vs. Temperature

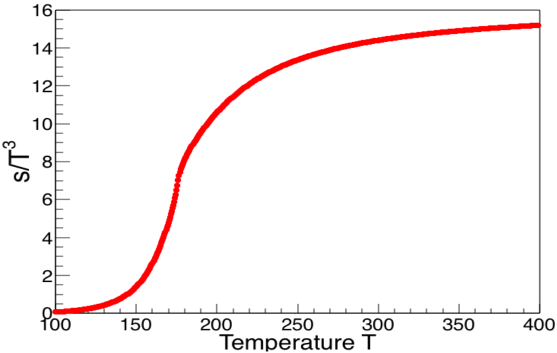


Figure : entropy density vs. T

# Results with constant Thermal width

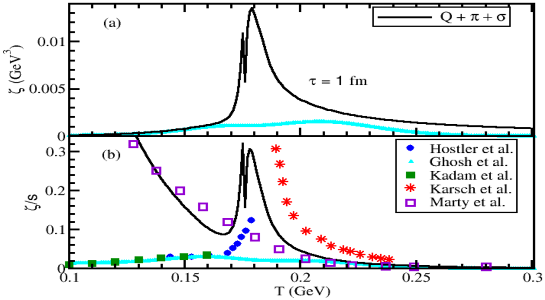


Figure : Temperature dependence of $\zeta$ (a) and $\zeta/s$ (b)

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# $c_s^2$ vs. Temperature

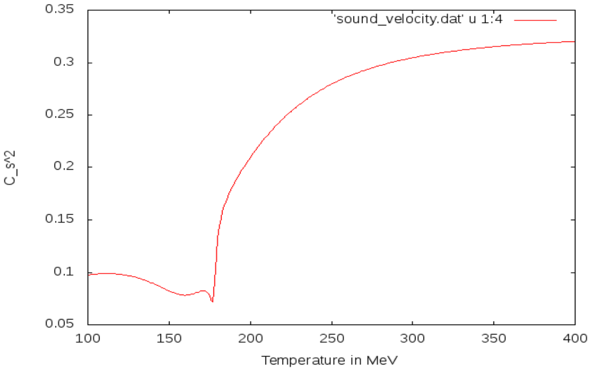



Figure : $C_s^2$ vs T

# $(\frac{1}{3} - c_s^2)$ vs. Temperature

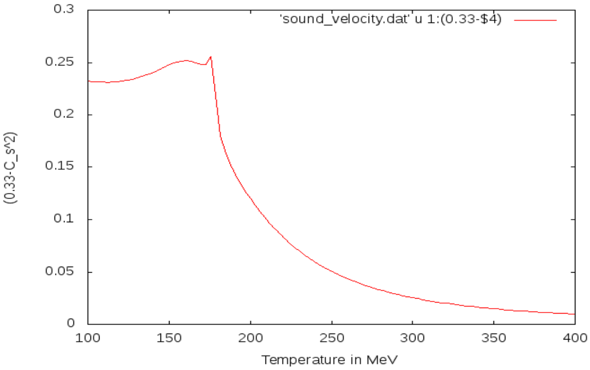
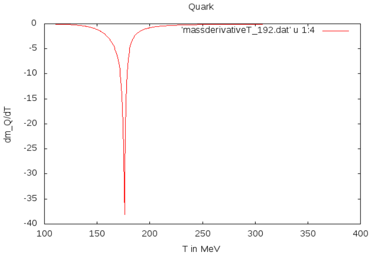
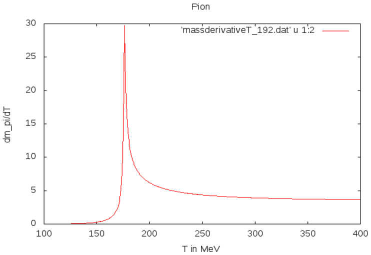
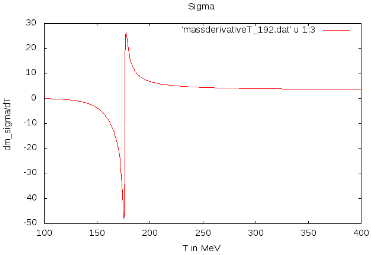



Figure : $(\frac{1}{3}C_s^2)$ vs T


Quark
'massderivativeT_192.dat' u 1:4
dm_Q/dT
0
-5
-10
-15
-20
-25
-30
-35
-40
100
150
200
250
300
350
400
T in MeV


Pion
'massderivativeT_192.dat' u 1:2
dm_pi/dT
30
25
20
15
10
5
0
100
150
200
250
300
350
400
T in MeV


Sigma
'massderivativeT_192.dat' u 1:3
dm_sigma/dT
30
20
10
0
-10
-20
-30
-40
-50
100
150
200
250
300
350
400
T in MeV

# Results with Temperature dependent thermal width

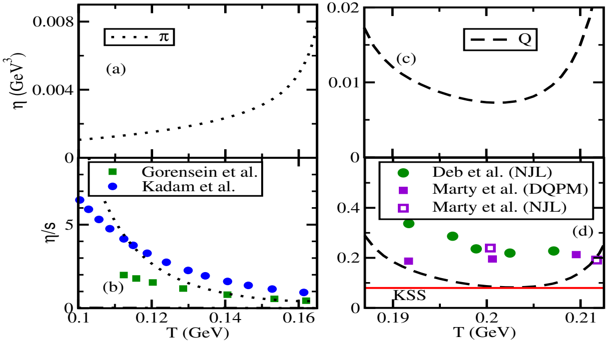



Figure : Temperature dependence of $\eta$ (a) and $\eta/s$ (b) for pion and quark using $\tau(T, K)$

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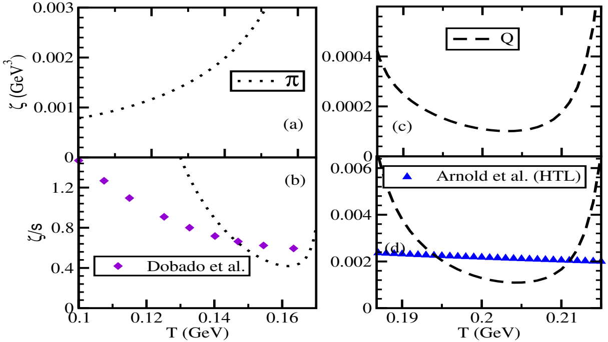


Figure : Temperature dependence of $\zeta$ (a) and $\zeta/s$ (b) of quark pion by using $\tau(T,k)$.

# Discussion

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• We have investigated the role of PQM dynamics in calculations of different transport coefficients like shear viscosity $\eta$, bulk viscosity $\zeta$ of quark and hadronic medium.
• Starting from standard expressions for transport coefficient with both constant and temperature dependant relaxation time the plots for transport coefficients are obtained which are in qualitative agreement with expected and available results.


THANK
YOU