

# Does the chiral magnetic effect affect the dynamic critical phenomena in QCD?

Sogabe Noriyuki (Keio University)  
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Collaborators: Masaru Hongo (RIKEN iTHESS)  
Naoki Yamamoto (Keio University)

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# Two goals of Heavy-ion collisions

The screenshot shows the homepage of the Beam Energy Scan Theory (BEST) Collaboration. At the top left is the Brookhaven National Laboratory logo. In the center, the text "Beam Energy Scan Theory (BEST) Collaboration" is displayed. At the top right is the U.S. Department of Energy logo. Below the header is a navigation bar with links: Home, Collaboration Members, Publications, News, Events, and Contacts. The "Home" link is highlighted with a blue background.

## Our Mission

The Beam Energy Scan Theory (BEST) Collaboration is a Topical Collaboration in Nuclear Theory, funded by the [US Department of Energy](#), [Office of Science](#), [Office of Nuclear Physics](#) for the period 2016-2020.

The BEST Collaboration, involving collaborators from two national laboratories and 11 universities, will construct and provide a theoretical framework for interpreting the results from the ongoing Beam Energy Scan program at the [Relativistic Heavy Ion Collider](#) (RHIC). The main goals of this program are to discover, or put constraints on the existence, of a [critical point in the QCD phase diagram](#), and to locate the onset of chiral symmetry restoration by observing correlations related to [anomalous hydrodynamic effects](#) in quark gluon plasma.

For this purpose, the BEST Collaboration is developing a set of theoretical tools, including hot-dense lattice QCD, initial state models, state-of-the-art hydrodynamic codes incorporating dissipation, hydrodynamic and critical fluctuations, and the effects of the chiral anomaly, as well as hadronic models of the final state of a heavy ion collision. These tools will be used to analyze RHIC Beam Energy Scan data.



## Upcoming Events

- ▶ [BEST Collaboration Annual Meeting](#), August 5-6, 2017, Stony Brook University, Stony Brook, New York, USA
- ▶ [Critical Point and Onset of Deconfinement \(CPOD-2017\)](#), August 7-11, 2017, Stony Brook University, Stony Brook, New York, USA
- ▶ [Phases of QCD and Beam Energy Scan](#)

<https://www.bnl.gov/physics/best/>

# Two goals of Heavy-ion collisions

- QCD critical phenomena
  - Universality class
  - Critical behavior
- Chiral transport phenomena
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Vortical Effect (CVE)

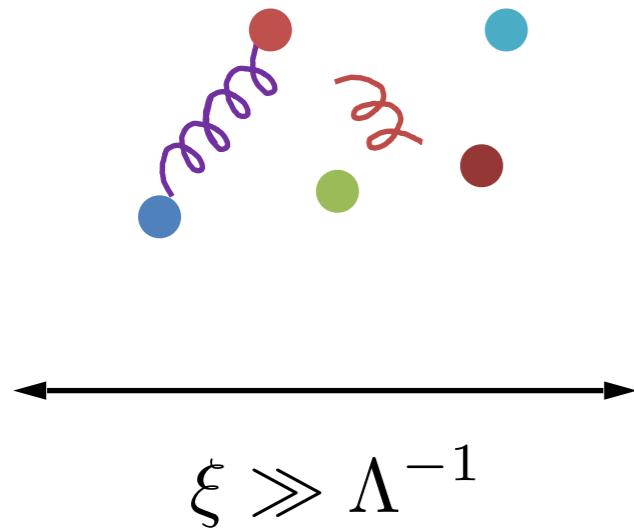
How about the interplay?

Does the CME affect the dynamic critical phenomena in QCD?

# Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

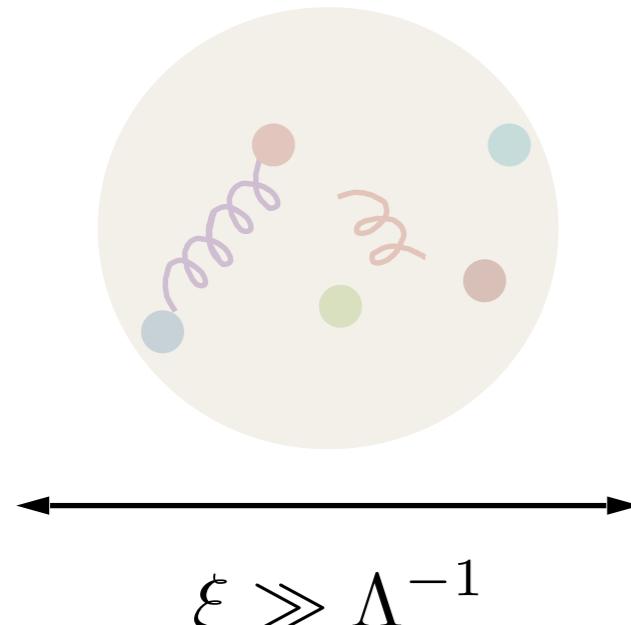
Microscopic theory



# Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Microscopic theory



Integrating out

Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved densities
- Nambu-Goldstone modes

$$\xi \gg \Lambda^{-1}$$

Same Symmetries

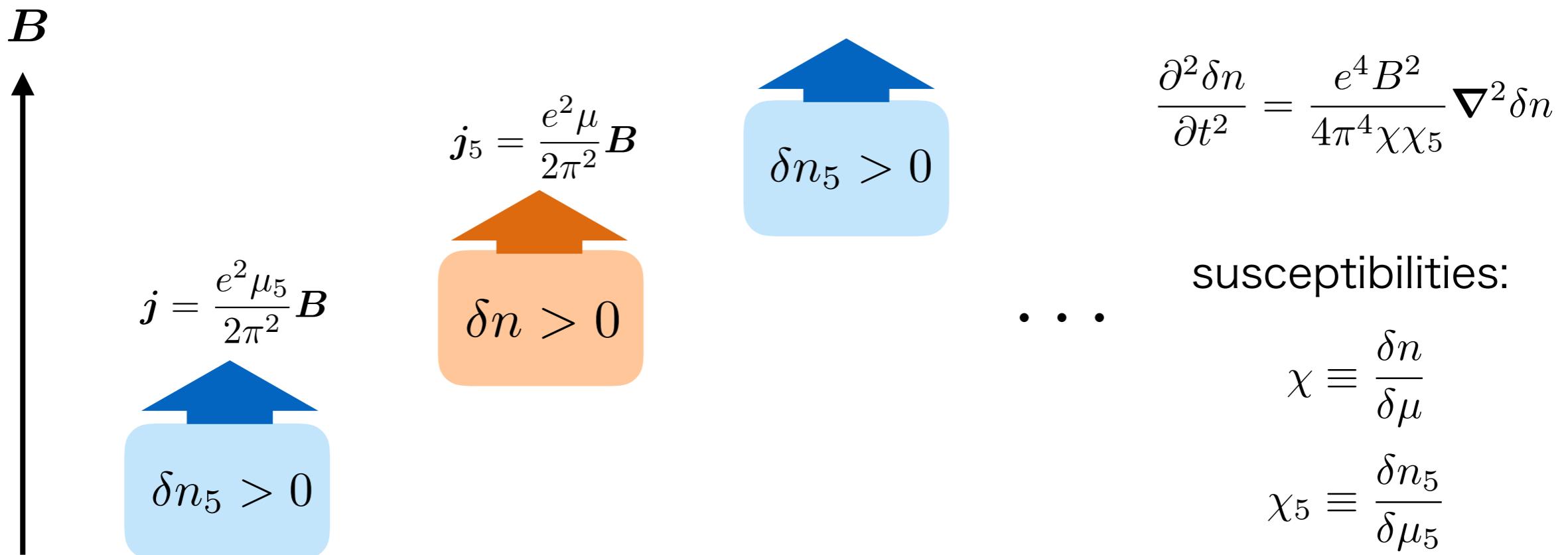
Gapless modes are important.

# Chiral Magnetic Wave

G. M. Newman (2006)

D. E. Kharzeev and H. Yee (2011)

Collective **gapless** density wave



affect dynamic critical phenomena?

# Outline

1

Setup

2

Static critical phenomena

Ginzburg-Landau theory

3

Dynamic critical phenomena

Langevin theory

# Setup

- 2 flavor QCD with **massless** u, d quarks at finite  $T, \mu_I, B$
- Chiral Symmetry

$$\begin{array}{ccc} B = 0 & & B \neq 0 \\ \text{SU}(2)_L \times \text{SU}(2)_R & \longrightarrow & \text{U}(1)_L^3 \times \text{U}(1)_R^3 \\ q_{L,R} \rightarrow e^{i\theta_{L,R}^a \tau^a} q_{L,R} & & q_{L,R} \rightarrow e^{i\theta_{L,R}^3 \tau^3} q_{L,R} \end{array}$$

- Second order chiral phase transition

# Hydrodynamic variables

- Order parameter

$$\phi_\alpha \equiv \begin{pmatrix} \sigma \\ \pi \end{pmatrix} \quad \begin{array}{l} \text{chiral condensate: } \sigma \equiv \bar{q}q \\ \text{neutral pion: } \pi \equiv \bar{q}i\gamma_5\tau^3q \end{array}$$

- Conserved densities

$$n_I \equiv \bar{q}\gamma^0\tau^3q \quad n_{I5} \equiv \bar{q}\gamma^0\gamma^5\tau^3q$$

- Note: Charged pions are **not** hydrodynamic  $m_{\pi^\pm} \propto \sqrt{e|B|}$

# Ginzburg-Landau (GL) theory

$$F = \int d\mathbf{r} \left[ \frac{r}{2}(\phi_\alpha)^2 + \frac{1}{2}(\nabla\phi_\alpha)^2 + \frac{1}{2\chi_I} n_I^2 + \frac{1}{2\chi_{I5}} n_{I5}^2 + u(\phi_\alpha)^2(\phi_\beta)^2 + \gamma n_I \phi_\alpha^2 \right]$$

- Near second-order phase transition  $\longrightarrow$  small order parameters
- Long-range behavior  $\longrightarrow$  derivative expansion
- QCD symmetries  $\longrightarrow$  constraints on the expansion
  - chiral symmetry and CPT symmetries

# Static critical phenomena

$$F = \int d\mathbf{r} \left[ \frac{r}{2}(\phi_\alpha)^2 + \frac{1}{2}(\nabla\phi_\alpha)^2 + \frac{1}{2\chi_I} n_I^2 + \frac{1}{2\chi_{I5}} n_{I5}^2 + u(\phi_\alpha)^2(\phi_\beta)^2 + \gamma n_I \phi_\alpha^2 \right]$$

Correlation functions:  $\langle \mathcal{O}[\phi, n_I, n_{I5}] \rangle = \frac{\int \mathcal{D}\phi \mathcal{D}n_I \mathcal{D}n_{I5} O[\phi, n_I, n_{I5}] e^{-F}}{\int \mathcal{D}\phi \mathcal{D}n_I \mathcal{D}n_{I5} e^{-F}}$

e.g. isospin susceptibility:

$$\chi_I^{\text{ren}} \sim \xi^{\frac{\alpha}{\nu}} \quad \frac{\alpha}{\nu} = \frac{\epsilon}{5} \quad (\epsilon \equiv 4 - d)$$

↑  
correlation length

critical exponents:  $\xi \sim \tau^\nu, \quad \chi_I \sim \tau^\alpha \quad \text{with} \quad \tau \equiv \frac{T - T_c}{T_c}$

# Langevin equation

$$\frac{\partial \phi_\alpha(\mathbf{r}, t)}{\partial t} = -g \int d\mathbf{r}' [\phi_\alpha(\mathbf{r}, t), n_{I5}(\mathbf{r}', t)] \frac{\delta F}{\delta n_{I5}(\mathbf{r}', t)} - \Gamma \frac{\delta F}{\delta \phi_\alpha(\mathbf{r}, t)} + \xi_\alpha(\mathbf{r}, t)$$

↑  
Poisson bracket                              ↑  
kinetic coefficient

$$[n_{I5}(\mathbf{r}, t), \phi_\alpha(\mathbf{r}', t)] = \varepsilon_{\alpha\beta} \phi_\beta \delta(\mathbf{r} - \mathbf{r}')$$

U(1)<sub>A</sub><sup>3</sup> algebra

$$\langle \xi_\alpha(\mathbf{r}, t) \xi_\beta(\mathbf{r}', t') \rangle = 2\Gamma \delta_{\alpha\beta} \delta^d(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

Fluctuation dissipation relation (FDR)

# Langevin equation

$$\frac{\partial n_{\text{I}}(\mathbf{r}, t)}{\partial t} = - \int d\mathbf{r}' [n_{\text{I}}(\mathbf{r}, t), n_{\text{I5}}(\mathbf{r}', t)] \frac{\delta F}{\delta n_5(\mathbf{r}', t)} + \lambda_{\text{I}} \nabla^2 \frac{\delta F}{\delta n_{\text{I}}(\mathbf{r}, t)} + \xi_{n_{\text{I}}}(\mathbf{r}, t)$$

↑  
Poisson bracket      ↑  
kinetic coefficient

$$[n_{\text{I}}(\mathbf{r}, t), n_{\text{I5}}(\mathbf{r}', t)] = C \mathbf{B} \cdot \nabla \delta(\mathbf{r} - \mathbf{r}')$$

$C$  : anomaly coefficient

Anomalous commutation relation

R. Jackiw and K. Johnson (1969)

S. L. Adler and D. G. Boulware (1969)

# Field theory of Langevin eqs.

P. C. Martin, E. D. Siggia, and A. Rose (1973)

$$\frac{\partial \psi_N(\mathbf{r}, t)}{\partial t} = \mathcal{F}_N[\{\psi_M\}] + \xi_N(\mathbf{r}, t) \quad \text{Langevin eq.}$$
$$\langle \xi_M(\mathbf{r}, t) \xi_N(\mathbf{r}', t') \rangle = L_{MN}(\nabla) \delta^d(\mathbf{r} - \mathbf{r}') \delta(t - t') \quad \text{FDR}$$
$$\psi_N \equiv \{\phi_\alpha, n_I, n_{I5}\}$$

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$$\psi_N \equiv \{\phi_\alpha, n_I, n_{I5}\}$$

$$\langle \mathcal{O}[\psi^\xi] \rangle = \mathcal{N} \int \mathcal{D}[\xi] \mathcal{O}[\psi^\xi] \exp \left[ -\frac{1}{4} \int dt \int d\mathbf{r} \xi_M L_{MN}^{-1} \xi_N \right]$$

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$$1 = \int \mathcal{D}[\psi] \prod_N \delta(\psi_N - \psi_N^\xi) = \int \mathcal{D}[\psi] \prod_N \prod_{\mathbf{r}, t} \delta \left( \frac{\partial \psi_N}{\partial t} - \mathcal{F}_N[\{\psi_M\}] - \xi_N \right)$$

replace  $\mathcal{O}[\psi^\xi]$  by  $\mathcal{O}[\psi]$   $\longrightarrow$  integrating out  $\xi_N$

# Field theory of Langevin eqs.

P. C. Martin, E. D. Siggia, and A. Rose (1973)

$$\frac{\partial \psi_N(\mathbf{r}, t)}{\partial t} = \mathcal{F}_N[\{\psi_M\}] + \xi_N(\mathbf{r}, t) \quad \text{Langevin eq.}$$

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$$\psi_N \equiv \{\phi_\alpha, n_I, n_{I5}\}$$

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$$1 = \int \mathcal{D}[\psi] \prod_N \delta(\psi_N - \psi_N^\xi) = \int \mathcal{D}[\psi] \prod_N \prod_{\mathbf{r}, t} \delta \left( \frac{\partial \psi_N}{\partial t} - \mathcal{F}_N[\{\psi_M\}] - \xi_N \right)$$

replace  $\mathcal{O}[\psi^\xi]$  by  $\mathcal{O}[\psi]$  → integrating out  $\xi_N$

$$\langle \mathcal{O}[\psi] \rangle = \mathcal{N}' \int \mathcal{D}[i\tilde{\psi}] \int \mathcal{D}[\psi] \mathcal{O}[\psi] \exp \left( -S[\{\tilde{\psi}_M\}, \{\psi_M\}] \right)$$

$$S[\{\tilde{\psi}_M\}, \{\psi_M\}] = \int dt \int d\mathbf{r} \left[ \tilde{\psi}_N \left( \frac{\partial \psi_N}{\partial t} - \mathcal{F}_N[\{\psi_M\}] \right) - \tilde{\psi}_M L_{MN}(\nabla) \tilde{\psi}_N \right]$$

$\tilde{\psi}_N \equiv \{\tilde{\phi}_\alpha, \tilde{n}_I, \tilde{n}_{I5}\}$

response field

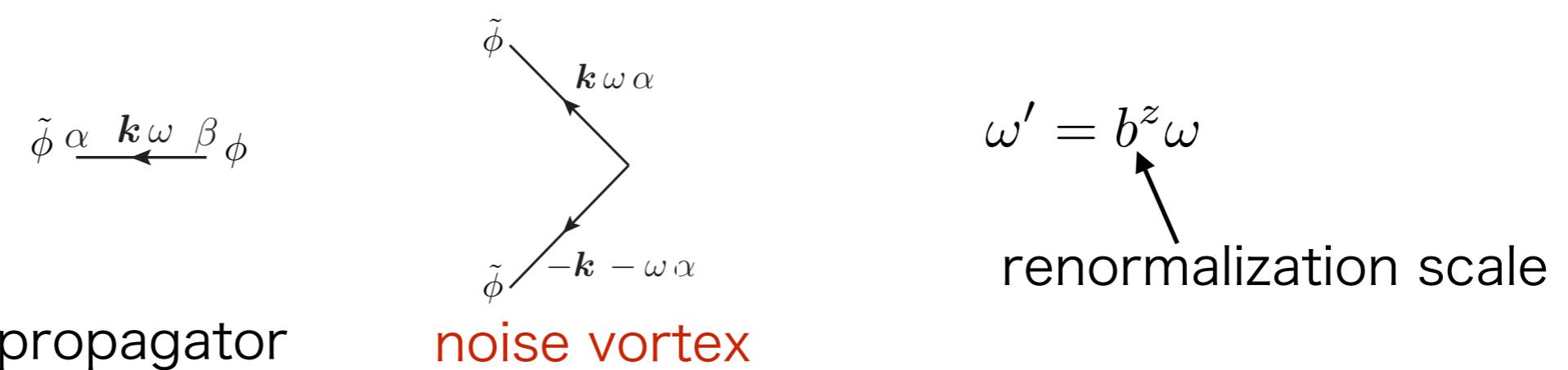
# Dynamic renormalization group (RG)

(Quantum) Field theory (QFT) manipulation

- Feynman rules
- Diagrammatic expansions
- RG equations for all parameters in the Langevin theory:

GL parameters, kinetic and anomaly coefficients

notes: difference from the usual QFT



# Dynamic universality class

CME **does not** affect the dynamic universality class

Anti-ferromagnetic class “Model G”

P. C. Hohenberg and B. I. Halperin (1977)

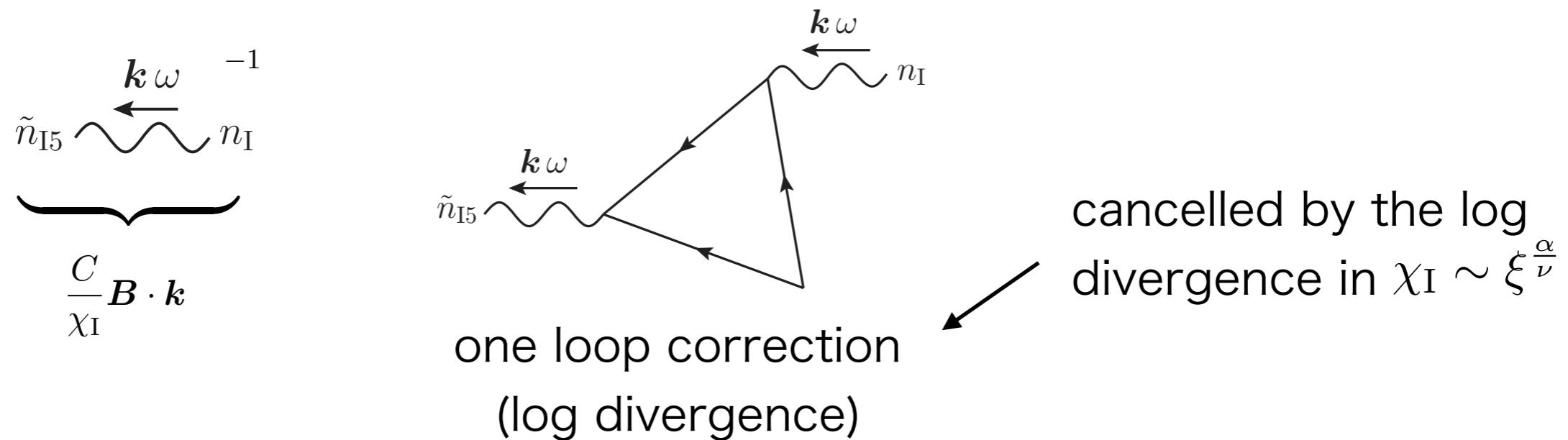
Dynamic critical exponent:  $z = \frac{d}{2}$

two-component order parameter with symmetry group  $U(1)_A^3$

c.f. QCD critical point (finite quark mass and finite  $\mu_B$ ) “Model H”

# Extended non-renormalization theorem

- Generally, no higher-order corrections to the CME coefficient
- How about near the chiral phase transition where  $\sigma$  is massless?



No loop corrections on CME coefficient near phase transition point

# New dynamic critical behavior

- Speed of the chiral magnetic wave:

from statics

$$c_s^2 \equiv \frac{C^2 B^2}{\chi_I \chi_{I5}} \sim \xi^{-\frac{\alpha}{\nu}}$$

$$\chi_I \sim \xi^{\frac{\alpha}{\nu}} \quad \frac{\alpha}{\nu} = \frac{\epsilon}{5}$$

Critical attenuation

- Near the QCD critical point?
- Possible signature of the QCD critical point in Heavy-ion collisions

# Summary

We study the second order chiral phase transition of two flavor QCD with massless quarks in the presence of the CME:

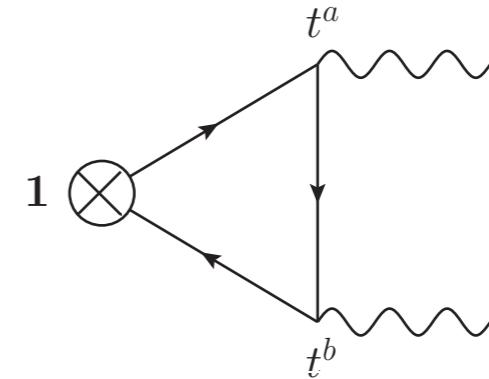
- Dynamic universality class
- CME (anomaly) coefficient
- Critical attenuation of the chiral magnetic wave:

$$c_s^2 \equiv \frac{C^2 B^2}{\chi_I \chi_{I5}} \sim \xi^{-\frac{\alpha}{\nu}} \quad \frac{\alpha}{\nu} = \frac{\epsilon}{5}$$

# Backup slides

# Chiral magnetic wave for Baryon charges

$$\frac{\partial n_B}{\partial t} + \nabla \cdot j_B = 0 \quad \text{CME}$$
$$\frac{\partial n_{B5}}{\partial t} + \nabla \cdot j_{B5} = \# \text{tr} G^{\mu\nu} \tilde{G}_{\mu\nu} \quad \text{CSE}$$



c.f. for isospin charges

$$\frac{\partial^2 \delta n_I}{\partial t^2} = \frac{e^4 B^2}{4\pi^4 \chi \chi_5} \nabla^2 \delta n_I$$

