The critical line of QCD at small baryon density

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based on C. B, M. D'Elia, F. Negro, F. Sanfilippo, K. Zambello 1805.02960
and previous works 1410.5758 and 1507.03571

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Outline

The deconfinement transition at zero density

The deconfinement/chiral symmetry restoration transition at vanishing baryon density has been extensively studied using Lattice QCD and its properties are by now well established.

It is not a "phase transition" but just a smooth analytical crossover; as a consequence all "critical properties" are observable dependent.

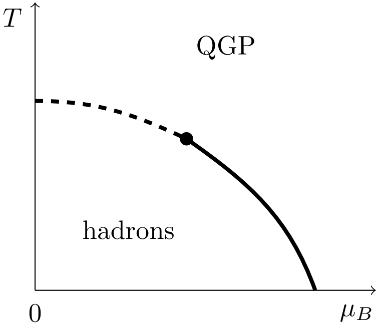
In the following all Lattice QCD results will be related to the chiral symmetry restoration aspects.

Critical temperature:

$T_c|_{\mathrm{BW}} = 152(5)\,\mathrm{MeV}$ Aoki et al. Phys. Lett. B **643**, 46 (2006)

$T_c|_{\mathrm{hotQCD}} = 154(9)\,\mathrm{MeV}$ Bazavov et al. Phys. Rev. D **85** 054503 (2012)

The "basic" phase diagram in the $T - \mu_B$ plane



- analytic crossover for $\mu = 0$ (no known symmetries to break, it would be a real transition for massless or infinitely massive quarks)
- first order transition for $T = 0$ (simple argument based on light particles counting)
- a second order transition somewhere in the middle

What do we know of the $T - \mu_B$ phase diagram? Very little...

To use Monte Carlo sampling methods the Euclidean action has to be positive, however the usual γ_5 hermiticity $\gamma_5 \not{D}(\mu)\gamma_5 = \not{D}(-\mu^*)^\dagger$ ensures positivity only for vanishing or imaginary values of μ: sign problem

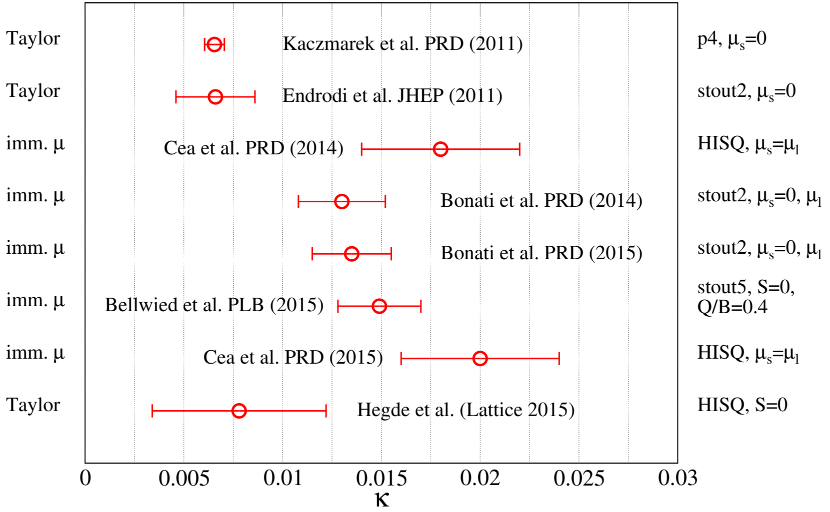
The region that can be reliably explored by Lattice QCD is that of "small" μ_B, where some workaround to the sign problem exist.

Since $Z(\mu_B) = Z(-\mu_B)$ we have

$$T_c(\mu_B) = T_c(0)\left(1 - \kappa\left(\frac{\mu_B}{T_c(0)}\right)^2 + c\left(\frac{\mu_B}{T_c(0)}\right)^4 + \cdots\right)$$

and the curvature κ can be determined using the computational methods available at present.

Various determinations of κ from LQCD



Analytic continuation method

For $\mu = i\mu^I$, $\mu^I \in \mathbb{R}$, there is no sign problem.

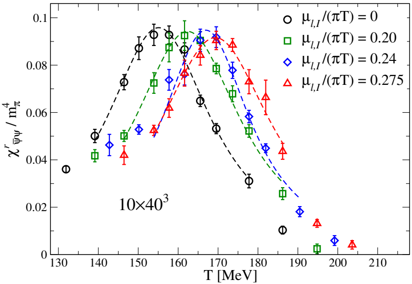
For each value of μ^I we can perform a scan in T and look for the transition using standard procedures (look for χ maxima or $\bar{\psi}\psi$ inflection points).

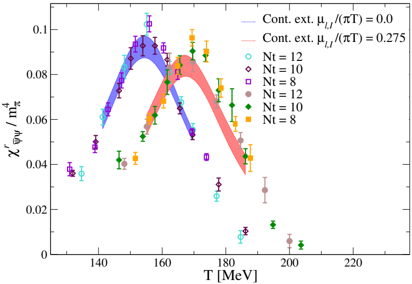
Two different procedures for the continuum limit

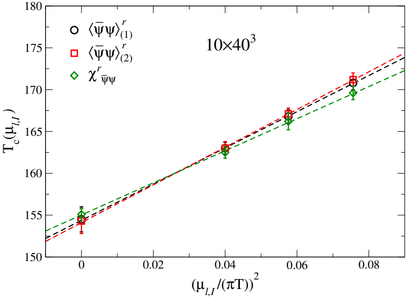
1. fix lattice spacing a
2. for each μ^I compute $T_c(\mu^I, a)$
3. compute the curvature at fixed a: κ_a
4. try several a values and extrapolate κ_a to continuum

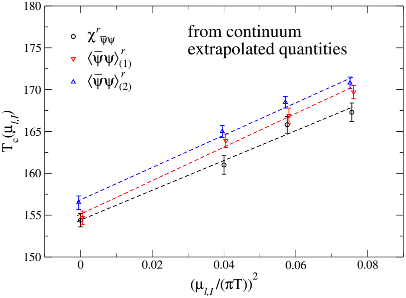
1. fix lattice spacing a
2. for each μ^I compute $\bar{\psi}\psi^r(\mu^I, a)$ and $\chi^r_{\bar{\psi}\psi}(\mu^I, a)$
3. try several a values and extrapolate $\bar{\psi}\psi^r(\mu^I, a)$ and $\chi^r_{\bar{\psi}\psi}(\mu^I, a)$ to continuum
4. find $T_c(\mu^I, a = 0)$ using the results of the previous point
5. compute κ

The two ways to the continuum limit









Imaginary chemical potential: systematics checklist

- finite volume: aspect ratio 4 was shown to be enough for finite volume effects to be smaller than statistical errors.
- continuum limit: the two different ways of extracting the continuum limit gave compatible results (the small discrepancies has been used as an estimate of the systematics)
- observable dependence: to locate the transition we used both the inflection point of $\bar{\psi}\psi$ (renormalized in two different ways) and the maximum of χ. Different results were compatible with each other and the (small) differences were used as systematics.
- μ^I-specific systematics: the dependence on the specific chemical potential setups and the possibility of contamination from higher order terms were investigated and the presence of significants systematics was excluded.

Taylor expansion method (1)

We have to estimate κ by using observables defined at $\mu = 0$ and there are several way of doing it.

If the transiton with $m_\ell \equiv m_u = m_d = 0$ is second order, since the baryon number does not break chiral symmetry, for $m_\ell \approx 0$ we can define the scaling variables

$$t \simeq \frac{1}{t_0}\left(\frac{T - T_c(0)}{T_c(0)} + \kappa\left(\frac{\mu_B}{T_c(0)}\right)^2\right) \quad h \simeq \frac{1}{h_0}\frac{m_\ell}{m_s}$$

and $\bar{\psi}\psi$ has the scaling form $\bar{\psi}\psi(t, h)$, thus (Kaczmarek et al. PRD (2011))

$$\kappa_1 = T_c(0)\,\frac{\frac{\partial}{\partial \mu_B^2}\bar{\psi}\psi}{\frac{\partial}{\partial T}\bar{\psi}\psi}\Bigg|_{\substack{\mu_B=0\\ T=T_c(0)}}$$

This is, strictly speaking, the curvature in the chiral limit.

Taylor expansion method (2)

One can alternatively define $T_c(\mu_B)$ by the equation

$$\bar{\psi}\psi(T_c(\mu_B), \mu_B) = \bar{\psi}\psi(T_c(0), 0)$$

from which (Endrodi et al. JHEP (2011))

$$\kappa_1 = -T_c(0)\, \frac{\mathrm{d}T_c(\mu_B)}{\mathrm{d}\mu_B^2}\bigg|_{\mu_B=0} = T_c(0)\, \frac{\frac{\partial}{\partial \mu_B^2}\bar{\psi}\psi}{\frac{\partial}{\partial T}\bar{\psi}\psi}\Bigg|_{\substack{\mu_B=0 \\ T=T_c(0)}}$$

that is identical to the expression by Kaczmarek et al. PRD (2011).

In Bonati et al. PRD (2014) it was verified by using analytic continuation that $T_c(\mu_B)$ defined in this way is consistent with the inflection point temperature of $\bar{\psi}\psi(T, \mu_B)$.

Taylor expansion method (3)

Define as usual the transition point $T_c(\mu_B)$ as the inflection point of $\bar{\psi}\psi(T, \mu_B)$ at fixed μ_B.

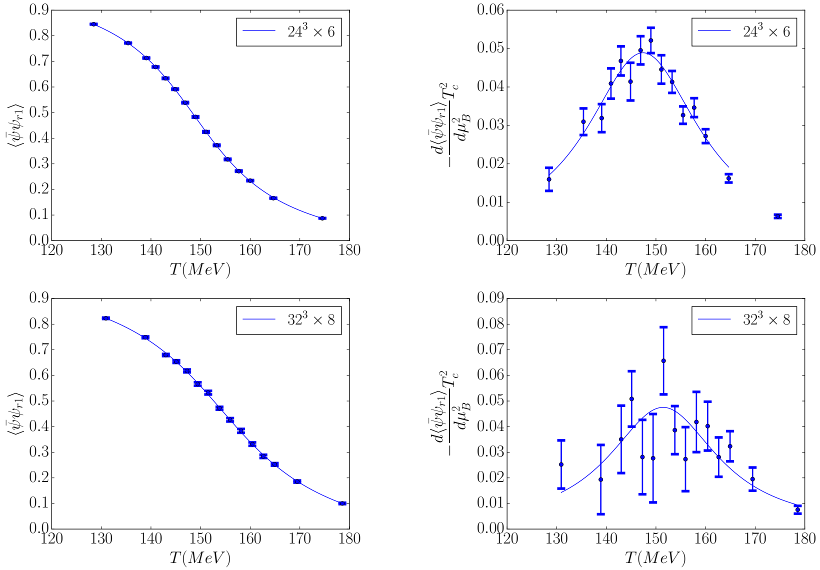
By developing $\bar{\psi}\psi(T, \mu_B)$ in $T - T_c(0)$ and μ_B one obtains

$$\kappa_2 = T_c(0) \left. \frac{\frac{\partial^2}{\partial T^2}\frac{\partial}{\partial \mu_B^2}\bar{\psi}\psi(T, \mu_B)}{\frac{\partial^3}{\partial T^3}\bar{\psi}\psi(T, \mu_B)} \right|_{\substack{\mu_B=0 \\ T=T_c(0)}}$$

This method is theoretically more solid than the other (if there is a transition it gets it) but requires higher derivatives of $\bar{\psi}\psi$.

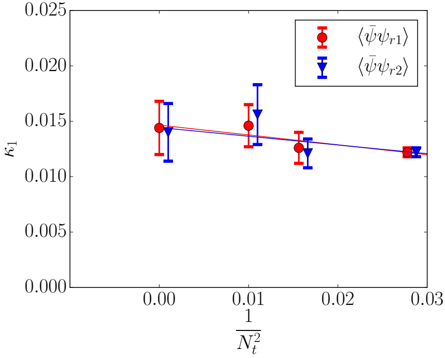
higher derivatives $\Rightarrow$ (much) higher statistics required
bad scaling with volume

An example of the problem for κ_2



Numerical results

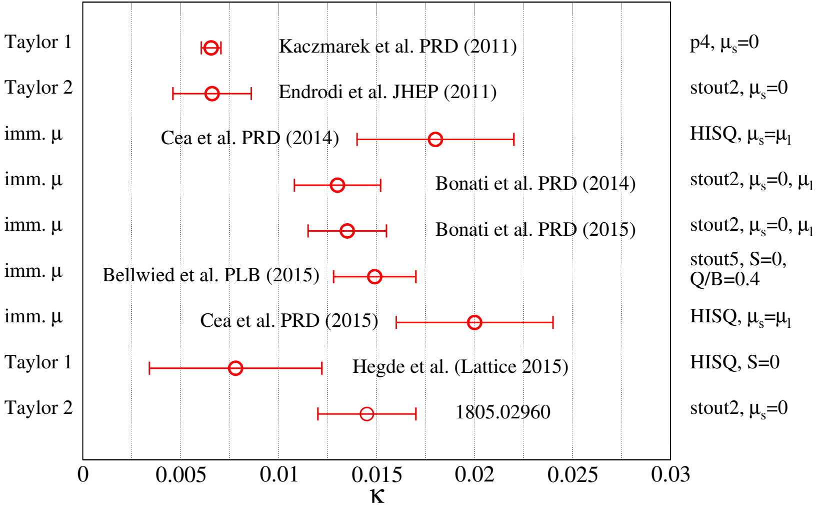
Lattice	$\kappa_1(\bar{\psi}\psi_{r1})$	$\kappa_2(\bar{\psi}\psi_{r1})$	$\kappa_1(\bar{\psi}\psi_{r2})$	$\kappa_2(\bar{\psi}\psi_{r2})$
$16^3 \times 6$	0.0122(5)	0.016(6)	0.0122(5)	0.016(6)
$24^3 \times 6$	0.0122(4)	0.015(4)	0.0122(4)	0.015(4)
$32^3 \times 8$	0.0126(14)	0.014(9)	0.0121(13)	0.012(8)
$40^3 \times 10$	0.0146(19)	-	0.0154(21)	-



- κ_1 is always compatible with κ_2
- no significant dependence on the renormalization scheme ($r1$, $r2$)
- very mild lattice artefacts

continuum value $\kappa = 0.0145(25)$

κ from LQCD



Conclusions

- In the past years consensus was reached on the value of $T_c(\mu_B = 0) \simeq 150\,\mathrm{MeV}$ for the (pseudo)critical temperature obtained from chiral observables.
- Results obtained in the last couple of years are converging to the value $\kappa \approx 0.015$ for the curvature of the (pseudo)critical line obtained from chiral observables.
- The tension between results obtained by analytic continuation and by Taylor expansion is disappearing.
- This is a (further) confirmation that the QCD phase diagram can be reliably studied for "small" μ_B using first principle lattice computations.

Thank you for your attention!

Backup slides with something more

A reminder on the quark chemical potentials

The relations between the conserved charges and the quark numbers are

$$
\begin{aligned}
B &= (N_u + N_d + N_s)/3 \\
Q &= (2N_u - N_d - N_s)/3 \\
S &= -N_s
\end{aligned}
$$

The quark chemical potentials are defined in such way that

$$B\mu_B + Q\mu_Q + S\mu_S = N_u\mu_u + N_d\mu_d + N_s\mu_s$$

thus

$$
\begin{aligned}
\mu_u &= \mu_B/3 + 2\mu_Q/3 \\
\mu_d &= \mu_B/3 - \mu_Q/3 \\
\mu_s &= \mu_B/3 - \mu_Q/3 - \mu_S
\end{aligned}
$$

Typical setups and strangeness neutrality

In (almost) all simulations $\mu_Q \equiv 0$ and one of the two following extreme cases is used:

$$\begin{aligned}
1) \quad & \mu_u = \mu_d = \mu_B/3; \quad \mu_s = 0 \\
2) \quad & \mu_u = \mu_d = \mu_B/3; \quad \mu_s = \mu_B/3
\end{aligned}$$

that correspond to

$$\begin{aligned}
1) \quad & \mu_S = \mu_B/3 \\
2) \quad & \mu_S = 0
\end{aligned}$$

If we want to impose strangeness neutrality ($\langle N_s \rangle = 0$) we need

$$0 = \frac{\partial \log Z(\mu_B, \mu_S)}{\partial \mu_S} \simeq \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_S}\right|_{\mu=0} \mu_S + \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_B}\right|_{\mu=0} \mu_B + \cdots$$

from which we get a relation between μ_B and μ_S.

Strangeness neutrality and Q/B ratio

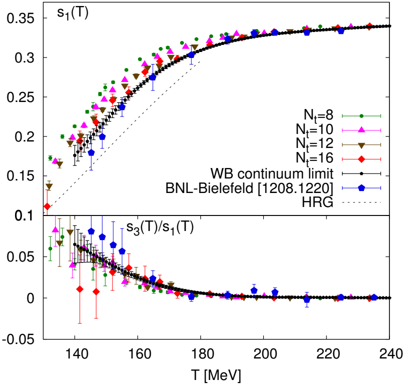
Explicitly one can write

$$\frac{\mu_S}{T} = s_1(T)\frac{\mu_B}{T} + s_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$

At $T \approx T_c$ we have $\mu_S \simeq \mu_B/4$ and thus $\mu_s \simeq \mu_B/12 = \mu_u/4$.

In a similar way μ_Q can be fixed by imposing $\langle N_Q \rangle = r\langle N_B \rangle$, where $r = Z/A \simeq 0.4$, obtaining:

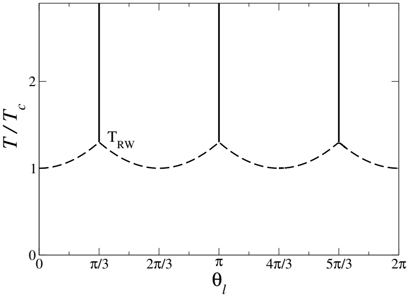
$$\frac{\mu_Q}{T} = q_1(T)\frac{\mu_B}{T} + q_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$



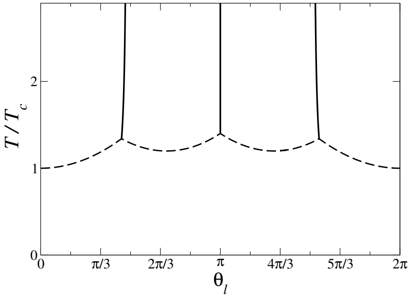
Bazavov et al. PRL (2012)

Borsányi et al. PRL (2013)

Phase diagram at imaginary chemical potential



$\mu_s = \mu_\ell,\ \theta_\ell = \mu_\ell^I / T$



$\mu_s = 0,\ \theta_\ell = \mu_\ell^I / T$

Lattice renormalizations

Renormalized chiral condensate:

- Cheng et al. PRD (2008)

$$\langle\bar{\psi}\psi\rangle^r_{(1)}(a,T) = \frac{\langle\bar{\psi}\psi\rangle_\ell(a,T) - \frac{2m_\ell}{m_s}\langle\bar{\psi}\psi\rangle_\ell(a,T)}{\langle\bar{\psi}\psi\rangle_\ell(a,T=0) - \frac{2m_\ell}{m_s}\langle\bar{\psi}\psi\rangle_\ell(a,T=0)}$$

- Endrodi et al. JHEP (2011)

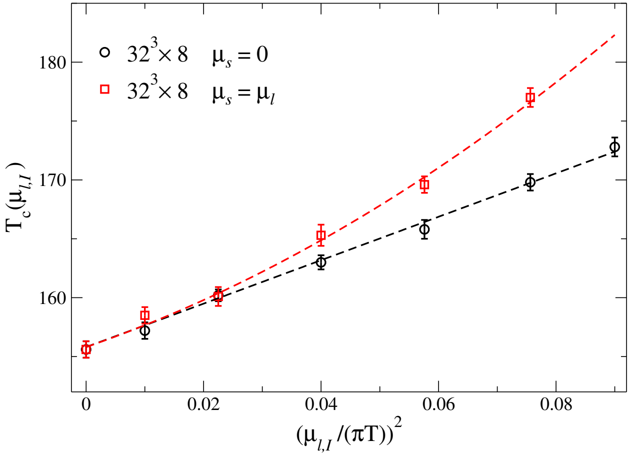
$$\langle\bar{\psi}\psi\rangle^r_{(2)}(a,T) = \frac{m_\ell}{m_\pi^4}\Big(\langle\bar{\psi}\psi\rangle_\ell(a,T) - \langle\bar{\psi}\psi\rangle_\ell(a,T=0)\Big)$$

Renormalized chiral susceptibility

- Aoki et al. JHEP (2009)

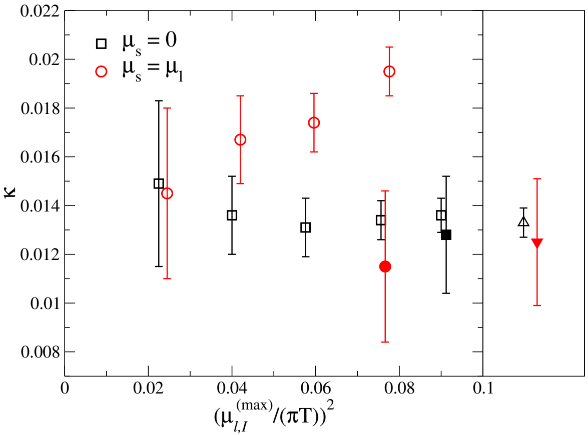
$$\chi^r_{\bar{\psi}\psi}(a,T) = m_\ell^2\Big(\chi_{\bar{\psi}\psi}(a,T) - \chi_{\bar{\psi}\psi}(a,T=0)\Big)$$

μ-systematics



Possible explanation for the different behaviours: the different location of the Roberge Weiss transition.

μ-systematics



Empty symbols: purely quadratic fit. Filled symbols: also quartic correction. Right panel: combined fit (i.e. fixing a common value for $T_c(0)$) to both data sets when a quartic correction is used for the $\mu_s = \mu_l$ data. The empty (filled) triangle corresponds to $\mu_s = 0$ ($\mu_s = \mu_l$).

Inflection point with Taylor method

We use

$$\bar{\psi}\psi(T,\mu) \simeq \bar{\psi}\psi(T,0) + \mu^2 \left. \frac{\partial \bar{\psi}\psi(T,\mu)}{\partial \mu^2} \right|_{\mu=0} \equiv A(T) + \mu^2 B(T)$$

and we search for the inflection point temperature $\frac{\partial^2}{\partial T^2}\bar{\psi}\psi(T,\mu) = 0$:

$$\begin{aligned} 0 = A''(T) + \mu^2 B''(T) \simeq A''(T_c(0)) + A'''(T_c(0))\big(T - T_c(0)\big) + \\ +\mu^2 \Big(B''(T_c(0)) + B'''(T_c)\big(T - T_c(0)\big)\Big) \, . \end{aligned}$$

Solving the equation and using $A''(T_c(0)) = 0$ one finds for $T_c(\mu)$ up to $\mathcal{O}(\mu^2)$ the expression

$$T_c(\mu) \simeq T_c(0) - \mu^2 \frac{B''(T_c(0))}{A'''(T_c(0))}$$