Fate of axial U(1) symmetry at two flavor chiral limit of QCD in finite temperature

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Thanks to

- Those who gave me useful information for this talk
    - Phillipe de Forcrand
    - Christian Lang
    - Gian Carlo Rossi
    - Peter Petreczky
    - Sayantan Sharma
    - Vicente Azcoiti
    - Bastian Brandt
- for useful discussion
    - Ryuichiro Kitano
    - Norikazu Yamada
- JLQCD members
    - Sinya Aoki
    - Guido Cossu
    - Shoji Hasihmoto
    - Hidenori Fukaya
    - Kei Suzuki ……

U(1) axial

$$\partial_\mu J_5^\mu = \frac{N_f}{32\pi^2} F\tilde{F}$$

- violated by quantum anomaly

$$\langle \partial_\mu J_5^\mu(x) \cdot O(0) \rangle = \frac{N_f}{32\pi^2} \langle F\tilde{F}(x) \cdot O(0) \rangle$$

up to contact terms

- at T=0, responsible for η' mass
    - non-trivial topology of gauge field

- at high T, this Ward-Takahashi identity is still valid

- however, if configurations that contribute to RHS is suppressed………

➡ the symmetry effectively recovers

◉ here N_f=2 (including N_f=2+1 with "2" driven to chiral limit)

Why bother ?

- **Because it is unsettled problem !**
- fate of $U(1)_A$ - analytic
  - Gross-Pisarski-Yaffe (1981) restores in high temperature limit
    - Dilute instanton gas
  - Cohen (1996)
    - measure zero instanton effect → restores
  - Lee-Hatsuda (1996)
    - zero mode does contributes → broken
  - Aoki-Fukaya-Tanigchi (2012)
    - QCD analysis (overlap) → restores w/ assumption (lattice)
  - Kanazawa-Yamamoto (2015)
    - EFT case study how restore / break
  - Azcoiti (2017)
    - case study how restore / break

Why bother ?

- **Because it is unsettled problem !**
- fate of $U(1)_A$ lattice
  - HotQCD (DW, 2012) broken
  - JLQCD (topology fixed overlap, 2013) restores
  - TWQCD (optimal DW, 2013) restores ?
  - LLNL/RBC (DW, 2014) broken
  - HotQCD (DW, 2014) broken
  - Dick et al. (overlap on HISQ, 2015) broken
  - Brandt et al. (O(a) improved Wilson 2016) restores
  - JLQCD (reweighted overlap from DW, 2016) restores
  - JLQCD (current: see Suzuki et al Lattice 2017) restores
  - Ishikawa et al (Wilson, 2017) at least Z_4 restores

Why bother ?

- **it may provide useful information on the phase transition**

- if the $\mathbf{U(1)_A}$ continue to be broken
  - $SU(2)_L \times SU(2)_R \simeq O(4)$ universality class for 2nd order
- if the $\mathbf{U(1)_A}$ recovers
  - $U(2)_L \times U(2)_R / U(2)_V$ for 2nd order
- provides crucial information on the universality class
- 1st order possible for both cases
  - though often discussed in context with $\mathbf{U(1)_A}$ **restoration**

Why bother ?

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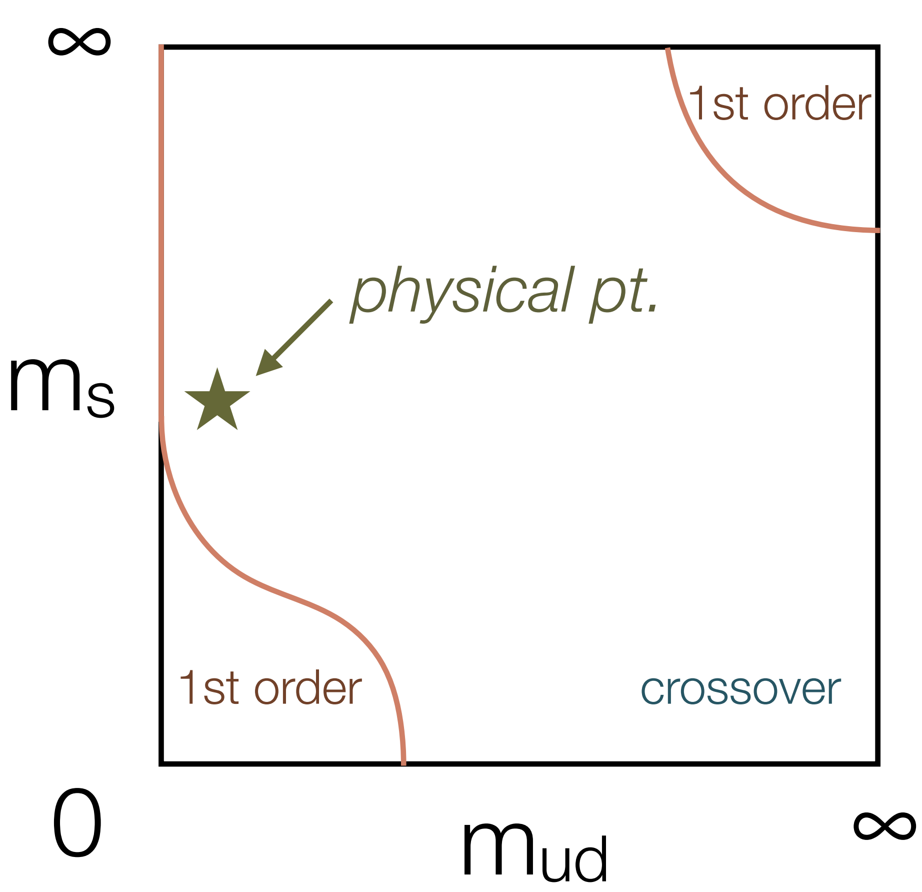
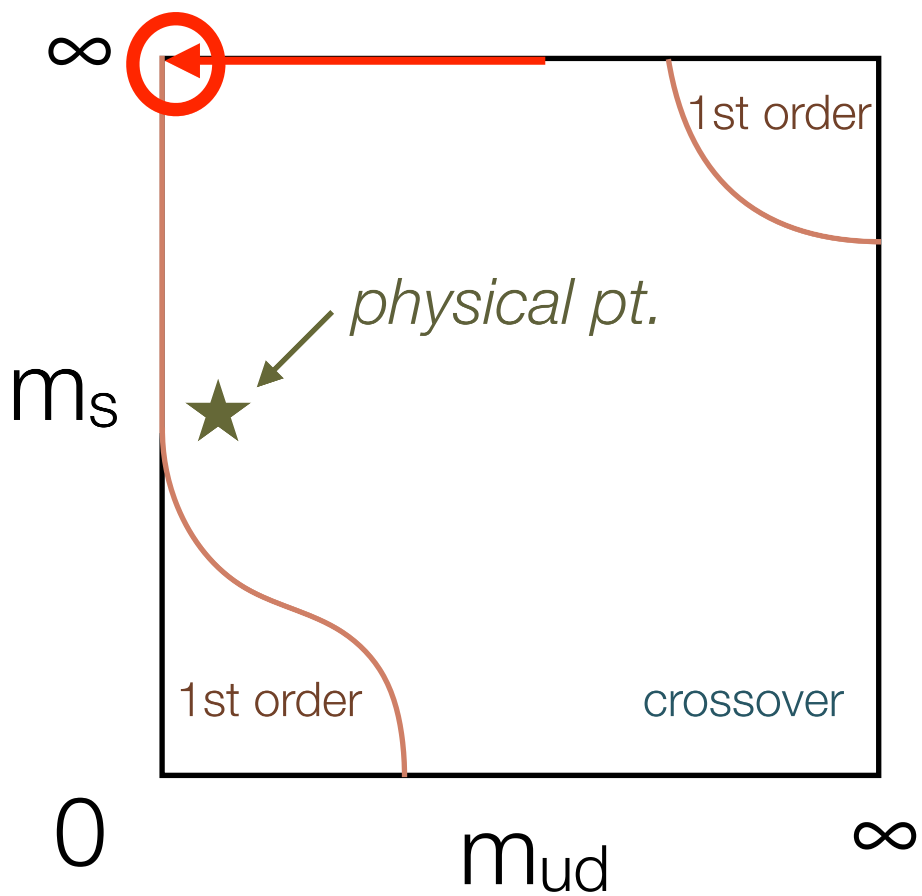
➡ **Columbia plot**



- Physical pt : crossover
  Wuppertal 2006
- Right upper corner : 1st order
  pure gauge
- other parts are less known

[original Columbia plot: Brown et al 1990]

Columbia plot: direct search of PT / scaling



- 2nd order
    - improved Wilson
        - WHOT-QCD Lat2016 (O(4) scaling)
        - Ejiri et al PRD 2016 [heavy many flavor]
- 1st oder
    - imaginary μ → 0
        - staggered   Bonati et al PRD 2014
        - Wilson   Phillipsen et al PRD 2016

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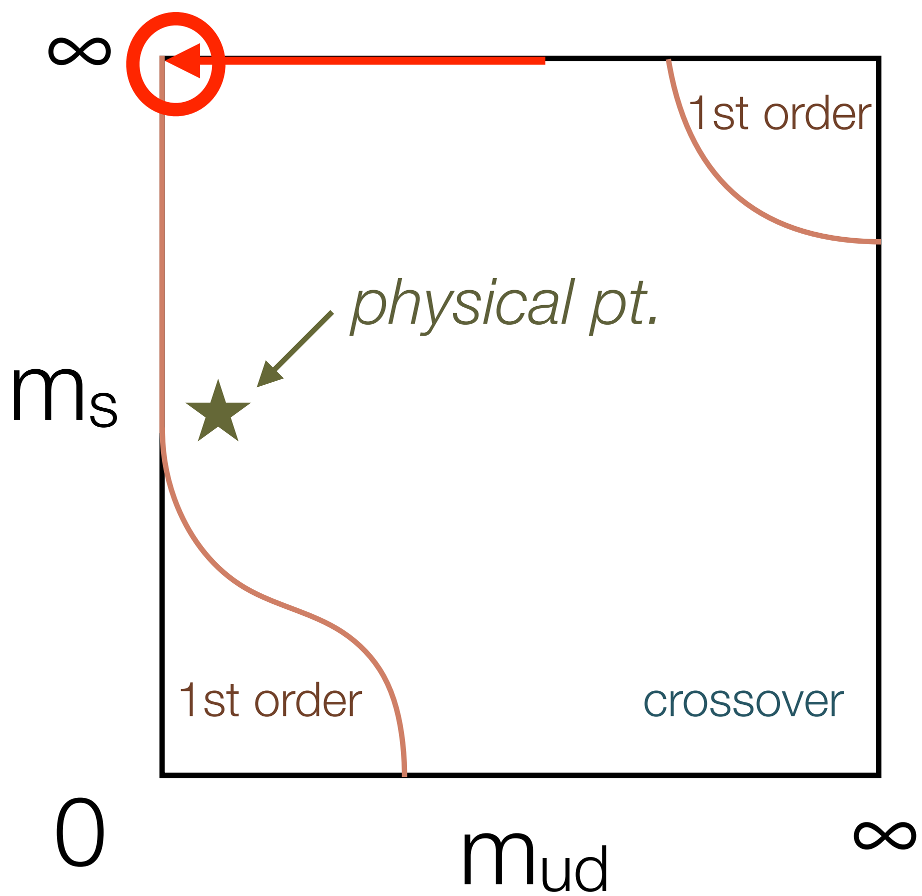

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external parameter
→ phase boundary
→ point of interest
➡ detour the demanding region

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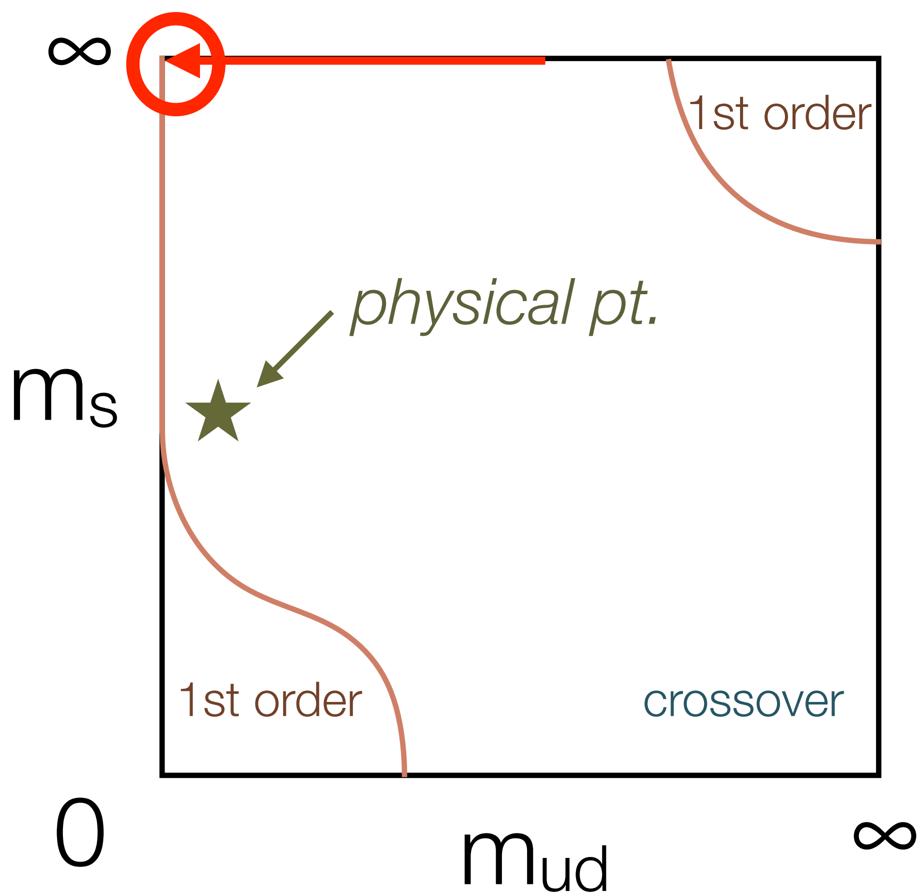

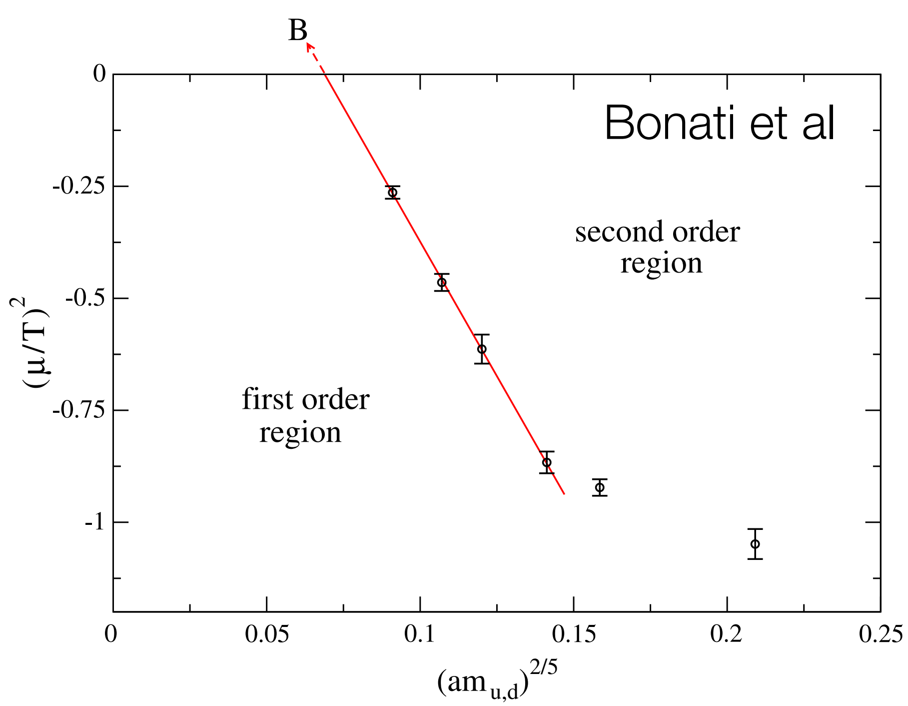

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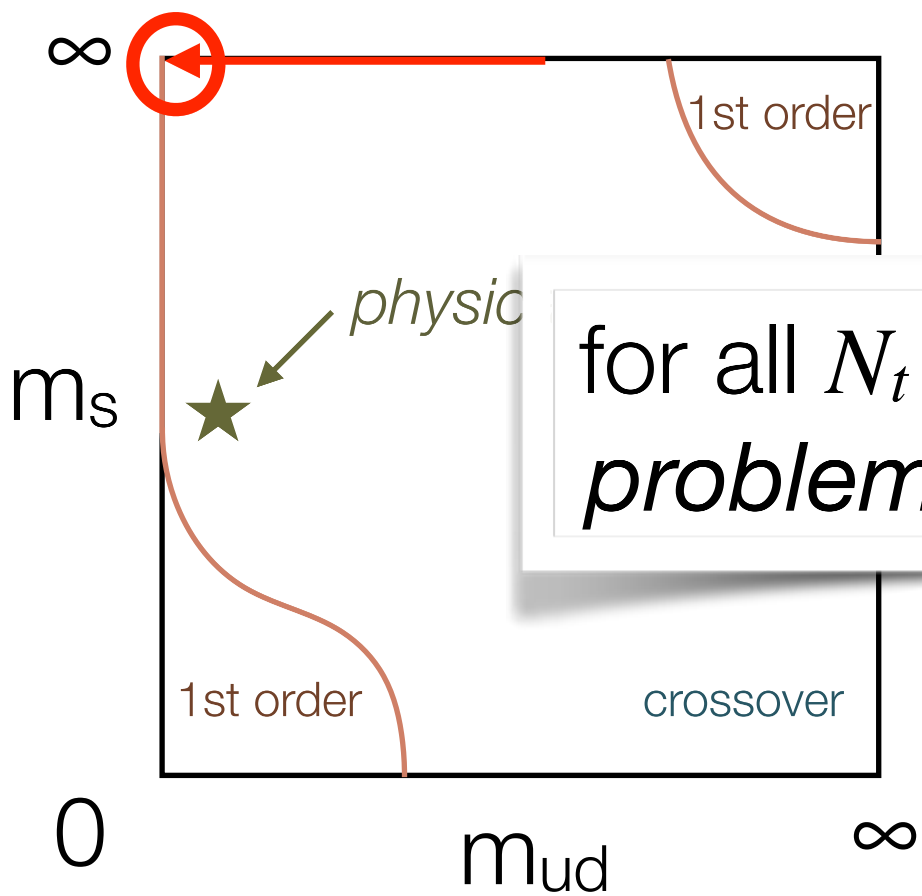
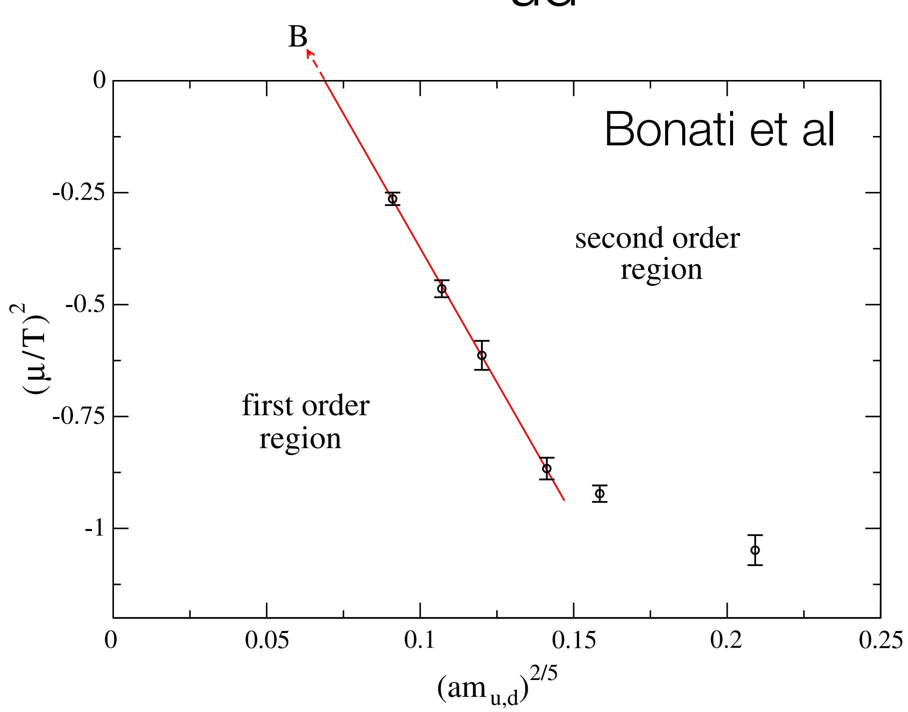


for all $N_t = 1/(aT)$ =4 or 6

problem not settled yet

- 2nd order
- on
- D Lat2016 (O(4) scaling)
- RD 2016 [heavy many flavor]
- imaginary μ → 0
  - staggered Bonati et al PRD 2014
  - Wilson Phillipsen et al PRD 2016



external parameter
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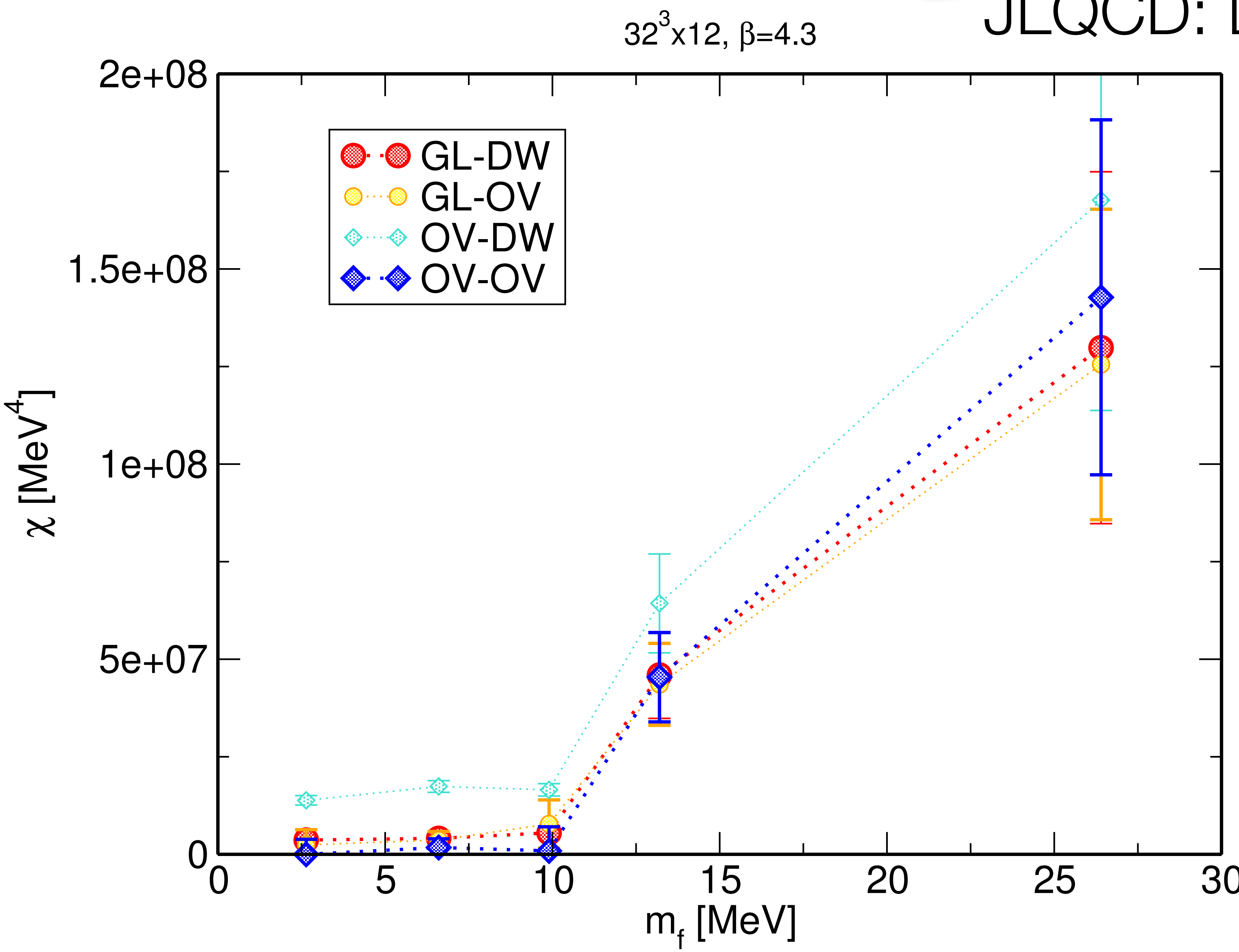
$\chi_t(m_f)$ for N_f=2 T=220 MeV

GL-DW	gluonic charge on DW
GL-OV	gluonic charge on OV
OV-	OV index on DW ensemble
OV-OV	OV index on OV ensemble

JLQCD: Lattice 2017



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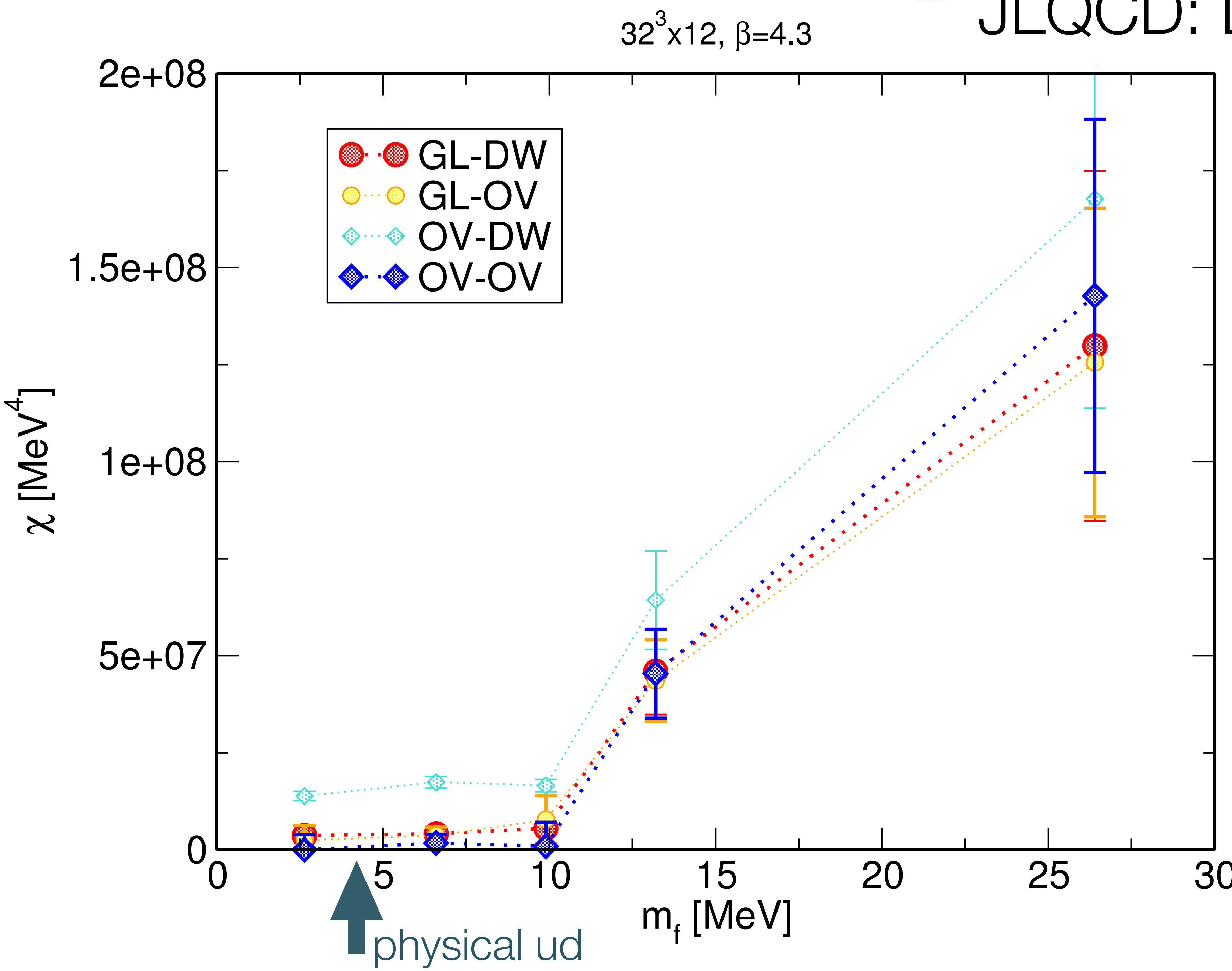

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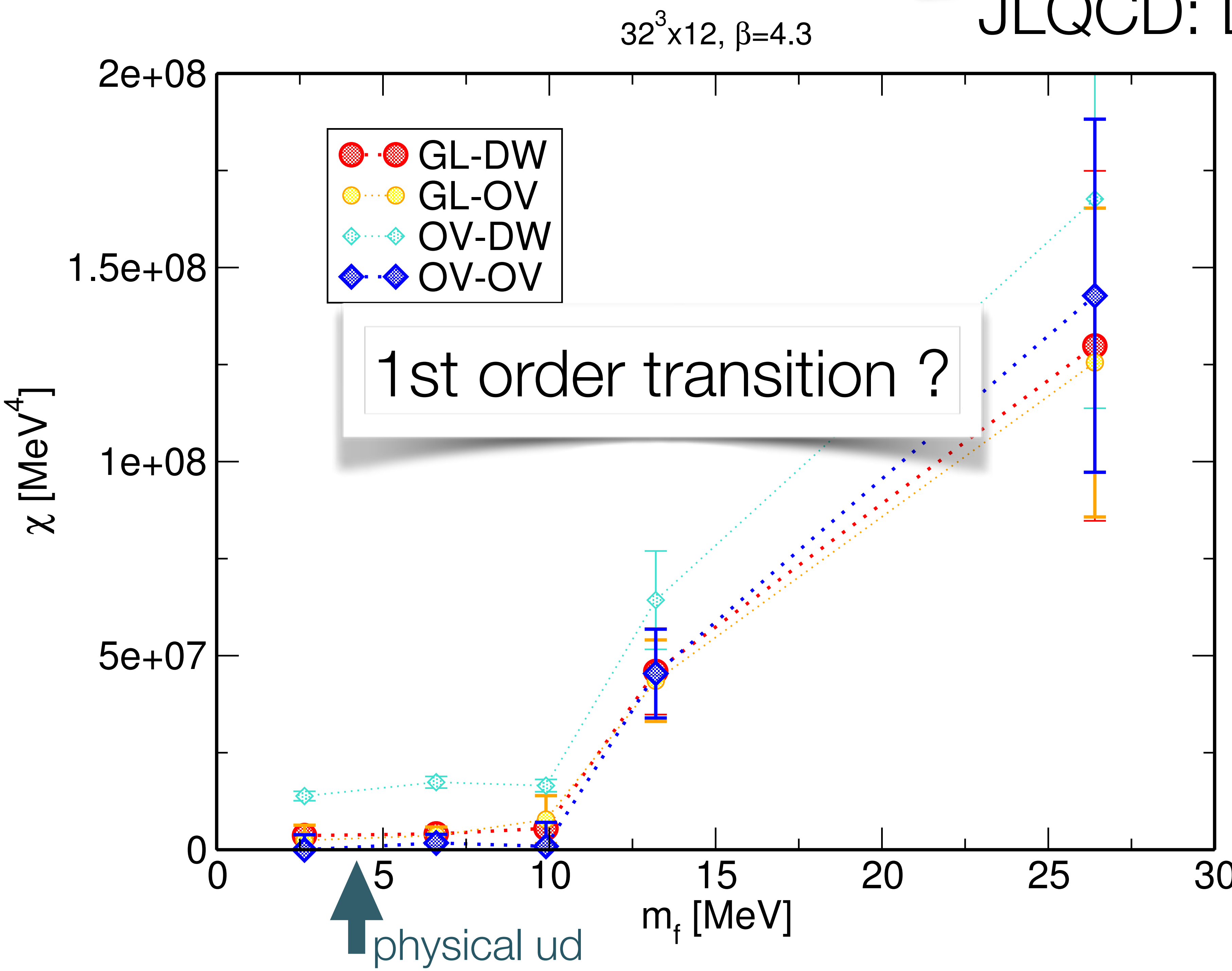

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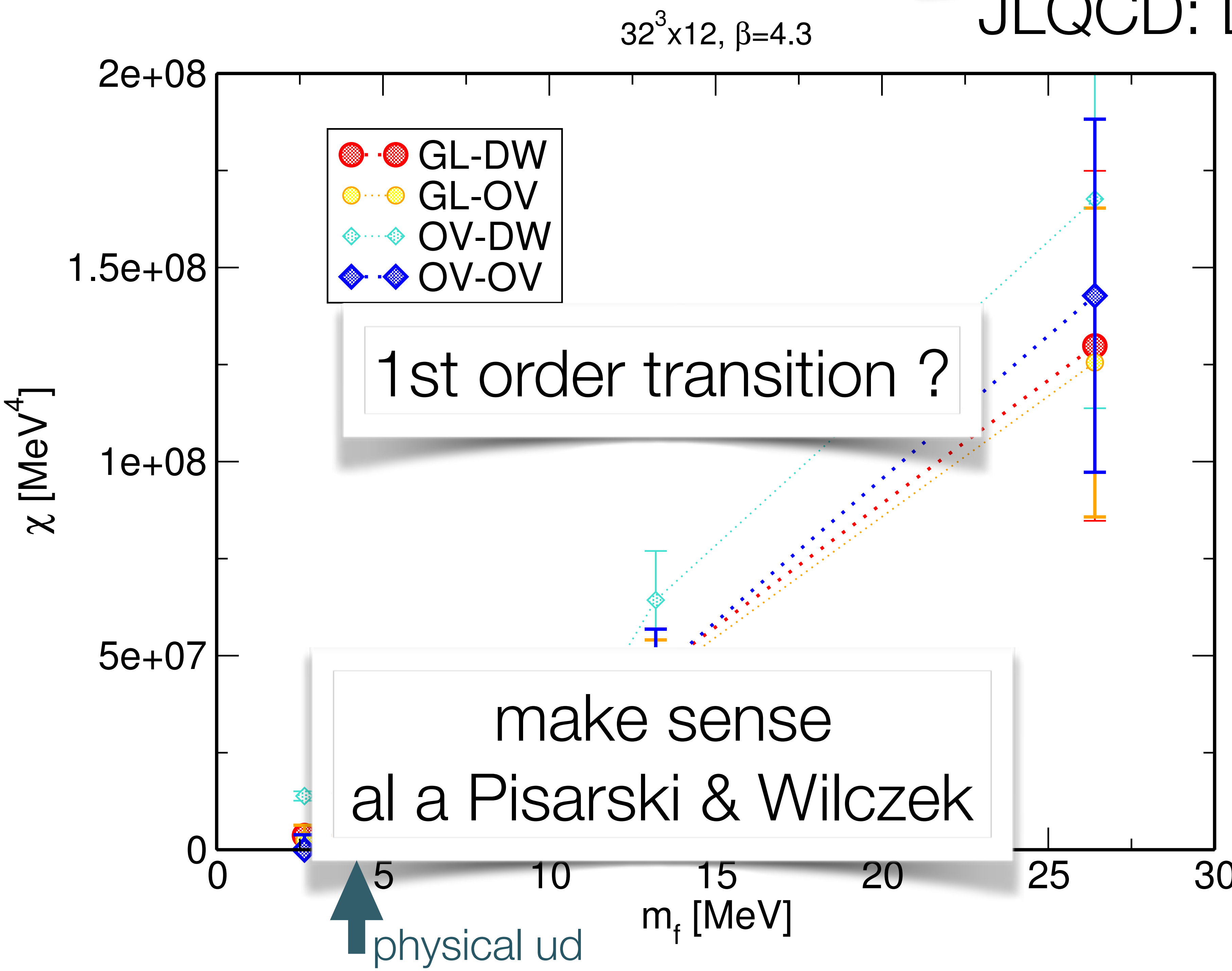

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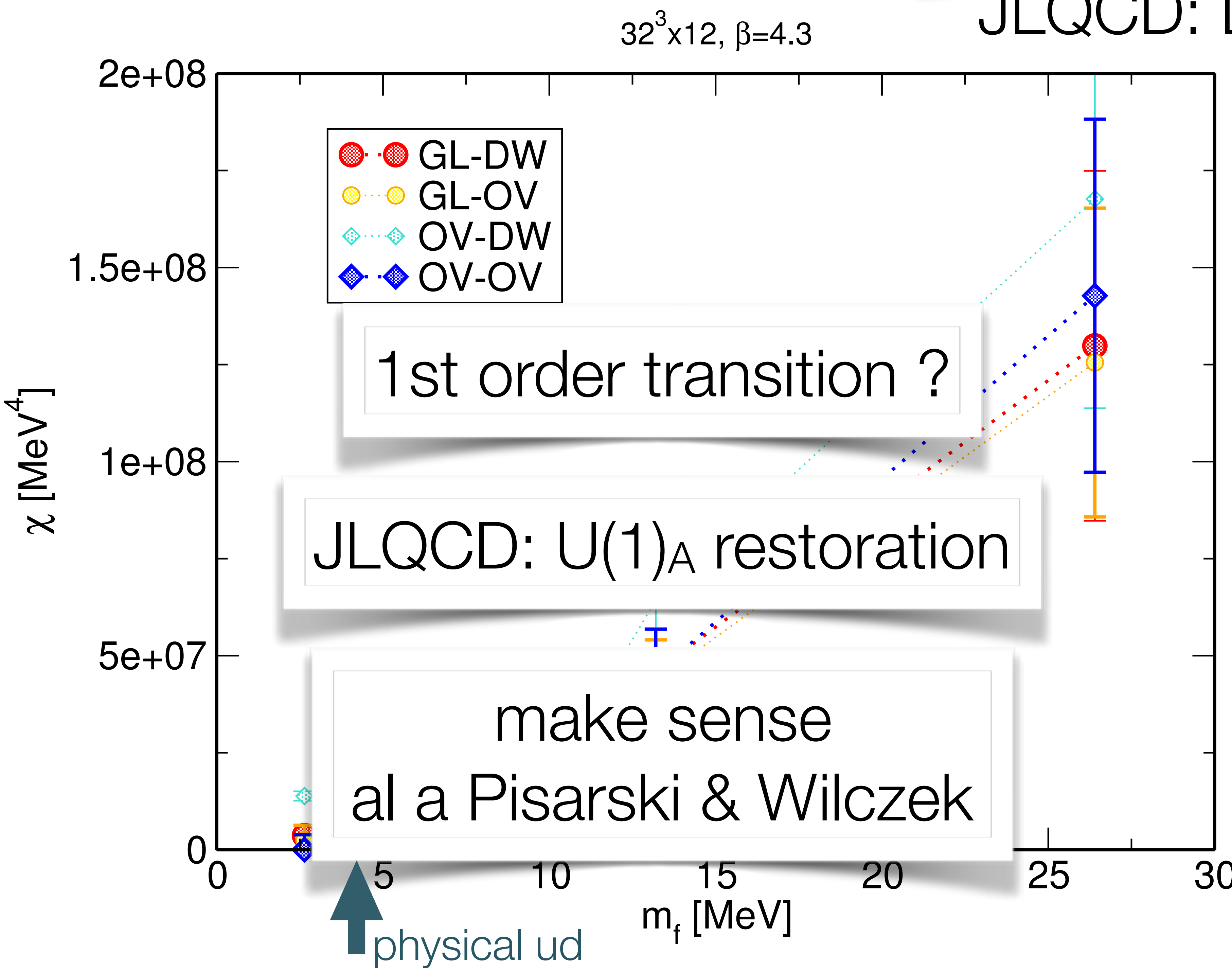


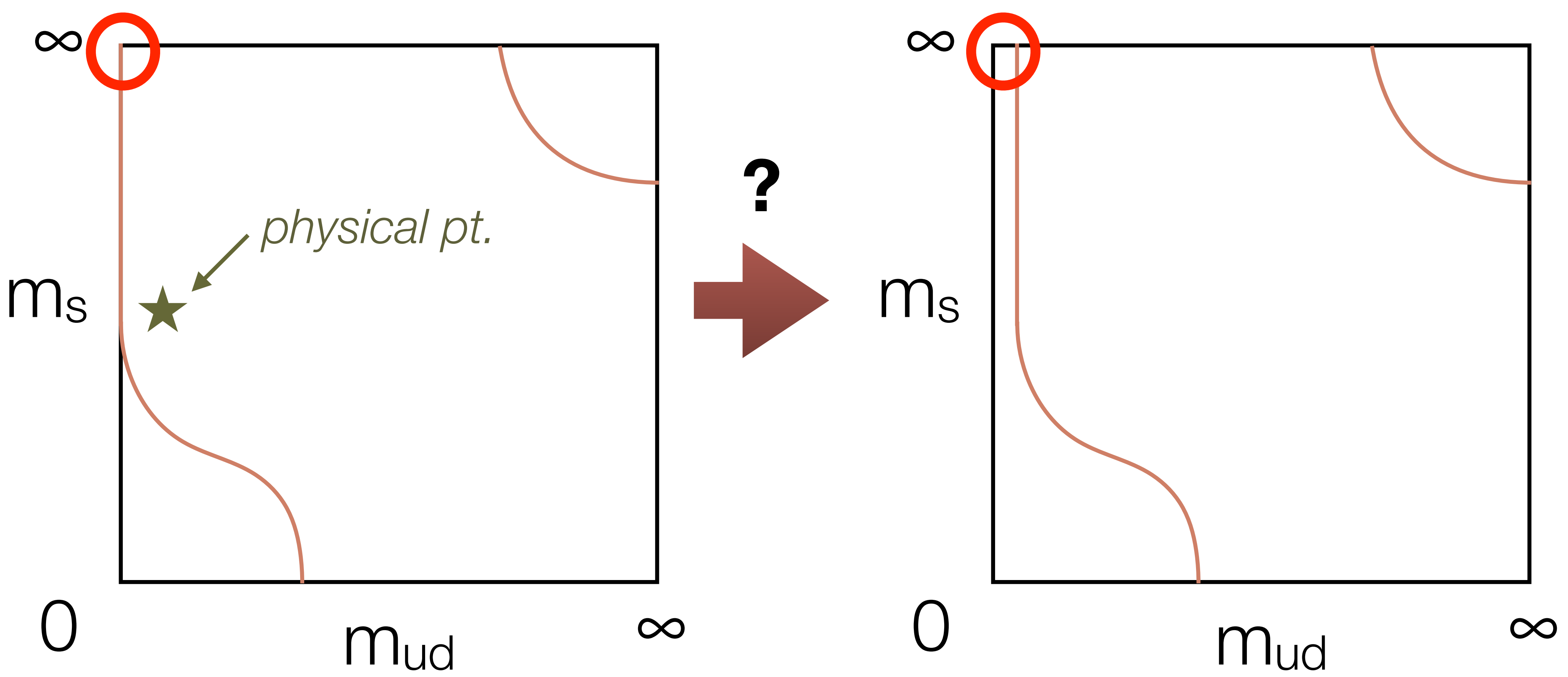
1st order transition ?

JLQCD: U(1)$_A$ restoration

make sense
al a Pisarski & Wilczek

if upper left corer is 1st order

- $0 \le m_f < m_c$: 1st oder
- might affect the physics around physical point



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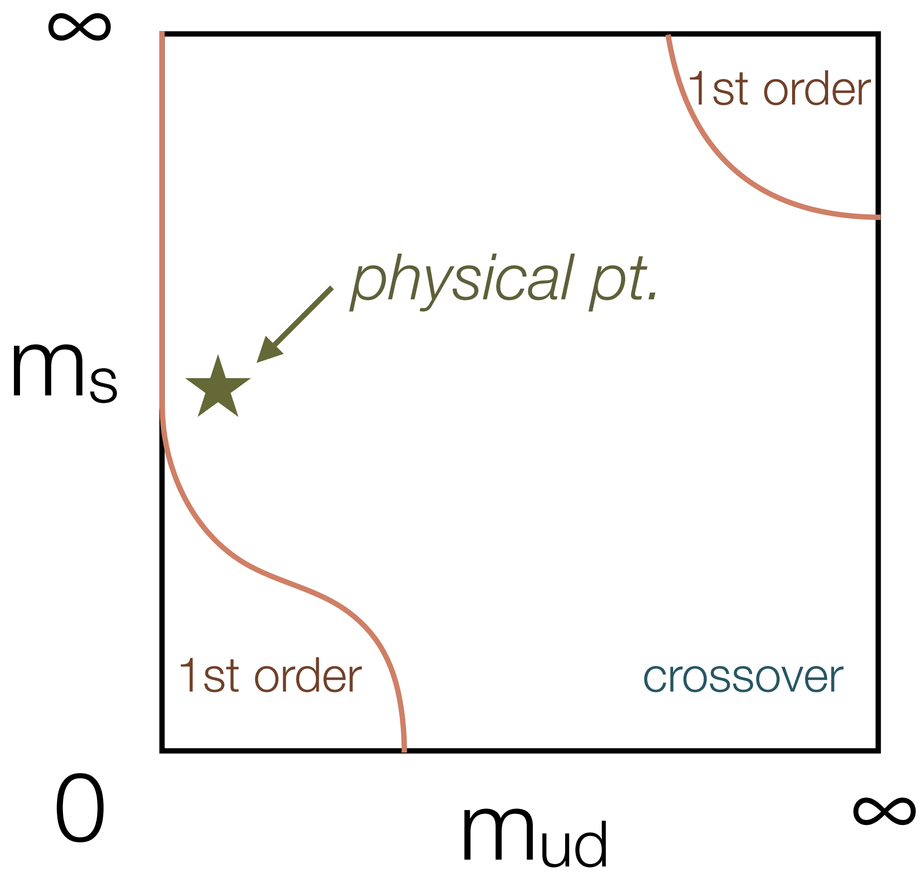

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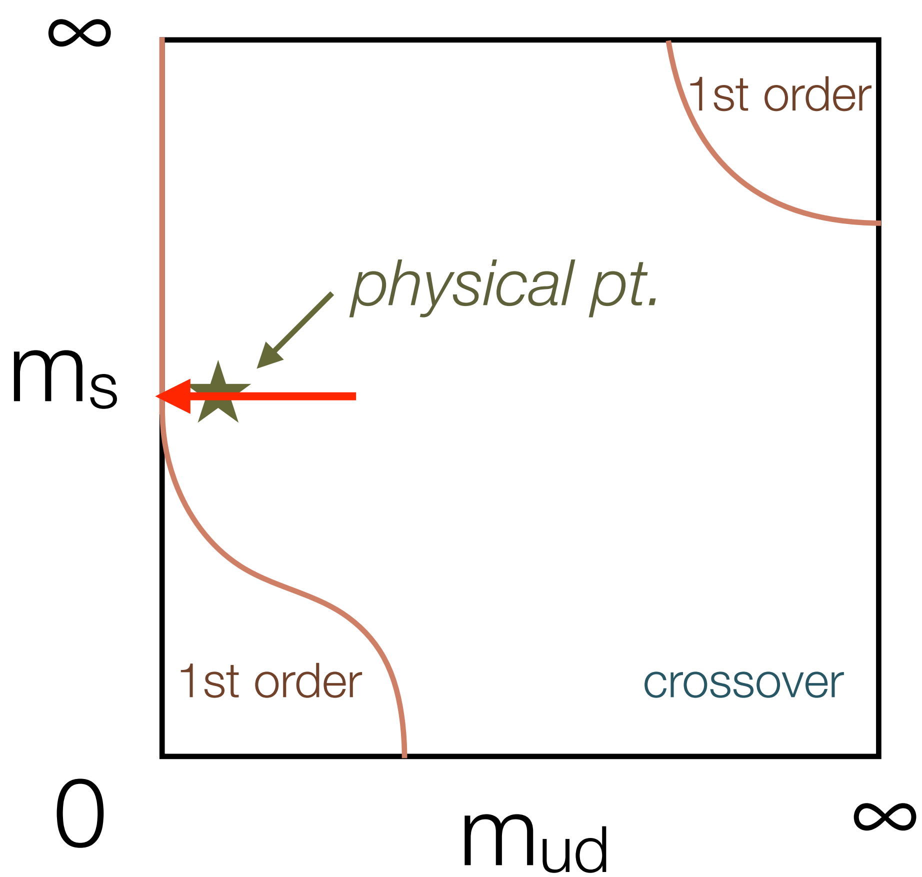

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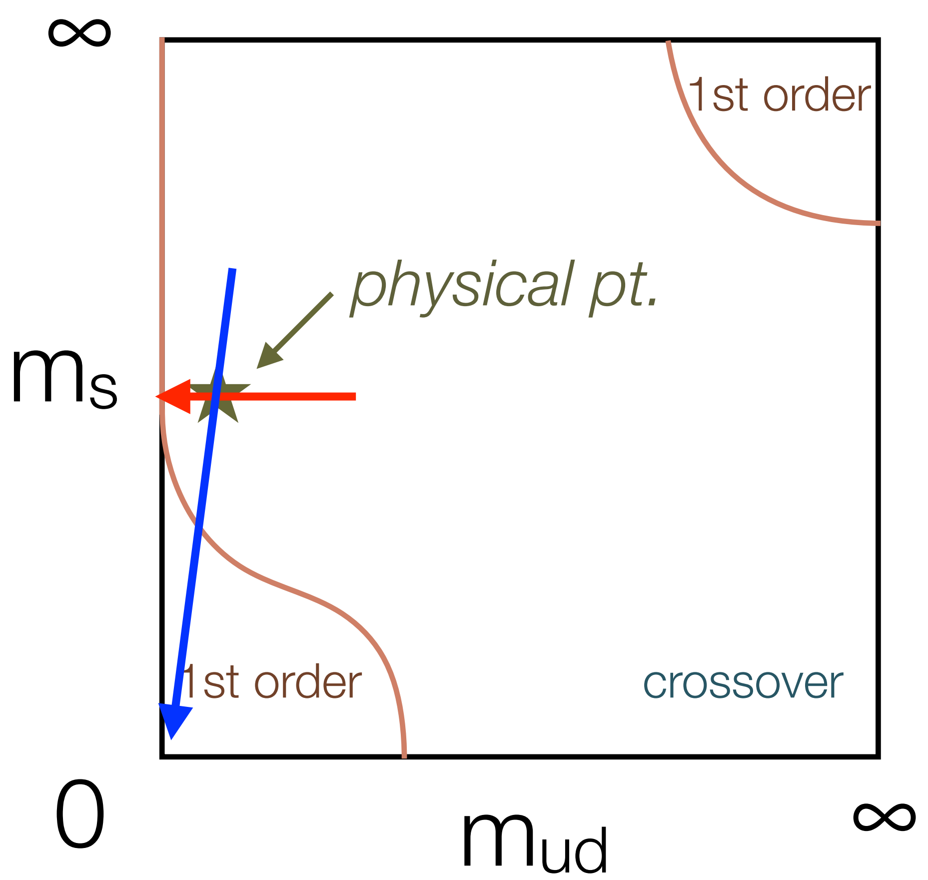

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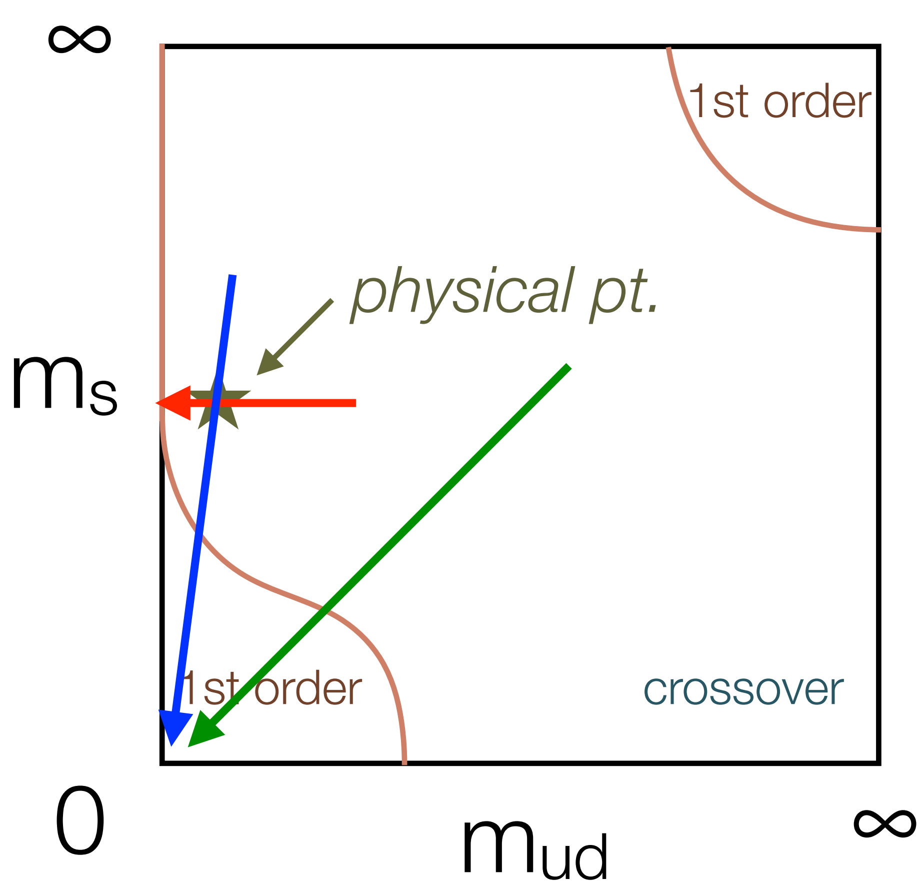


N_f=2+1 or 3

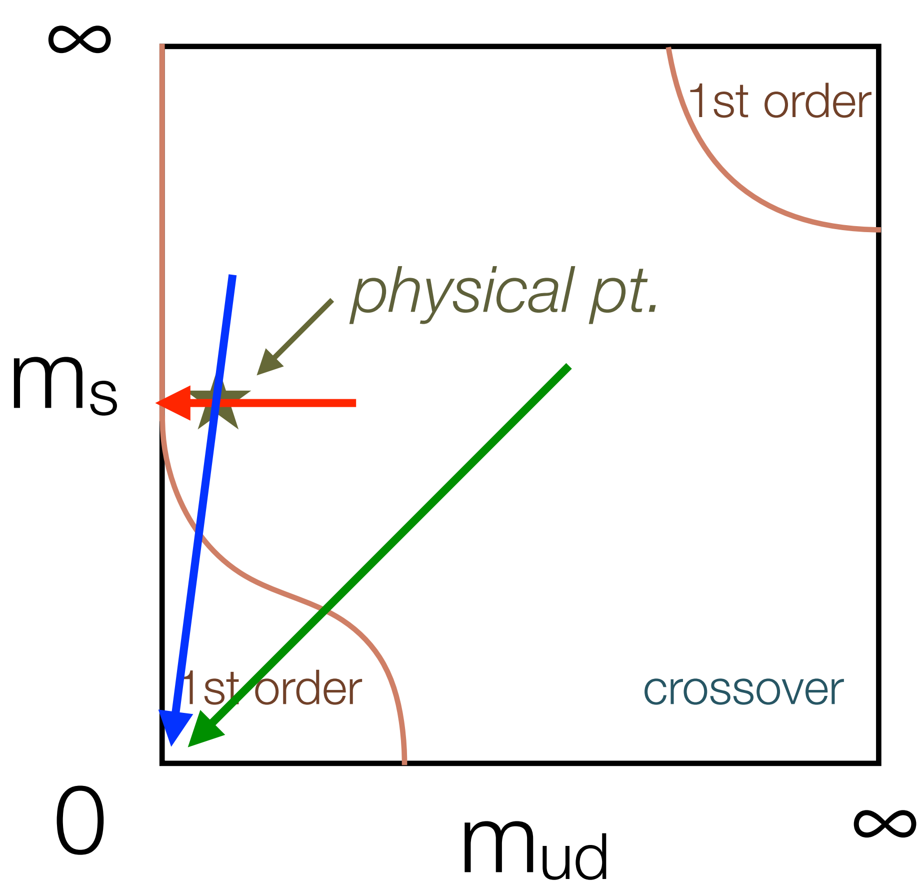
- either
  - no PT found
  - 1st order region
    - **shrinks** as $a \to 0$

      with both staggered and Wilson
    - or even disappear ?
- *for more information see eg*
  - Meyer Lattice 2015
  - Ding Lattice 2016
  - de Forcrand

    "Surprises in the Columbia plot"

    (Lapland talk 2018)

Columbia plot: direct search of PT / scaling



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Understanding of the diagram being changed a lot

  - Ding Lattice 2016
  - de Forcrand

    “Surprises in the Columbia plot”

    (Lapland talk 2018)

Why bother ?

- **in relation with "extended symmetry"**
  - spin-chiral symmetry for vector and scalar props. at high T
    - $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$
      - C. Rohrhofer et al., PRD17 [1707.01881]
      - C. Lang [1803.08693]
  - original discussion on this symmetry: Glozman et al
    - for the T=0 but low-mode subtracted Dirac operator

Why bother ?

- **axion cosmology scenario may fail for $U(1)_A$ restoration**

  due to vanishing / suppressed topological susceptivility

  - $\chi_t|_{m=0} = 0$ & $d^n\chi_t / dm^n|_{m=0} = 0$ Aoki-Fukaya-Tanigchi

    ➡ $\chi_t = 0$ for small non-zero m OR

    ➡ exponential decay for $T>T_c$

$$\chi_t(T) \sim \begin{cases} m_q \Lambda_{\mathrm{QCD}}^3, & T < T_c, \\ m_q^2 \Lambda_{\mathrm{QCD}}^2 e^{-2c(m_q)T^2/T_c^2}, & T > T_c, \end{cases}$$

$$c(m_q) \to \infty \text{ as } m_q \to 0,$$

  - axion mass and decay constant: $\chi_t = m_a^2 f_a^2$

  ➡ **axion window can possibly be closed**

Kitano-Yamada JHEP [1506.00370]

- see also for θ=π QCD non-standard case with rich implications

Di Vecchia et al. JHEP [1709.00731]

$U(1)_A$ restoration or not

- need to make sure if not comparing apples and oranges…
- key points
  - systematics effects of lattice discretization under control ?
  - ud chiral limit of
    - $N_f=2$ QCD or
    - $N_f=2+1$ QCD → strange quark mass effect !
  - discussing $m_{ud} \to 0$ or just around physical ud mass
  - discussing $X = 0$? or $X \simeq 0$?

a U(1)$_A$ order parameter

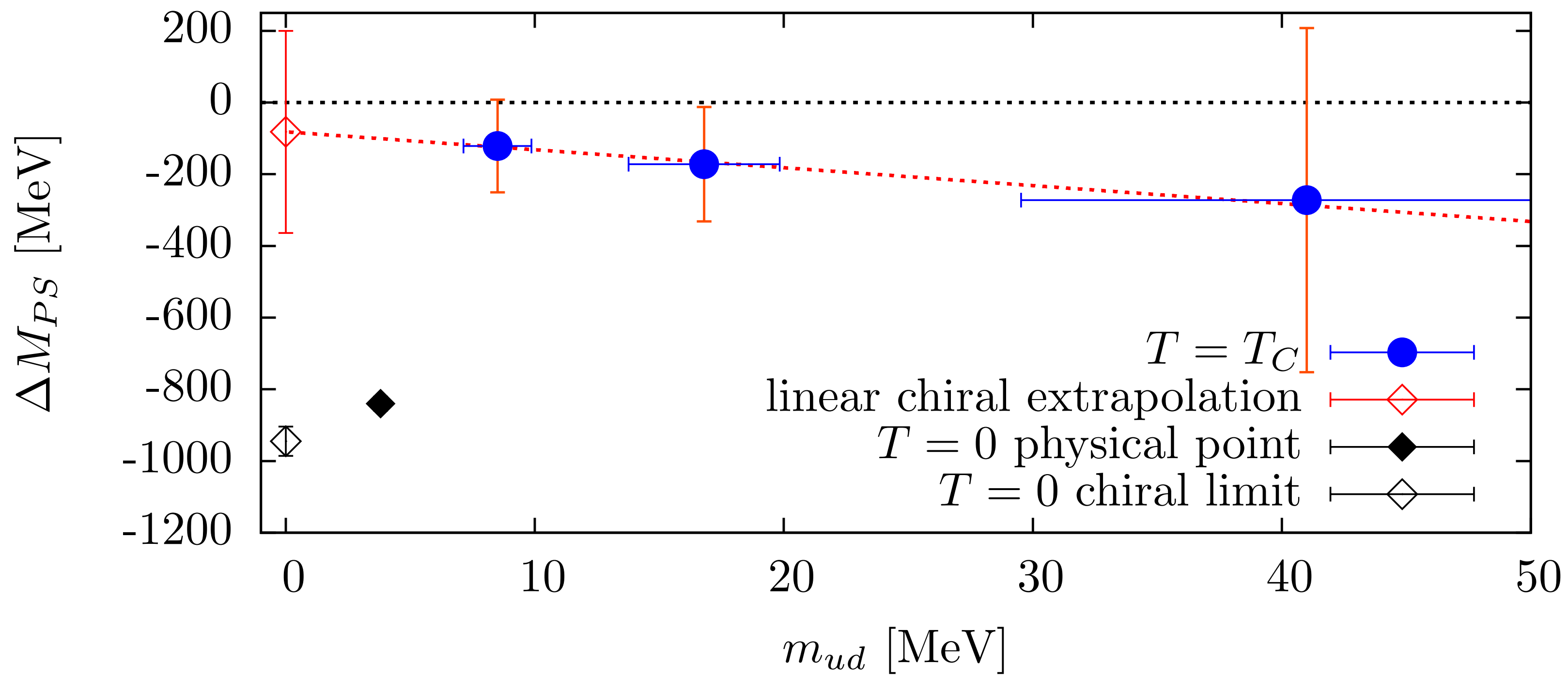
- symmetry in switching flavor non-singlet pseudoscalar and scalar
- order parameter:

$$\Delta_{\pi-\delta} = \int d^4x[\langle \boldsymbol{\pi}^a(x)\boldsymbol{\pi}^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle],$$

➡ 0 for U(1)$_A$ restoration

- as a result, screening masses for these channel will degenerate
    - not a sufficient condition for U(1)$_A$ restoration

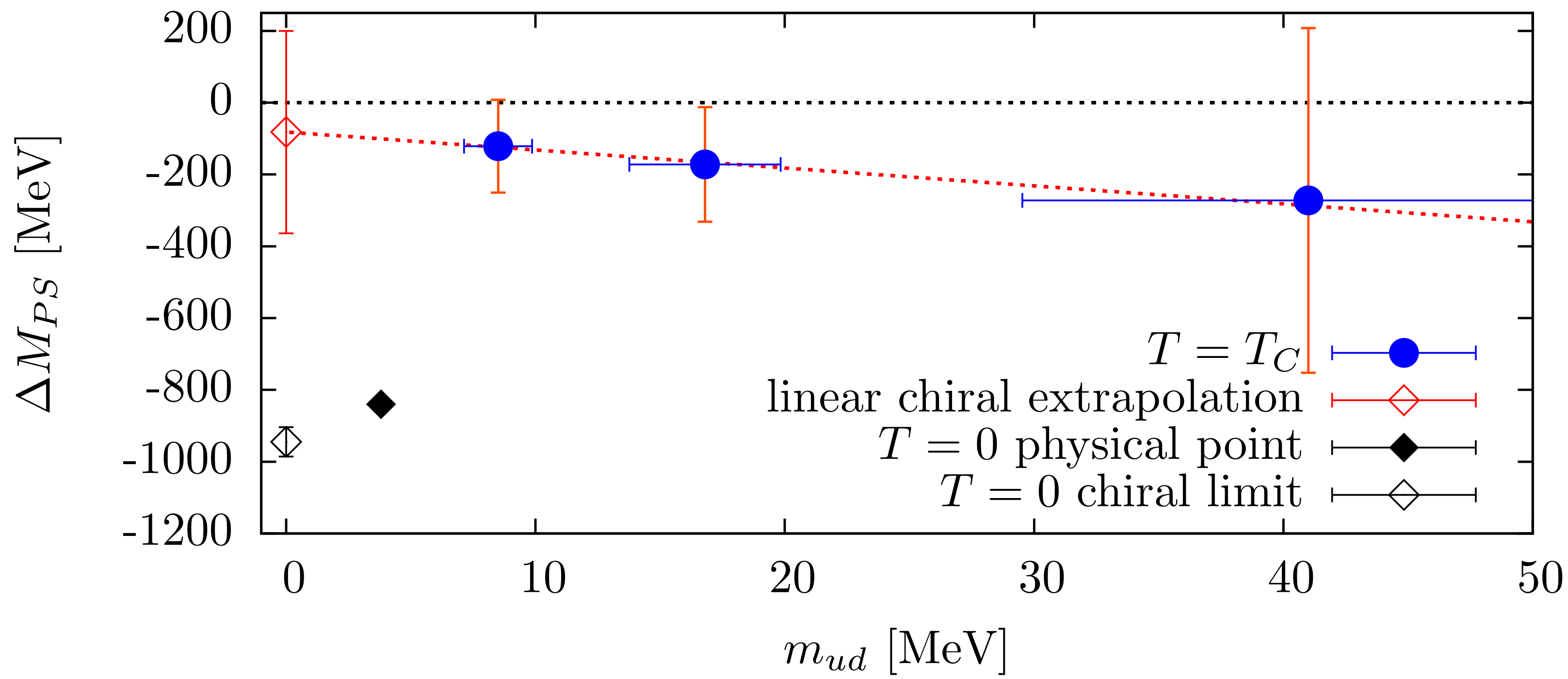
screening mass from O(a) improved Wilson f N_f=2



Brandt et al JHEP [1608.06882]

screening mass from O(a) improved Wilson f N_f=2

- mass difference between π and δ



Brandt et al JHEP [1608.06882]

- $N_t = 1/(aT) = 16$ - quite fine lattice
- $T=T_c$ - **on top of transition temperature**

only one existing study for N_f=2

- $\Delta M_{PS} = 0$ (with a sizable error) → consistent with $U(1)_A$ restoration

relation with Dirac eigenmode spectrum ρ(λ)

$$-\langle \overline{q}q \rangle = \lim_{m \to 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

$$\Delta_{\pi - \delta} = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \to \sim \rho'(0)$$

relation with Dirac eigenmode spectrum ρ(λ)

- chiral condensate : order parameter of $SU(2)_A$

$$-\langle \overline{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi\rho(0)$$

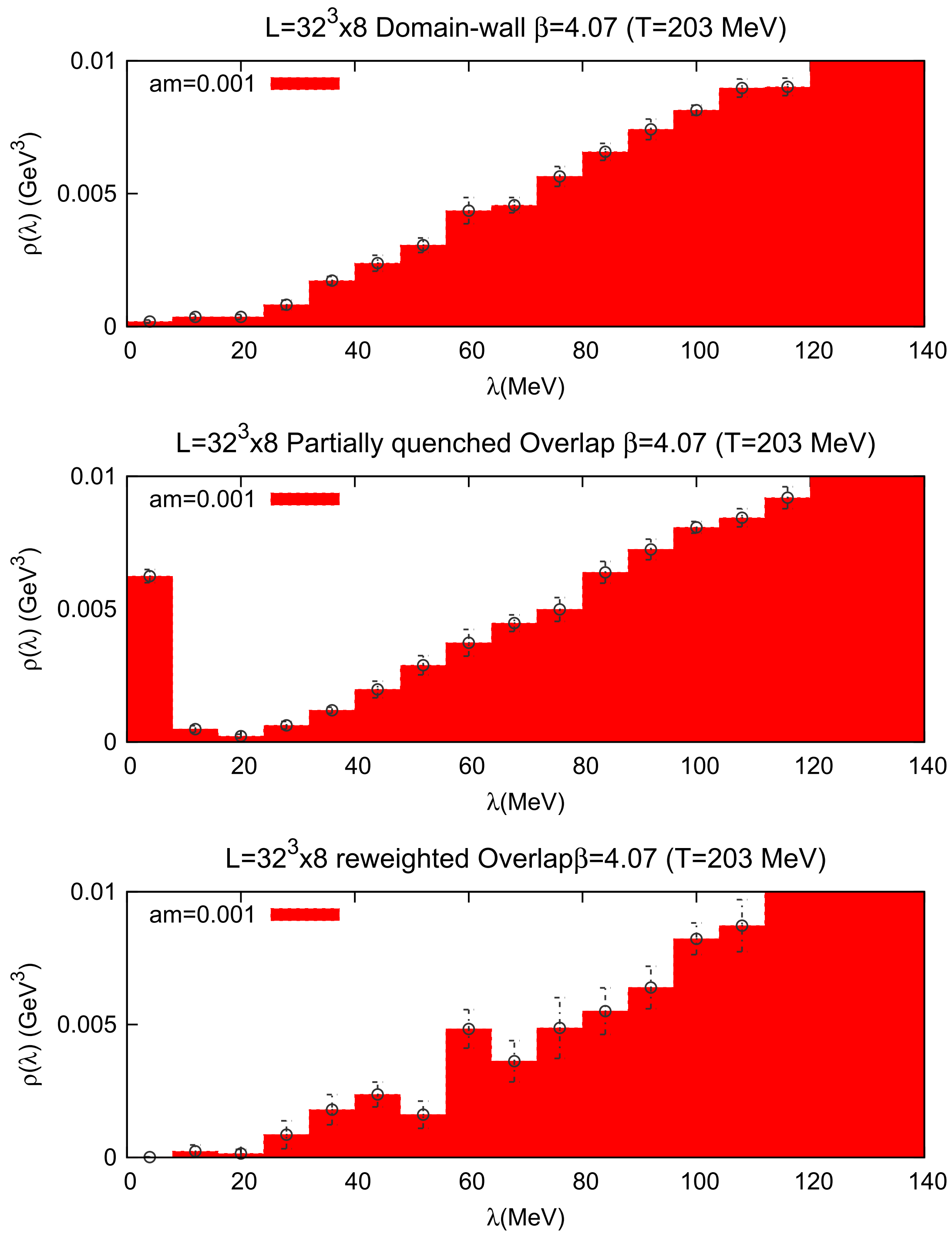
- $U(1)_A$:

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \to\sim \rho'(0)$$

very roughly speaking

  - very sensitive to the spectrum near λ=0
  - overlap fermion, able to distinguish zero/nonzero modes, is ideal

JLQCD 16: H_{OV}, H_{DW} spectrum: above T_c N_f=2



- DW: Domain wall fermion sea
- OV: Overlap valence
    - exact “chiral symmetry”
- reweighting to OV

[JLQCD 2016 Tomiya et al]

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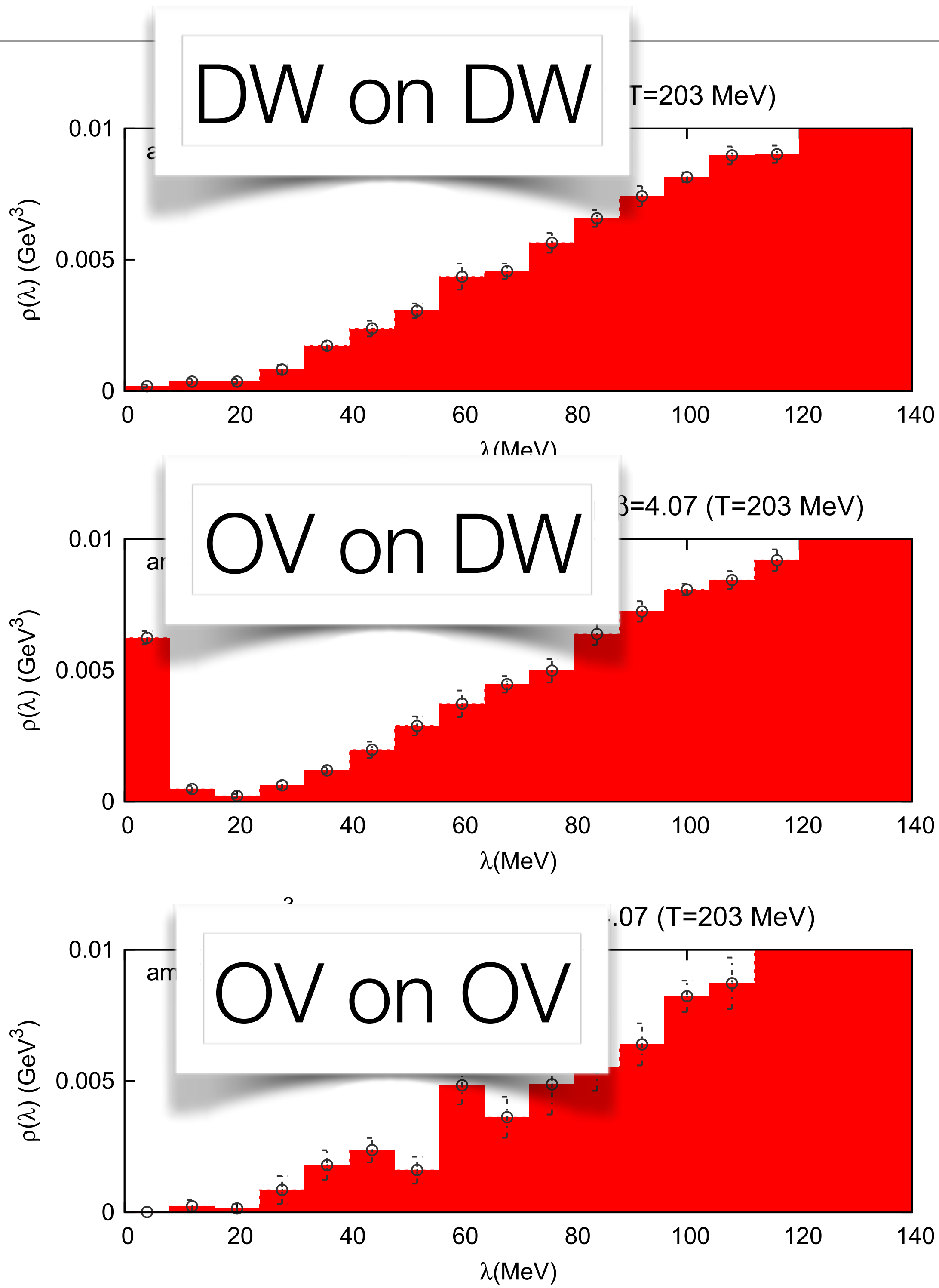

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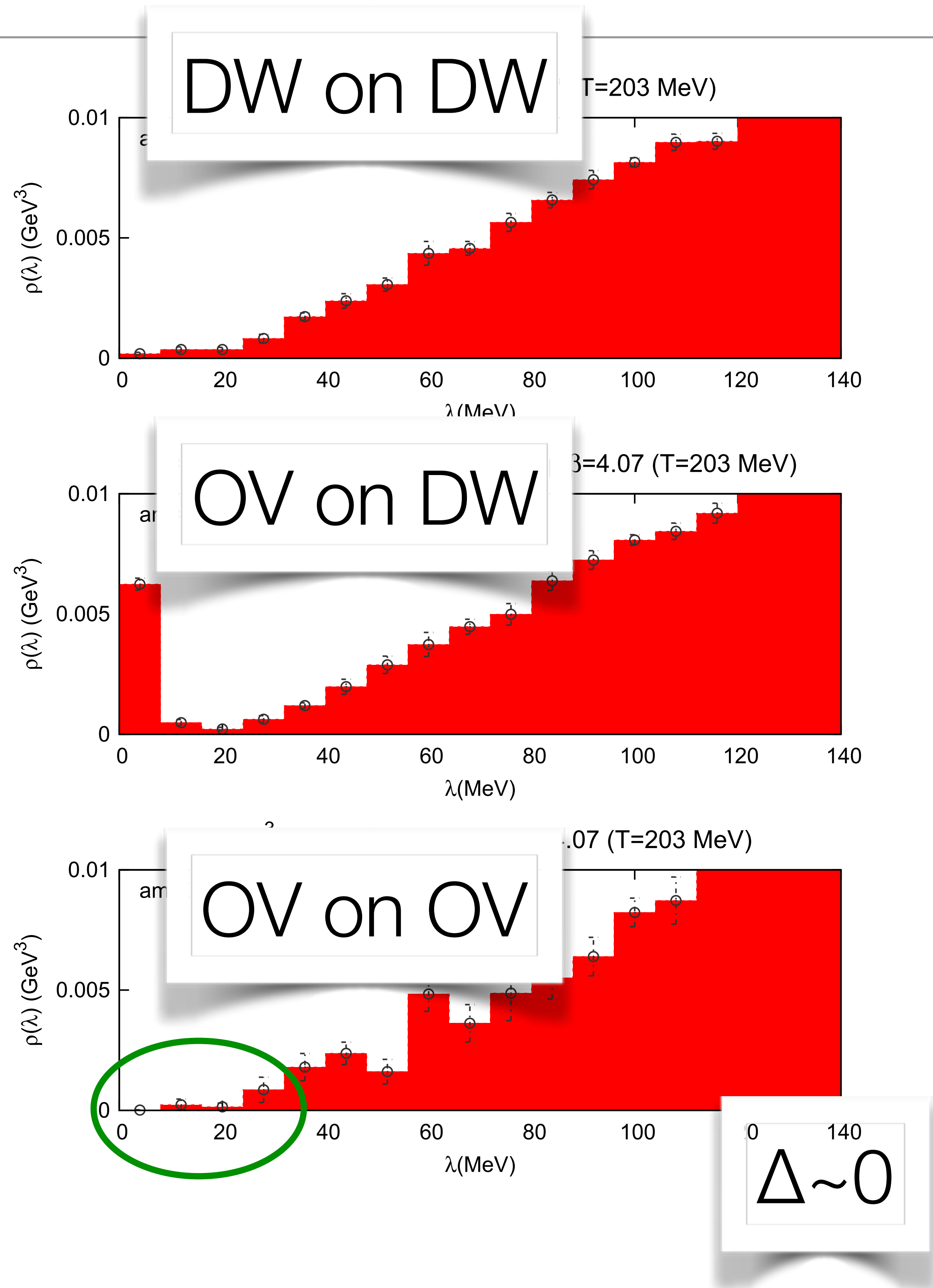

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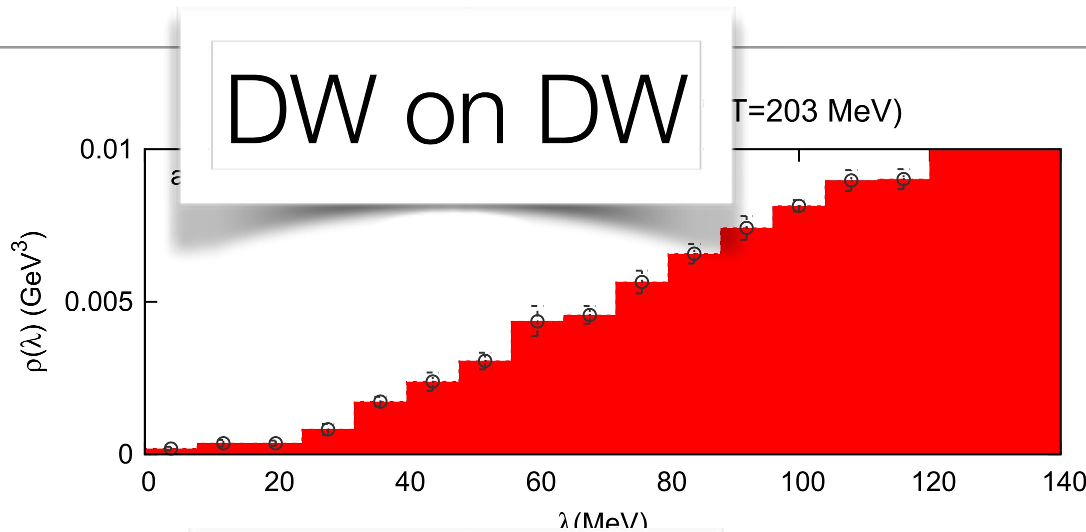

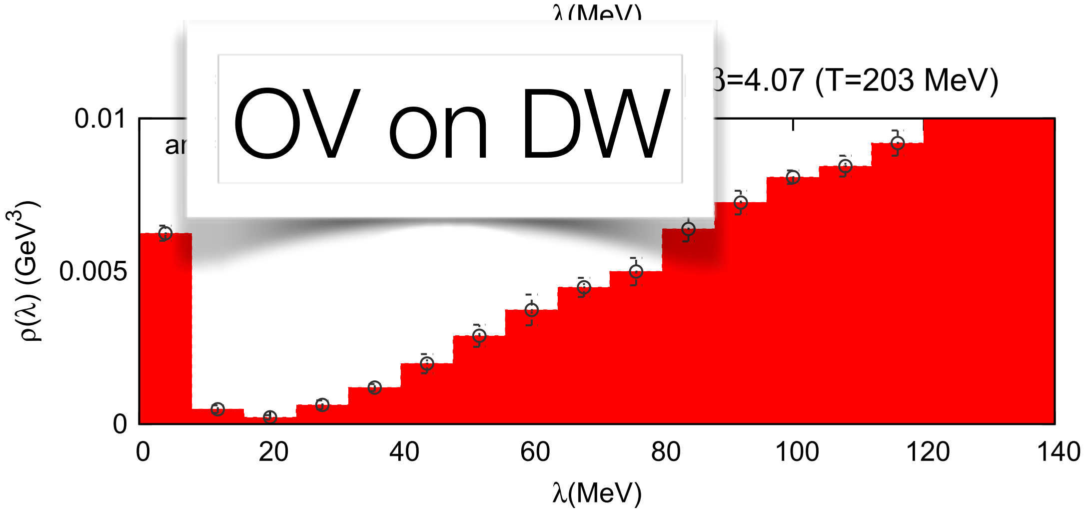

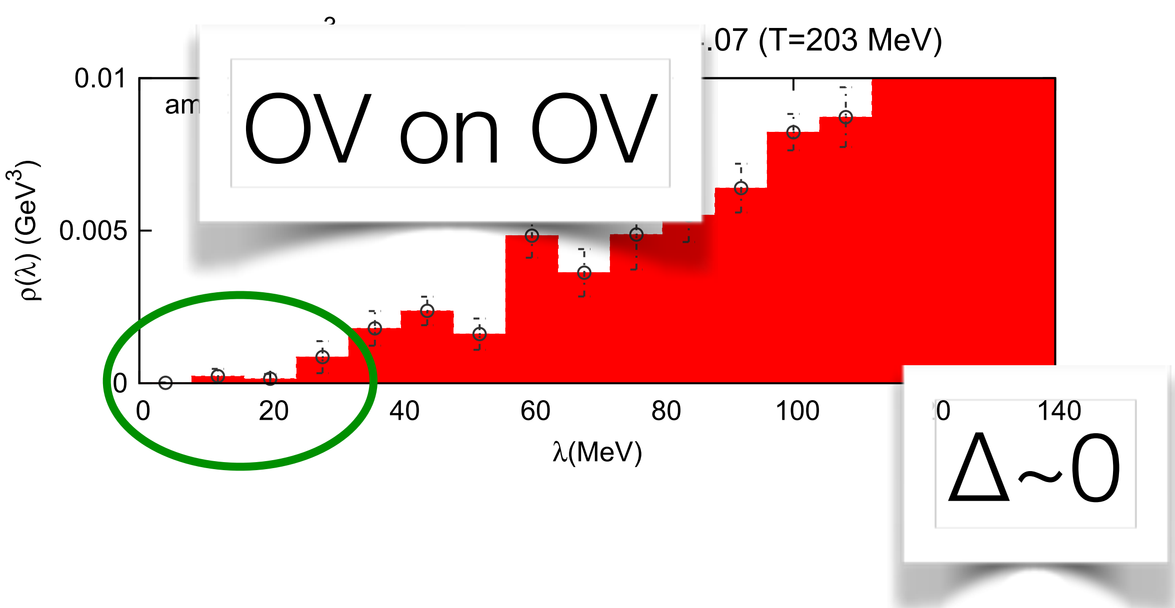

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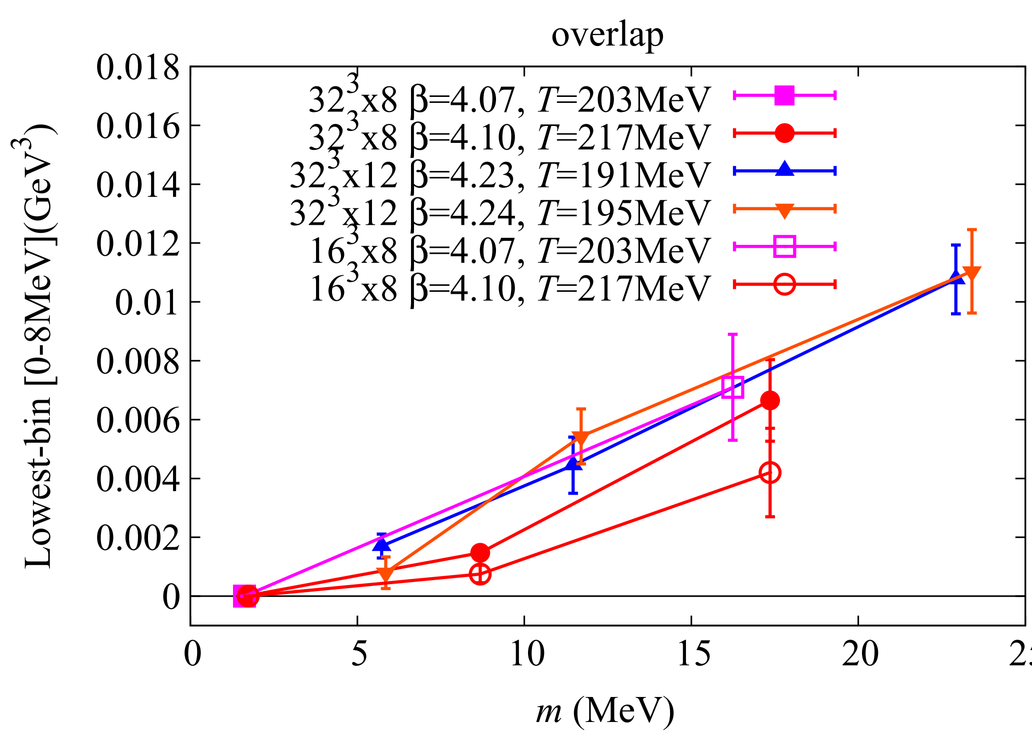
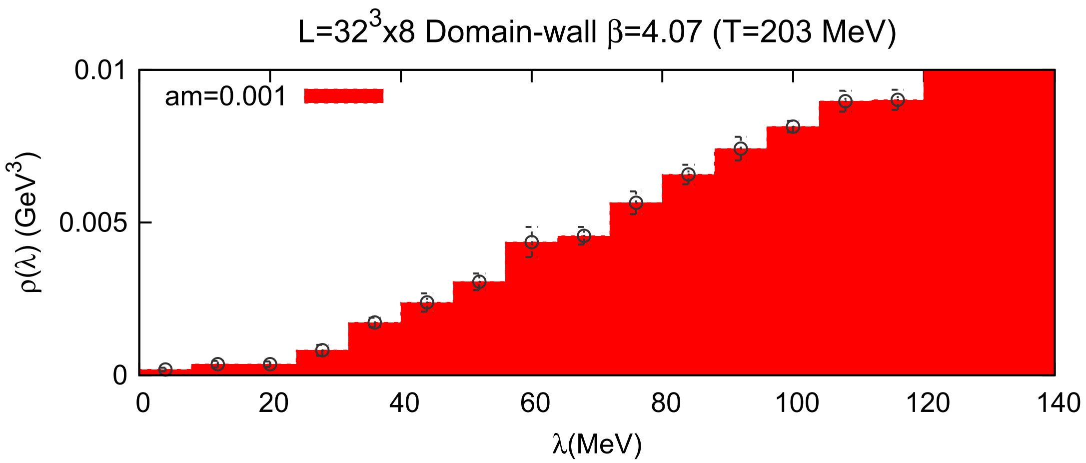
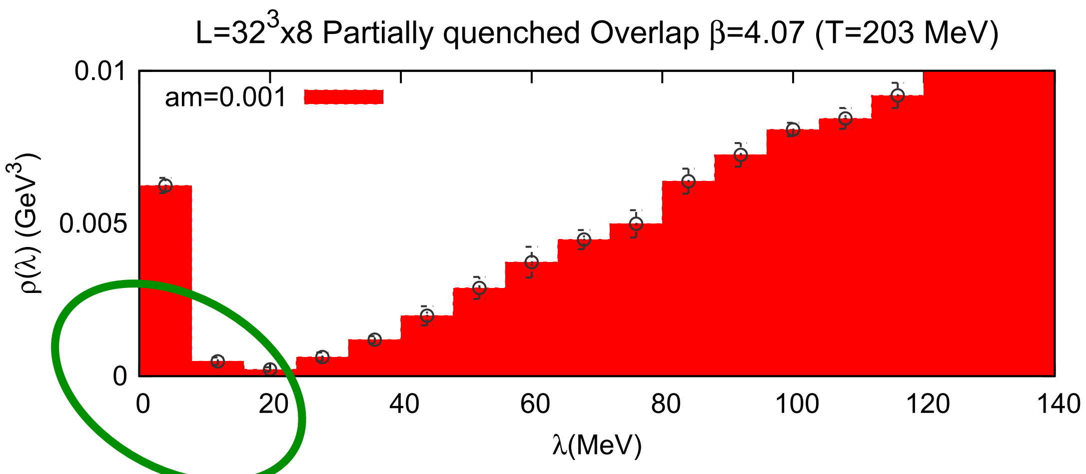


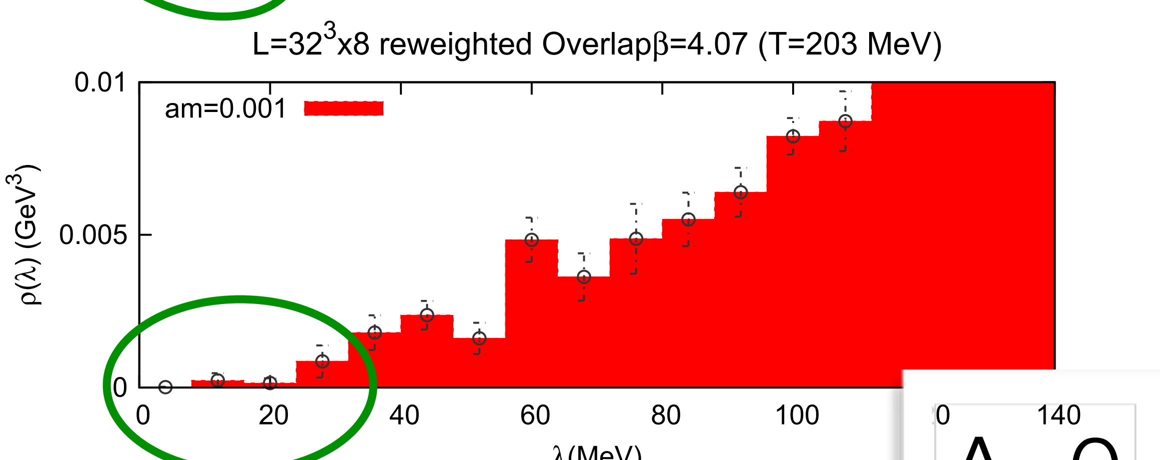
Lowest bin→0
consistent with $SU_A(2)$ restoration

[JLQCD 2016 Tomiya et al]

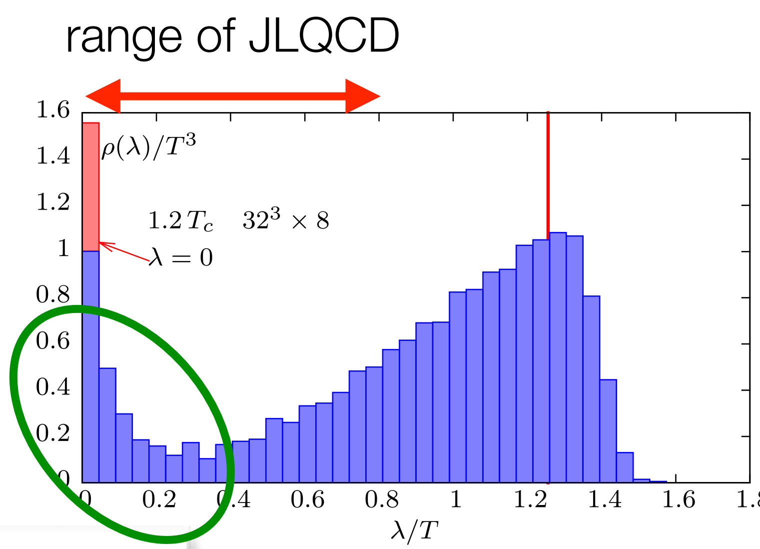
Comparison: unitary <-> partially quench







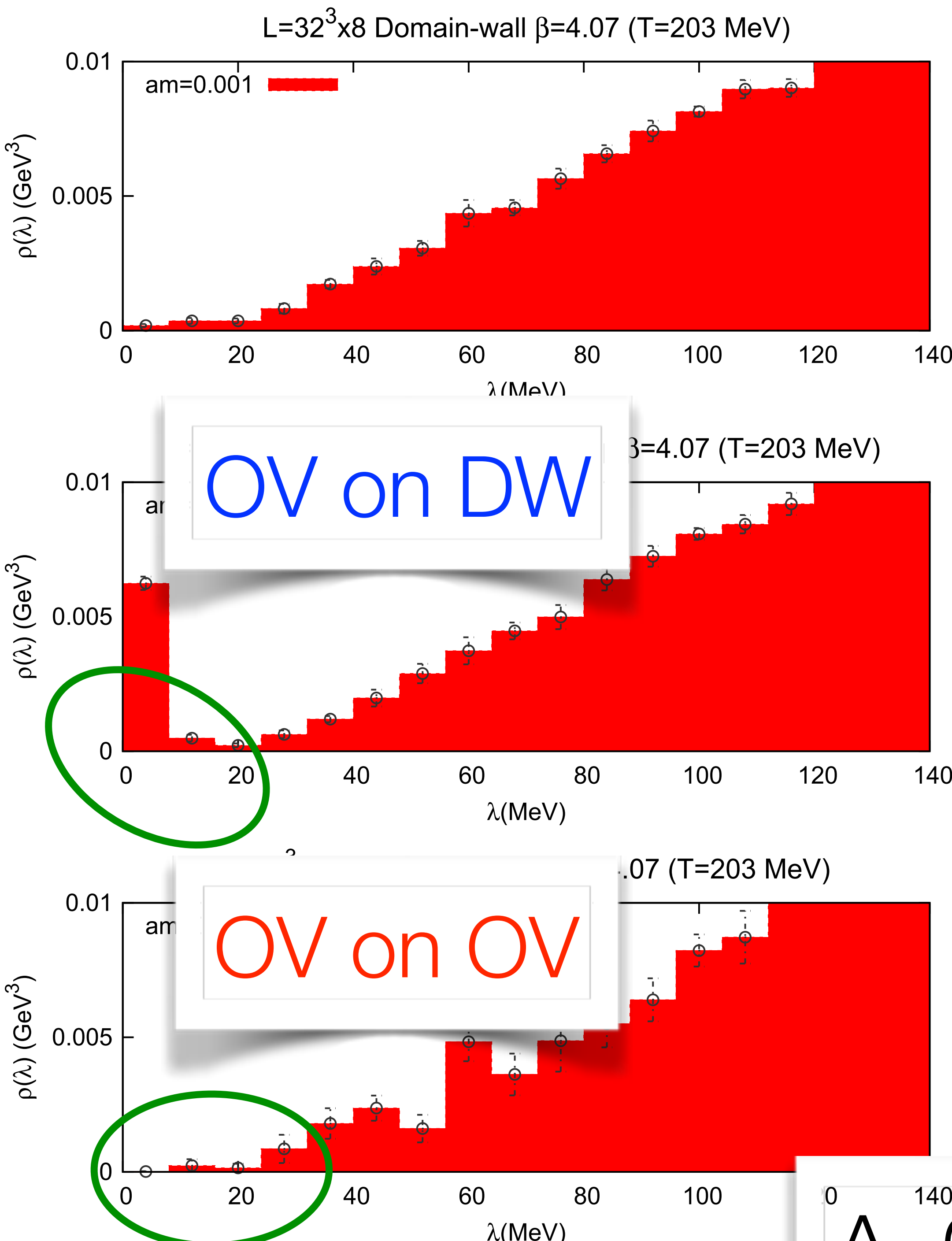
Δ~0



Δ>0

Dick et al PRD [1502.06190]

Comparison: unitary <-> partially quench

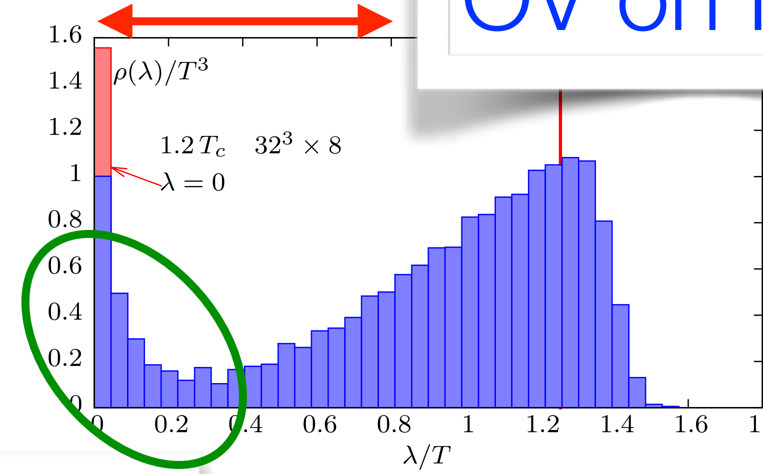


OV on DW

OV on OV

Δ~0

range of JLQCD



OV on HISQ

Δ>0

Dick et al PRD [1502.06190]

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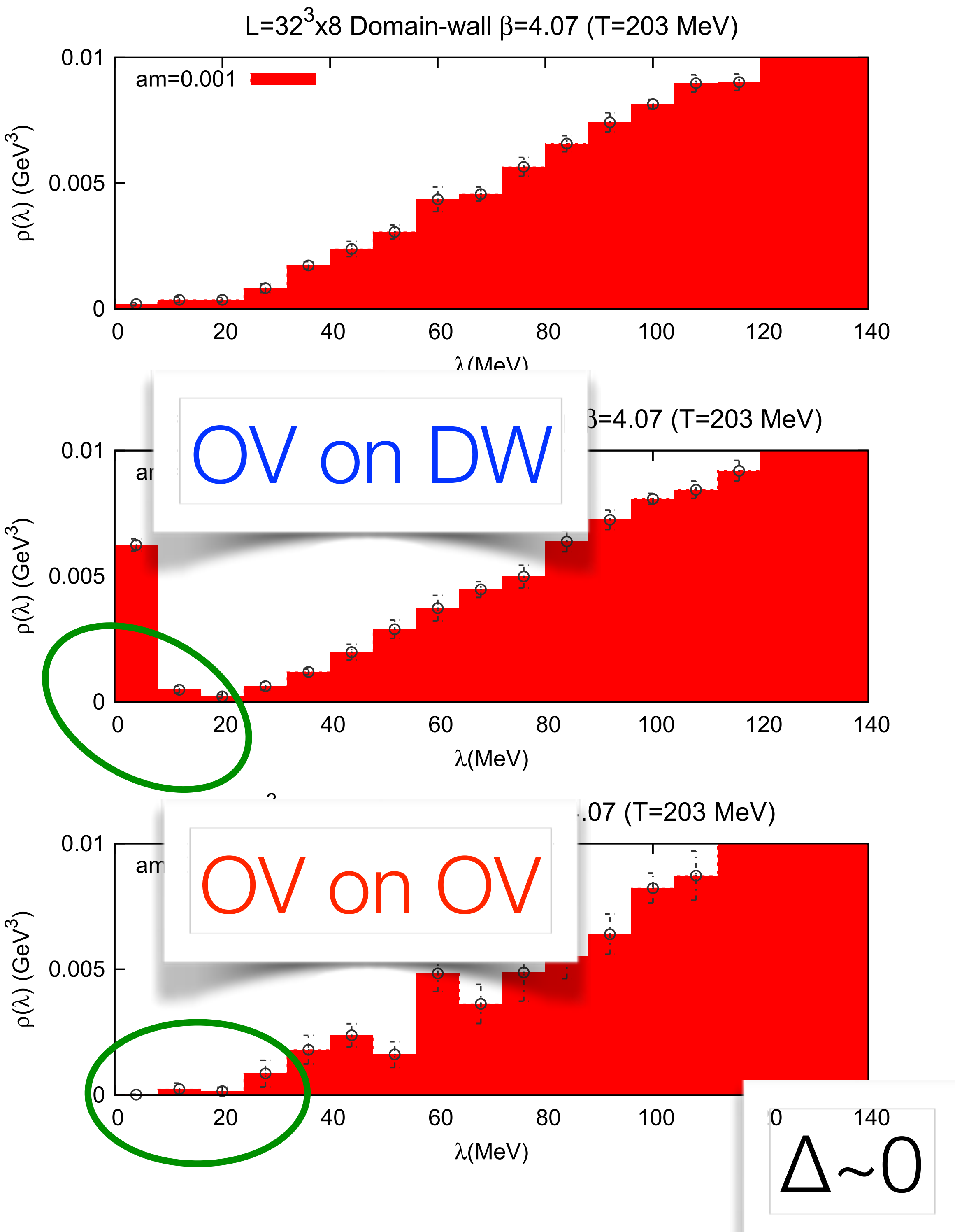
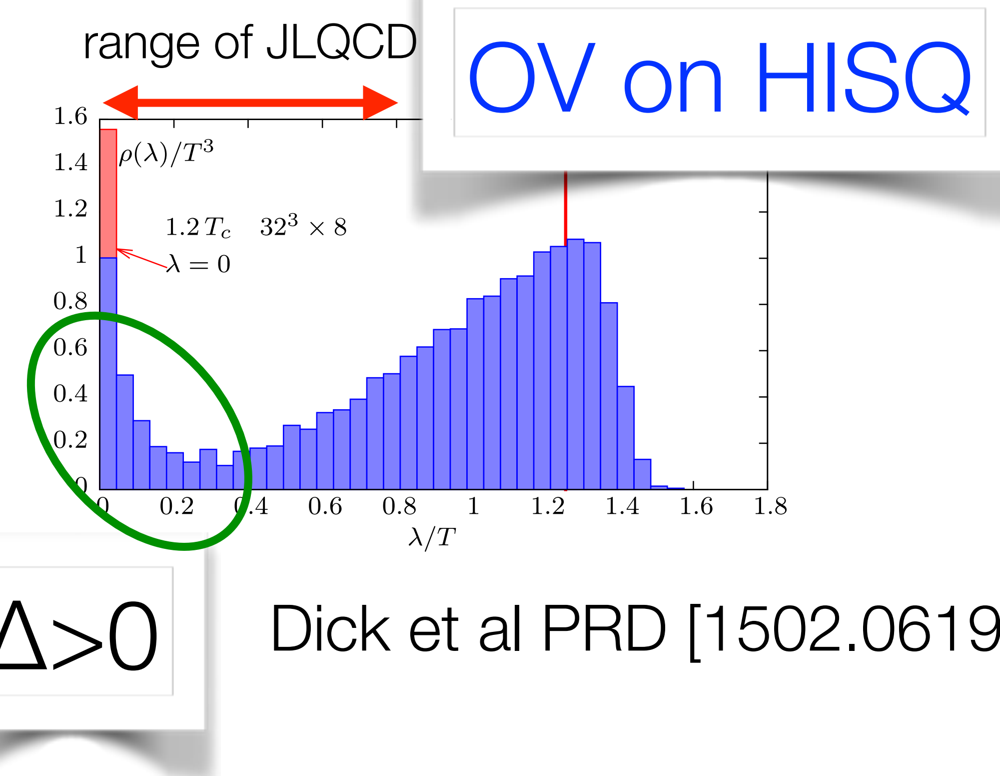
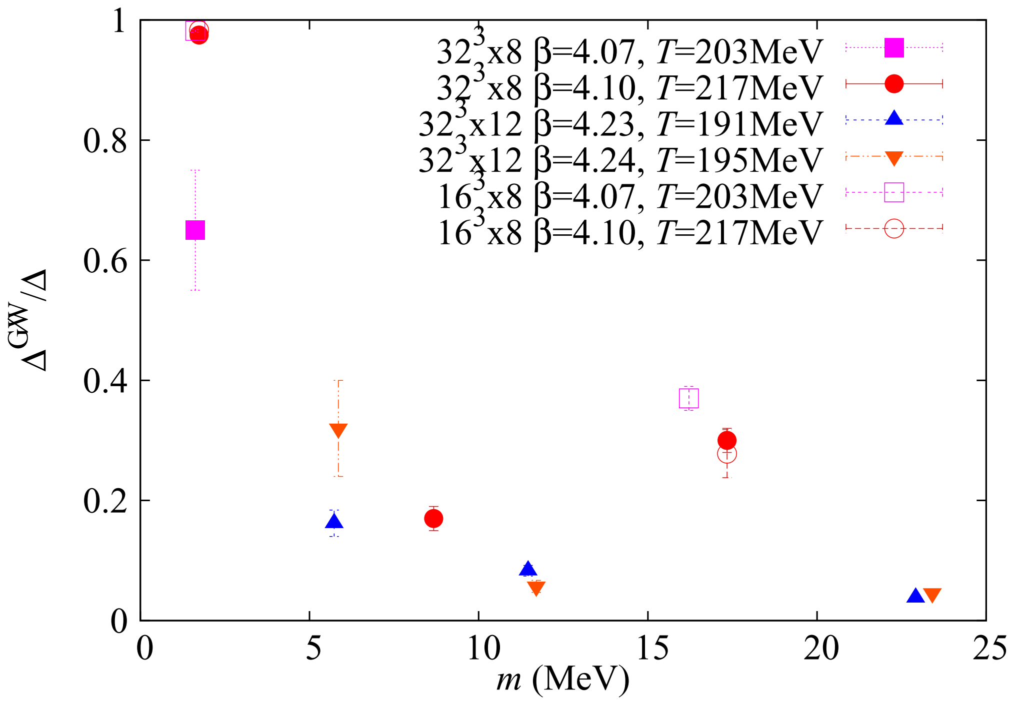




Dick et al PRD [1502.06190]

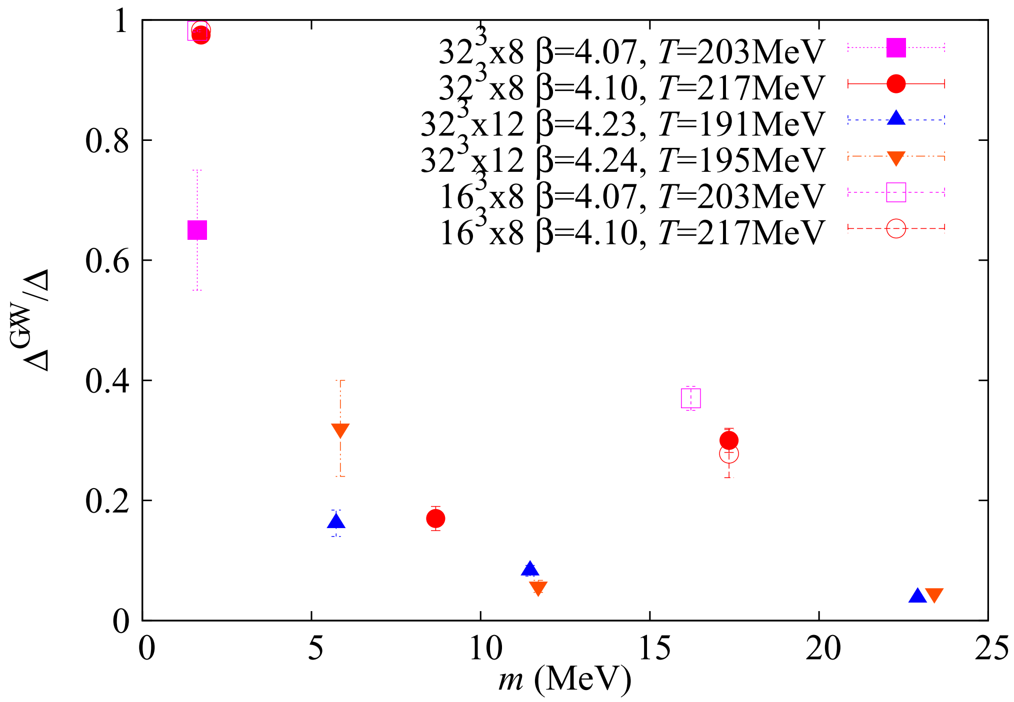
Partially quench effect needs to be investigated

U(1)$_A$ residual chiral symmetry br. of DWF



U(1)$_A$ residual chiral symmetry br. of DWF

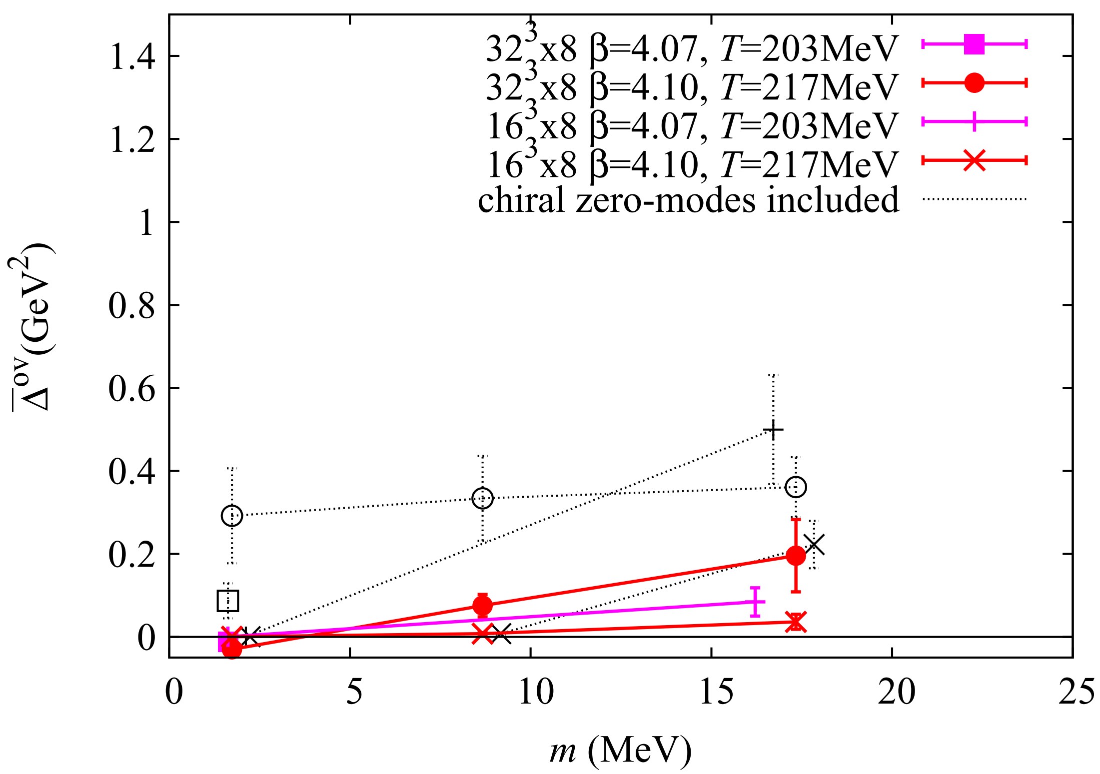
- fraction of Δ from residual chiral symmetry breaking [JLQCD]

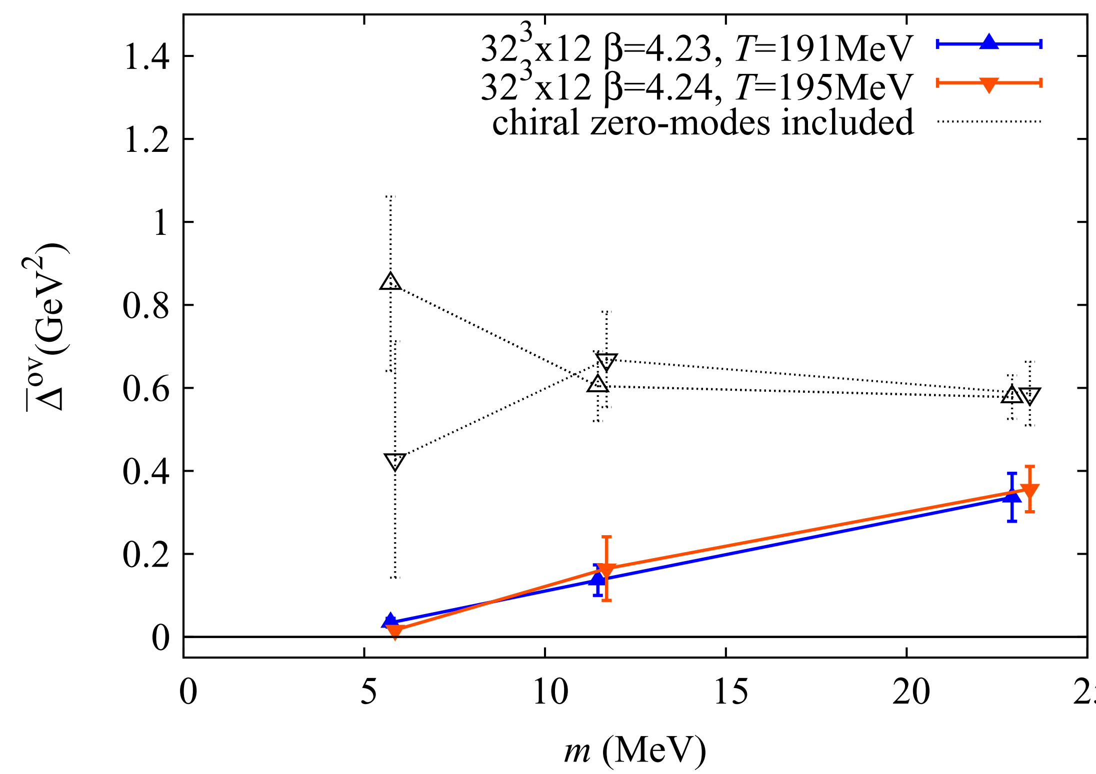


- residual breaking, which is small in terms of m_{res}

dominates the U(1)$_A$ br.

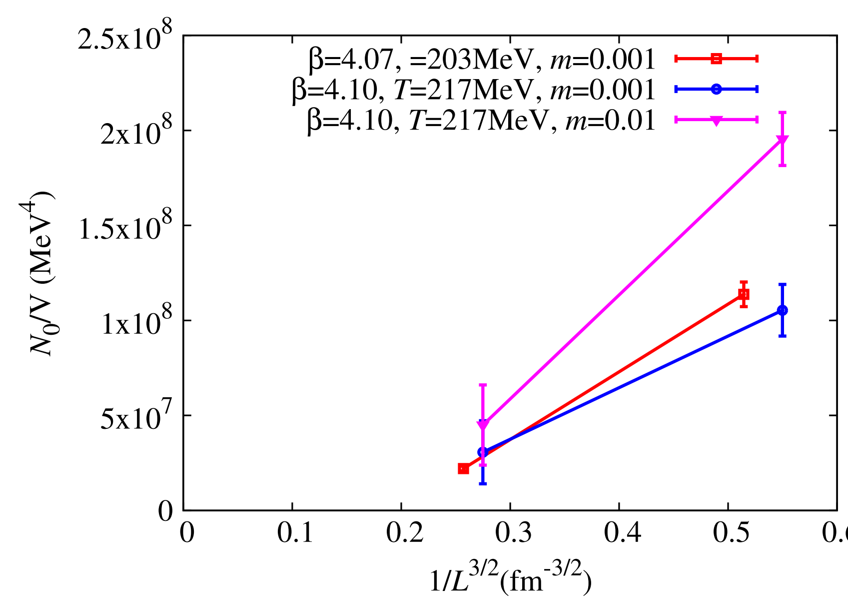
JLQCD 16: $U_A(1)$ susceptibility: T=190-220 MeV



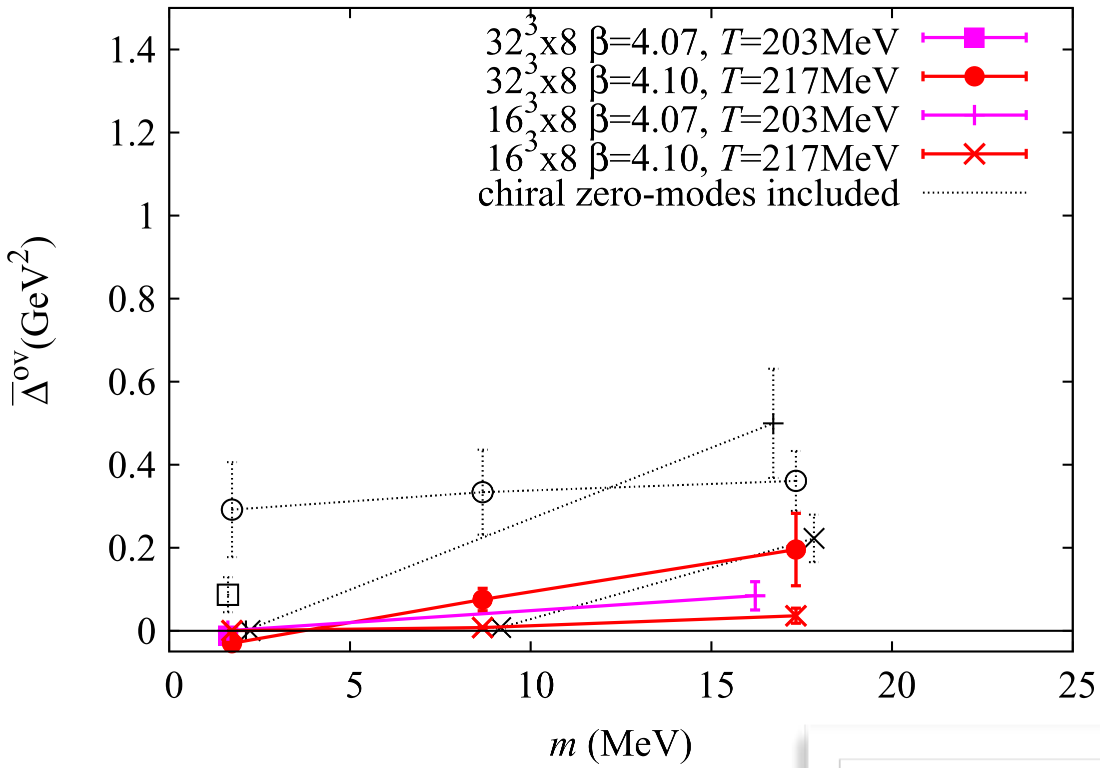


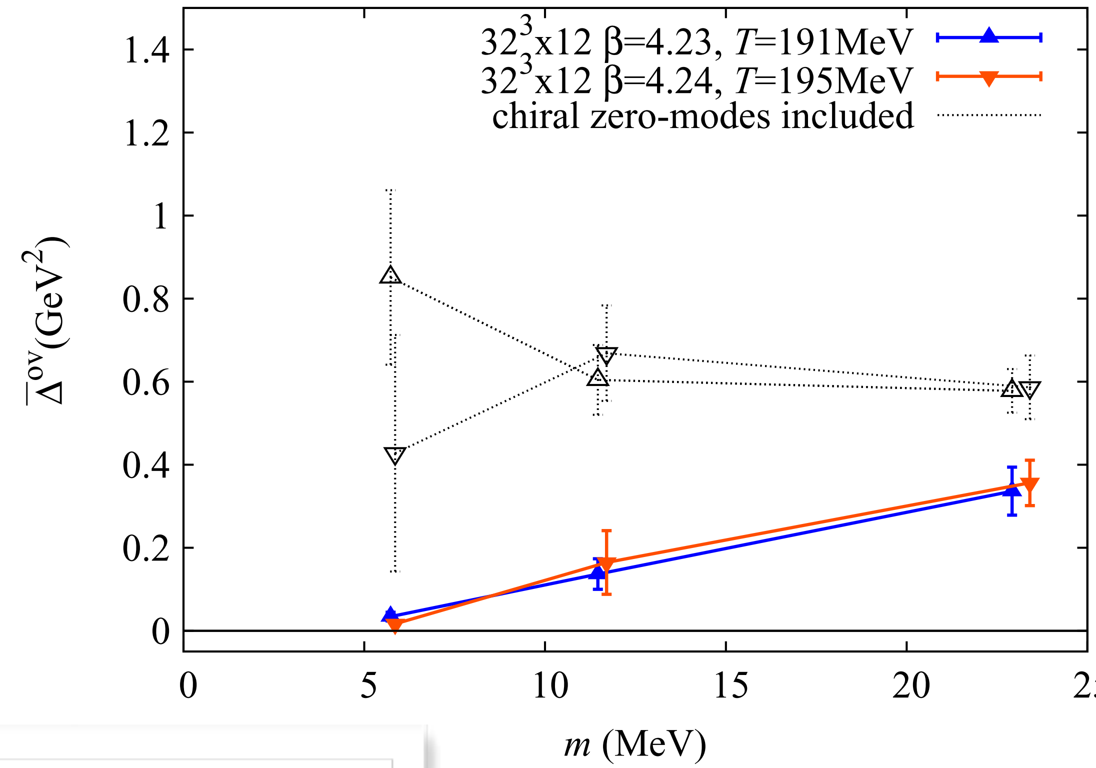
$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2}.$$

zero mode effect



JLQCD 16: $U_A(1)$ susceptibility: T=190-220 MeV

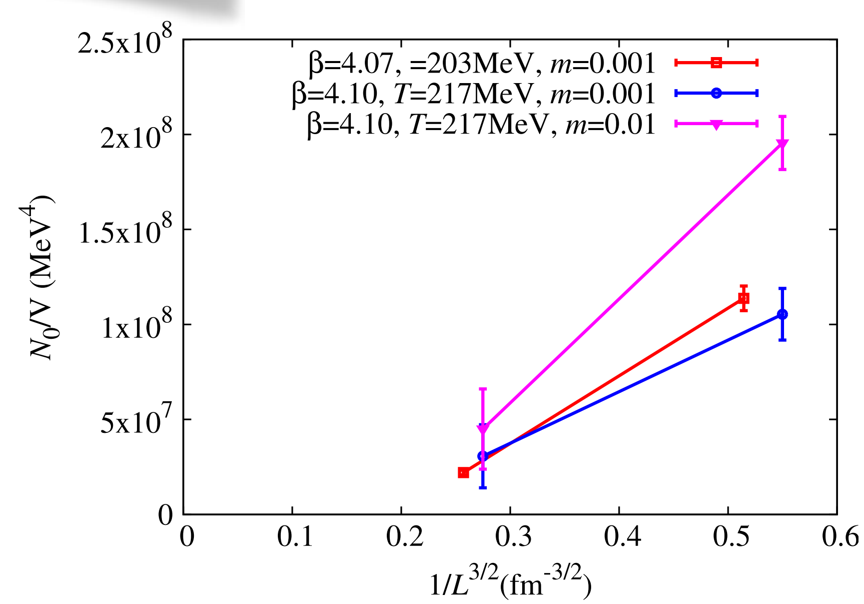




seemingly $\Delta \to 0$

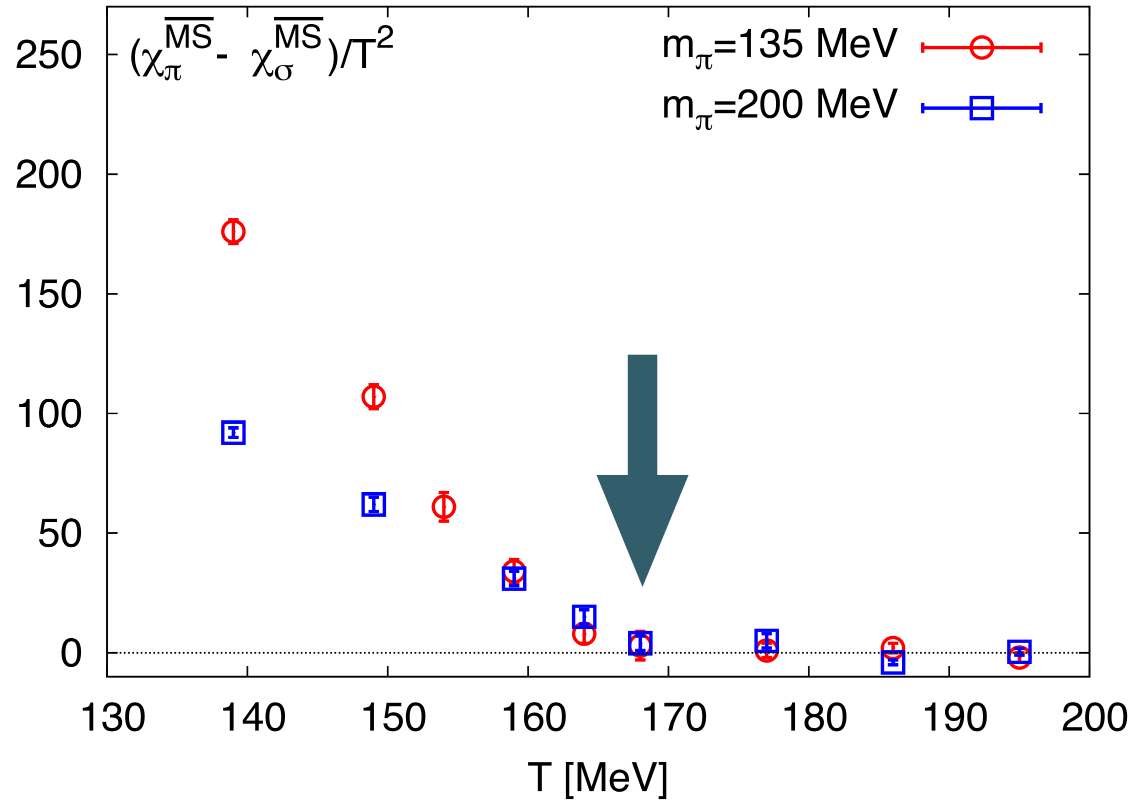
$$\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta} \equiv \Delta^{\mathrm{ov}}_{\pi-\delta} - \frac{2N_0}{Vm^2}.$$

zero mode effect

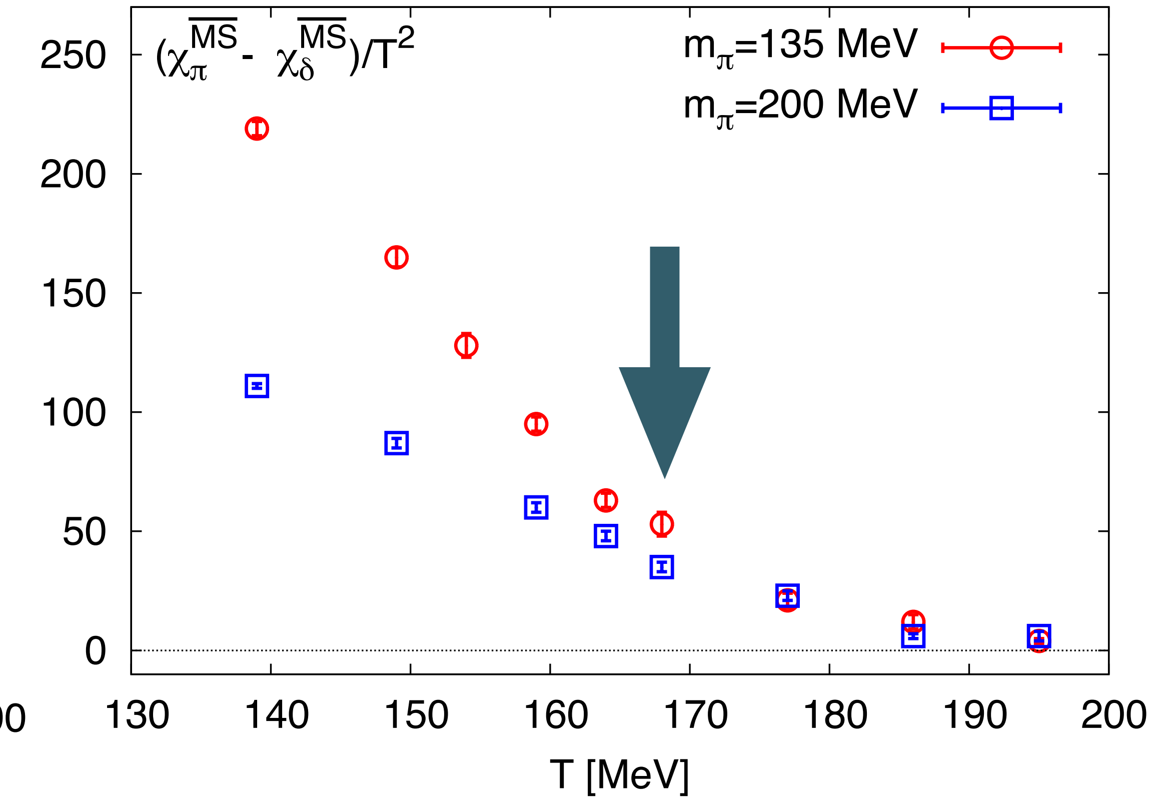


HotQCD 2014: DWF N_f=2+1

$SU(2)_A$



$U(1)_A$



[figures from Ding Lattice 2016]

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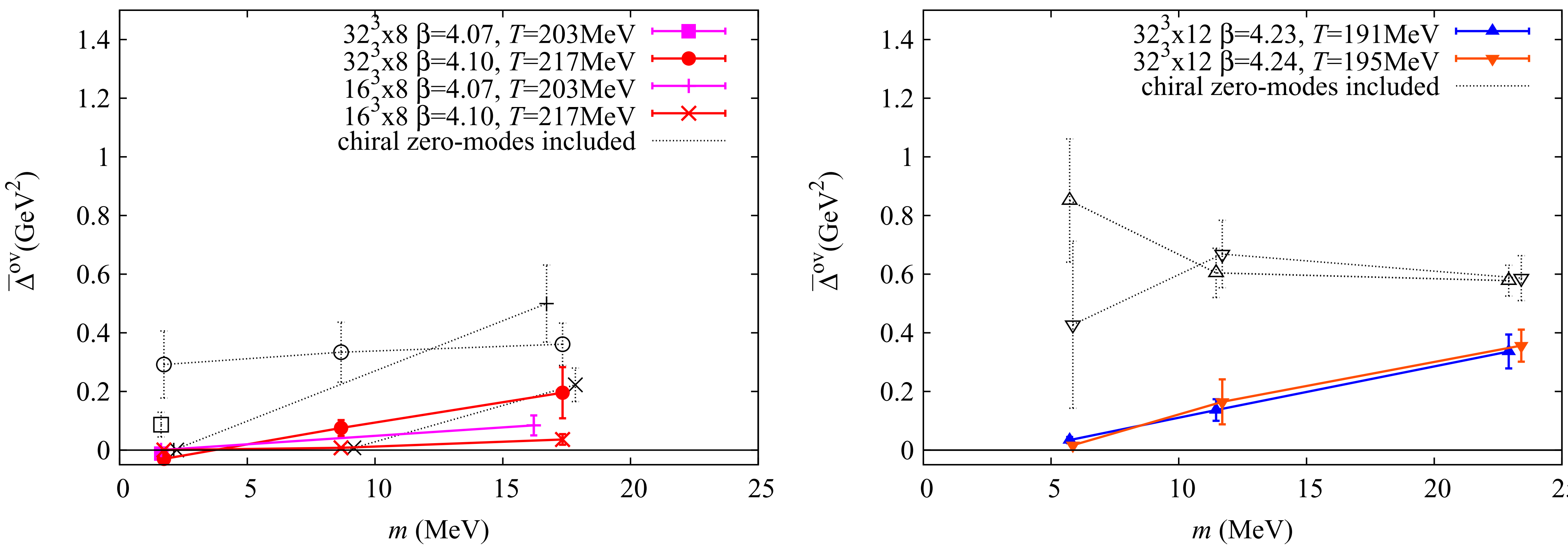
JLQCD 16: $U_A(1)$ susceptibility



is this showing really, exactly $\Delta \to 0$?

update available
closer to continuum limit

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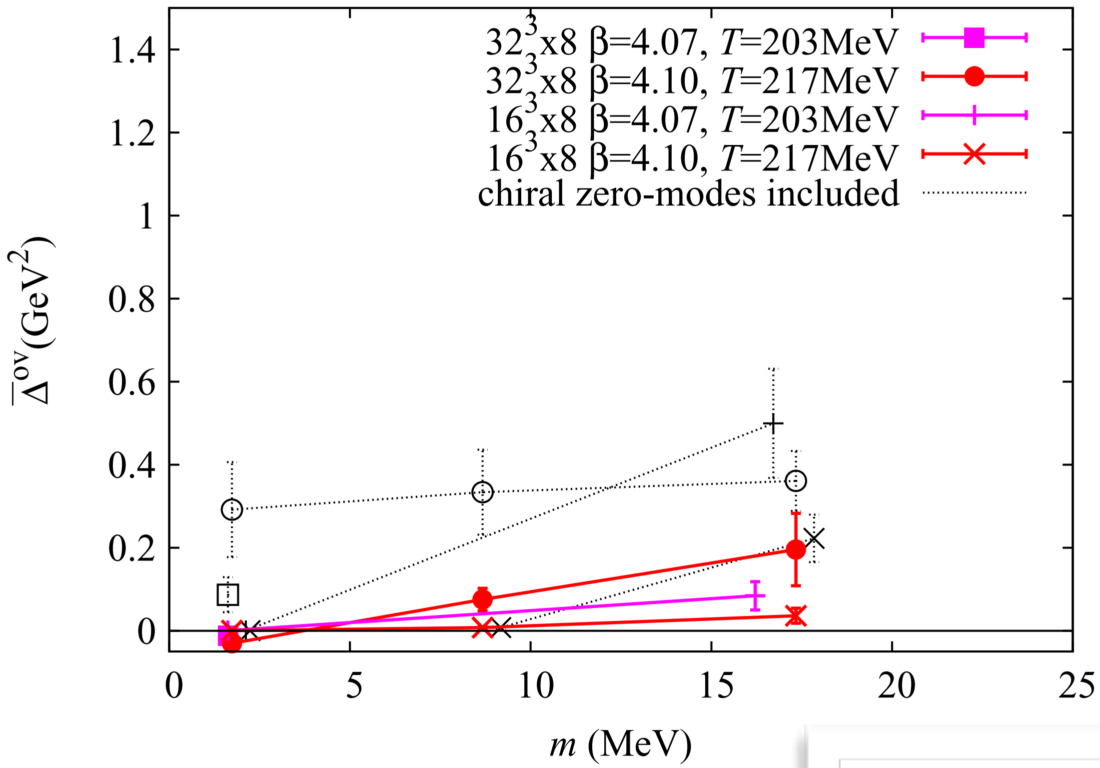

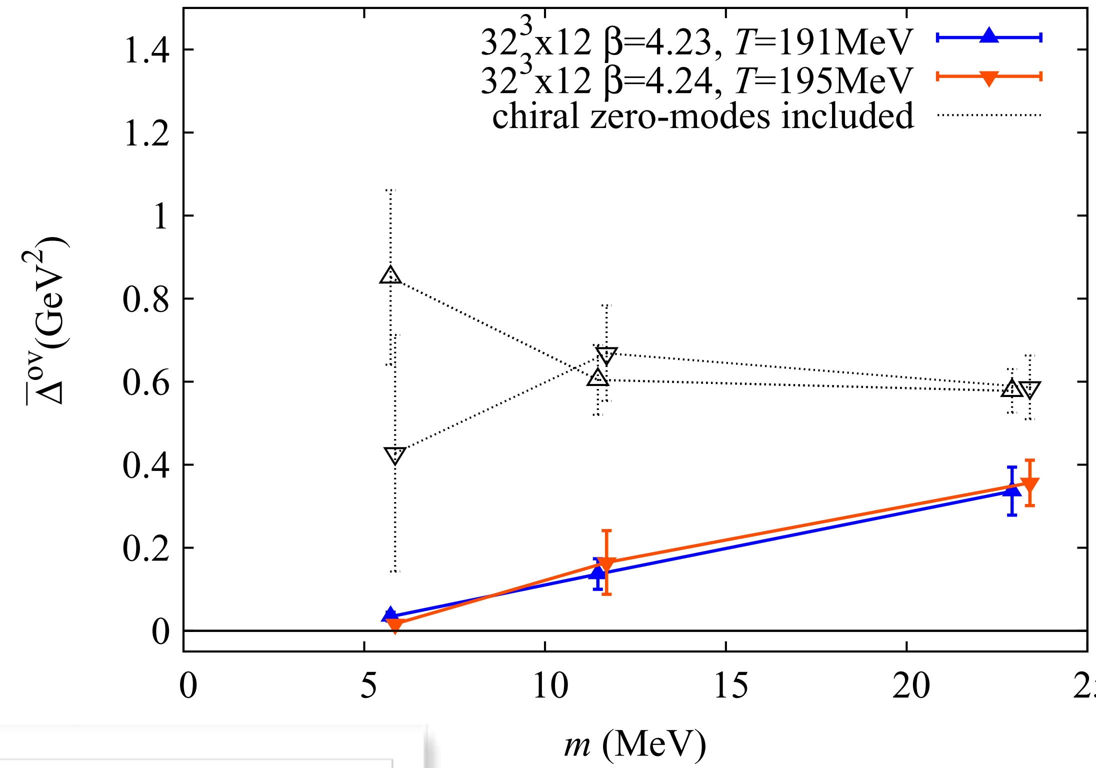

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update available
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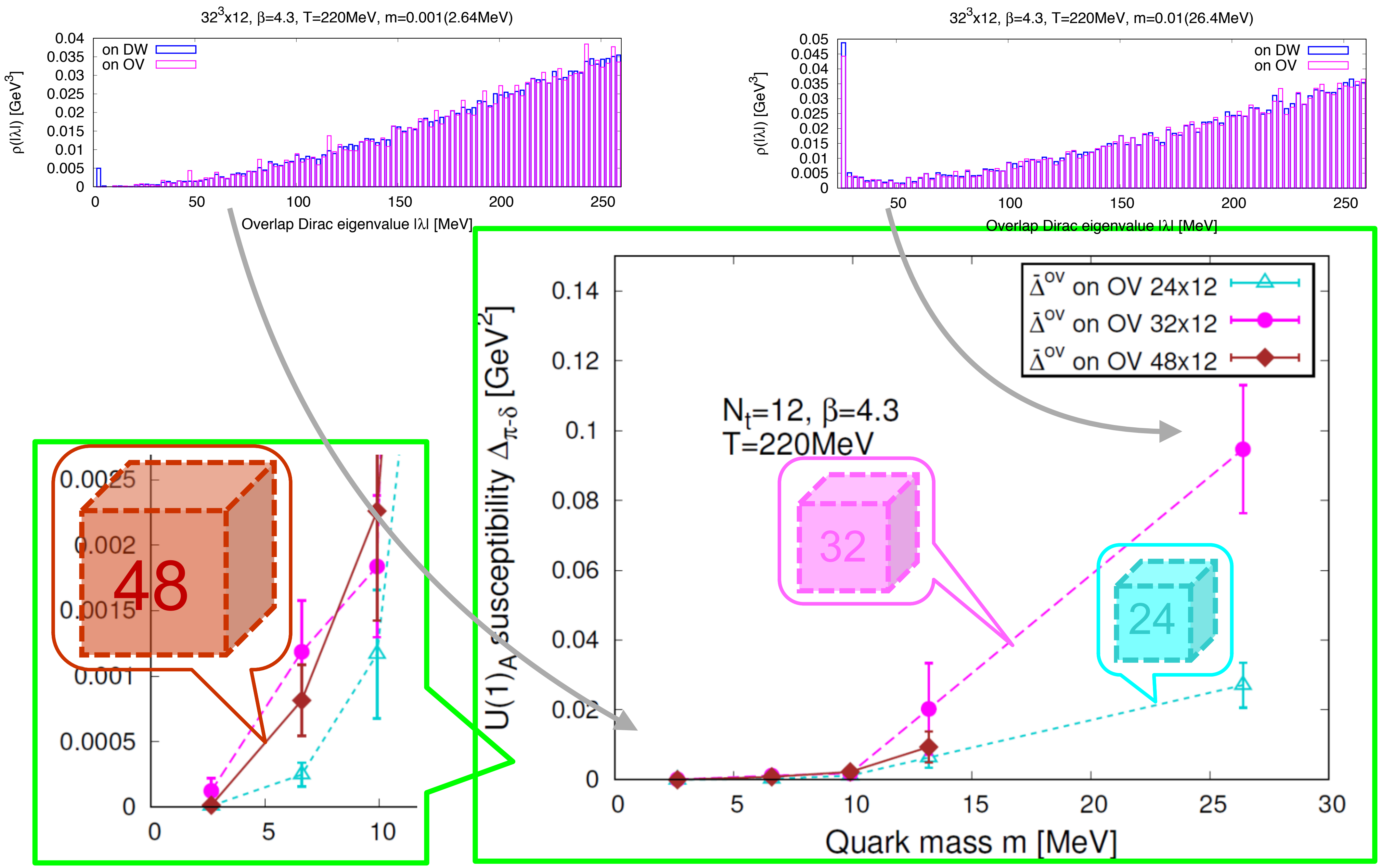
U(1)$_A$ susceptibility   N$_f$=2

[JLQCD preliminary]

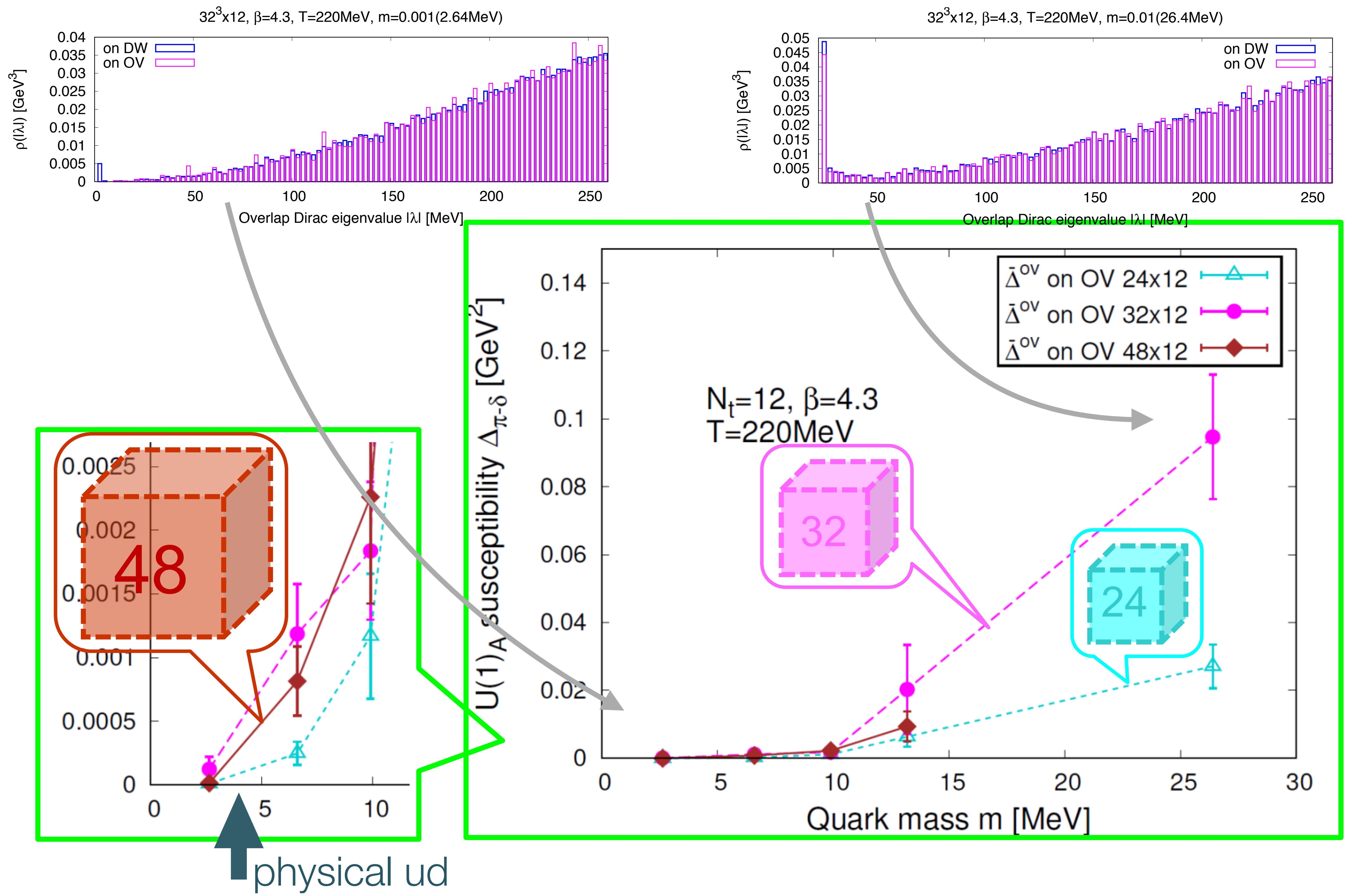


seemingly vanishing as m→0

Suzuki, JPS 2018.3

U(1)$_A$ susceptibility   N$_f$=2

[JLQCD preliminary]

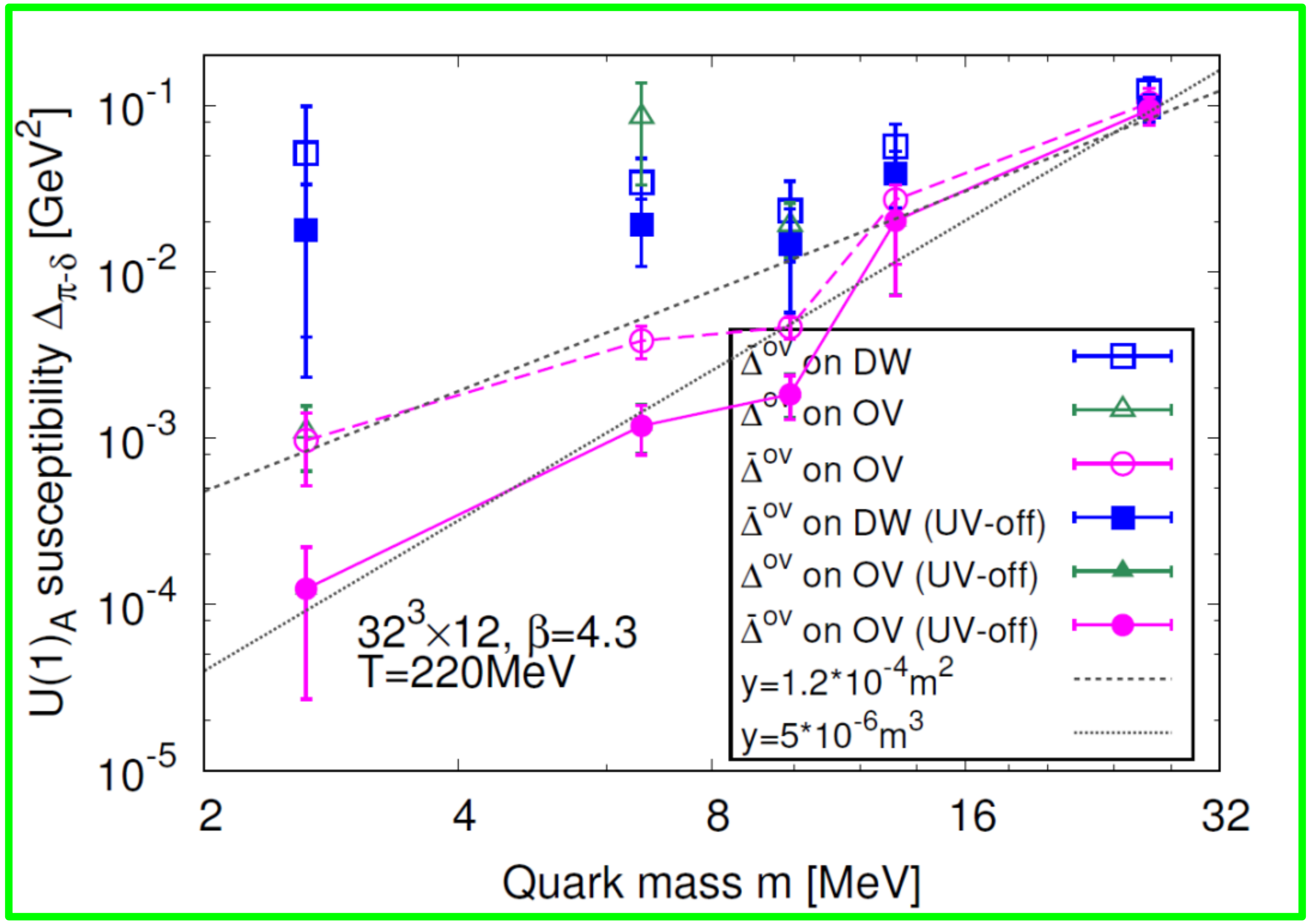


seemingly vanishing as m→0

Suzuki, JPS 2018.3

$U(1)_A$ susceptibility $N_f=2$

[JLQCD preliminary]



seemingly vanishing as m→0,
more evident in log-log prot

Analytic works

- Aoki-Fukaya-Taniguchi
  - QCD with OV regulator
  - assuming analyticity of $\rho(0)$
- $f_A \to 0$: $U(1)_A$ br. parameter
- $\chi_{top} = 0$ for $0<m<m_c$

- Kanazawa-Yamamoto
  - assuming $f_A \neq 0$
  - expansing free energy in m
- discussing
  - finite m and V effect
  - contributions of topological sectors

Kanazawa - Yamamoto

- assuming $f_A \neq 0$
- expansing free energy in m

$$Z(T, V_3, M) = \exp\left[-\frac{V_3}{T} f(T, V_3, M)\right],$$

$$f(T, V_3, M) = f_0 - f_2 \,\mathrm{tr}\, M^\dagger M - \underline{f_A(\det M + \det M^\dagger)} + \mathcal{O}(M^4),$$

$$M \to e^{-2i\theta_A} V_L M V_R^\dagger \qquad \det M \to \mathrm{e}^{4i\theta_A} \det M$$ breaks $U(1)_A$

other terms are invariant under $U(1)_A$
all invariant under SU(2)LxR

- to study topological sectors

$$M \to M\,\mathrm{e}^{i\theta/N_f} \qquad Z_Q(T, V_3, M) \equiv \oint \frac{\mathrm{d}\theta}{2\pi}\,\mathrm{e}^{-iQ\theta}\, Z(T, V_3, M\mathrm{e}^{i\theta/2}).$$

$$= \mathrm{e}^{-V_4[f_0 - f_2(m_u^2 + m_d^2)]} \oint \frac{\mathrm{d}\theta}{2\pi}\,\mathrm{e}^{-iQ\theta}\,\mathrm{e}^{2V_4 f_A m_u m_d \cos\theta}$$

$$= \mathrm{e}^{-V_4[f_0 - f_2(m_u^2 + m_d^2)]}\, I_Q(2V_4 f_A m_u m_d),$$

$$\Delta_{\pi-\delta} = \sum_{Q=-\infty}^{\infty} \frac{Z_Q}{Z} P_Q \qquad P_Q = 8 f_A \frac{I_Q'(2V_4 f_A m^2)}{I_Q(2V_4 f_A m^2)}$$

Kanazawa - Yamamoto: $U(1)_A$ br. scenario

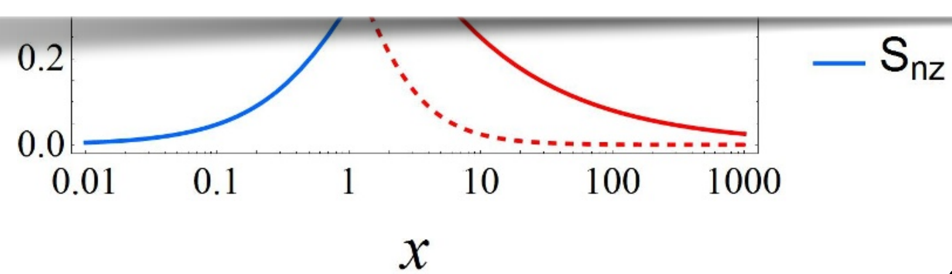
KY tells

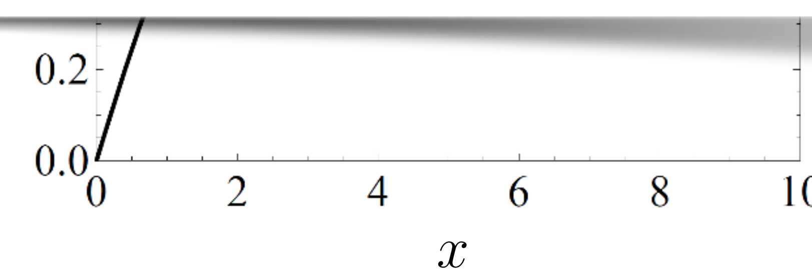
- fixed topology gives wrong result at small V
- adding all Q sector or large enough volume necessary

$$\Delta_{\pi-\delta} = \sum_{Q=-\infty}^{\infty} \frac{Z_Q}{Z} P_Q \qquad P_Q = 8 f_A \frac{I'_Q(2V_4 f_A m^2)}{I_Q(2V_4 f_A m^2)}$$

JLQCD

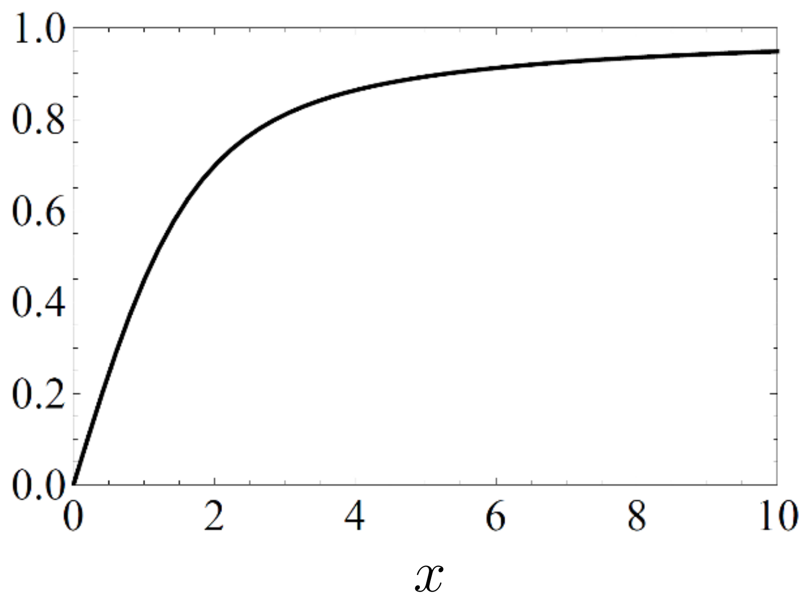
- does not fix topology (DW)
- zero-mode subtraction may have similar effect to fix Q=0
    - for smallest m: actually effectively fixed to Q=0





$$x = 2V_4 f_A m^2$$

compare with JLQCD Δ with non-zero modes



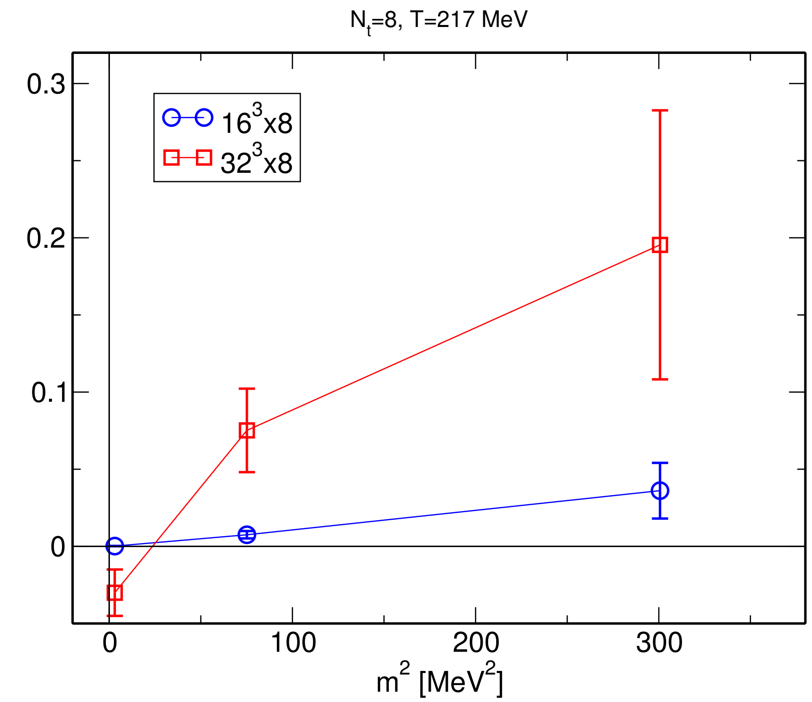
$$x = 2V_4 f_A m^2$$

fix V: Δ→0 as m² for m→0
even for $U(1)_A$ br. case

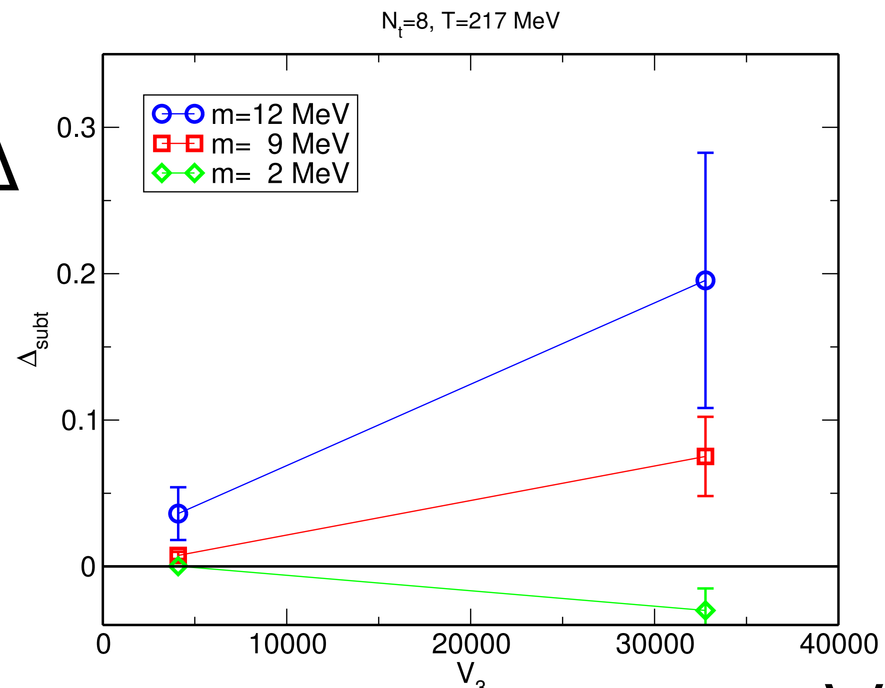
fix m : Δ∝V

→ NOT inconsistent with JLQCD results

[JLQCD 2016 Tomiya et al]



Δ

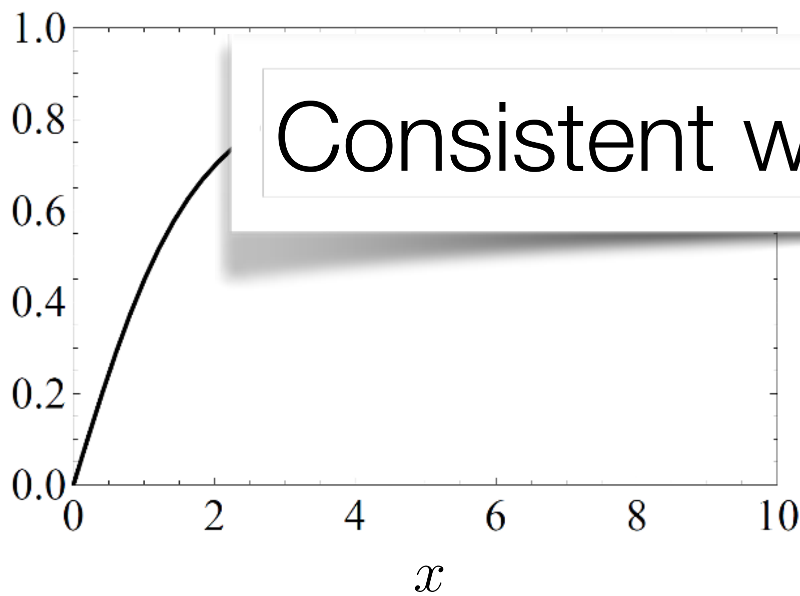


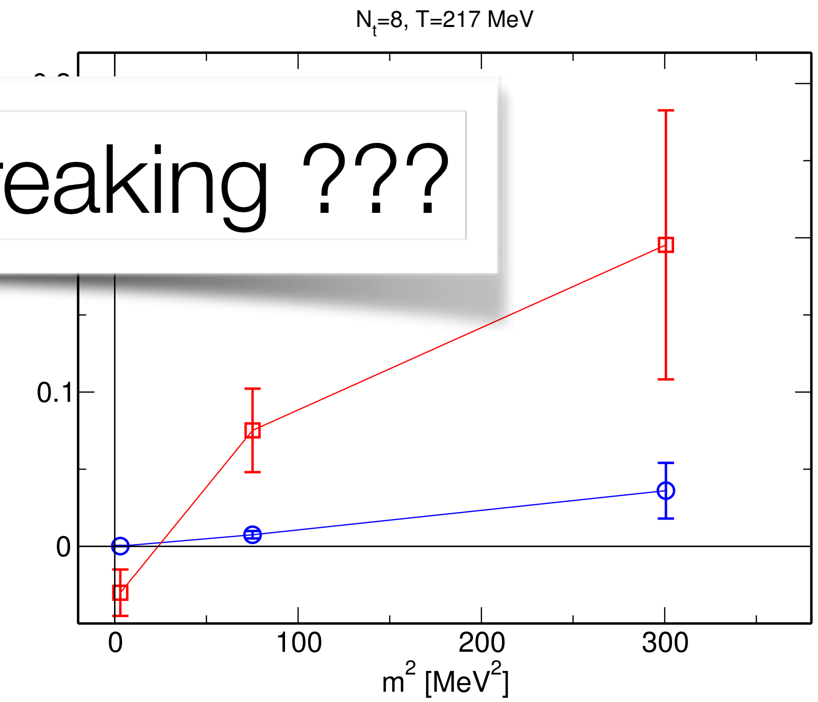
V

compare with JLQCD Δ with non-zero modes

[JLQCD 2016 Tomiya et al]

Consistent with U(1)A breaking ???





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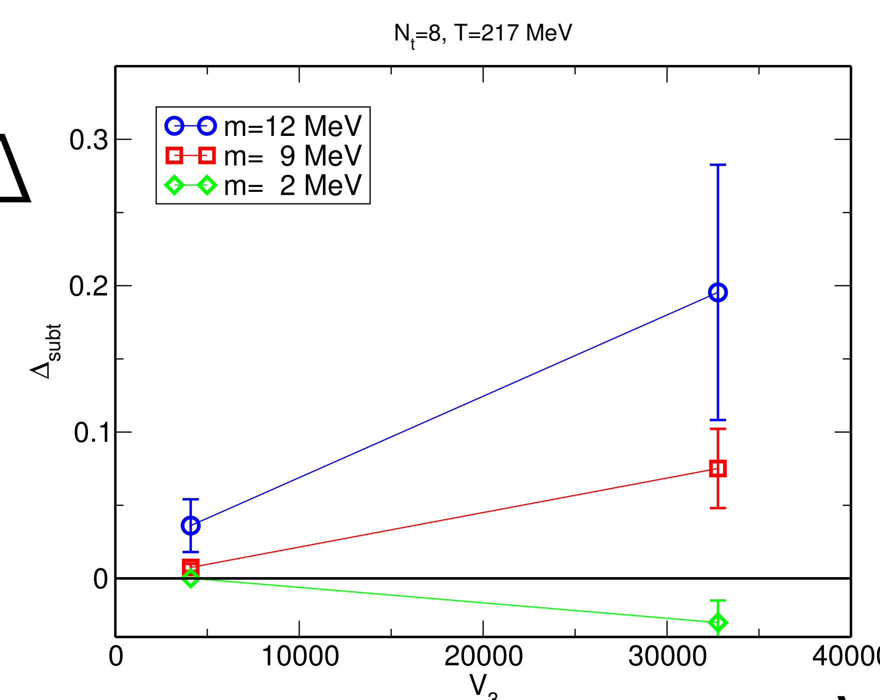
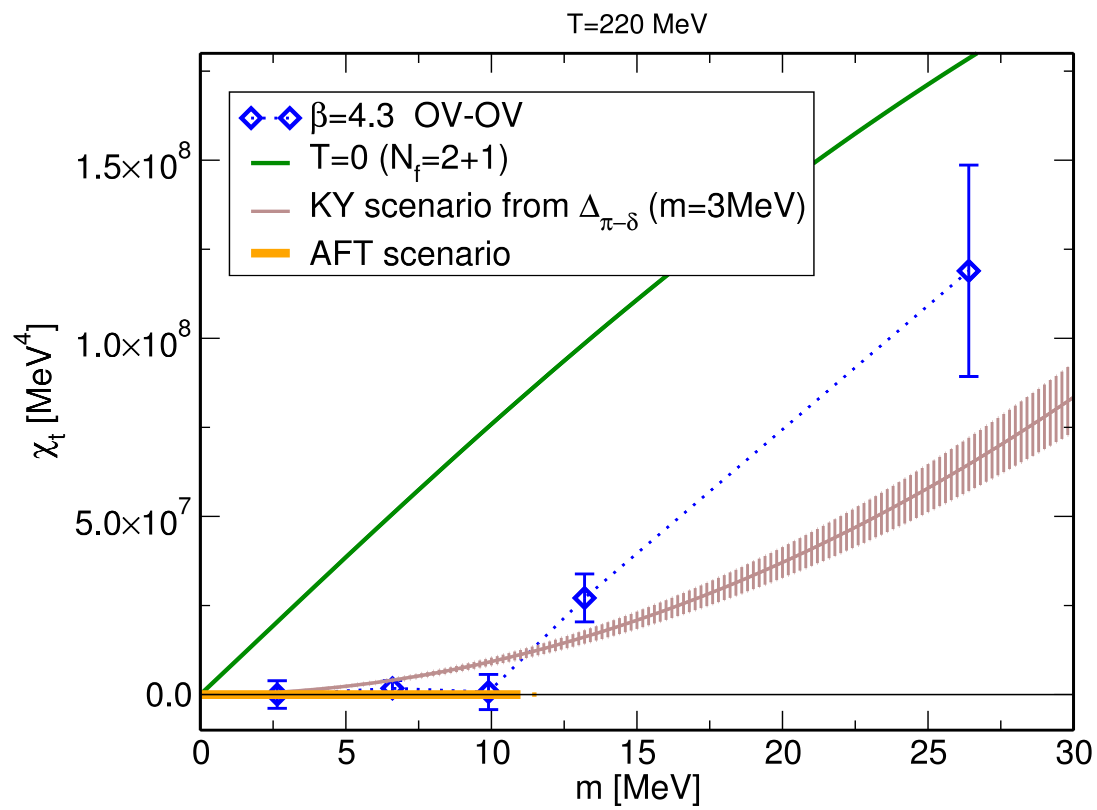
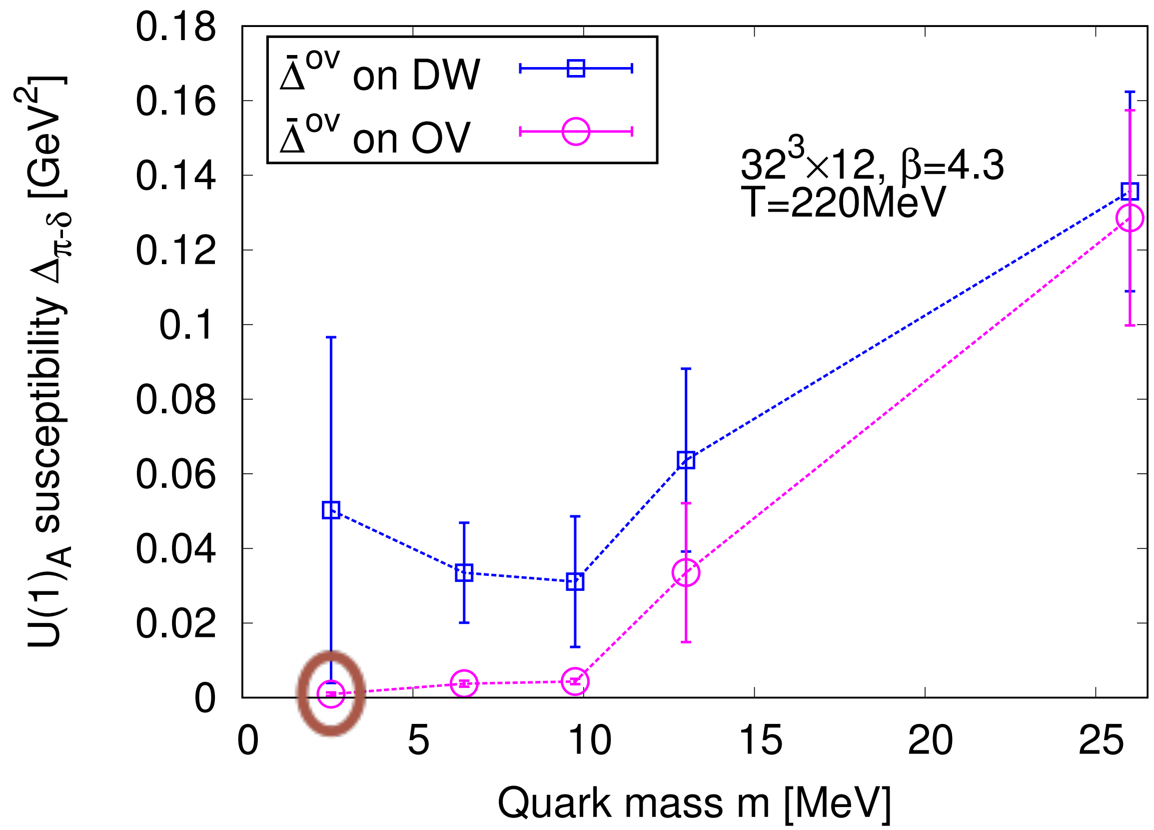
Δ



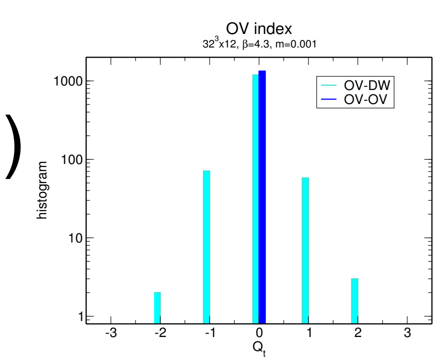
V

competing scenarios for χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV



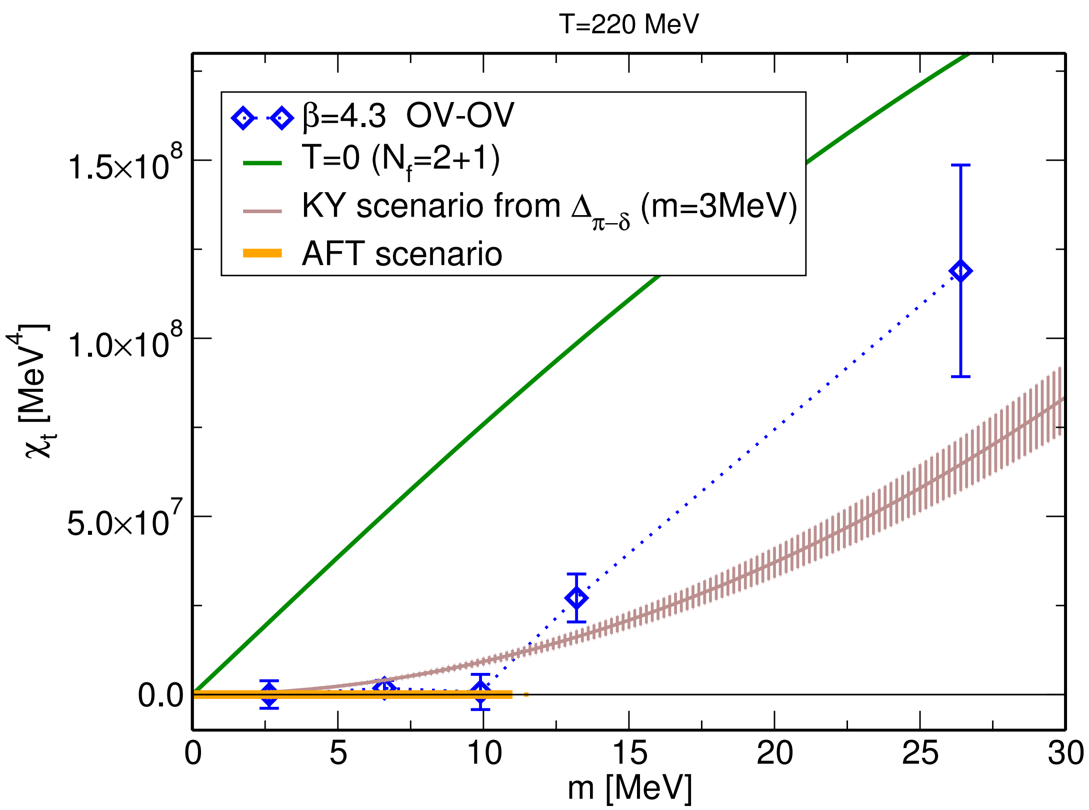


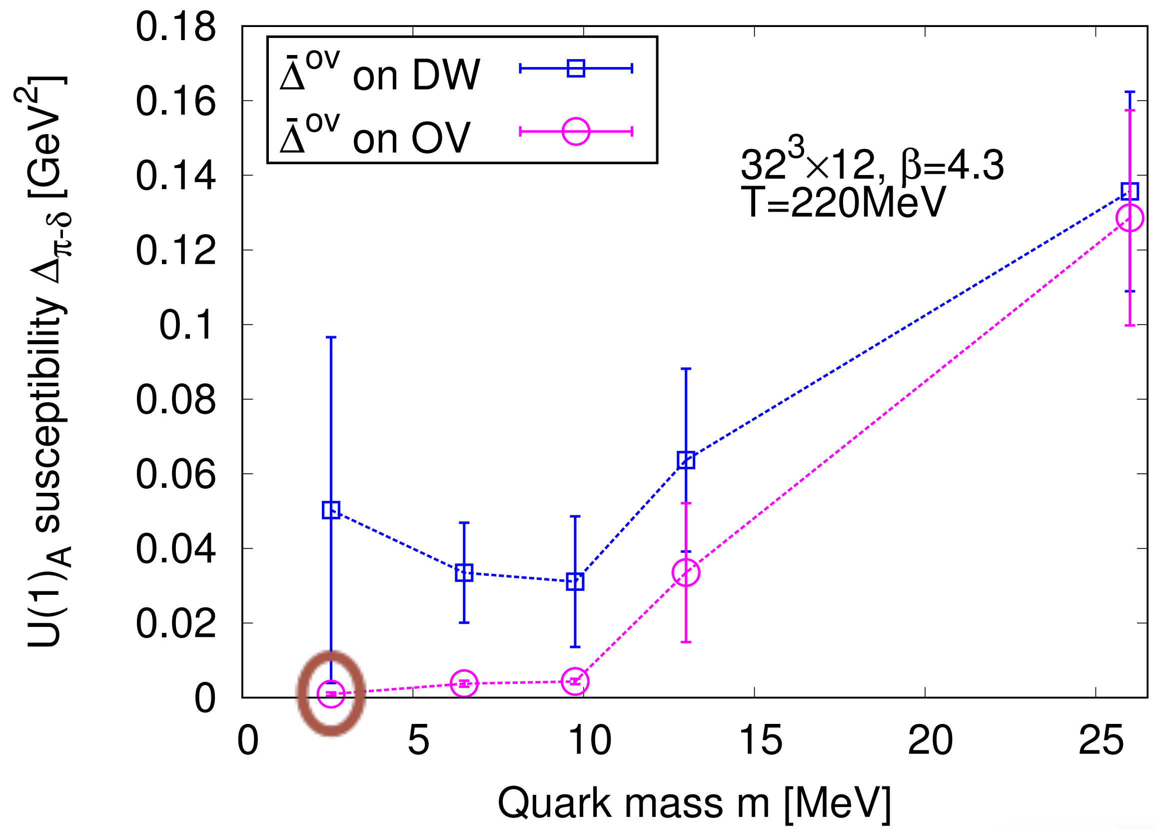
- KY scenario [Kanazawa, Yamamoto 2016]
  - $\Delta_{\pi\text{-}\delta}$: including zero mode cont. is proper
  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\Delta_{\pi\text{-}\delta} \simeq 8\, V\, f_A^2\, m^2$ for Q=0 sector (for $2V f_A m^2 < 1$)



- $\Delta_{\pi\text{-}\delta}$ @ lightest point only from Q=0
- $\chi_t = 2\, f_A\, m^2$
- tension at m≥10 MeV χ_t sudden growth

competing scenarios for
χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV





- KY scenari
- Volume study would be useful to check this
  - $\Delta_{\pi\text{-}\delta}$: inc… proper
  - $\Delta_{\pi\text{-}\delta}$ = const >0
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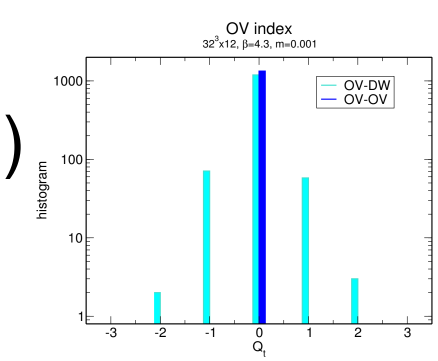
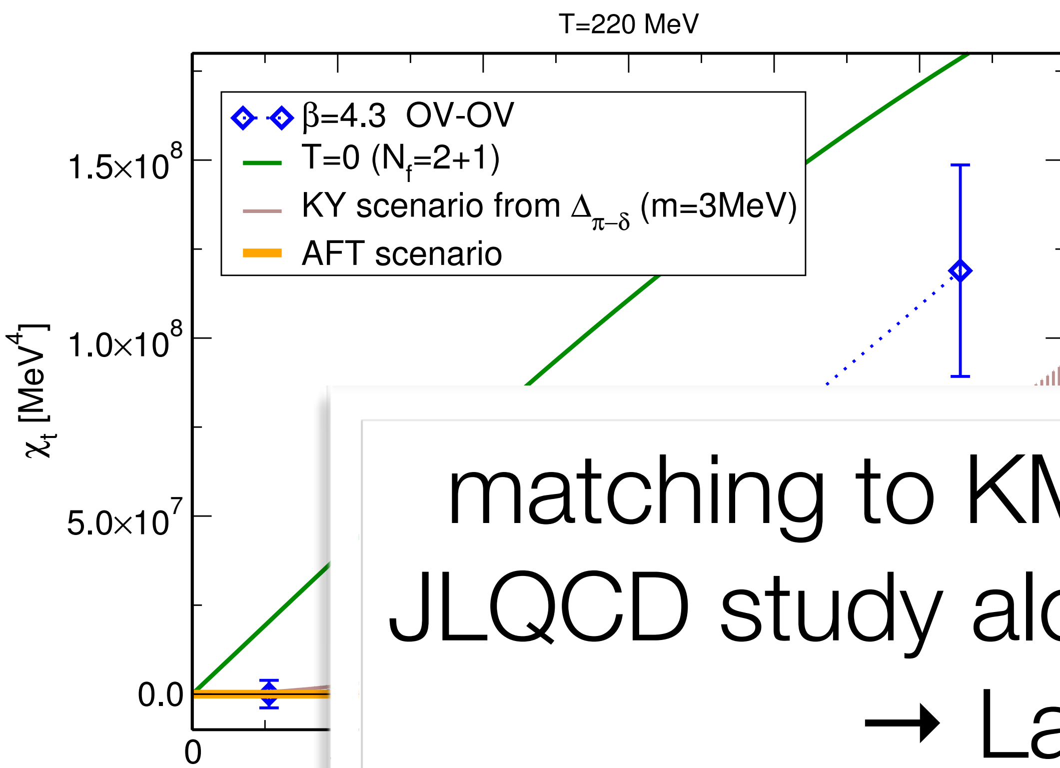
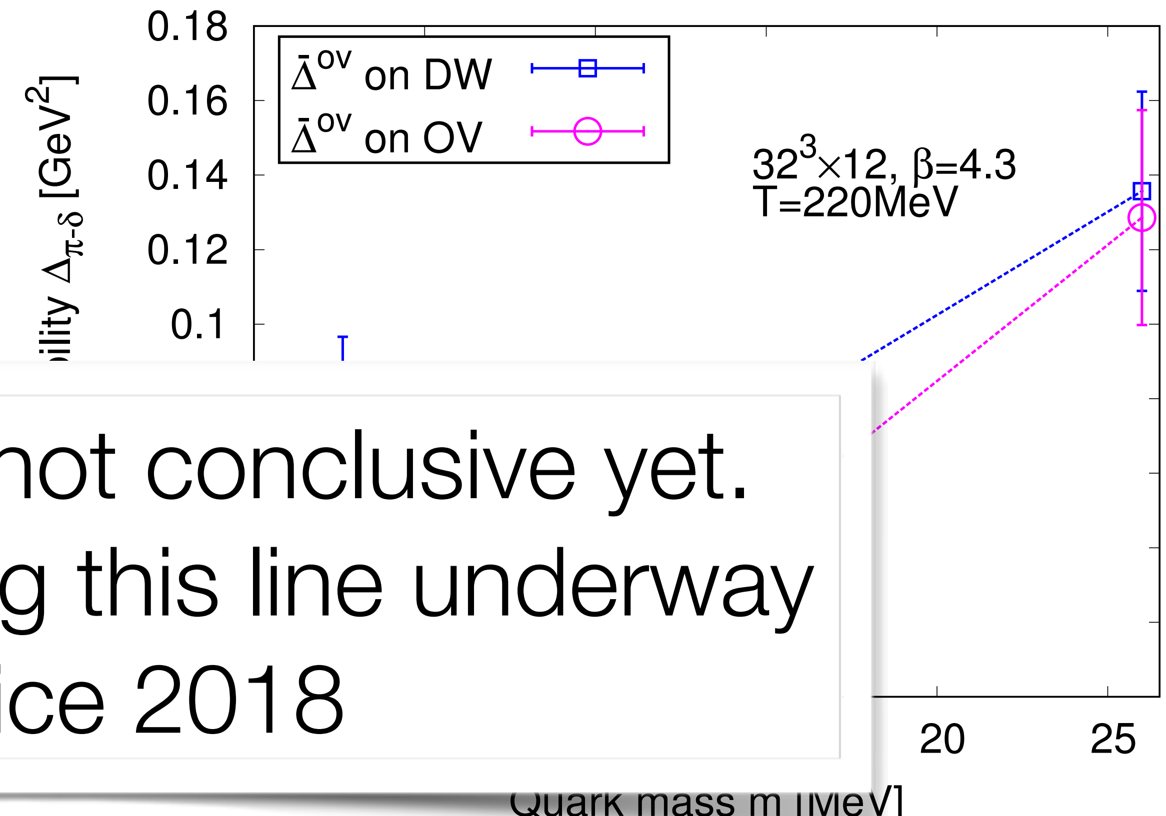


- $\Delta_{\pi\text{-}\delta}$ @ lightest point only from Q=0
- $\chi_t = 2\, f_A\, m^2$
- $\boxed{\text{tension at } m \geq 10 \text{ MeV} \quad \chi_t \text{ sudden growth}}$

competing scenarios for χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV





matching to KM not conclusive yet.
JLQCD study along this line underway
→ Lattice 2018

Volume study would be useful to check this

- KY scenari
  - $\Delta_{\pi\text{-}\delta}$: inc
  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\boxed{\Delta_{\pi\text{-}\delta} \simeq 8\, V\, f_A^2\, m^2}$ for Q=0 sector (for $2V f_A m^2 < 1$)
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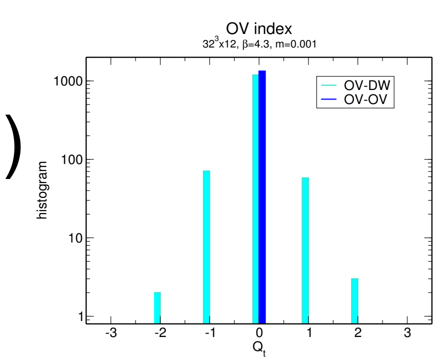

Why bother ?

- **Because it is unsettled problem !**
- fate of $U(1)_A$ lattice

		N_f
HotQCD (DW, 2012)	broken	2+1
JLQCD (topology fixed overlap, 2013)	restores	2
TWQCD (optimal DW, 2013)	restores ?	2
LLNL/RBC (DW, 2014)	broken	2+1
HotQCD (DW, 2014)	broken	2+1
Dick et al. (overlap on HISQ, 2015)	broken	2+1
Brandt et al. (O(a) improved Wilson 2016)	restores	2
JLQCD (reweighted overlap from DW, 2016)	restores	2
JLQCD (current: see Suzuki et al Lattice 2017)	restores	2
Ishikawa et al (Wilson, 2017)	at least Z_4 restores	2

Summary

Summary

- the status of the fate of $U(1)_A$ is still unclear at least to me
- So far
  - $N_f=2+1$ studies suggest $U(1)_A$ breaking
  - $N_f=2$ studies suggest $U(1)_A$ restoration
- needs to be carefully check these lattice technique / property
  - partially quenching
  - residual chiral symmetry breaking
  - other possible source of systematic error
  - finite volume effect should be checked for zero-mode subtracted $\Delta_{\pi\text{-}\delta}$

    ➡ JLQCD
- More study needed !

Thank you very much for your attention !

Lattice framework

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- DW→OV reweighting

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$$\langle \mathcal{O} \rangle_{\mathrm{ov}} = \frac{\langle \mathcal{O} R \rangle_{\mathrm{DW}}}{\langle R \rangle_{\mathrm{DW}}},$$

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$$\langle \mathcal{O} \rangle_{\rm ov} = \frac{\langle \mathcal{O} R \rangle_{\rm DW}}{\langle R \rangle_{\rm DW}},$$

$$R \equiv \frac{\det[H_{\rm ov}(m)]^2}{\det[H_{\rm DW}^{\rm 4D}(m)]^2} \times \frac{\det[H_{\rm DW}^{\rm 4D}(1/4a)]^2}{\det[H_{\rm ov}(1/4a)]^2}.$$

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$$D_{ov} = \underbrace{\frac{1}{2} \sum_{\lambda_i < \lambda_{th}} (1 + \gamma_5 {\rm sgn} \lambda_i) \, |\lambda_i\rangle\langle\lambda_i|}_{\text{Exact low modes}} + D_{DW}^{4D} \underbrace{\left(1 - \sum_{\lambda_i < \lambda_{th}} |\lambda_i\rangle\langle\lambda_i|\right)}_{\text{High modes}},$$

Lattice framework

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- DW→OV reweighting

λ for $H_M = \gamma_5 \dfrac{\alpha D_W}{2 + D_W}$

$$\langle \mathcal{O} \rangle_{\mathrm{ov}} = \frac{\langle \mathcal{O} R \rangle_{\mathrm{DW}}}{\langle R \rangle_{\mathrm{DW}}},$$

$$R \equiv \frac{\det[H_{\mathrm{ov}}(m)]^2}{\det[H_{\mathrm{DW}}^{\mathrm{4D}}(m)]^2} \times \frac{\det[H_{\mathrm{DW}}^{\mathrm{4D}}(1/4a)]^2}{\det[H_{\mathrm{ov}}(1/4a)]^2}.$$

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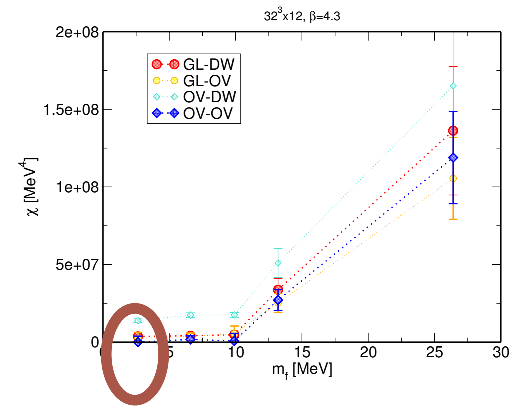
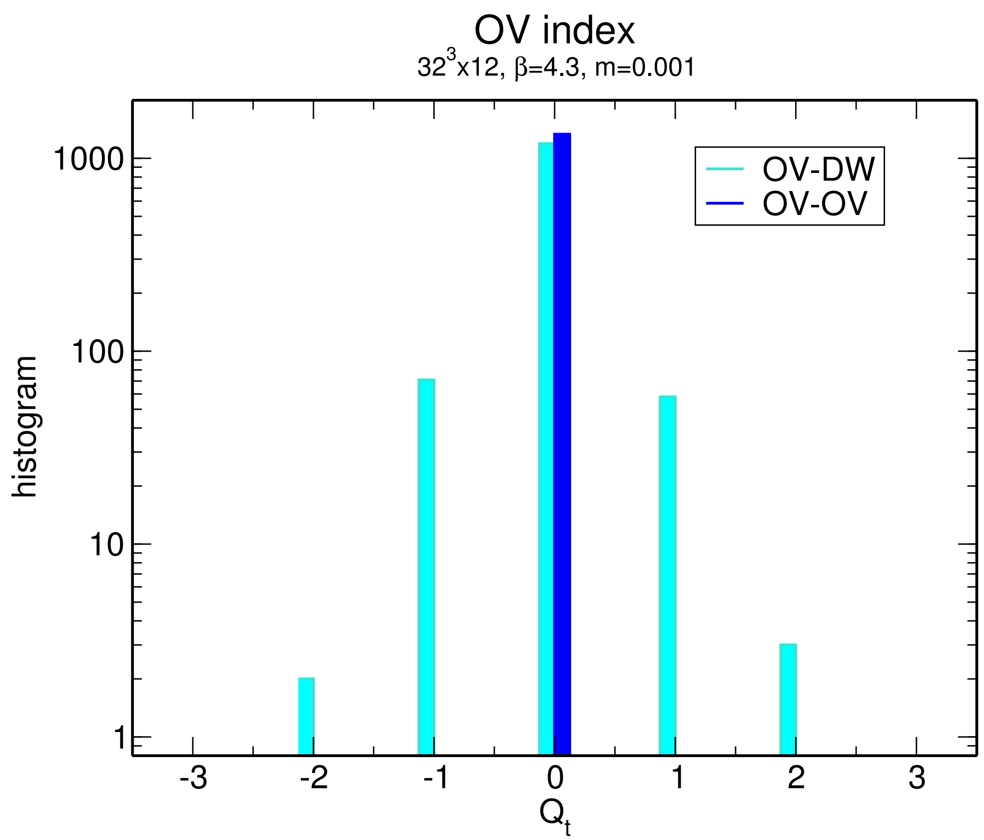
resolution of susceptibility (ex: m=0.001)





Effective number of statistics

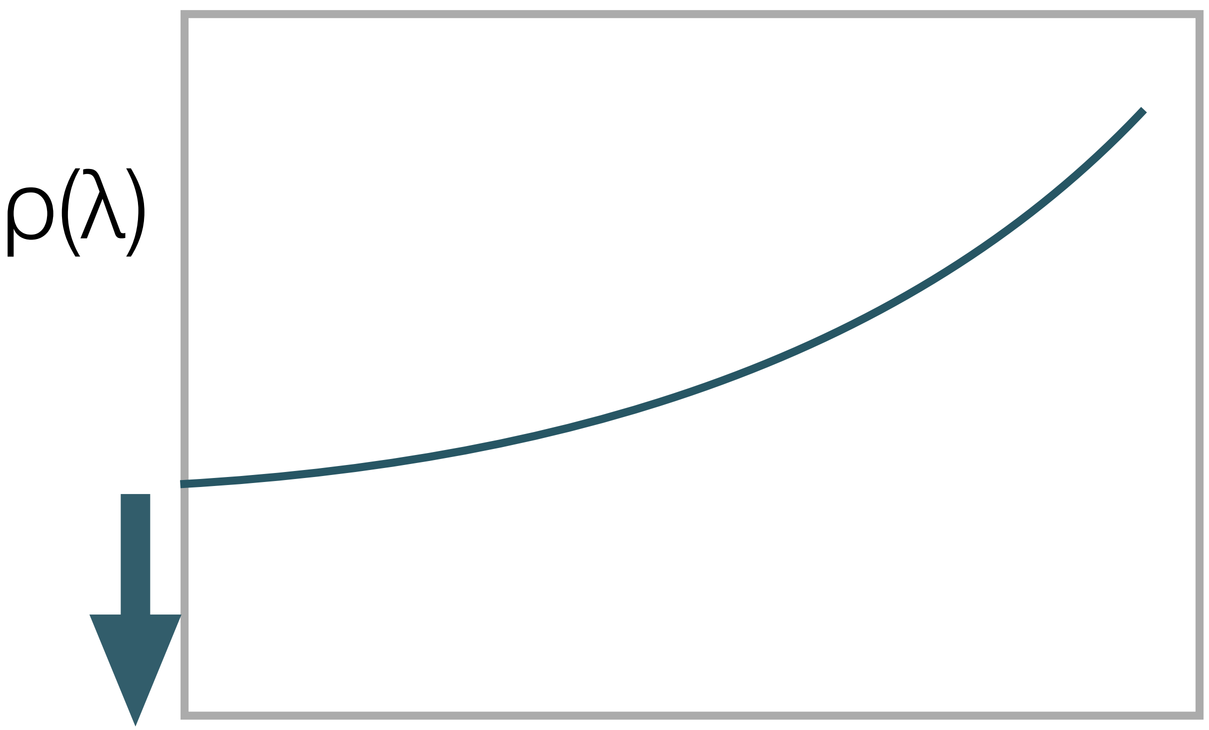
- decreases with reweighting
- $N_{eff}=N_{conf}$ $\langle R\rangle/R_{max}$
- $N_{conf}=1326 \rightarrow N_{eff} = 32$

null measurement of topological excitation after reweighting

- does not readily mean $\chi_t=0$: (this case $\langle Q^2\rangle=4(4)\ \mathrm{x}10^{-6}$)
- there must be a resolution of χ_t under given statistics
  - [resolution of $\langle Q^2\rangle$] = $1/N_{eff}$
- shall take the "statistical error" of $\langle Q^2\rangle = \max(\Delta\langle Q^2\rangle, 1/N_{eff})$

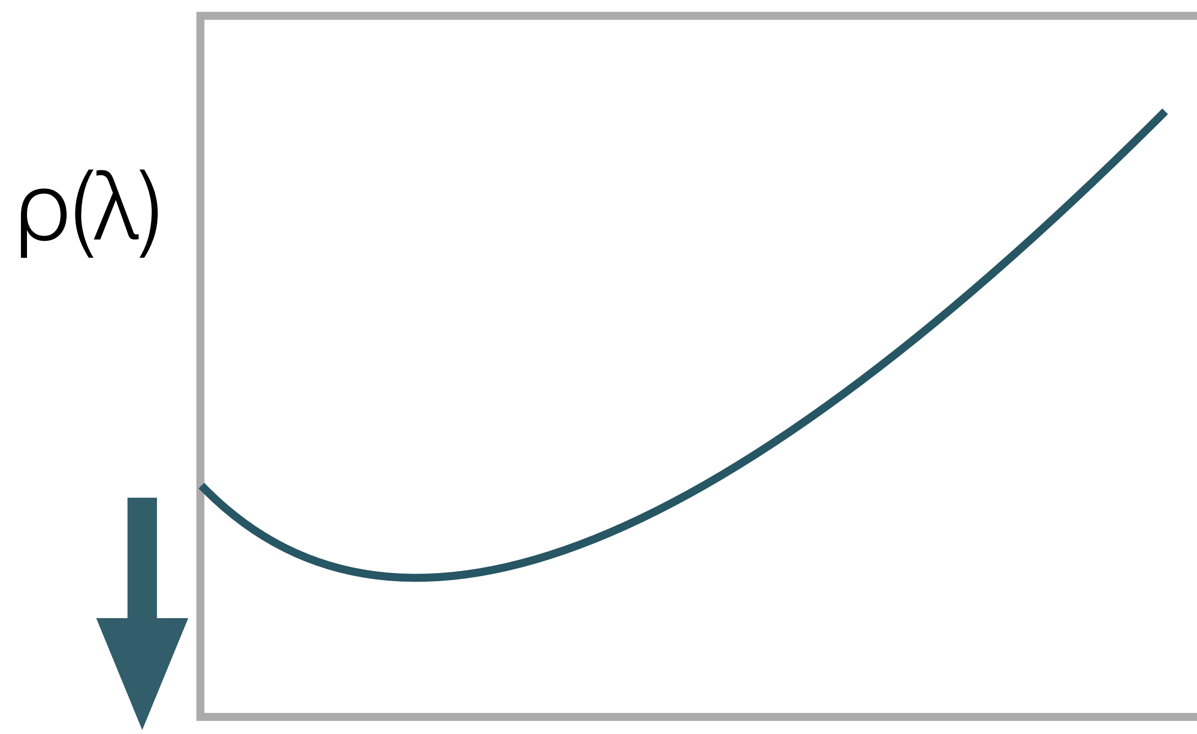
simply speaking,   in the $m \to 0$ limit

- $U(1)_A$ restores if



with $\rho(0) \to 0$ and $\rho'(0) \to 0$

- and not if



with $\rho(0) \to 0$ and $\rho'(0) \neq 0$

- non-analyticity at $\lambda \to 0$ required