

Pushing the HTL theory with effective field theory techniques

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eXtreme QCD
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Outline

- Breakdown of perturbation theory: HTLs and kinetic theory
- The OSEFT and rationale behind
- OSEFT Lagrangian and propagators
- OSEFT at work: one-loop photon polarization tensor (power corrections to the HTL)
- OSEFT at work: chiral kinetic theory

Breakdown of perturbation theory

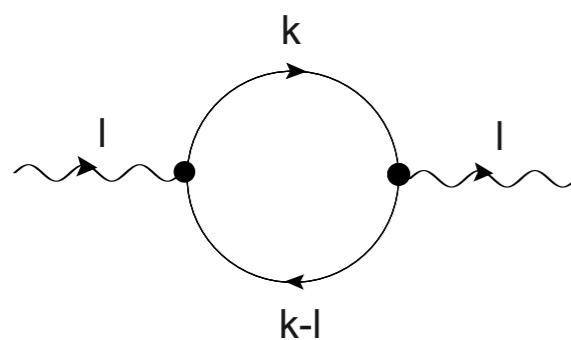
Braaten and Pisarski; Frenkel and Taylor, 90'

At high temperature: two relevant scales

$$g \ll 1$$

$$\left. \begin{array}{l} \text{hard} \sim T \\ \text{soft} \sim gT \end{array} \right\}$$

One-loop thermal corrections **hard thermal loops (HTLs)**
as relevant as the tree amplitudes for **soft momenta**
(and they arise from **hard loop momenta**)



$$\Pi_{\text{HTL}}(l) \sim g^2 T^2$$

$$\frac{\Pi_{\text{HTL}}(l)}{l^2} \sim 1$$

for soft momentum

and have to be **resummed** into effective vertices and propagators

HTLs and transport theory

Blaizot and Iancu, '94
Kelly, Liu, Lucchesi, CM, '94

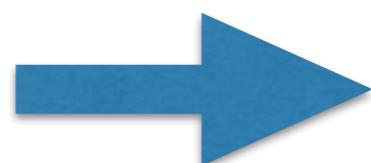
HTLs can be described with simple transport equations

hard scales



on-shell quasiparticles

soft scales

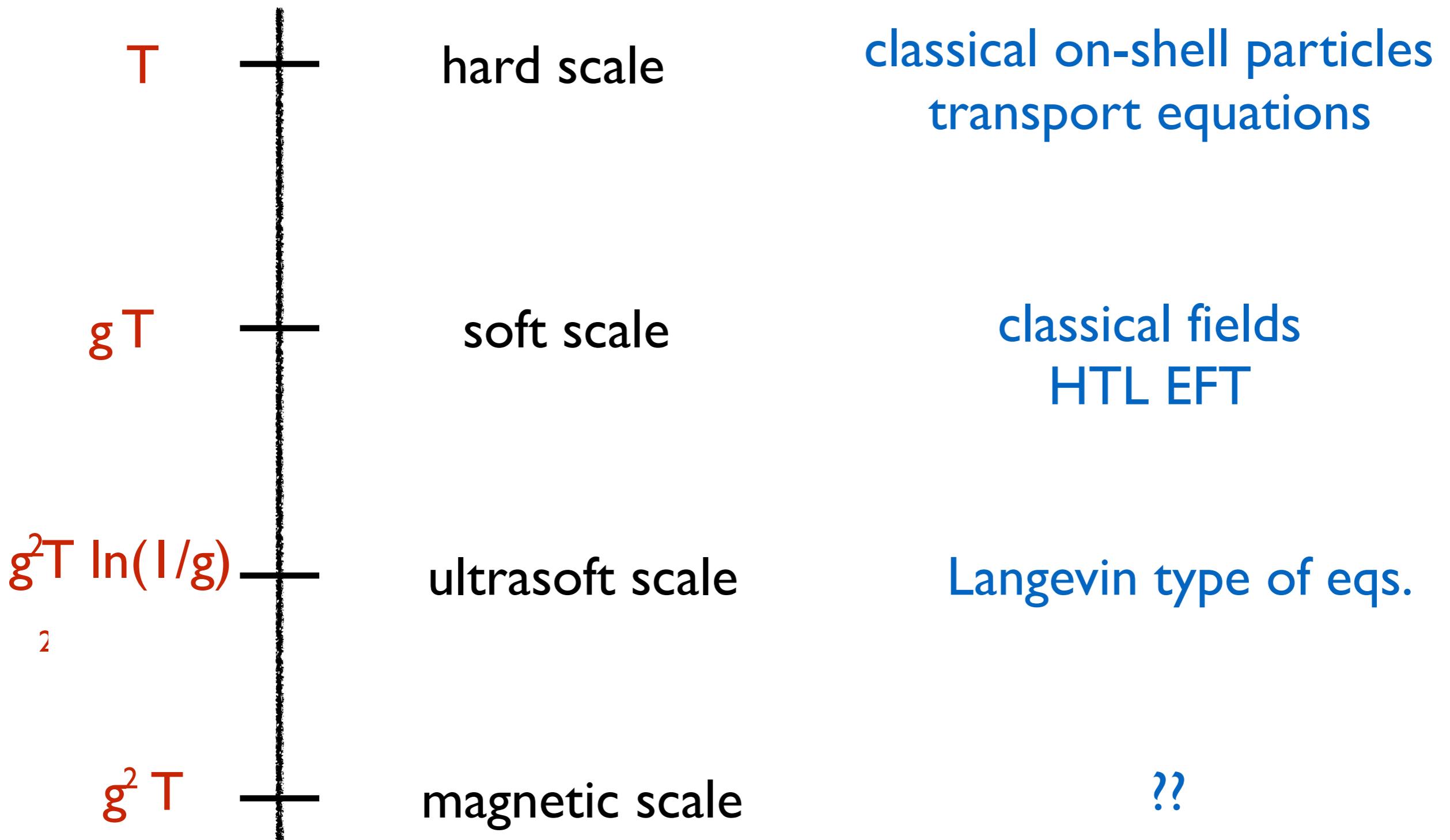


classical fields

$$n_B(p_0) = \frac{1}{e^{P_0/T} - 1} \sim \frac{T}{p_0}$$

Hot QCD plasmas

$$g \ll 1$$



Can we describe with better accuracy the physics described by every perturbative scale in the hot plasma?

ON-SHELL EFFECTIVE FIELD THEORY

- improve the treatment of the hard scales
(and thus also of the soft scales)
- get corrections to transport equations

we are inspired by many successful examples of EFT for QED and QCD: HDET, NRQED/QCD, LEET, SCET, etc,

OSEFT

Physical phenomena dominated by on-shell degrees of freedom

QED, $m=0$ (but it can be generalized)

OS fermion

$$p^\mu = p v^\mu$$

$$v^2 = 0$$

Almost OS fermion

$$q^\mu = p v^\mu + k^\mu$$

$$v^\mu = (1, \mathbf{v})$$

residual momentum $\mathbf{k} \ll \mathbf{p}$

Almost OS antifermion

$$q^\mu = -p \tilde{v}^\mu + k^\mu$$

$$\tilde{v}^\mu = (1, -\mathbf{v})$$

OSEFT Lagrangian

$$\mathcal{L} = \sum_{p,\mathbf{v}} \mathcal{L}_{p,\mathbf{v}}, \quad \mathcal{L}_{p,\mathbf{v}} = \bar{\psi}_{\mathbf{v}} \gamma \cdot iD\psi_{\mathbf{v}}, \quad iD_{\mu} = i\partial_{\mu} + eA_{\mu}$$

$$\psi_{\mathbf{v}} = e^{-ipv \cdot x} \left(P_v \chi_v(x) + P_{\tilde{v}} H_{\tilde{v}}^{(1)}(x) \right) + e^{ip\tilde{v} \cdot x} \left(P_{\tilde{v}} \xi_{\tilde{v}}(x) + P_v H_v^{(2)}(x) \right)$$

with particle/antiparticle projectors

$$P_v = \frac{1}{2} \gamma \cdot v \gamma_0$$

$$P_{\tilde{v}} = \frac{1}{2} \gamma \cdot \tilde{v} \gamma_0.$$

OSEFT Lagrangian

Integrate out the H fields (=solve its classical eqs. of motion)

$$\begin{aligned}\mathcal{L}_{p,v} = & \chi_v^\dagger(x) \left(i v \cdot D + i \cancel{D}_\perp \frac{1}{2p + i \tilde{v} \cdot D} i \cancel{D}_\perp \right) \chi_v(x) \\ & + \xi_{\tilde{v}}^\dagger(x) \left(i \tilde{v} \cdot D + i \cancel{D}_\perp \frac{1}{-2p + i v \cdot D} i \cancel{D}_\perp \right) \xi_{\tilde{v}}(x)\end{aligned}$$

$$P_\perp^{\mu\nu} = g^{\mu\nu} - \frac{1}{2} (v^\mu \tilde{v}^\nu + v^\nu \tilde{v}^\mu) \quad D_\perp^\mu = P_\perp^{\mu\nu} D_\nu$$

Particle/antiparticle fields are totally decoupled,
but there's a symmetry between the particle/antiparticle L

$$v^\mu \Leftrightarrow \tilde{v}^\mu \quad p \Leftrightarrow -p$$

OSEFT Propagators

Real Time Formalism

The momentum p acts as a **chemical potential** for the fermion quantum fluctuations from the lowest order Lagrangian

$$S(k) = P_v \gamma_0 \left[\begin{pmatrix} \frac{1}{v \cdot k + i\epsilon} & 0 \\ 0 & \frac{1}{v \cdot k - i\epsilon} \end{pmatrix} + 2\pi i \delta(v \cdot k) \begin{pmatrix} n_f(p + k_0) & n_f(p + k_0) \\ -1 + n_f(p + k_0) & n_f(p + k_0) \end{pmatrix} \right]$$

This propagator might be also deduced from the full propagator, after expanding for large p $q^\mu = p v^\mu + k^\mu$

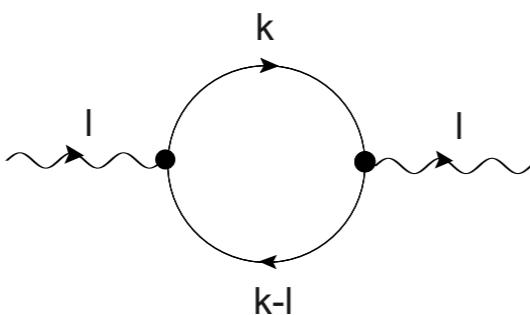


this brings an additional p dependence, not contained in the L , of the propagators

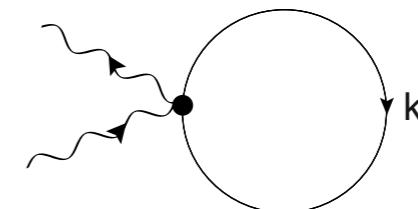
The machinery is (almost) ready for Feynman loop computations!

- Interaction vertices $\sim (\text{momentum})^n/p^m$
- Propagators with p dependence (dispersion rules and “chemical potential”)

Retarded photon polarization tensor



Bubble



Tadpole

Two topologies: the two are needed to respect gauge invariance at every order (Ward Identity)

Tadpoles: they give account of fermion-photon interactions mediated by an off-shell antifermion in QED

Perform the k_0 integral, and re-express the resulting integral in terms of the original variable

$$q^\mu = p v^\mu + k^\mu$$

$$\sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3}$$

$$p = q - k_{\parallel,\mathbf{q}} + \frac{\mathbf{k}_{\perp,\mathbf{q}}^2}{2q} + \mathcal{O}\left(\frac{1}{q^2}\right),$$

$$\mathbf{v} = \hat{\mathbf{q}} - \frac{\mathbf{k}_{\perp,\mathbf{q}}}{q} - \frac{\hat{\mathbf{q}}\mathbf{k}_{\perp,\mathbf{q}}^2 + 2k_{\parallel,\mathbf{q}}\mathbf{k}_{\perp,\mathbf{q}}}{2q^2} + \mathcal{O}\left(\frac{1}{q^3}\right)$$

$$n_f(p) = n_f(q) + \frac{dn_f}{dq} \left(-k_{\parallel}^{\mathbf{q}} + \frac{\mathbf{k}_{\perp,\mathbf{q}}^2}{2q} \right) + \frac{1}{2} \frac{d^2n_f}{dq^2} k_{\parallel,\mathbf{q}}^2 +$$

$n=1$, HTLs are recovered

$$\Pi_{(1)}^{\mu\nu}(l) = 4e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left\{ \frac{dn_f}{dq} \left(\delta^{\mu 0} \delta^{\nu 0} - l_0 \frac{v_{\mathbf{q}}^\mu v_{\mathbf{q}}^\nu}{v_{\mathbf{q}} \cdot l} \right) + \mathcal{O}\left(\frac{1}{q^2}\right) \right\}$$

$$v_{\mathbf{q}}^\mu \equiv (1, \hat{\mathbf{q}})$$

$n=2$, both tadpoles and bubble vanish after angular integration!

$$\begin{aligned} \Pi_{b,(2)}^{\mu\nu}(l) &= e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{q} \frac{dn_f}{dq} \left\{ l_{\parallel,\mathbf{q}} \left(l_{\perp,\mathbf{q}}^\mu v_{\mathbf{q}}^\nu + l_{\perp,\mathbf{q}}^\nu v_{\mathbf{q}}^\mu \right) \frac{1}{v \cdot l} \right. \\ &\quad \left. + v_{\mathbf{q}}^\mu v_{\mathbf{q}}^\nu \left(\frac{l_{\perp,\mathbf{q}}^2 - 2l_{\parallel,\mathbf{q}}^2}{v_{\mathbf{q}} \cdot l} + \frac{l_{\perp,\mathbf{q}}^2 l_{\parallel,\mathbf{q}}}{(v_{\mathbf{q}} \cdot l)^2} \right) + \mathcal{O}\left(\frac{1}{q}\right) \right\} \end{aligned}$$

Non-vanishing though in presence of chiral imbalance!

We have carried out the same computation in QED to the same accuracy to **match** our OSEFT results **and check** the consistency of the approach

The computation in QED requires to expand for large internal loop the integrand of the Feynman diagram:

we recognize the structures seen in the OSEFT computation

counterterms to eliminate UV, and also to reproduce finite local pieces of QED

$$\mathcal{L}_{c.t.} = -\frac{Z(\alpha, \epsilon)C(\alpha, \mu)}{2}F_{0i}F^{0i} - \frac{Z'(\alpha, \epsilon)C'(\alpha, \mu)}{4}F_{ij}F^{ij}$$

$$Z = Z' = Z_{QED} = 1 - \frac{2}{3\epsilon}\frac{\alpha}{\pi}$$

$$C = 1 + \frac{\alpha}{\pi}C^{(1)} \quad , \quad C' = 1 + \frac{\alpha}{\pi}C'^{(1)}$$

$$C^{(1)} = 0$$

$$C'^{(1)} = \frac{2}{3} \left(\ln \frac{\sqrt{\pi}T}{2\mu} - \frac{\gamma}{2} - 1 \right)$$

In the MS scheme, for $\mu = \frac{\sqrt{\pi}}{2} T e^{-1-\gamma/2}$

$$\Pi_{\text{total},(3)}^L(l_0, \mathbf{l}) = \frac{\alpha}{\pi} \left[\mathbf{l}^2 - \frac{1}{3} l_0^2 + \frac{1}{6} \frac{l_0}{|\mathbf{l}|} (l_0^2 - 3\mathbf{l}^2) \left(\ln \left| \frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right| - i\pi \Theta(|\mathbf{l}|^2 - l_0^2) \right) \right],$$

$$\Pi_{\text{total},(3)}^T(l_0, \mathbf{l}) = \frac{\alpha}{\pi} \left[\frac{1}{2} l_0^2 - \frac{2}{3} \mathbf{l}^2 + \frac{1}{6} \frac{l_0^4}{\mathbf{l}^2} - \frac{1}{12} \frac{l_0^3}{|\mathbf{l}|^3} \left(l_0^2 + 2\mathbf{l}^2 - 3 \frac{\mathbf{l}^4}{l_0^2} \right) \left(\ln \left| \frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right| - i\pi \Theta(|\mathbf{l}|^2 - l_0^2) \right) \right]$$

which corrects the HTL result

$$\Pi_{\text{total},(1)}^L(l_0, \mathbf{l}) = m_D^2 \left(\frac{l_0}{2|\mathbf{l}|} \left(\ln \left| \frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right| - i\pi \Theta(|\mathbf{l}|^2 - l_0^2) \right) - 1 \right)$$

$$\Pi_{\text{total},(1)}^T(l_0, \mathbf{l}) = -m_D^2 \frac{l_0^2}{2|\mathbf{l}|^2} \left[1 + \frac{1}{2} \left(\frac{|\mathbf{l}|}{l_0} - \frac{l_0}{|\mathbf{l}|} \right) \left(\ln \left| \frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right| - i\pi \Theta(|\mathbf{l}|^2 - l_0^2) \right) \right]$$

New pieces: perturbative corrections to the soft propagation.

For soft momentum $l \sim eT$

$$\Pi^{(1-loop)} \sim \alpha T^2 + \alpha l^2$$

↑ ↑
HTL new piece!

competes with 2-loops coming from hard scales for soft momenta

$$\Pi^{(2-loops)} \sim \alpha^2 T^2$$

Nothing new: loop expansion \neq perturbative expansion

$$\Pi^{\mu\nu}_{(1)}(L)=2e^2\nu^{3-d}\int\frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q}\left(\frac{v^\mu v^\nu L^2}{(v\cdot L)^2}-\frac{v^\mu L^\nu+v^\nu L^\mu}{v\cdot L}+g^{\mu\nu}\right)$$

$$\Pi^{\mu\nu}_{(3)}(L)=2e^2\nu^{3-d}\int\frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q^3}\frac{L^4}{4(v\cdot L)^2}\left(\frac{v^\mu v^\nu L^2}{(v\cdot L)^2}-\frac{v^\mu L^\nu+v^\nu L^\mu}{v\cdot L}+g^{\mu\nu}\right)$$

Effective Lagrangian

Carignano, CM, Soto, '18

$$\mathcal{L}_{\text{HTL}}^{(1)} = \frac{e^2}{2} \int \frac{d^3 q}{(2\pi)^3} \left\{ \frac{2n_F(q)}{q} \left(F_{\rho\alpha} \frac{v^\alpha v^\beta}{(v \cdot \partial)^2} F_\beta^\rho \right) - \frac{2(n_F(q) + n_B(q))}{q} \left(\bar{\psi} \frac{v \cdot \gamma}{(iv \cdot D)} \psi \right) \right\}$$

In d spatial dimensions

$$v^\mu = q^\mu / |\mathbf{q}|$$

$$\mathcal{L}_{\text{HTL}}^{(3)\gamma} = \frac{e^2 \nu^{3-d}}{4} \int \frac{d^d q}{(2\pi)^d} \frac{1 - 2n_F(q)}{q^3} \left\{ F_{\rho\alpha} \frac{v^\alpha v^\beta}{(v \cdot \partial)^4} \partial^4 F_\beta^\rho \right\}$$

$$\mathcal{L}_{\text{HTL}}^{(3)\psi} = \frac{e^2 \nu^{3-d}}{4} (d-1) \left[\int \frac{d^d q}{(2\pi)^d} \frac{n_F(q) + n_B(q)}{q^3} \left\{ \bar{\psi} D^2 \frac{v \cdot \gamma}{(iv \cdot D)^3} D^2 \psi \right\} \right.$$

$$\left. + \int \frac{d^d q}{(2\pi)^d} \frac{1 + 2n_B(q)}{2q^3} \left\{ \bar{\psi} \left(D^2 (iD \cdot \gamma) \frac{1}{(iv \cdot D)^2} + + \frac{1}{(iv \cdot D)^2} (iD \cdot \gamma) D^2 \right) \psi \right\} \right] + \mathcal{O}(e^3)$$

OSEFT in an arbitrary frame

- Introduce a frame vector

$$p \rightarrow u \cdot p \quad \gamma_0 \rightarrow u \cdot \gamma \quad u^\mu = \frac{v^\mu + \tilde{v}^\mu}{2}$$

$$v^2 = \tilde{v}^2 = 0 , \quad v \cdot \tilde{v} = 2 \quad u \cdot v = 1 , \quad u^2 = 1$$

$$\begin{aligned} \mathcal{L} &= \bar{\chi}_v(x) \left(i v \cdot D + i \not{D}_{\perp} \frac{1}{2E + i \tilde{v} \cdot D} i \not{D}_{\perp} \right) \frac{\not{\psi}}{2} \chi_v(x) \\ &+ \bar{\xi}_{\tilde{v}}(x) \left(i \tilde{v} \cdot D + i \not{D}_{\perp} \frac{1}{-2E + i v \cdot D} i \not{D}_{\perp} \right) \frac{\not{\psi}}{2} \xi_{\tilde{v}}(x) \end{aligned}$$

$$P_v = \frac{1}{2} \not{\psi} \not{\psi} = \frac{1}{4} \not{\psi} \not{\psi} \quad P_{\tilde{v}} = \frac{1}{2} \not{\psi} \not{\psi} = \frac{1}{4} \not{\psi} \not{\psi}$$

Sum over velocity directions of SCET L!

Reparametrization Invariance

same as in SCET

Manohar et al, 2002

Apparent breaking of Lorentz Invariance

$$\{v_\mu M^{\mu\nu}, \tilde{v}_\mu M^{\mu\nu}\}$$

$$\psi_{v,\tilde{v}}(x) = \psi'_{v',\tilde{v}'}(x)$$

type I

$$\begin{cases} v^\mu \rightarrow v^\mu + \lambda_\perp^\mu \\ \tilde{v}^\mu \rightarrow \tilde{v}^\mu \end{cases}$$

type II

$$\begin{cases} v^\mu \rightarrow v^\mu \\ \tilde{v}^\mu \rightarrow \tilde{v}^\mu + \epsilon_\perp^\mu \end{cases}$$

type III

$$\begin{cases} v^\mu \rightarrow (1 + \alpha)v^\mu \\ \tilde{v}^\mu \rightarrow (1 - \alpha)\tilde{v}^\mu \end{cases}$$

$$v \cdot \lambda_\perp = v \cdot \epsilon_\perp = \tilde{v} \cdot \lambda_\perp = \tilde{v} \cdot \epsilon_\perp = 0.$$

On-shell and residual parts of the momenta change

$$\delta_{(\text{I})}\mathcal{L}_{p,v} = \delta_{(\text{II})}\mathcal{L}_{p,v} = \delta_{(\text{III})}\mathcal{L}_{p,v} = 0$$

OSEFT used to derive CKT

Derive a transport equation in the EFT

$$S_{E,v}(x, y) = -\langle \bar{\chi}_v(y) \chi_v(x) \rangle$$

$$S = \sum_{\chi=\pm} P_\chi J_\chi^\rho \gamma_\rho , \quad P_\chi = \frac{1 + \chi \gamma_5}{2}$$

$$J_\chi^\rho = v^\rho G_\chi$$

$$G_{E,v}(x, y) = \langle \bar{\chi}_v(y) \frac{\tilde{\psi}}{2} \chi_v(x) \rangle$$

Wigner transform (gauge cov. modified)

$$X = \frac{1}{2}(x + y) , \quad s = x - y$$

$$\bar{G}_v(X, k) = \int d^4s e^{ik \cdot s} U\left(X, X + \frac{s}{2}\right) G_v\left(X + \frac{s}{2}, X - \frac{s}{2}\right) U\left(X - \frac{s}{2}, X\right)$$

- Deduce both on-shell conditions and dynamical eqs at every order in the I/E expansion
- Perform the Wigner transformation and a gradient expansion $\partial_X \ll \partial_s$
- Project over chiralities
- Return to the original variables

$$G_{E,v}^\chi(X, k) = a\delta_+(K^\chi) f_{E,v}^\chi(X, k)$$

Collisionless chiral transport equation

$$\left(v_\mu^q \left(1 - \frac{e}{2E_q^2} S_\chi^{\alpha\beta} F_{\alpha\beta} \right) + u^\mu \frac{e}{2E_q^2} S_\chi^{\alpha\beta} F_{\alpha\beta} - \frac{e}{2E_q^2} S_\chi^{\mu\nu} F_{\nu\rho} (2u^\rho - v_q^\rho) \right) \Delta_\mu f(X, q) \delta_+(Q) = 0$$

$$E_q = u \cdot q$$

Only corrections up to order n=2

$$v_q^\mu = \frac{q^\mu}{E_q} \quad \delta((q^2 - eS_\chi^{\mu\nu} F_{\mu\nu})) \theta(E_q) \equiv \delta_+(Q)$$

$$S_\chi^{\mu\nu} = \chi \frac{\epsilon^{\alpha\beta\mu\nu} u_\beta q_\alpha}{2(q \cdot u)}$$

$$\Delta^\mu \equiv \partial_X^\mu - eF^{\mu\nu}(X)\partial_{q,\nu}$$

Side jumps derived from RI

$$v^\mu \rightarrow v^\mu$$

$$\tilde{v}^\mu \rightarrow \tilde{v}^\mu + \epsilon_\perp^\mu$$

$$E \rightarrow E + \frac{1}{2}(\epsilon_\perp \cdot p)$$

$$\chi_v(x) \rightarrow \left(1 + \frac{1}{2}\not{\epsilon}_\perp \frac{1}{2E + i\tilde{v} \cdot D} i\not{D}_\perp\right) \chi_v(x)$$

from type II transformations

$$(f^\chi(X, q))' \rightarrow f^\chi(X, q) - \frac{1}{E_q} S_\chi^{\mu\nu} \epsilon_\nu^\perp \Delta_\mu f^\chi(X, q) + \mathcal{O}(\epsilon_\perp^2, \frac{1}{E_q^2})$$

Outlook

- OSEFT: tool to treat systematically the hard degrees of freedom in hot plasmas - with all the advantages of the EFTs
- OSEFT also helpful for transport theory