Spectral function for over-occupied gluo-dynamics from real-time lattice simulations

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Kirill Boguslavski

European Research Council
Established by the European Commission
Supporting top researchers from anywhere in the world

Project: CGCglasmaQGP

Talk based on:

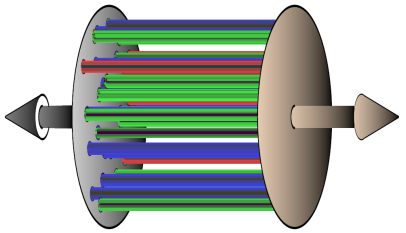
KB, Aleksi Kurkela, Tuomas Lappi and Jarkko Peuron,
arXiv:1804.01966

Table of Contents

Motivation

Strong Yang-Mills fields (for $g \ll 1$)*

In heavy-ion collisions:
formation and dynamics of Quark-Gluon Plasma



Glasma
From: Gelis et al (2010),

i. Initial stage: *Glasma* with $\langle A_\mu A_\mu \rangle \sim 1/g^2$ at $p \sim Q_s$
ii. Still *far from equilibrium*: distribution $f \sim 1/g^2$ for $p \lesssim Q_s$
iii. In *thermal* equilibrium: $f \gg 1$ for soft modes $p \ll T$

Real-time, highly occupied situations at weak coupling
⇒ Classical approximation for field dynamics

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)

Controlled way to study initial stages in detail?
Spectral properties?

* $Q_s \gg \Lambda_{\mathrm{QCD}}$ assumed

Motivation

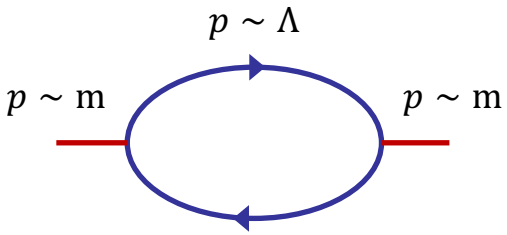
Hard (Thermal) Loop formalism (HTL)

Diagrammatically:

- Soft modes $p \sim m$, hard loops $p \sim \Lambda \gg m$
- Scale separation assumed, expand in $m/\Lambda \ll 1$
- Details of $f(t, p)$ "hidden" in few parameters (m, T_*, …)

Braaten, Pisarski (1990); Blaizot, Iancu (2002)

Diagrammatic example



Lattice simulations of HTL in real time: (sphalerons, instabilities, transport, …)

- Additionally explicit separation between soft gauge fields ($m \ll 1/a_s$) and hard particles ($\Lambda \gg 1/a_s$), (e.g., coupled via Vlasov-Wong equations)

Hu, Muller, Moore (1997, 1998); Bodeker, Moore, Rummukainen (2000); Dumitru, Nara, Schenke, Strickland, Greiner (2008, 2009); Attems, Rebhan, Strickland (2008, 2013)

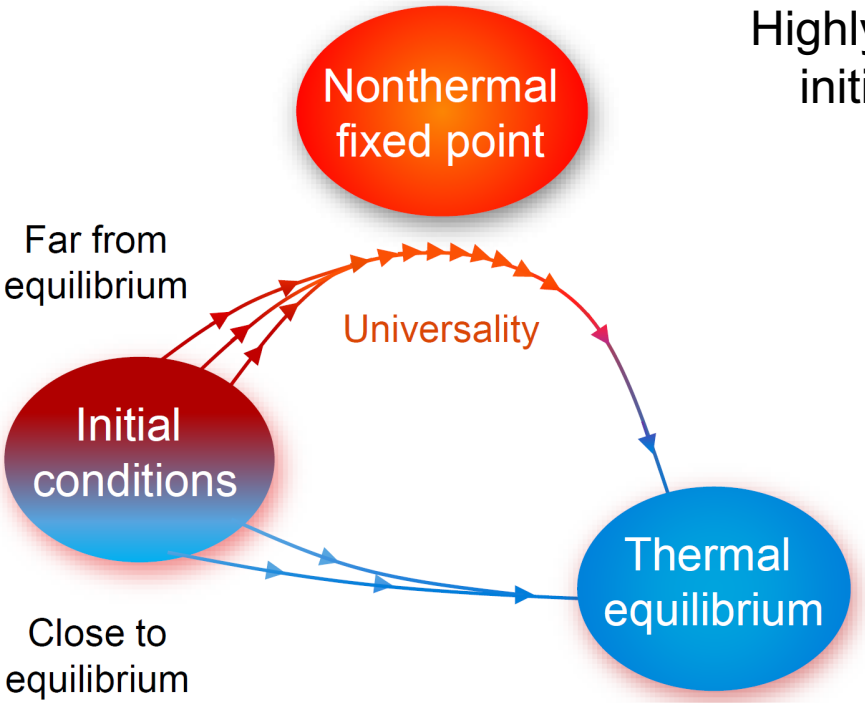
Idea here: all scales on *same lattice*, $m/\Lambda \sim 1$ also possible!

Motivation

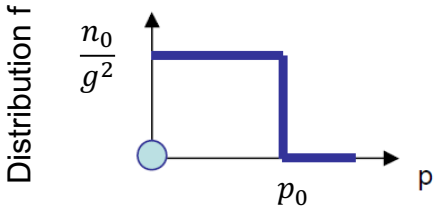
Objectives of this work

- Study *spectral* and *statistical* functions $\rho(t, \omega, p)$ and $F(t, \omega, p)$
- Develop *non-perturbative approach* for strong fields
  (not based on $m/\Lambda \ll 1$, an explicit separation of hard and soft modes or analytical continuation)
- *Quantify* to what extent HTL at LO is a good approximation of soft modes for systems with $m/\Lambda \ll 1$
- Extract quantities *exceeding LO HTL* that are hard to compute diagrammatically. Application: damping rates $\gamma_{T/L}(p)$.
- A *way of obtaining* $\rho(t, \omega, p)$ for nonthermal and thermal systems?

A suitable system



Highly occupied initial state:



$$f(t_0, p) \sim \frac{n_0}{g^2}\,\Theta(p_0 - p)$$

Nonthermal fixed point (NTFP)

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

(Single-particle) Distribution function: $f(t,p) = t^{\alpha}\, f_S\left(t^{\beta} p\right)$

NTFP: Micha, Tkachev, *PRD* 70, 043538 (2004); Berges, Rothkopf, Schmidt, *PRL 101, 041603 (2008)*

Universality: Berges, KB, Schlichting, Venugopalan, *PRL 114, 061601 (2015)*

A suitable system

Self-similar attractor

Far-from-equilibrium, isotropic, overoccupied $f \sim 1/g^2$:

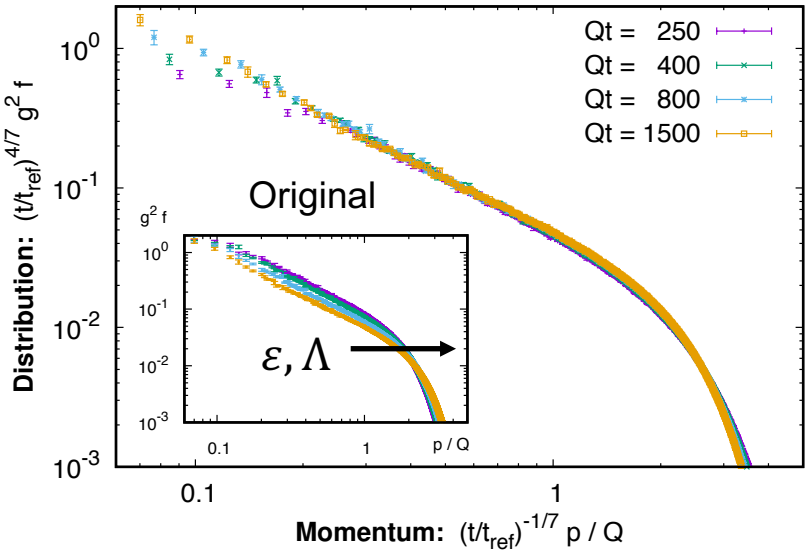
- Undergoes a *cascade of energy* ε to the UV in a *self-similar* regime

$$f(t,p) = t^{\alpha}\, f_S\left(t^{\beta} p\right)$$

- *Universal* scaling exponents

$$\alpha = -4/7 \text{ and } \beta = -1/7$$

- Hard scale $\Lambda \sim Q(Qt)^{-\beta}$ grows



Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014)

Q: constant scale from energy density $g^2\varepsilon \sim Q^4$

$$Qt_{\mathrm{ref}} = 1500$$

A suitable system

Scale separation increases

- Our choice for initial conditions (IC):

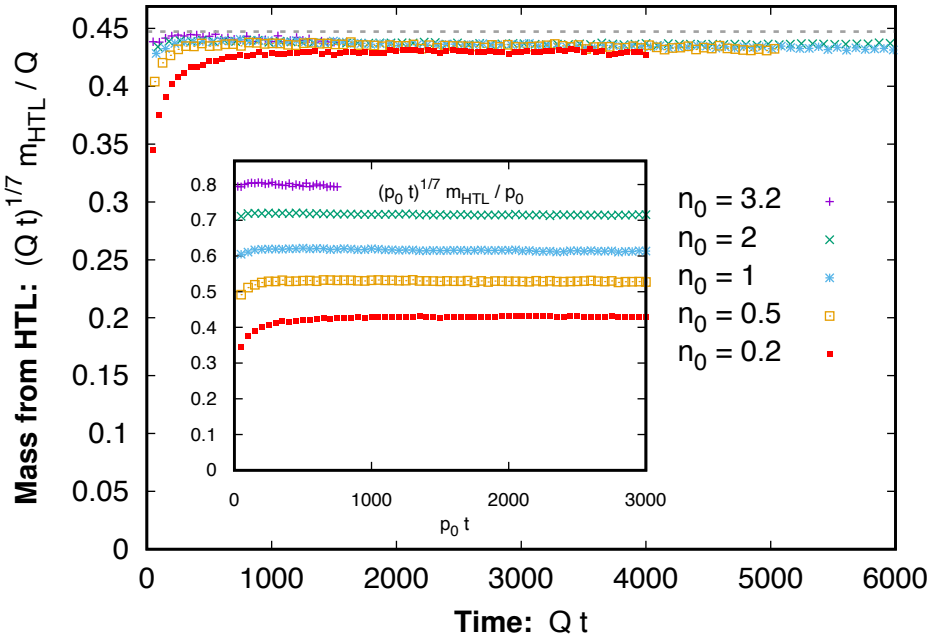
$$f(t=0,p) = \frac{n_0}{g^2}\,\frac{p_0}{p}\,e^{-p^2/2p_0^2}$$

- Evolution becomes *independent* of n_0 (insensitive to IC)
- Depends only on energy density ($\sim Q^4$)
- *Scale separation grows* with time as

$$m/\Lambda \sim (Qt)^{-2/7}$$

⇒ HTL should be applicable

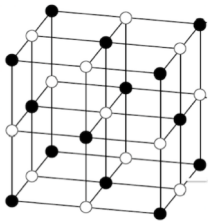
Mass computation (soft scale m)



Definition: $m_{\mathrm{HTL}}^2 \approx 2N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3}\,\frac{g^2\,f(t,p)}{p}$

Computational method

Classical-statistical lattice simulations



- $SU(N_c)$ *gauge theory* with $N_c = 2$ in temporal $A_0 = 0$ gauge
- Large occupancies $f(p \sim \Lambda) \gg 1 \;\Rightarrow$ dynamics approx. by *classical EOM* in link and chromo-electric fields U_i, E_i on 3D spatial lattice
- Initialization:

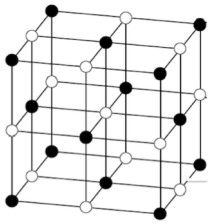
$$\langle |A(t=0,\boldsymbol{p})|^2 \rangle \sim \frac{f(t=0,p)}{p}, \qquad \langle |E(t=0,\boldsymbol{p})|^2 \rangle \sim p\, f(t=0,p)$$

then $A_i(0,\boldsymbol{p}) \to U_i(0,\boldsymbol{x})$, $E_i(0,\boldsymbol{p}) \to E_i(0,\boldsymbol{x})$ and restore Gauss law

Remark: Gauge-invariant initial conditions possible by starting with (classical) thermal system, then gauge cooling until $\Lambda \ll 1/a_s$ (a_s: lattice spacing)

Computational method

Computation of retarded propagator G_R



Linear response theory:

- Perturb system at $t = t_{\mathrm{pert}}$ with a source $j_a^k(t, \boldsymbol{p}) = j_{0,a}^k(\boldsymbol{p})\, \delta(t - t_{\mathrm{pert}})$
- Linear reponse $\boxed{\langle a_j(t, \boldsymbol{p})\rangle = \int dt' G_{R,jk}(t, t', \boldsymbol{p})\, j^k(t', \boldsymbol{p})}$

Our numerical approach:

- Split gauge field $A_j(t, \boldsymbol{x}) \mapsto A_j(t, \boldsymbol{x}) + a_j(t, \boldsymbol{x})$, solve *newly developed equations* for $a_j(t, \boldsymbol{x})$ (also: restore Gauss law)

Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*

- Perturb system at $t = t_{\mathrm{pert}}$ in Coulomb gauge $\partial_j A_j = 0$ with a source

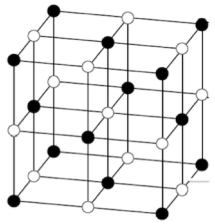
$$\langle j_{0,a}^k(\boldsymbol{p})\, (j_{0,b}^l(\boldsymbol{q}))^*\rangle_j = \delta_{a,b}\; V \delta_{\boldsymbol{p},\boldsymbol{q}}\, v_k(\boldsymbol{p})\, v_l^*(\boldsymbol{q})$$

Polarization
Transverse: $\boldsymbol{v} \perp \boldsymbol{p}$,
Longitudinal: $\boldsymbol{v} = \boldsymbol{p}$

- Obtain $\boxed{G_{R,kl}(t, t_{\mathrm{pert}}, \boldsymbol{p}) = \frac{1}{(N^2-1)V} \langle \langle a_k^b(t, \boldsymbol{p})\rangle\, (j_{0,l}^b(\boldsymbol{p}))^*\rangle_j}$

Computational method

Computation of spectral function ρ



Finally the spectral function:

- Quantum-mechanical definition: $\rho_{jk}\left(t, t_{\mathrm{pert}}, \boldsymbol{x}-\boldsymbol{x}'\right) = i\langle[A_j^b(t,\boldsymbol{x}), A_k^b(t_{\mathrm{pert}},\boldsymbol{x}')]\rangle$
- Relation (used here): $G_{R,jk} = \theta\left(t - t_{\mathrm{pert}}\right)\rho_{jk}$
- Use *isotropy*, *homogeneity*, etc. to improve the statistical uncertainty
- Transverse polarization: $\rho_T(\boldsymbol{p}) = v_j^*(\boldsymbol{p})\rho^{jk}(\boldsymbol{p})v_k(\boldsymbol{p})$ with $\boldsymbol{v} \perp \boldsymbol{p}$
- Longitudinal polarization: $\rho_L(\boldsymbol{p}) = p_j\,\rho^{jk}(\boldsymbol{p})\,p_k\,/\,p^2$

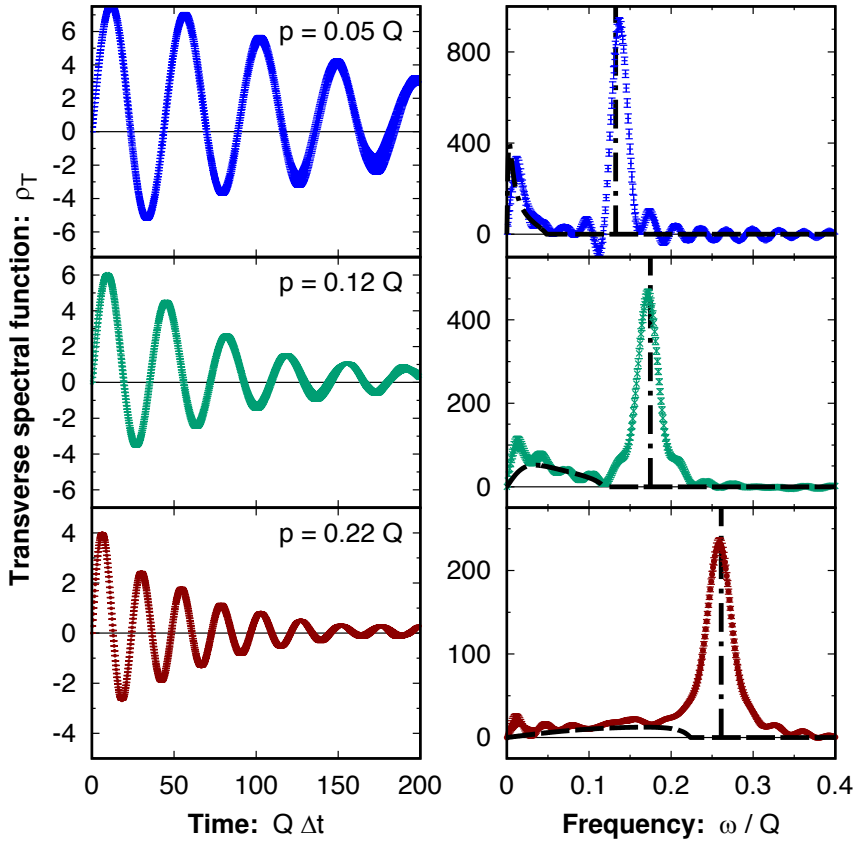
Extracted spectral function vs. HTL predictions

Transverse spectral function ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t_{\mathrm{pert}} \gg \Delta t$

- black dashed lines: HTL at LO
- for $|\omega| \leq p$: *Landau cut*
- damped oscillations, Lorentz peaks: *existence of quasi-particles* with dispersion $\omega_T(p)$ and damping rate $\gamma_T(p)$.

Similarly for ρ_L (in backup)



Good agreement with HTL !

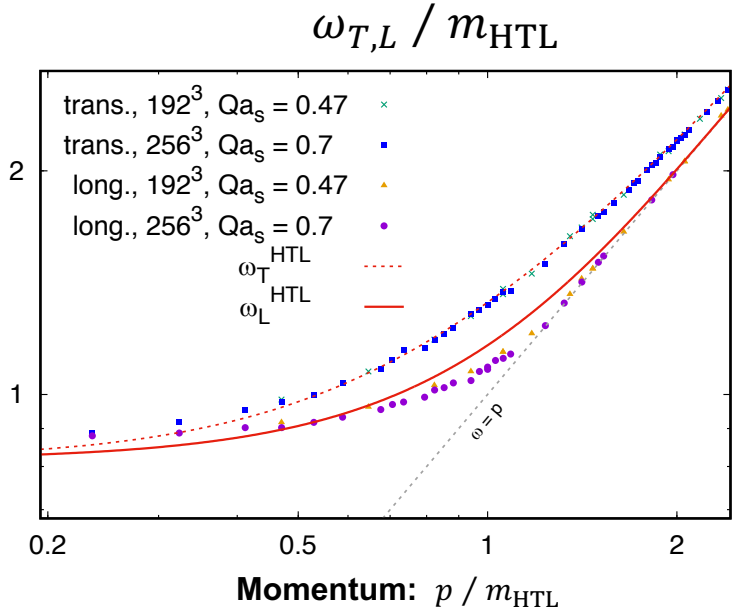
Extracted spectral function vs. HTL predictions

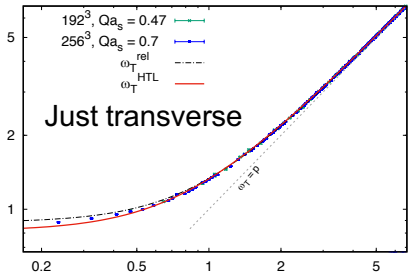
Extracted dispersion relations $\omega_{T,L}(p)$

- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- For plasmon frequency $\omega_{\mathrm{pl}} = \omega(p=0)$, asymptotic mass m_∞ (gap at $p \to \infty$), we get

$$m_\infty/\omega_{\mathrm{pl}} = 0.96 \pm 0.03$$

- *Different from HTL* at LO: $\sqrt{2/3} \approx 0.82$
- Same dispersion for *different* t_{pert} , n_0 (→ Backup)





Remark: $\omega_T(p)$ also compatible with $\omega_T^{\mathrm{rel}} = \sqrt{m_\infty^2 + p^2}$

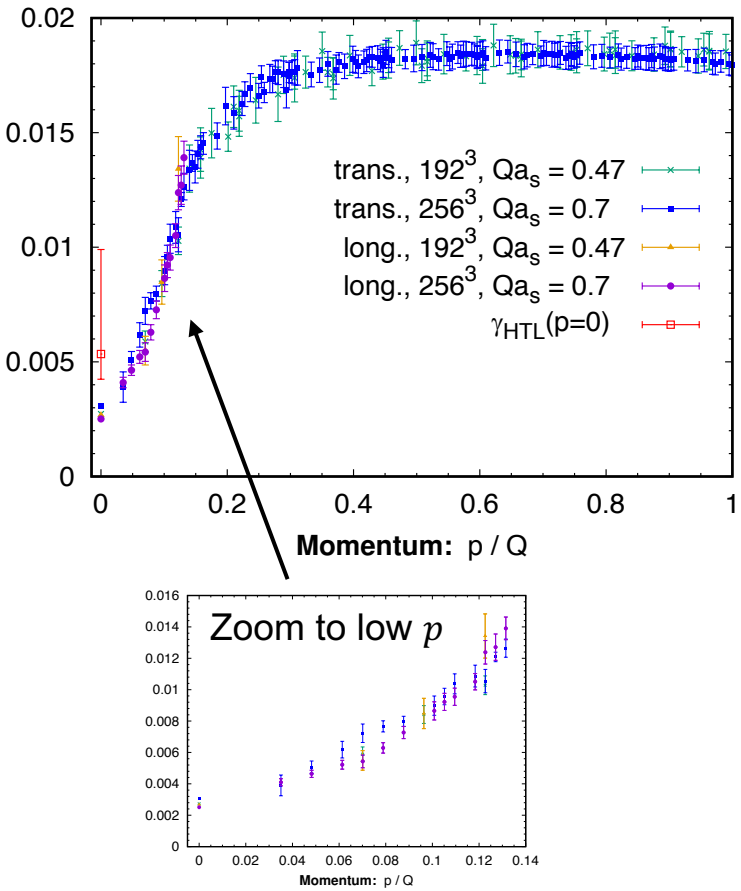
Extracted spectral function vs. HTL predictions

Extracted damping rates $\gamma_{T,L}(p)$

- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$ Braaten, Pisarski, *PRD 42, 2156 (1990)*
- "Isotropic" $\boxed{\gamma_T \approx \gamma_L}$ for $p \lesssim m$

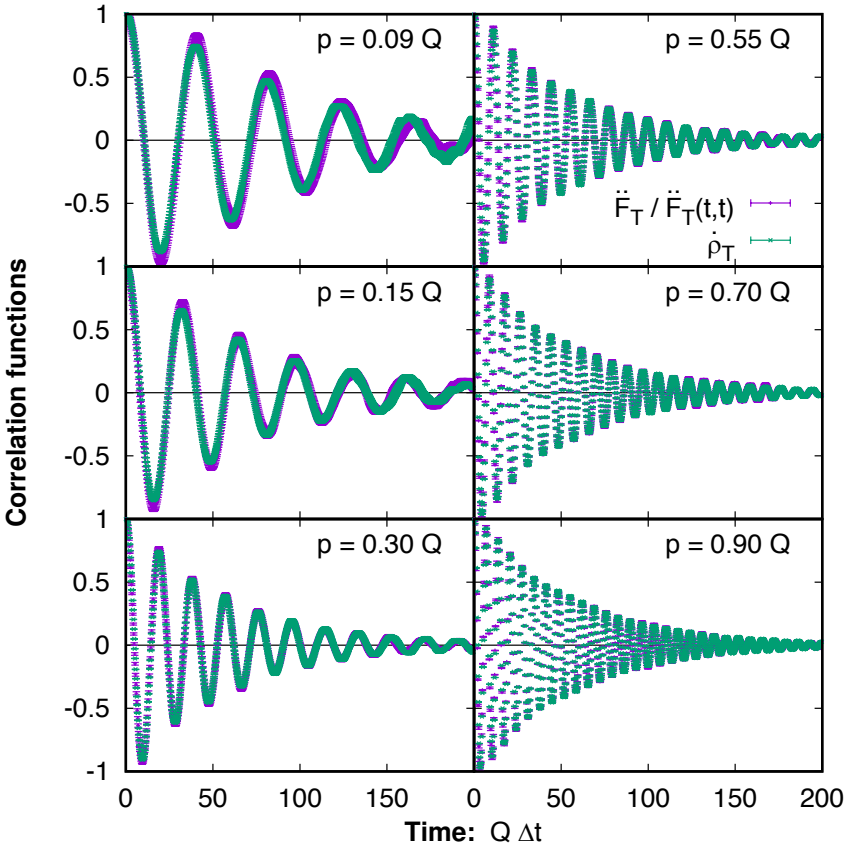
- $\gamma_{T,L} \,/\, \gamma_{\mathrm{HTL}}(p=0)$ as function of p/m_{HTL} is *not sensitive to* t_{pert} (→ Backup)

$\gamma_{T,L}(p) \,/\, Q$



Extracted spectral function vs. HTL predictions

Statistical correlation function F



Remarks: $\dot{\rho}_T = \partial_t \rho_T$, $\dot{\rho}_T(t, \Delta t = 0, p) = 1$

(Classical) definition:

$$\ddot{F}^{jk}(t, \Delta t, \boldsymbol{p}) = \langle E^j(t, \boldsymbol{p})\, E^{*,k}(t', \boldsymbol{p}) \rangle$$

(As always: $\Delta t = t - t'$, Fourier transform to ω)

(Thermal) fluctuation-dissipation relation:

$$\ddot{F}_T(t, \omega, p) \,/\, T = \dot{\rho}_T(t, \omega, p)$$

Used to estimate thermal spectral function

Example: G. Aarts, *PLB 518, 315 (2001)*

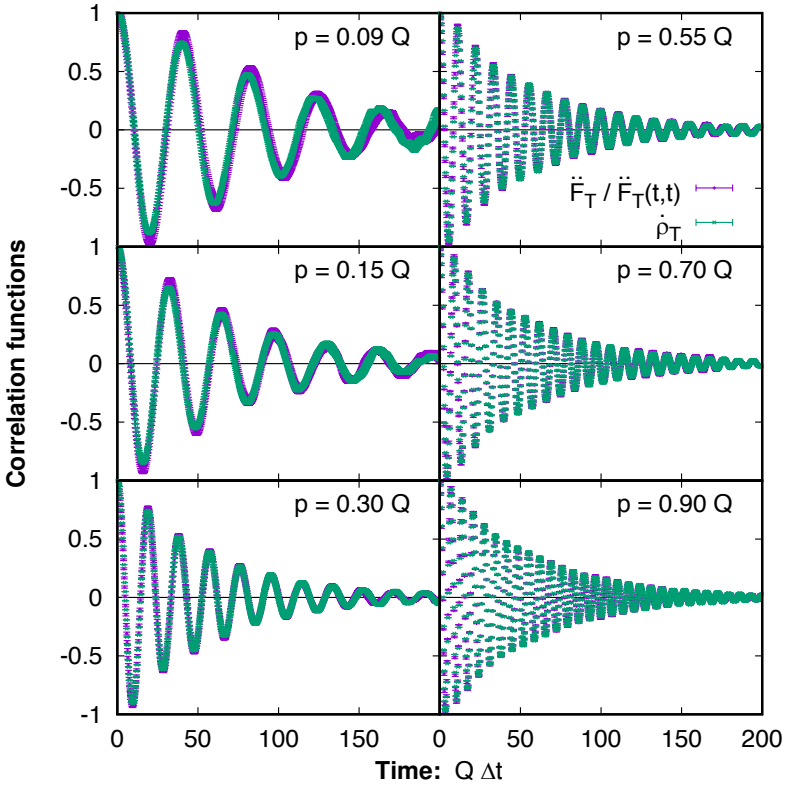
We *extract $\ddot{F}$, $\dot{\rho}$ independently*, we observe a generalized fluctuation-dissipation relation:

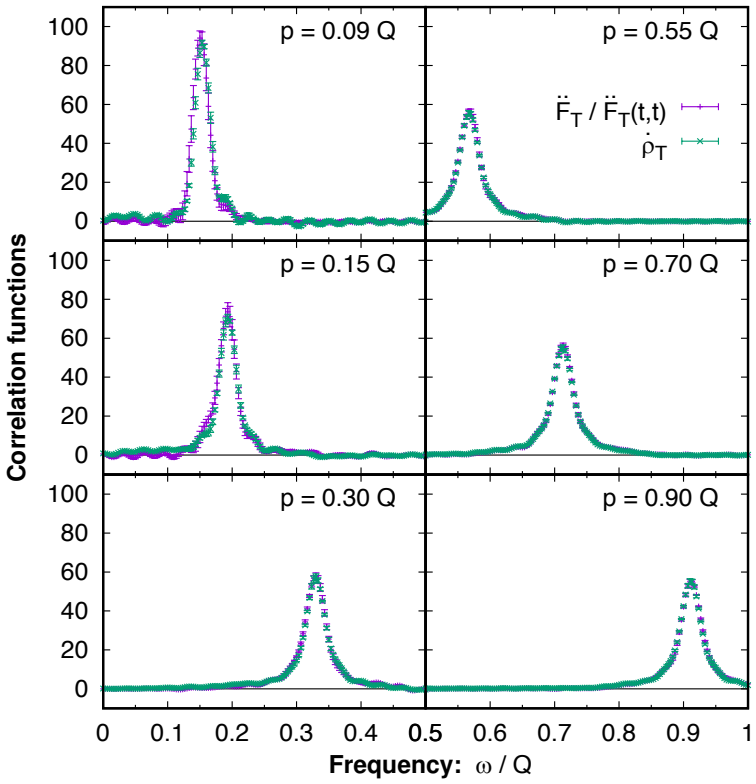
$$\frac{\ddot{F}_T(t, \Delta t, p)}{\ddot{F}_T(t, \Delta t = 0, p)} = \dot{\rho}_T(t, \Delta t, p)$$

Extracted spectral function vs. HTL predictions

Observation: $\frac{\ddot{F}_T(t,\Delta t,p)}{\dot{\rho}_T(t,\Delta t,p)} = \frac{\ddot{F}_T(t,\omega,p)}{\dot{\rho}_T(t,\omega,p)} = \ddot{F}_T(t,\Delta t=0,p)$

$\ddot{F}_T$ and $\dot{\rho}_T$ have same functional form in Δt or ω!





<u>Remark</u>: Similarly for longitudinal but with larger statistical error for $\ddot{F}_L$ in our simulations

Extracted spectral function vs. HTL predictions

In detail: deviations from HTL

- *HTL expectation* for $\mathrm{p} \ll \Lambda$ in gray, e.g.

$$\ddot{F}_T(t,\omega,p) \,/\, T_* = \dot{\rho}_T(t,\omega,p)$$

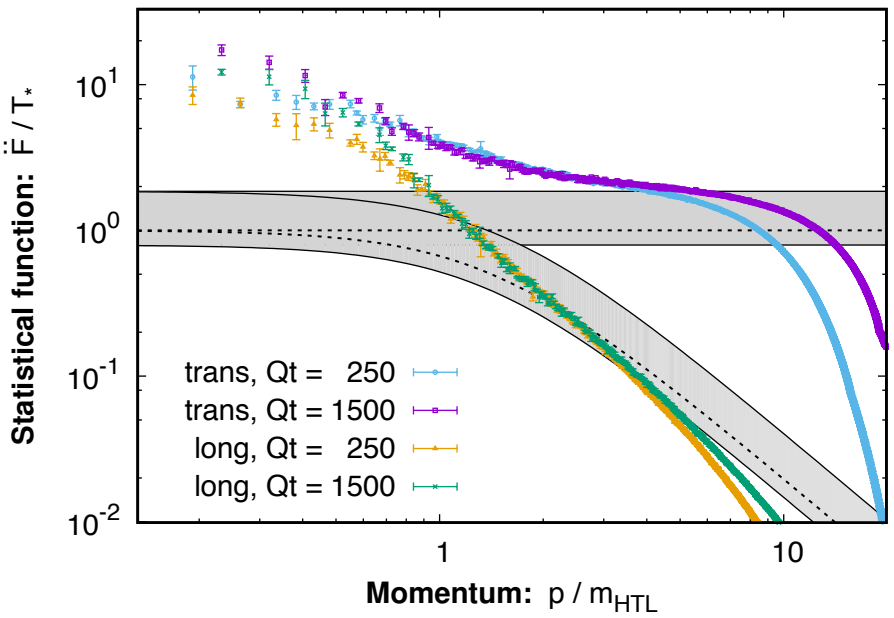
$$\Rightarrow \quad \ddot{F}_T(t,\Delta t = 0,p) = T_*$$

Arnold, Moore, Yaffe, *JHEP 01, 030 (2001)*

- With effective temperature

$$g^2 T_* \approx \frac{\int \mathrm{d}^3p\ (g^2 f)^2}{2\int \mathrm{d}^3p\ (g^2 f)/p} \sim \gamma_{\mathrm{HTL}}(p=0)$$

Statistical function: $\ddot{F}_{T,L}(t,\Delta t = 0,p) \,/\, T_*$



Deviations:

- p-dependence even for $\mathrm{p} \ll \Lambda$
- *Enhancement* of $\ddot{F}_{T,L}$ for $p \lesssim m$ visible

$\Rightarrow$ *Scale separation* not good enough? Effect of *magnetic scale*?

Conclusion

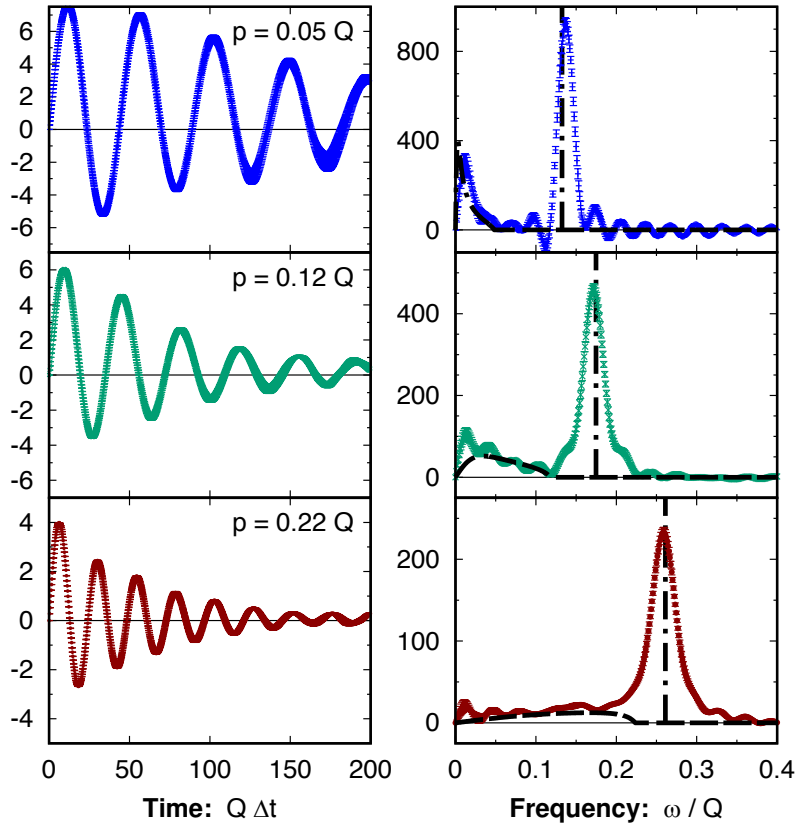
a. We *developed a non-perturbative numerical approach* based on classical lattice simulations, linearized equations and linear response theory to extract $\rho_{T,L}$ in over-occupied gauge systems (no $m/\Lambda \ll 1$ needed)

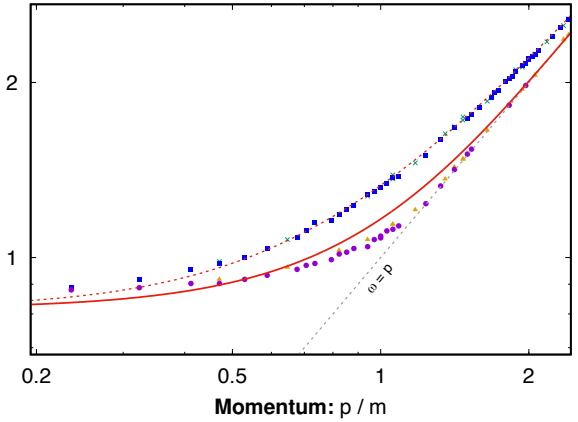
b. HTL at LO *can describe* many observables but some *deviations* also found; effects beyond: *computation of damping rates* $\gamma_{T,L}(p)$

c. Suggestion: Results relevant also *for thermal state*

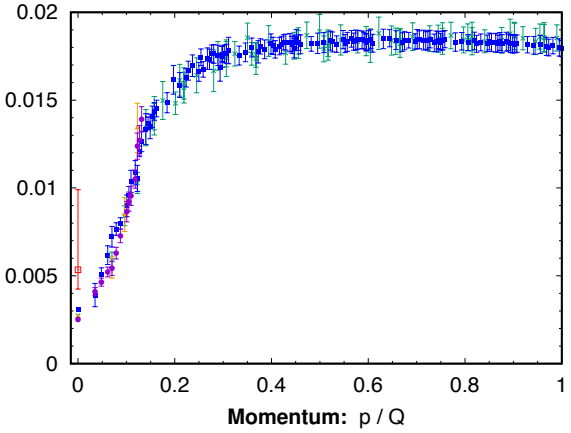
Outlook: We aim to use these techniques to study for instance:

- *Transport* coefficients, jet quenching?
- *Anisotropy*, plasma instabilities
- *Expanding* Glasma for initial stages in heavy-ion collisions





Thank you for your attention!



BACKUP SLIDES

Introduction

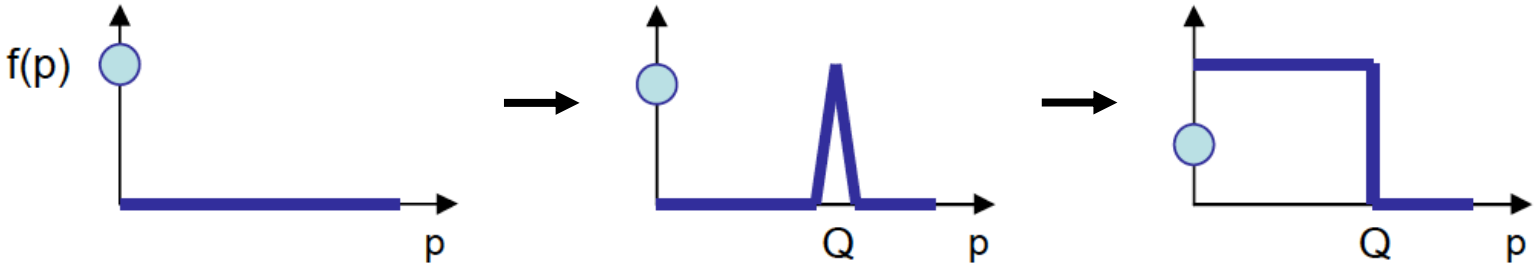
Highly occupied systems

Example: cosmological reheating after inflation ($\lambda\phi^4$ model)

Large initial condensate $\phi_0 \sim 1/\sqrt{\lambda}$
+ vacuum fluctuations

Instabilities
(e.g. parametric resonance)

Highly occupied system
created with $f(p \leq Q) \sim 1/\lambda$



(Single-particle) Distribution function: $f(p) = \frac{\mathrm{d}^3 N}{\mathrm{d}^3 p}$

Similarly, highly occupied systems found in heavy-ion collisions.

Nonthermal fixed points

Scaling region (of a nonthermal fixed point)

Self-similar evolution of distribution function f

$$f(t,p) = t^{\alpha}\, f_S\left(t^{\beta} p\right)$$

with scaling behavior of typical scales $p_{\mathrm{typ}} \sim t^{-\beta}$, $f(p_{\mathrm{typ}}) \sim t^{\alpha}$

Classification: universality classes far from equilibrium

Via scaling exponents α, β and the scaling function $f_s(x)$

NTFP	Close to 2nd order PT
Time scale t	Inverse reduced temp. T_c / (T-T_c)
Self-similar evolution	Critical slowing down, power laws
Scaling exponents & function	Critical exponents & surface

Computational method

What is gauge invariant?

- *Equations of motion* of background (BG) field and linearized fluctuations
- *Initial conditions for the BG* can, in principle, be set gauge-invariantly (starting with classical thermal system $T \gg 1/a_s$, gauge cool until $\Lambda \ll 1/a_s$)
- *Correlation function* $\langle \langle a_k^b(t,\boldsymbol{x})\rangle\, U_0(t,t_{\rm pert},\boldsymbol{x})\, j_{0,l}^b(\boldsymbol{x})\rangle_j$, where $U_0(t,t_{\rm pert},\boldsymbol{x})$ is Wilson line, i.e., product of U_0 links. In our framework:
  - $A_0 = 0 \;\Rightarrow\; U_0 = 0$
  - Use source only for one momentum $\boldsymbol{p}$

  $\Rightarrow$ $\boxed{\langle \langle a_k^b(t,\boldsymbol{x})\rangle\, U_0(t,t_{\rm pert},\boldsymbol{x})\, j_{0,l}^b(\boldsymbol{x})\rangle_j \sim\; G_{R,kl}\big(t,t_{\rm pert},\boldsymbol{p}\big)}$, a *gauge-inv. observable*!

  Remark: But it corresponds to G_R only in temporal gauge
- *Gauge-dependent*: The choice of the source $j_{0,l}^b(\boldsymbol{x})$

Extracted spectral function vs. HTL predictions

Lorentz peak

Formula:

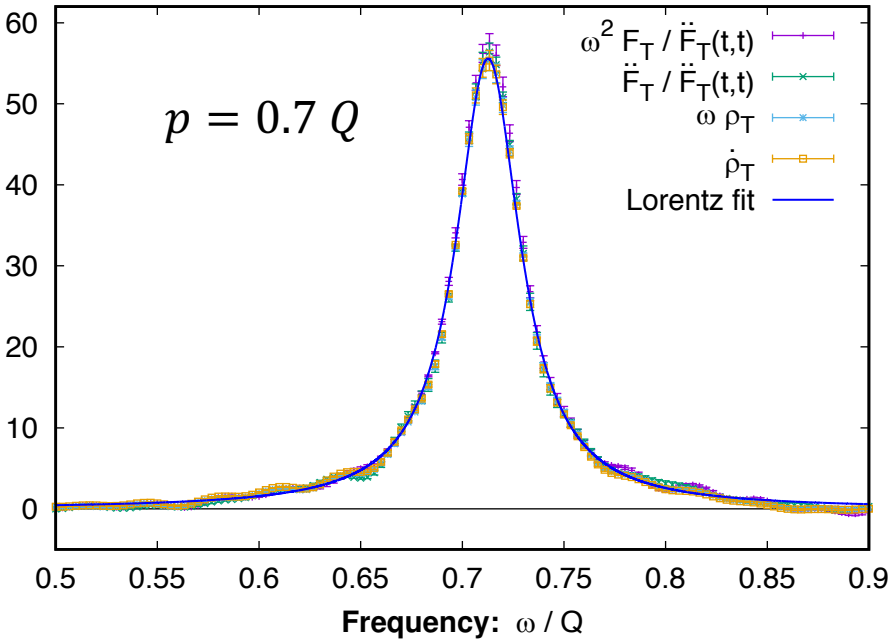
$$\dot{\rho}_T \approx \frac{A}{\pi} \frac{\gamma(p)}{\left(\omega - \omega(p)\right)^2 + p^2}, \quad \text{with } A \leq 1$$

$$(\dot{\rho} = \partial_t \rho \simeq \omega \rho)$$

Sum rules:

$$\int_{-\infty}^{\infty} \mathrm{d}\omega \; \rho_T(t, \omega, p) = 0$$

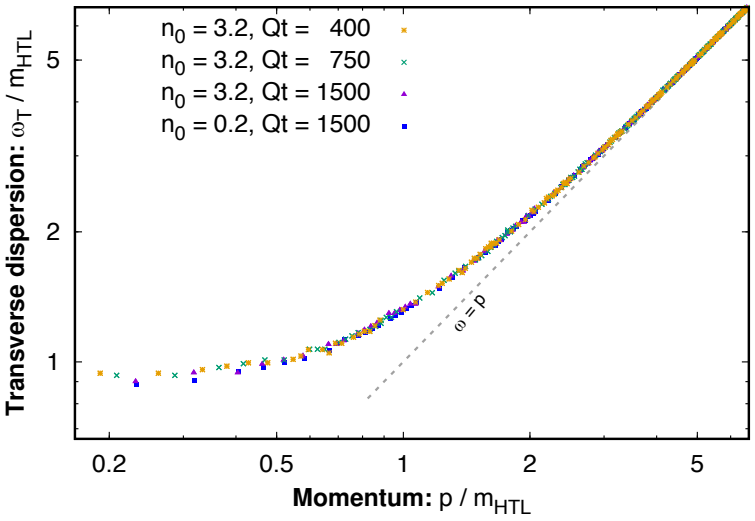
$$\int_{-\infty}^{\infty} \mathrm{d}\omega \; \dot{\rho}_T(t, \omega, p) = 1$$



The second sum rule results from $\dot{\rho}_T \sim i\langle[A(t), E(t')]\rangle$ and the uncertainty principle.

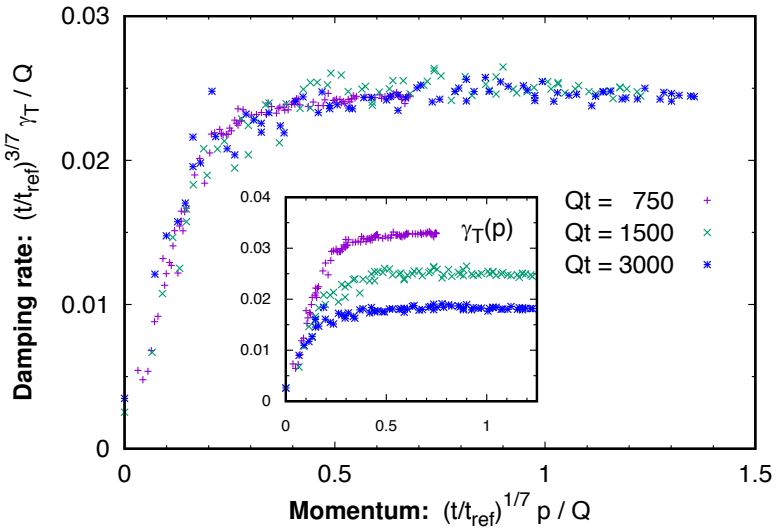
Extracted spectral function vs. HTL predictions

Variation of initial parameters



$$m_{\mathrm{HTL}}(p=0) \sim Q(Qt)^{-1/7}$$

Scaling for $\omega_{T,L} \,/\, m_{\mathrm{HTL}}$ as function of p/m_{HTL}



$$\gamma_{\mathrm{HTL}}(p=0) \sim g^2 T_* \sim Q(Qt)^{-3/7}$$

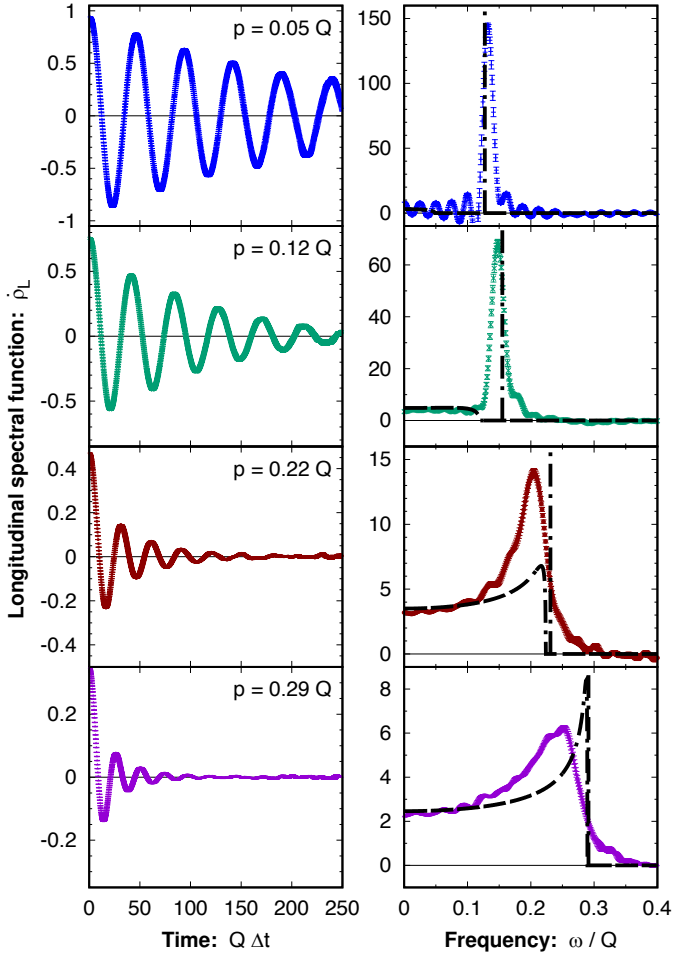
Scaling for $\gamma_{T,L} \,/\, \gamma_{\mathrm{HTL}}(p=0)$ as function of p/m_{HTL}

Extracted spectral function vs. HTL predictions

Longitudinal spectral function ρ_L

$\dot{\rho}_L(\simeq \omega\rho_L)$ as function of $\Delta t = t - t'$ (left) or ω (right) at late time $t_{\mathrm{pert}} \gg \Delta t$

- Similar as for ρ_T, *existence of quasi-particles* with $\omega_L(p)$ and $\gamma_L(p)$ …
- … but for $p \gtrsim m \approx 0.15\ Q$:
  - Quasiparticle peak suppressed exponentially
  - Landau cut dominates oscillations
  - And smeared around light cone



Extracted spectral function vs. HTL predictions

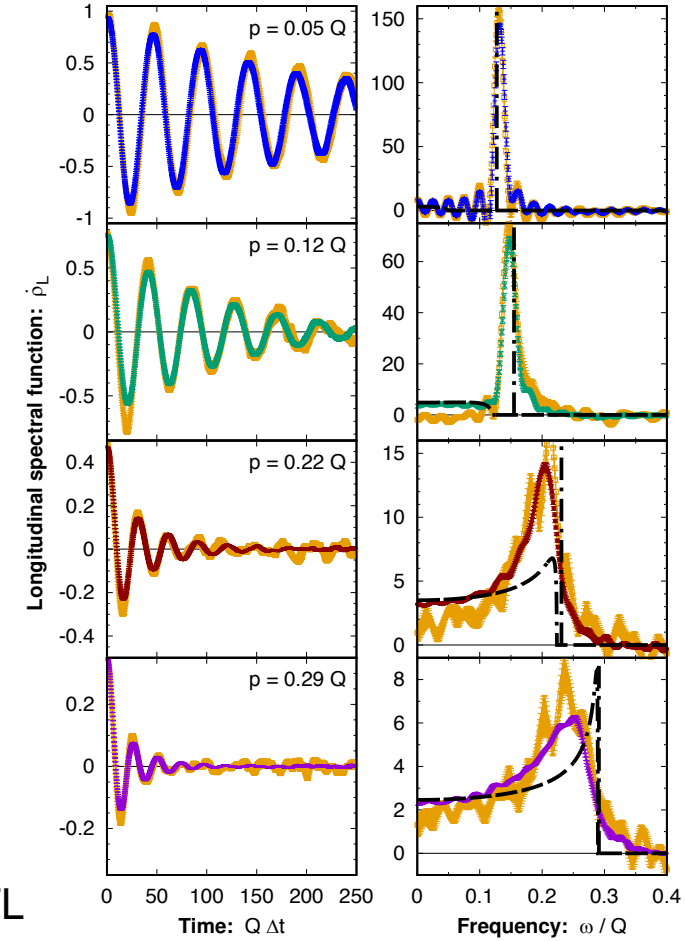
Longitudinal statistical function $\ddot{F}_L$

Orange curves are normalized $\ddot{F}_L(\simeq \omega^2 F_L)$ as functions of $\Delta t = t - t'$ or ω, also $\dot{\rho}_L$

- *Generalized fluctuation-dissipation relation:*

$$\frac{\ddot{F}_L(t,\Delta t,p)}{\dot{\rho}_L(t,\Delta t,p)} = \frac{\ddot{F}_L(t,\omega,p)}{\dot{\rho}_L(t,\omega,p)} = \frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)}$$

- Higher statistical uncertainty because $\ddot{F}^{jk}(t,t',\boldsymbol{p})$ has both polarizations, the transverse one leads to additional noise



Remark: sum rule $\dot{\rho}_L(t,\Delta t=0,p) = \frac{2m^2}{2m^2+p^2}$ from HTL