

Measuring the Rate of Isotropization of Quark-Gluon Plasma Using Rapidity Correlations

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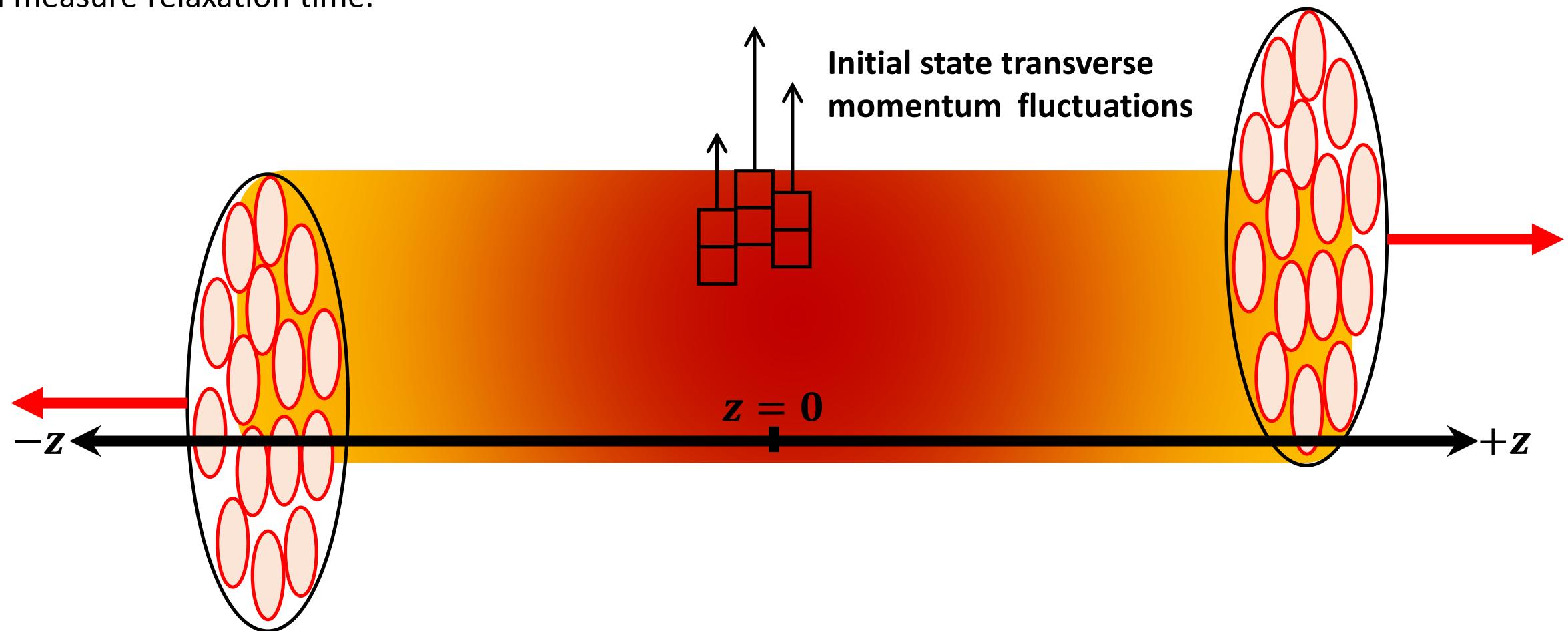
Fluctuations in Nuclear Collisions

Damping of transverse flow fluctuations can be used to measure viscosity.

Gavin & Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302

Rapidity dependence of flow fluctuations can measure relaxation time.

Gavin, GM, Zin, Phys. Rev. C94 (2016) no.2, 024921

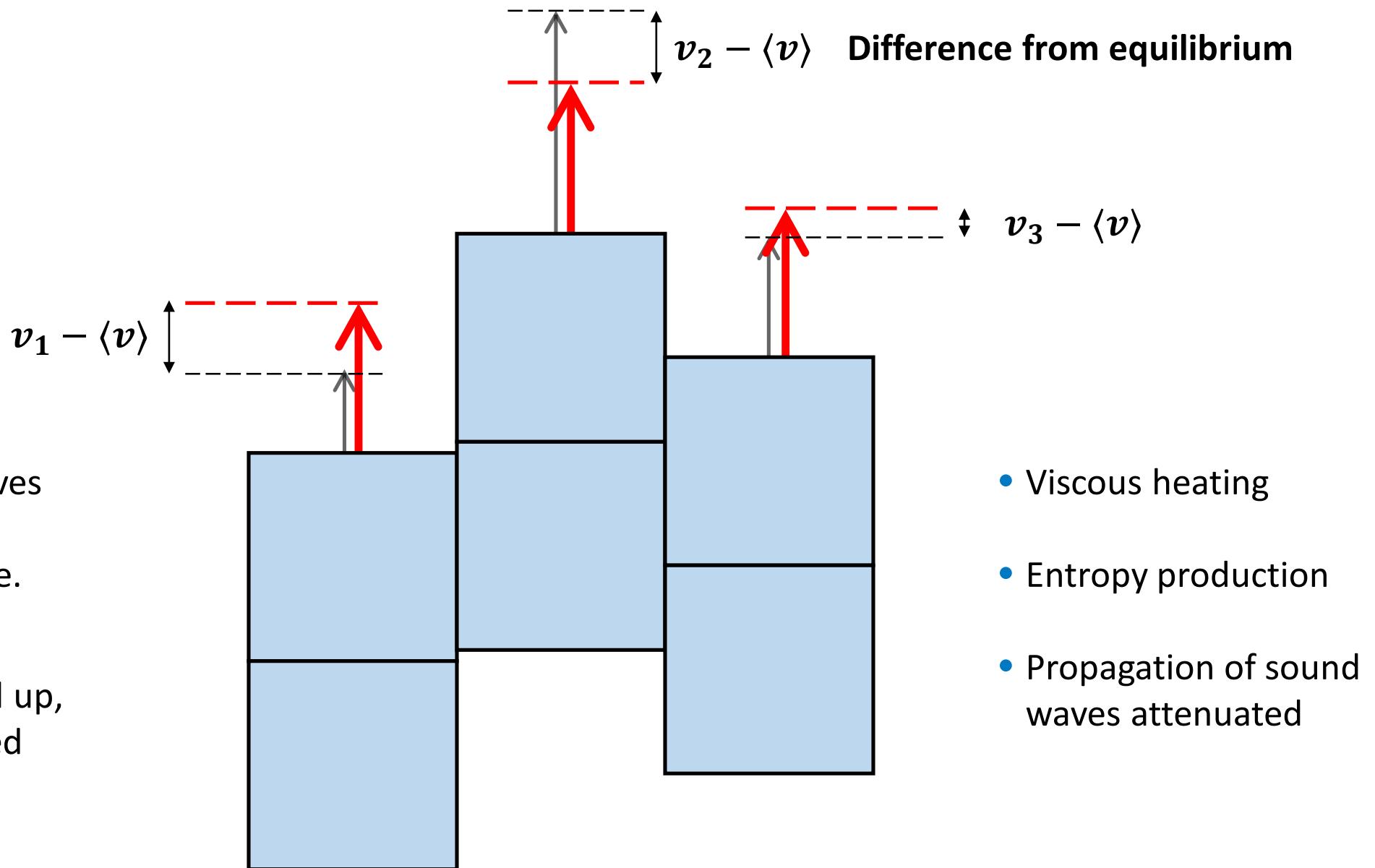


Fluctuations from Equilibrium

$$T_{zr} = -\eta \frac{\partial v_r}{\partial z}$$

Shear viscosity drives the fluid velocities toward the average.

Slow cells are sped up, fast cells are slowed down.



Momentum in Fluctuating Hydrodynamics

- Momentum current – small fluctuations $M_i \equiv T_{0i} - \langle T_{0i} \rangle$
- Momentum conservation
linearized Navier-Stokes $\frac{\partial M_i}{\partial t} + \nabla_i p = \frac{\eta/3 + \zeta}{sT} \nabla_i (\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i$
- Helmholtz decomposition $\vec{M} = \vec{g} + \vec{h}$
- “longitudinal” mode (curl free part) $\vec{\nabla} \times \vec{h} = 0$
- “transverse” mode (divergence free part) $\vec{\nabla} \cdot \vec{g} = 0$

The Shear Mode and Noise

- Momentum conservation
linearized Navier-Stokes

$$\partial_i M_i + \nabla_i p = \frac{\eta/3 + \zeta}{sT} \nabla_i (\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i$$



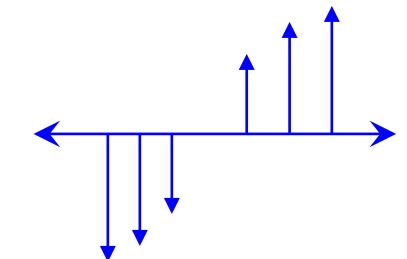
- Viscous diffusion of divergence free modes

$$\vec{\nabla} \times (\star) \rightarrow \quad \partial_t \vec{g} = \nu \nabla^2 \vec{g}$$

- kinematic viscosity $\nu = \eta/Ts$

- Dissipative parts modified by noise

$$T_{ji}^{diss} \approx -\nu \nabla_j g_i + \text{noise}$$



$$\partial_t g_i = \nu \nabla^2 (g_i + \text{noise})$$

People are working on this problem in general. See:

Akamatsu, Mazeliauskas, Teaney,

Ohnishi, Kitazawa, Asakawa,

Stephanov, Kapusta, Mueller,

Plumberg, Pratt, Young, Schlichting

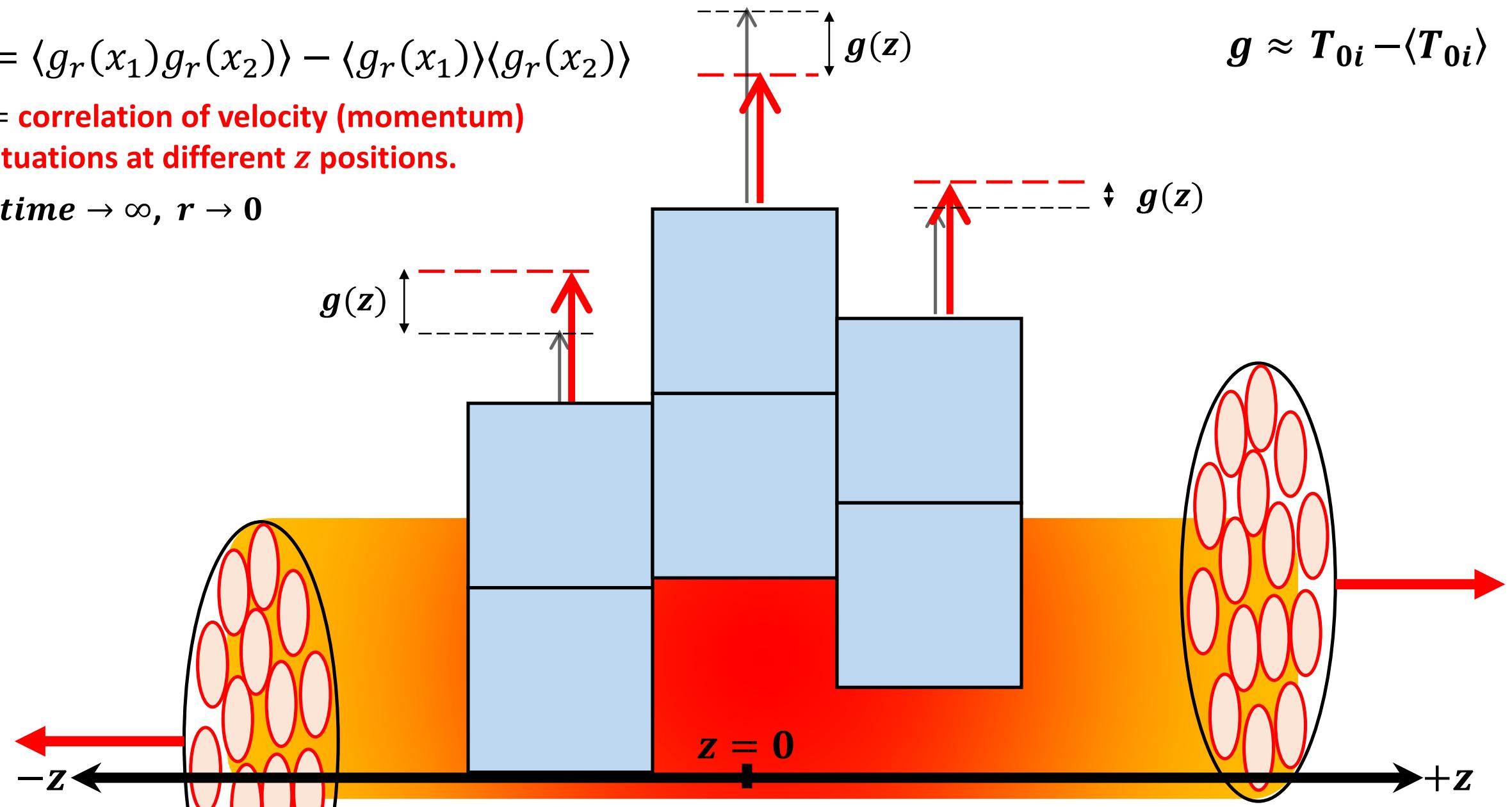
Comparing Changes in Fluctuations with Position

$$r = \langle g_r(x_1)g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

$$g \approx T_{0i} - \langle T_{0i} \rangle$$

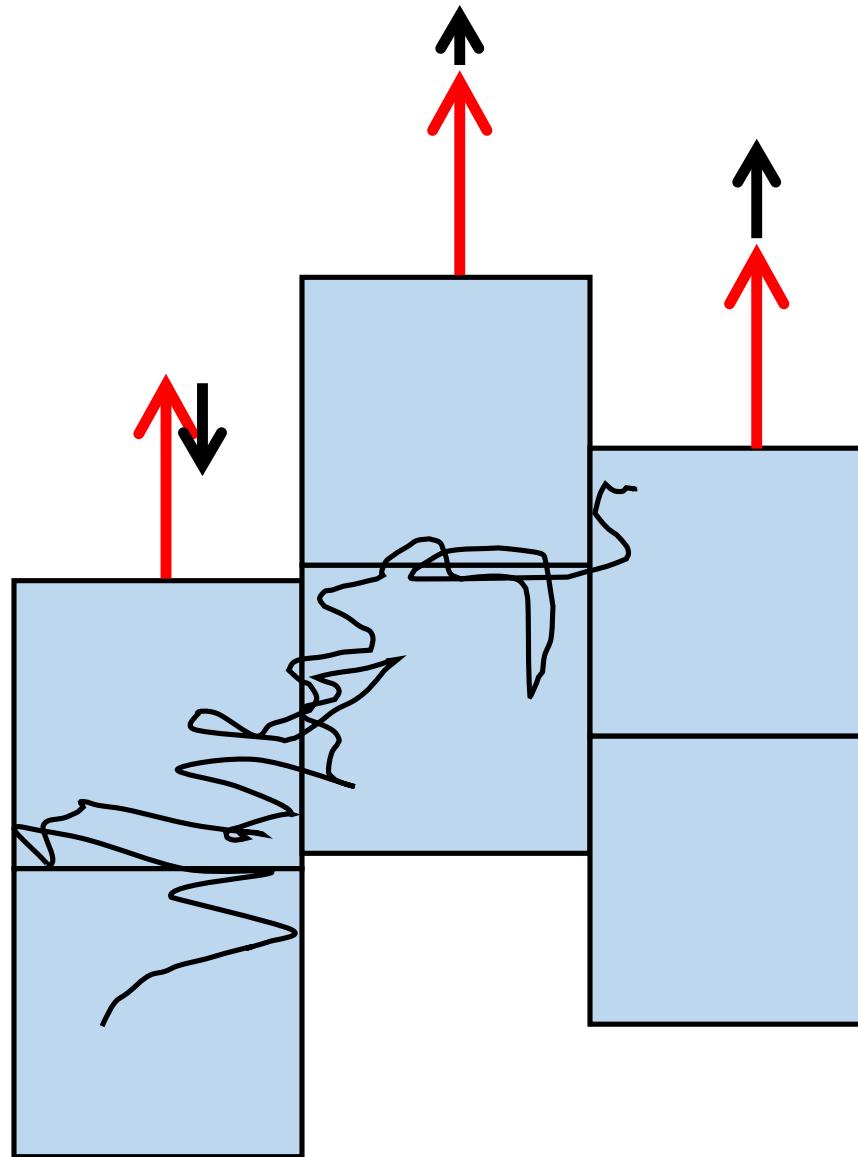
r = correlation of velocity (momentum)
fluctuations at different z positions.

As *time* $\rightarrow \infty$, $r \rightarrow 0$



Fluctuations from Noise

- No noise:
as $\text{time} \rightarrow \infty, r \rightarrow 0$
- Including noise:
as $\text{time} \rightarrow \infty, r \rightarrow r_{eq}$
- The difference
 $\Delta r = r - r_{eq}$
still satisfies a
diffusion equation



Diffusion of Momentum Correlations

- Momentum flux density correlation function

$$r = \langle g_r(x_1)g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

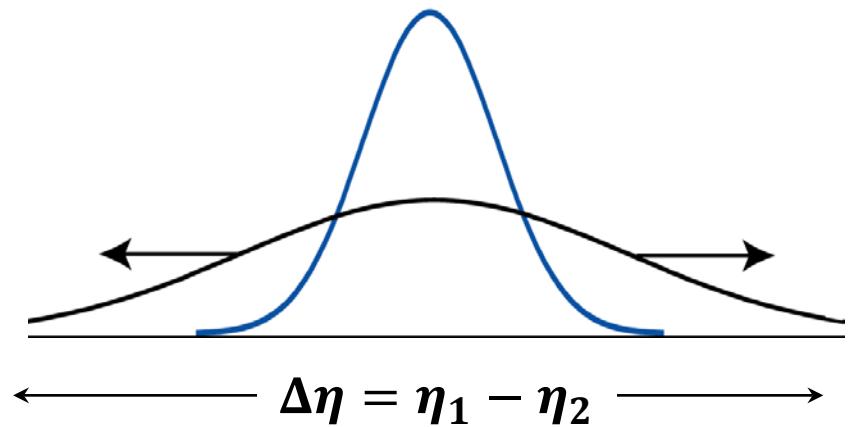
- The difference $\Delta r = r - r_{eq}$ still satisfies a diffusion equation

$$\left(\frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right) \Delta r = 0$$

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

- fluctuations diffuse through volume, driving $r \rightarrow r_{eq}$

width in relative spatial rapidity grows from initial value σ_0



Measuring Correlations

- Momentum flux density correlation function

$$r = \langle g_r(x_1)g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

- The difference $\Delta r = r - r_{eq}$ still satisfies a diffusion equation

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Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

- observable: $C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{pairs} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2 = \frac{1}{\langle N \rangle^2} \int \Delta r \, dx_1 \, dx_2$

- assumptions:

- 1) only “transverse” shear modes
- 2) proper-time freeze out

Abdel-Aziz & Gavin., PRL 97 (2006) 162302; PR C70 (2004) 034905
Pratt, Schlichting, Gavin, Phys. Rev. C 84 (2011) 024909

Measuring Correlations

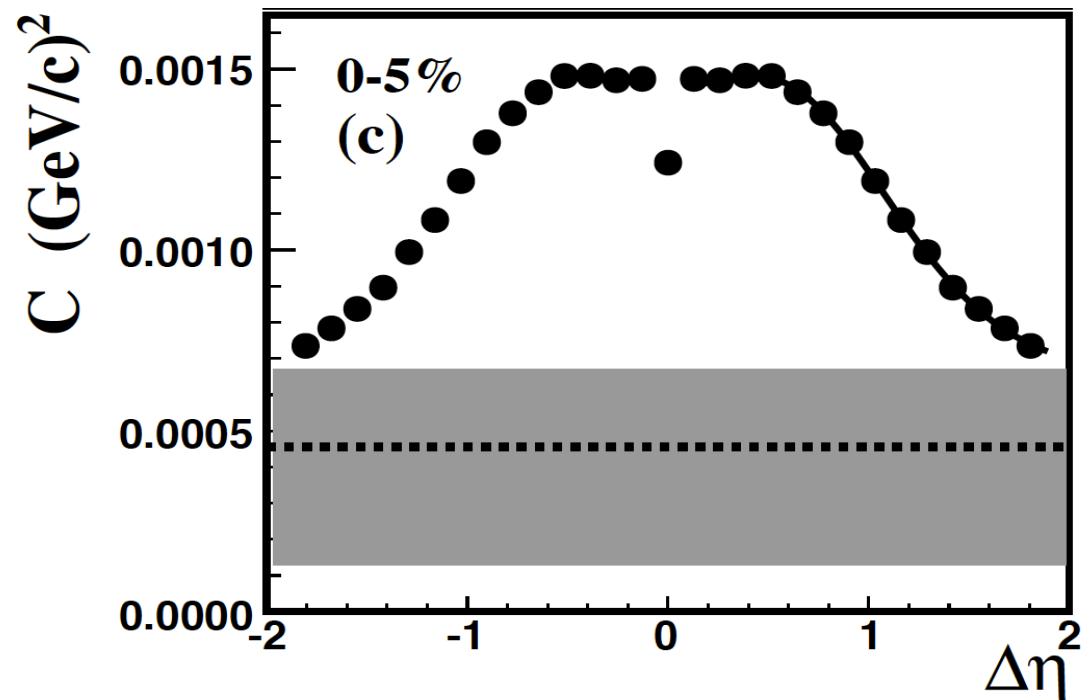
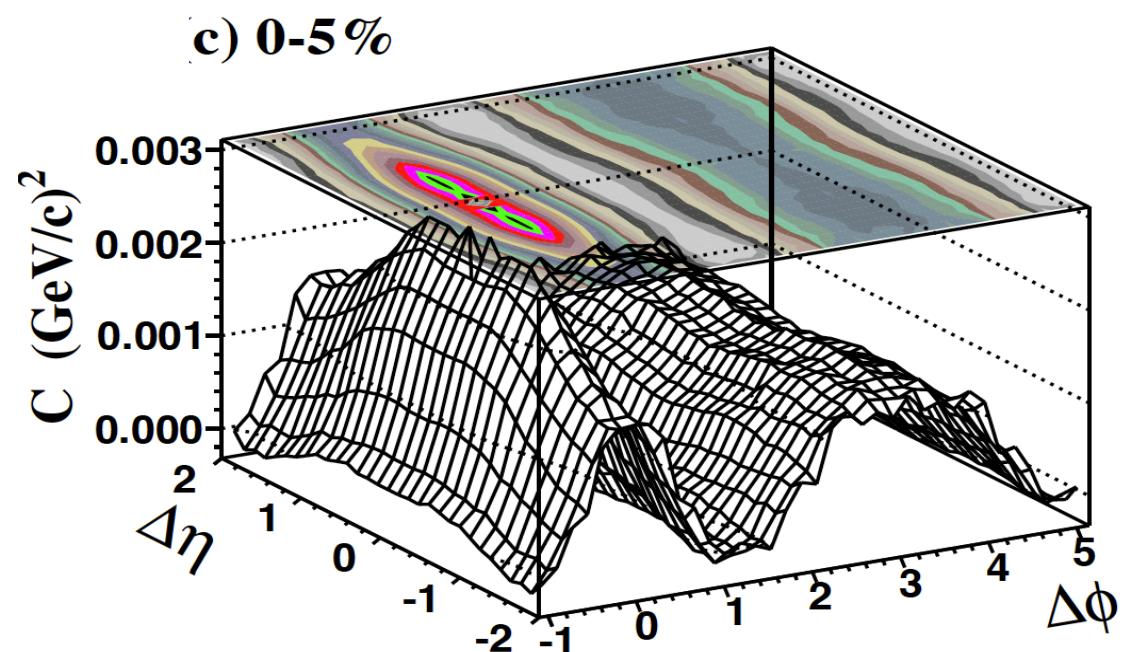
STAR Phys.Lett. B704 (2011) 467-473 arXiv:1106.4334

measured: rapidity width of near side peak

- fit peak + constant offset
 - offset is ridge, i.e., long range rapidity correlations
 - report RMS width of the peak
-
- **find:** width increases in central collisions

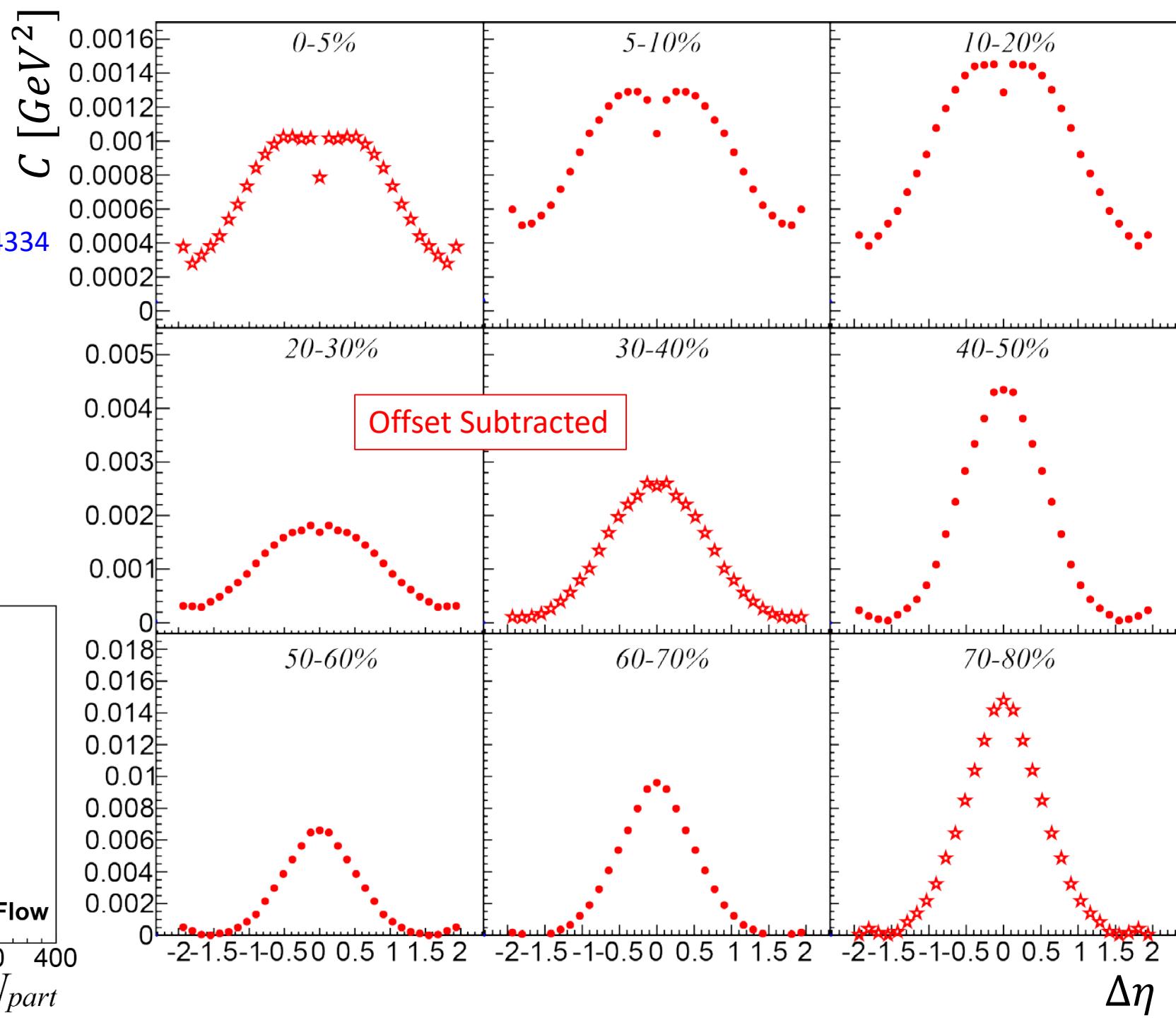
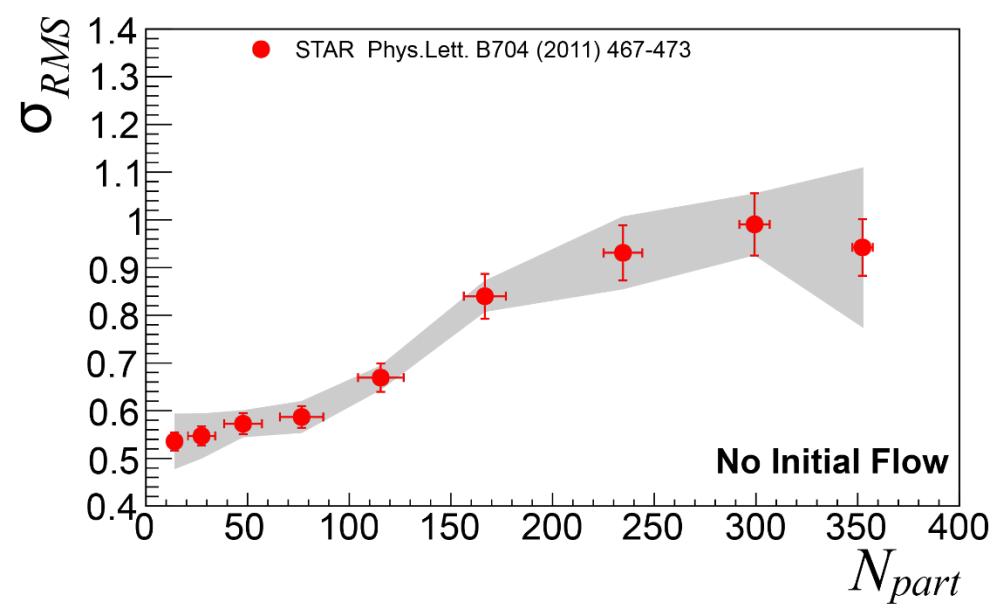
$$\sigma_{central} = 1.0 \pm 0.2$$

$$\sigma_{peripheral} = 0.54 \pm 0.02$$



Longitudinal p_T Fluctuations

STAR Phys.Lett. B704 (2011) 467-473 arXiv:1106.4334



First Order Diffusion

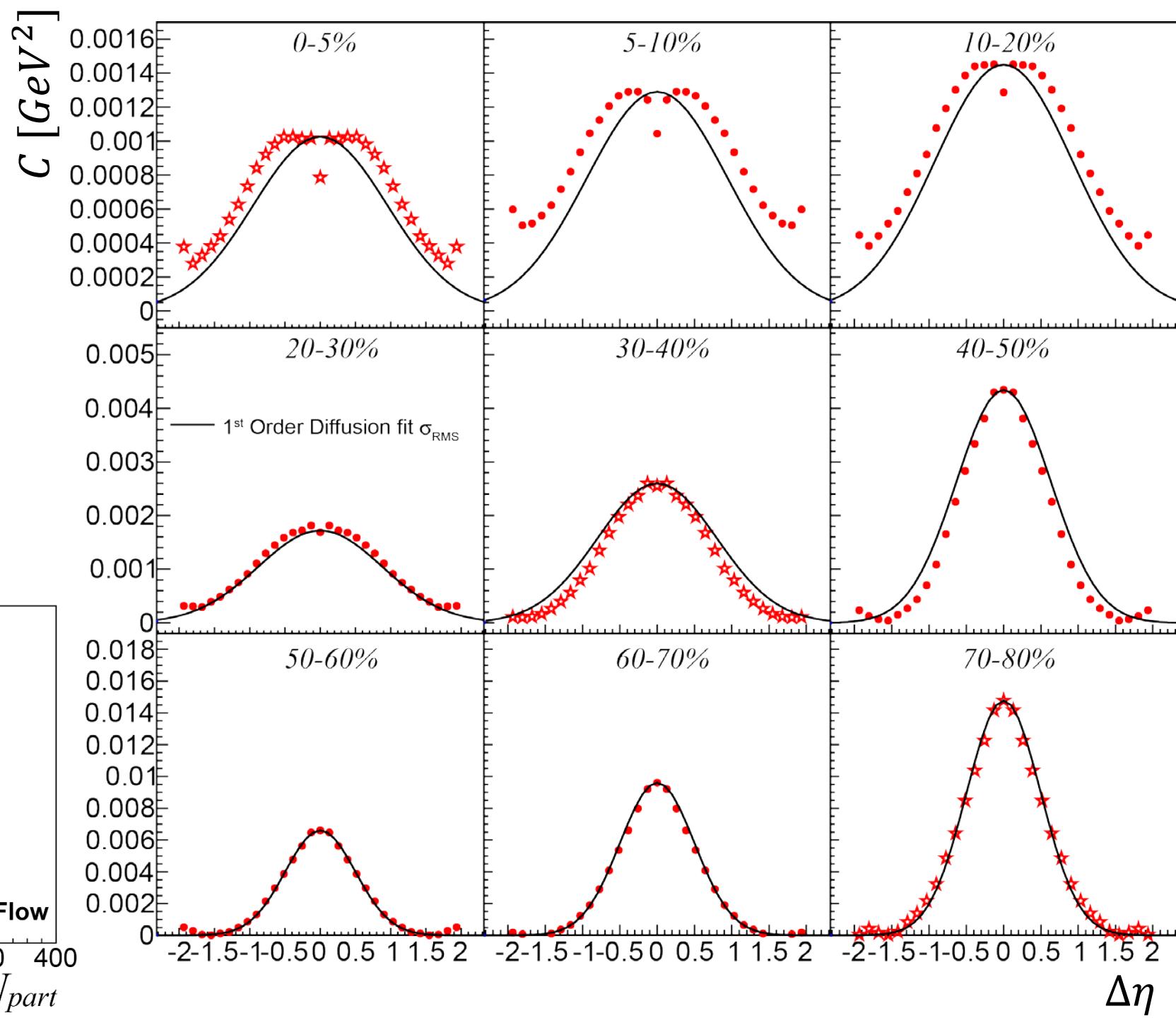
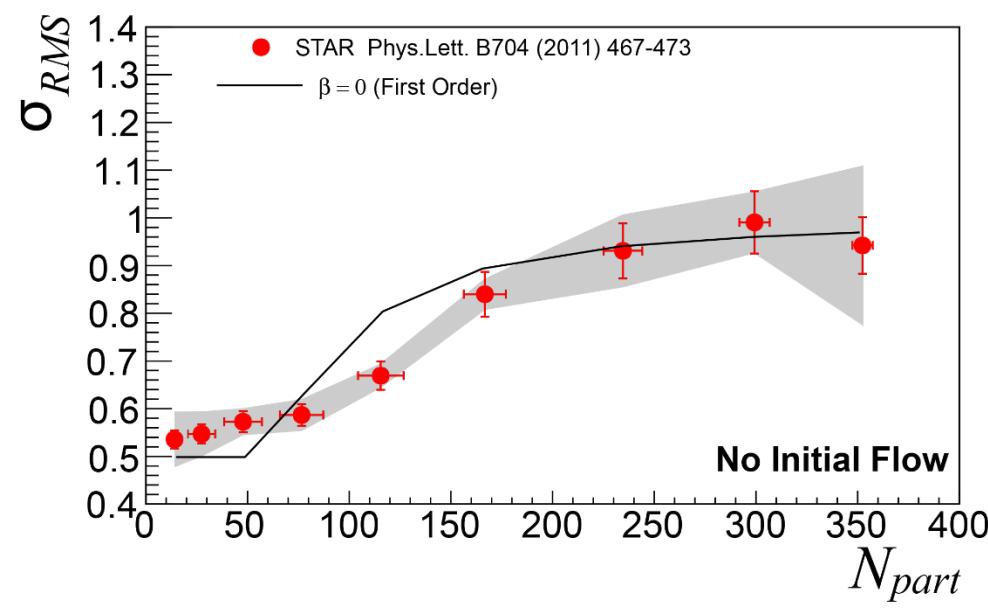
$$\eta/s = 1/4\pi$$

$$v = \frac{\eta}{T_F s} = \text{constant}$$

$$T_F = 143 \text{ MeV}$$

$$\tau_0 = 0.6 \text{ fm}$$

$$\tau_{F,\text{central}} = 10 \text{ fm}$$



Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2} \frac{\partial^2}{\partial \tau^2} + \left(1 + \frac{\kappa \tau_\pi}{\tau} \right) \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \Delta r = 0$$

Second Order Diffusion Equation for Correlations

$$\left[\tau_\pi \frac{\partial^2}{2 \partial \tau^2} + \left(1 + \frac{\kappa \tau_\pi}{\tau} \right) \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \Delta r = 0$$

Relaxation Time
 $\tau_\pi = \beta \nu$

Heating from
gradients in velocities

Kinematic Viscosity
 $\nu = \eta / Ts$

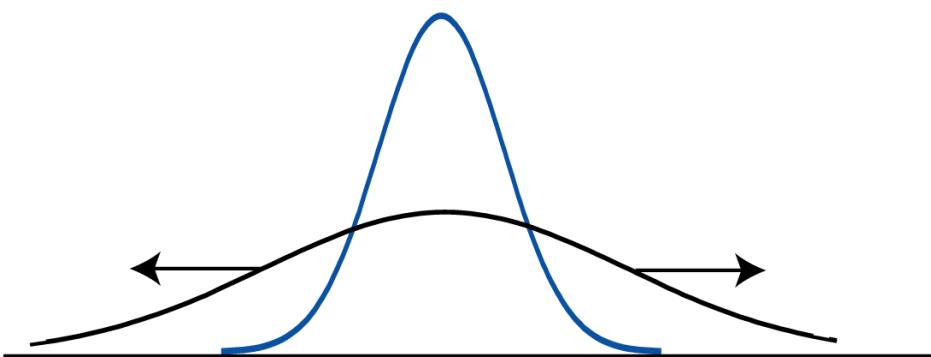
Second Order Diffusion Equation for Correlations

$$\left[\frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \Delta r = 0$$

Diffusion Equation

Diffusion (1st Order)

- Gaussian peak spreads
- tails violate causality



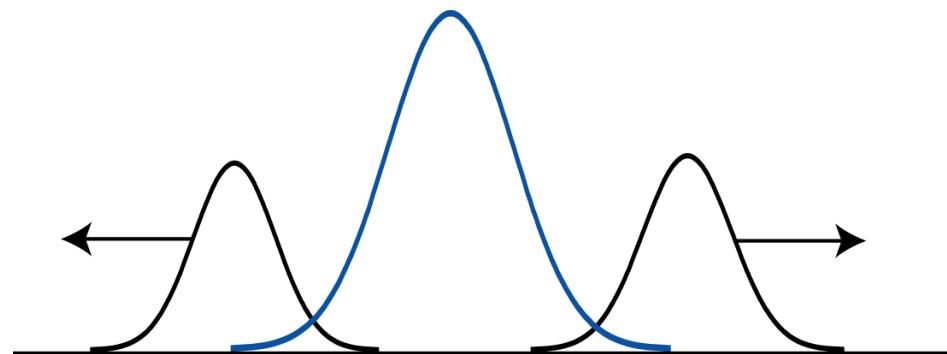
Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2} \frac{\partial^2}{\partial \tau^2} \begin{array}{c} \text{diagonal hatching} \\ \hline \end{array} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \Delta r = 0$$

Wave Equation

Wave propagation

- peak splits into left and right traveling pulses
- propagation speed c_s

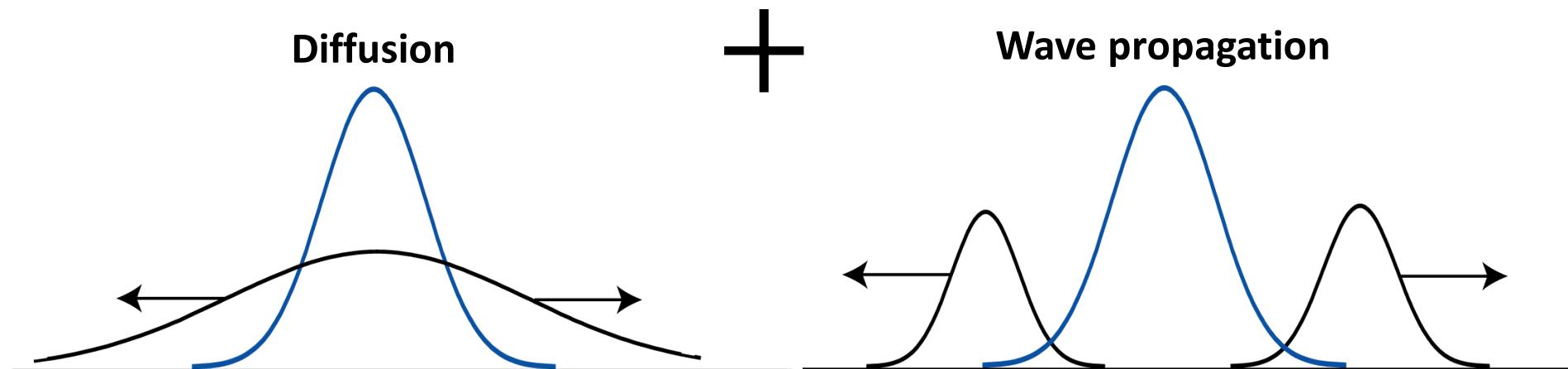


Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2} \frac{\partial^2}{\partial \tau^2} + \left(1 + \frac{\kappa \tau_\pi}{\tau} \right) \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \Delta r = 0$$

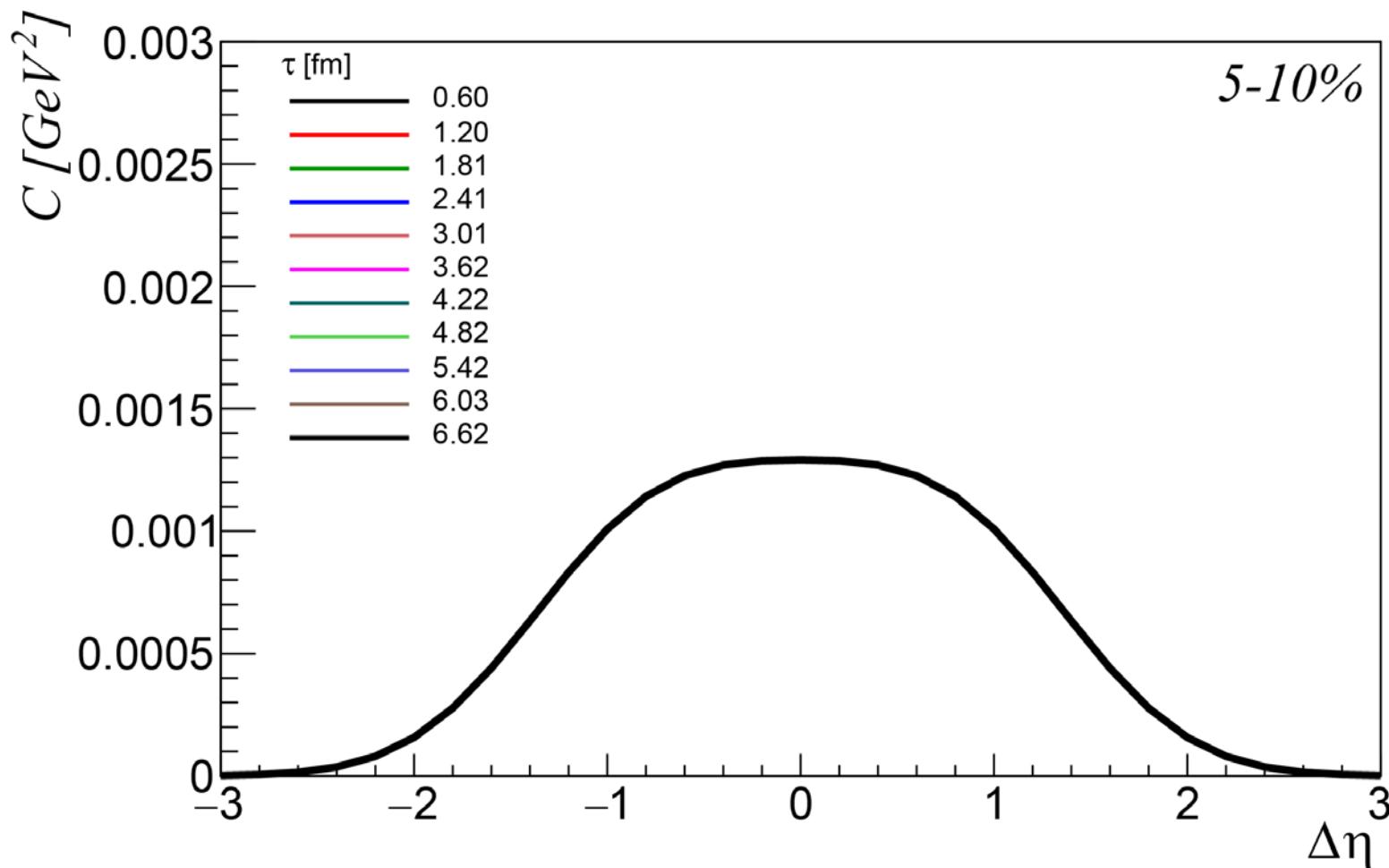
Relaxation Time $\tau_\pi = \beta \nu$

- $\beta = 0$, diffusion only, waves move at infinite speed
- $\beta = 5$, predicted by kinetic theory of gas of massless particles
- Larger β means slower wave → slower changes in fluctuation correlations



The Effect of τ_π

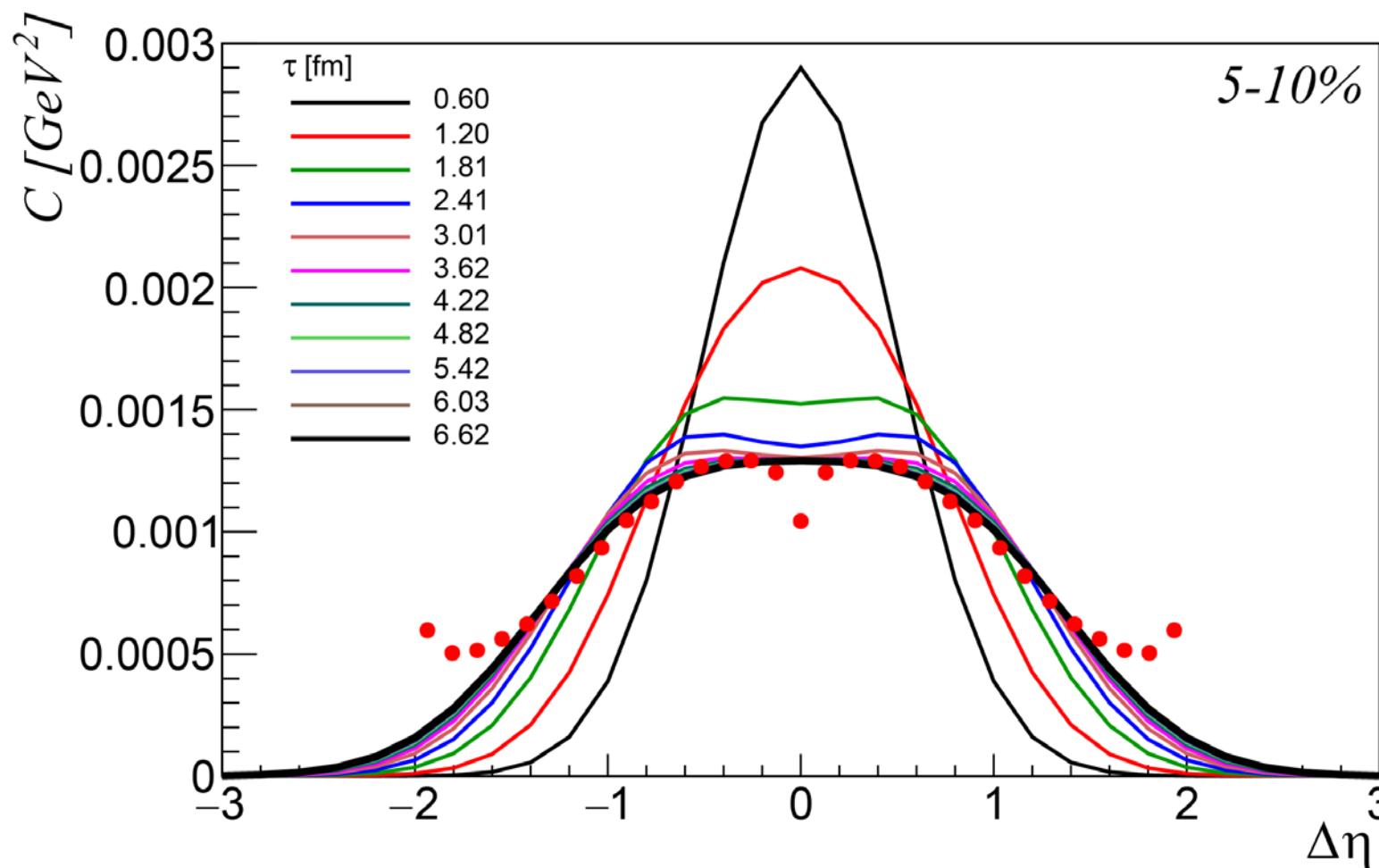
$$\left[\frac{\tau_\pi}{2} \frac{\partial^2}{\partial \tau^2} + \left(1 + \frac{\kappa \tau_\pi}{\tau} \right) \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(2 \frac{\partial^2}{\partial \Delta\eta^2} + \frac{1}{2} \frac{\partial^2}{\partial \eta_a^2} \right) \right] \Delta r = 0$$



$\kappa = 0$
 $\beta = 10$
 $\eta/s = 1/4\pi$
 $\nu = \frac{\eta}{T_F S} = constant$
 $T_F = 143 MeV$
 $\tau_0 = 0.6 fm$
 $\tau_{F,central} = 10 fm$
No initial flow

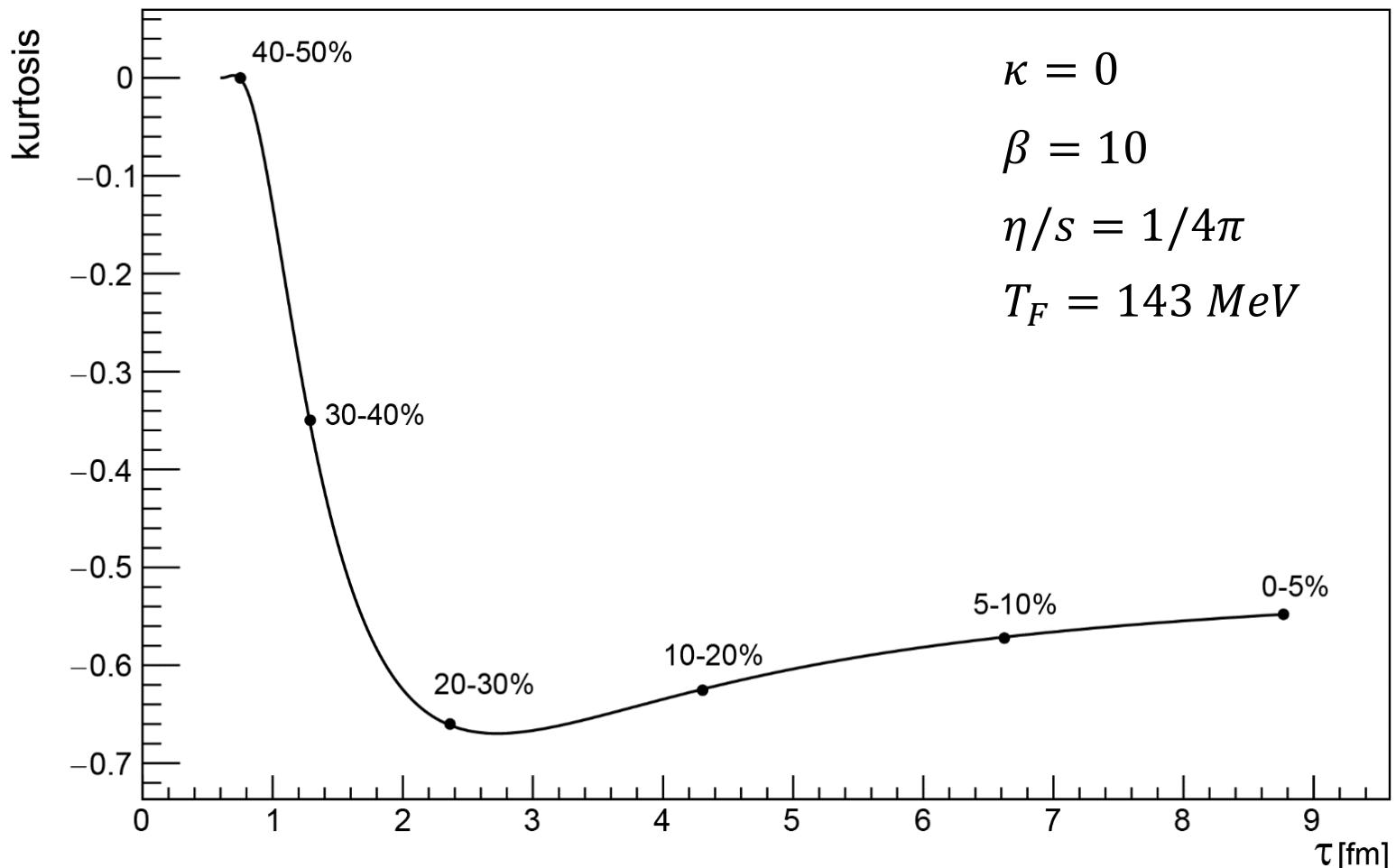
Comparison to Experiment

$$\left[\frac{\tau_\pi}{2} \frac{\partial^2}{\partial \tau^2} + \left(1 + \frac{\kappa \tau_\pi}{\tau} \right) \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2} \left(2 \frac{\partial^2}{\partial \Delta\eta^2} + \frac{1}{2} \frac{\partial^2}{\partial \eta_a^2} \right) \right] \Delta r = 0$$



Wave and Diffusion Competition

- For constant viscosity and $\kappa = 0$ every centrality follows the same time evolution
- Wave effects happen early and rapidly, diffusion influences show up later
- For this specific set of calculation conditions 20-30% collisions freeze out near the maximum (visible) effect of the wave behavior



Second Order Diffusion

$$\kappa = 0$$

$$T_F = 143 \text{ MeV}$$

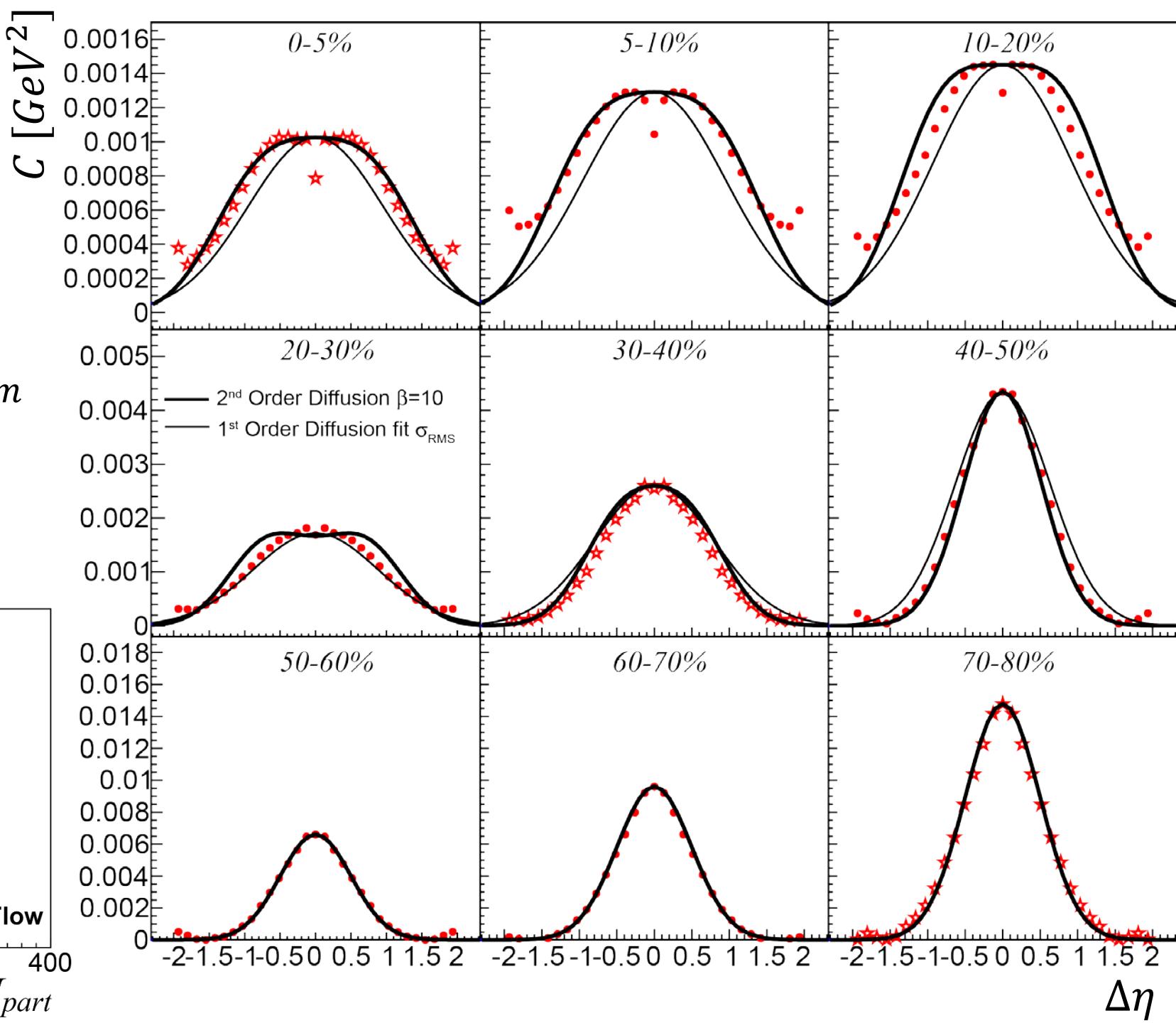
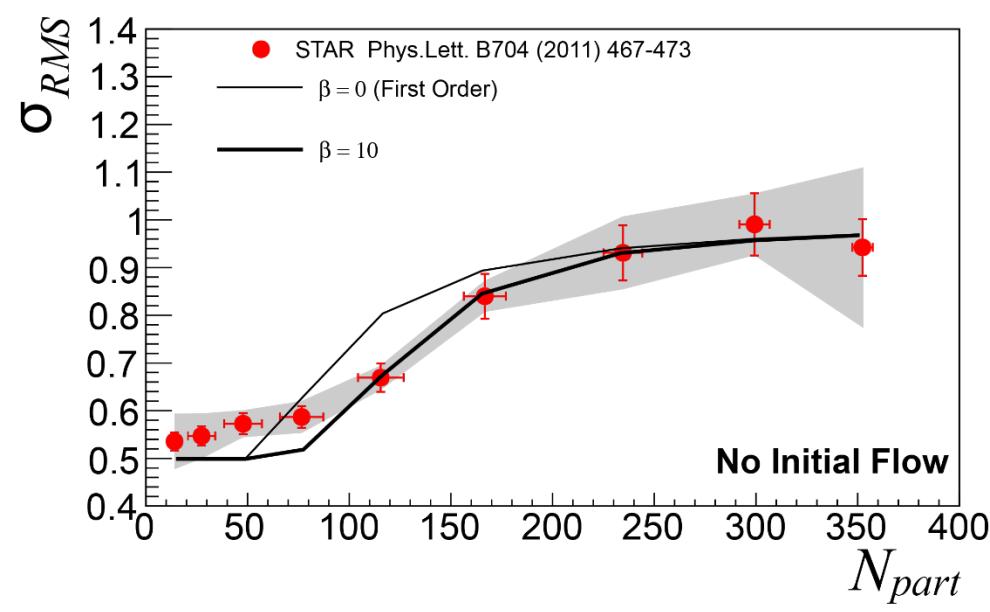
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Evolving Parameters

- Entropy production due to viscous heating and longitudinal expansion.

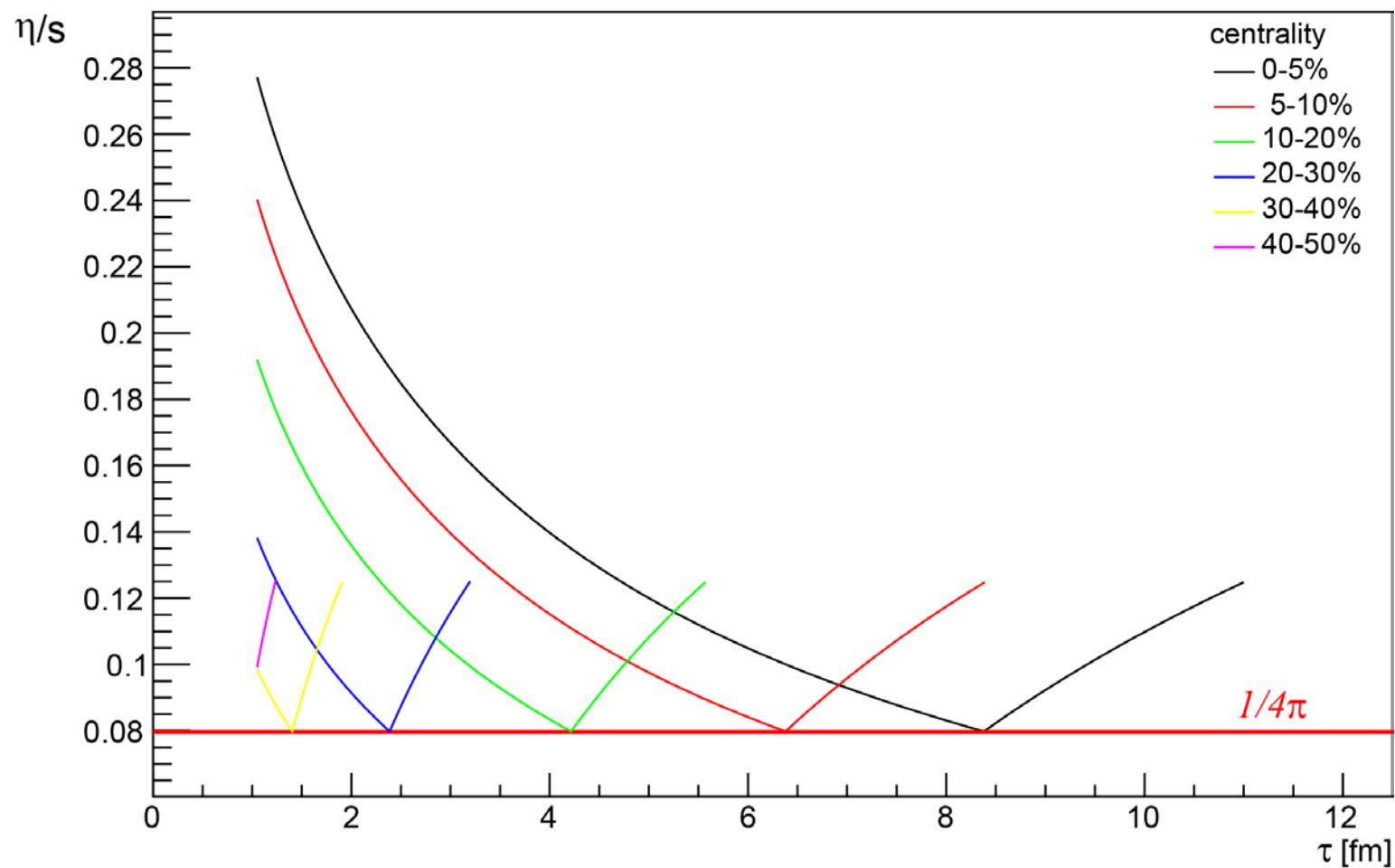
$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

- Relaxation equation. Causality delays heating.

$$\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi} \left(\Phi - \frac{4\eta}{3\tau} \right) - \frac{\kappa}{\tau} \Phi$$

- Coefficient associated with the gradient of speeds of fluid cells

$$\kappa = \frac{1}{2} \left\{ 1 + \frac{d \ln(\tau_\pi / \eta T)}{d \ln \tau} \right\}$$



η/s parameterization follows Phys. Rev. C86 (2012) 014909

Evolving Parameters

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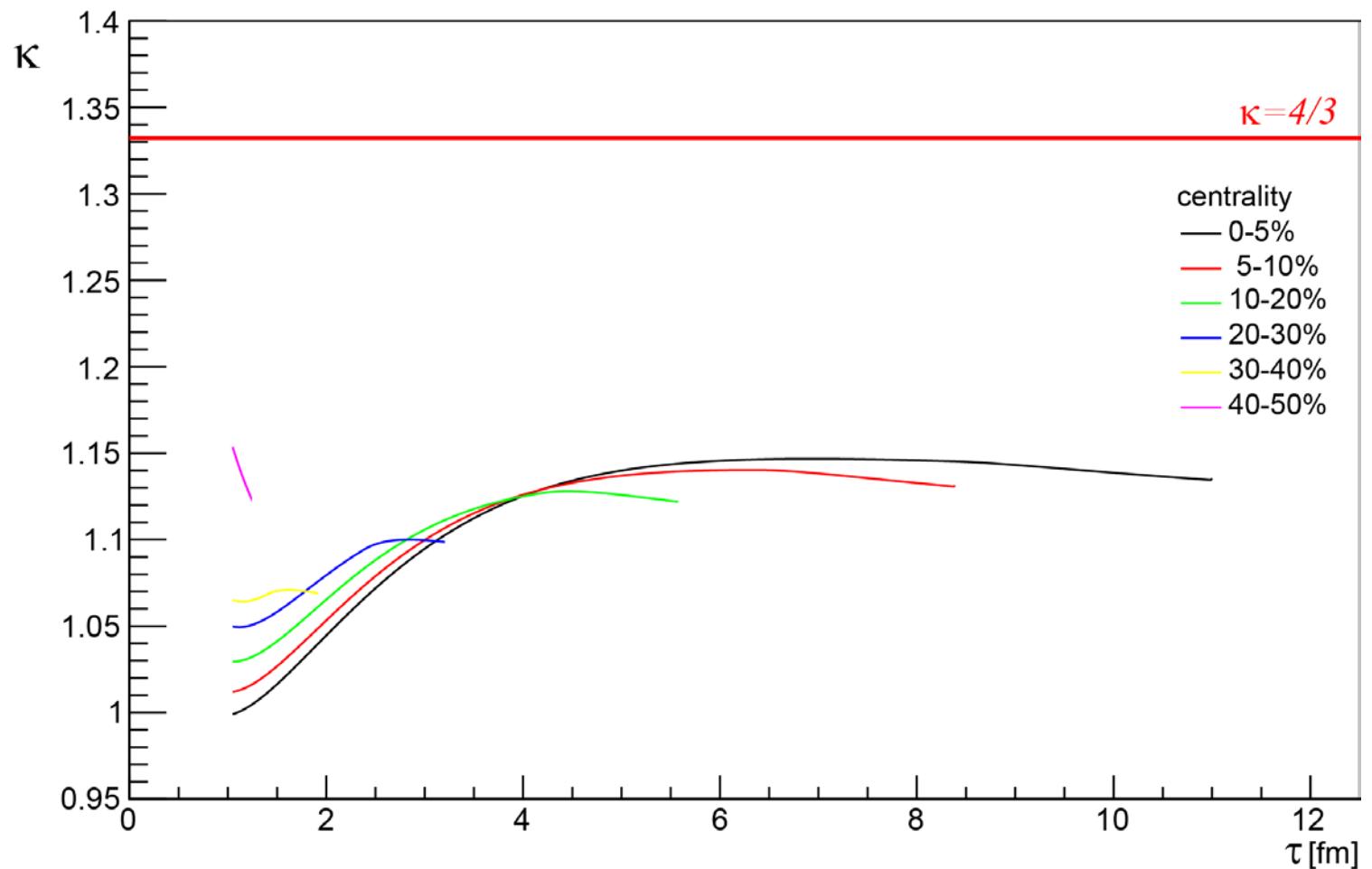
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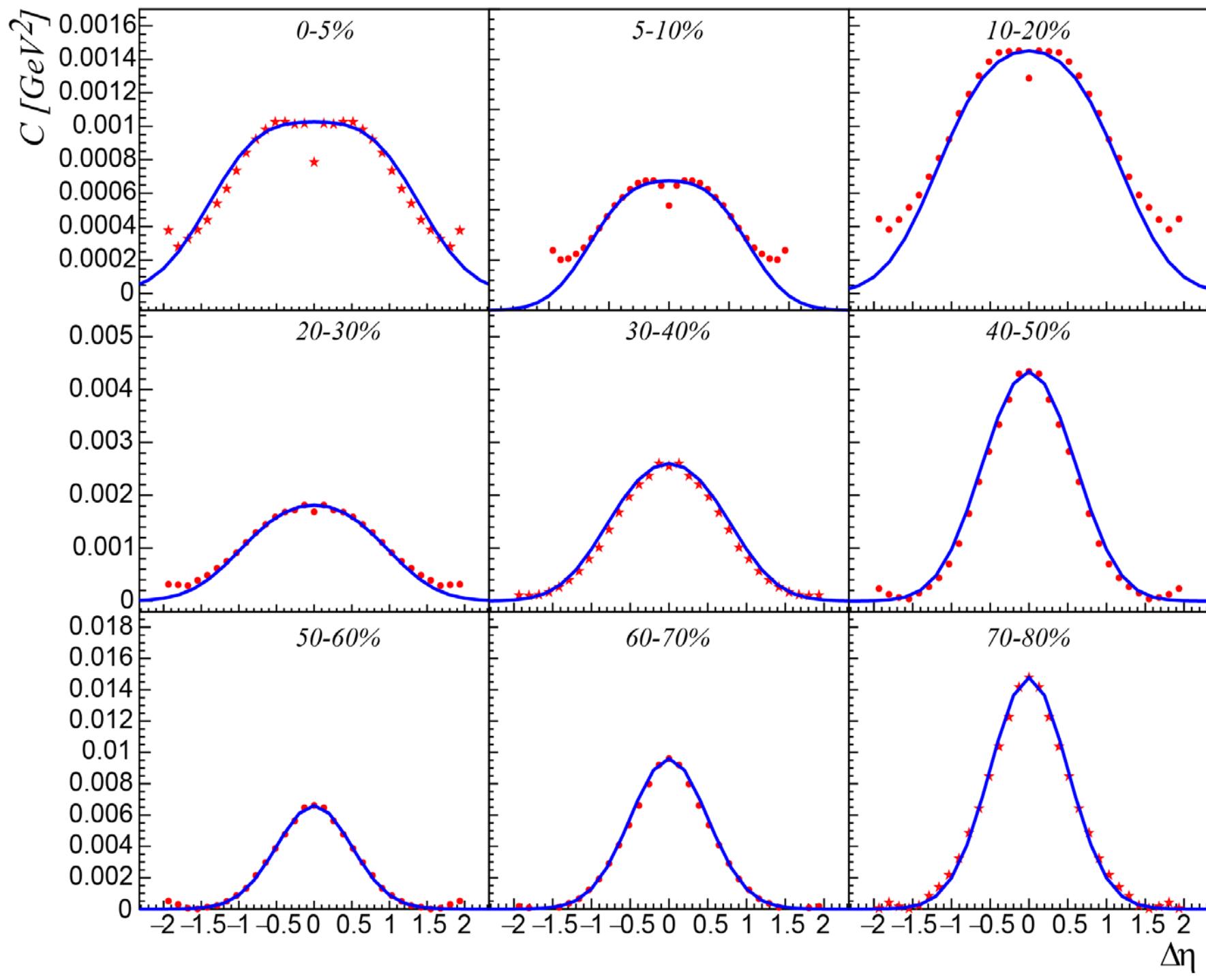
Evolving Parameters

$$\beta = 5.5$$

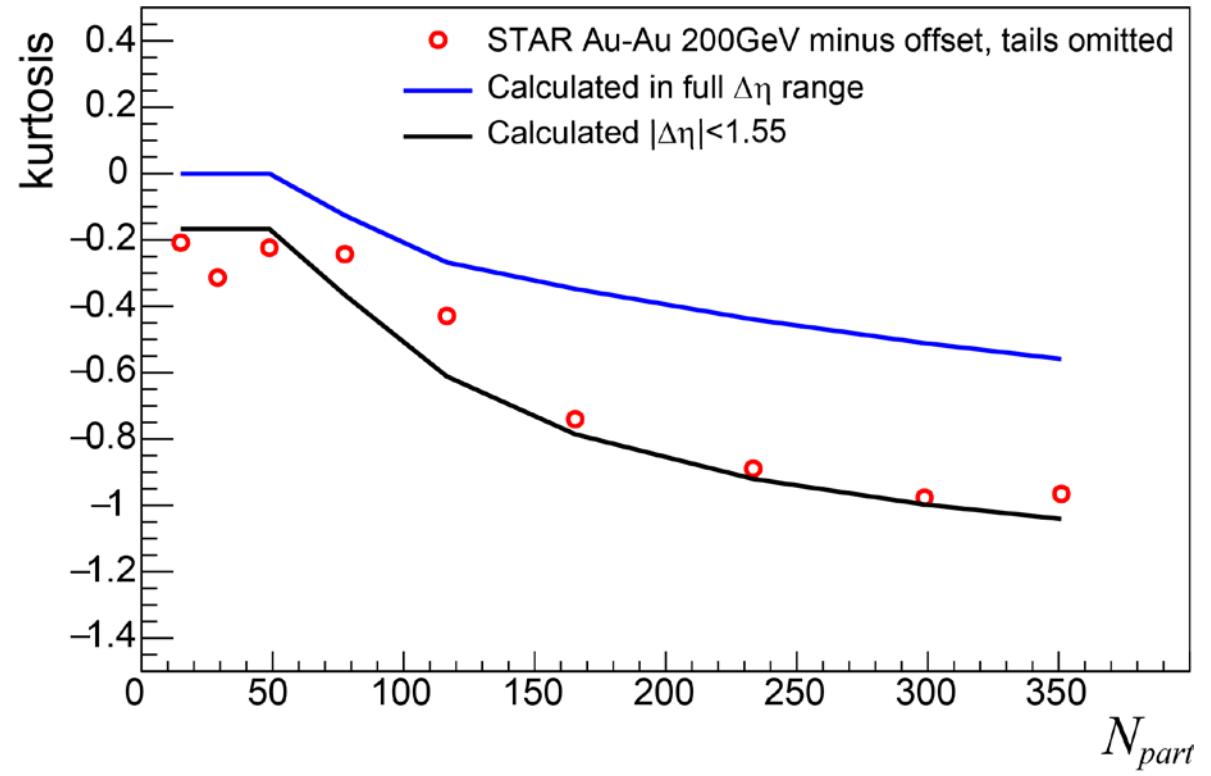
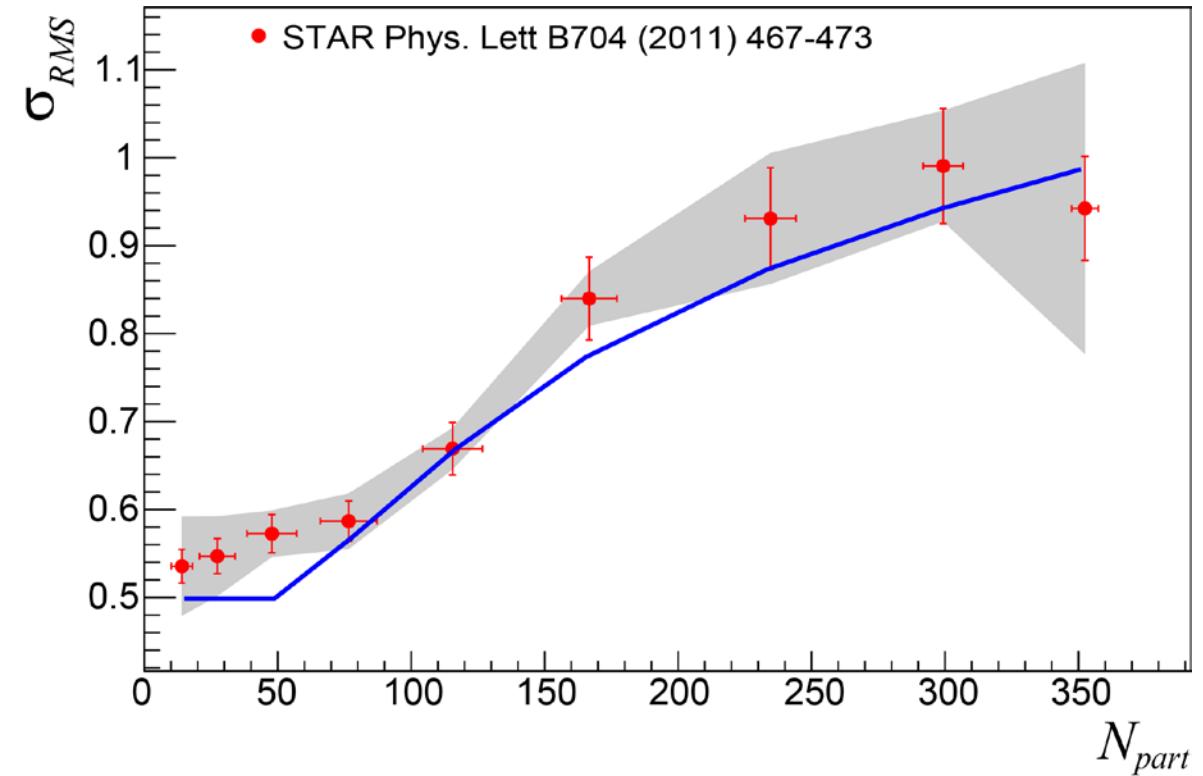
$$T_F = 150 \text{ MeV}$$

$$\tau_0 = 1.05 \text{ fm}$$

$$\tau_{F,central} = 12.5 \text{ fm}$$



Width and Kurtosis



Kurtosis with Evolving Parameters

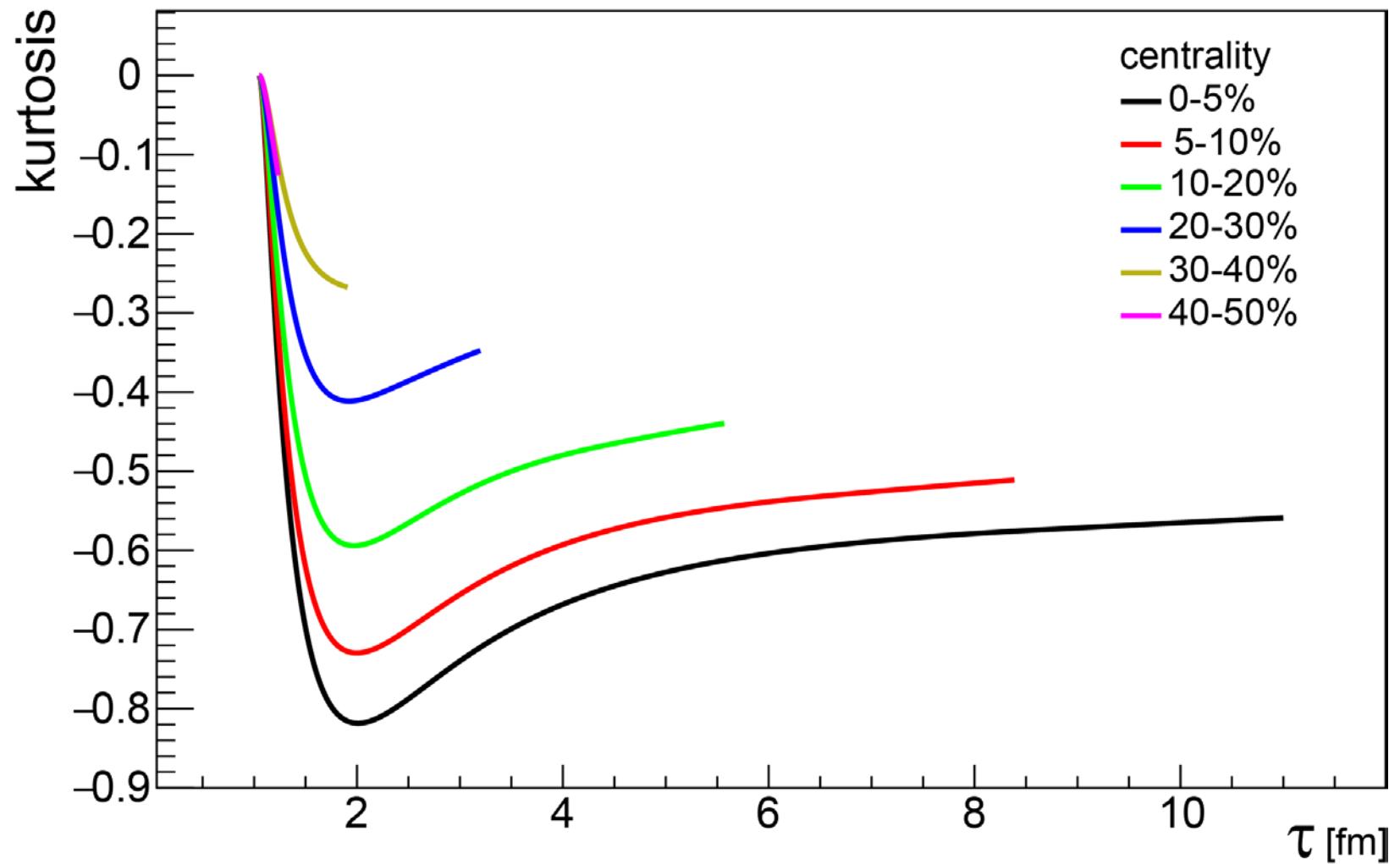
$$\beta = 5.5$$

$$T_F = 150 \text{ MeV}$$

$$\tau_0 = 1.05 \text{ fm}$$

$$\tau_{F,central} = 12.5 \text{ fm}$$

With initial flow



Realistic Limits on $\beta = \tau_\pi/\nu$

$T_F = 150 \text{ MeV}$

$\tau_0 = 1.05 \text{ fm}$

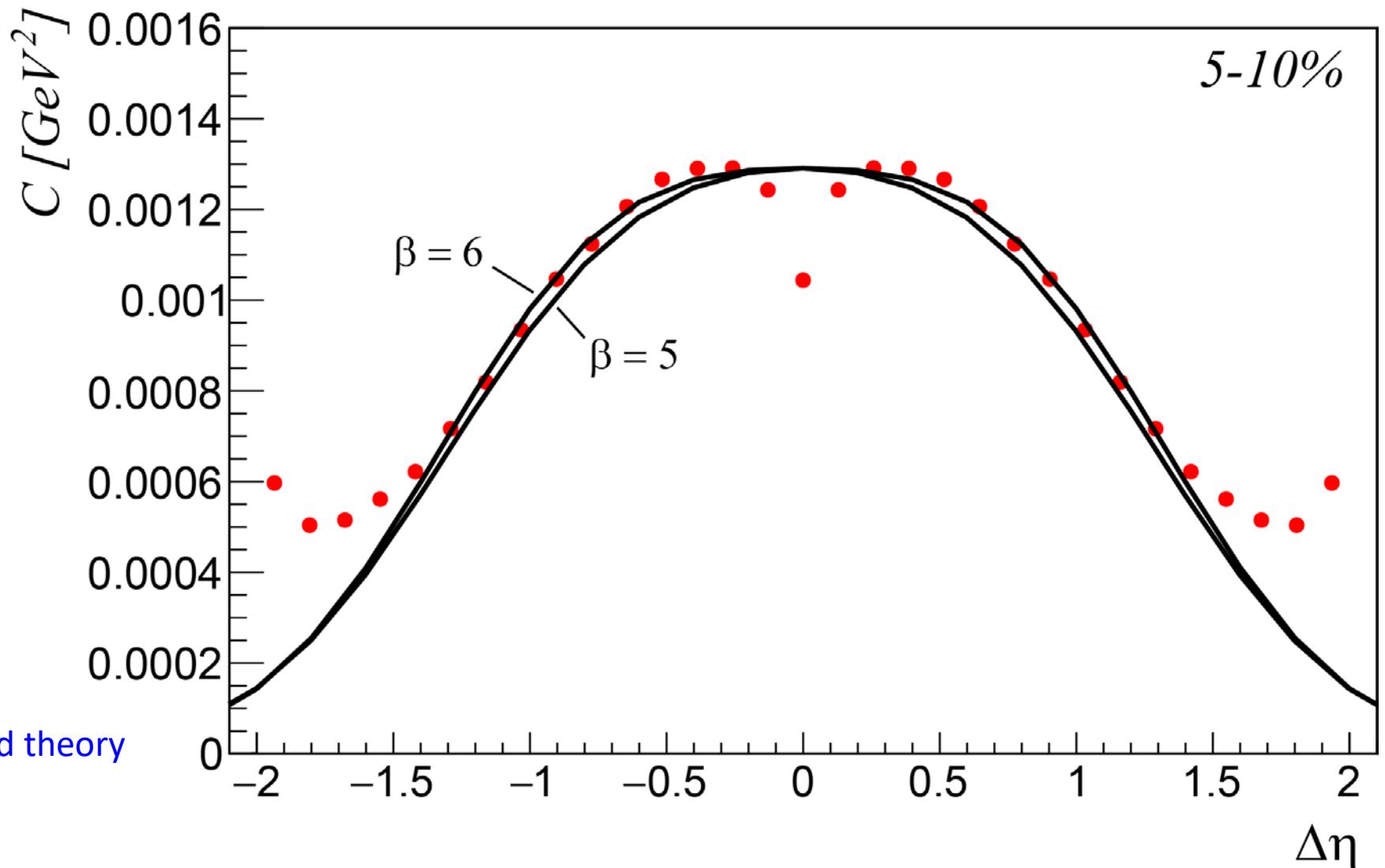
$\tau_{F,central} = 12.5 \text{ fm}$

With initial flow

Jeon & Czajka massless scalar field theory

$\tau_\pi/\nu = 5 - 7$

Phys. Rev. C95 (2017) no.6, 064906



Summary

Hydro formulation: only transverse modes effect these correlations

- Wavelike propagation of fluctuations important at early times
- Diffusive effects show up as waves attenuate and separate

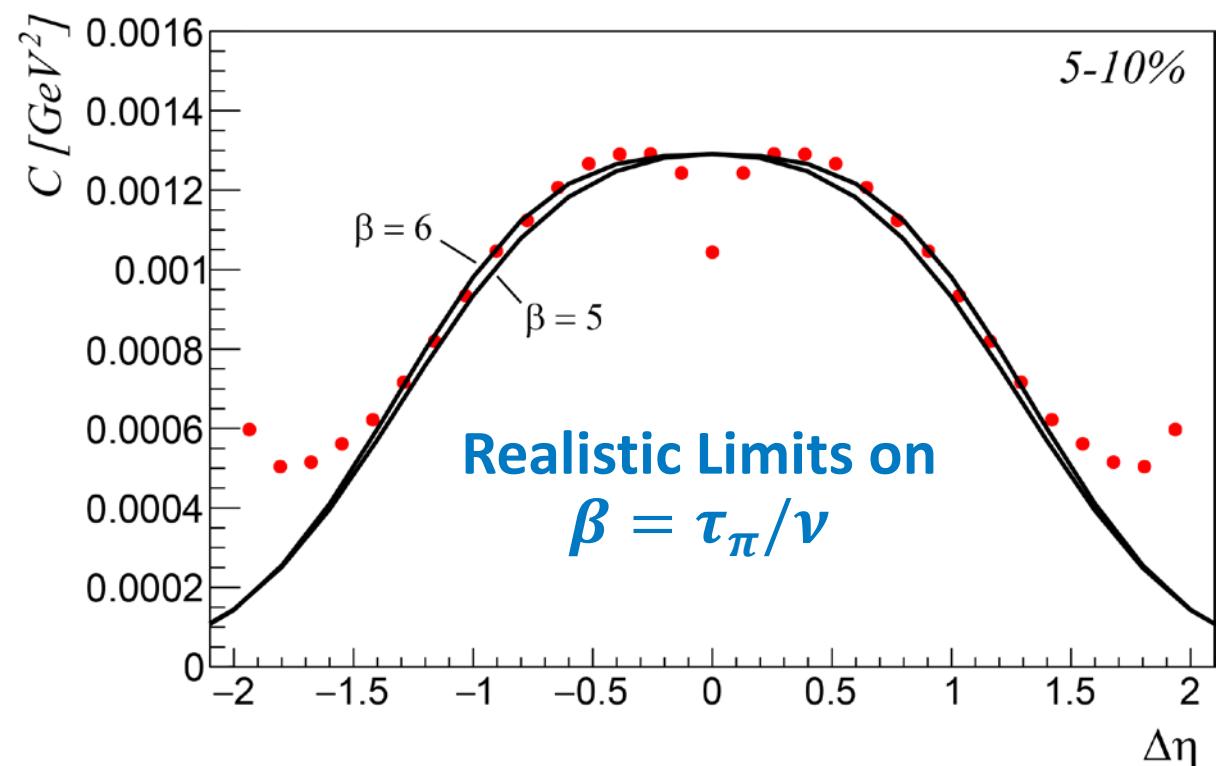
Causality shapes the rapidity dependence of correlations

$\tau_\pi = \beta\nu \rightarrow$ double humps

$\nu = \frac{\eta}{Ts} \rightarrow$ width

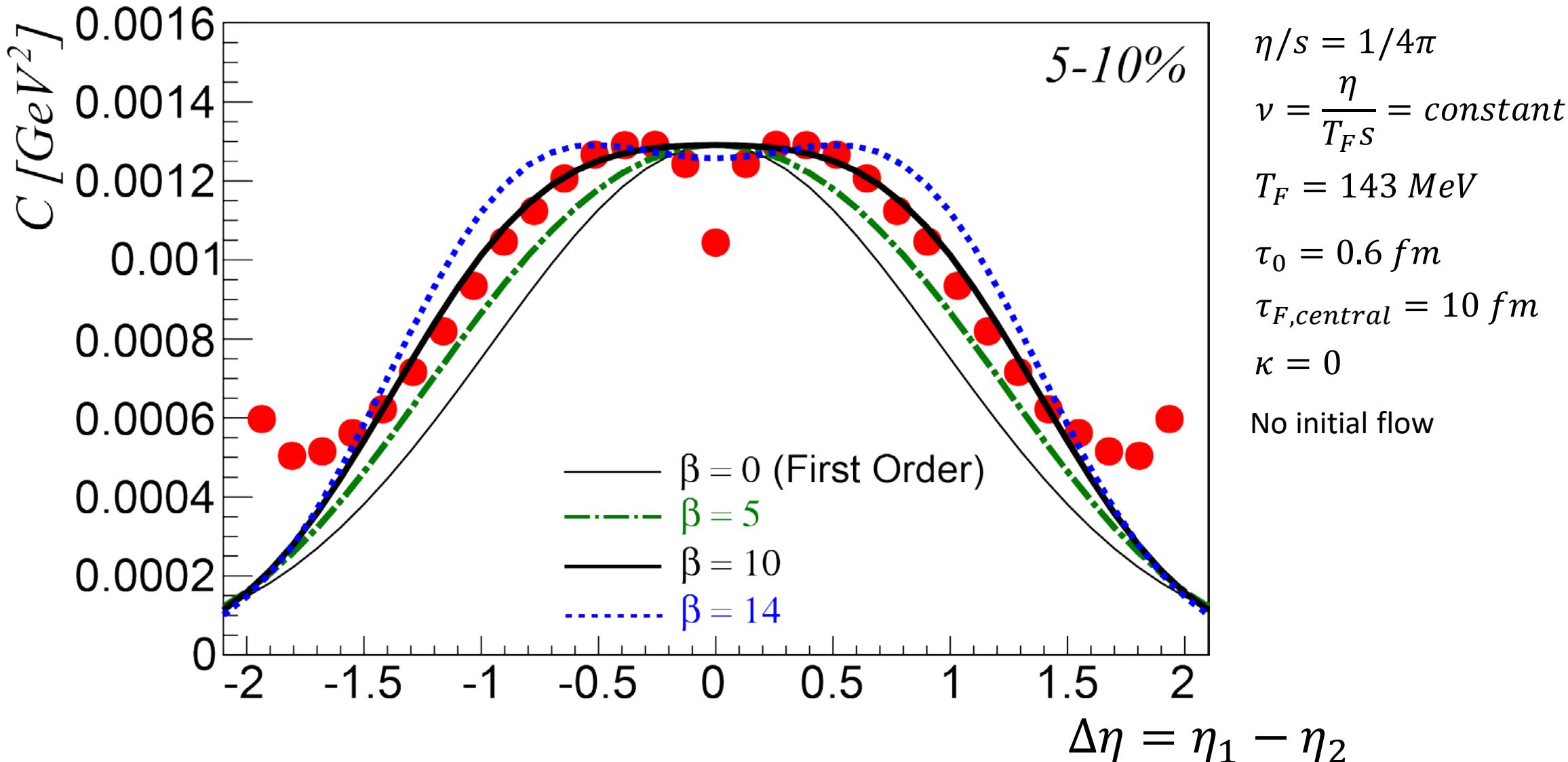
Open Questions

- Influence of sound and heat modes on observables
- Charge balancing, resonances, jets, HBT



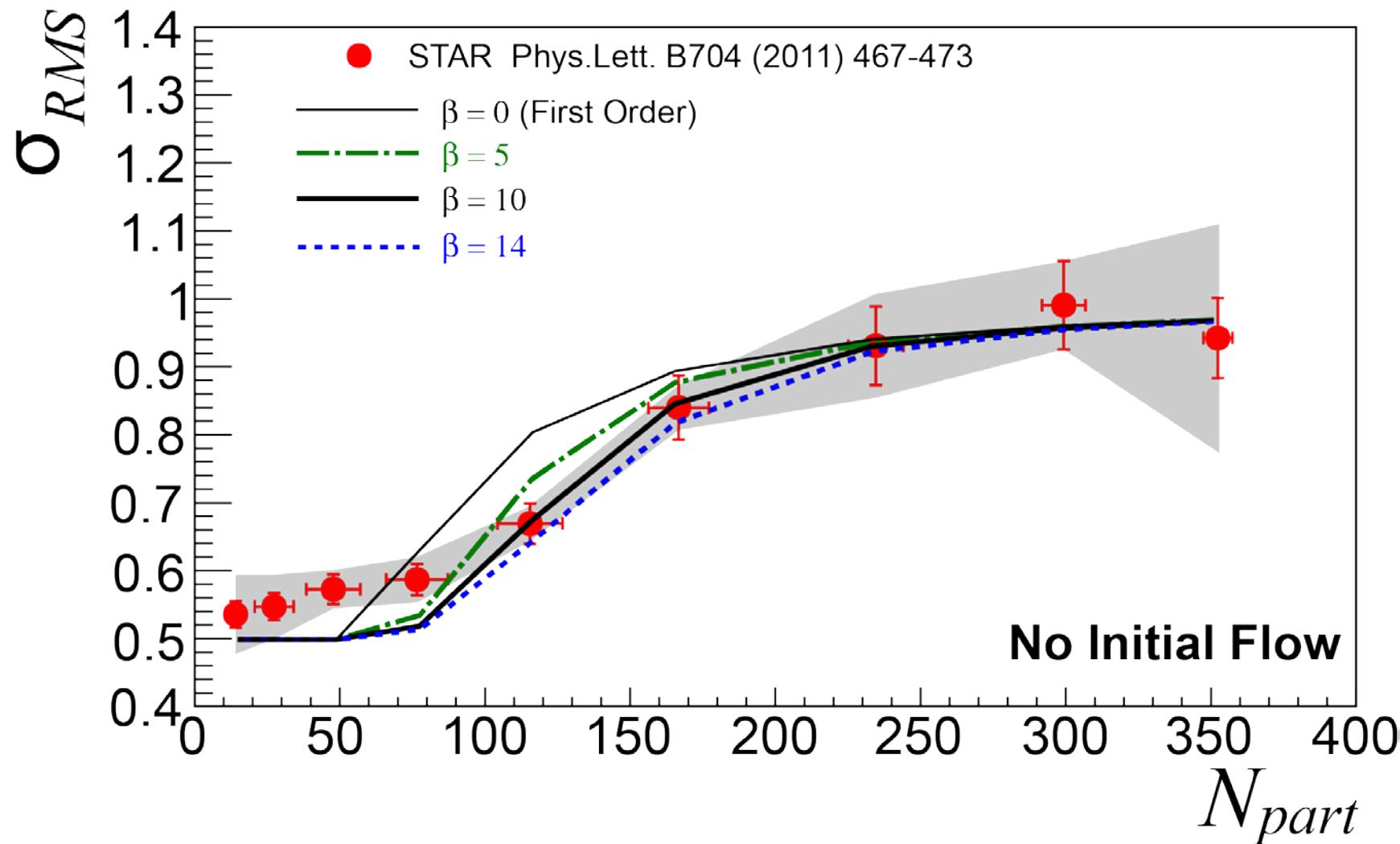
What the Experiment Can Show Us

Data from the STAR Collaboration,
Phys.Lett. B704 (2011) 467



What the Experiment Can Show Us

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$$\eta/s = 1/4\pi$$

$$\nu = \frac{\eta}{T_F s} = \text{constant}$$

$$T_F = 143 \text{ MeV}$$

$$\tau_0 = 0.6 \text{ fm}$$

$$\tau_{F,central} = 10 \text{ fm}$$

$$\kappa = 0$$

No initial flow

The stress energy tensor relaxes due to shear viscous forces. In second order hydrodynamics (we will keep only shear contributions)

$$\Delta_\alpha^\mu \Delta_\beta^\nu D \Pi^{\alpha\beta} = -\frac{1}{\tau_\pi} (\Pi^{\mu\nu} - S^{\mu\nu}) - \kappa \nabla_\alpha u^\alpha \Pi^{\mu\nu}$$

$$D = u^\mu \partial_\mu$$

$$\nabla_\mu = \partial_\mu - u_\mu u^\nu \partial_\nu$$

$$S^{\mu\nu} = \eta \left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha \right)$$

Here we require $\partial_\mu (T_{ideal}^{\mu i} + \Pi^{\mu i}) = 0$ leading to Israel-Stewart equations.

Schematically $\frac{\partial}{\partial t} \Pi^{\mu i} \sim -\frac{1}{\tau_\pi} (\Pi^{\mu\nu} - S^{\mu\nu})$