

Relativistic perfect-fluid dynamics with spin

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Introduction

A new framework for relativistic hydrodynamics with spin is proposed. It is based on the conservation laws for charge, energy, momentum, and angular momentum. The conservation laws lead to hydrodynamic equations for the charge density, local temperature, and fluid velocity, as well as for the spin tensor. The resulting set of differential equations extend the standard picture of perfect-fluid hydrodynamics with a conserved entropy current in a minimal way [1, 2].

Local distribution functions for spin-1/2 particles

- Starting point [3]

$$f_{rs}^+(x, p) = \frac{1}{2m} \bar{u}_r(p) X^+ u_s(p), \quad f_{rs}^-(x, p) = -\frac{1}{2m} \bar{v}_s(p) X^- v_r(p)$$

$$X^\pm = \exp [\pm \xi(x) - \beta_\mu(x)p^\mu] M^\pm, \quad M^\pm = \exp \left[\pm \frac{1}{2} \omega_{\mu\nu}(x) \Sigma^{\mu\nu} \right]$$

with $\beta^\mu = u^\mu/T$, $\xi = \mu/T$, $\Sigma^{\mu\nu} = (i/4)[\gamma^\mu, \gamma^\nu]$

- $\omega_{\mu\nu}$ analogue to EM field-strength tensor $F_{\mu\nu} = E_\mu u_\nu - E_\nu u_\mu + \epsilon_{\mu\nu\beta\gamma} u^\beta B^\gamma$

$$\omega_{\mu\nu} \equiv k_\mu u_\nu - k_\nu u_\mu + \epsilon_{\mu\nu\beta\gamma} u^\beta \omega^\gamma$$

- Assumptions: $k \cdot \omega = 0$, and $k \cdot k - \omega \cdot \omega \geq 0$ (ζ is real)

$$M^\pm = \cosh(\zeta) \pm \frac{\sinh(\zeta)}{2\zeta} \omega_{\mu\nu} \Sigma^{\mu\nu}, \quad \zeta \equiv \frac{1}{2} \sqrt{k \cdot k - \omega \cdot \omega}$$

- ζ imaginary studied in [4]

Basic conservation laws

- Charge current [5]

$$N^\mu = \int \frac{d^3 p}{2(2\pi)^3 E_p} p^\mu [\text{tr}_4(X^+) - \text{tr}_4(X^-)] = n u^\mu$$

$$n = 4 \cosh(\zeta) \sinh(\xi) n_{(0)}(T) = \underbrace{(e^\zeta + e^{-\zeta})}_{\text{spin-up + spin-down}} \underbrace{(e^\xi - e^{-\xi})}_{\text{particles - antiparticles}} n_{(0)}(T)$$

- Energy-momentum tensor [5]

$$T^{\mu\nu} = \int \frac{d^3 p}{2(2\pi)^3 E_p} p^\mu p^\nu [\text{tr}_4(X^+) + \text{tr}_4(X^-)] = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

$$\varepsilon = 4 \cosh(\zeta) \cosh(\xi) \varepsilon_{(0)}(T), \quad P = 4 \cosh(\zeta) \cosh(\xi) P_{(0)}(T)$$

- Entropy current

$$s = u_\mu S^\mu = \frac{\varepsilon + P - \mu n - \Omega w}{T}$$

Ω defined through the relation $\zeta = \Omega/T$, and

$$w = 4 \sinh(\zeta) \cosh(\xi) n_{(0)}$$

- New thermodynamic variable Ω ("Spin chemical potential")

$$s = \frac{\partial P(T, \mu, \Omega)}{\partial T} \Big|_{\mu, \Omega}, \quad n = \frac{\partial P(T, \mu, \Omega)}{\partial \mu} \Big|_{T, \Omega}, \quad w = \frac{\partial P(T, \mu, \Omega)}{\partial \Omega} \Big|_{T, \mu}$$

- Conservation of energy and momentum

$$\partial_\mu T^{\mu\nu} = 0 \implies T \partial_\mu(s u^\mu) + \mu \partial_\mu(n u^\mu) + \Omega \partial_\mu(w u^\mu) = 0 \quad (1)$$

Charge conservation

$$\partial_\mu(n u^\mu) = 0. \quad (2)$$

Perfect fluid \Rightarrow entropy conservation \Rightarrow we demand extra conservation law

$$\partial_\mu(w u^\mu) = 0 \quad (3)$$

Self-consistent with entropy conservation $\partial_\mu(s u^\mu) = 0$

- Eqs. (1), (2), (3) are the hydrodynamic spin background equations

Spin dynamics

- Total angular momentum tensor ("orbital" + "spin")

$$J^{\lambda, \mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + S^{\lambda, \mu\nu}$$

Conservation of energy and momentum

$$\partial_\lambda J^{\lambda, \mu\nu} = 0 \implies \partial_\lambda S^{\lambda, \mu\nu} = T^{\mu\nu} - T^{\mu\nu}$$

In our case $T^{\mu\nu}$ is symmetric, i.e.,

$$\partial_\lambda S^{\lambda, \mu\nu} = 0$$

- For the spin tensor we use [6]

$$S^{\lambda, \mu\nu} = \int \frac{d^3 p}{2(2\pi)^3 E_p} p^\lambda \text{tr}_4 [(X^+ - X^-) \Sigma^{\mu\nu}] = \frac{w u^\lambda}{4\zeta} \omega^{\mu\nu}$$

Rescaled spin-polarization tensor $\bar{\omega}^{\mu\nu} = \omega^{\mu\nu}/(2\zeta)$

$$u^\lambda \partial_\lambda \bar{\omega}^{\mu\nu} = \frac{d \bar{\omega}^{\mu\nu}}{d\tau} = 0$$

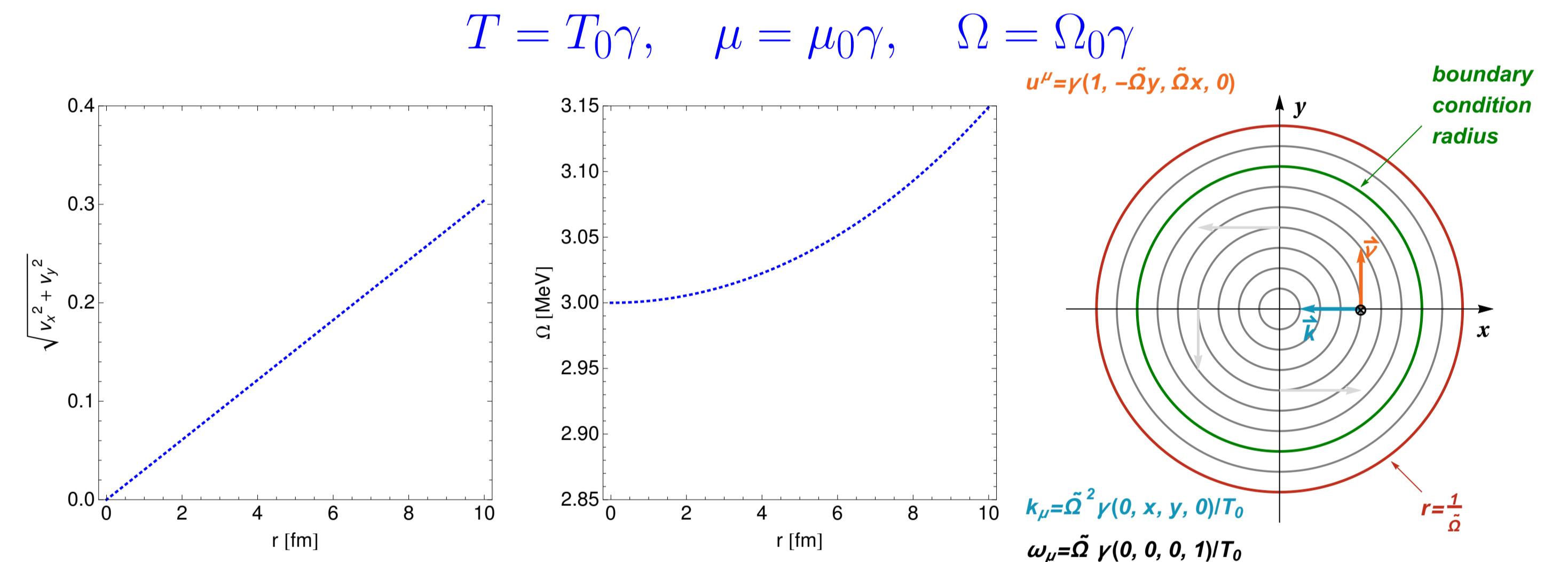
Global equilibrium with rotation

- Rigid rotor: $u^0 = \gamma$, $u^1 = -\gamma \tilde{\Omega} y$, $u^2 = \gamma \tilde{\Omega} x$, $u^3 = 0$

$\tilde{\Omega}$ is a constant, $\gamma = 1/\sqrt{1 - \tilde{\Omega}^2 r^2}$, $r^2 = x^2 + y^2$

Due to limiting light speed, $0 \leq r \leq R < 1/\tilde{\Omega}$

- Solution for hydrodynamic spin background:

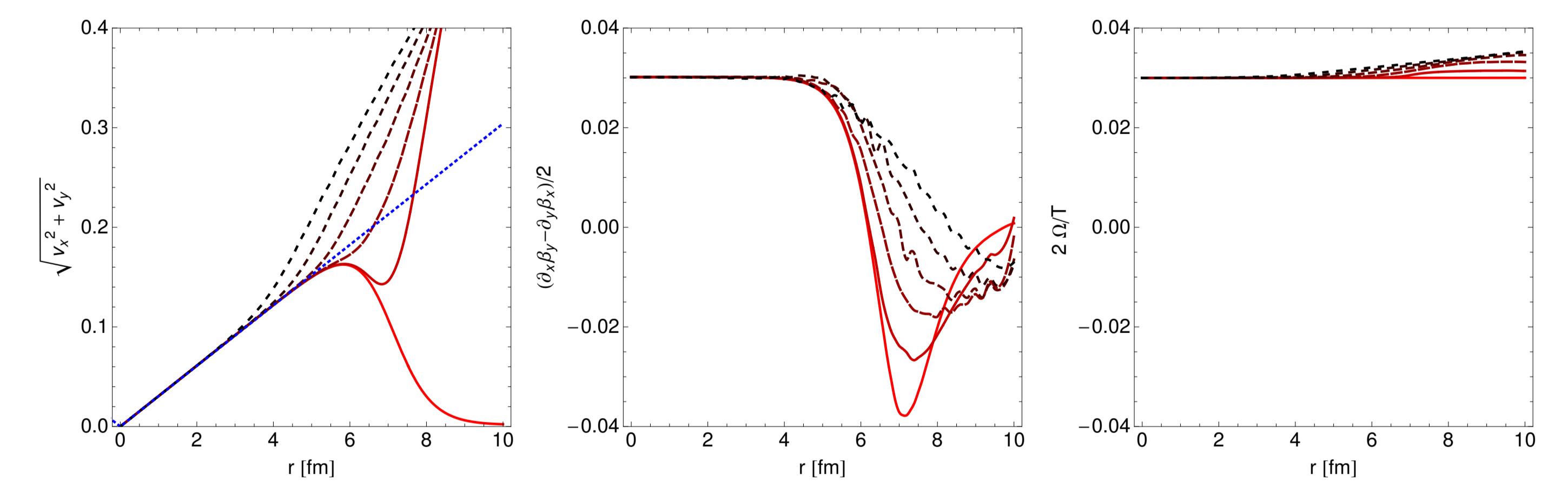


- Polarized vortex:

$$\omega_{xy} = -\omega_{yx} = \tilde{\Omega}/T_0, \quad (\omega_{\mu\nu} = 0 \text{ otherwise}) \quad \text{with} \quad \tilde{\Omega} = 2\Omega_0$$

$\omega_{\mu\nu} = \text{thermal vorticity: } \omega_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$

Isolated vortex



Time increases by 2 fm: red \rightarrow black lines

- $\omega_{\mu\nu} \neq$ thermal vorticity

Outlook

- Dissipative effects
- Spin-orbit interaction \Rightarrow Asymmetric $T^{\mu\nu}$

References

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