Deutsche Forschungsgemeinschaft

ISOQUANT SFB1225

In-medium heavy quark potential for quarkonium as an open quantum system

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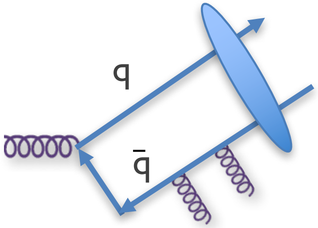
References:

with Y. Burnier and O.Kaczmarek	JHEP 1512 (2015) 101, JHEP 1610 (2016) 032
with S. Kajimoto, Y. Akamatsu, M. Asakawa	PRD97 (2018) 014003
with P. Petreczky and J. Weber	in preparation
with B. Krouppa and M. Strickland	PRD97 (2018) 016017

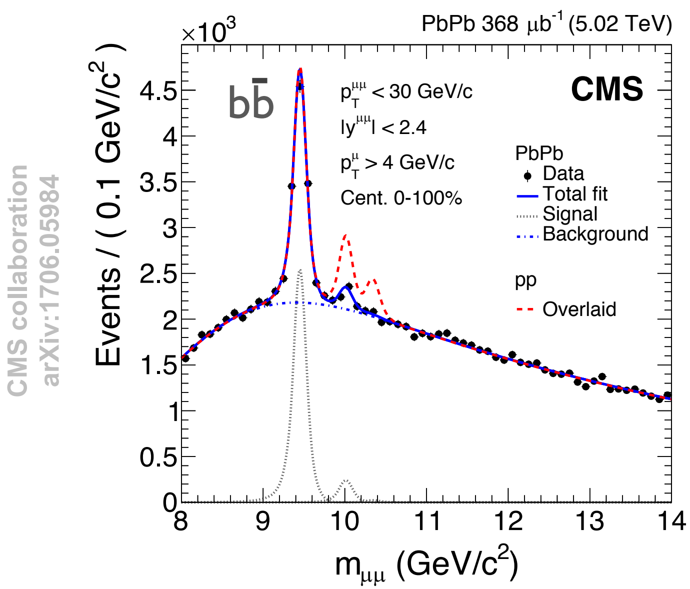
Motivation: Heavy-ion collisions

- Hard probe: susceptible to medium but distinguishable from it $Q_{probe} > T_{med}$

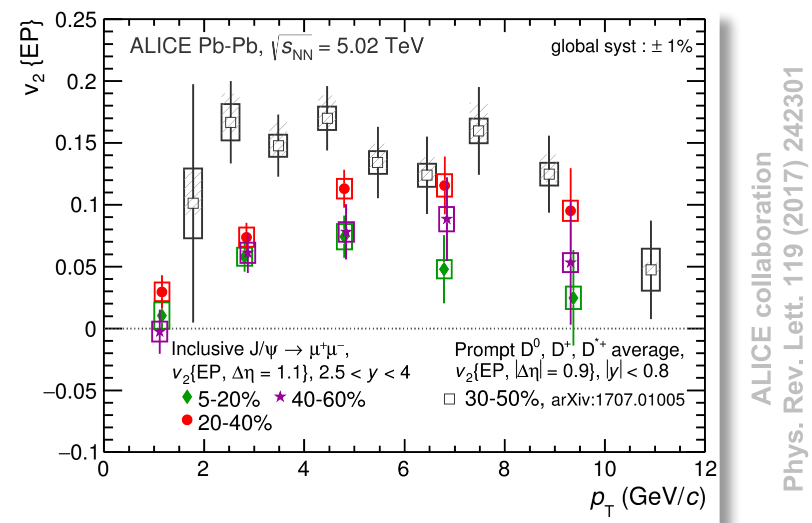


Quarkonium separation of scales: $M_Q > T_{med}$, $M_Q > \Lambda_{QCD}$

In vacuum: m^{Υ}=9.460GeV, Γ^{Υ} = 54(1)keV; $m^{J/\psi}$=3.096 GeV, $\Gamma^{J/\psi}$ = 93(3)keV



bb: samples nonequilibrium evolution



cc: thermal probe of the late stages

- Goal: Description of the (non-)equilibrium evolution of QQ via an in-medium potential

Outline

- Motivation
- Current state of the T>0 heavy quark potential from the lattice
- Application of the potential to Bottomonium evolution
- $Q\bar{Q}$ real-time evolution as open quantum system
- Summary

Heavy quark potential from EFT

- The era of model potentials is finally over: ***genuine QCD definition*** available
- pNRQCD ***effective field theory***: $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1$
  - Describes ***thermal QQ*** as singlet and octet wavefunctions: $\psi_S(R,t), \psi_O(R,t)$

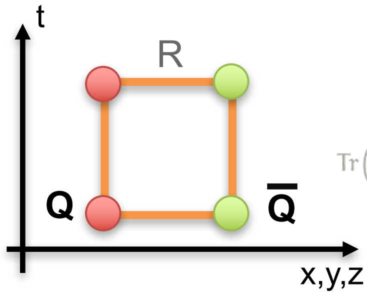
$$i\partial_t \psi_S = \Big(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\Big)\psi_S$$

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423
Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

  - Derived from QCD: V^{QCD} as Wilson coefficient determined via matching at m=∞

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} \in \mathbb{C}$$

Im[V]: Laine et al. JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008



$$W_\square(R,t) = \mathrm{Tr}\Big(\exp\Big[-i\int_\square dx^\mu A_\mu(x)\Big]\Big)$$

- Challenge: real-time definition not directly evaluable in lattice QCD simulations

Extracting V^{QCD} from the lattice

- How to connect to the Euclidean domain: **spectral functions**

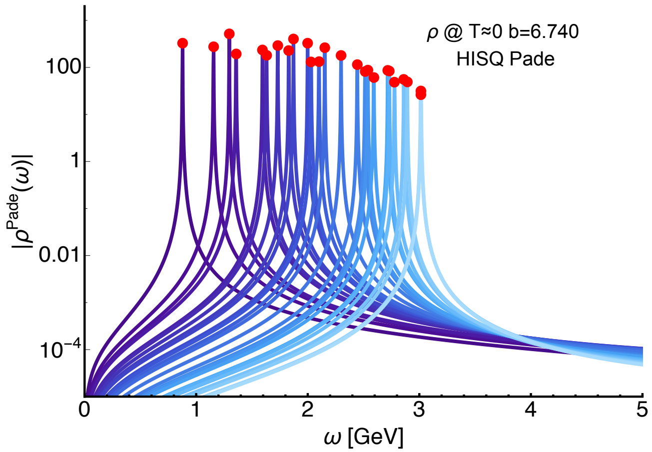
A.R., T.Hatsuda & S.Sasaki
PRL 108 (2012) 162001

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

Spectral Decomposition

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)}$$

A.R. & TUM-QCD collaboration
(In preparation)



Spectral Reconstruction

- In case of sizable $\Delta W/W=10^{-2}$ statistical uncertainty in $W_\square$: **Bayesian inference**

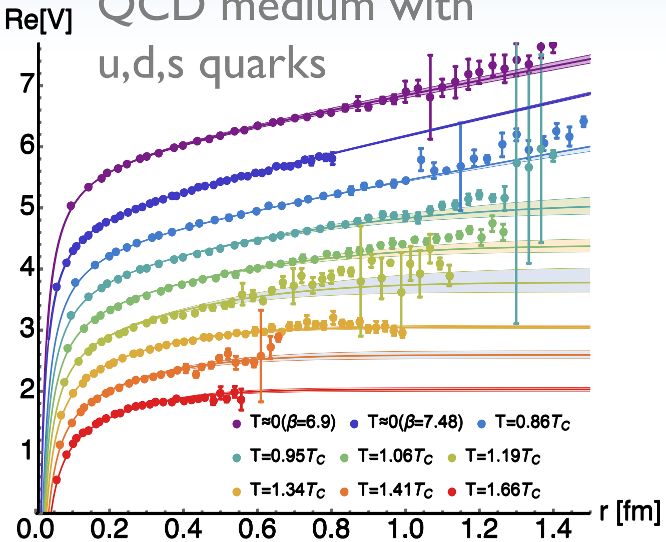
incorporate prior information to regularize the inversion task (BR method)

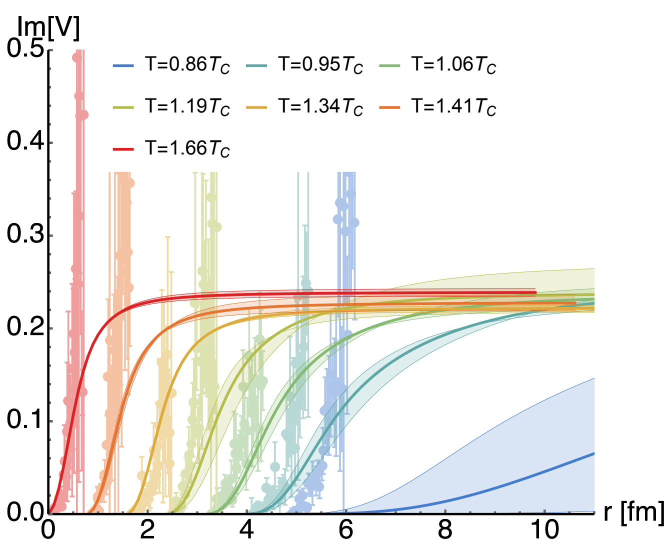
- In case of small $\Delta W/W=10^{-3}$ statistical uncertainty in $W_\square$ also **Pade approximation**

exploit the analyticity of the Wilson correlator to extract spectra

T>0 static potential from the lattice

QCD medium with u,d,s quarks



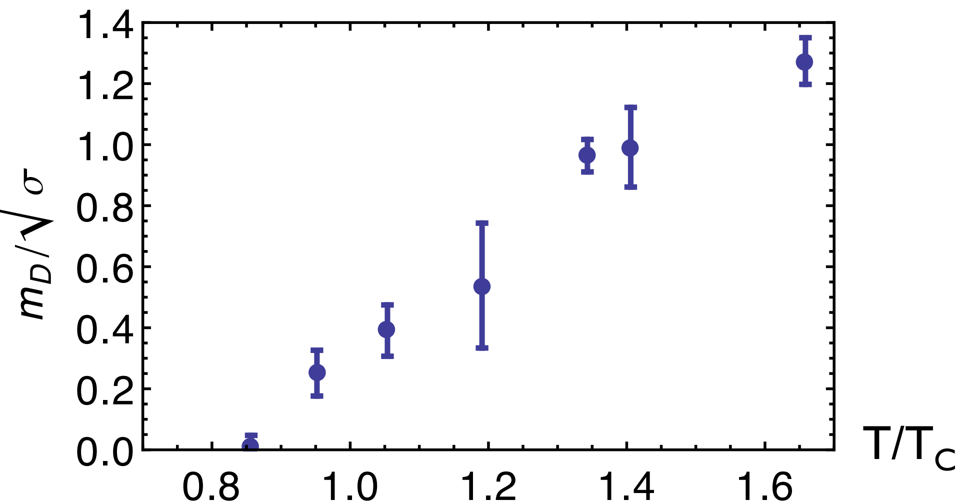


Y.Burnier, O.Kaczmarek, A.R. PRL 114 (2015) 082001

Nf=2+1, 48^3x12, asqtad action, m_π~300MeV

- Best estimate from N_f=2+1 lQCD at m_π~300MeV
- At T~0 Re[V] on the lattice well described by naïve Cornell ansatz: V= -α/r + σr + c
- Analytic parametrization from a generalized gauss law with a single T-dep. parameter m_D

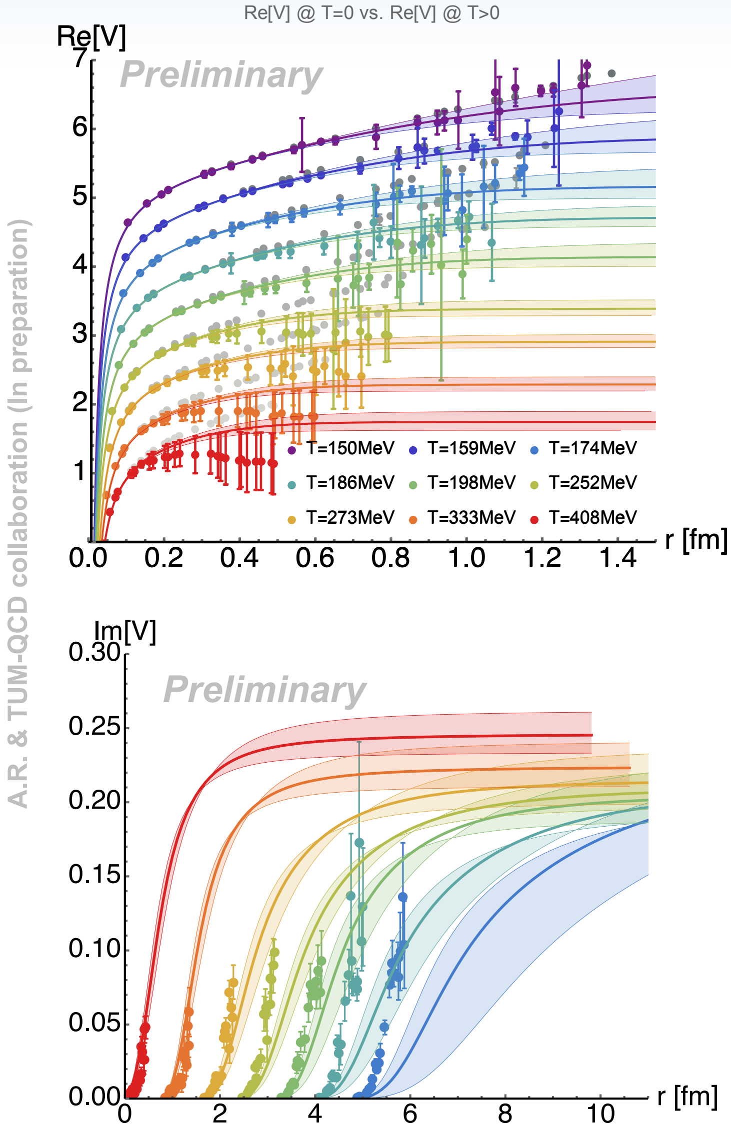
Y. Burnier, A.R. PLB753 (2016) 232



- In absence of continuum limit: manual corrections

$$m^\pi_{\rm lat} > m^\pi_{\rm phys} \qquad \sigma^{Q\bar{Q}}_{\rm lat} \neq \sigma^{Q\bar{Q}}_{\rm phys} \qquad T^c_{\rm lat} > T^c_{\rm phys}$$

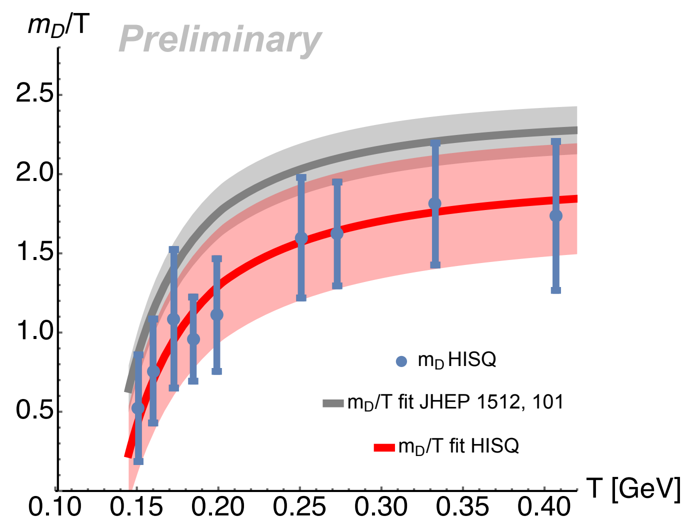
Towards a genuine continuum limit



- Together with TUM-QCD collaboration progress by use of more realistic lattices (larger T range):

$$T^c_{lat} = 159\text{MeV} \qquad m^\pi_{lat} = 160\text{MeV} \qquad 48^3\text{x}12$$

- Order of magnitude better statistics allows spectral reconstruction via Pade approximation



- Towards continuum limit with upcoming TUM-QCD 48^3x16 ensembles (results eta. at QM18)

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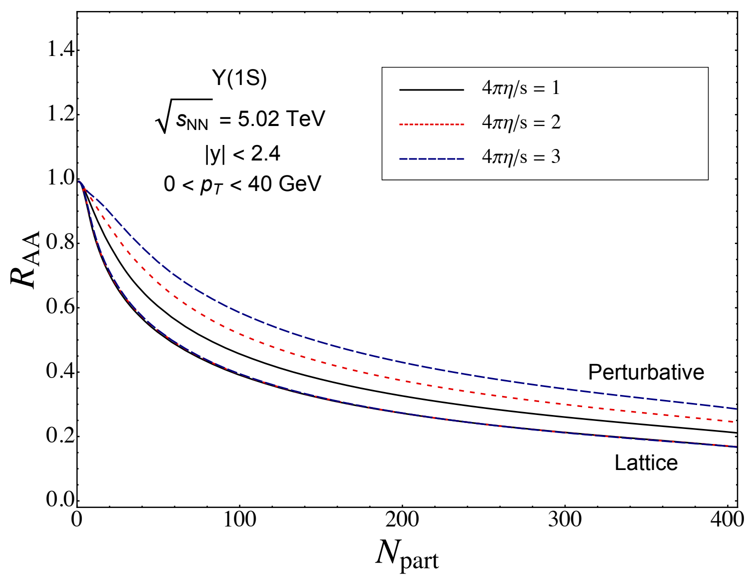
Bottomonium in HIC

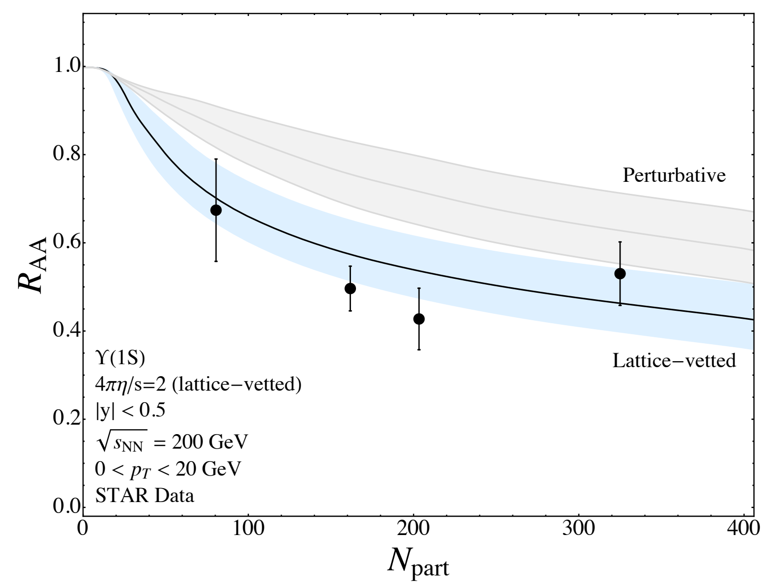
- Current state-of-the-art setup to describe bottomonium has three ingredients:

Anisotropic Hydro ($m_D(\tau,x)$)	&	Glauber initial pure state QQ prod.	&	V_{QQ} in Schrödinger Eq. E_{bind} and $\Gamma(\tau,x)$

Folding together over real-time evolution gives R_{AA}

- Intent: remove modelling input (color singlet free energies + pert. ImV)
- Larger Im[V] in the lattice vetted potential leads to stronger suppression





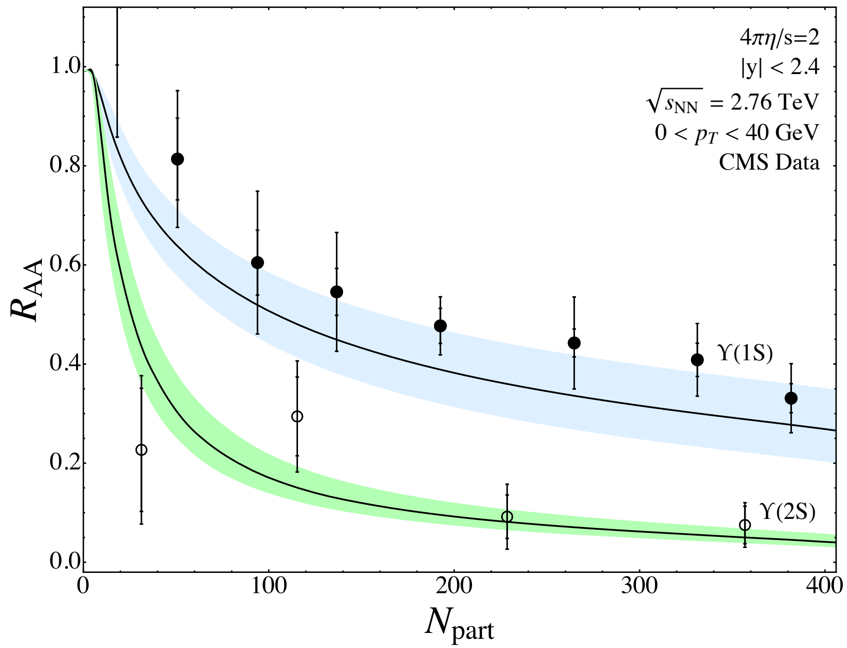
B. Kroppa, A.R., M. Strickland
PRD97 (2018) 016017

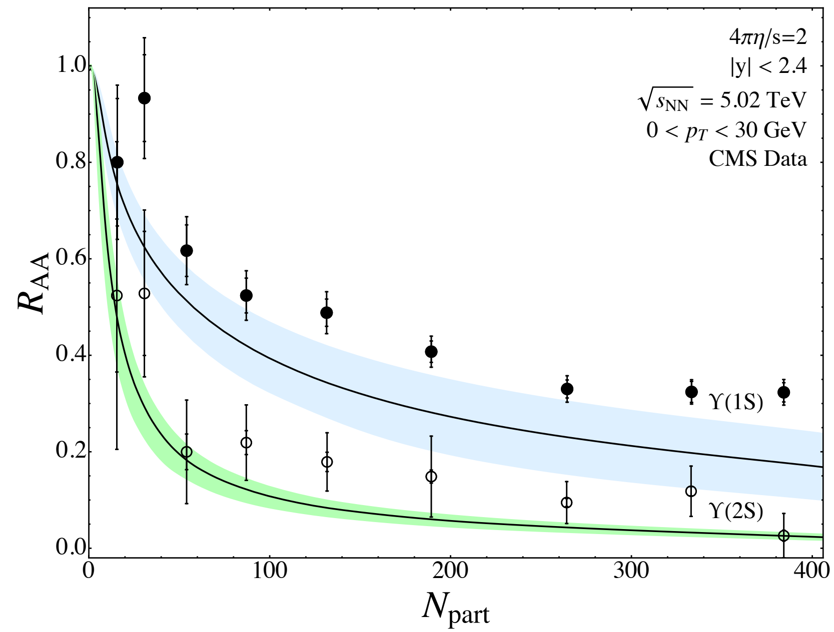
- V^{QCD} form and temperature dependence significantly reduce η/s dependence

Bottomonium at LHC energies

- At 2.76TeV slightly low on Υ(1S), excellent agreement with Υ(2S)

Abstract submitted to QM2018





B. Kroupa, A.R., M. Strickland PRD97 (2018) 016017

- At 5.02TeV both Υ(1S) and Υ(2S) underestimated
- Two possible underlying reasons:

Onset of bottomonium regeneration (work in progress with M. Strickland) vs. Naïve Schrödinger equation evolution inaccurate (work in progress with Y. Akamatsu/M. Asakawa)

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The open quantum systems picture

- Is solving the Schrödinger eq. with complex V^{QCD} appropriate for unequilibrated bb?
- Need a general approach to couple quarkonium to a thermal medium
  - Overall system is closed, hermitean Hamiltonian: Schrödinger equation time evolution

$$H = H_{Q\bar{Q}} \otimes I_{med} + I_{Q\bar{Q}} \otimes H_{med} + H_{int} \qquad \frac{d}{dt}\rho(t) = -i[H, \rho]$$

  - Interested in the dynamics of the reduced QQbar system only

$$\rho_{Q\bar{Q}}(t) = \mathrm{Tr}_{med}\left[\rho(t)\right] \qquad \frac{d}{dt}\rho_{Q\bar{Q}}(t) = ?$$

- Derivation via path integral formalism: Feynman-Vernon influence functional

For details see Y. Akamatsu, Phys.Rev. D87 (2013) 4, 045016 and arXiv:1403.5783

$$\rho(t,x,y,X,Y) = \int dx_0 dy_0 dX_0 dY_0\, \rho(0,x_0,y_0,X_0,Y_0) \int_{x_0,y_0,X_0,Y_0}^{x,y,X,Y} \mathcal{D}[\bar{x},\bar{y},\bar{X},\bar{Y}] e^{iS[\bar{x},\bar{X}]-iS[\bar{y},\bar{Y}]}$$

$$\rho_{Q\bar{Q}}(t,x,y) = \int dX dY \delta(X-Y) \rho(t,x,y,X,Y)$$

$$\rho_{Q\bar{Q}}(t,x,y) = \int dx_0 dy_0\, \rho_{Q\bar{Q}}(0,x_0,y_0) \int_{x_0,y_0}^{x,y} \mathcal{D}[\bar{x},\bar{y}] e^{iS_{Q\bar{Q}}[\bar{x}]-iS_{Q\bar{Q}}[\bar{y}]+iS_{IF}[\bar{x},\bar{y}]}$$

all information about medium - QQ interaction

Unravelling the master equation

- Use scale separation: $m_Q > T > \Lambda_{QCD}$ heavy mass & weak coupling approximation

$$S_{IF} \approx S_{pot}\left[\mathrm{Re}[V]\right] + S_{fluct}\left[\mathrm{Im}[V]\right] + S_{diss}\left[\mathrm{Im}[V]\right] + S_{corr}$$

- In QM language corresponds to Markovian evolution by Lindblad equation

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_{i=1}^{N} \gamma_i \left(L_i \rho_{Q\bar{Q}} L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho_{Q\bar{Q}} - \frac{1}{2} \rho_{Q\bar{Q}} L_i^\dagger L_i \right)$$

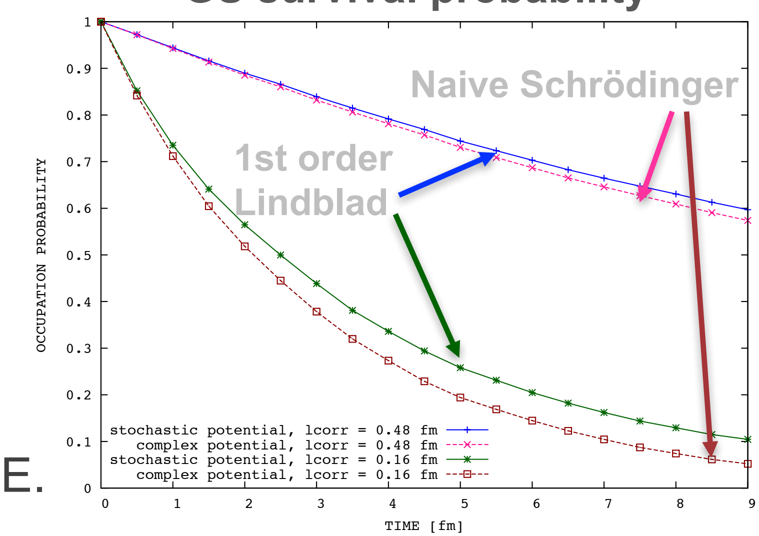
- L, $L^\dagger$ can be expressed in terms of Re[V] and Im[V] but e.o.m cannot be unravelled into a simple deterministic Schrödinger equation for ψ_{QQ}

- Naïve Schroedinger vs. 1st order gradient exp.

$$i\partial_t \psi_{Q\bar{Q}}(t) = \exp\Big[it\Big(\underbrace{-\frac{\nabla^2}{2M} + \mathrm{Re}[V]}_{S_{pot}} + \underbrace{\eta(t)}_{S_{fluct}}\Big)\Big]\psi_{Q\bar{Q}}(0)$$

$$i\partial_t \langle \psi_{Q\bar{Q}}(t) \rangle = \left(-\frac{\nabla^2}{2M} + \mathrm{Re}[V] - i\mathrm{Im}[V] \right) \langle \psi_{Q\bar{Q}}(t) \rangle$$

GS survival probability



- Clear difference, survival probability of ground state systematically underestimated by naïve S.E.

S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

Towards realistic simualtions

- Use scale separation: $m_Q > T > \Lambda_{QCD}$ heavy mass & weak coupling approximation

$$S_{IF} \approx S_{pot}\left[Re[V]\right] + S_{fluct}\left[Im[V]\right] + S_{diss}\left[Im[V]\right] + S_{corr}$$

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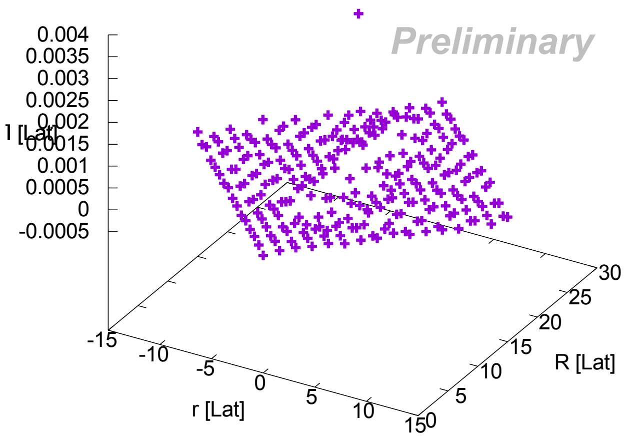
- Outlook: tackling the challenge how to unravel the full S_{IF} including dissipation?
  - Well known in condensed matter theory: quantum state diffusion allows to write down a non-linear stochastic Schrödinger equation (high cost in sampling)
  - Brute force: solve the 6d Lindblad master equation directly in the reduced relative coordinates (r,R) of the heavy quarkonium system. (high cost due to dimensionality)



Summary

- ***QCD derived static potential*** V_{QQ} for in-medium heavy-QQ available
- ***Lattice QCD progress*** in determining Re & Im of V_{QQ}
  - Full QCD results available from the lattice with manual continuum correction
    Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101
  - Ongoing work towards continuum limit on realistic HISQ lattices from TUM-QCD
    P. Petreczky, A.R., J. Weber in progress
- Promising results on ***quarkonium phenomenology*** from the potential
  - R_{AA} ***of Y(1S, 2S)*** in HIC promising for STAR, at LHC question of regeneration?
    B. Krouppa, A.R., M. Strickland PRD97 (2018) 016017
- Conceptual progress in ***open-quantum systems*** description of QQ
  - Consistent potential based real-time evolution beyond the naïve Schrödinger equation
    S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003 & in preparation

Thank you for your attention