Fictions, fluctuations and mean fields

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Constraining the QCD Phase Boundary with Data from Heavy Ion Collisions

February 12, 2018, GSI, Darmstadt

in collaboration with Peter Petreczky, arXiv:1708.00879

The speaker has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 665778 via the National Science Center, Poland, under grant Polonez DEC-2015/19/P/ST2/03333

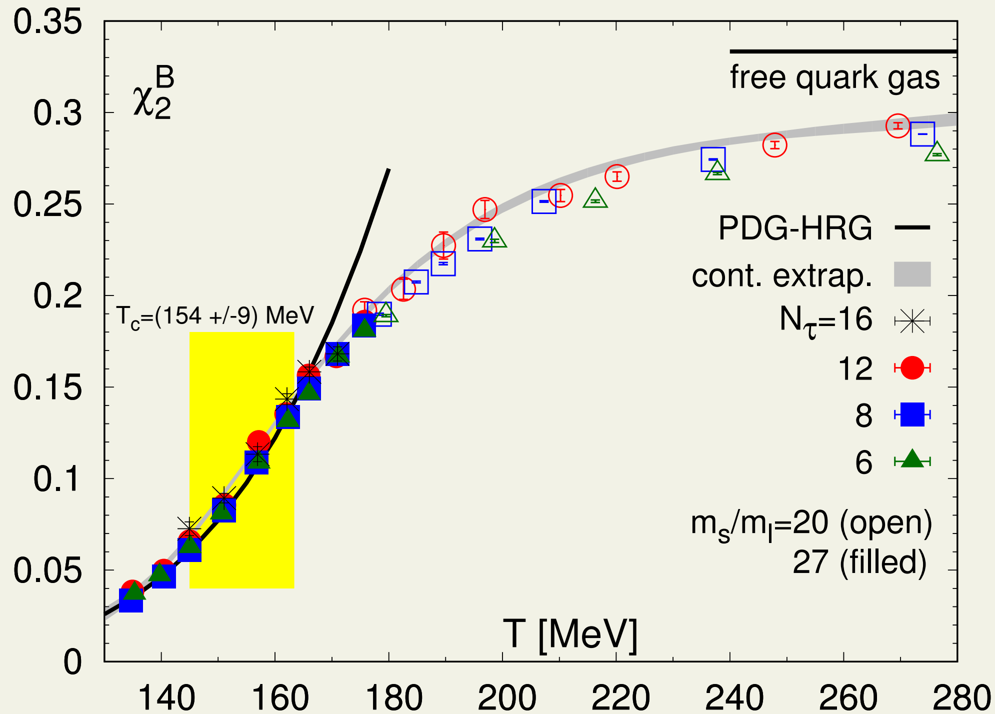
Fiction, noun

A fictitious particle, i.e. a particle predicted by some model without solid empirical evidence for its existence

Fluctuations of conserved charges

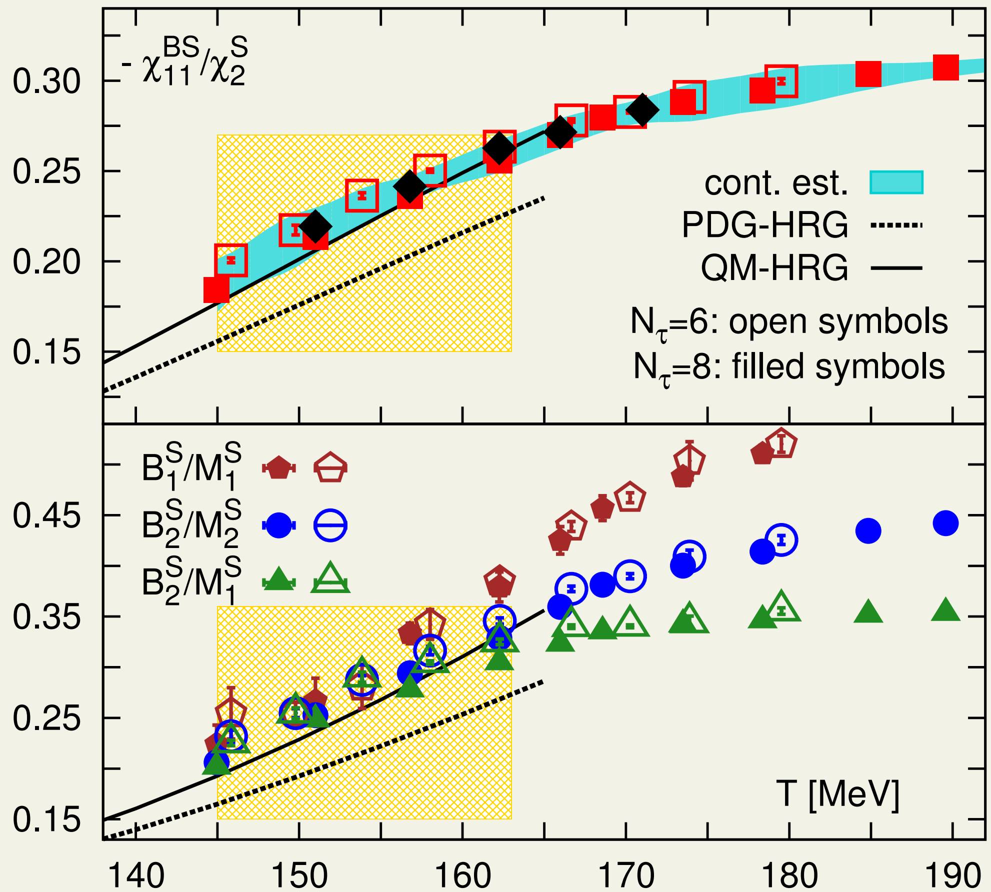
$$\chi_n^X = T^n \frac{\partial^n P/T^4}{\partial \mu_X^n}\bigg|_{\mu_X=0}$$

$$\chi_{nm}^{XY} = T^{n+m} \frac{\partial^{n+m} P/T^4}{\partial \mu_X^n \partial_Y^m}\bigg|_{\mu_X=0,\, \mu_Y=0}$$



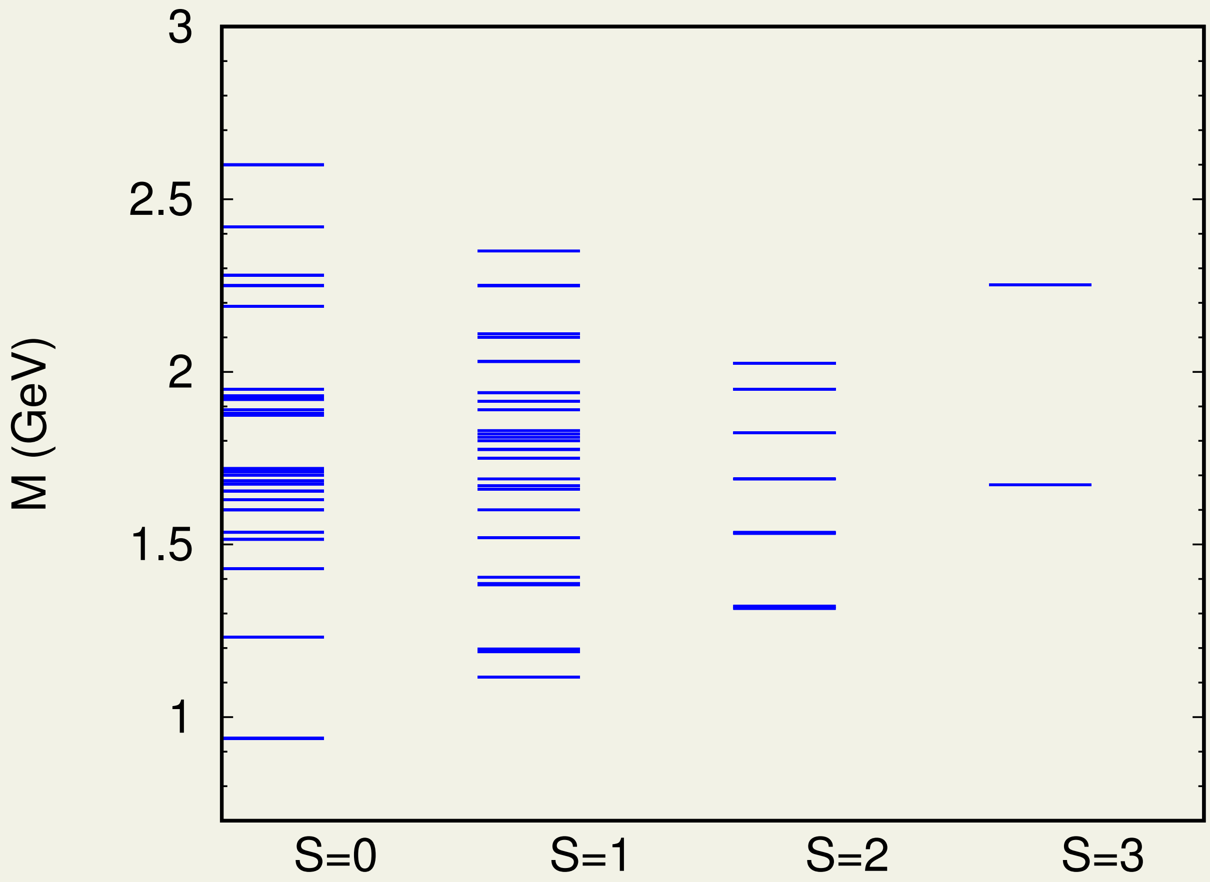
Bazavov et al., PRD95, 054504 (2017)

More resonances?



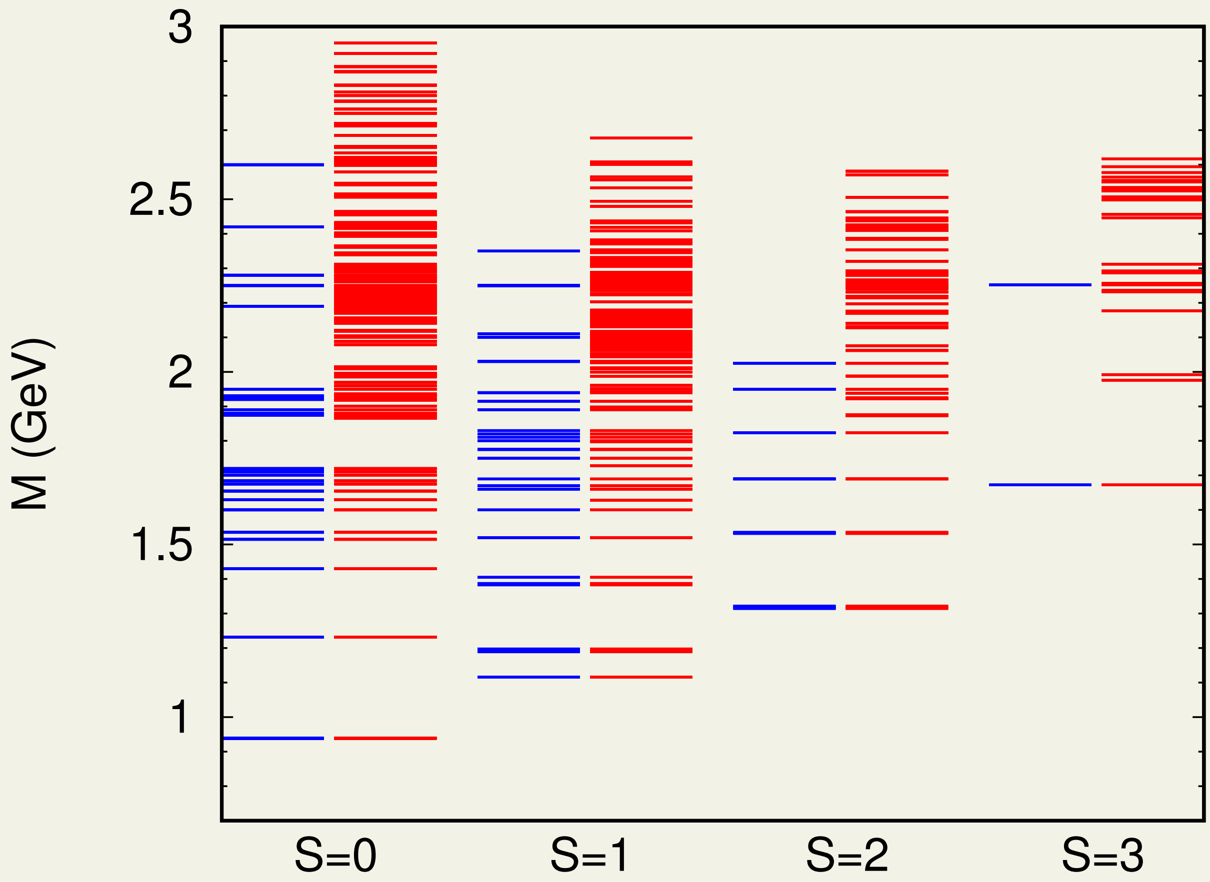
Bazavov et al., PRL113, 072001 (2014)

Baryon spectrum



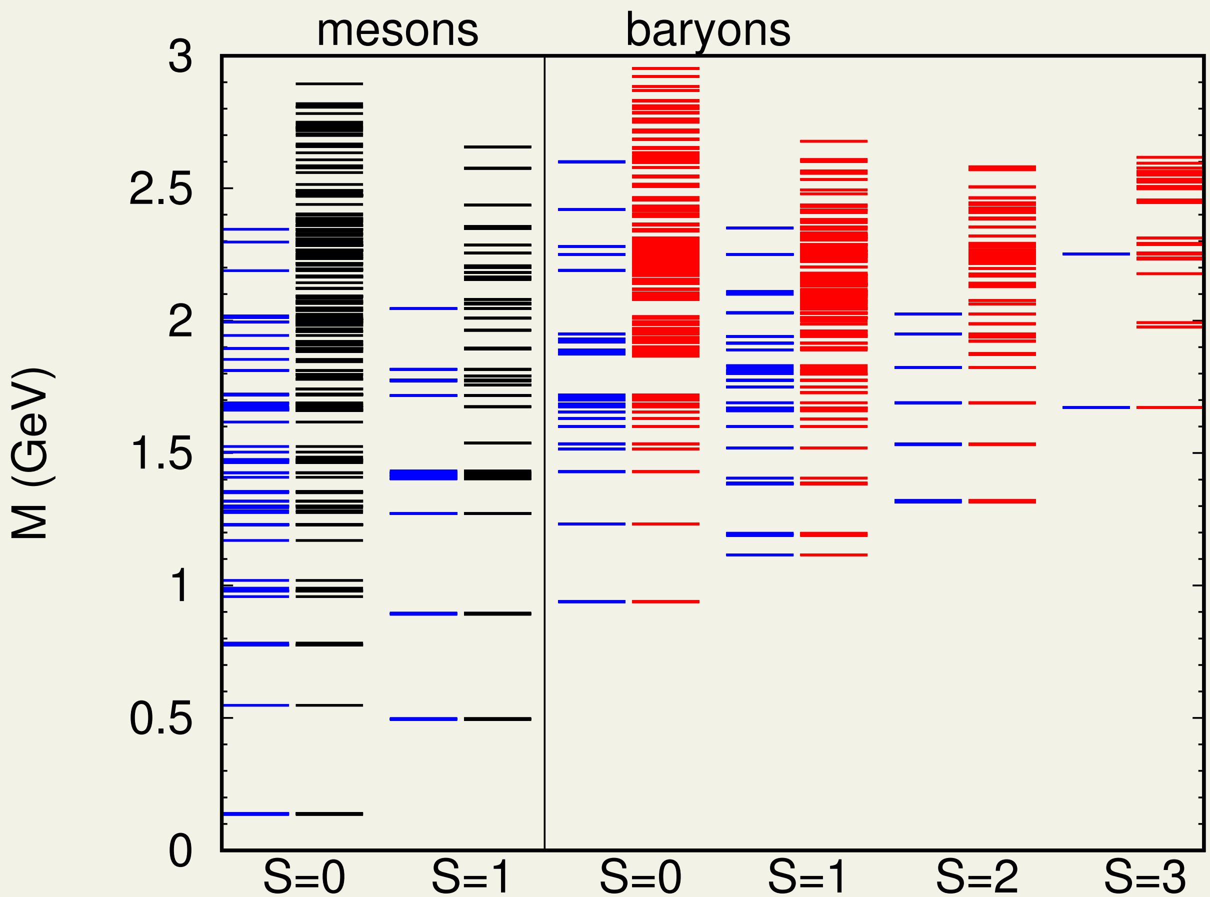
Blue: Particle Data Group

Baryon spectrum



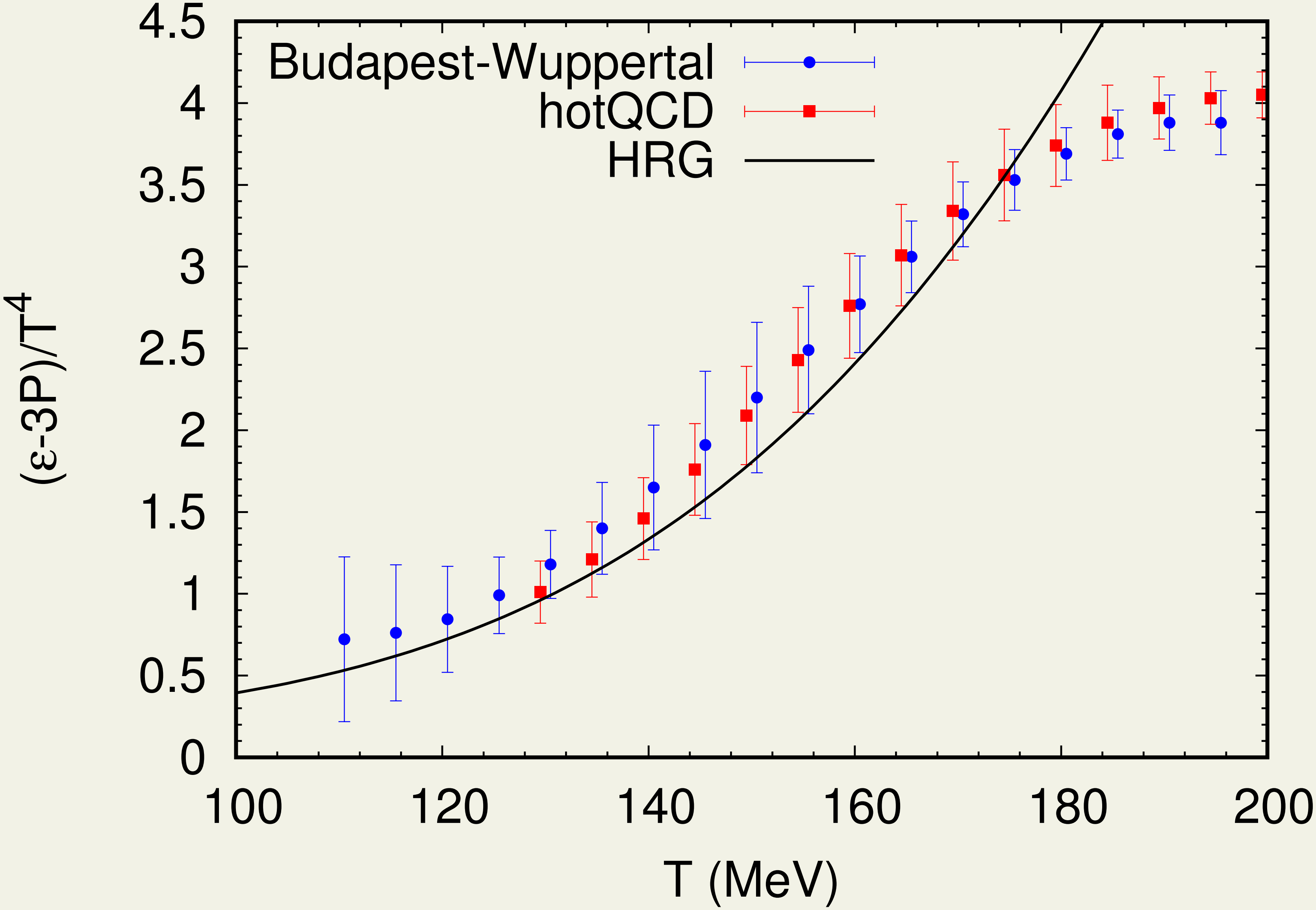
Blue: Particle Data Group
Red: PDG + Löring et al., EPJA10, 395 (2001) & EPJA10, 447 (2001)

Hadron spectrum



Blue: Particle Data Group
Red: PDG + Löring et al., EPJA10, 395 (2001) & EPJA10, 447 (2001)
Black: PDG + Ebert et al., PRD79, 114029 (2009)

Trace anomaly



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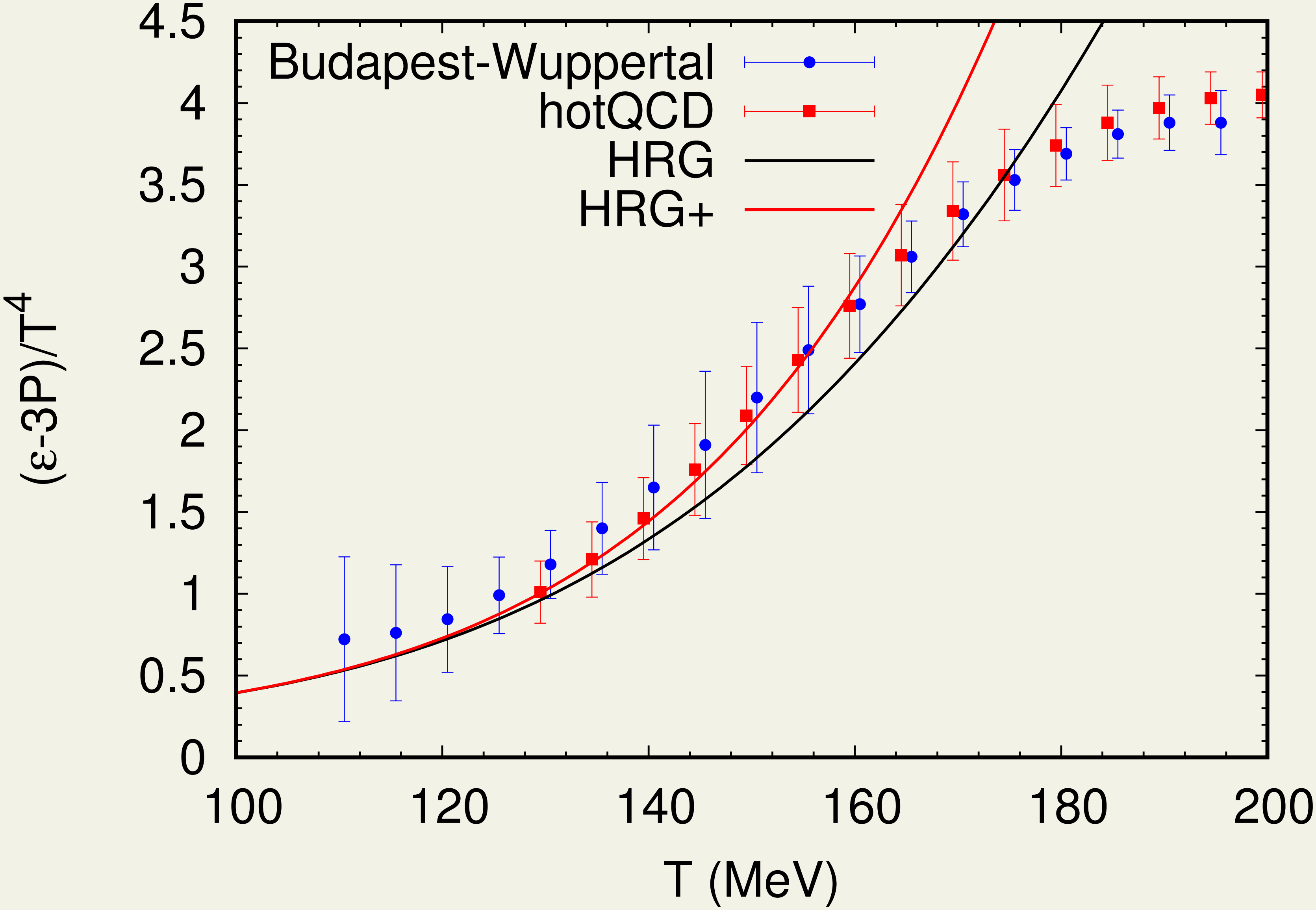
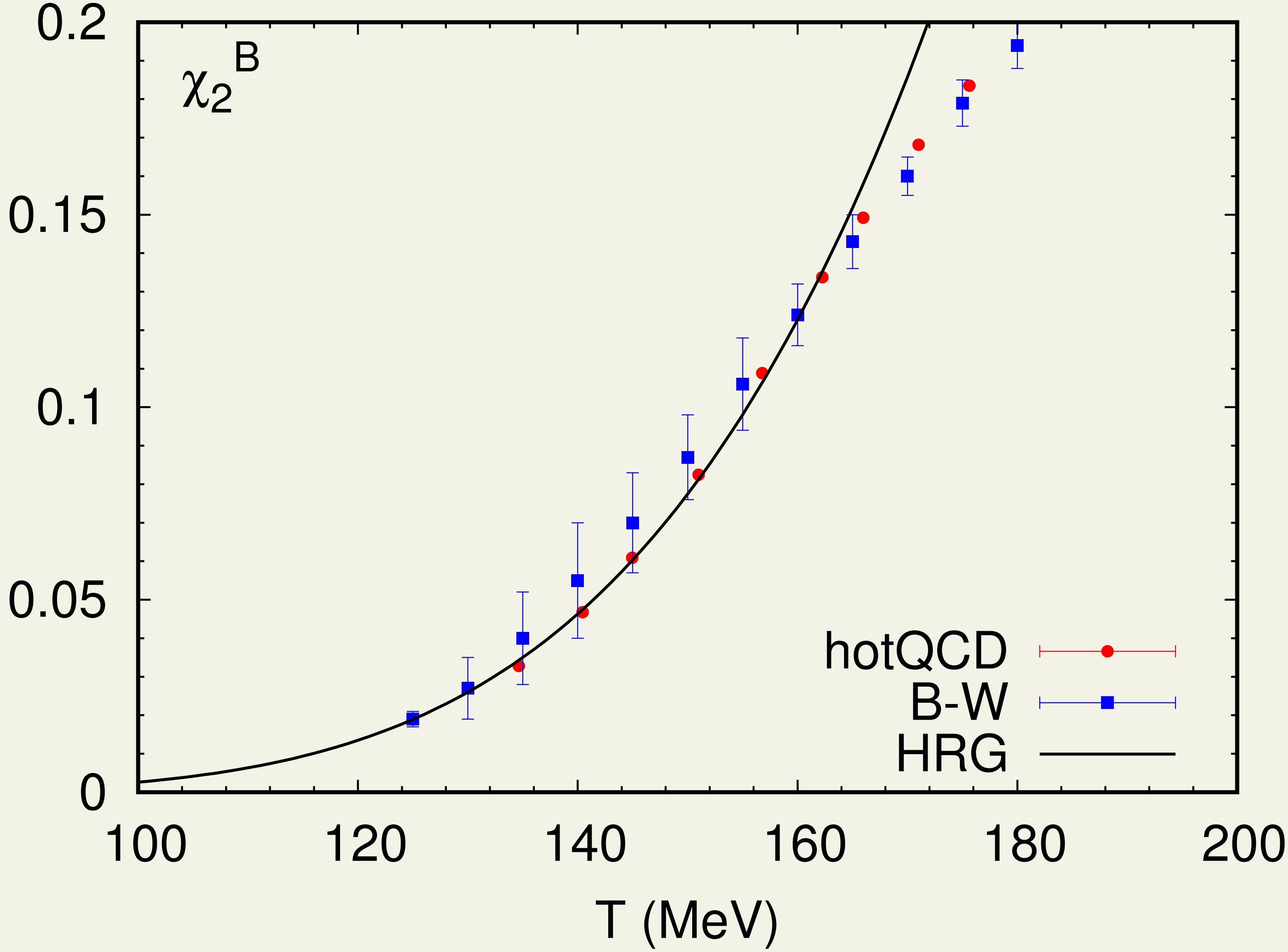


χ_B^2



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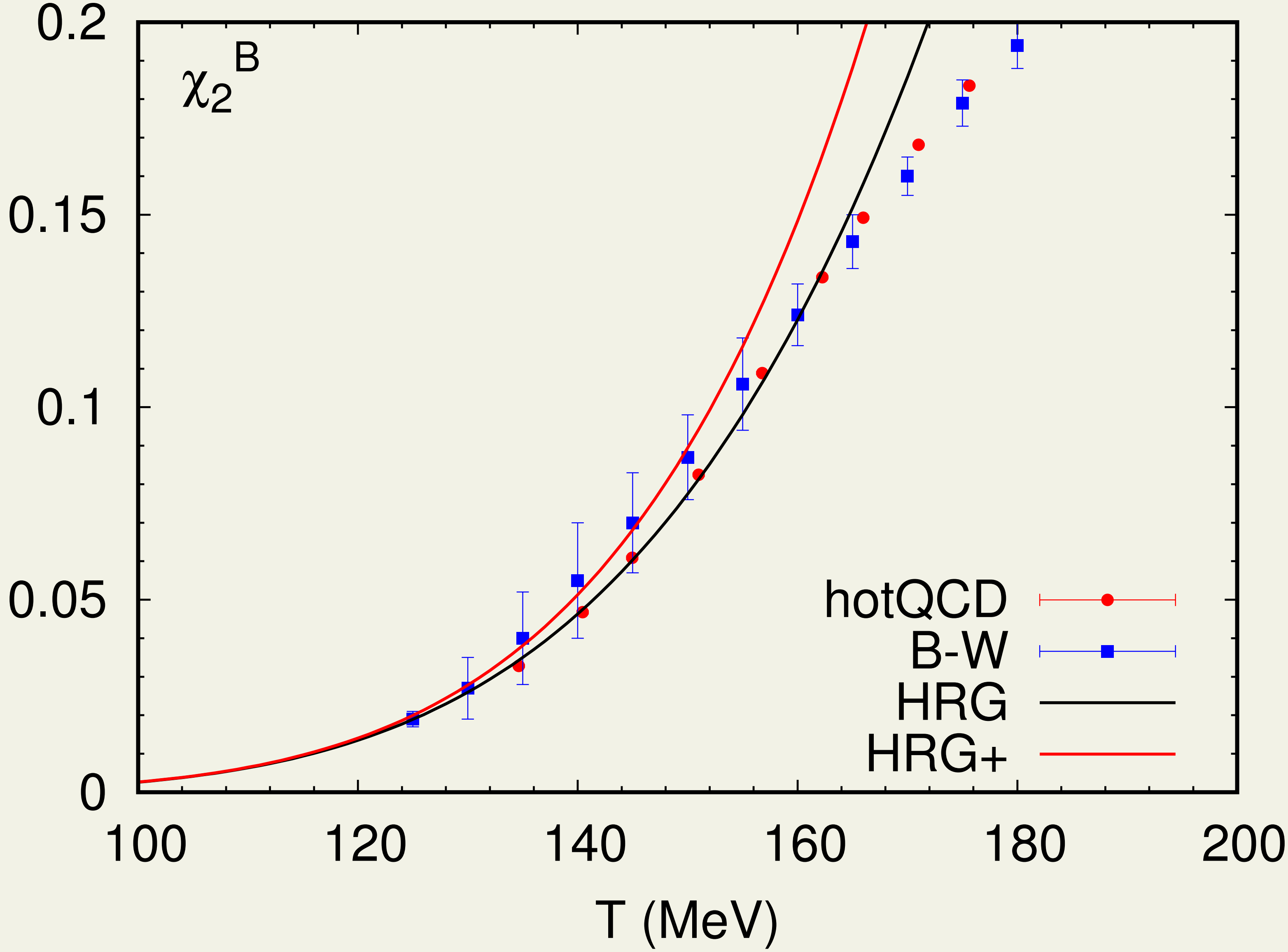

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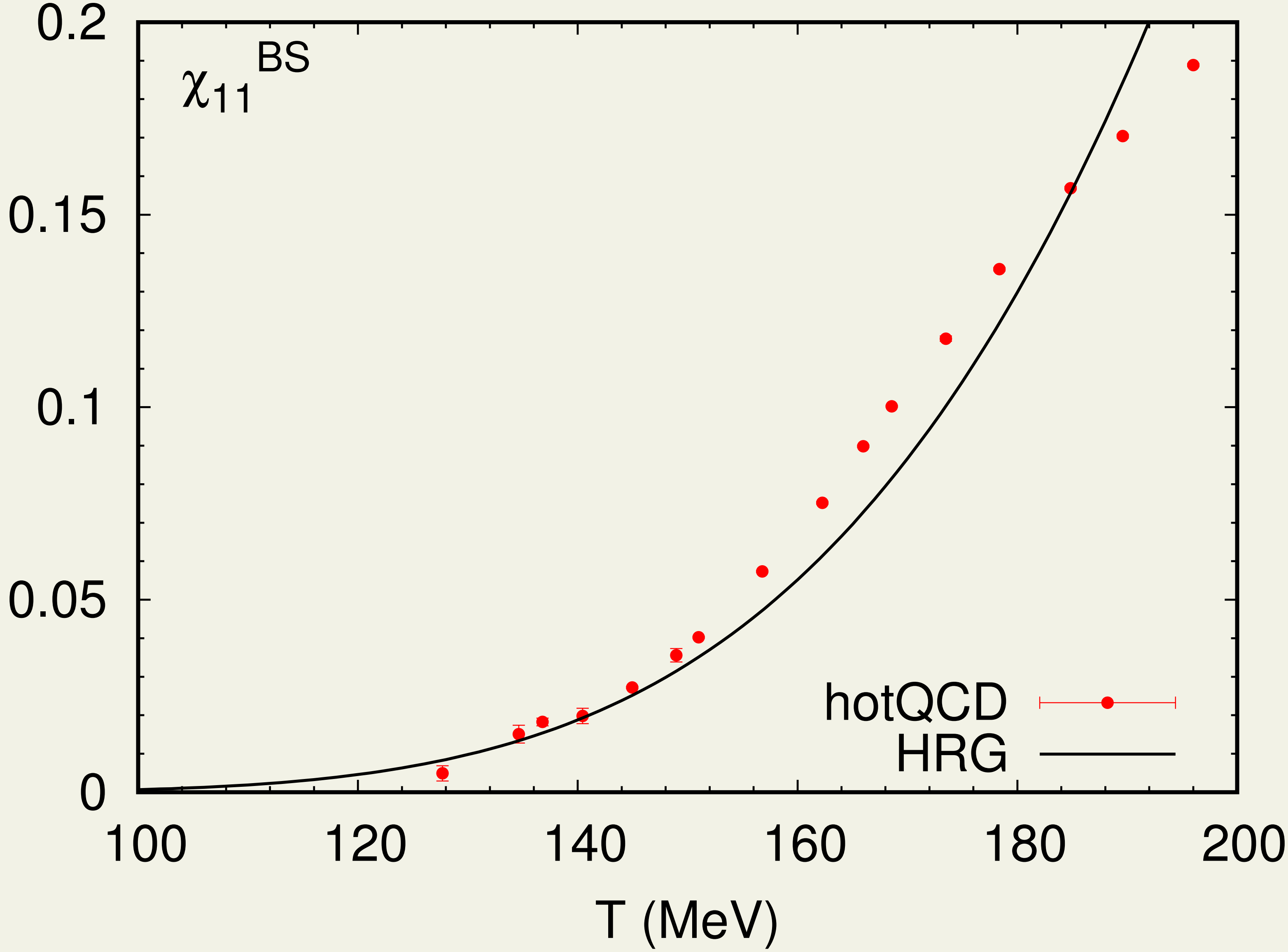
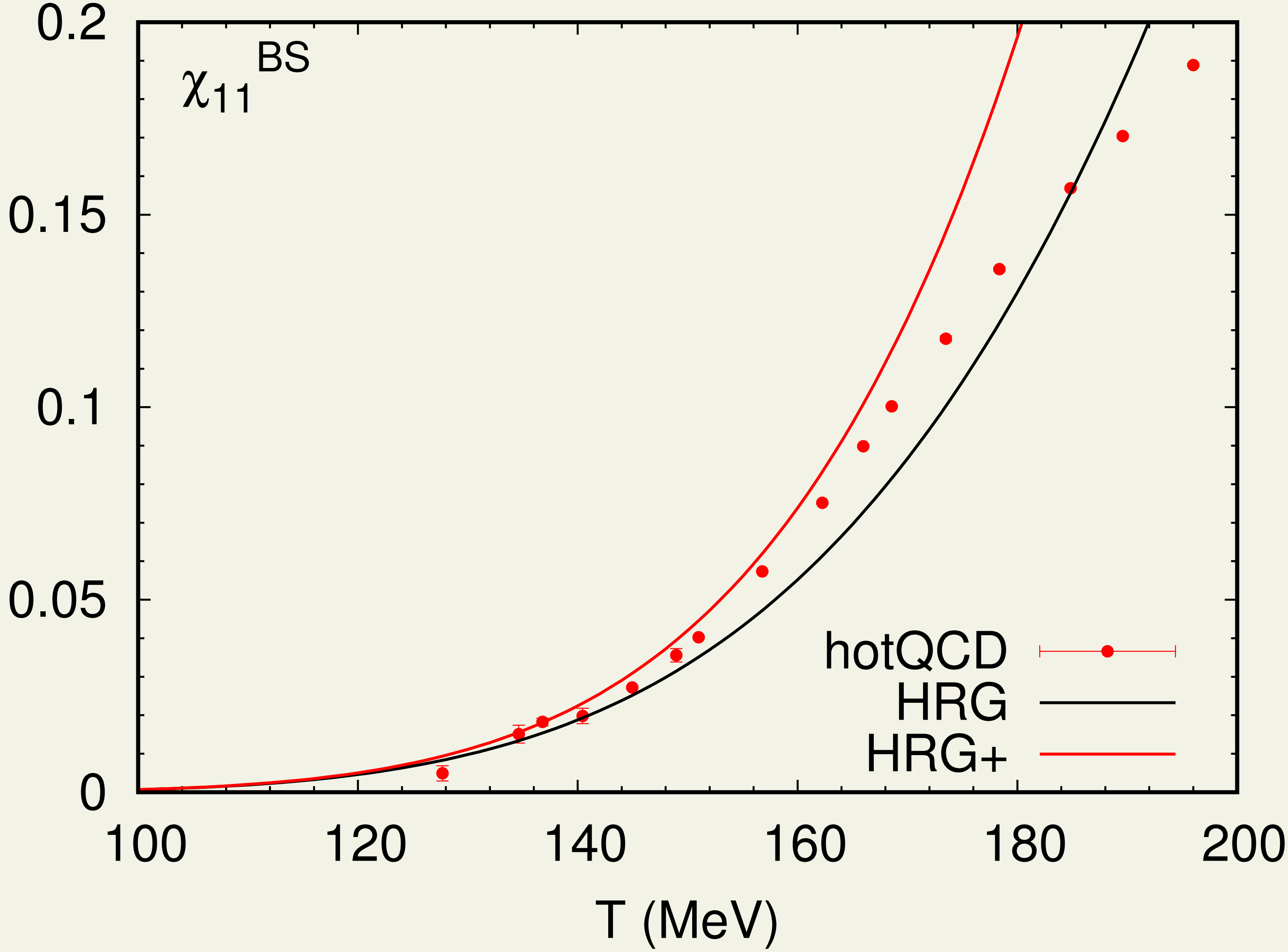
T (MeV)
χ2B
hotQCD
B-W
HRG
HRG+

χ_{BS}^{11}



χ^{11}_{BS}



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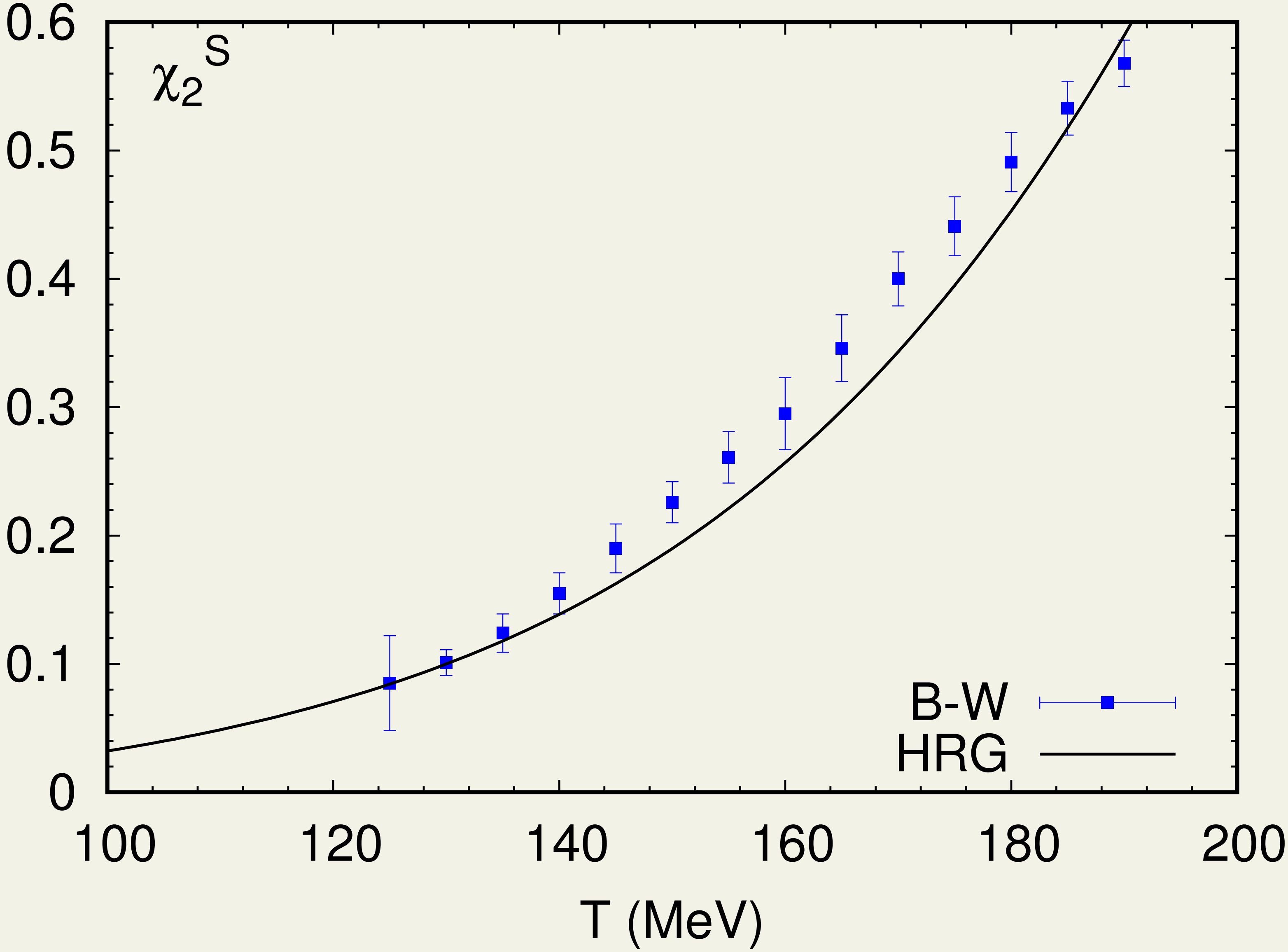
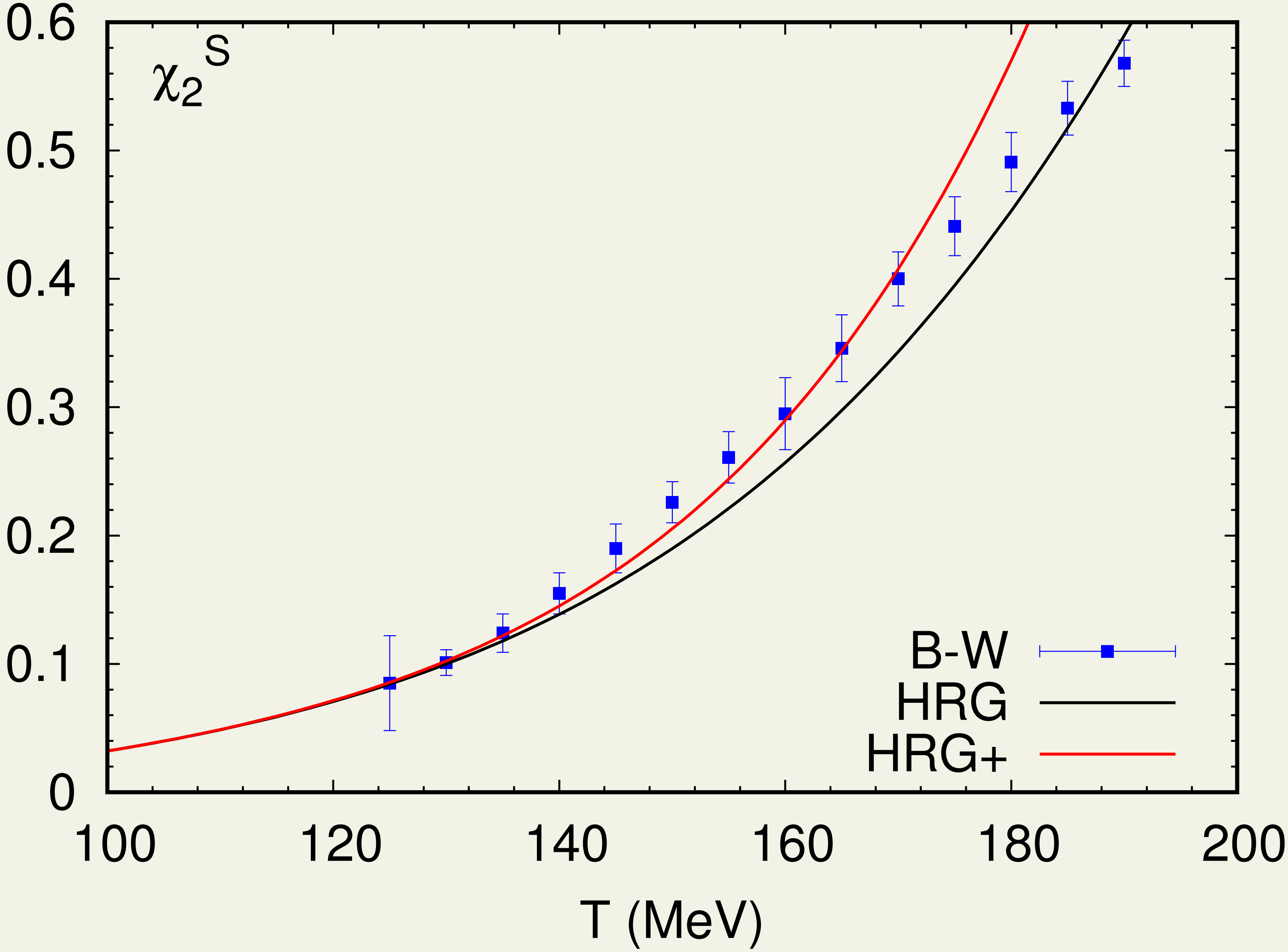
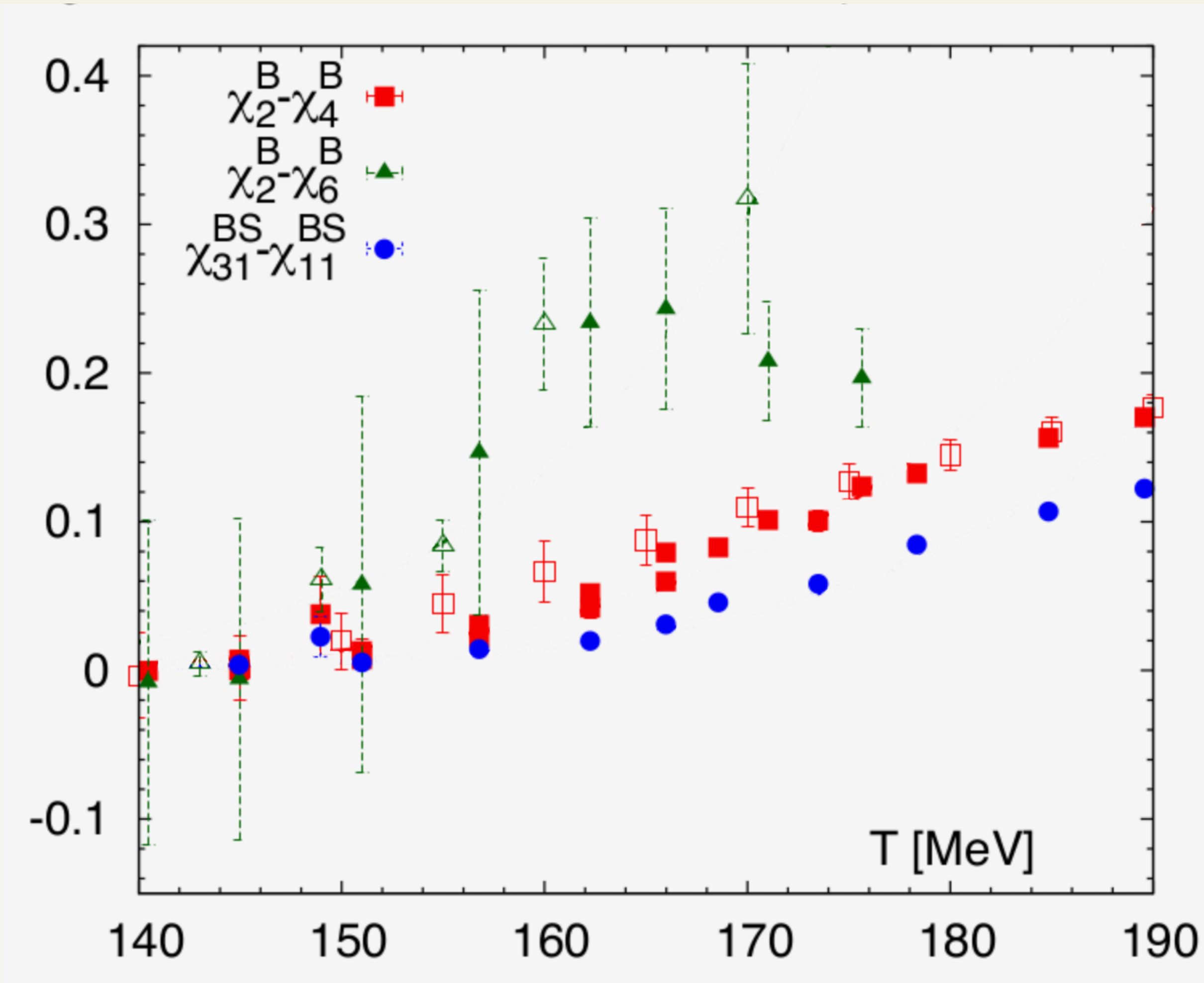


χ^2_S



Differences of fluctuations



Filled symbols: HISQ

Bazavov et al.,

PRL111, 082301 (2013)

PRD95, 054504 (2017)

Open symbols: stout

4th order

Bellwied et al.,

PRD92, 114505 (2015)

6h order

D'Elia et al,.

PRD95, 094503 (2017)

- **These zero in Boltzmann approximation**

Virial expansion

$$P = P^{\mathrm{ideal}} + T \sum_{ij} b_2^{ij}(T) e^{\beta\mu_i} e^{\beta\mu_j}$$

b_2^{ij} **can be related to the S-matrix of scattering of particles** i **and** j

- $\pi\pi$, πN, **etc. scatterings dominated by resonance formation**
- **no resonances in** NN **scatterings**

Virial expansion in nucleon gas

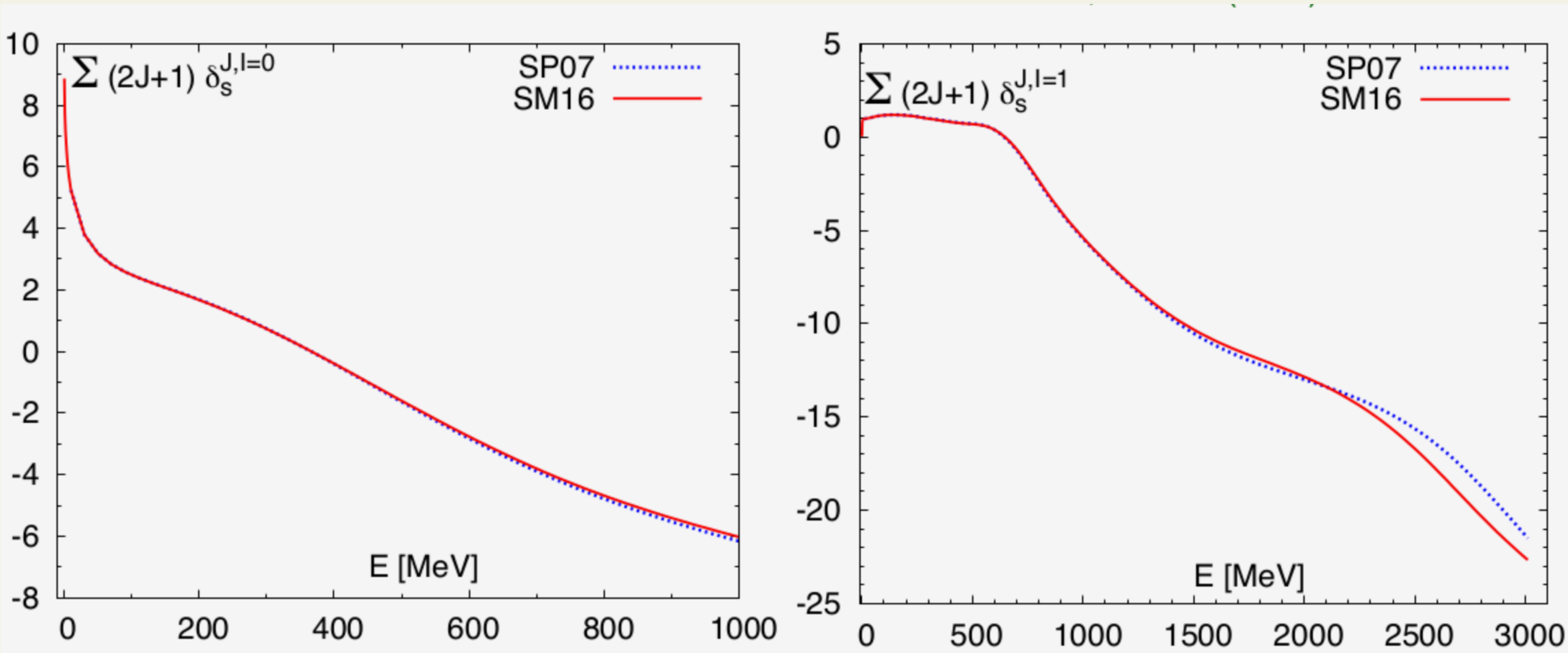
$$
\begin{aligned}
P(T,\mu) &= P_0(T)\cosh(\beta\mu) + 2b_2(T)\,T\cosh(2\beta\mu) \\
P_0(T) &= \frac{4m^2T^2}{\pi^2}K_2(\beta m)
\end{aligned}
$$

$$
b_2(T) = \frac{2T}{\pi^3}\int_0^\infty \mathrm{d}E\left(\frac{mE}{2}+m^2\right)K_2\left(2\beta\sqrt{\frac{mE}{2}+m^2}\right)\frac{1}{4i}\mathrm{Tr}[S^\dagger\frac{\mathrm{d}S}{\mathrm{d}E}-\frac{\mathrm{d}S^\dagger}{\mathrm{d}E}S]
$$

Virial expansion in nucleon gas

Elastic part of the S-matrix from scattering phase shift:

$$\frac{1}{4i}\mathrm{Tr}[S^\dagger \frac{\mathrm{d}S}{\mathrm{d}E} - \frac{\mathrm{d}S^\dagger}{\mathrm{d}E} S] \to \sum_s \sum_J (2J+1) \left(\frac{\mathrm{d}\delta_s^{J,I=0}}{\mathrm{d}E} + 3\frac{\mathrm{d}\delta_s^{J,I=1}}{\mathrm{d}E} \right)$$



Workman et al., PRC94, 065203 (2016); Arndt et al., PRC76, 025209 (2007)

Repulsive mean field

Assume: interactions reduce single partice energy by $U = K n_b$ where n_b is single nucleon density (Olive, NPB190, 483 (1981))

$$n_b = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}$$

Small $\mu \Rightarrow \beta K n_b \ll 1$ and

$$\begin{aligned} n_b &\approx n_b^0(1 - \beta K n_b^0) \Rightarrow \\ P(T,\mu) &= T(n_b + n_{\bar{b}}) - \frac{K}{2}\left((n_b^2)^2 + (n_{\bar{b}}^0)^2\right) \end{aligned}$$

or

$$P(T,\mu) = P_0(T)\left(\cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m)\cosh(2\beta\mu)\right)$$

Virial expansion vs. mean field

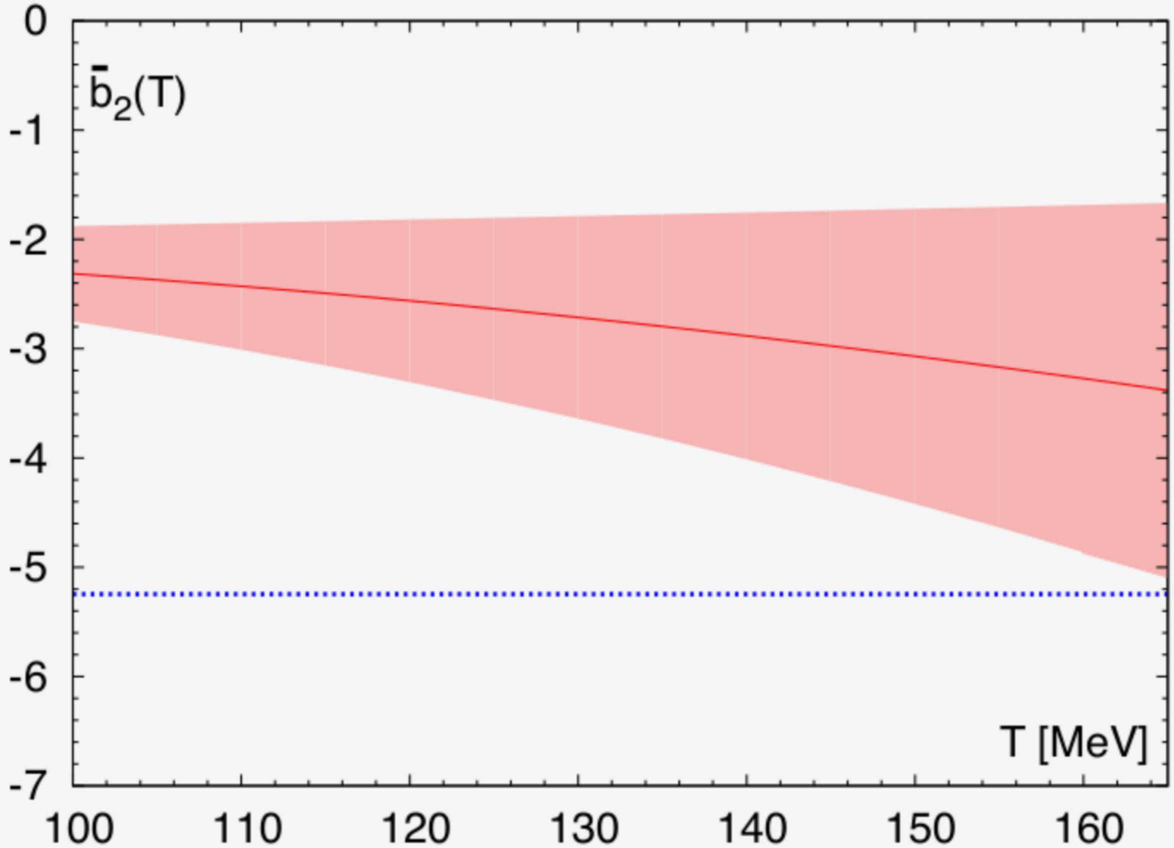
Repulsive mean field

$$P(T,\mu) = P_0(T)\times \left(\cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m)\cosh(2\beta\mu)\right)$$

Virial expansion

$$P(T,\mu) = P_0(T)\times \left(\cosh(\beta\mu) - \bar{b}_2(T) K_2(\beta m)\cosh(2\beta\mu)\right)$$

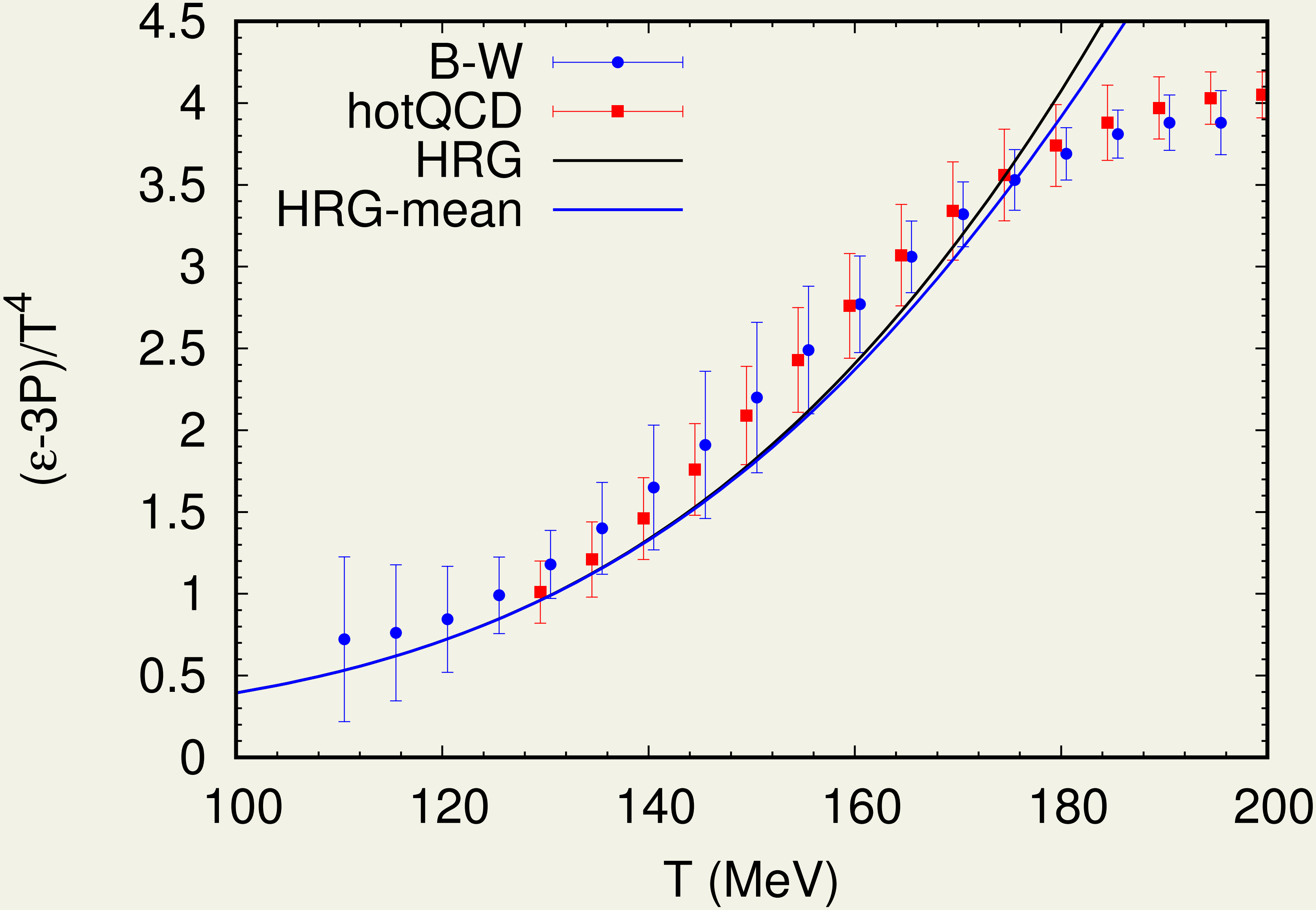
where $\bar{b}_2 = \frac{2Tb_2(T)}{P_0(T)K_2(\beta m)}$



In-elastic interactions become important for $E > 400$ MeV $\Rightarrow$ use σ_{el}/σ_{tot} to estimate the uncertainties in $b_2(T)$ due to these effects

$-\frac{KM^2}{\pi^2}$ for typical phenomenological value $K = 450$ MeV fm^3

Trace anomaly



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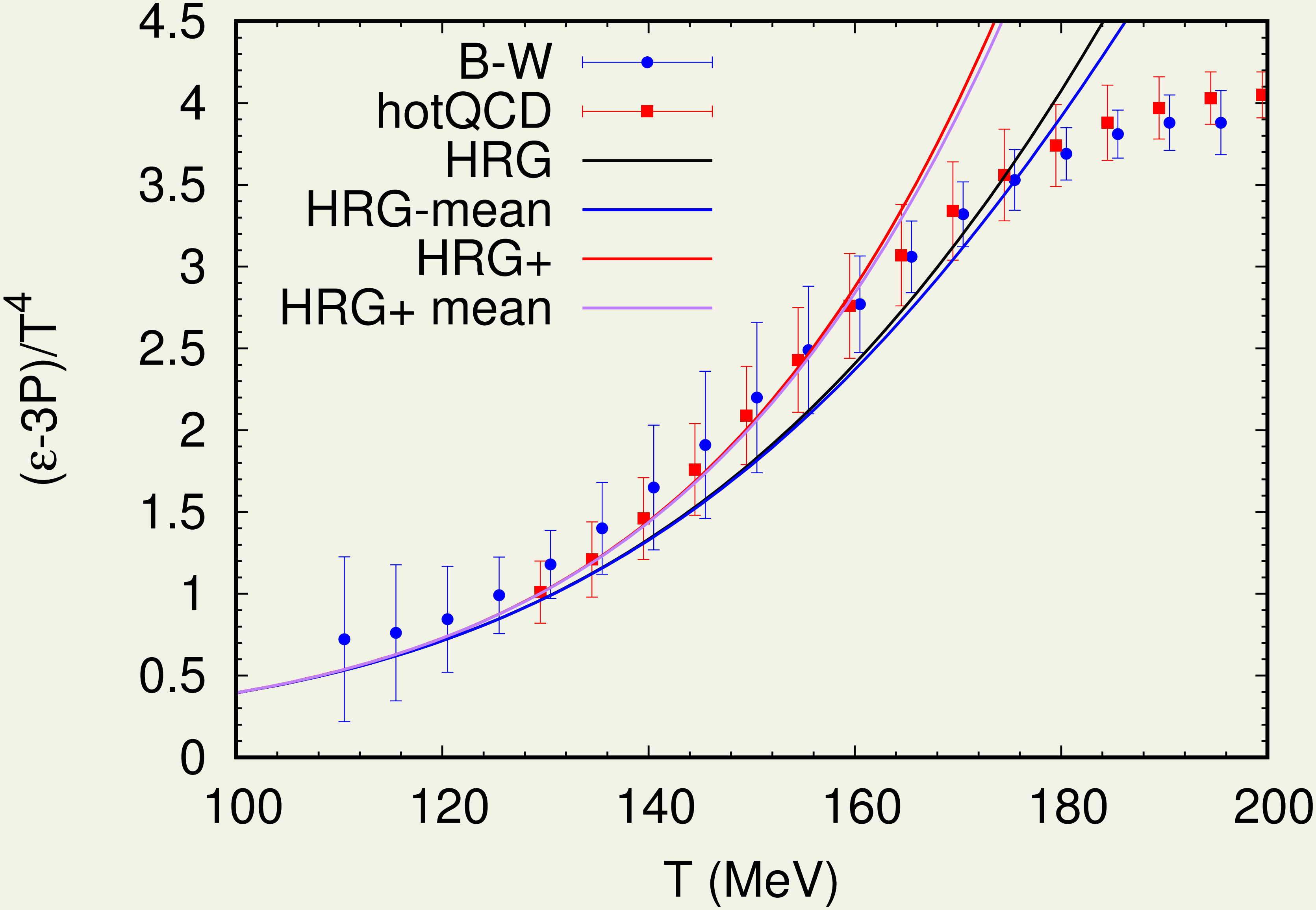
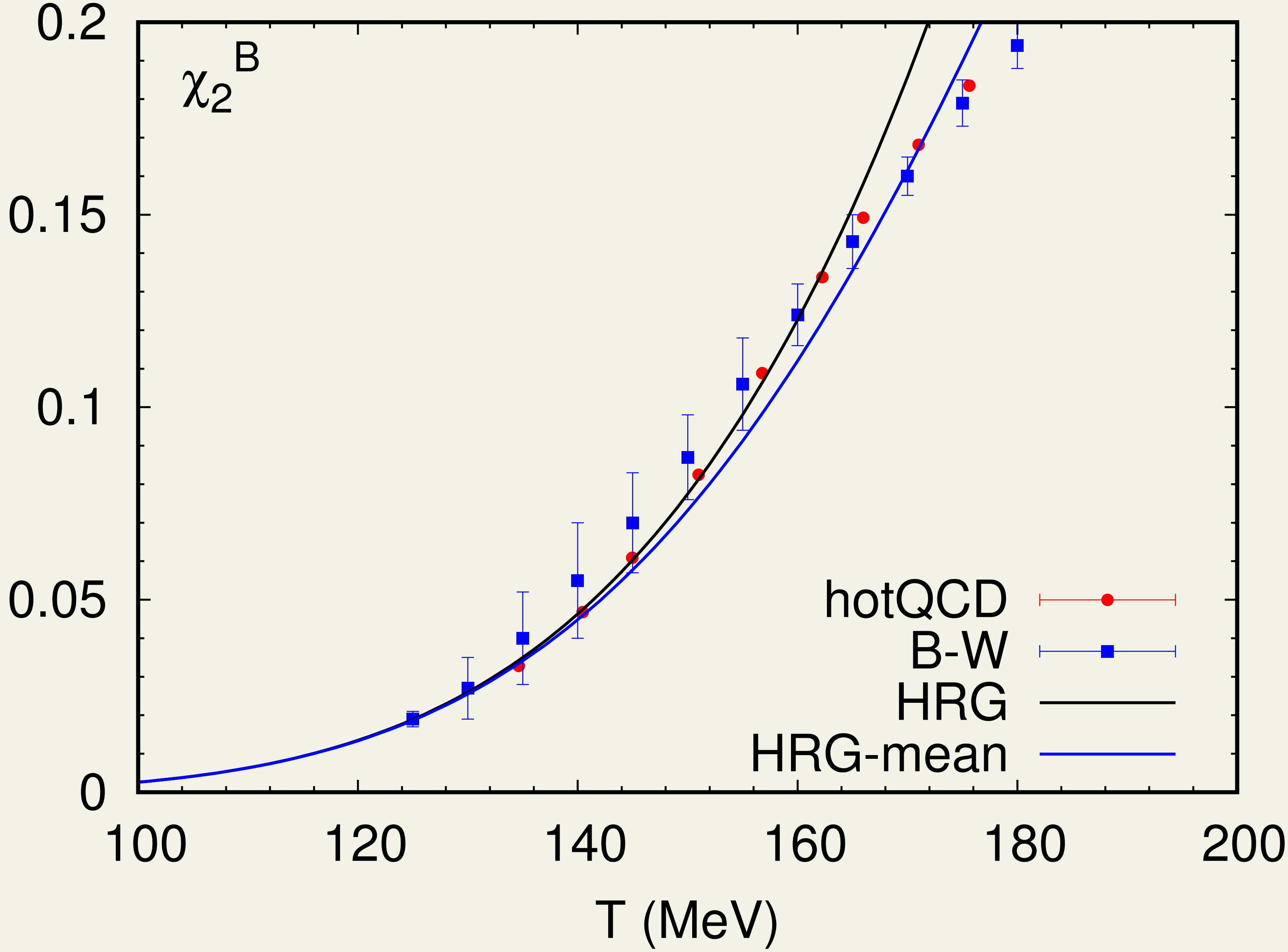


χ^2_B



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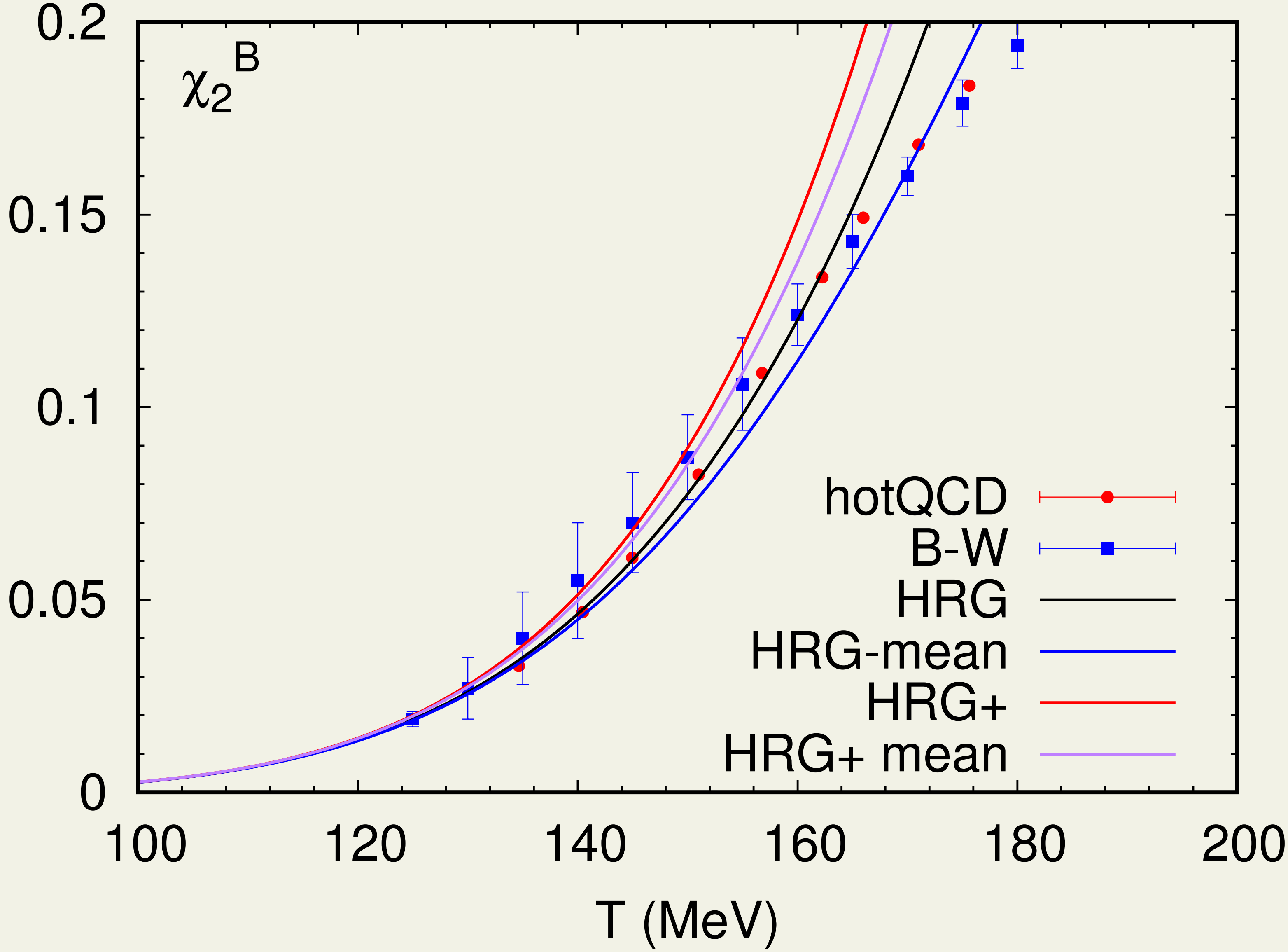

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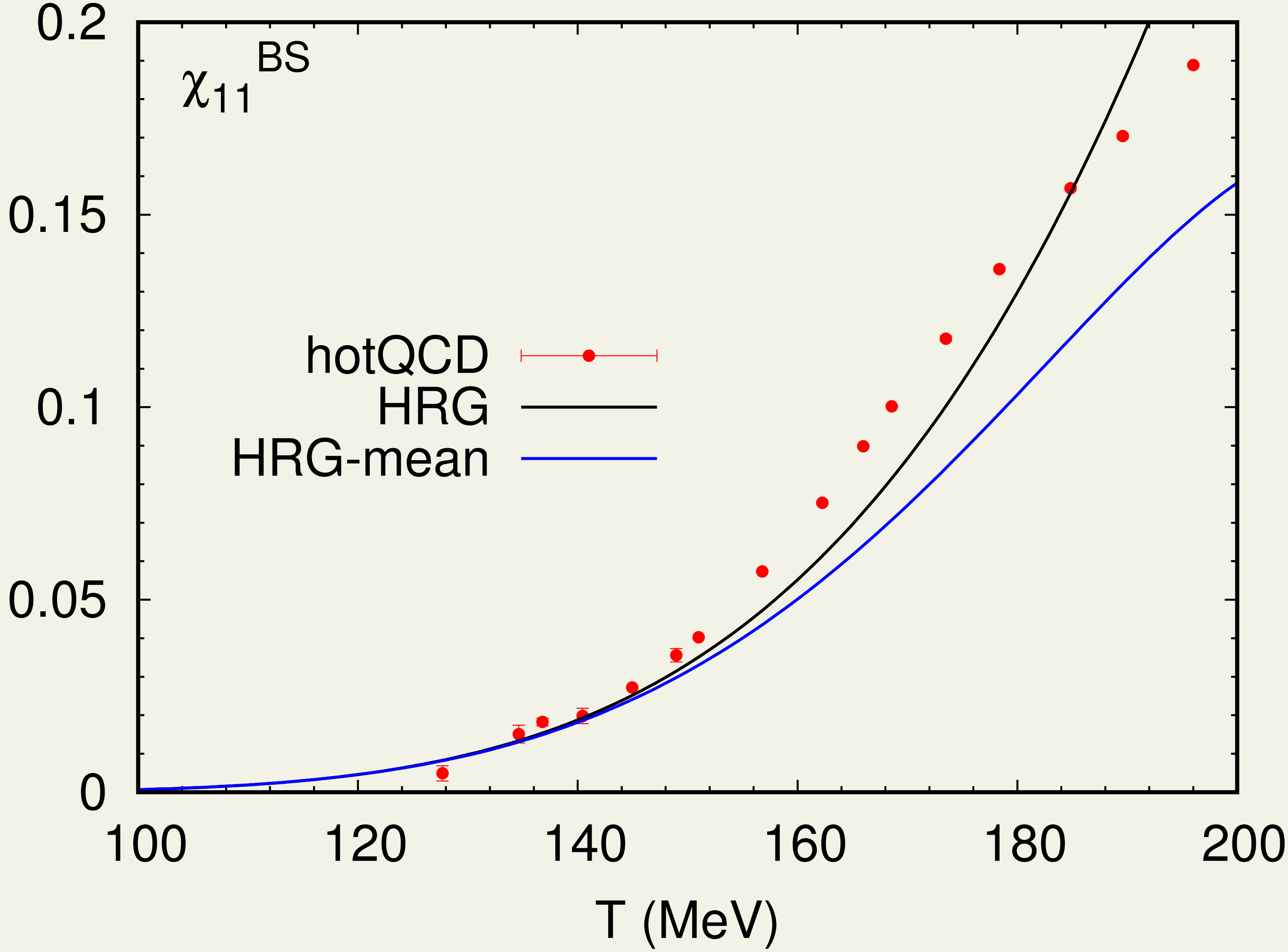

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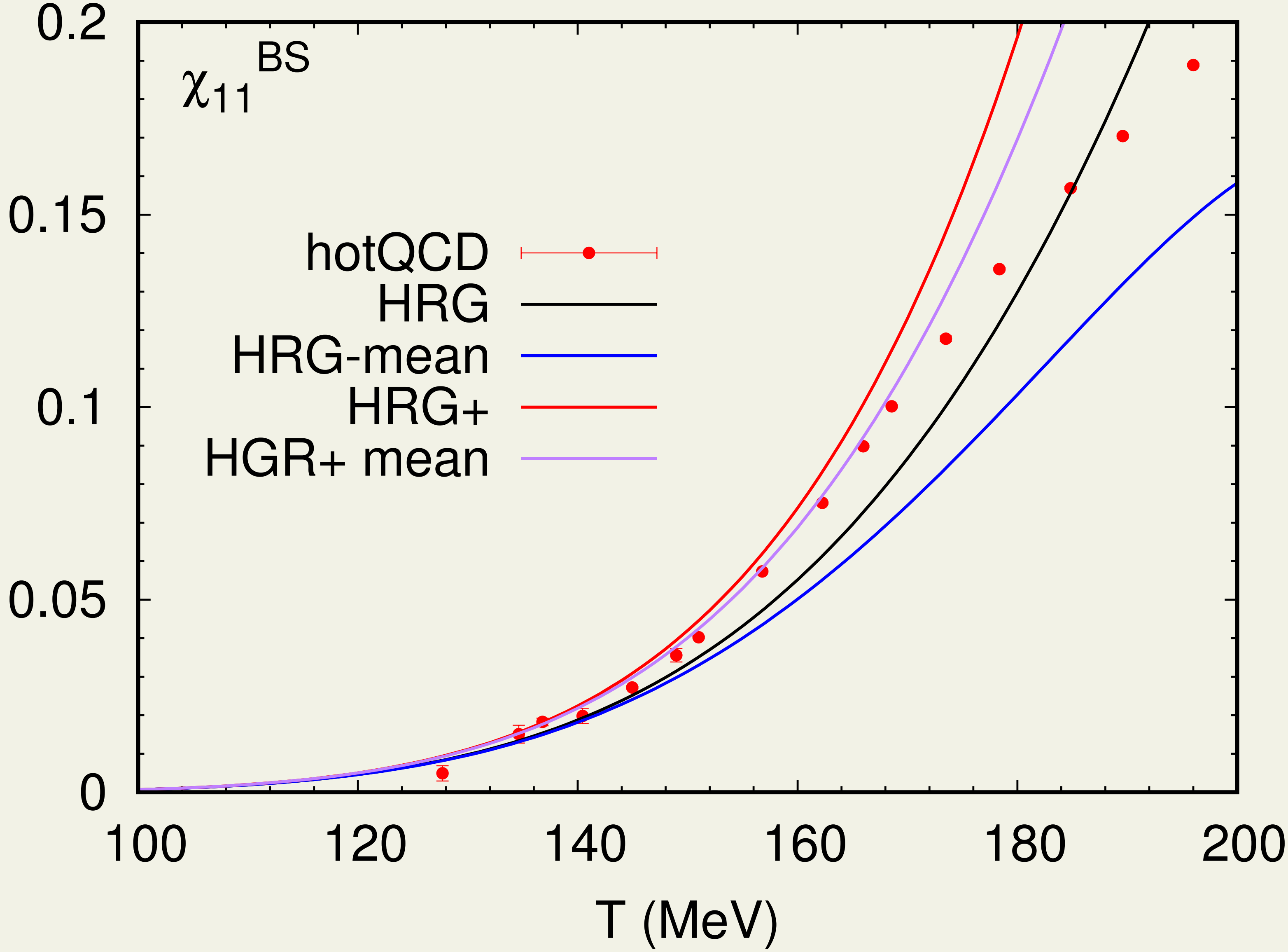

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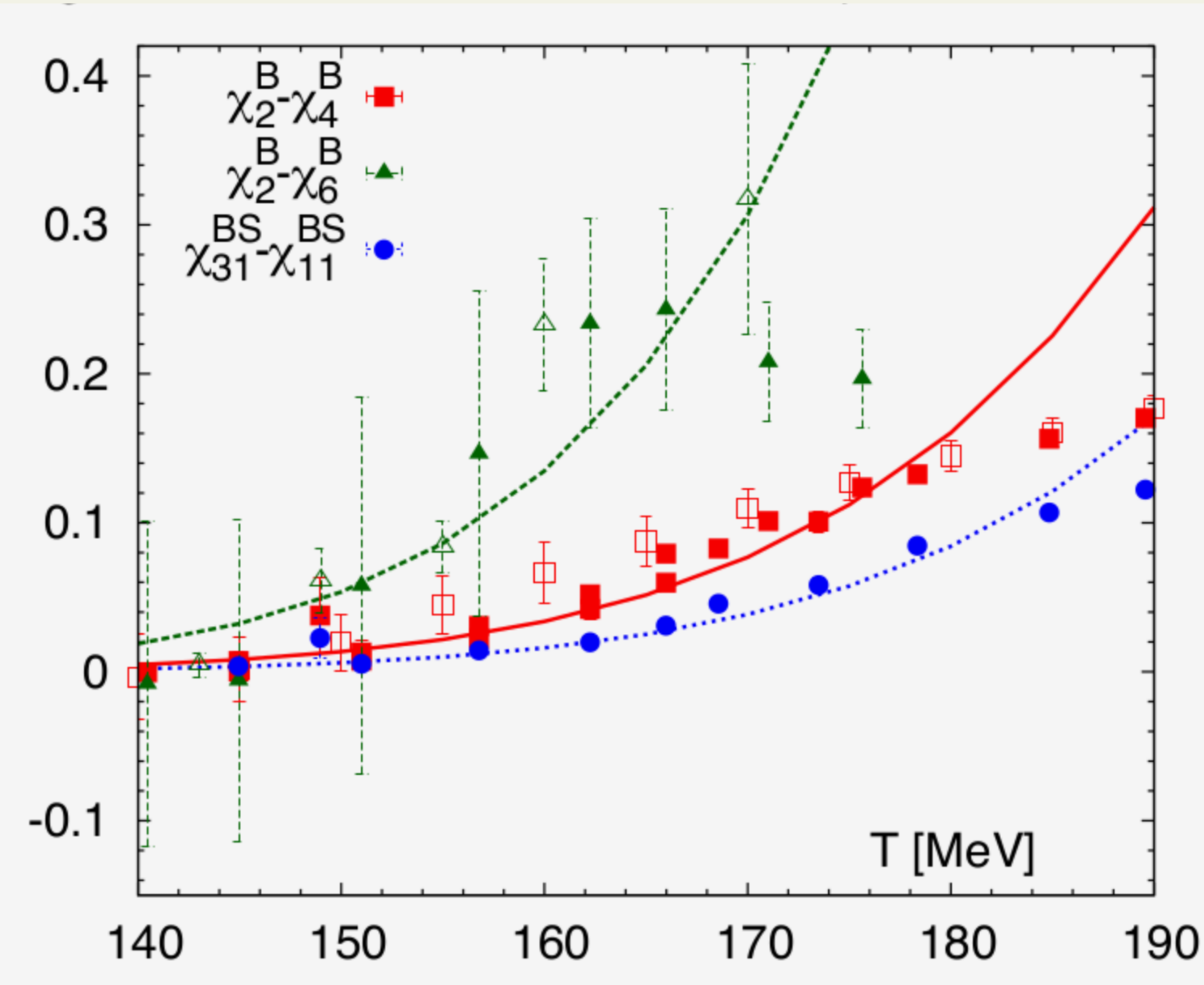
Differences of fluctuations



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- **These zero in Boltzmann approximation**
- **Repulsive interactions create similar differences**

Summary

- **lattice QCD indicates there are more resonances than observed**
  - **inclusion of quark model states improves the fit to some, and weakens the fit to some observables**

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- lattice QCD indicates there are more resonances than observed
  - inclusion of quark model states improves the fit to some, and weakens the fit to some observables
- repulsive mean field can describe the differences between baryonic fluctuations of different orders
- mean field strength can be constrained by phase shifts

Hadron Resonance Gas with mean field

Assume: only members of baryon octet and decuplet repel each other

$$P(T,\mu) = Tn - \frac{K}{2}\left((n^0_{od})^2 + (n^0_{\bar{o}d})^2\right)$$

where

$$n_{od}(T) = \frac{T}{2\pi^2}\sum_i g_i m_i^2 K_2(\beta m_i)$$

$i = N, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$

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$$\begin{aligned}
\chi_n^B &= \chi_n^{B(0)} - 2^n \beta^4 K (n_{od}^0)^2 \\
\chi_{n1}^{BS} &= \chi_{n1}^{BS(0)} + 2^{n+1}\beta^5 K n_{od}^0 (P_B^{S1} + 2P_B^{S2} + 3P_B^{S3})
\end{aligned}$$