Modeling chiral criticality and its consequences for heavy-ion collisions

Gabor Almasi

MTA Wigner RCP, Budapest

GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt

Exploring the phase diagram

I. Heavy-ion collisions

- Experimental data on the freeze-out line

II. Lattice QCD

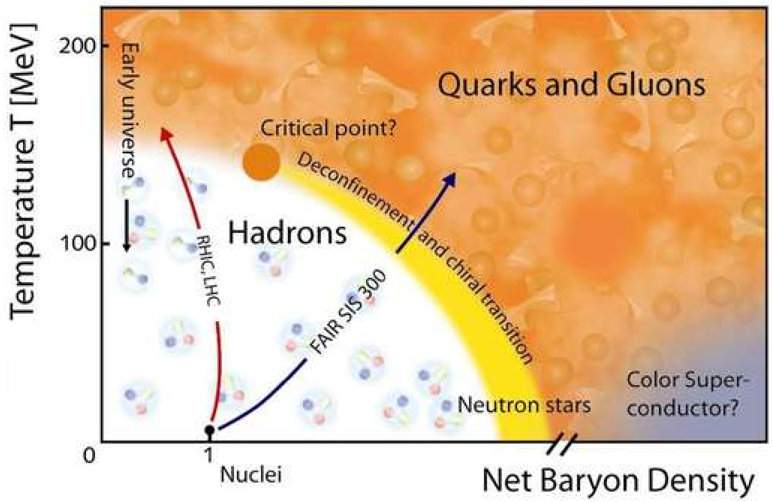
- First principle calculations
- Sign problem: difficult to explore $\mu \neq 0$

III. Effective models

- Same universality class as QCD
- Hard to make quantitative predictions

IV. Functional methods

- Apply methods developed for effective models to QCD



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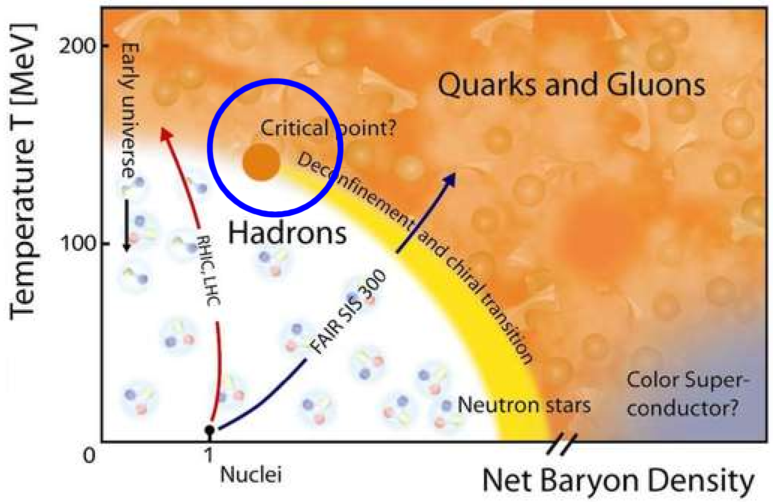
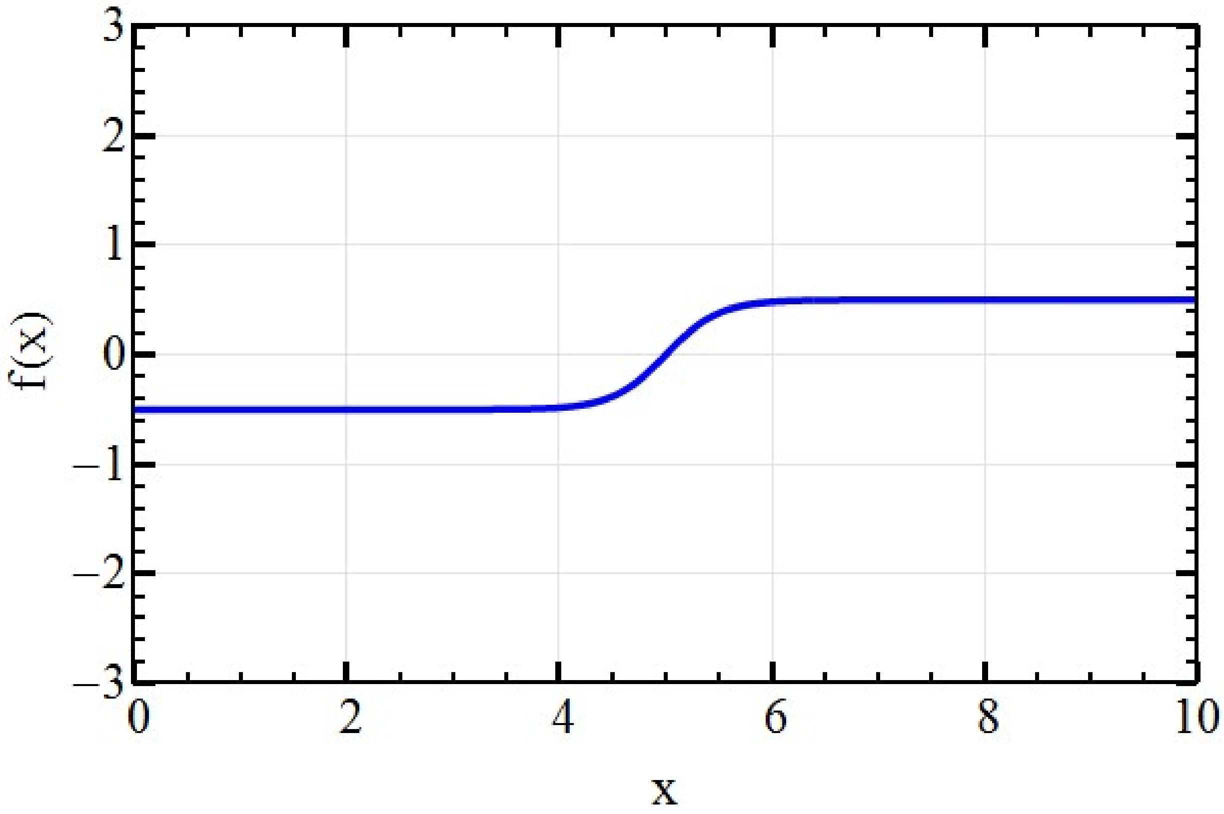


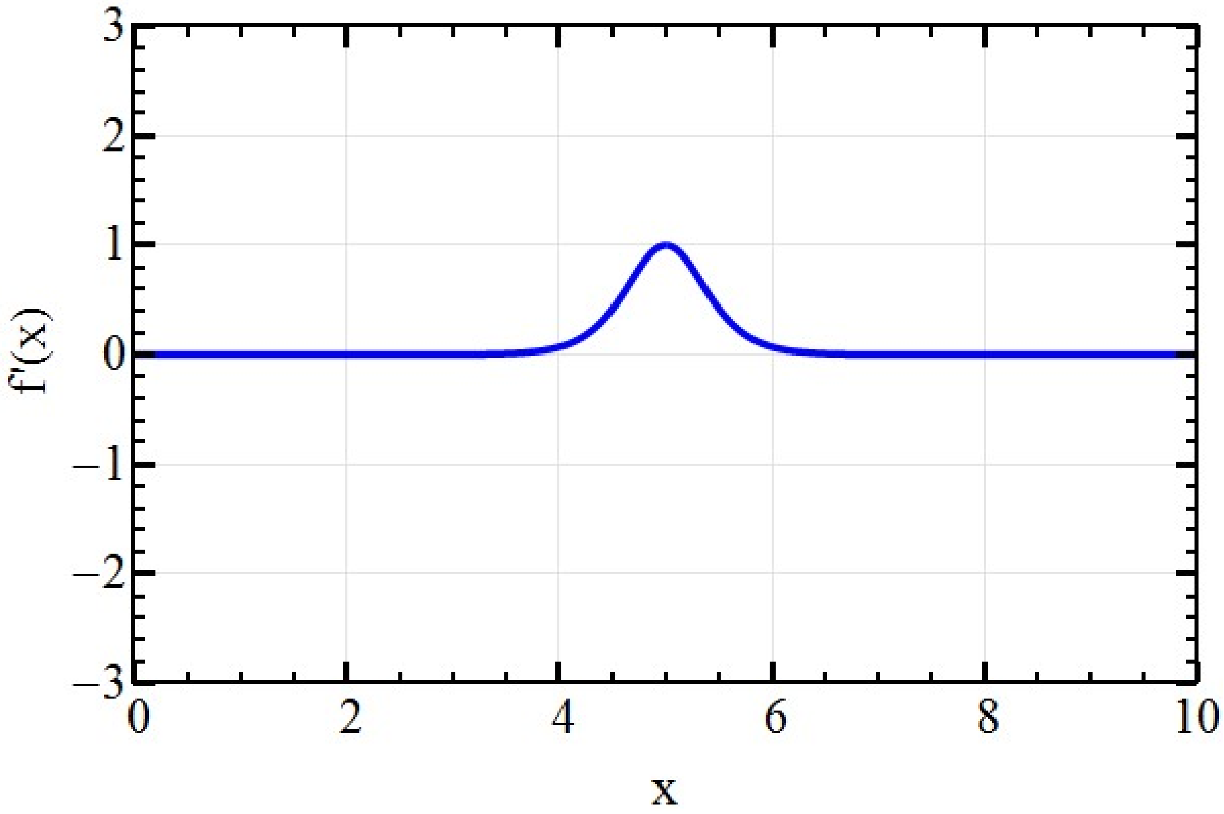
What signals the CEP?

Fluctuations ~ derivatives of the generating functional



Higher derivatives – stronger signal at the transition

Fluctuations ~ derivatives of the generating functional



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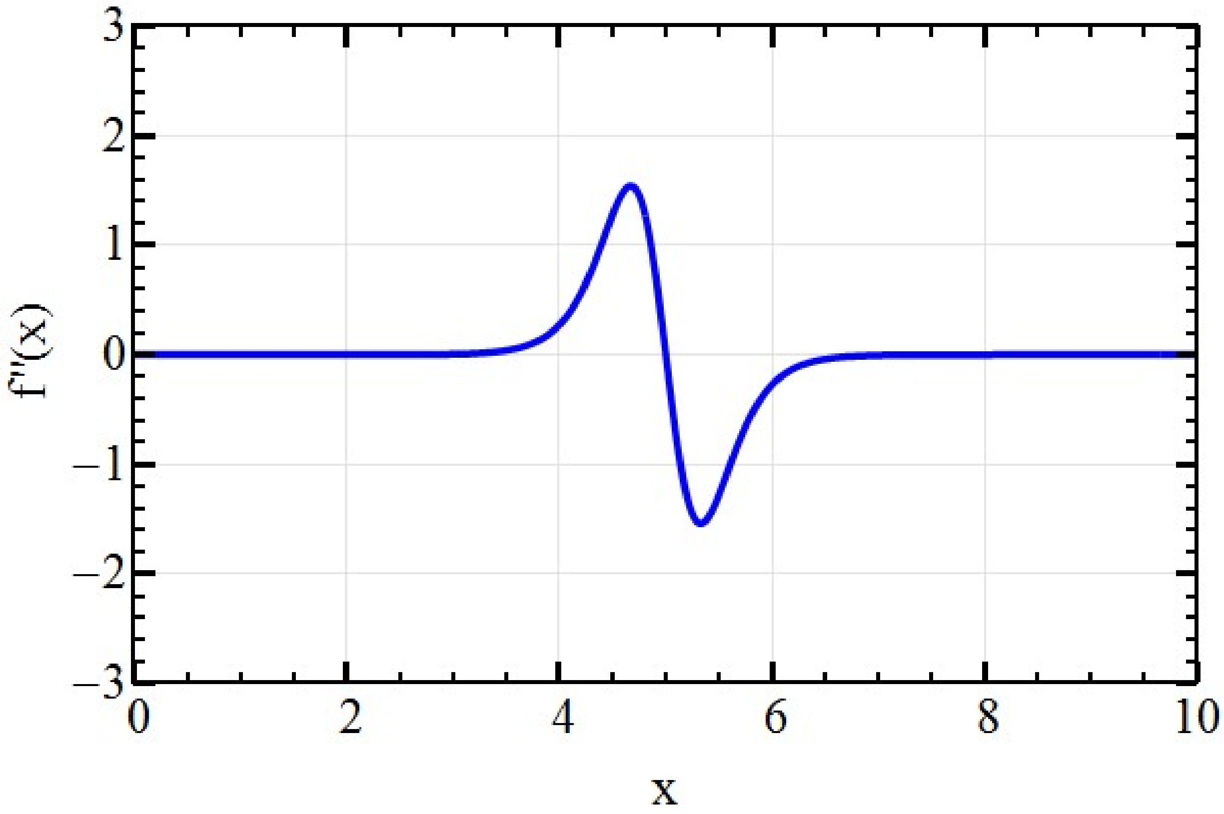

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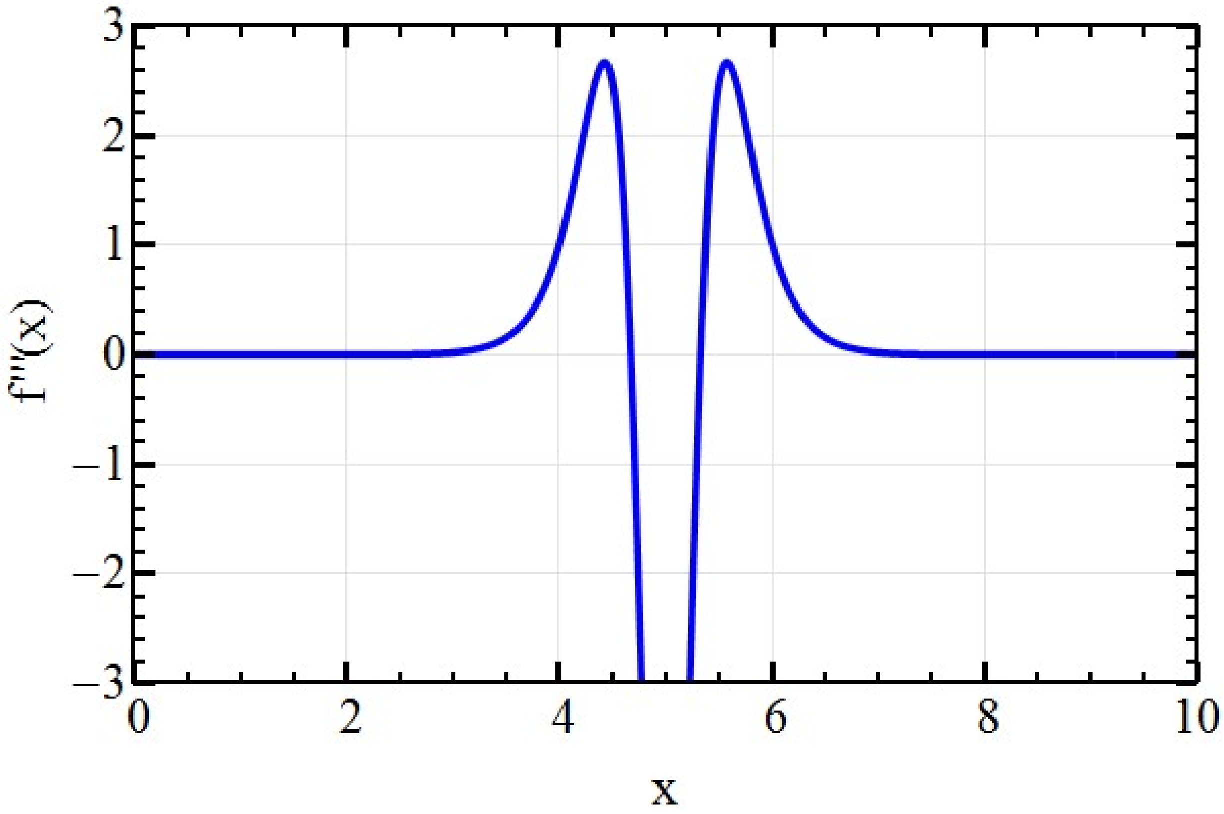

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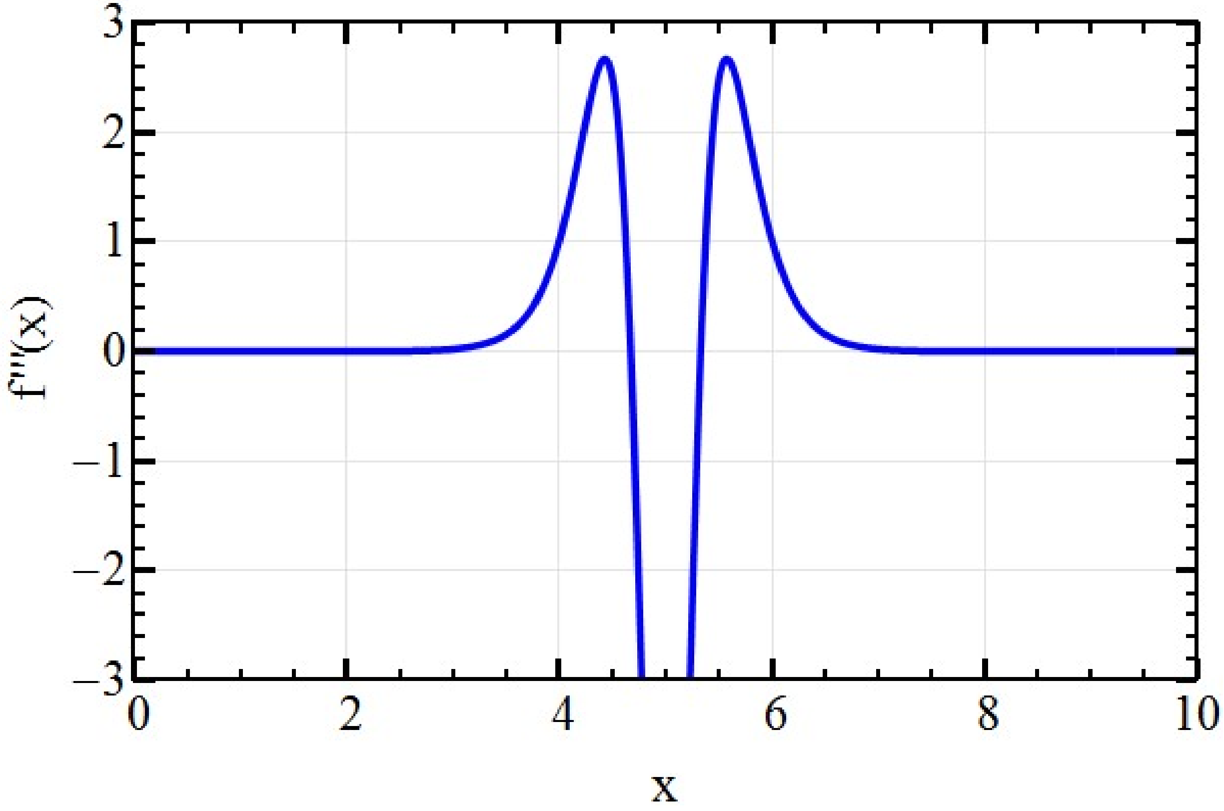


Higher derivatives – stronger signal at the transition

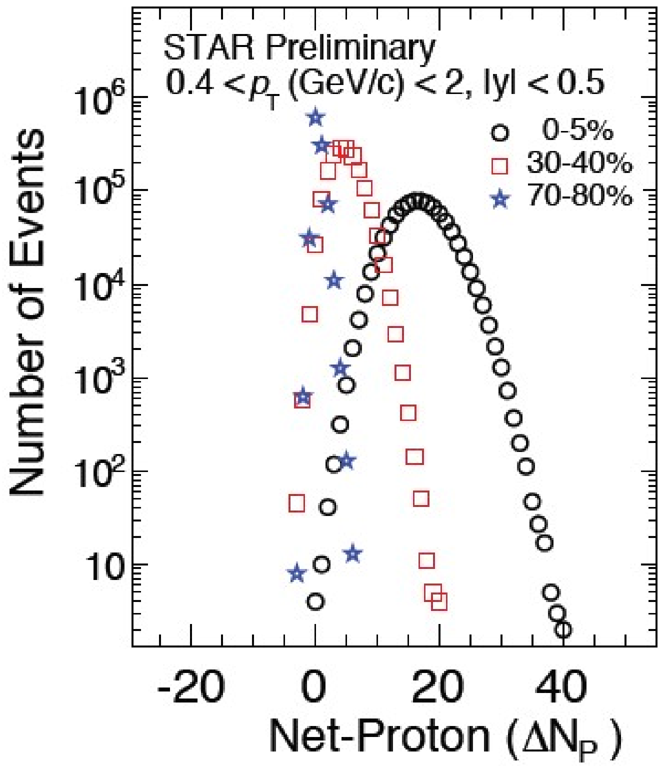
Helps to suppress background

Locating the critical endpoint

Critical point: enhanced fluctuations!

Experiment:

Measurement of **net proton distribution**



STAR Coll. PRL112 (2014)

Theory:

Calculation of **cumulants of baryon number**

$$\chi_B^n = T^{n-4} \frac{\partial^n P(\mu_B, T)}{\partial \mu_B^n}$$

P: pressure

T: temperature

μ_B: baryon chemical potential

Translation between theory and experiment

Cumulants in the function of moments:

$$\chi^1 = \frac{1}{VT^3}\langle N\rangle \qquad \chi^2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle$$

$$\chi^3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle \qquad \chi^4 = \frac{1}{VT^3}(\langle(\Delta N)^4\rangle\text{-}3\langle(\Delta N)^2\rangle)$$

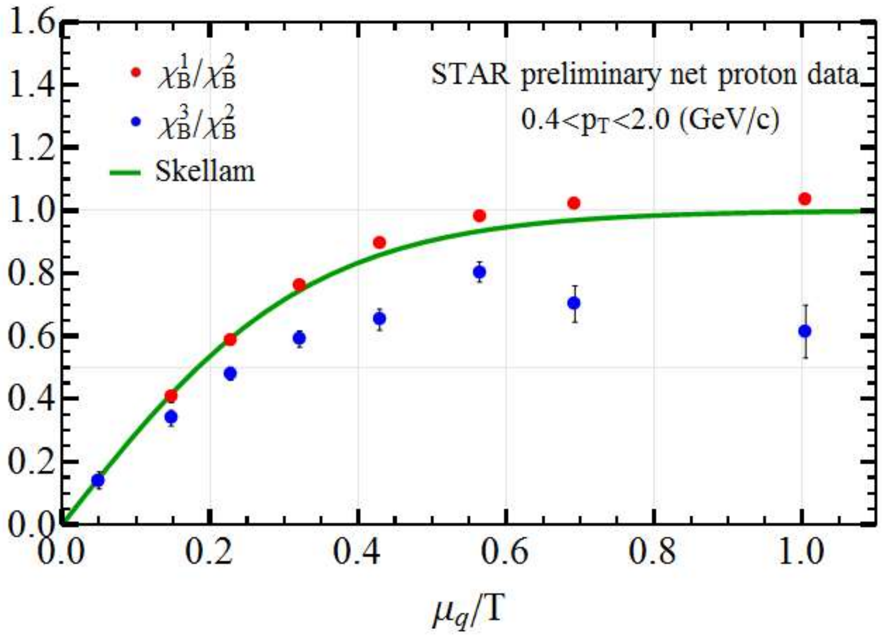
To cancel the volume dependence:

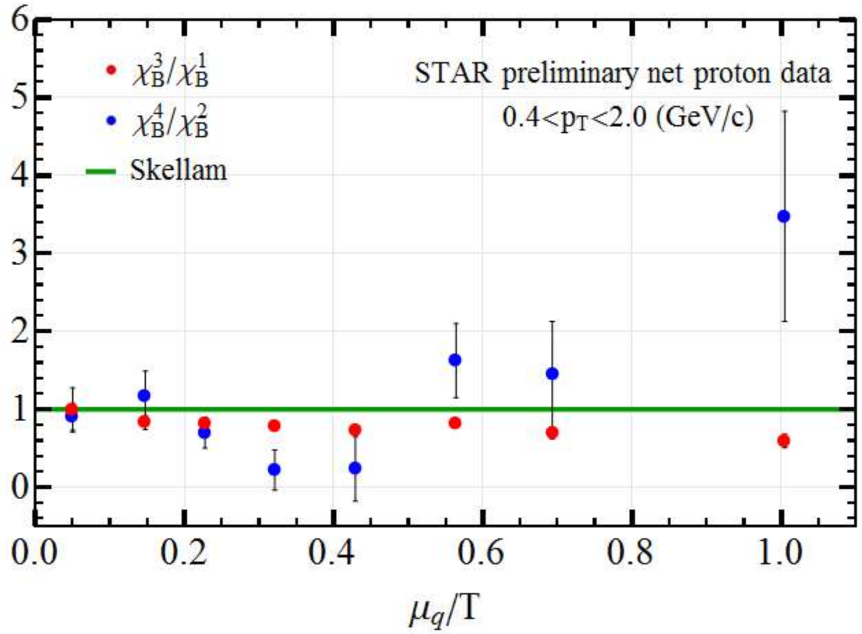
$$\chi^1/\chi^2 = \frac{M}{\sigma^2} \qquad \chi^3/\chi^2 = S\sigma$$

$$\chi^4/\chi^2 = \kappa\sigma^2 \qquad \chi^3/\chi^1 = S\sigma^3/M$$

M: Mean σ: Variance S: Skewness κ: Kurtosis

STAR Beam Energy Scan (BES)





Baseline: Skellam distribution

$$\chi^{2k+1}/\chi^{2l} = \tanh(\mu_B/T)$$

$$\chi^{2k}/\chi^{2l} = \chi^{2k+1}/\chi^{2l+1} = 1$$

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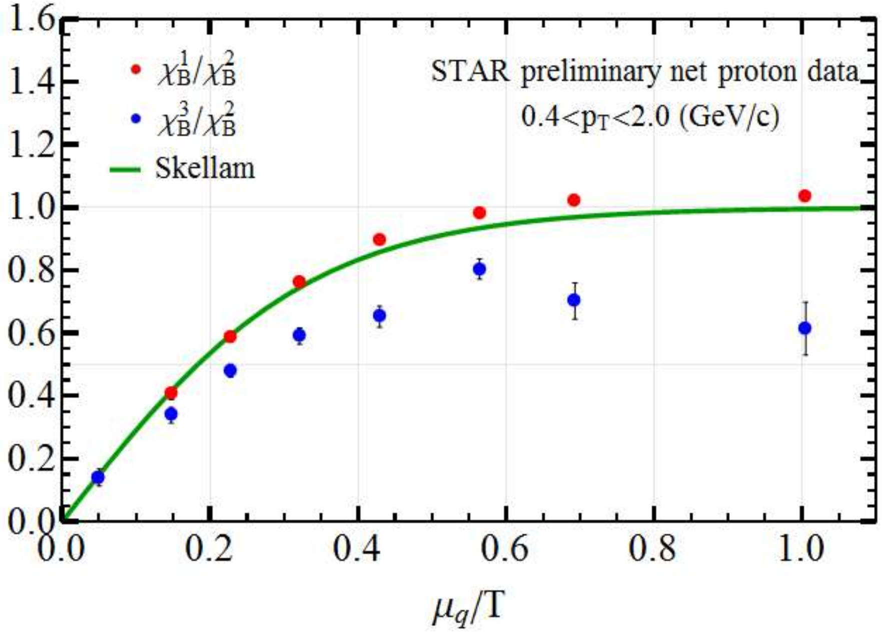

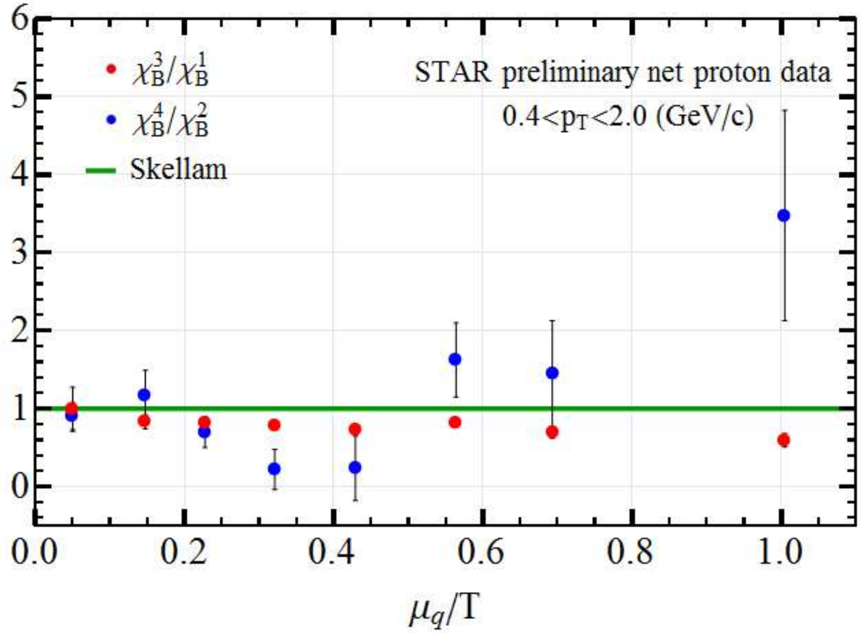

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Deviations → Critical endpoint?

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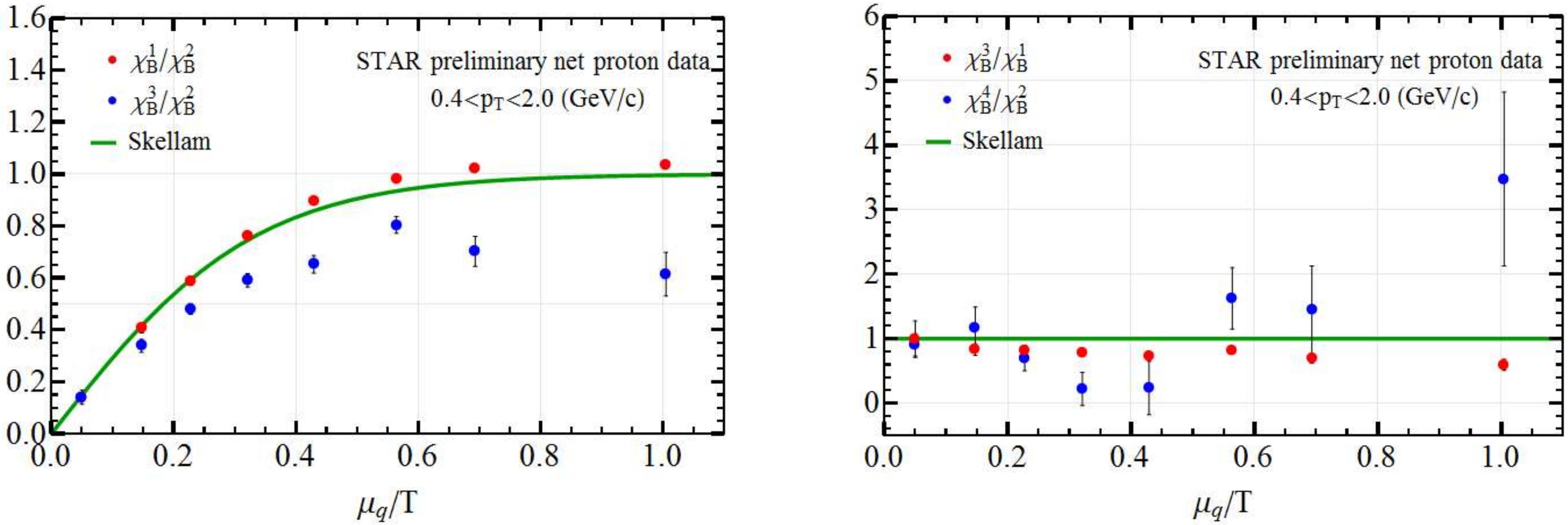

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Deviations → Critical endpoint?

Check: effective models

Polyakov-quark-meson (PQM) model

$$\mathcal{L} = \overline{q}\left[iD_\mu\gamma^\mu - g(\sigma + i\gamma_5\vec{\tau}\,\vec{\pi})\right]q + \frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\pi\right)^2 - U(\sigma,\vec{\pi}) - U_P\left(T,\ell,\overline{\ell}\right)$$

with

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - H\sigma, \qquad D_\mu = \partial_\mu + iA_\mu$$

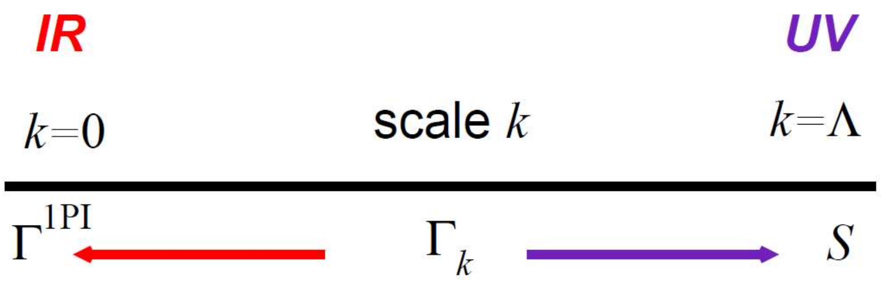
- Low energy effective theory of QCD
- Degrees of freedom: light quarks, pions, sigma meson
- Chiral symmetry breaking and restoration
- Statistical confinement: **Polyakov-loop** suppresses the single quark fluctuations at low temperatures
- Same universality class as QCD
- Functional renormalization group (FRG)

Functional Renormalization Group (FRG)

Scale dependent regulation of modes:

$$Z_k[J] = \int D\Phi \, e^{-S[\Phi]+\int_x \Phi(x)J(x) - \Delta S_k[\Phi]}$$

$$\Delta S_k[\Phi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d} \Phi(-q) R_k(q) \Phi(q)$$



Effective average action: $\Gamma_k[\phi] = \sup_J \left(\int_x J(x)\phi(x) - \log Z_k[J] \right) - \Delta S_k[\phi]$

Scale evolution governed by the Wetterich equation:

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\int_x \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \partial_k R_k$$

Typical regulator (Litim):

$$R_k(q) = \left(k^2 - q^2\right)\theta\left(k^2 - q^2\right)$$

Coming up

- Critical fluctuations and the CEP

  G. Almasi, B. Friman, K. Redlich, PRD 96, 014027 (2017)

- Finite volume studies

  G. Almasi, R. Pisarski, V. Skokov, PRD 95, 056015 (2017)

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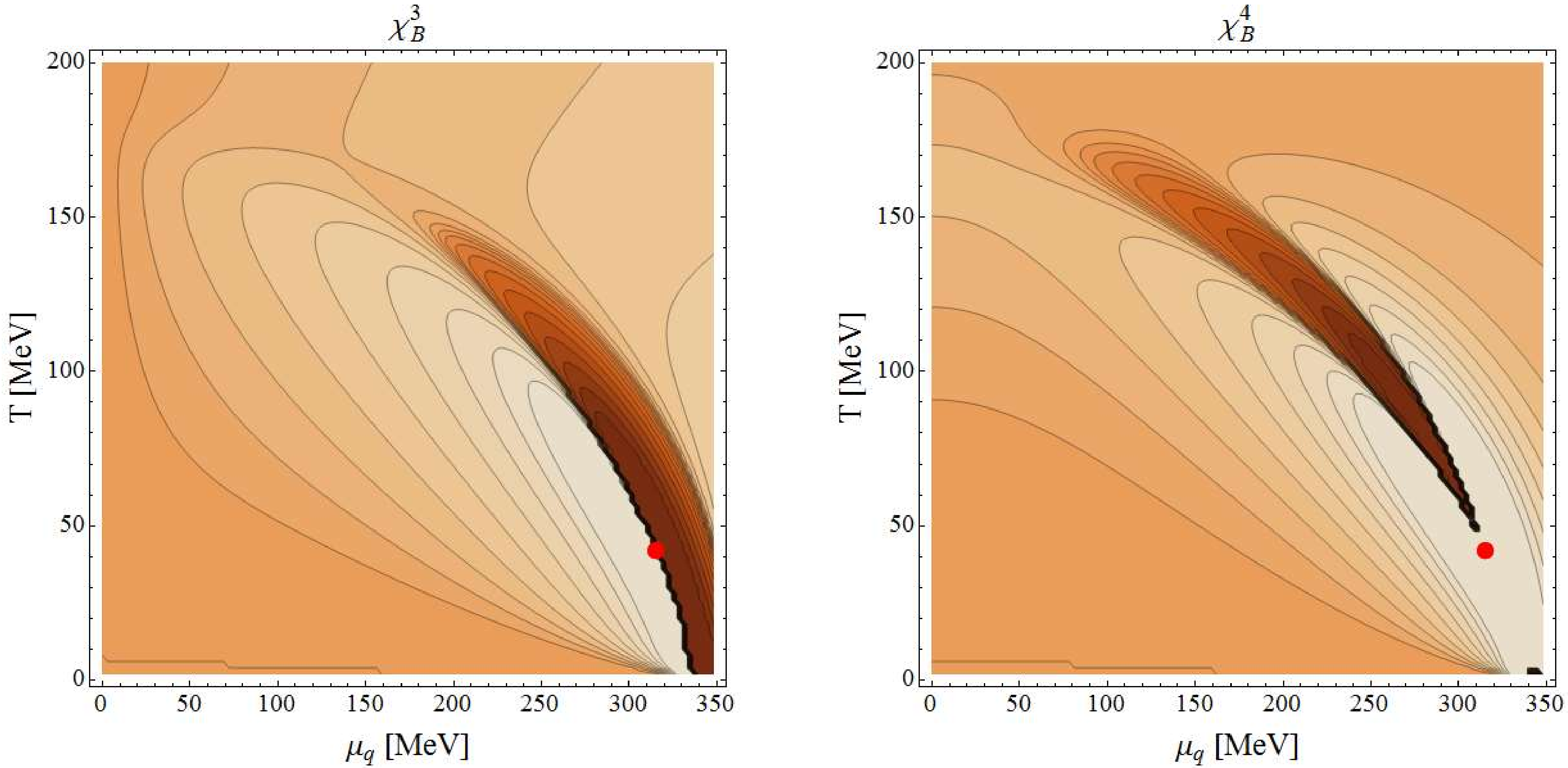
Cumulants in effective models (PQM-MF)



$\mu_q = \mu_B/3$ quark chemical potential

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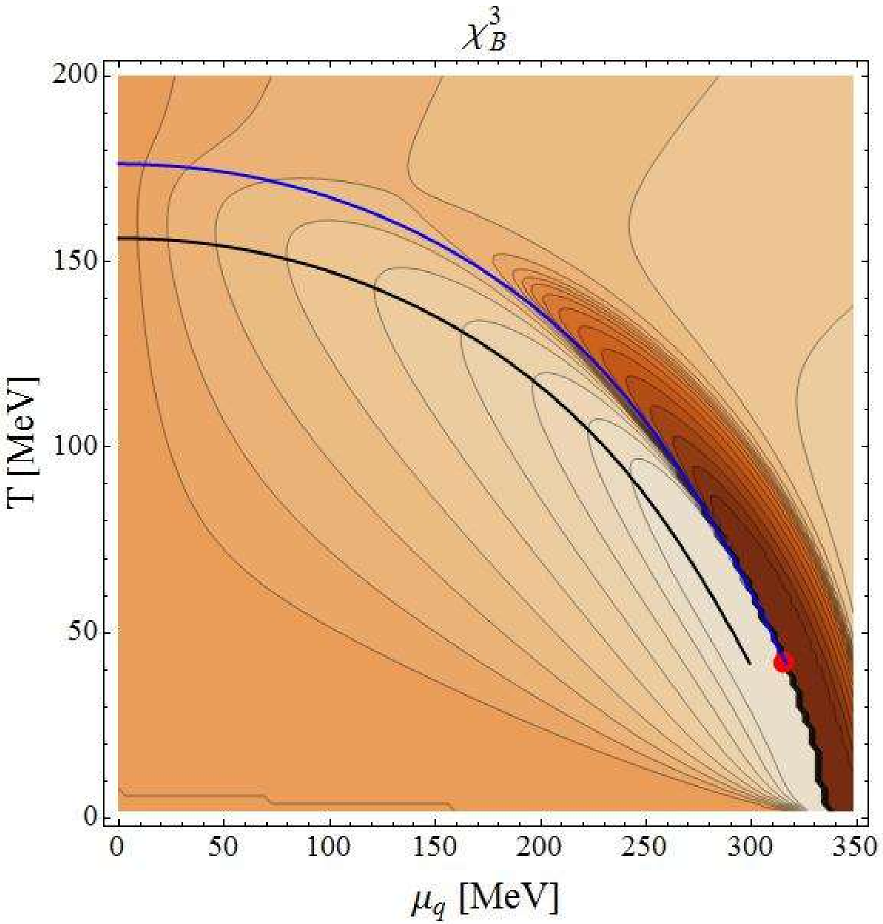

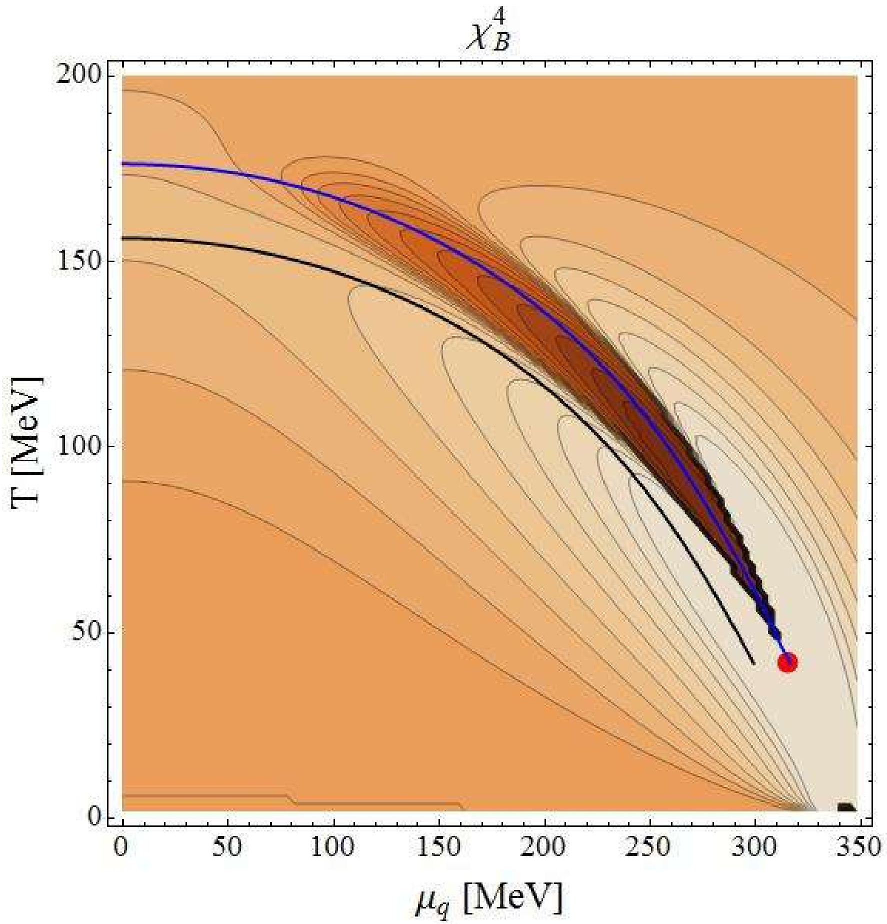

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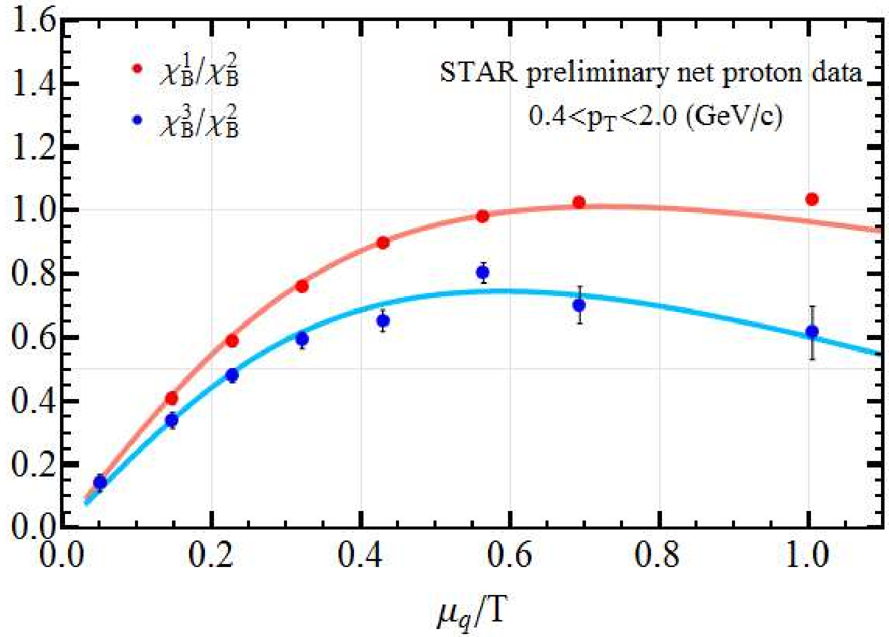
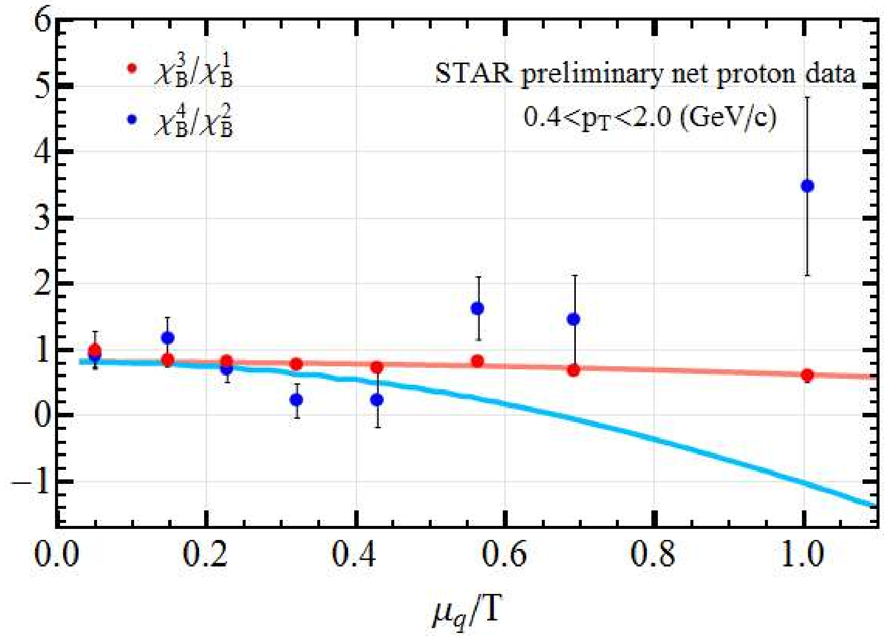
Consistency of the data





- Freeze-out line fitted to reproduce χ_B^3/χ_B^1
- All other cumulant ratios are calculated
- Agreement at small μ: signature of equilibrium?

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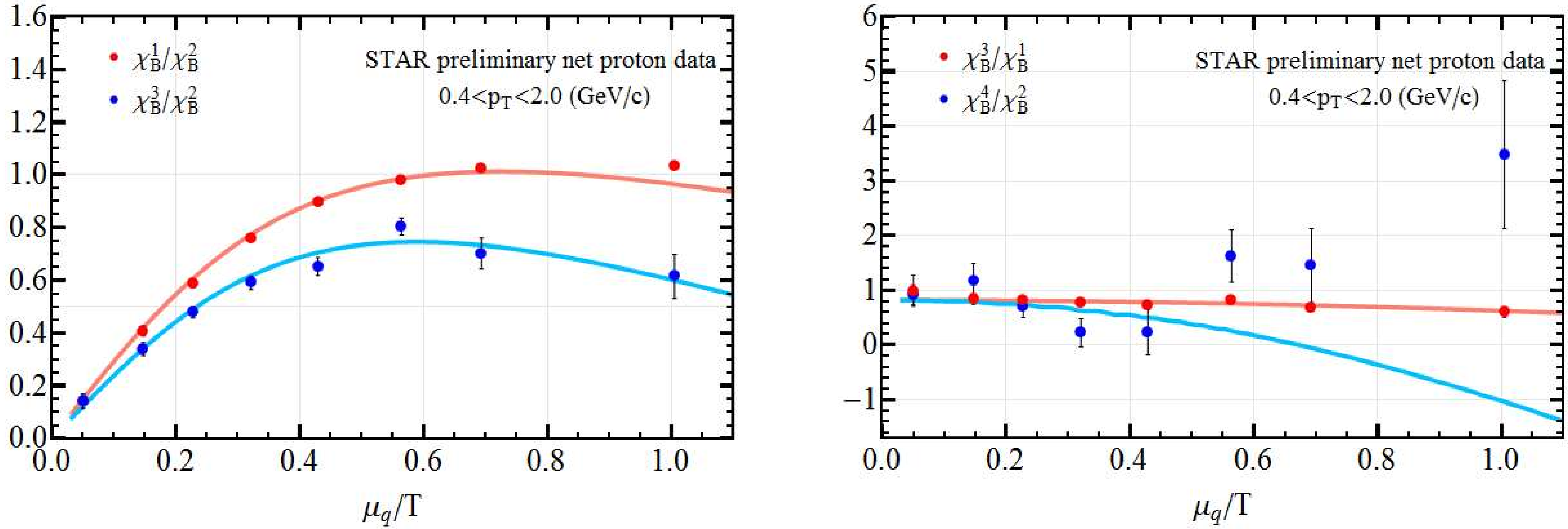

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Critical endpoint? χ_B^4/χ_B^2 data not understood

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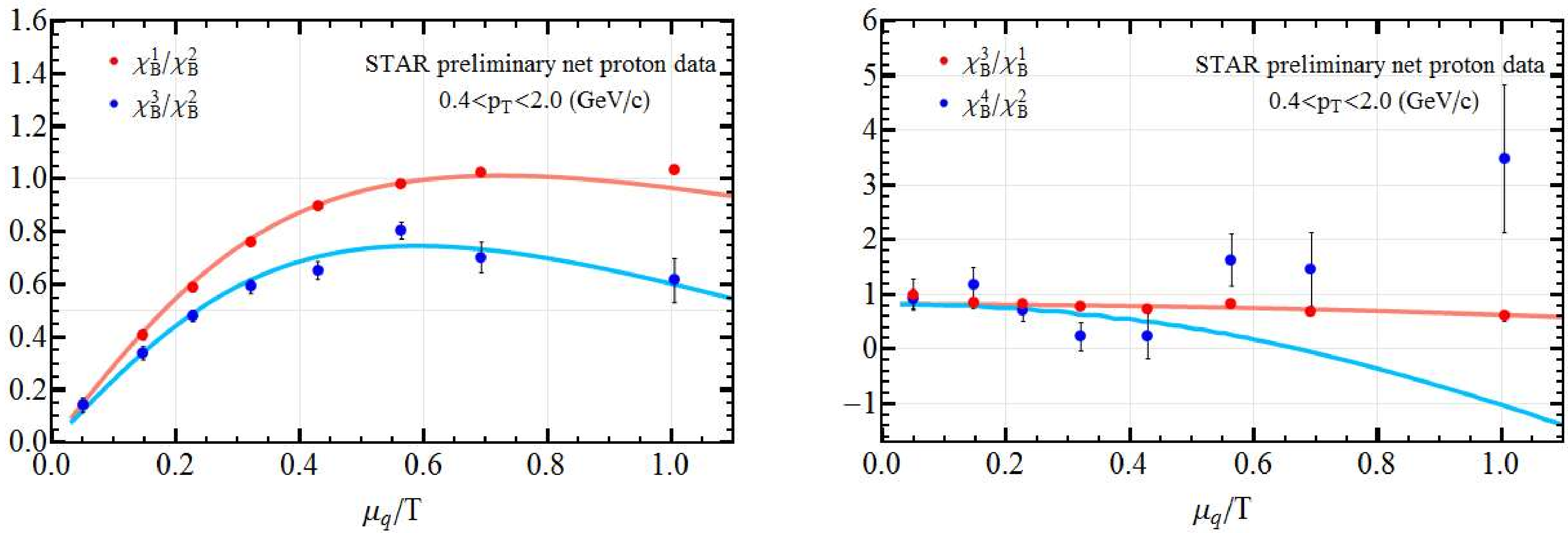

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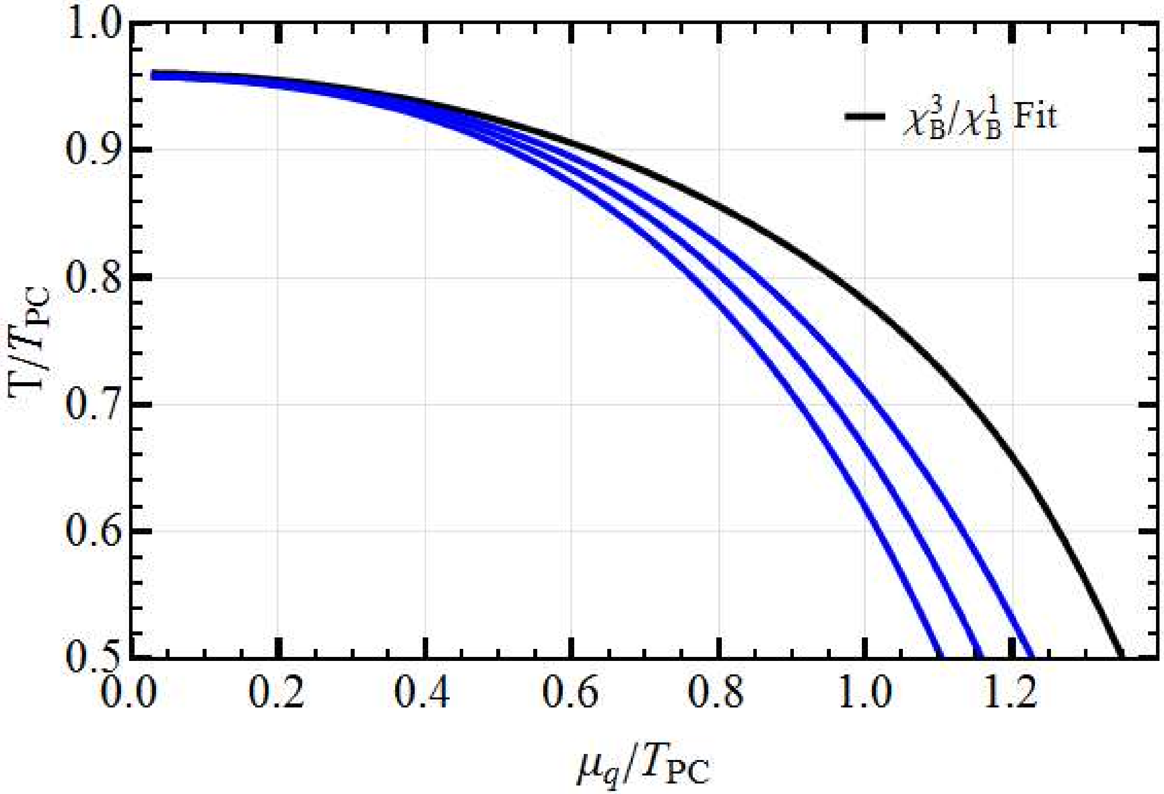
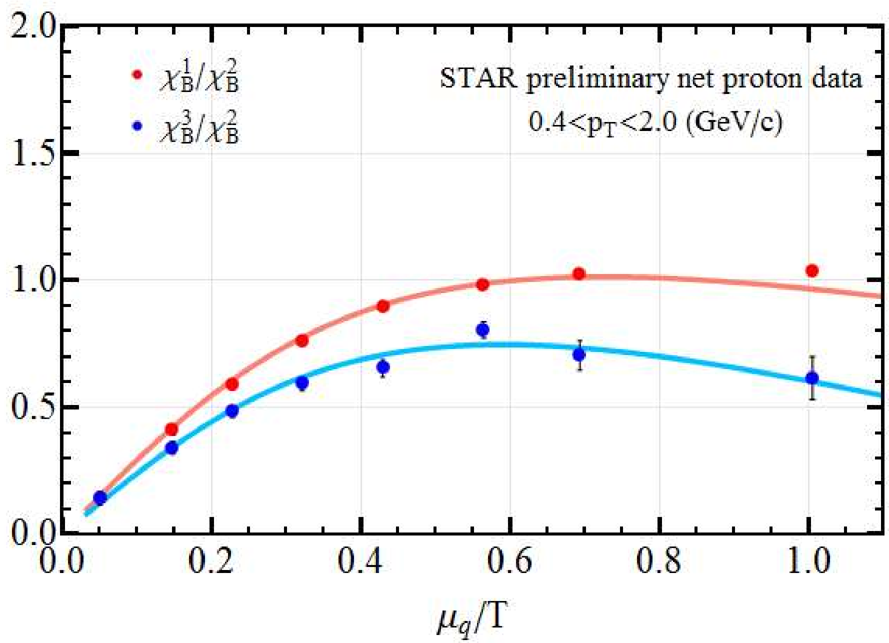
  Critical endpoint? χ_B^4/χ_B^2 data not understood
- Vector repulsion doesn't help either

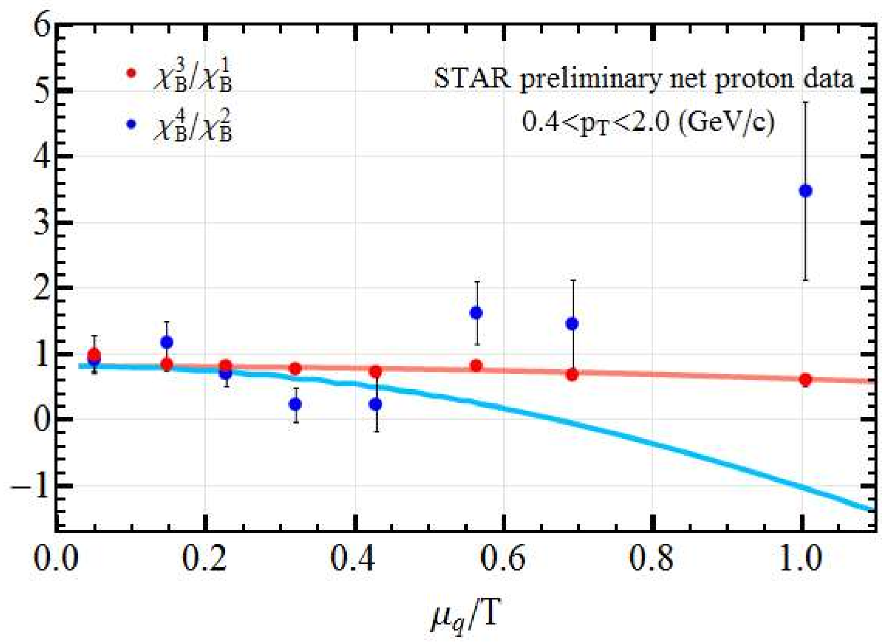
Reproducing χ_B^4/χ_B^2?



- Can we reproduce the χ_B^4/χ_B^2 data on some line in the phase diagram?

Consistency of data II.





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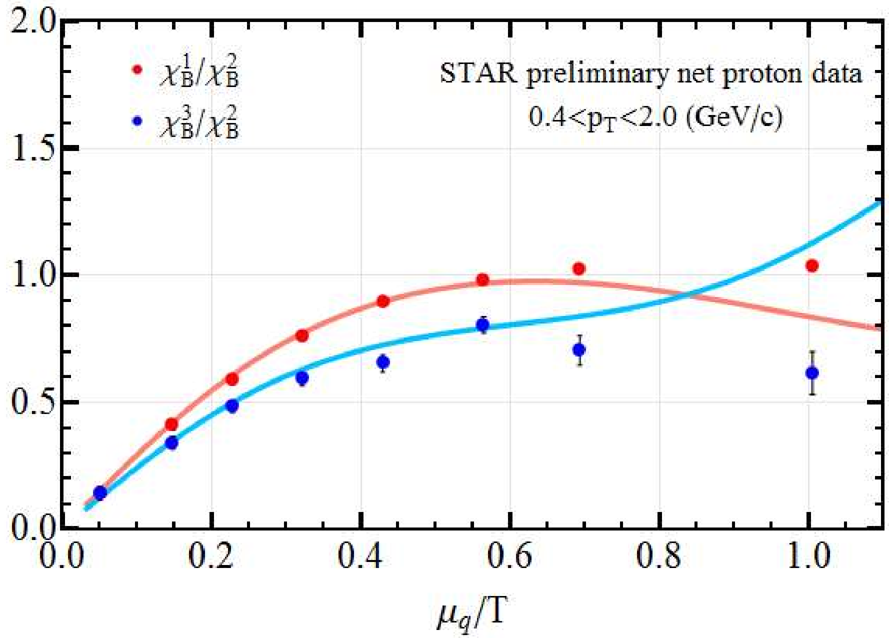

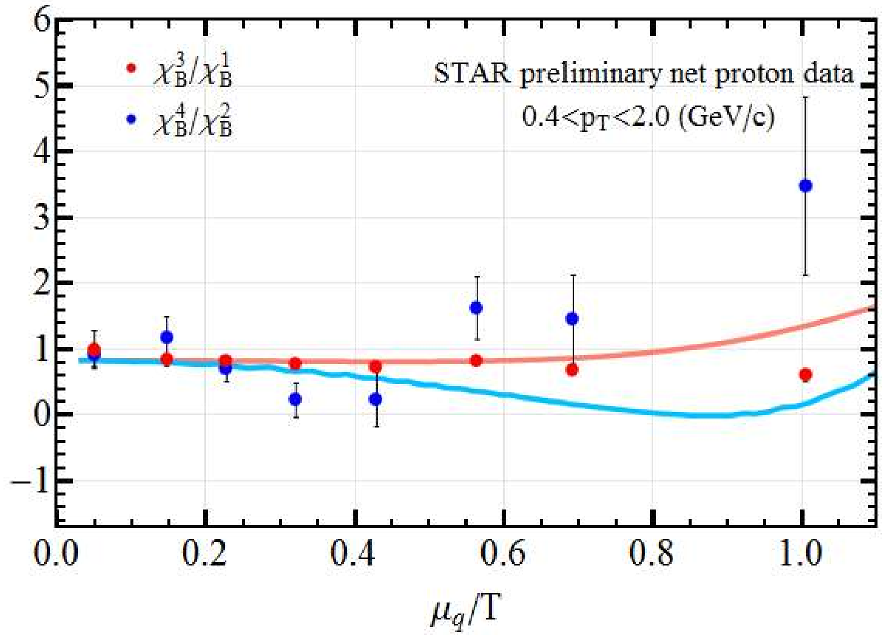

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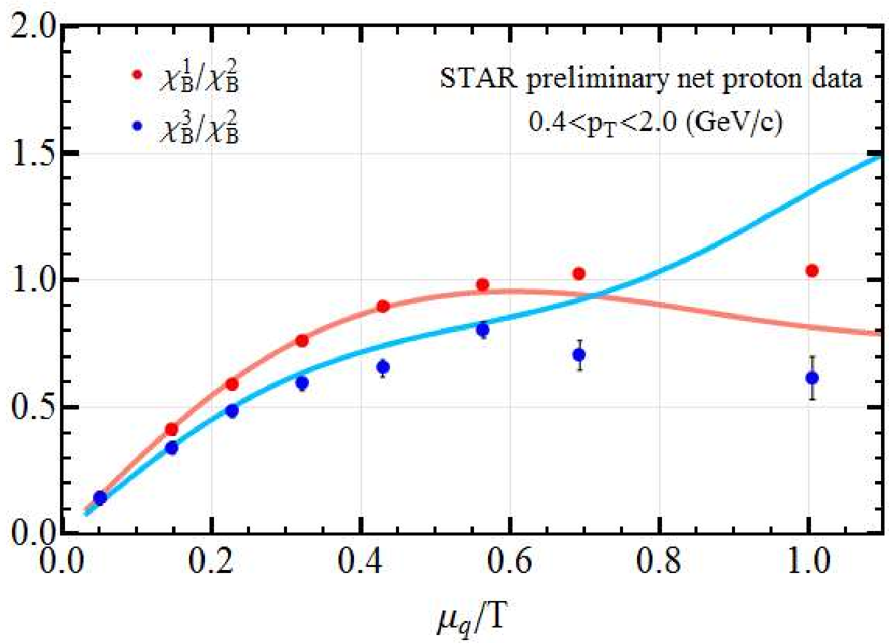

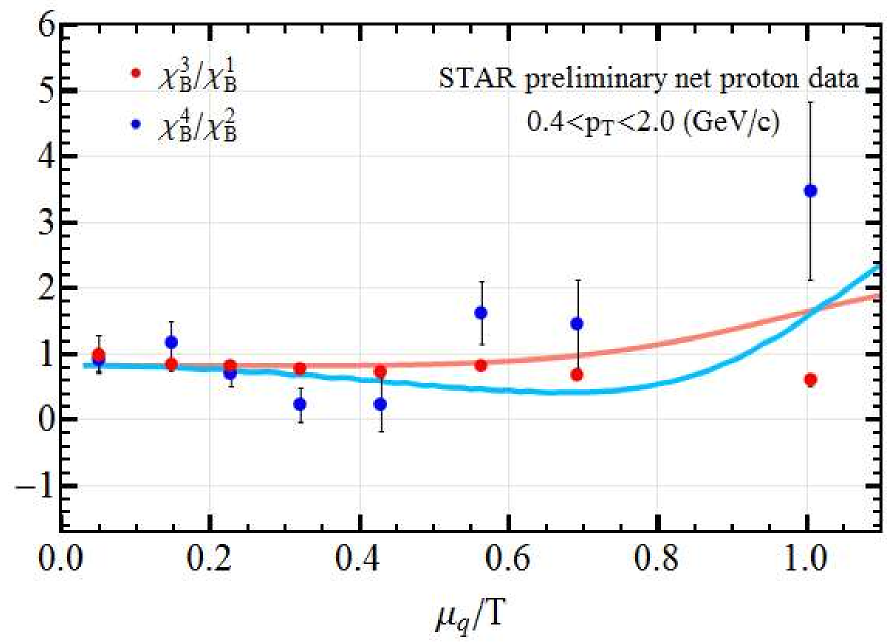

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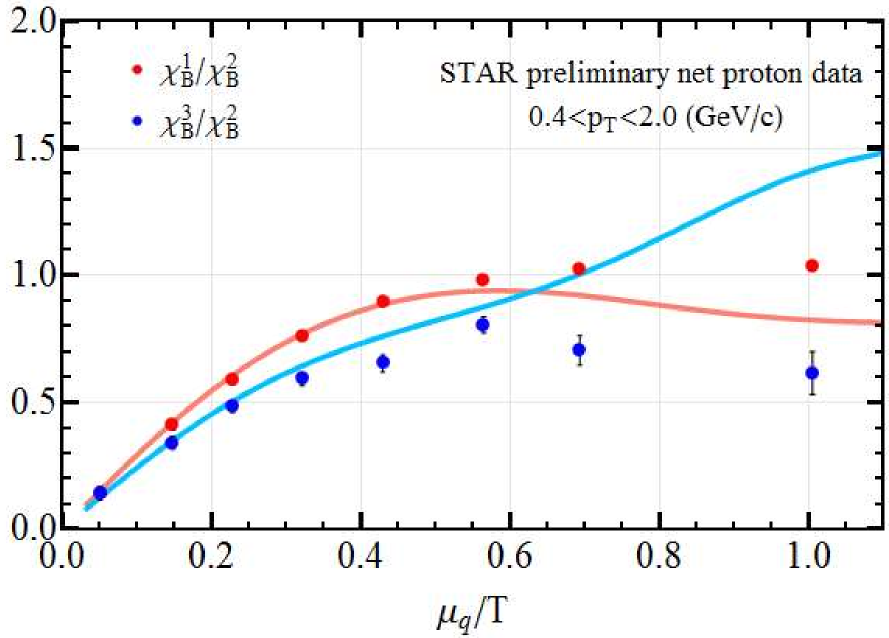
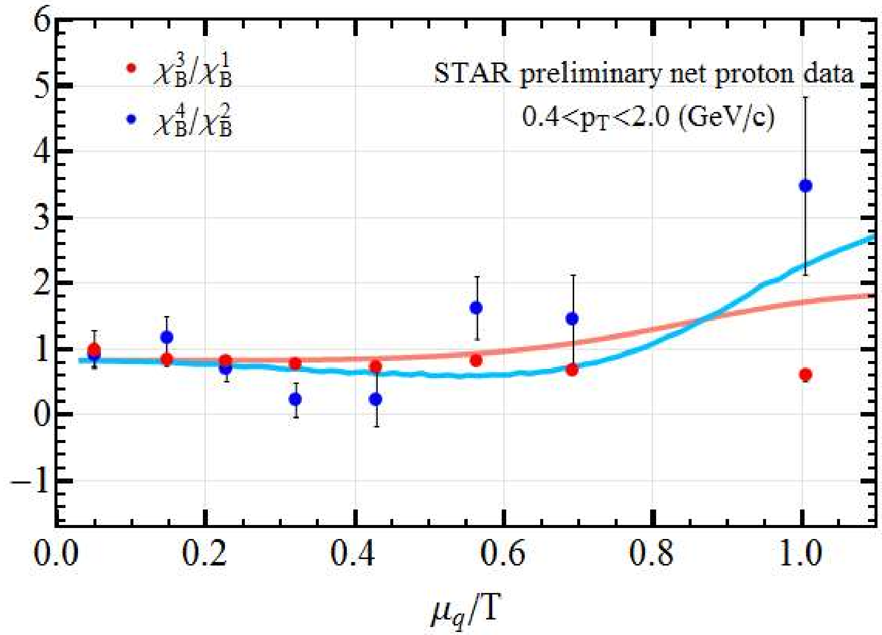




χ_B^4/χ_B^2 can be qualitatively reproduced

Other ratios are inconsistent

Coming up

- Critical fluctuations and the CEP

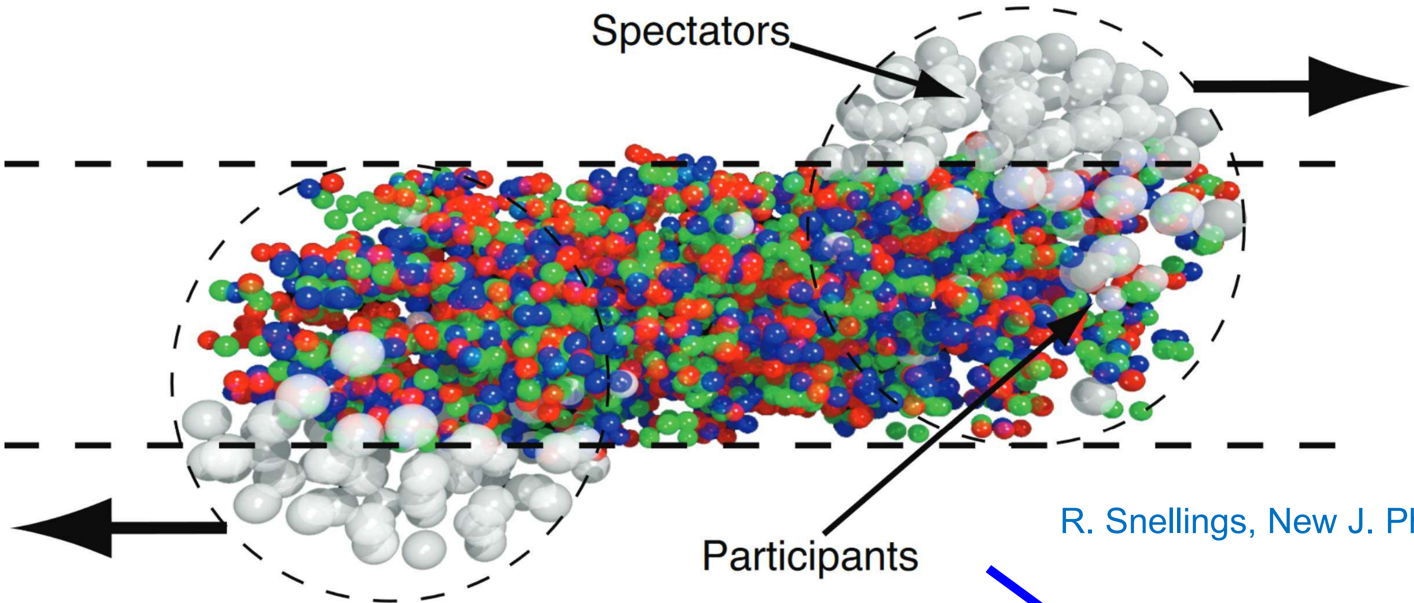
  G. Almasi, B. Friman, K. Redlich, PRD 96, 014027 (2017)

- Finite volume studies

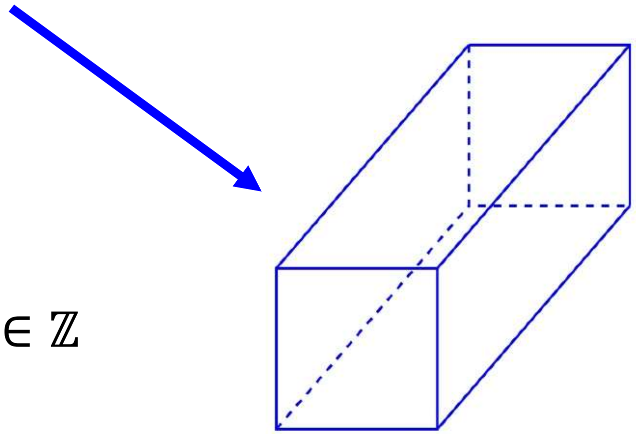
  G. Almasi, R. Pisarski, V. Skokov, PRD 95, 056015 (2017)

Quark Meson model in finite volume

Motivation: Heavy-ion collisions!



R. Snellings, New J. Phys. 13 (2011)



Implementation: periodic boundary conditions
momenta of particles are quantized

$$k_x = \frac{2\pi n_x}{L_x}, \quad k_y = \frac{2\pi n_y}{L_y}, \quad k_z = \frac{2\pi n_z}{L_z}, \quad n_x, n_y, n_y \in \mathbb{Z}$$

Quark Meson model in finite volume

Finite volume: momentum integrals are replaced by summation

$$\int d^3q \rightarrow \frac{1}{L^3} \sum_{n_x, n_y, n_z}$$

Litim regulator

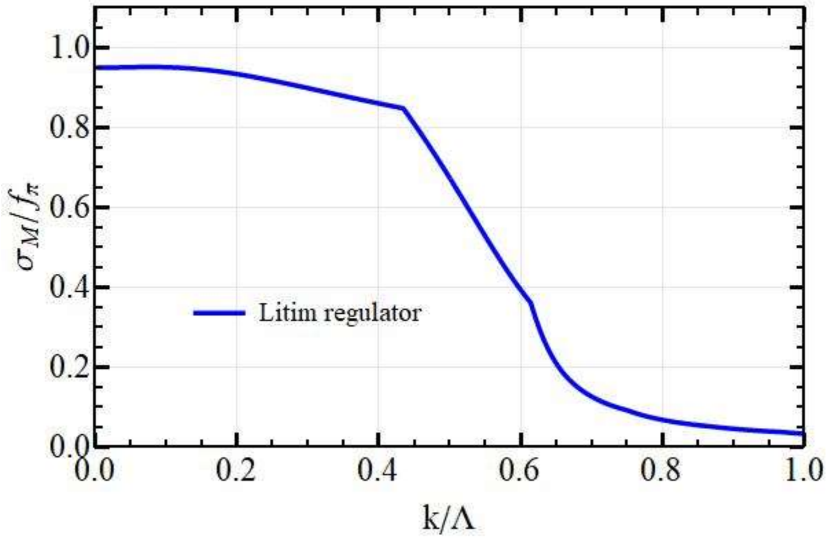
$$R_k\,(q) = \left(k^2 - q^2\right)\theta\left(k^2 - q^2\right)$$

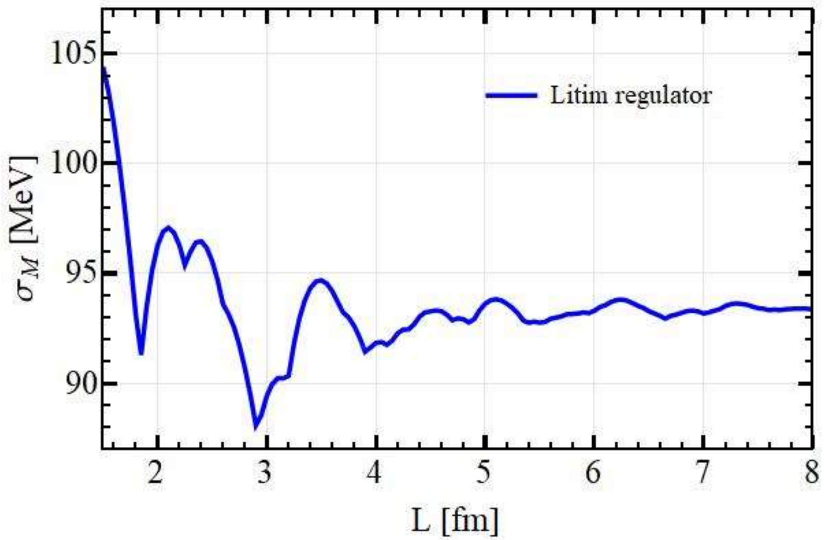
not ideal for finite volume studies:

- Flow derivative discontinuous
- Wiggles appear

Our solution: exponential regulator

$$R_k\,(q) = \frac{q^2}{\exp(q^2/k^2) - 1}$$





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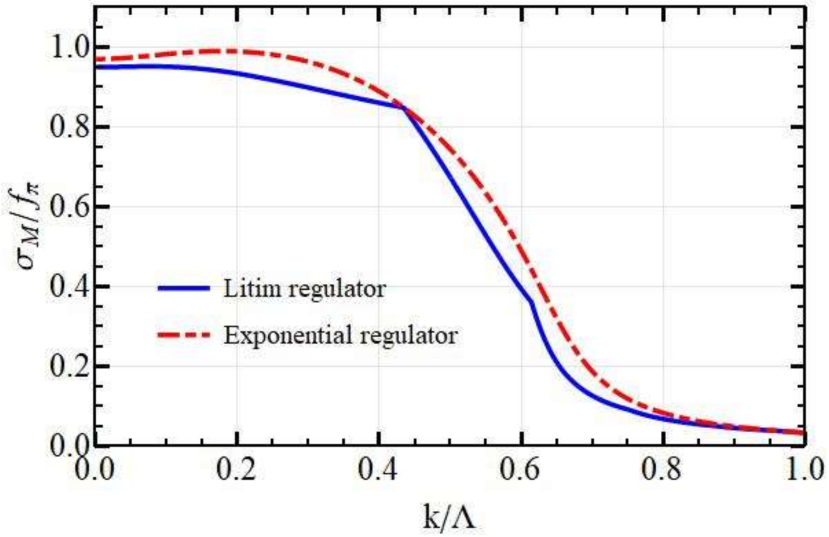
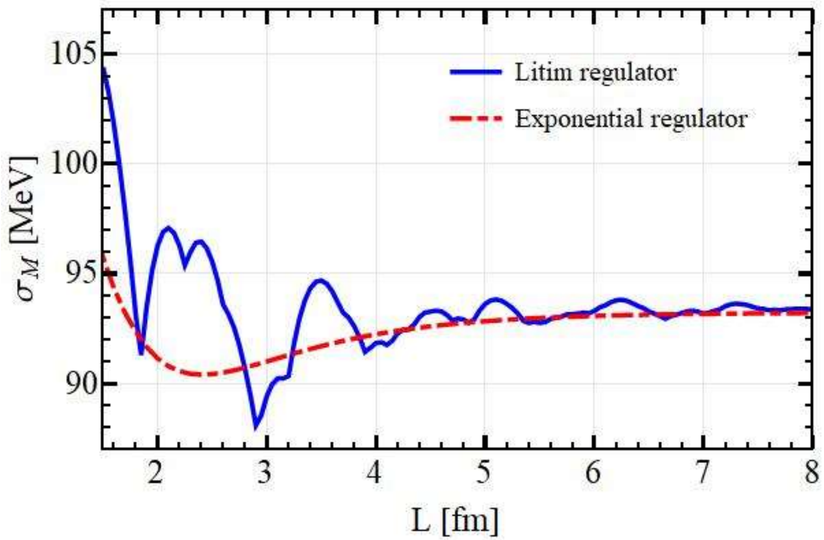




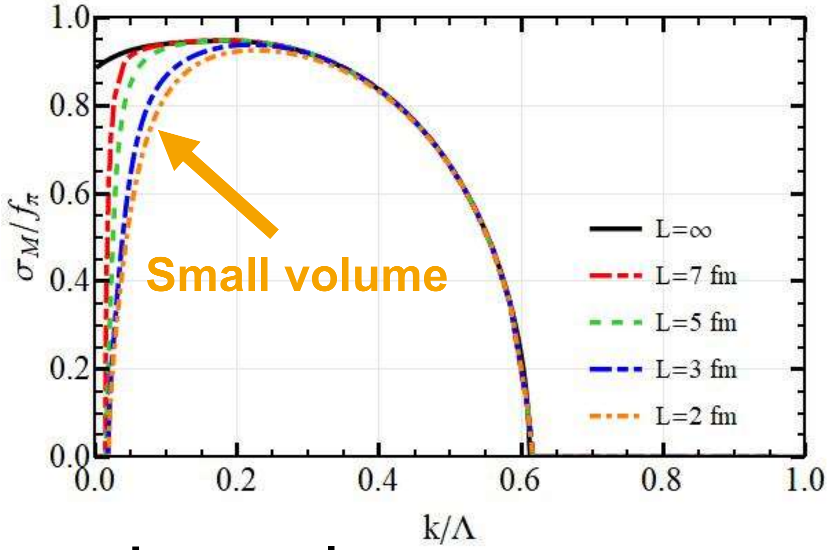
Finite volume – unphysical pion masses

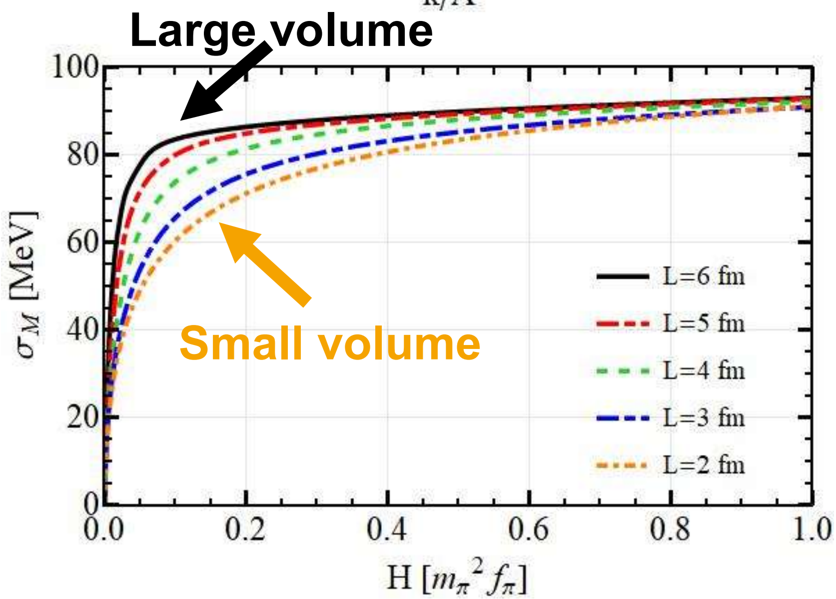
Chiral limit:

- No spontaneous symmetry breaking
- Nontrivial minimum evolves at intermediate RG scales
- Volume independence at large RG scales

Finite values of pion mass:

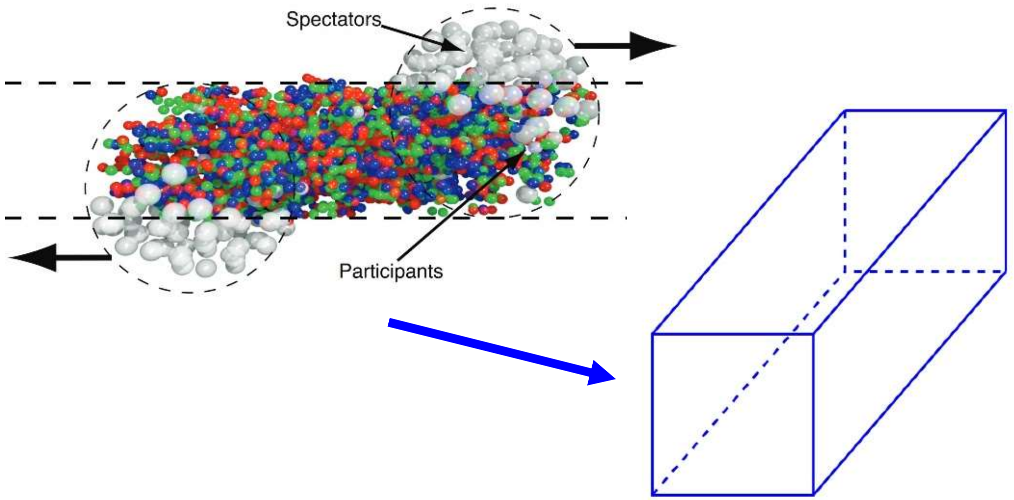
- At large pion mass tiny difference
- Smaller pion mass → larger finite volume effects

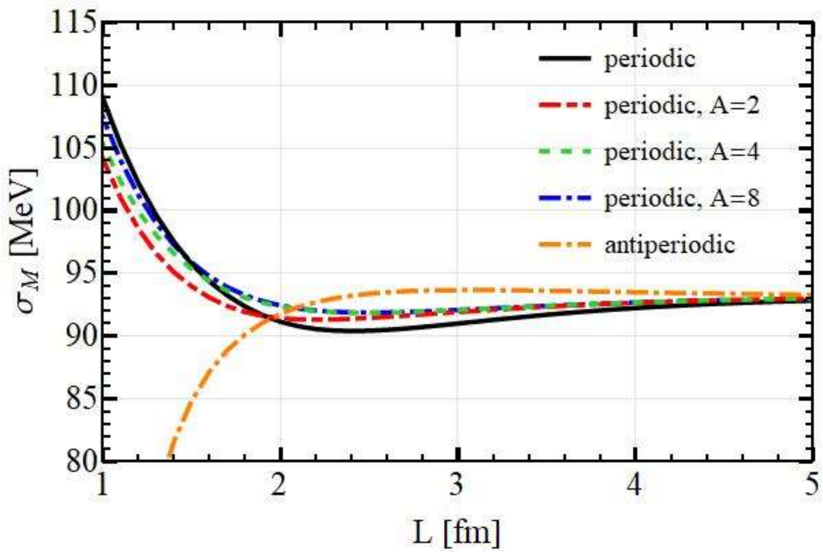


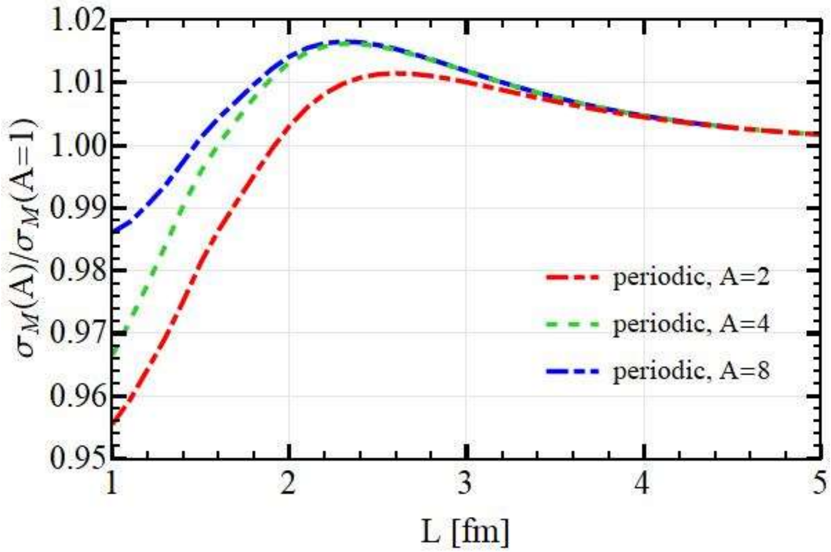


Finite volume – anisotropy

Motivation:

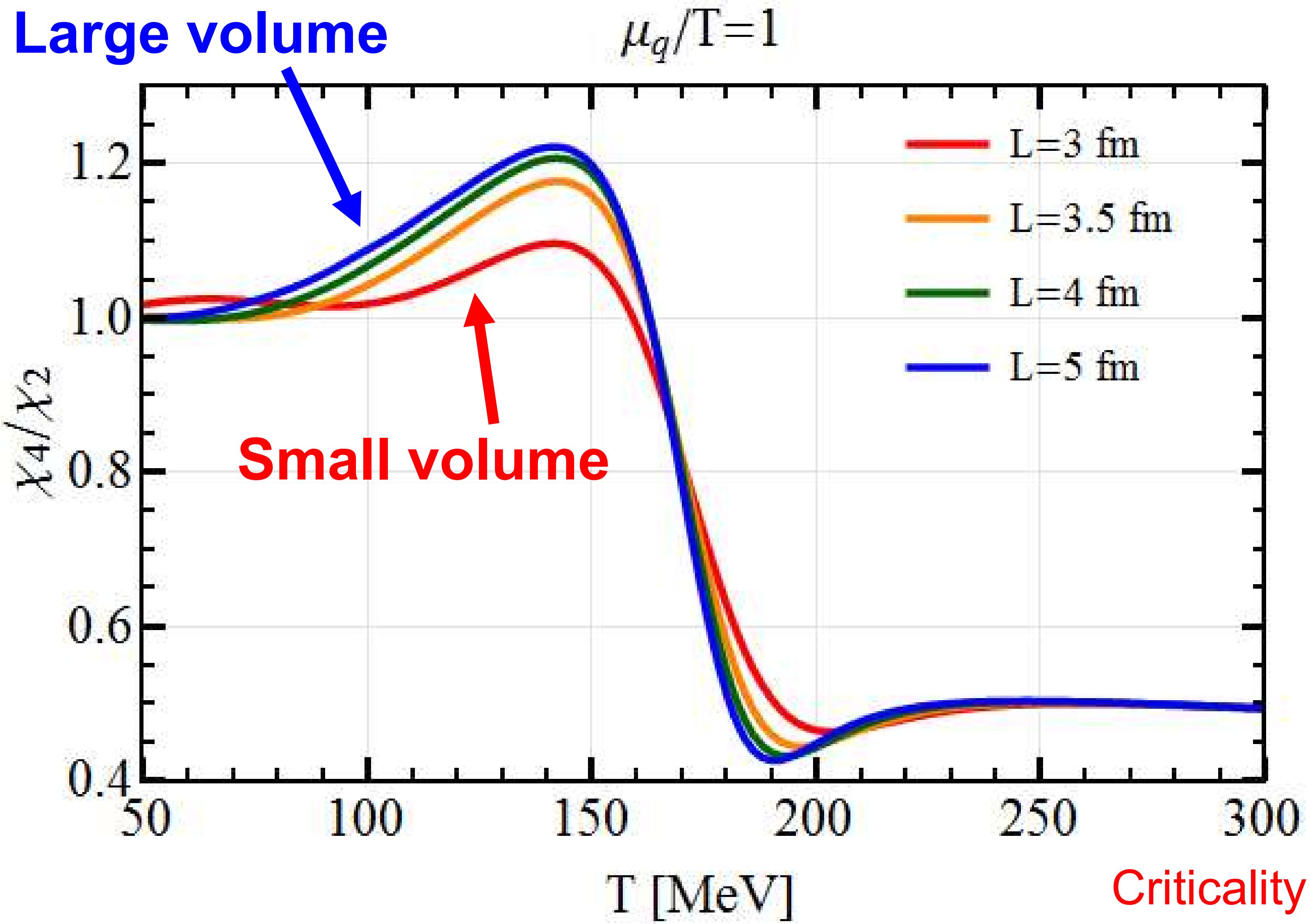






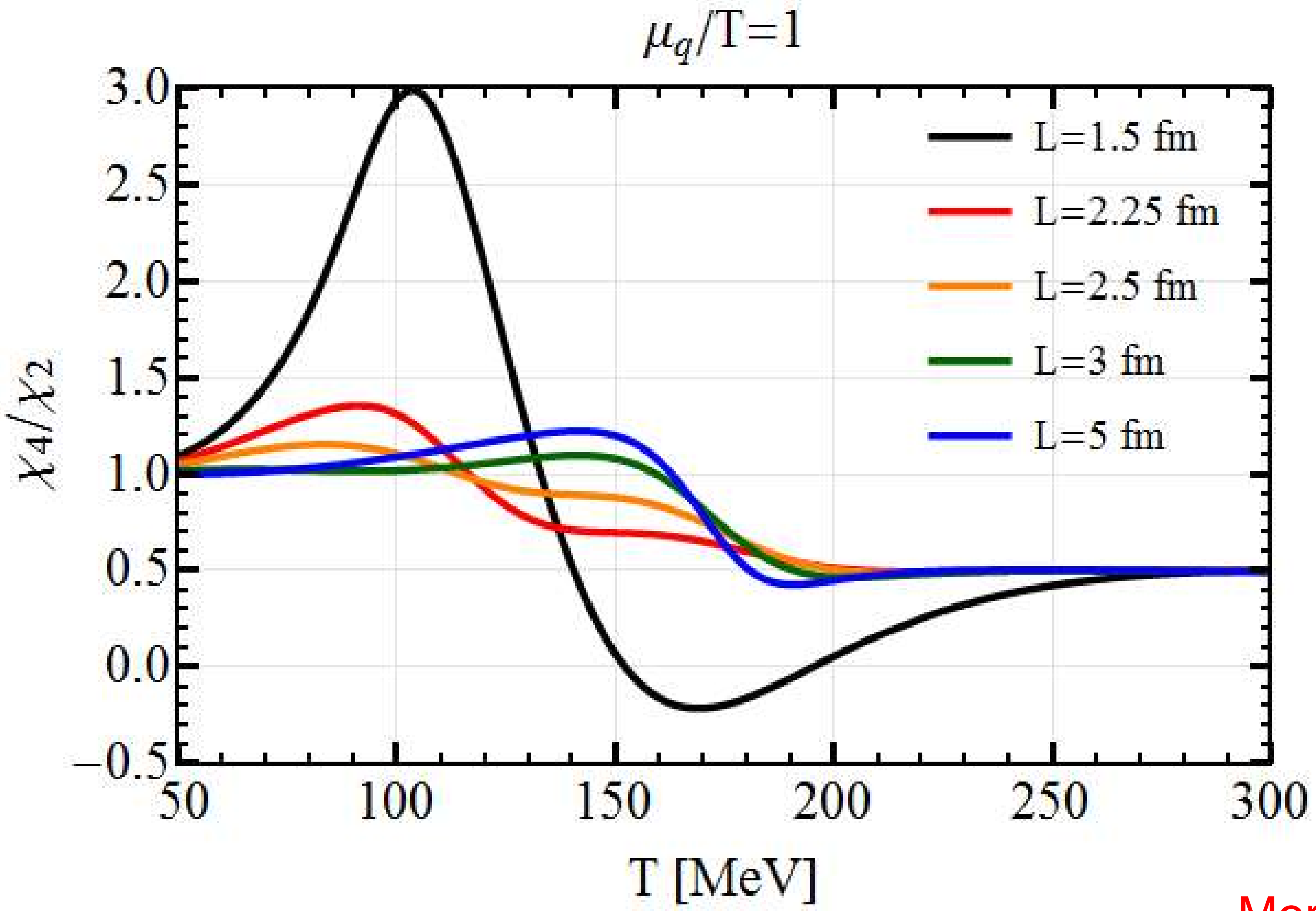
- Anisotropic box: $L_z = A\,L_x = A\,L_y$
- Small L behavior strongly depends on the applied boundary conditions
- Qualitatively similar behavior to isotropic box

Cumulants in finite volume



Criticality suppressed?

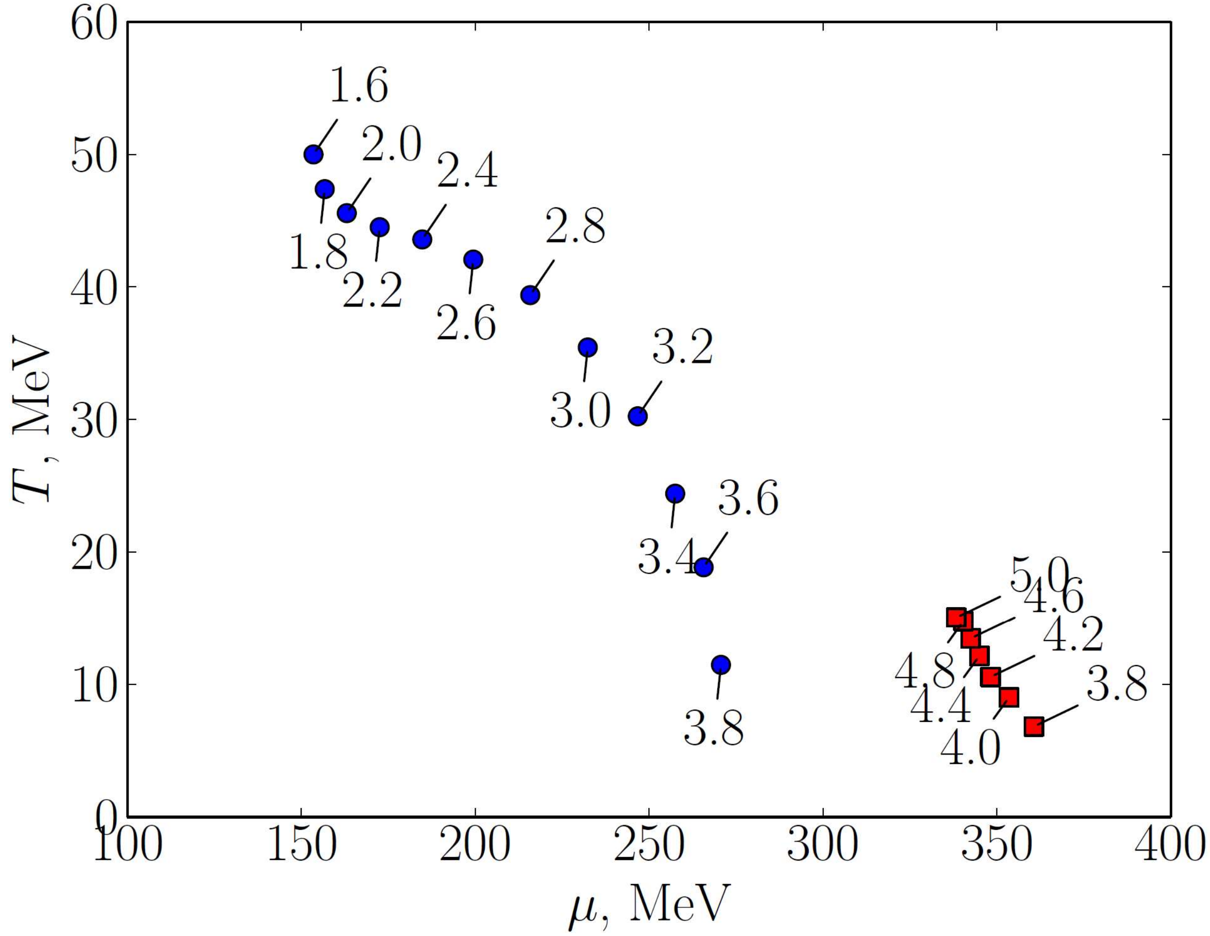
Cumulants in finite volume – small volumes



More complicated!

Apparent critical points (ACPs)

No critical point in finite volume



- ACP: local maximum of chiral susceptibility
- ACP1: close to infinite volume CEP
- ACP2: in very small volumes due to zero mode

Summary

I. PQM model

- Calculations are possible anywhere in the phase diagram
- Same universality class as QCD
- Baryon number cumulants calculated

II. Comparison to experiment

- 3 cumulant ratios qualitatively understood, χ_B^4/χ_B^2 not
- Many effects to consider
- Inclusion of vector interaction does not help

III. Finite volume

- No spontaneous symmetry breaking
- Behavior of cumulants far from trivial

Backup

Comparing theory to experiment…

Theory	Experiment
▶ Homogeneus system	▶ Inhomogenities
▶ Infinite matter	▶ Finite size effects
▶ Grand canonical ensemble	▶ Global conservation laws
▶ Information about particles of all momenta	▶ Momentum space cuts, finite efficiency
▶ Static	▶ Rapidly changing

Inclusion of vector interaction

$$\mathcal{L} = \mathcal{L}_{PQM} - g_\omega \bar{q}\omega_\mu \gamma^\mu q + \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Mean field approximation in ω :$\langle\omega_0\rangle \neq 0$

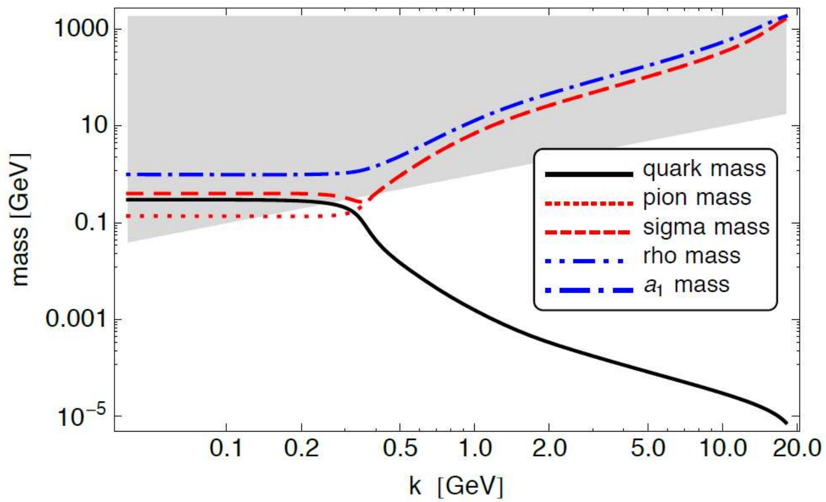
$$P(T,\mu) = P_{PQM}(T,\mu_{eff}) + \frac{g_\omega^2}{2m_\omega^2} n_{PQM}^2(T,\mu_{eff}),$$

$$\mu_{eff} \equiv \mu - g_\omega\langle\omega_0\rangle$$

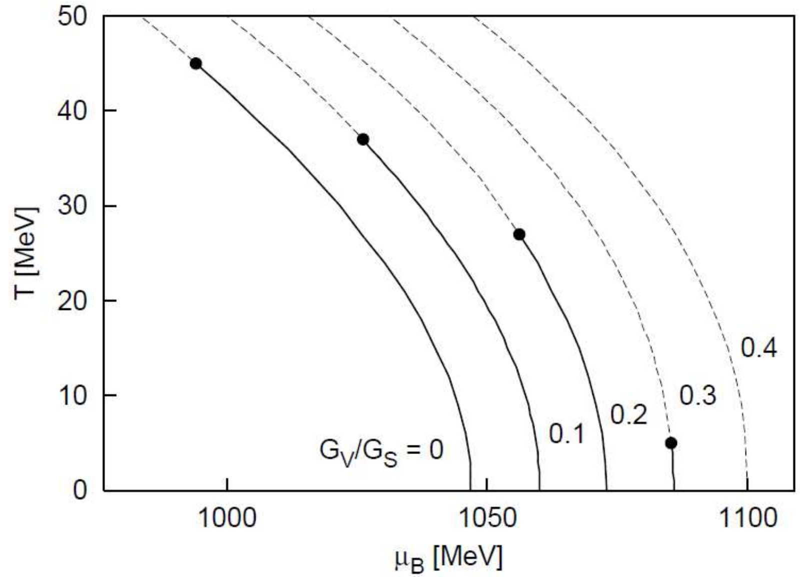
$$\langle\omega_0\rangle = \frac{g_\omega}{m_\omega^2} n_{PQM}(T,\mu_{eff})$$

Main effects:

- Shift in chemical potential: $\mu \to \mu_{eff}$
- CEP to lower T, higher μ

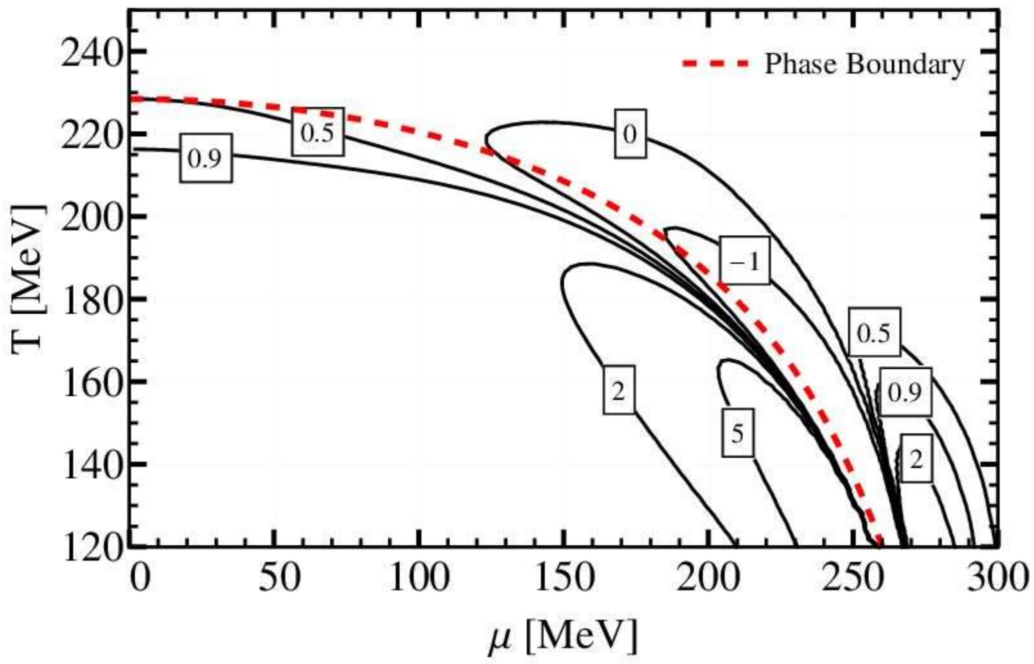


Fabian Rennecke, PRD92 (2014)

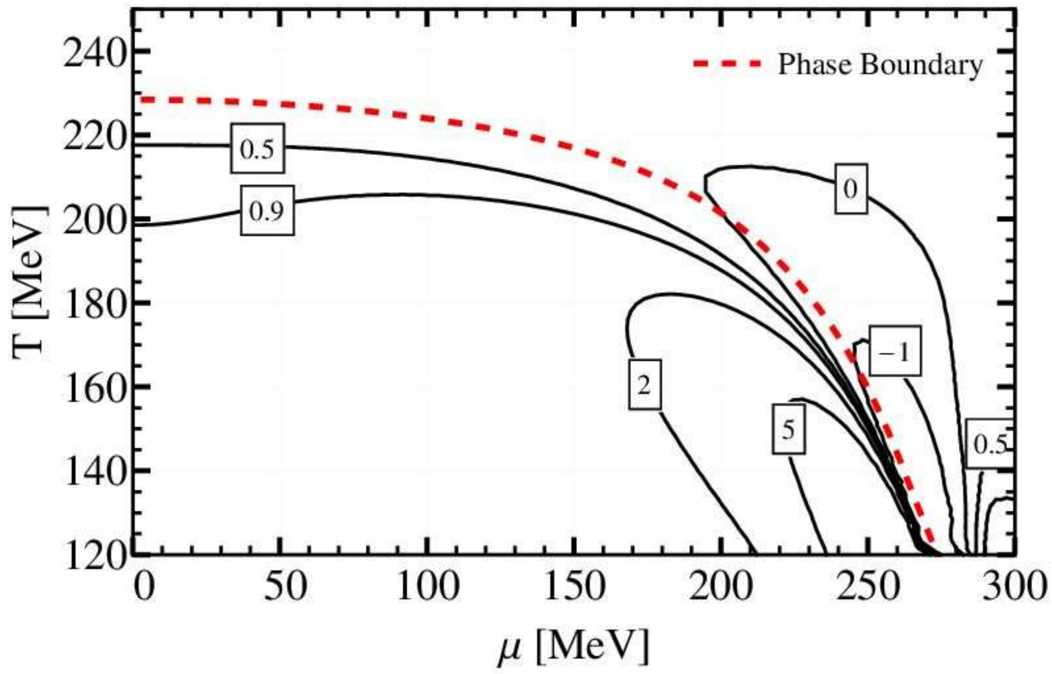


M. Kitazawa et al, Prog.Theor.Phys. 108 (2002)

Effect of the repulsive vector interaction

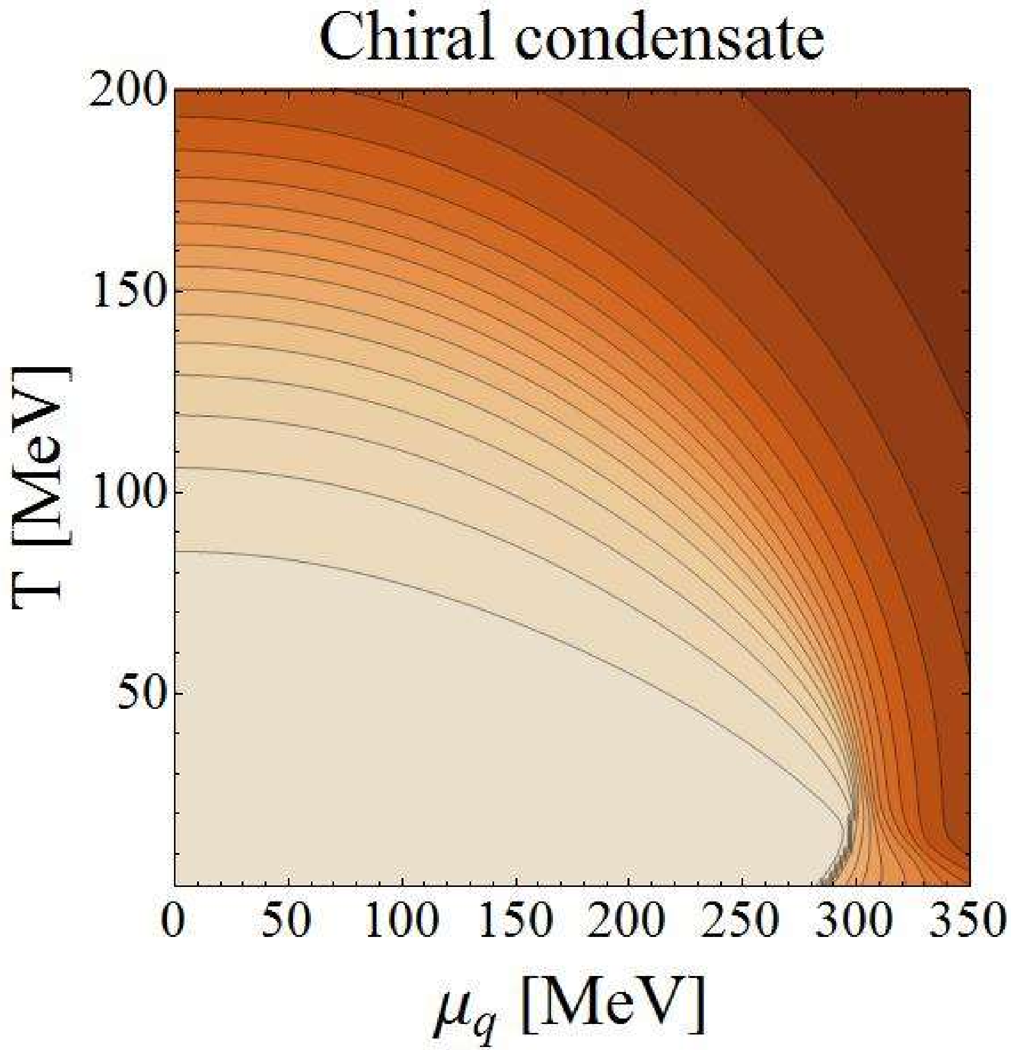


χ_B^4/χ_B^2 No vector interaction



χ_B^4/χ_B^2 Strong vector interaction

Phase diagram in PQM-FRG



- Spontaneous chiral symmetry breaking ✓
- Crossover-transition ✓
- 1st order transition ✓
- CEP ✓