FLUCTUATIONS OF CONSERVED CHARGES IN THE CANONICAL ENSEMBLE

confronting experimental results with theory

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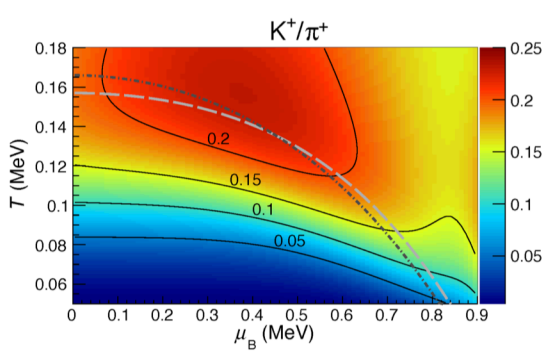
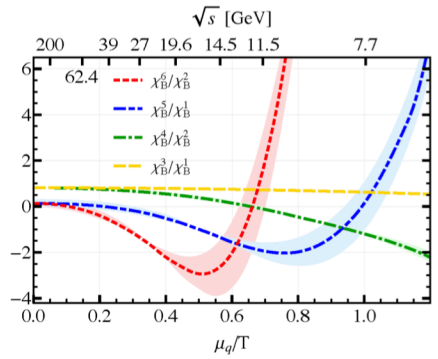
EMMI WORKSHOP

Constraining QCD Phase Boundary with Data from Heavy Ion Collisions

GSI, Darmstadt, Germany
February 12-14, 2018

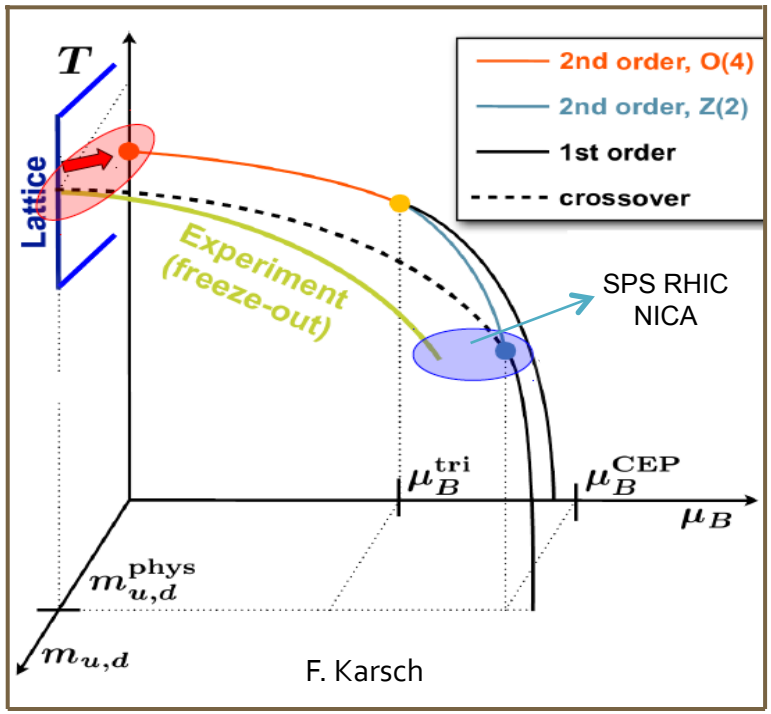




- Motivation
- Experimental results
  - NA61/SHINE, STAR, ALICE
- Fluctuations in CE
- Summary

BMBF Forschungsschwerpunkt ALICE Experiment 202

The Ultimate Goal



- To probe the structure of strongly interacting matter
  - Locate phase boundaries
  - Search for critical phenomena
  - …

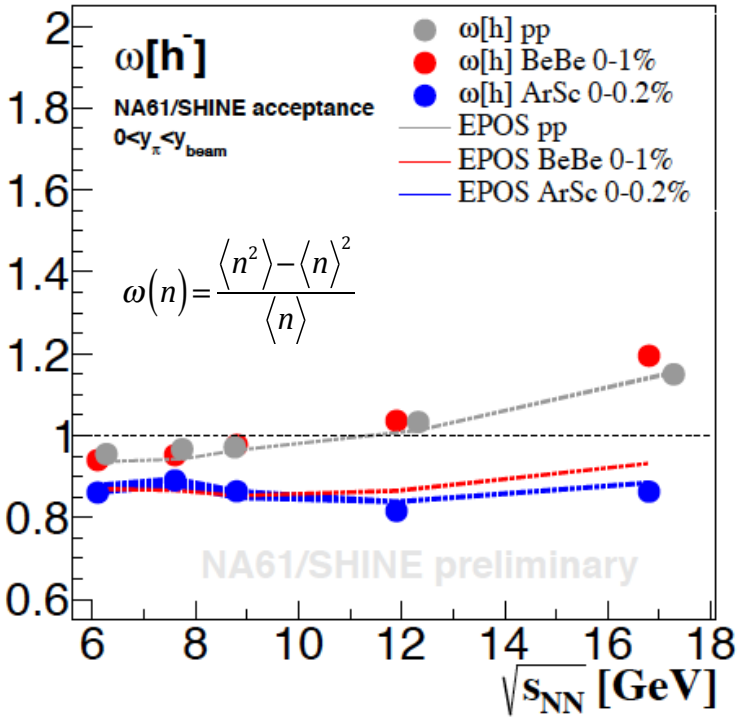
E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi}{V}, \quad \chi = -\frac{1}{V}\frac{\partial V}{\partial P}$$

fingerprints of criticality for $m_{u,d} = 0$ survive at crossover with $m_{u,d} \neq 0$

A. Bazavov et al., Phys.Rev. D85 (2012) 054503

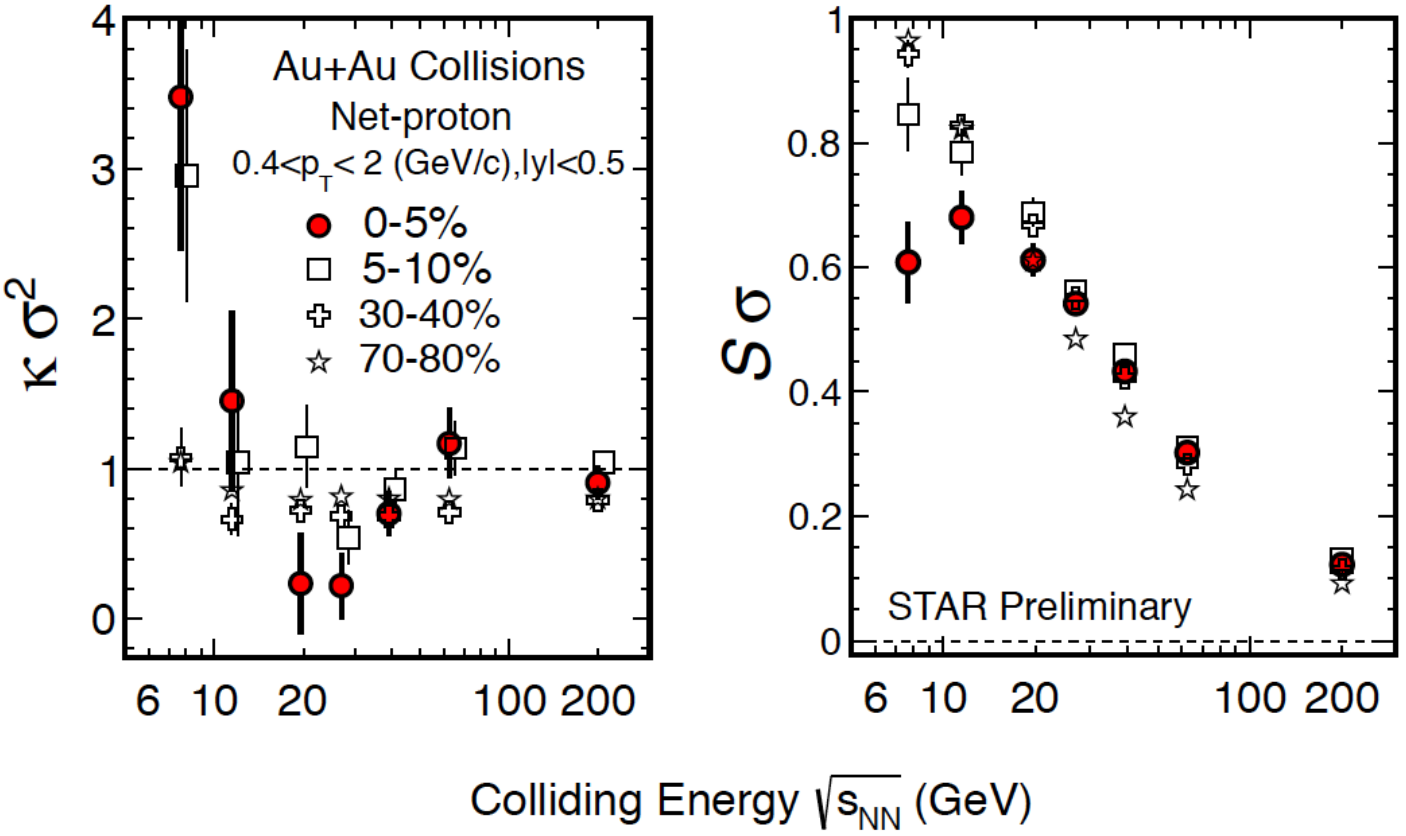
Results from NA61/SHINE



- Similar energy dependence for p+p and Be+Be
- Lower values for Ar+Sc (conservation laws)
- Deviation of Be+Be results from EPOS values
- No anomalies in the energy dependence

More details in presentation by M. Gazdzicki

Results from STAR

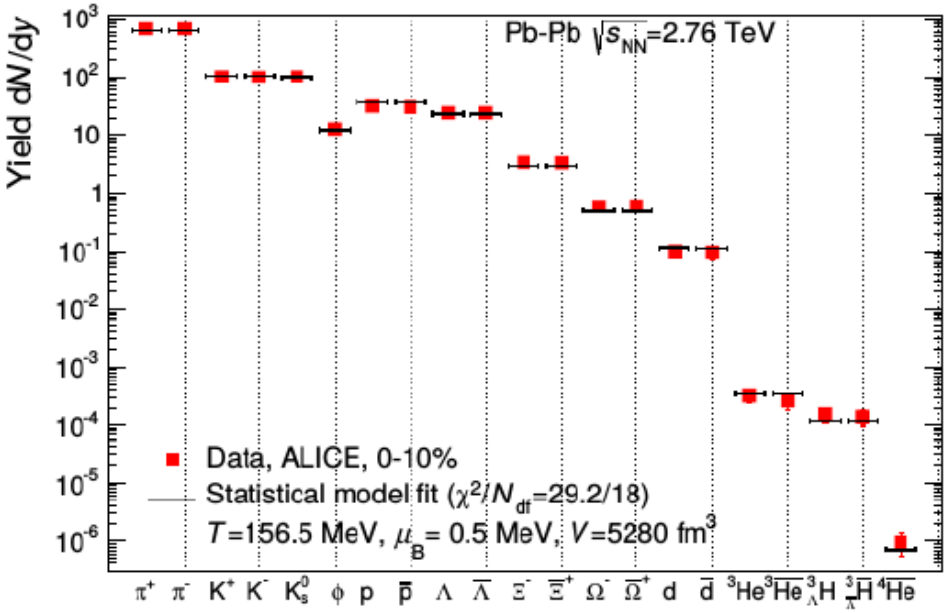


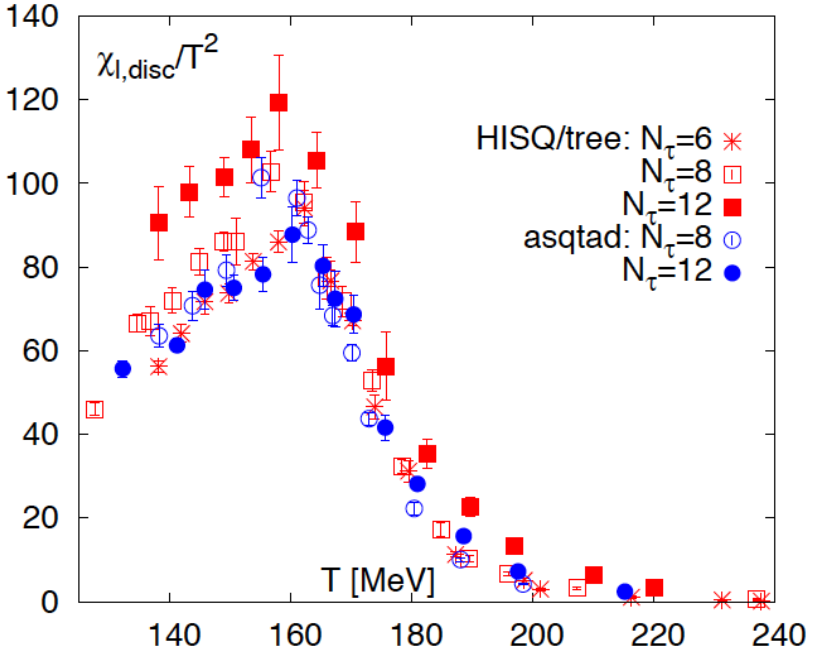
- Close to unity for peripheral collisions
- Below 39 GeV hints for a non-monotonic behavior
- ***More statistics and precise control of systematics are needed to explore this region***

Drop at 7.7 GeV for central events

X. Luo, PoS CPOD2014, 019 (2015)
STAR: PRL 112, 032302 (2014)

Criticality at crossover





$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

freeze-out at the phase boundary

$$T_c^{lattice} = 154 \pm 9\, MeV, \quad T_{fo}^{ALICE} = 156 \pm 3 MeV$$

ALICE, PLB 726 (2013) 610
J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich
J. Phys. Conf. Ser. 509 (2014) 012019

A. Bazavov et al., Phys.Rev. D85 (2012) 054503

y axis: 9 orders of magnitude, works in the energy range spanned by 3 orders of magnitude

Bridge from experiment to theory

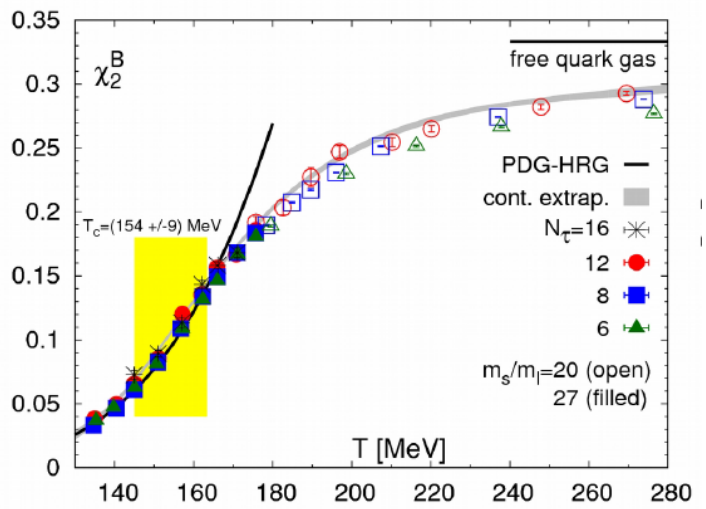
for a thermal system in a fixed volume V within the Grand Canonical Ensemble

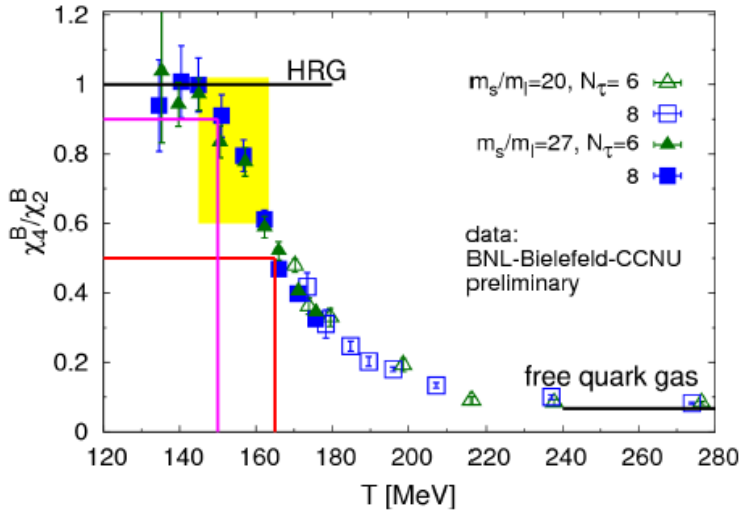
$$\hat{\chi}_2^B = \frac{\left\langle \Delta N_B^2 \right\rangle - \left\langle \Delta N_B \right\rangle^2}{VT^3} = \frac{\kappa_2\left(\Delta N_B\right)}{VT^3}$$

$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial\left(\mu_N/T\right)^n} \qquad \frac{P}{T^4} = \frac{1}{VT^3}\ln Z\left(V,T,\mu_{B,Q,S}\right)$$

- *In experiments*
  - *Volume (participants) fluctuates from E-to-E*
  - *Global conservation laws are important*

$$\frac{\kappa_4\left(\Delta N_B\right)}{\kappa_2\left(\Delta N_B\right)} \equiv \gamma_2\sigma^2 \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \qquad \frac{\kappa_3\left(\Delta N_B\right)}{\kappa_2\left(\Delta N_B\right)} \equiv \gamma_1\sigma \neq \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$





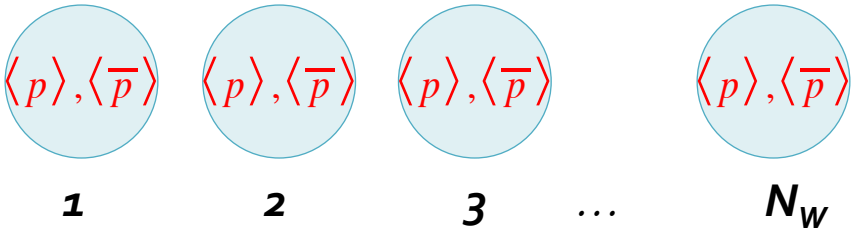
smaller than in HRG for T > 150 MeV

F. Karsch, QM17, arXiv:1706.01620
O. Kaczmarek, QM17, arXiv:1705.10682

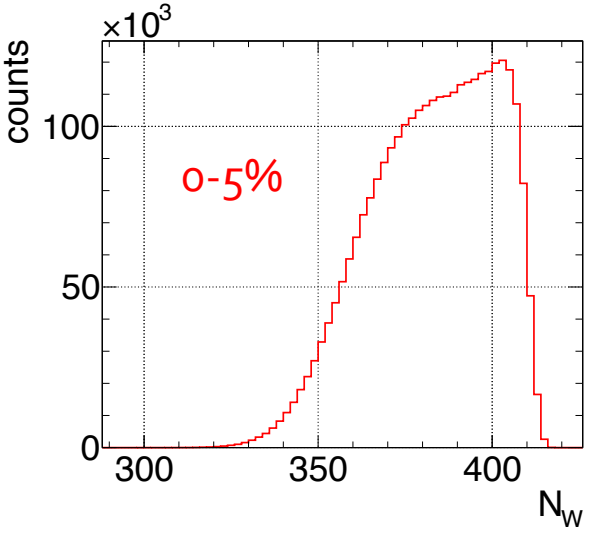
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

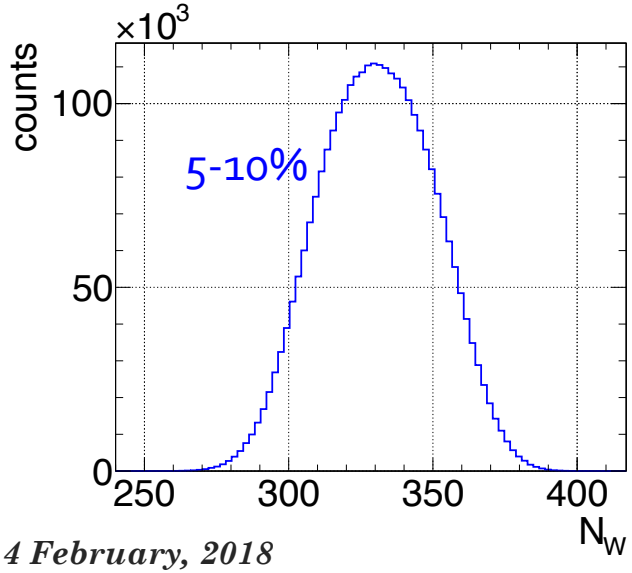
P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

Participant (volume) fluctuations



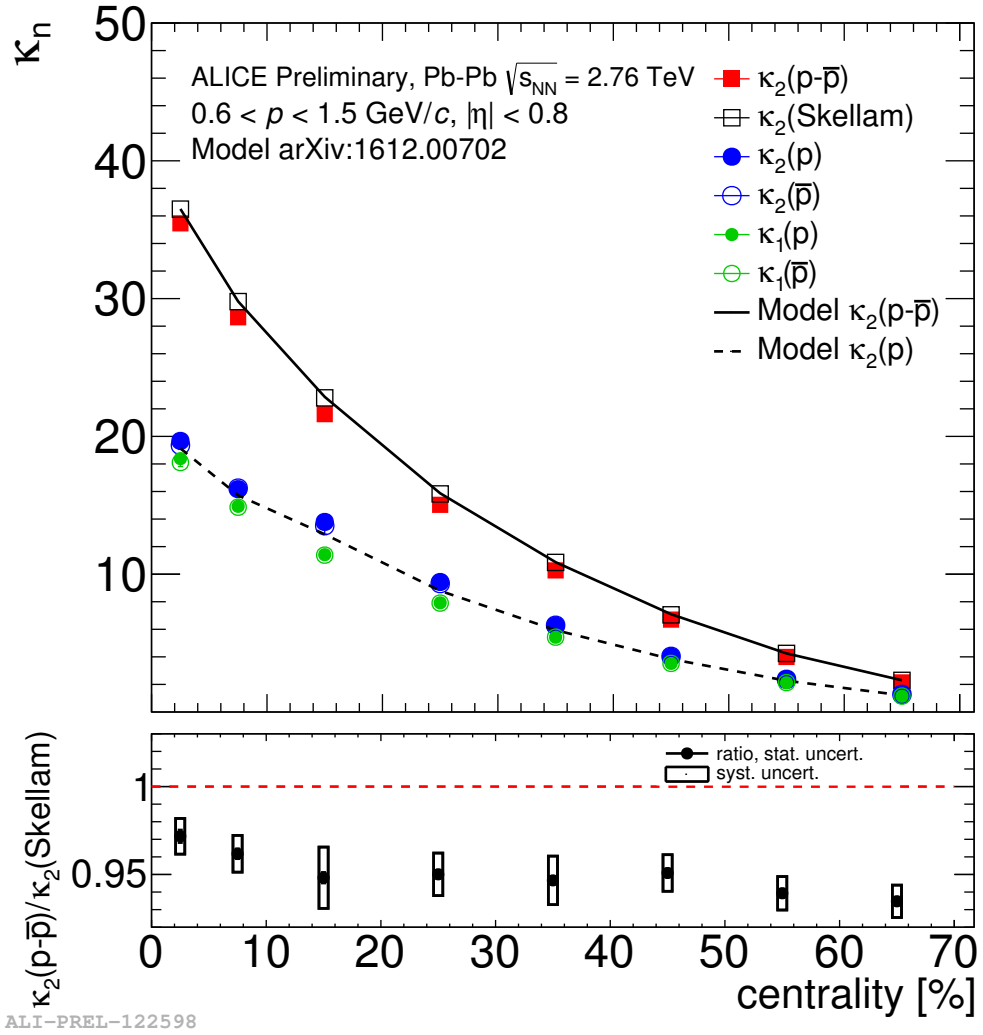
- N_W *fluctuates with MC Glauber initial conditions*
- *Particles are produced from each source (GCE)*
- *Inputs:*
  - *Mean proton multiplicities* $\langle p \rangle$, $\langle \overline{p} \rangle$
  - *Centrality selection like in experimental data*





P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

Results from ALICE



A. R., QM2017, arXiv:1704.05329

$$\kappa_2\left(N_B - N_{\bar{B}}\right) = \kappa_2\left(N_B\right) + \kappa_2\left(N_{\bar{B}}\right) - 2\left(\left\langle N_B N_{\bar{B}}\right\rangle - \left\langle N_B\right\rangle\left\langle N_{\bar{B}}\right\rangle\right)$$

<participants> vanishes at LHC

$$\kappa_2\left(N_B - N_{\bar{B}}\right) = \left\langle N_W\right\rangle \kappa_2\left(n_B - n_{\bar{B}}\right) + \left\langle n_B - n_{\bar{B}}\right\rangle^2 \kappa_2\left(N_W\right)$$

from single participant — volume fluct.

Input to the Model:

$\kappa_1(p)$, $\kappa_1(\bar{p})$

centrality selection procedure

Predictions:

——— $\kappa_2(p-\bar{p})$

– – – – – $\kappa_2(p)$

Second cumulants of net-particles at LHC are not affected by participant fluctuations

easy control of systematics

Acceptance dependence

Contribution from global baryon number conservation

$$\frac{\kappa_2\left(p-\bar{p}\right)}{\kappa_2\left(Skellam\right)}=1-\alpha \qquad \alpha=\frac{\left\langle p\right\rangle^{measured}}{\left\langle B\right\rangle^{4\pi}}$$

P. Braun-Munzinger, A. R., J. Stachel,
arXiv:1612.00702, NPA 960 (2017) 114

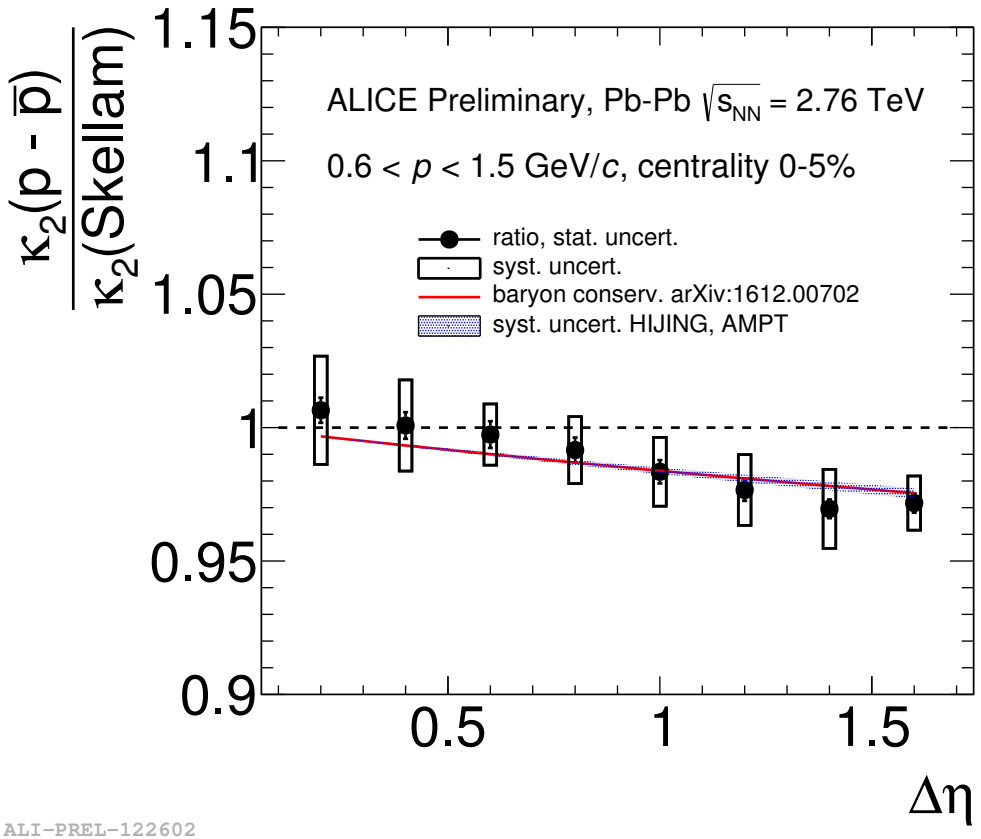
Inputs for $\left\langle B\right\rangle^{acc}$ from:

Phys. Lett. B 747, 292 (2015)
P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel

extrapolation from $\left\langle B\right\rangle^{acc}$ to $\left\langle B\right\rangle^{4\pi}$

using HIJING and AMPT models

A. R., QM2017, arXiv:1704.05329



The deviation from Skellam is due to the global baryon number conservation.

effects of conservation laws for higher moments

+

volume fluctuations

Our approach: MC sampling of canonical ensemble

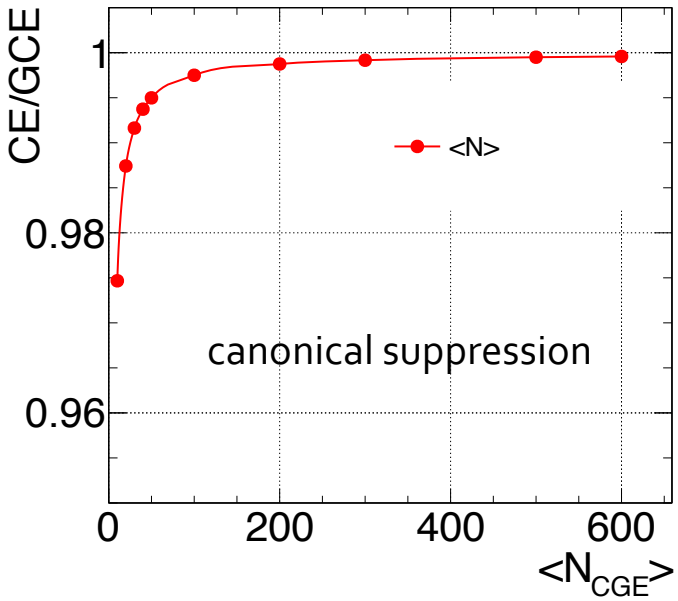
Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{\left(\lambda_B z\right)^{N_B}}{N_B!}\frac{\left(\lambda_{\bar{B}} z\right)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh\left(\mu/T\right)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{\left(\lambda_B z\right)^{N_B}}{N_B!}\frac{\left(\lambda_{\bar{B}} z\right)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta\left(N_B-N_{\bar{B}}-B\right)=I_B\left(2z\right)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

$$\left\langle N_{B,\bar{B}}\right\rangle_{GCE}=\lambda_{B,\bar{B}}\frac{\partial\ln Z_{GCE}}{\partial\lambda_{B,\bar{B}}}=e^{\pm\frac{\mu}{T}}z,\quad z=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}$$

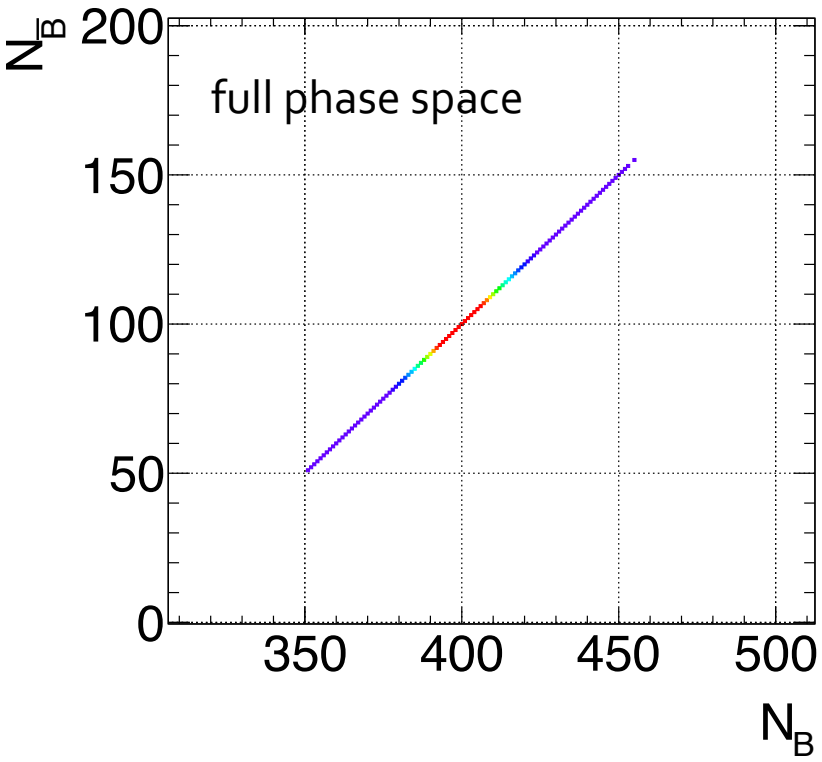
$$\left\langle N_{B,\bar{B}}\right\rangle_{CE}=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\frac{I_{B\mp1}\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}{I_B\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}$$

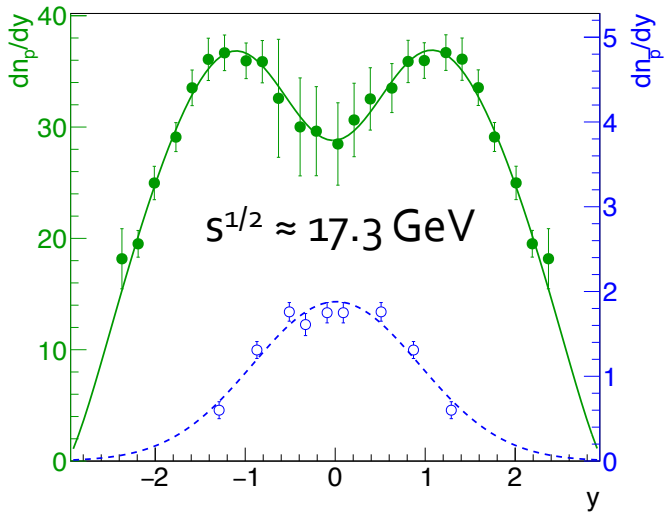


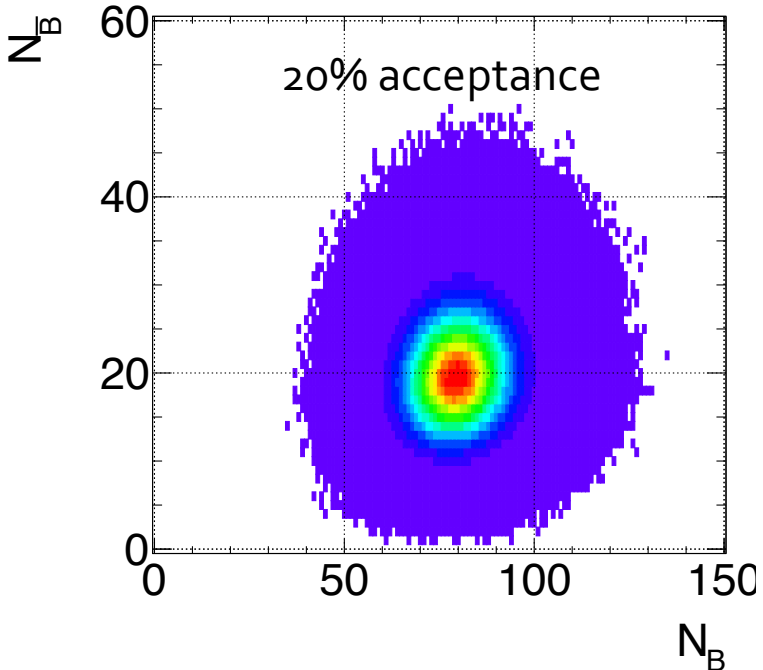
R. Hagedorn, K. Redlich Z. Phys. 27, 1985
V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich, V. Skokov, NPA 880 (2012)
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

Baryon number conservation (implementation)

$$N_B = 400, \quad N_{\bar{B}} = 100$$



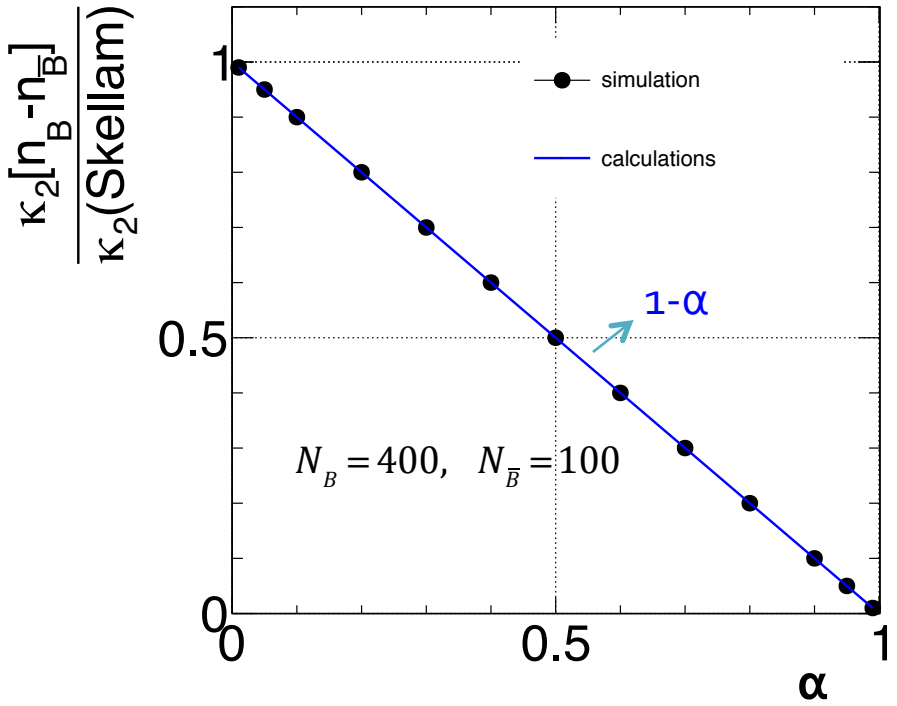


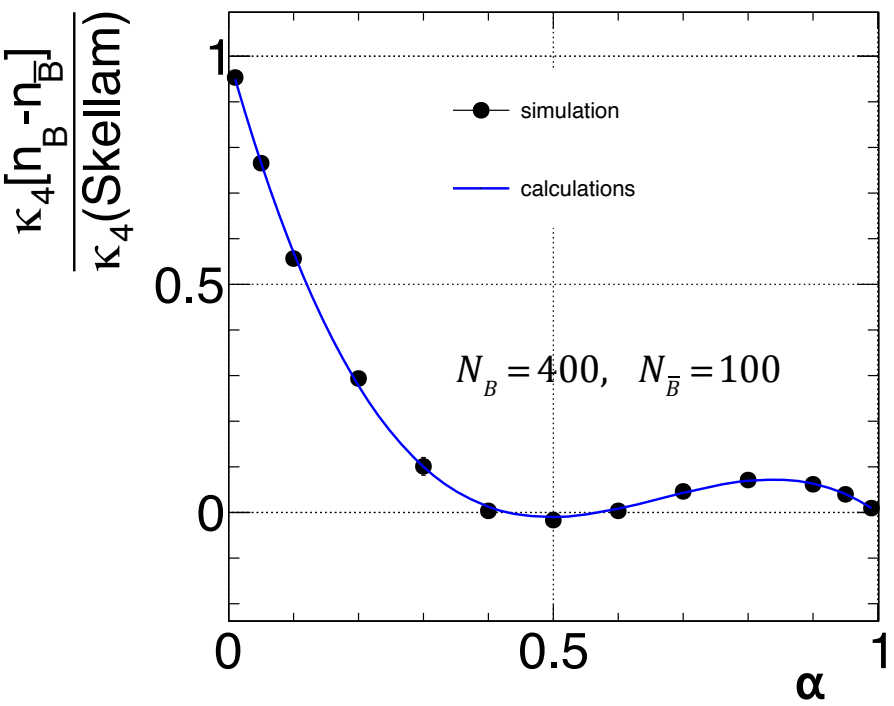


- **Fluctuations of net-baryons appear only inside finite acceptance**

P. Braun-Munzinger, A. R., J. Stachel, in preparation

Importance of acceptance

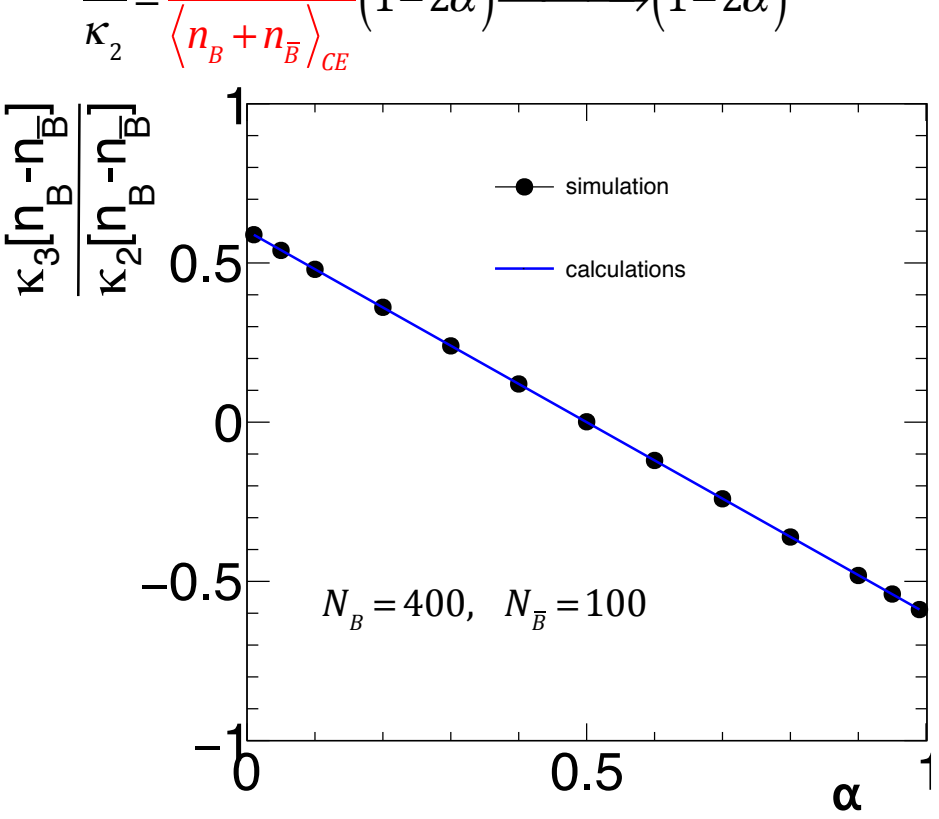


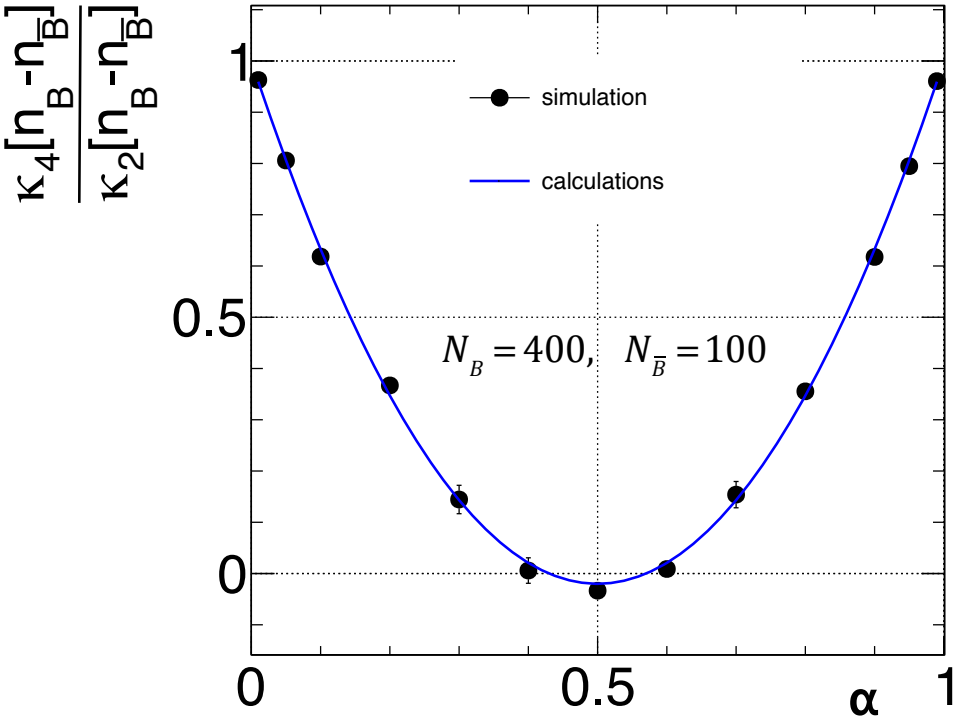


- Approach to independent Poisson (Skellam) for a small acceptance
- Approach to zero for full acceptance
- Acceptance is more crucial for 4th cumulant

Third and fourth cumulants

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{B}} \to 0 \rangle} (1-2\alpha)$$

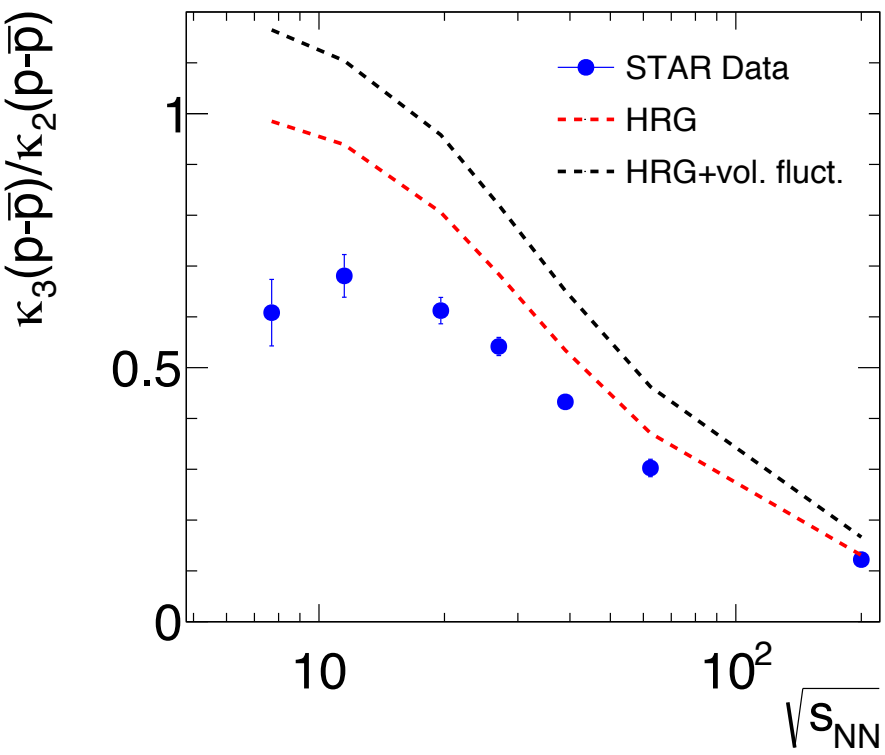


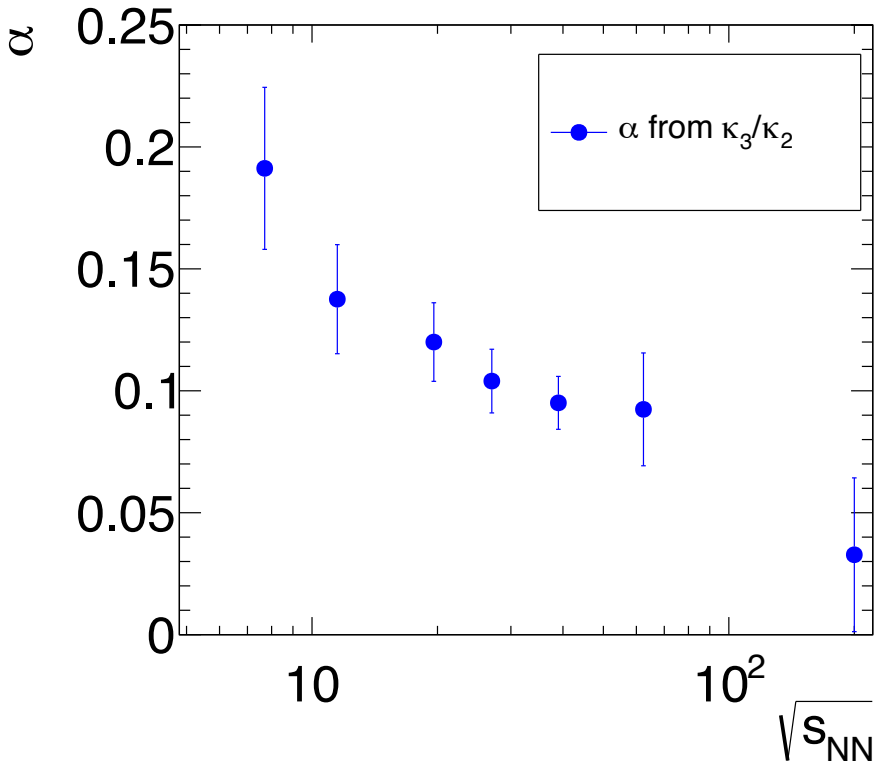


$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle N_B + N_{\bar{B}} \rangle_{CE}} \left[\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_{\bar{B}} \rangle_{CE} \right]\right)$$

Also cf. A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

Extracting the α parameter

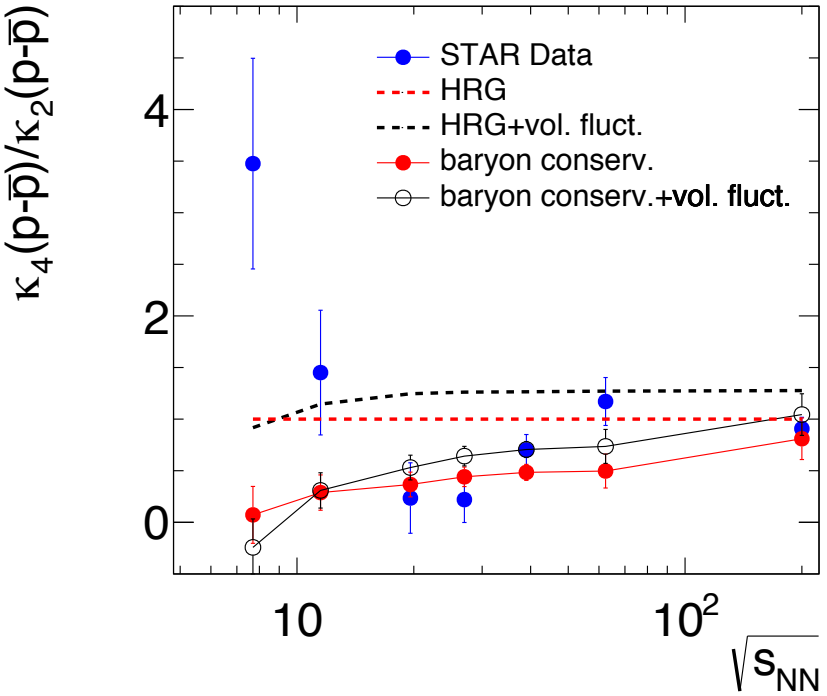




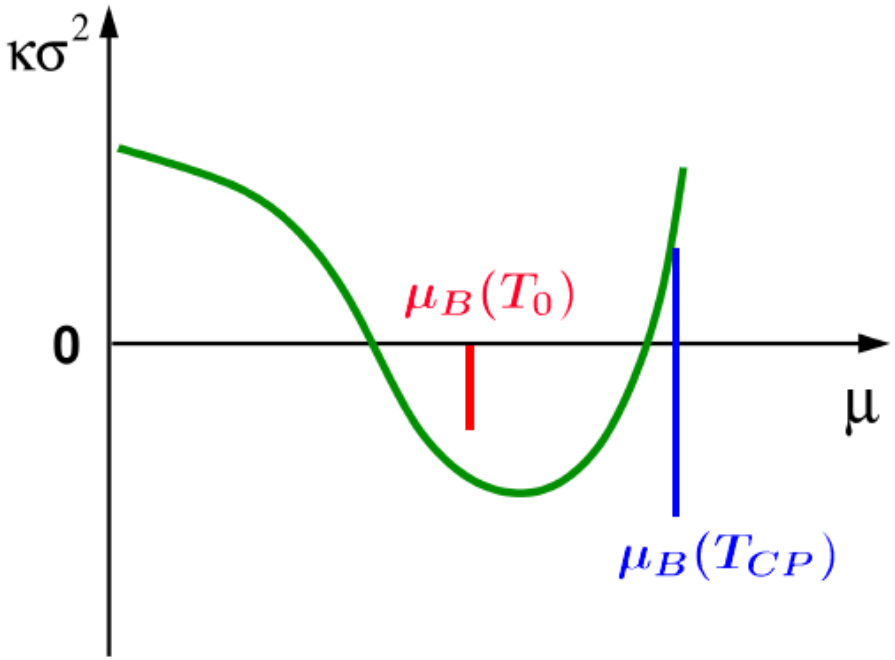
$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{B}} \to 0 \rangle} (1-2\alpha)$$

Modeling κ_4/κ_2

$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle N_B + N_{\bar{B}} \rangle_{CE}}\left[\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_{\bar{B}} \rangle_{CE}\right]\right)$$



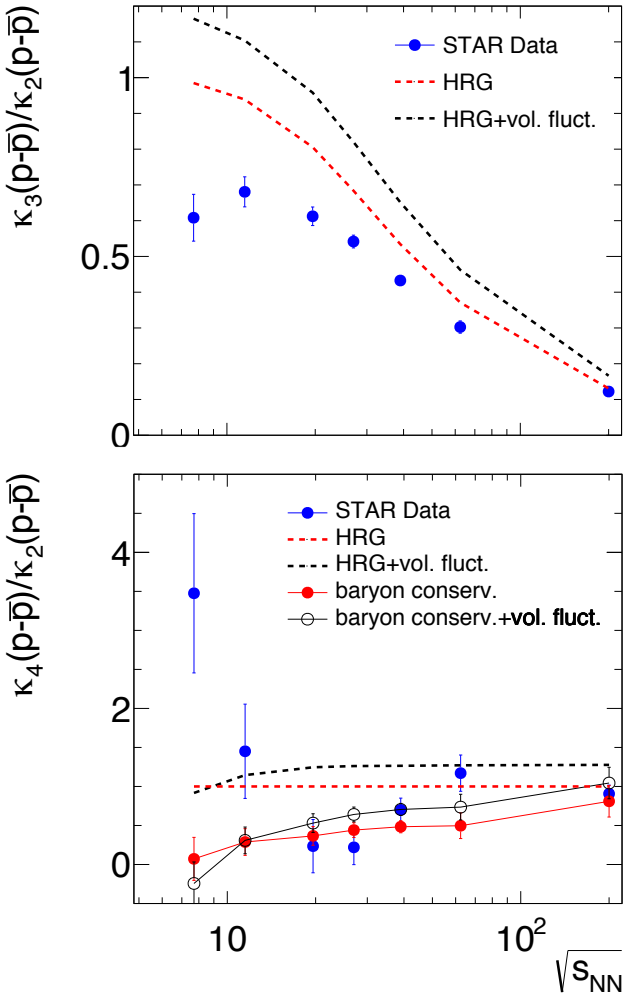
Above 11.5 GeV the STAR data
is consistent with baryon number conservation
+ vol. fluctuations



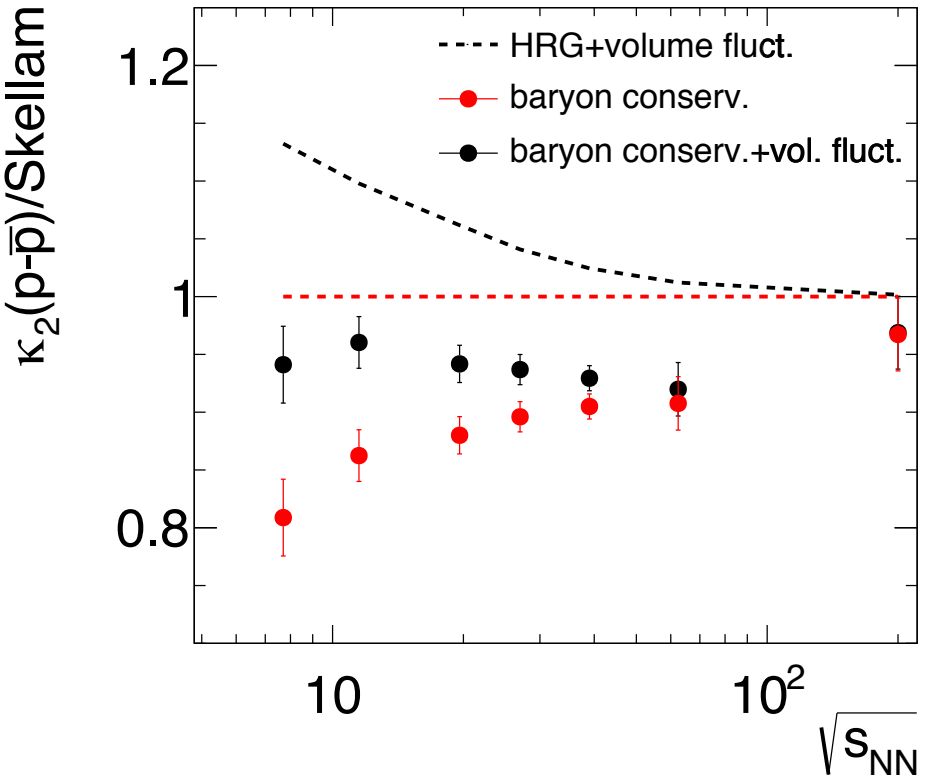
M. Stephanov, PRL 107, 052301 (2011)
F. Karsch, Wuhan 2012

A dip in the excitation function
seems to be generic

Complete picture



our prediction for κ_2/Skellam



P. Braun-Munzinger, A. R., J. Stachel, in preparation

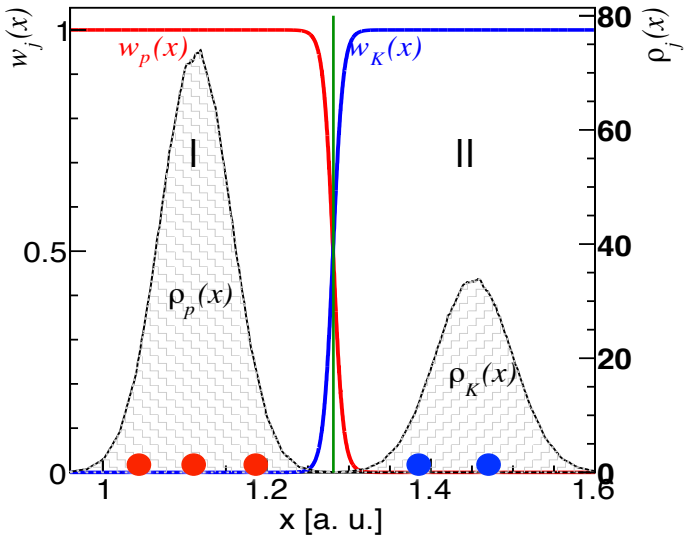
Summary

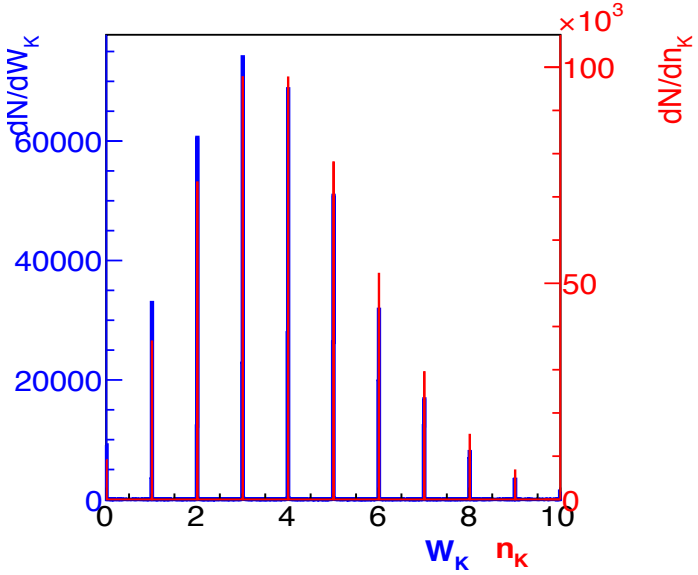
- No anomalies are observed in the energy dependence of fluctuations for p+p, Be+Be and Ar+Sc measured by NA61/SHINE
- The measured second cumulants of net-protons at ALICE are, after accounting for baryon number conservation, in agreement with the corresponding second cumulants of the Skellam distribution.
- Above 11.5 GeV the STAR DATA on κ_4/κ_2 is in agreement with the baryon number conservation + volume fluctuations
- The energy dependence of κ_2/Skellam is predicted within the canonical ensemble calculations + volume fluctuations

The analysis of higher cumulants are ongoing in ALICE, which is extremely important for understanding the nature of transition at vanishing μ_B

BONUS SLIDES

Analysis method





single event example : 3 protons, 2 kaons

traditional approach

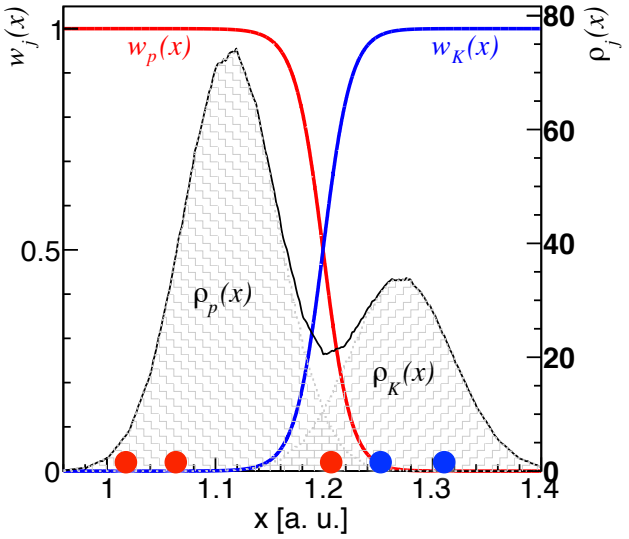
measurement in region I
count as proton
measurement in region II
count as kaon

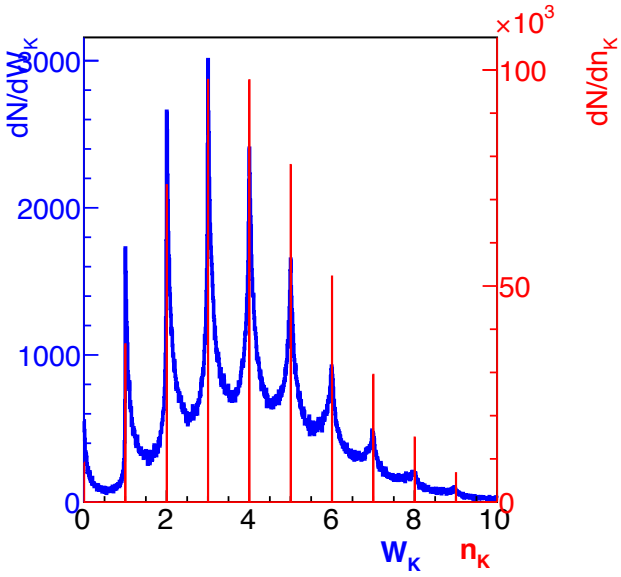
Identity method approach

$$w_p(x_i) = \frac{\rho_p(x_i)}{\rho_p(x_i)+\rho_K(x_i)} \qquad W_p = \sum_{i=1}^{5} w_p(x_i)$$

$$w_K(x_i) = \frac{\rho_K(x_i)}{\rho_p(x_i)+\rho_K(x_i)} \qquad W_K = \sum_{i=1}^{5} w_K(x_i)$$

Analysis method





$$\langle \vec{W} \rangle = A \times \langle \vec{N} \rangle$$

Fully defined by dE/dx fit functions

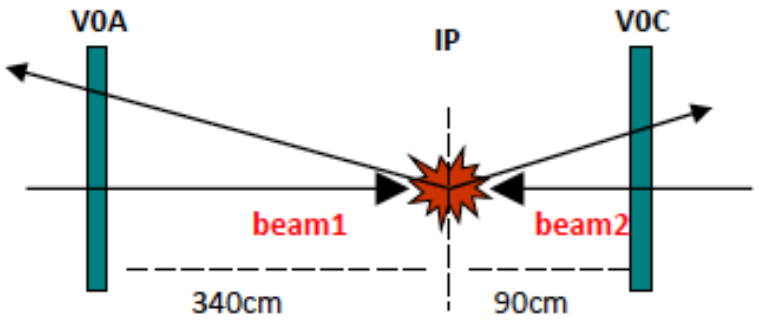
Identity method, basic idea:

$$\langle \vec{N} \rangle = A^{-1} \langle \vec{W} \rangle$$

M. Gazdzicki et al., PRC 83, 054907 (2011)
M. I. Gorenstein, PRC 84, 024902 (2011)
A. Rustamov, M. I. Gorenstein, PRC 86, 044906 (2012)

Used in NA49, NA61/SHINE, ALICE

Centrality determination matters

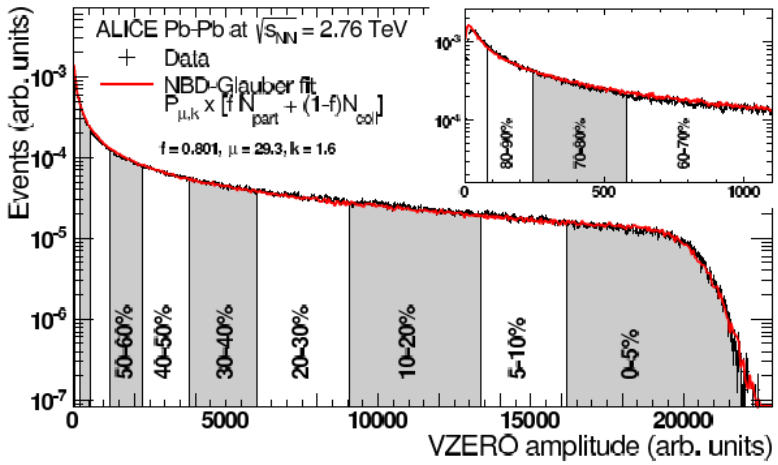


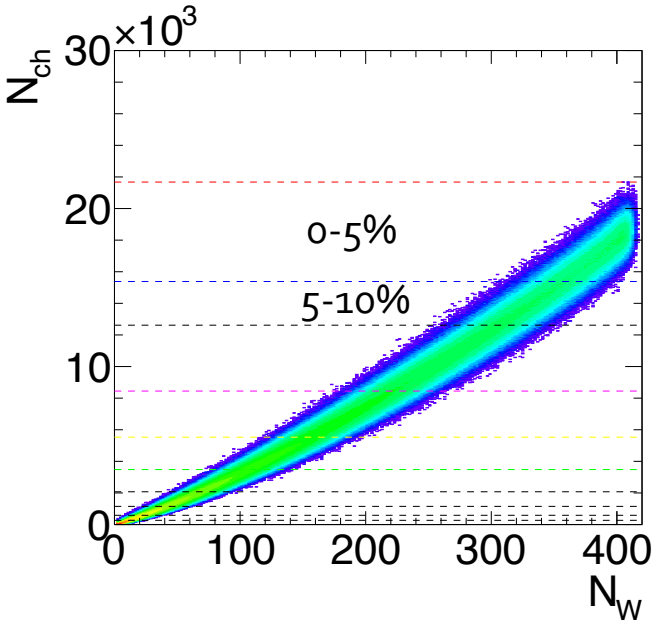
$2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$

ALICE Phys.Rev. C88 (2013) no.4, 044909

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{\left(\mu/k\right)^n}{\left(\mu/k+1\right)^{n+k}}$$

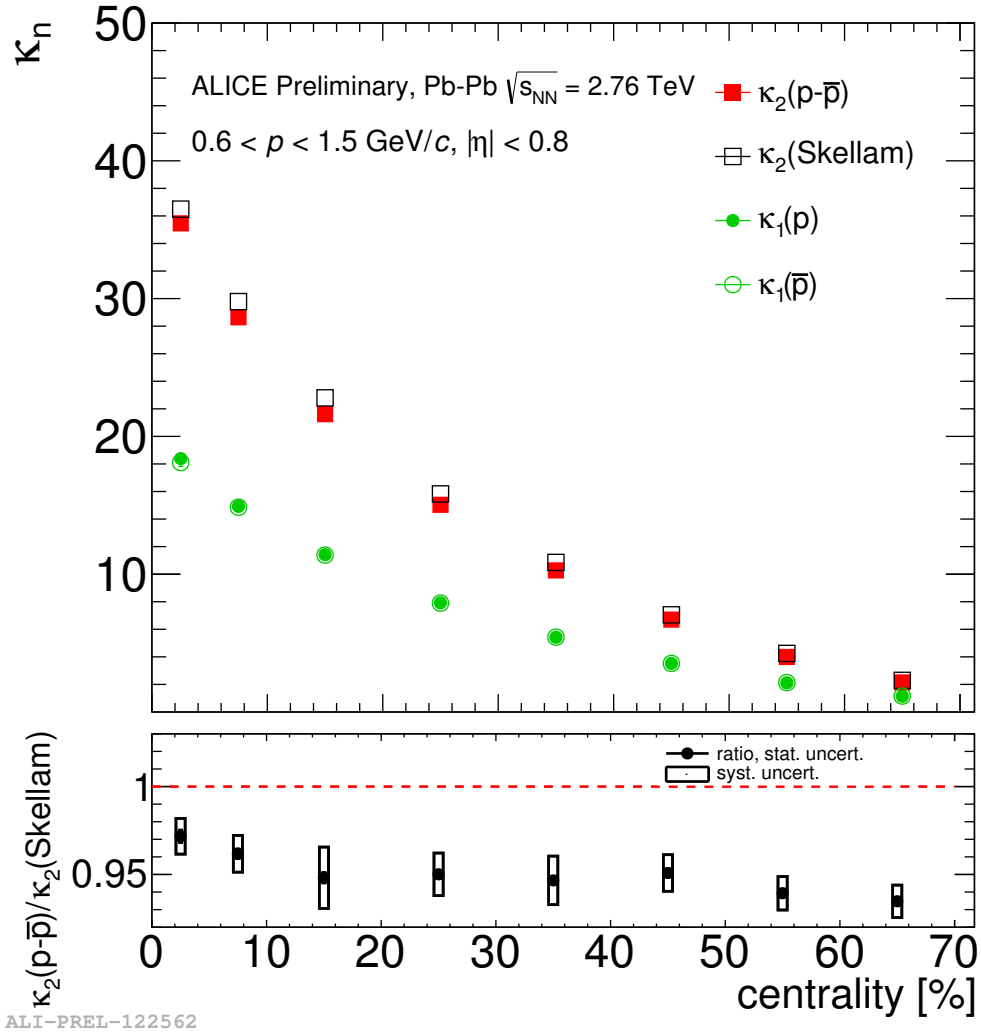
$$N = fN_W + (1-f)N_{coll}$$





P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1612.00702, NPA in print

Net-protons



$$\kappa_2\left(p-\bar{p}\right)=\kappa_2\left(p\right)+\kappa_2\left(\bar{p}\right)-2\left(\left\langle p\bar{p}\right\rangle-\left\langle p\right\rangle\left\langle \bar{p}\right\rangle\right)$$

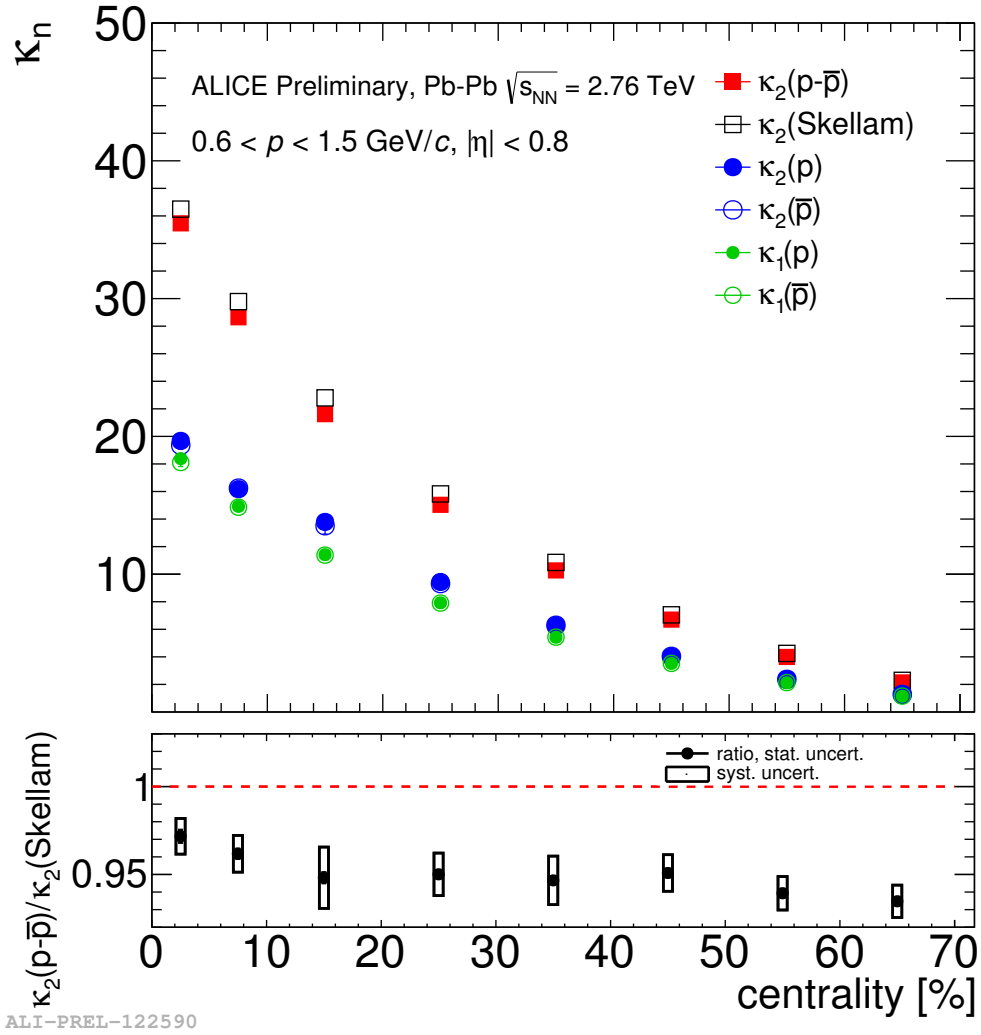
correlation term

$$\kappa_2\left(\text{Skellam}\right)=\kappa_1\left(p\right)+\kappa_1\left(\bar{p}\right)$$

● + ○ = □ ≠ ■

- correlation term?
- non Poisson (anti)protons?

Net-protons, protons, antiprotons



$$\kappa_2\left(p-\bar{p}\right)=\kappa_2\left(p\right)+\kappa_2\left(\bar{p}\right)-2\left(\left\langle p\bar{p}\right\rangle-\left\langle p\right\rangle\left\langle \bar{p}\right\rangle\right)$$

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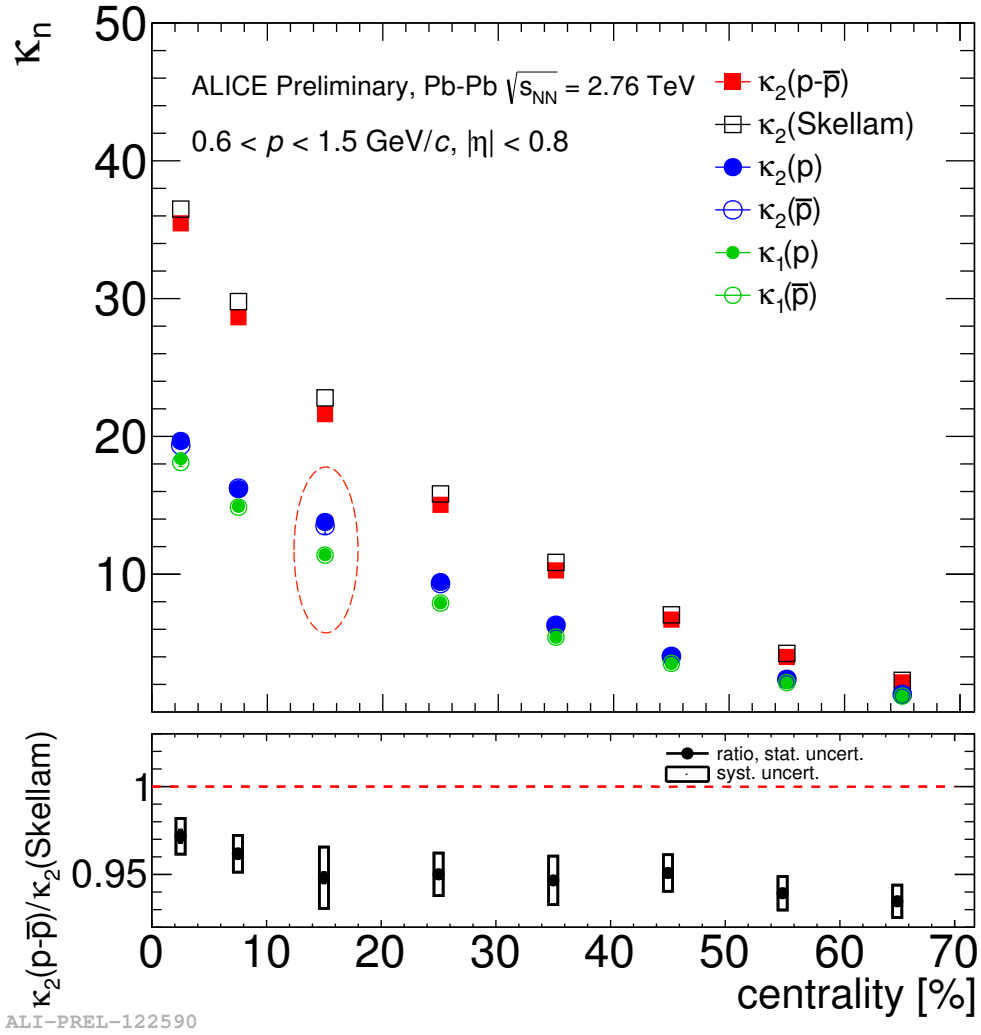
Net-protons, protons, antiprotons



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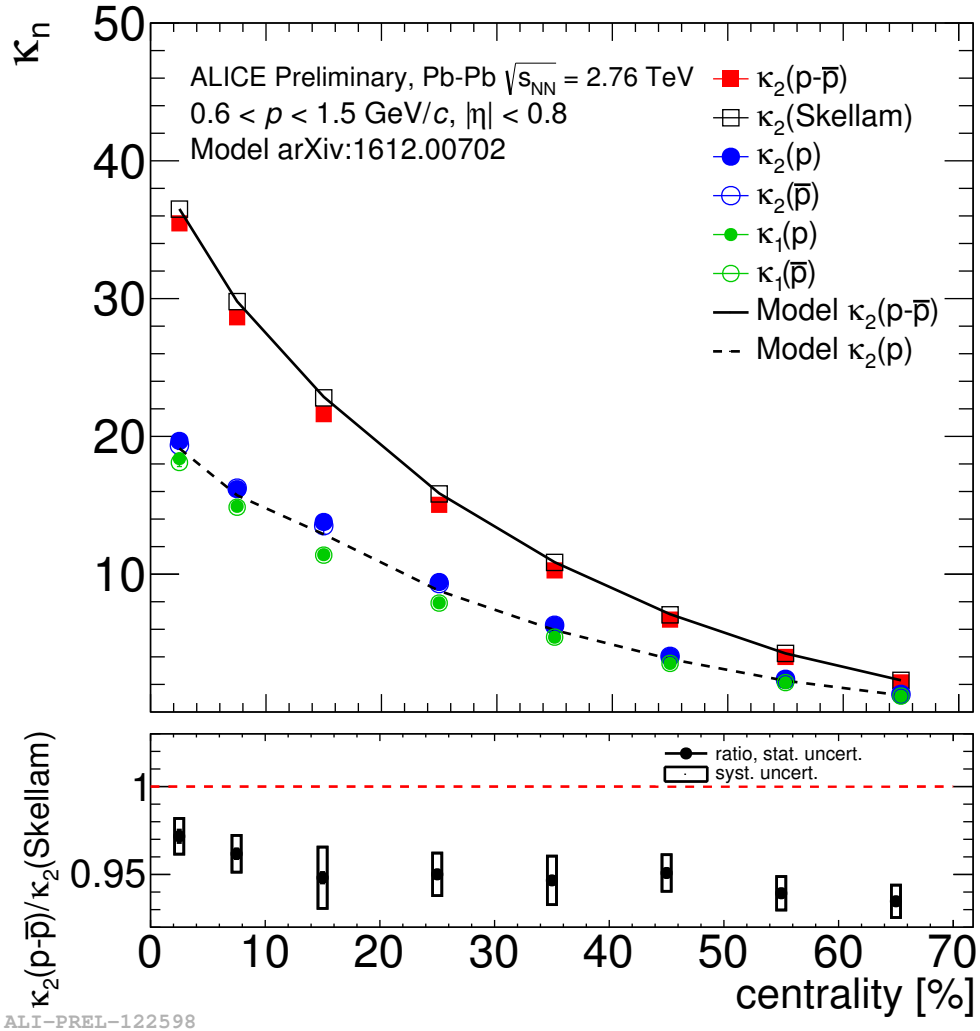
● + ○ = □ ≠ ■

- correlation term?
- non Poisson (anti)protons?

●, ○ ≠ ●, ○

- more evident in the third centrality class
- participant fluctuations ?

Results from ALICE



Analysis of higher cumulants is ongoing!

$$\kappa_2\left(n_B - n_{\bar{B}}\right) = \kappa_2\left(n_B\right) + \kappa_2\left(n_{\bar{B}}\right) - 2\left(\left\langle n_B n_{\bar{B}}\right\rangle - \left\langle n_B\right\rangle\left\langle n_{\bar{B}}\right\rangle\right)$$

Input to the Model

$\kappa_1\left(p\right)$, $\kappa_1\left(\bar{p}\right)$

centrality selection procedure

Predictions

——— $\kappa_2\left(p-\bar{p}\right)$

– – – – – $\kappa_2\left(p\right)$

participants — vanishes at LHC

$$\kappa_2\left(N_B - N_{\bar{B}}\right) = \left\langle N_W\right\rangle \kappa_2\left(n_B - n_{\bar{B}}\right) + \left\langle n_B - n_{\bar{B}}\right\rangle^2 \kappa_2\left(N_W\right)$$

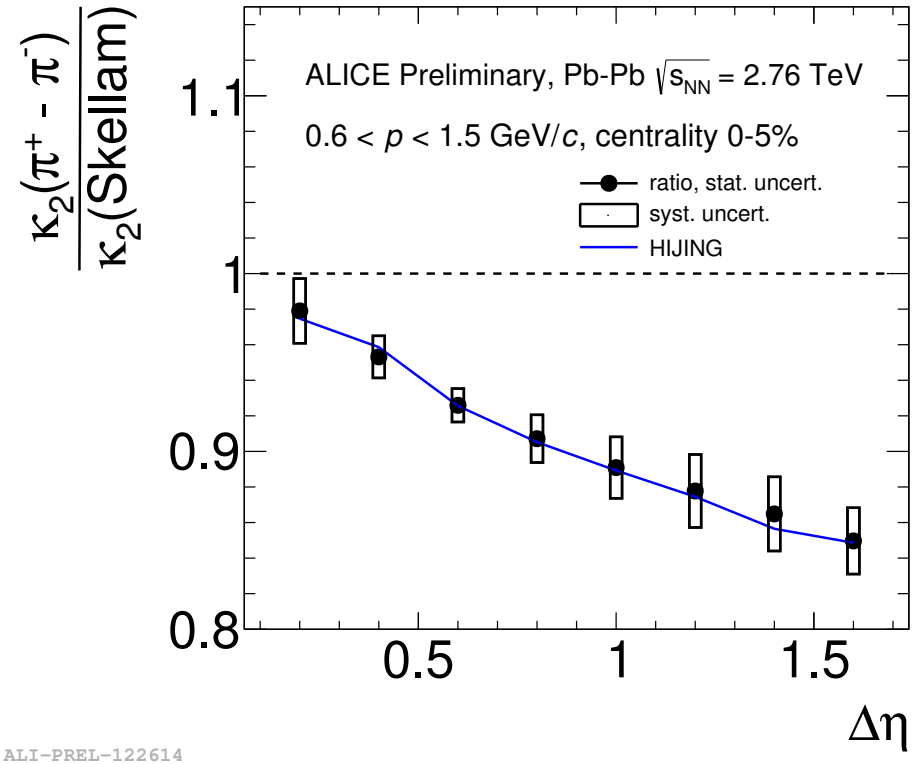
from single participant

Second cumulants of net-particles at LHC are not affected by participant fluctuations
easy control of systematics

A. R., QM2017, arXiv:1704.05329

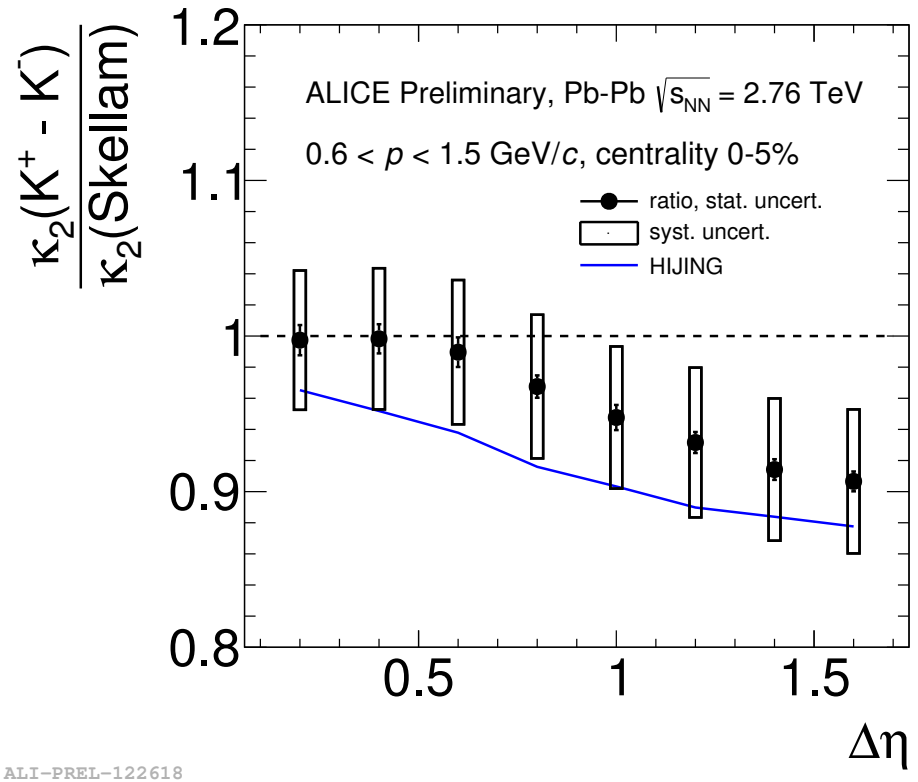
Net-pions and Net-kaons

net-pions



perfect agreement with HIJING

net-kaons



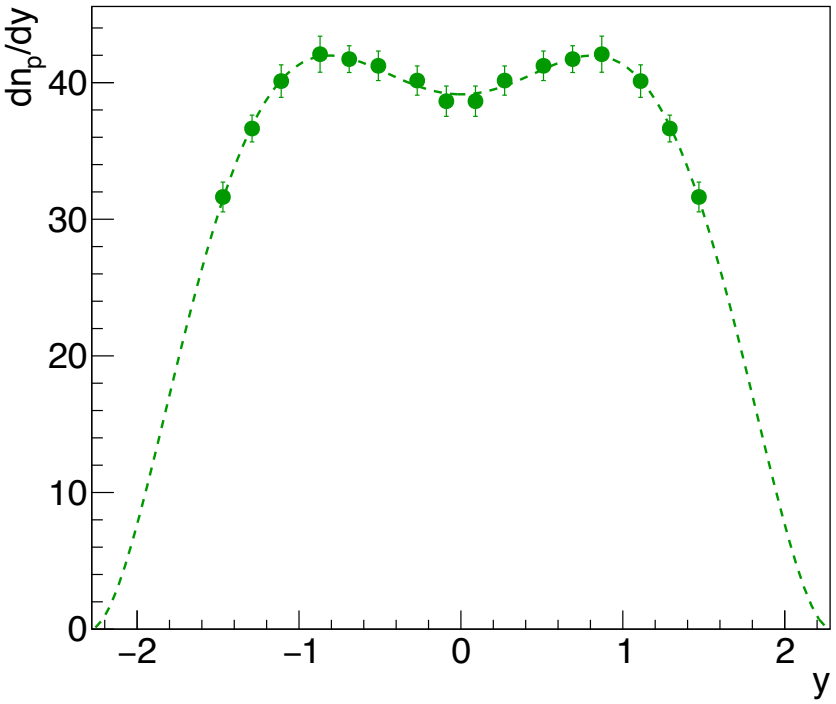
reasonable agreement with HIJING

resonance pion and kaon production is likely to explain the measured trend

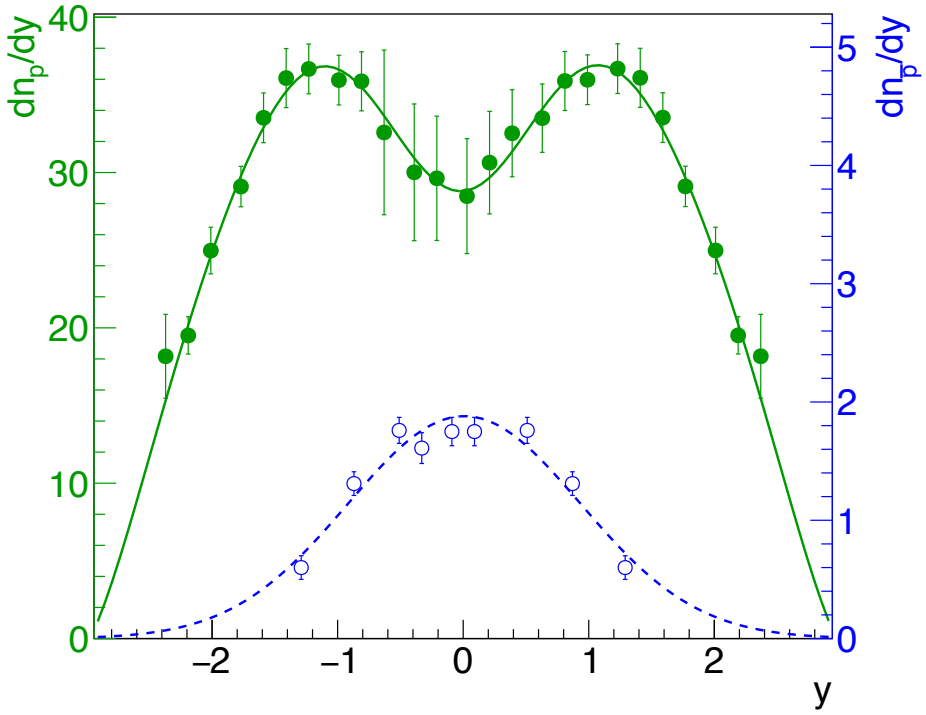
Warning: Skellam is not a proper baseline for net-pions and net-kaons

Folding with acceptance

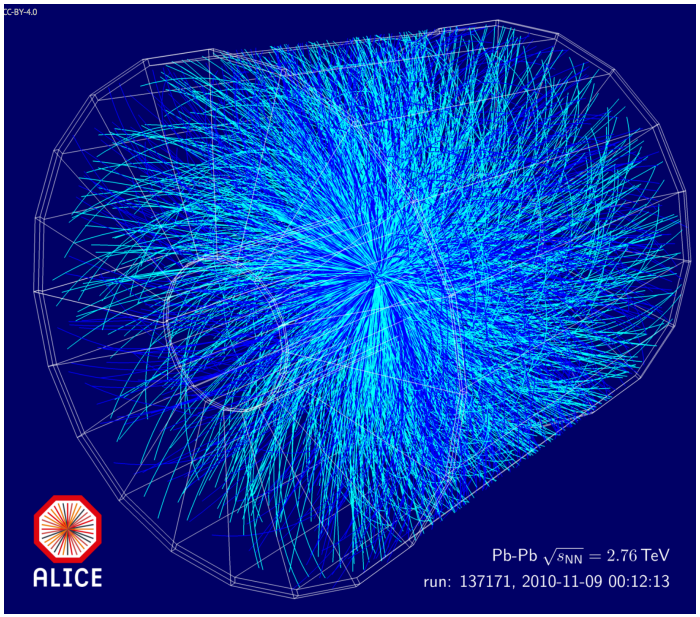
40 GeV lab momentum, $s^{1/2} \approx 8.8$ GeV

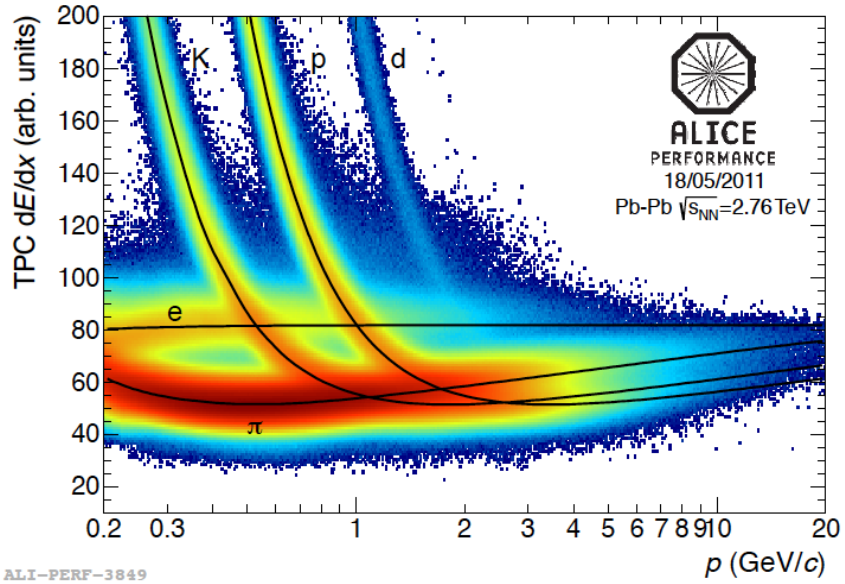


158 GeV lab momentum, $s^{1/2} \approx 17.3$ GeV



ALICE Pb-Pb data at $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$





Acceptance selection:

$$0.6 < p < 1.5\,\mathrm{GeV}/c, \quad |\eta| < 0.8$$

Centrality selection:

charged particle multiplicities in

$$2.8 < \eta < 5.1 \quad \text{and} \quad -3.7 < \eta < -1.7$$

Analysis technique: Identity Method