

# Polyakov loop fluctuations in the presence of external field

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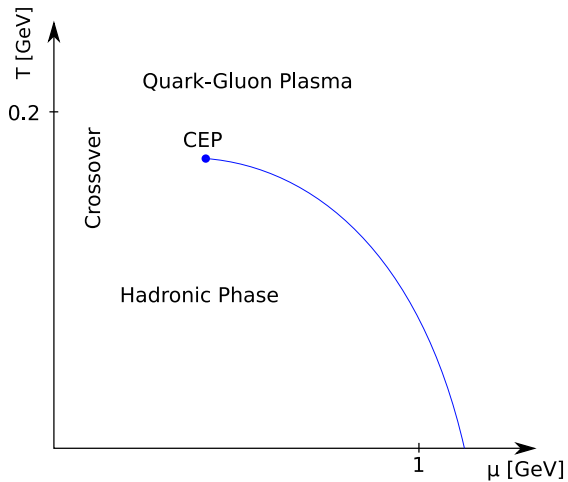
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Pure gauge: deconfinement  $\rightarrow$  1st order

- ▶ Spontaneous  $Z_3$  symmetry breaking
- ▶ Polyakov loop  $\rightarrow$  order parameter

QCD: deconfinement  $\rightarrow$  crossover

- ▶  $Z_3$  symmetry  $\rightarrow$  explicitly broken
- ▶ Polyakov loop  $\rightarrow$  approximate order parameter

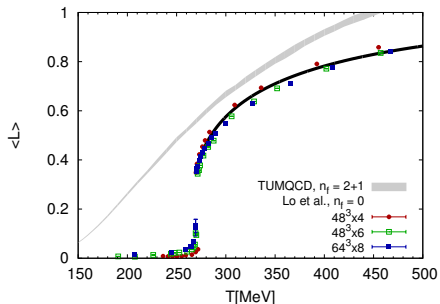


Figure: Polyakov loop in pure gauge<sup>1</sup> (colored points+black line) and in 2+1 QCD<sup>2</sup> (gray band)

<sup>1</sup>P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, *Phys. Rev. D* **88**, 014506 (2013)

<sup>2</sup>A. Bazavov, N. Brambilla, H.-T Ding, P. Petreczky, H. -P. Schadler, A. Vairo and J. H. Weber, *Phys. Rev. D* **93**, 114502 (2016).

Polyakov loop susceptibility  $\rightarrow$  probes fluctuations of order parameter

$$\chi_A = V \langle |L|^2 \rangle_c = V (\langle |L||L| \rangle - \langle |L| \rangle^2)$$

Additional susceptibilities

- ▶ Real (longitudinal)

$$\chi_L = V \langle x^2 \rangle_c = V (\langle x^2 \rangle - \langle x \rangle^2)$$

- ▶ Imaginary (transverse)

$$\chi_T = V \langle y^2 \rangle_c = V (\langle y^2 \rangle - \langle y \rangle^2)$$

Polyakov loop in LQCD  $\rightarrow$  renormalization scheme dependent

Ratios of susceptibilities  $\rightarrow$  alternative probes of deconfinement<sup>1</sup>

$$R_A = \frac{\chi_A}{\chi_L} \qquad R_T = \frac{\chi_T}{\chi_L}$$

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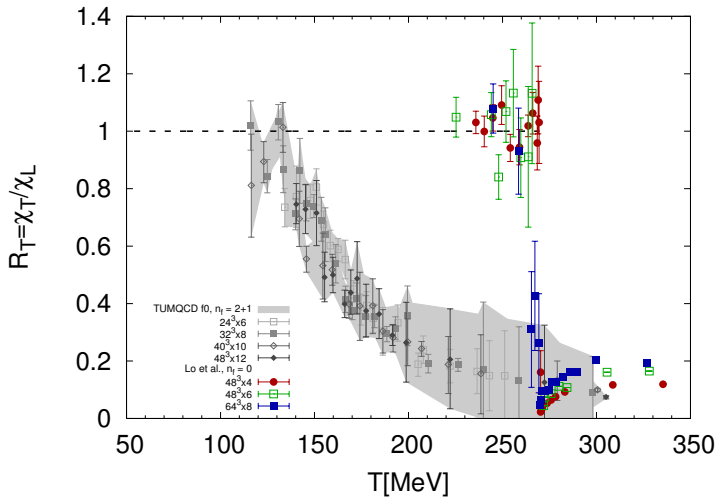


Figure: Lattice data on the  $R_T$  ratio in pure gauge<sup>1</sup> (colored points) and 2+1 QCD<sup>2</sup> (gray band).

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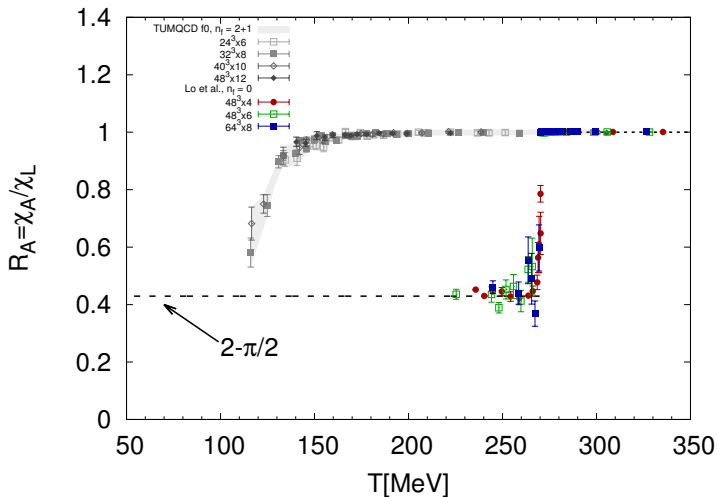


Figure: Lattice data on the  $R_A$  ratio in pure gauge<sup>1</sup> (colored points) and 2+1 QCD<sup>2</sup> (gray band).

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Color group integration

$$Z = \int dx dy e^{-\beta U}, \quad \langle O \rangle = \frac{1}{Z} \int dx dy O(x, y) e^{-\beta U}$$

Gaussian Approximation

$$U = \alpha T^4 |L||L| = \alpha T^4 (x^2 + y^2)$$

► Susceptibilities

$$\langle |L|^2 \rangle_c = \left(2 - \frac{\pi}{2}\right) \frac{1}{2\alpha \sqrt{T^3}}, \quad \langle x^2 \rangle_c = \langle y^2 \rangle_c = \frac{1}{2\alpha \sqrt{T^3}}$$

► Ratios

$$R_T = \frac{\chi_T}{\chi_L} = 1, \quad R_A = \frac{\chi_A}{\chi_L} = 2 - \frac{\pi}{2} \approx 0.43$$

Gaussian model  $\rightarrow$  No spontaneous symmetry breaking

Generalization  $\rightarrow$  to capture details of phase transition

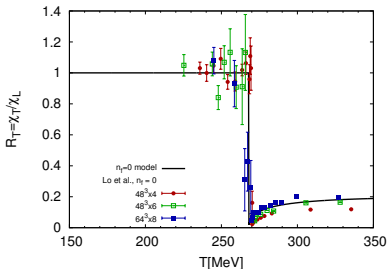
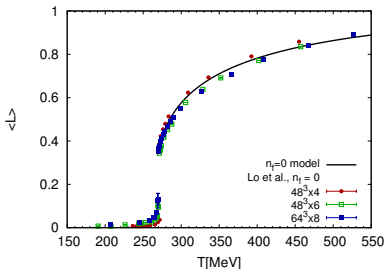
$$\mathcal{U} \rightarrow \mathcal{U} = \mathcal{U}_{Gluon} + \mathcal{U}_{Int}$$

- ▶  $\mathcal{U}_{Gluon}$   $\rightarrow$  Pure gauge potential
- ▶  $\mathcal{U}_{Int}$   $\rightarrow$  Quark-gluon interaction

$\mathcal{U}_{Gluon}$ : Pure gauge potential

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)L\bar{L} + b(T)\ln M_H(L, \bar{L}) + \frac{1}{2}c(T)(L^3 + \bar{L}^3) + d(T)(L\bar{L})^2$$

► Reproduces pure gauge  $P$ ,  $L$ ,  $\chi_L$  and  $\chi_T$

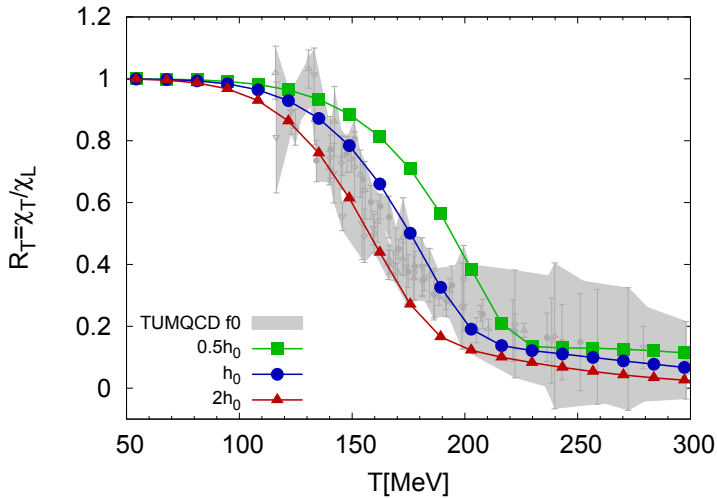


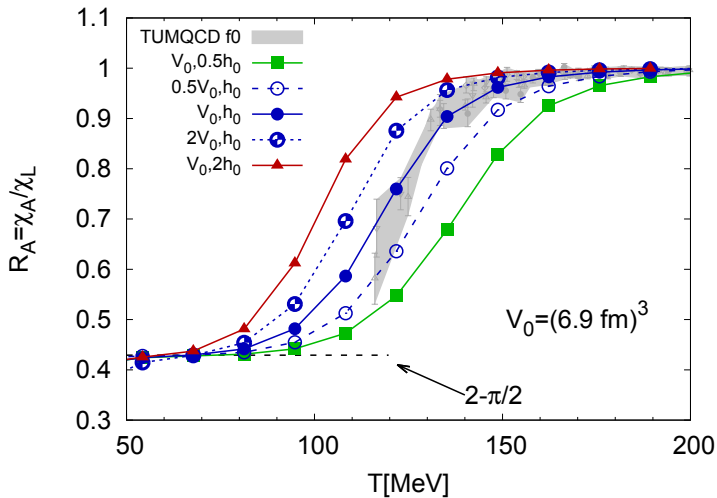
$\mathcal{U}_{Int}$ : Quark-gluon interaction

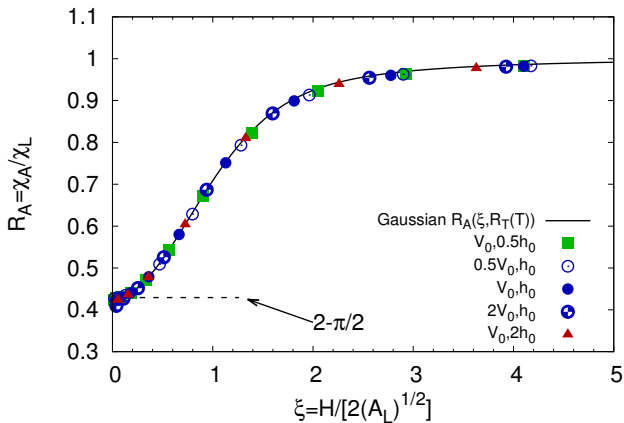
$$\frac{\mathcal{U}_{Int}}{T^4} = - \underbrace{[2h_q(T, m_l) + h_q(T, m_s)]}_h \times,$$

$$h_q(T, m) = \frac{6}{\pi^2 T^3} \int_0^\infty dk k^2 \frac{e^{-E/T} + e^{-2E/T}}{1 + e^{-3E/T}}, \quad E = \sqrt{k^2 + m^2}$$

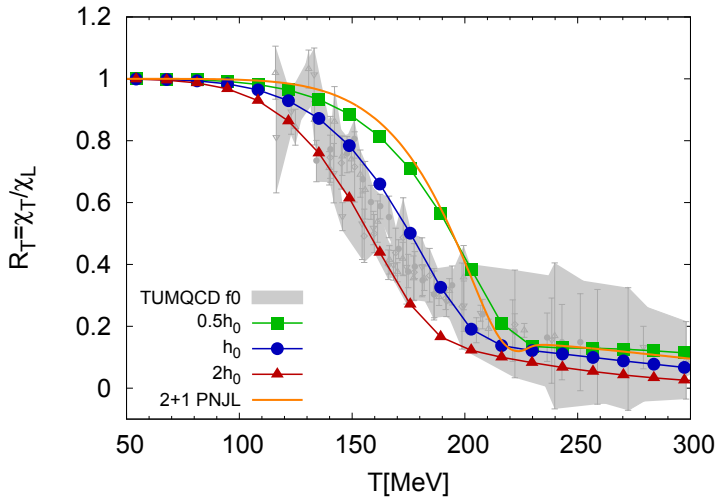
- ▶ External field coupled to Polyakov loop
- ▶  $m_l = 5 \text{ MeV}$ ,  $m_s = 20m_l = 100 \text{ MeV}$







$$\mathcal{A}_L = \frac{VT^3}{2\chi_L(T, h=0)}, \quad \mathcal{H} = VT^3 \left[ h + \frac{\langle L \rangle(T, h=0)}{\chi_L(T, h=0)} \right]^{-1}$$



## Conclusions

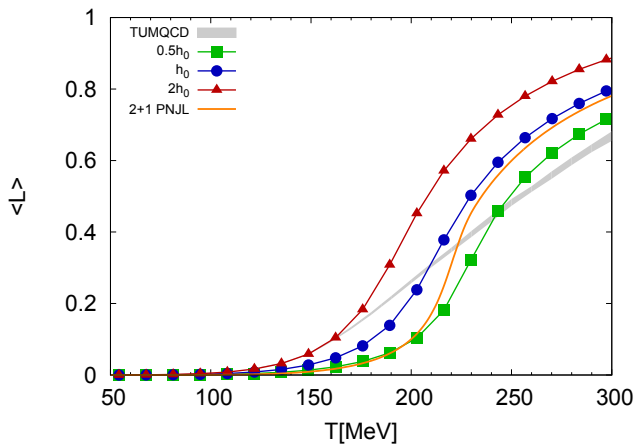
- ▶ Ratios of Polyakov loop susceptibilities are sensitive to deconfinement and explicit symmetry breaking strength
  - ▶ Useful for constraining effective models<sup>1</sup>
- ▶ The model discussed here captures lattice trends for  $R_A$  and  $R_T$  ratios
  - ▶ Model ratios connected by  $h$
  - ▶ We reveal scaling properties of  $R_A$  ratio
- ▶  $R_T$  ratio may be used to estimate the strength of explicit symmetry breaking

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<sup>1</sup>Unfortunately, current LQCD results suffer from renormalization scheme dependence



# Model Polyakov loop



# Effective gluon potential $\mathcal{U}_G$

Parametrization of the effective potential:

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)L\bar{L} + b(T)\ln M_H(L, \bar{L}) + \frac{1}{2}c(T)(L^3 + \bar{L}^3) + d(T)(L\bar{L})^2$$

▶  $a(T)$ ,  $c(T)$  and  $d(T)$  of the form:

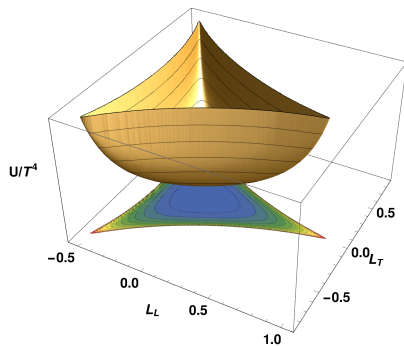
$$x(t \equiv T/T_c) = \frac{x_1 + x_2/t + x_3/t^2}{1 + x_4/t + x_5/t^2}$$

▶  $b(T)$  of the form:

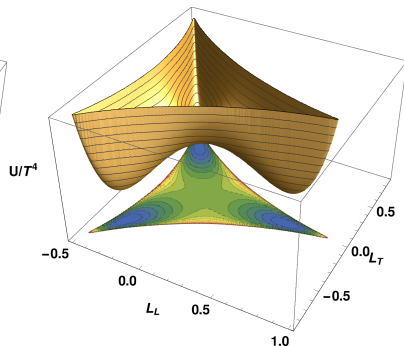
$$b(t \equiv T/T_c) = x_1 t^{-x_4} (1 - e^{x_2/t^{x_3}})$$

Parameters  $x_n$  can be found in: *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, 074502 (2013)

# Effective gluon potential $\mathcal{U}_G$



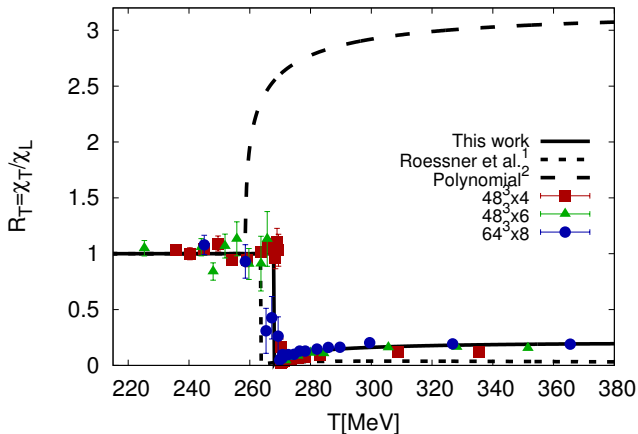
$T < 270 \text{ MeV}$



$T > 270 \text{ MeV}$

Figure: Shape of effective potential used in model calculations in confined (left panel) and deconfined (right panel) phases.

# $R_T$ for different potentials



<sup>1</sup> S. Roessner, C. Ratti and W. Weise, *Phys. Rev. D* **75**, 034007 (2007)

<sup>2</sup> Potential: C. Ratti, M. A. Thaler and W. Weise, *Phys. Rev. D* **73**, 014019 (2006).

parametrization: A. V. Friesen, Y. L. Kalinovsky and V. D. Toneev, *Int. J. Mod. Phys. A* **27**, 1250013 (2012)

# Matching to Gaussian model

The general form of Gaussian distribution function:

$$Z = \int dL_L dL_T \exp \left[ -\mathcal{A}_1 L_L^2 - \mathcal{A}_2 L_T^2 + \mathcal{H} L_L \right]$$

- ▶ In this case  $R_A(\mathcal{A}_1, \mathcal{A}_2, \mathcal{H}) = R_A(\xi, R_T)$ , with

$$\xi = \mathcal{H} / (2\sqrt{\mathcal{A}_1}), \quad R_T = \mathcal{A}_1 / \mathcal{A}_2.$$

- ▶ The  $R_A$  ratio  $\rightarrow$  exact:

$$R_A(\xi, R_T) = 1 + R_T + 2\xi^2 - \frac{2}{\pi^2} R_T e^{-2\xi^2} [\mathcal{I}(\xi, R_T)]^2,$$
$$\mathcal{I}(\xi, R_T) = \int_{-\infty}^{\infty} dx e^{-x^2 + 2\xi x} \frac{x^2}{2R_T} \left[ K_0\left(\frac{x^2}{2R_T}\right) + K_1\left(\frac{x^2}{2R_T}\right) \right]$$

The effective scaling variable  $\xi \rightarrow$  Gaussian matching:

$$\mathcal{A}_{1,2} = VT^3 [2\chi_{L,T}(T, h=0)]^{-1}, \quad \mathcal{H} = VT^3 \left[ h + \frac{\langle L \rangle(T, h=0)}{\chi_L(T, h=0)} \right]^{-1}$$