

# Parity Doubling of Baryons in QCD Thermodynamics

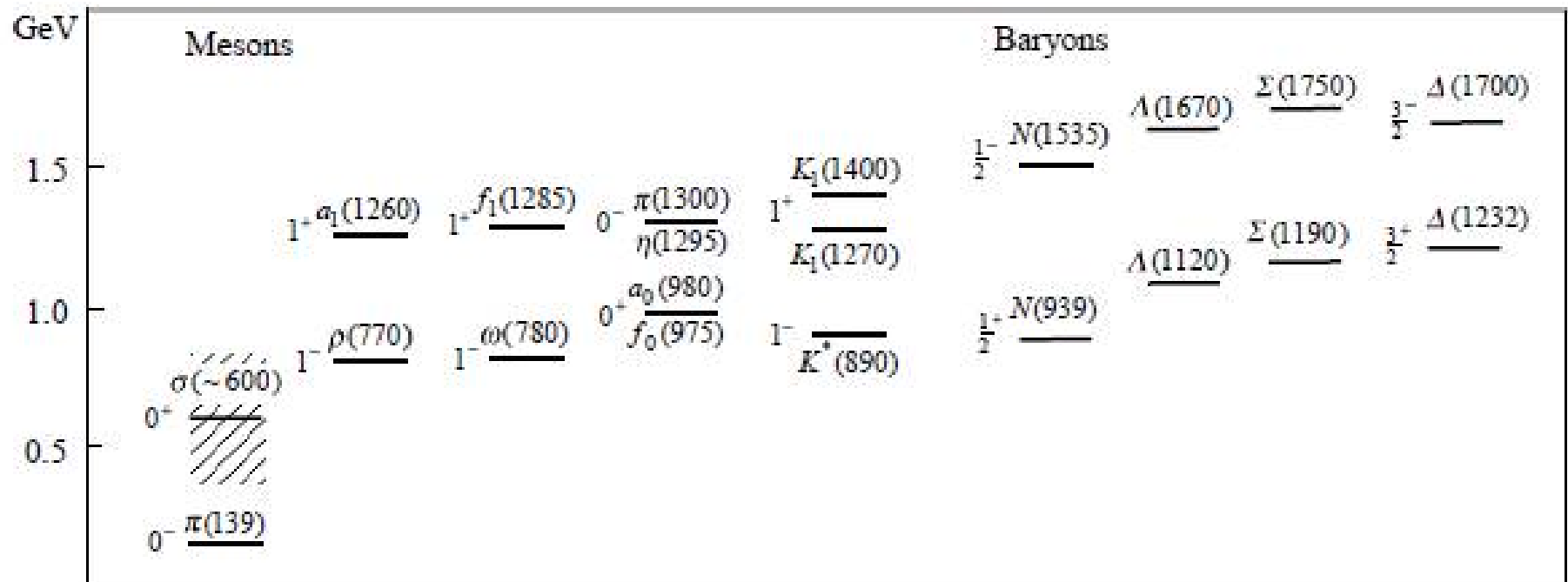
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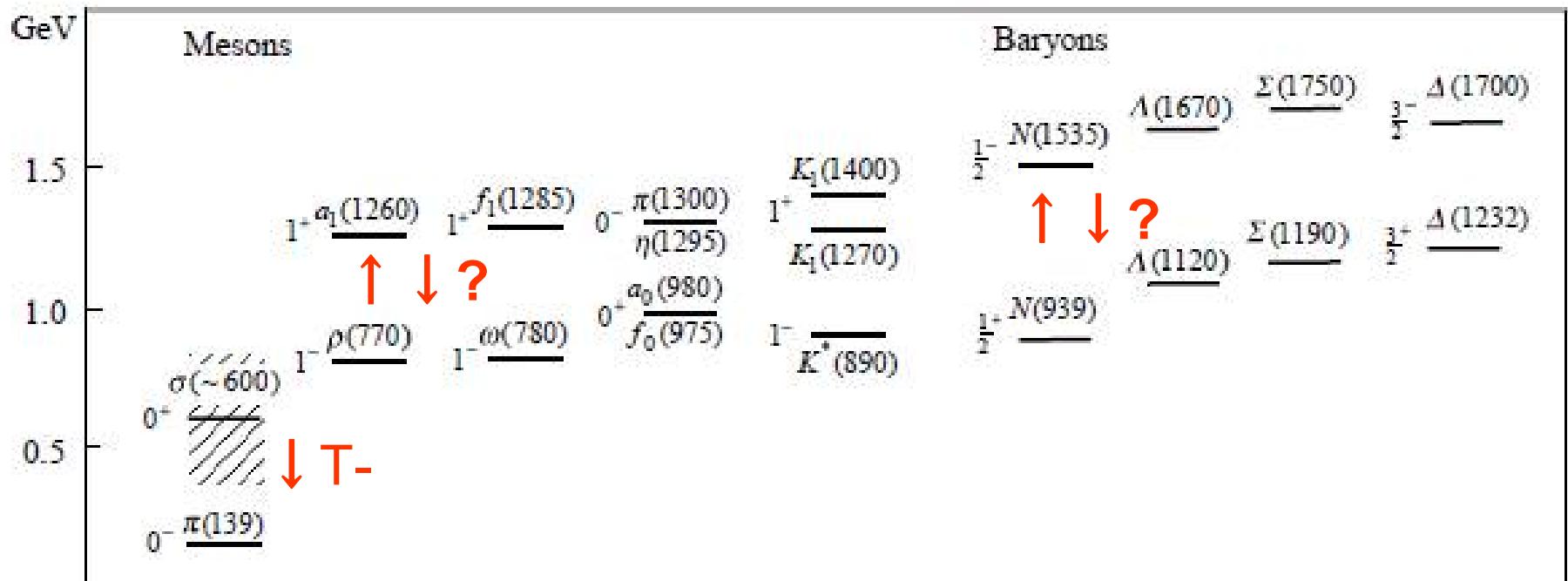
# Spectra in a chirally broken world

- Lowest pseudo-scalar mesons as NG bosons
- Mass splitting between positive and negative parity hadrons



# Spectra in a chirally restored world

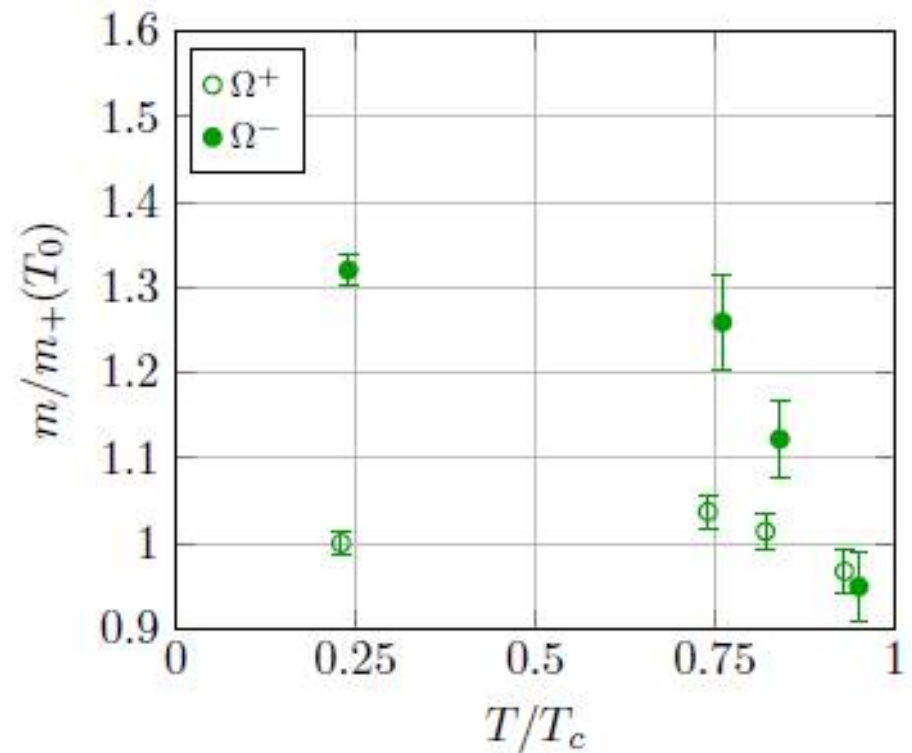
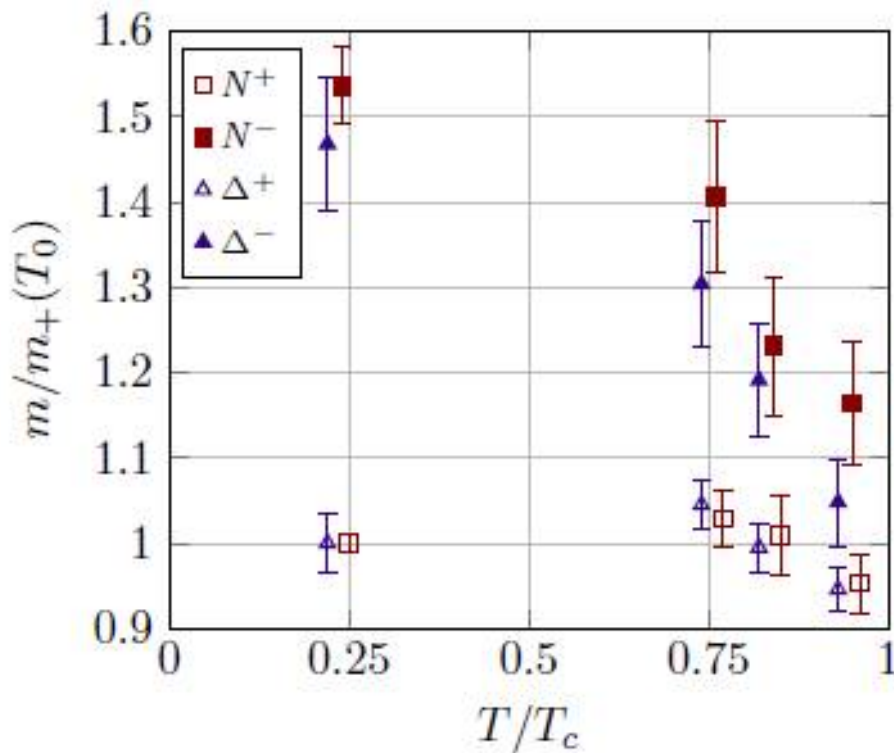
- ❑ Lowest scalar meson  $\rightarrow$  O(4) vector with pion
- ❑ Parity partners degenerate  $\rightarrow$  chiral partners
- [mq  $\approx$  0: helicity eigenstates  $\approx$  parity eigenstates]



# Lattice QCD tells us ...

## □ Temporal correlations in baryonic channels

[FASTSUM Coll., 2015-17:  $m_{\pi} \approx 400$  MeV,  $m_K \approx 500$  MeV, Wilson fermions,  $T_{ch} = 185$  MeV]



vs.  $M_n \approx 3 \times M_q$

# Nucleon mass vs. CSB

- Gell-Mann—Levy model/conventional LSM

$$\mathcal{L}_{\text{GL}} = i\bar{N}\not{\partial}N - g\bar{N}(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})N + \mathcal{L}_{\text{meson}}$$

$$\psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R \quad m_N = g\langle\sigma\rangle$$

- When CS restored  $\rightarrow$  massless nucleon

- How to introduce a mass term?

# Non-SCB mass of nucleons

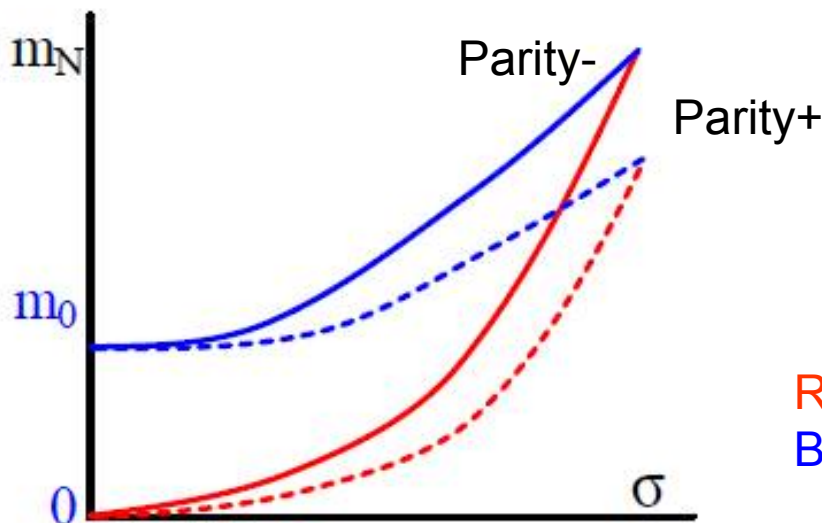
□ SU(2) chiral transformation of 2 nucleons

→ how to assign 2 indep. rotation to them?

$$\psi_{1L} \rightarrow g_l \psi_{1L}, \quad \psi_{1R} \rightarrow g_r \psi_{1R} \sim \psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2)$$

$$\psi_{2L} \rightarrow g_r \psi_{2L}, \quad \psi_{2R} \rightarrow g_l \psi_{2R} \sim \psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0)$$

$$\mathcal{L}_m = m_0 (\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2) \Rightarrow m_{N\pm} = \frac{1}{2} \left[ \sqrt{c_1 \sigma^2 + 4m_0^2} \mp c_2 \sigma \right]$$



[DeTar-Kunihiro, 1989]

# Origin of the survival mass?

❑ Emergence of a scale in QCD → trace anomaly

$$\partial_\mu J^\mu = T^\mu_\mu \propto \langle H | G^2 | H \rangle$$

❑ in hot matter [Miller, 2007: lattice QCD EoS]

$$\langle G^2 \rangle_{T_{\text{chiral}}} \simeq \frac{1}{2} \langle G^2 \rangle_{T=0}$$

❑ in nuclear matter [Cohen et al., 1995: Feynman-Hellmann theorem & low-density approx.]

$$\begin{aligned} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} - \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\text{vac}} \\ = -\frac{8}{9} (M_N - \sigma_N - S) \rho_N + \dots \quad 5\% \text{ smaller} \end{aligned}$$

# How large is $m_0$ ?

- ❑ Vacuum:  $m_0 = 270 - 460 \text{ MeV}$  [DeTar-Kunihiro, Nemoto et al. , Gallas et al.]
- ❑ Finite  $m_0$ : dominated by color-magnetic gluon
  - VEV of dilaton  $\rightarrow m_0$  [CS et al.]
- ❑ Nuclear matter
  - Ground state: binding energy, normal NM density
  - Preferred  $m_0 \approx 500\text{-}800 \text{ MeV}$  (w/ and w/o 4Q)  
[Zschesche et al. 2007, Gallas et al. 2011]
- ❑ LQCD/FASTSUM: near  $T_{ch}$ , zero chem.pot.  
$$m_0(\text{octet,decuplet}) \leq m_+(T=0)$$

**$m_0 \approx \text{a few } \Lambda_{\text{qcd}}$**

**Mass difference: weak  $m_0$ -dep.**

# Parity doubling of baryons

- Baryon octet and decuplet with finite  $m_0$
- Consistent with established phenomenology:
  - ✓ Gell-Mann-Okubo mass formula

$$\frac{3}{4}m_{\Lambda} + \frac{1}{4}m_{\Sigma} - \frac{1}{2}(m_N + m_{\Xi}) = 0$$

- ✓ Gell-Mann's equal spacing rule

$$m_{\Sigma^*} - m_{\Delta} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Omega} - m_{\Xi^*}$$

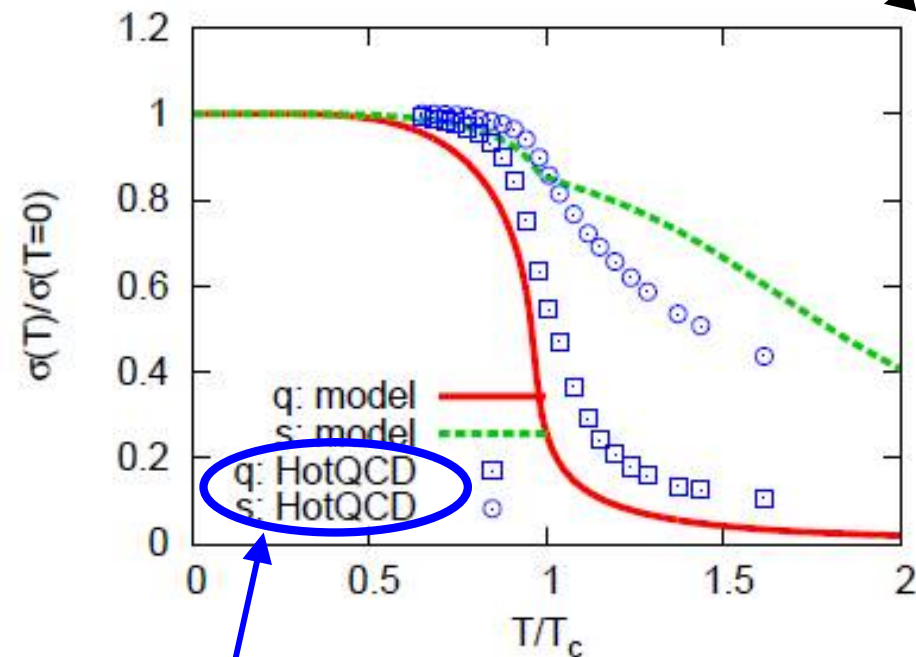
- Mass relations [CS, NPA ('18)]

$$M_B(\sigma_q, \sigma_s; a, b, m_0)$$

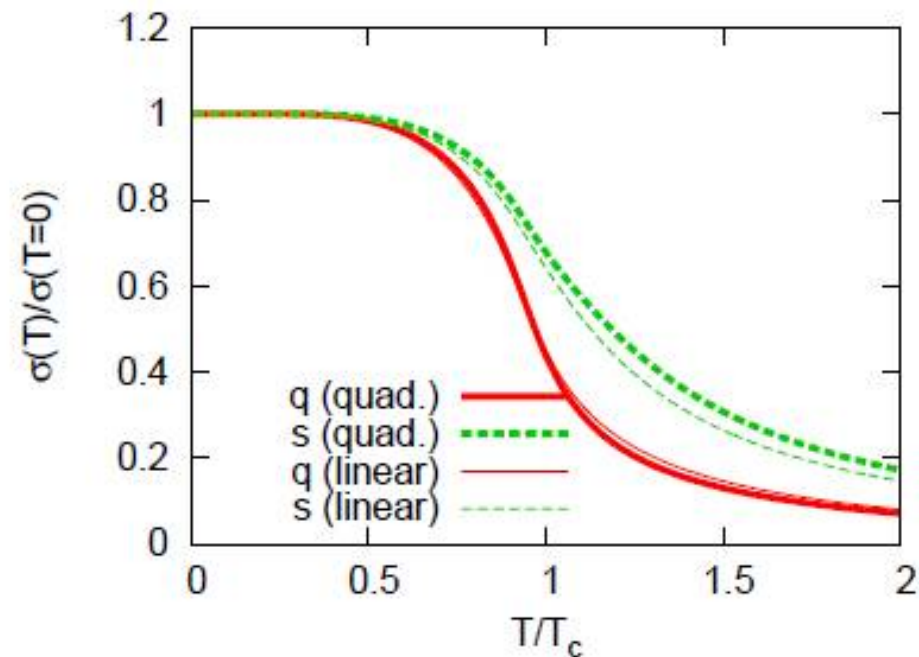
Light-quark condensate    Strange-quark condensate

# Chiral condensates

- ❑ Quark condensates from a model vs. LQCD
- ❑ Pion mass dependence:  $m_{\pi} = 140, 400 \text{ MeV}$

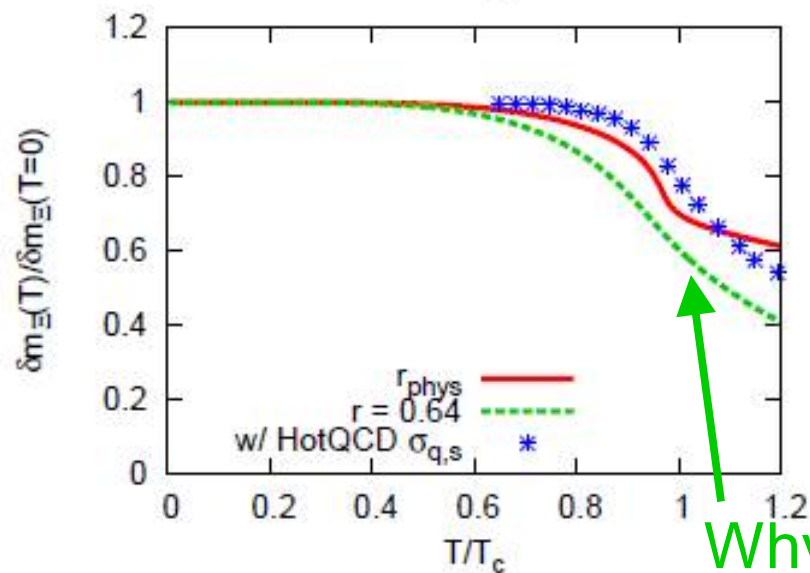
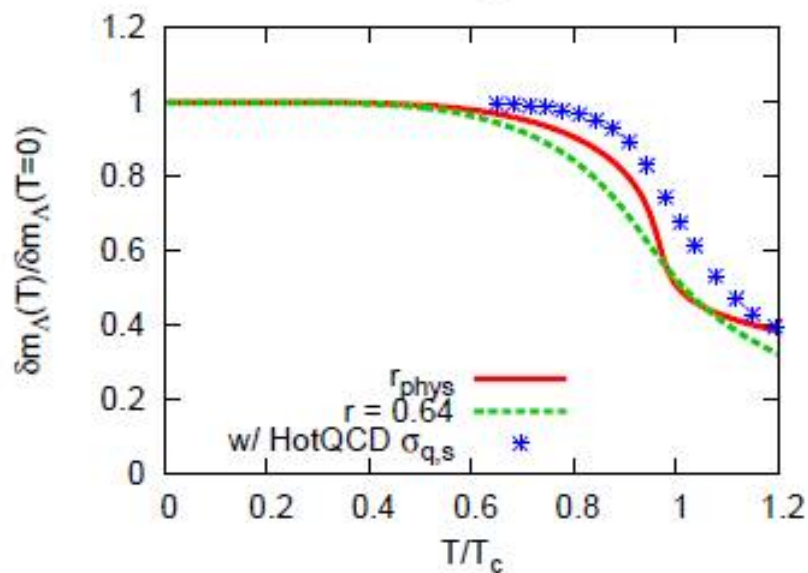
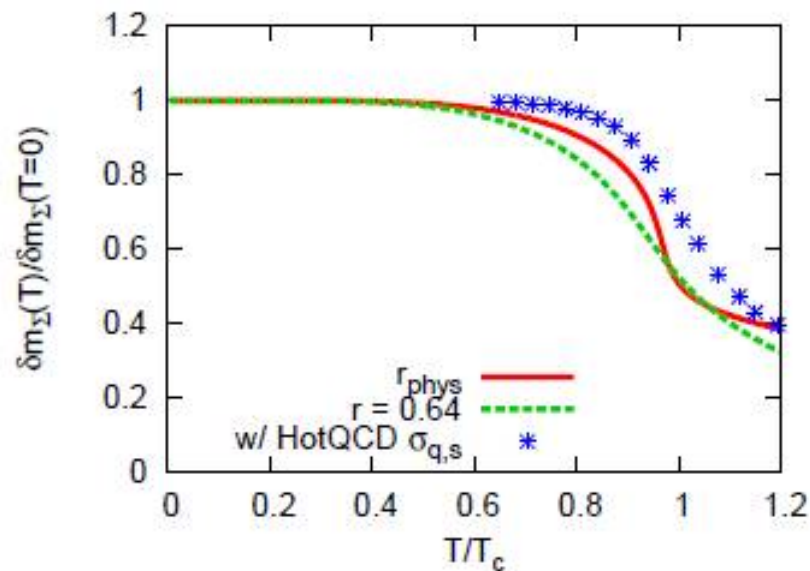
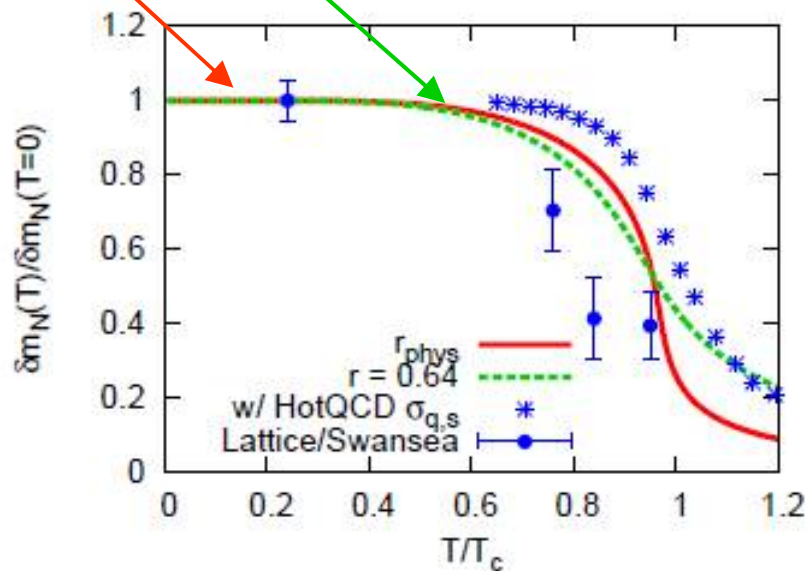


Lattice QCD w/ physical  $m_{\pi}$



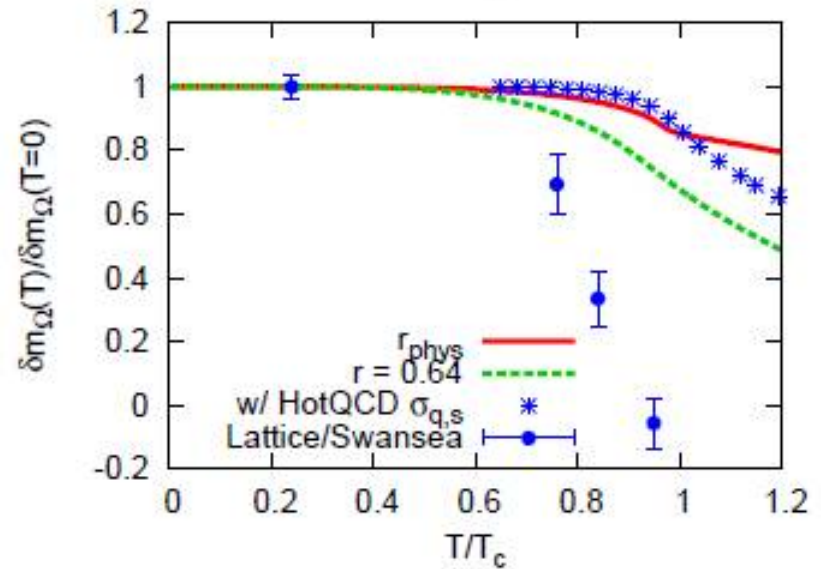
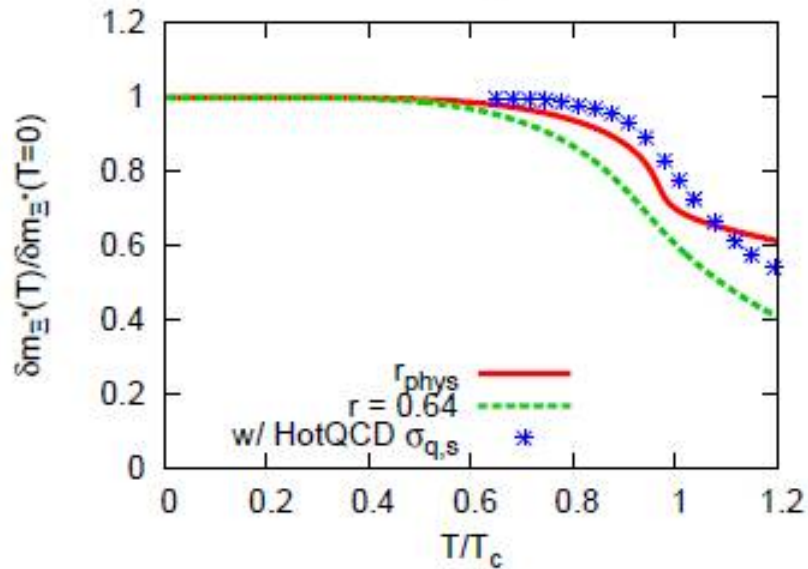
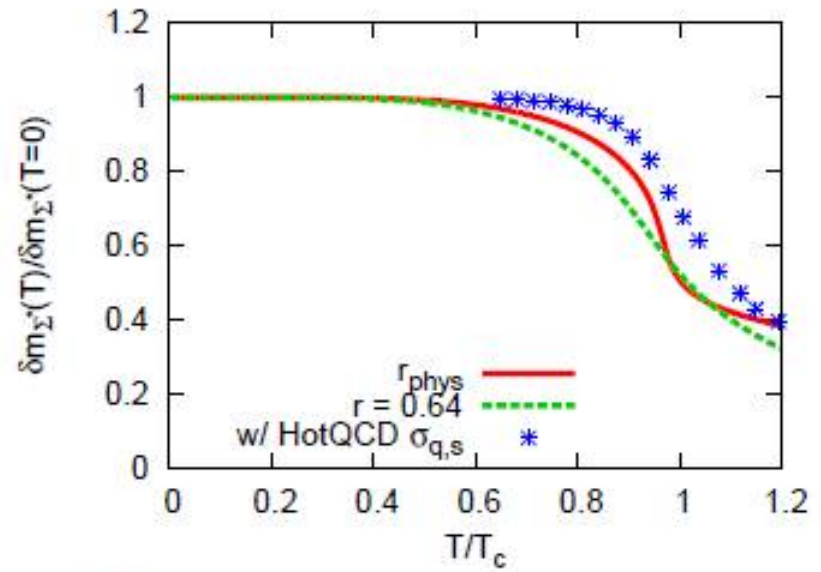
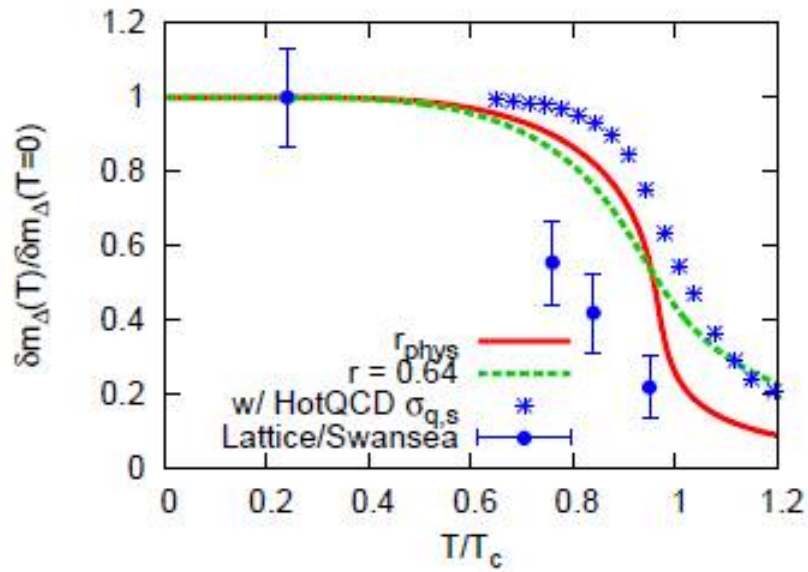
$M_{\pi} = 140 \text{ MeV}$   
 $400 \text{ MeV}$

# Mass splitting: octet



Why?

# Mass splitting: decuplet



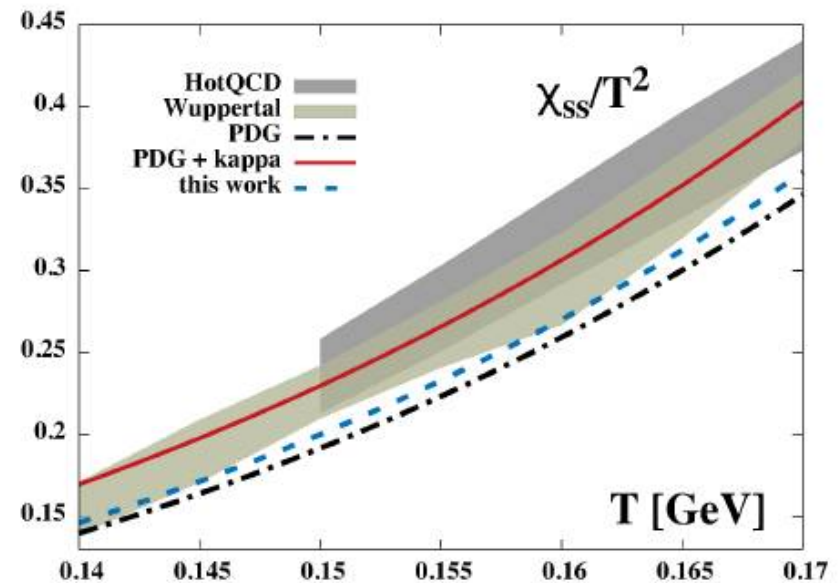
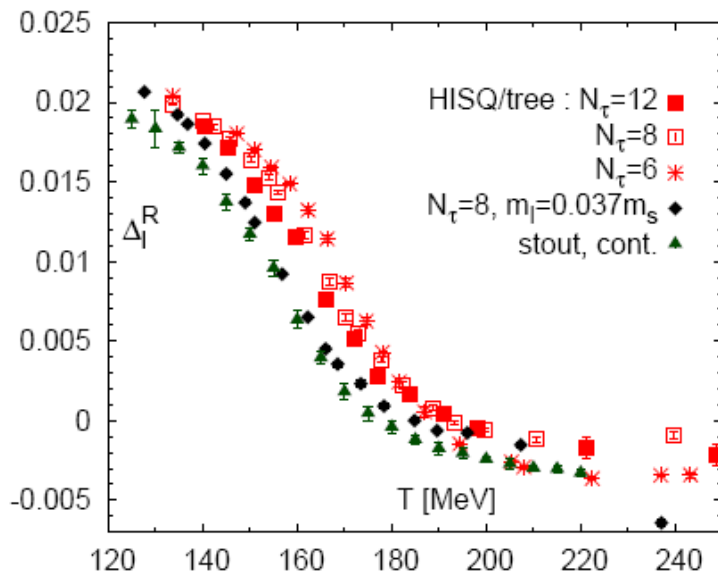
# Remarks

- ❑ Nucleons and deltas:  $\delta m$  drops substantially.
- ❑ Much milder trend in hyperons: **s-quark effect**
- ❑ Heavier  $m_{\pi} \rightarrow m_K$ :  $\delta m(u,d) \approx \delta m(s)$ 
  - more explicit breaking but closer to SU(3)
  - FASTSUM's setup: **SU(3)** rather than SU(2+1)
    - when  $m_{\pi} \nearrow$ , hyperon-  $\delta m \searrow$  --- **1<sup>st</sup> order**
  - Still, Omega-baryon mass needs to be understood.
  - Missing piece(s)? --- **the onset of deconfinement**
- ❑ Need simulations w/ physical  $m_{\pi}$  & other fermions (vs. Wilson)

**Any imprint in EoS?**

# Signal of chiral symmetry restoration

- ❑ Lattice QCD shows clearly  $\langle qq\bar{q} \rangle$  dropping!
- ❑ More deviation from HRG in higher-order fluctuations  $\rightarrow$  Missing states? Interactions? and/or in-medium effects?



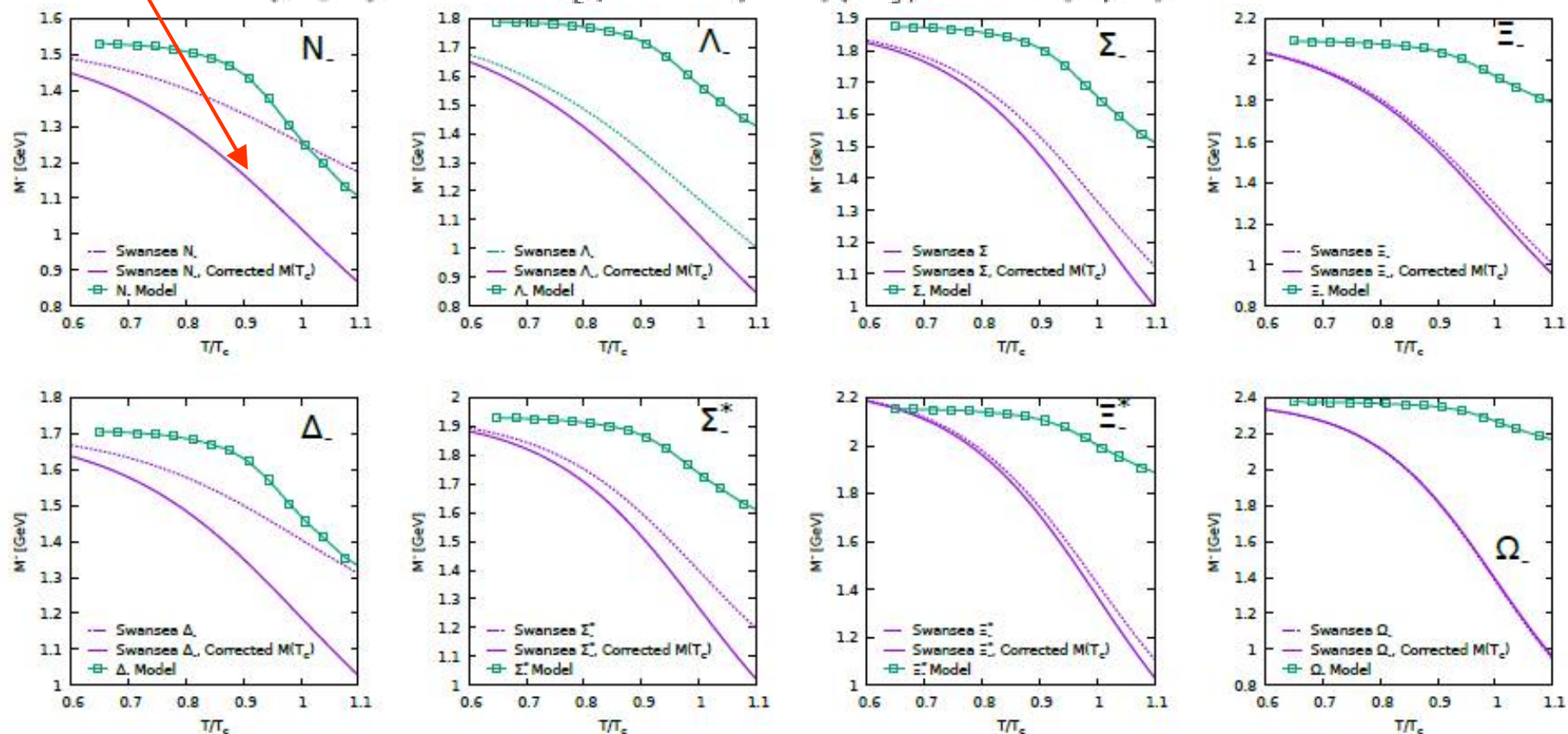
$$M_-^{\text{lat}}(T_c) \times \frac{M_+^{\text{PDG}}}{M_+^{\text{lat}}}$$

# In-medium HRG

□ T-dep. motivated by Lattice findings [Aarts et al.]

$$M^-(T) = M^-(T=0)\omega(T, b) + M^-(T_c)(1 - \omega(T, b))$$

$$\omega(T, b) = \tanh[(1 - T/T_c)/b] / \tanh(1/b)$$

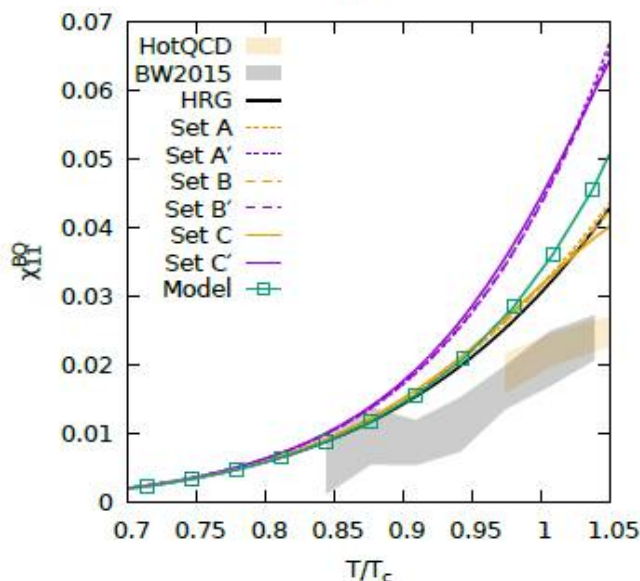
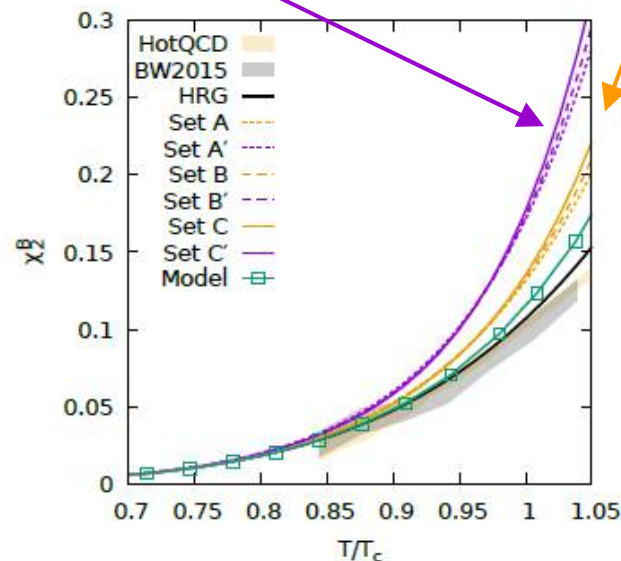


[Morita et al., arXiv:1711.10779 [hep-ph]]

corrected  $M_-^{\text{lat}}(T_c)$

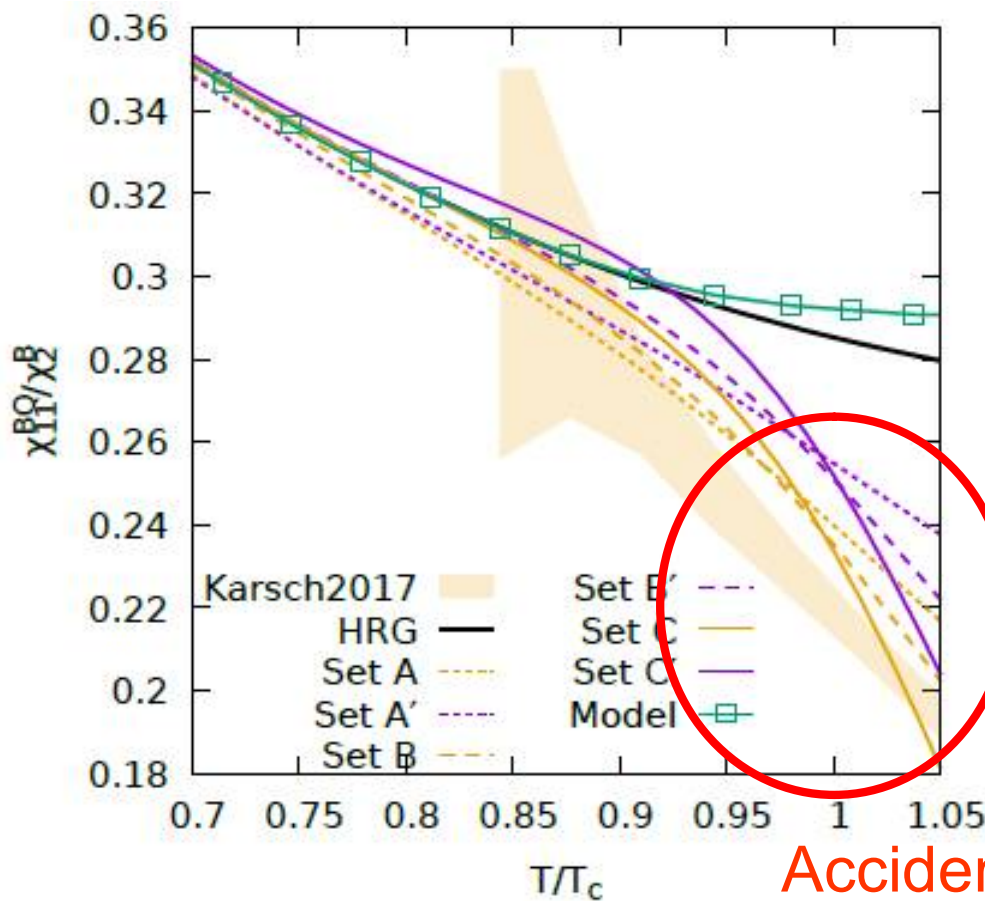
uncorrected

# Fluctuations of net-baryon number



$$\chi_{ijk}^{BQS} \equiv$$

$$\frac{\partial^{i+j+k}}{\partial^i(\mu_B/T) \partial^j(\mu_Q/T) \partial^k(\mu_S/T)} p(T, \mu_B, \mu_Q, \mu_S) / T^4$$



Accidental!

[Morita et al., arXiv:1711.10779 [hep-ph]]

# What is missing? --- finite width

□ Thermodynamics of broad resonances

→ S matrix approach [Dashen, Ma and Bernstein, 1969]

- Grand canonical potential

$$\Omega = \Omega_0 + \Omega_{\text{int}}$$
$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left[ S^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S \right]_c$$

- Leading contribution: 2-body [Beth-Uhlenbech, 1937]

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E)$$

Phase shift

Dynamical information

# What is missing? --- finite width

□  $K^0^*/\kappa$  (800) meson: chiral partner of kaon

NOTE: omitted from PDG summary table

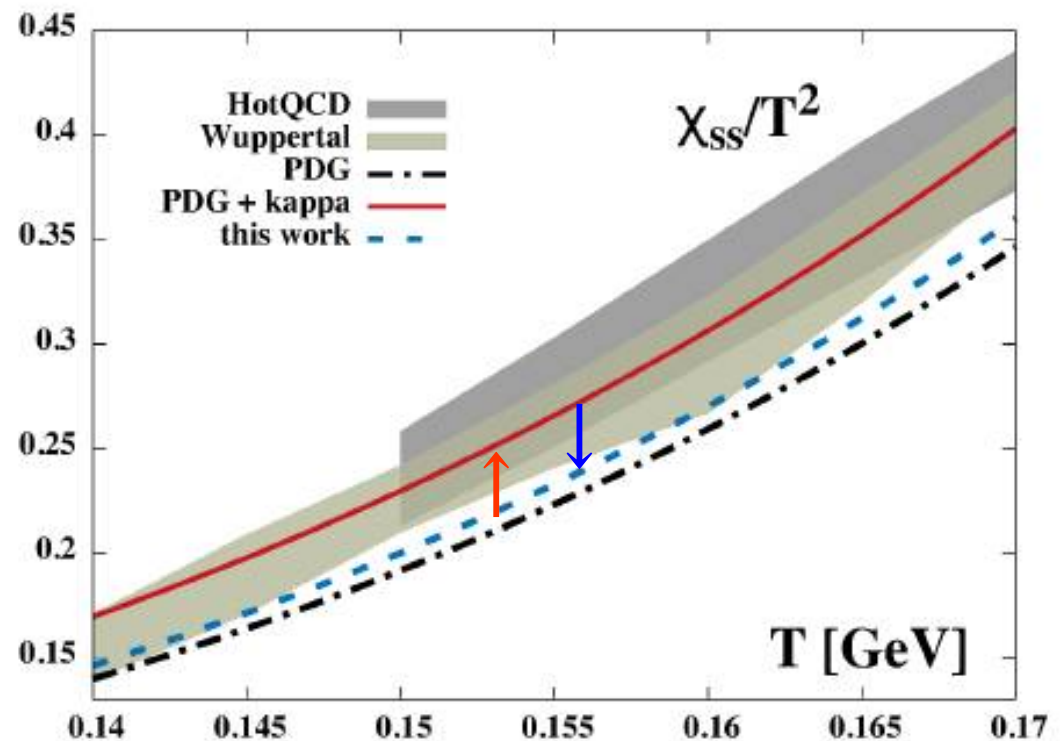
□ S matrix approach [Friman et al. 2015]

✓ Empirical  $\pi$ -K phase shift from experiment

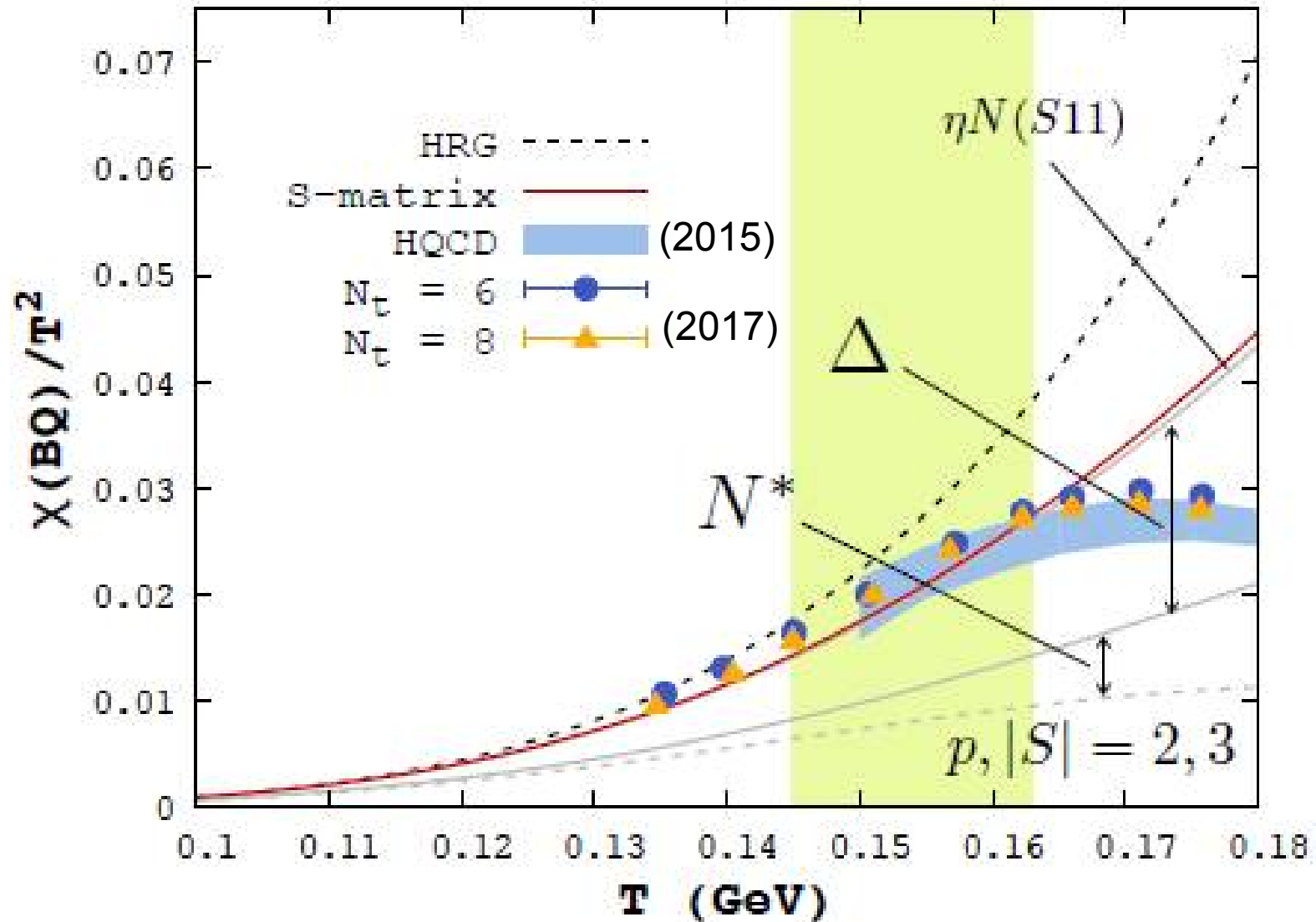
$$\Omega = \Omega_\pi + \Omega_K + \Omega_{\text{int}}$$

cf. HRG

$$\Omega = \Omega_\pi + \Omega_K + \Omega_{\text{res}}$$



# Pi-Nucleon system



# Summary

□ Emergent parity-doubling structure as a manifestation of restored chiral symmetry

### Lessons:

- ◆ Naive “in-medium HRG” does not work.
- ◆ Effect of resonance widths – beyond HRG
- ◆ Survival mass  $\approx$  chromo-magnetic sector
- ◆ Higher-lying states near QCD p.t.
- ◆ Interplay between CSB and confinement
- ◆ Toward more realistic description of QCD