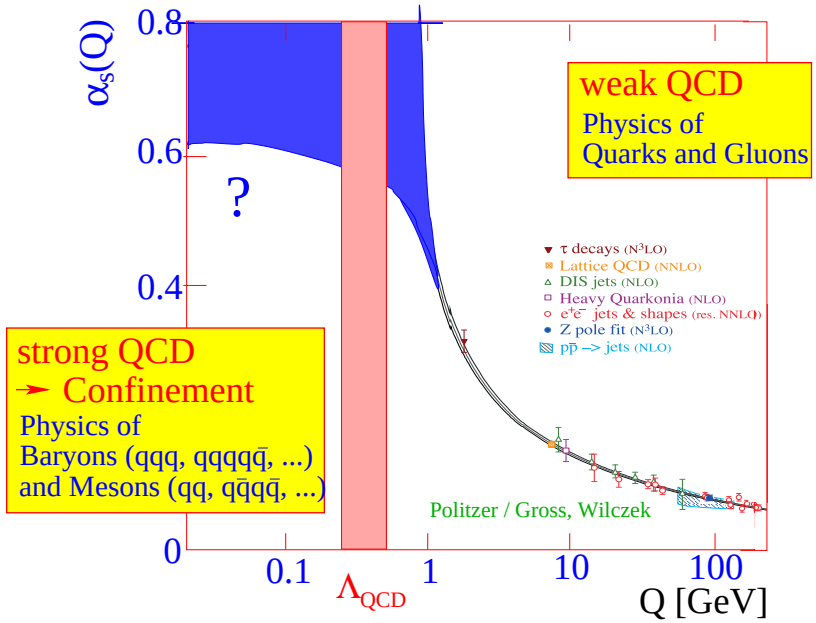
Lecture series on
Exotic Mesons
Part I: Effective Field theories and their application to $D_s(2317)$

Christoph Hanhart

Forschungszentrum Jülich

Setting the stage ...



→ While QCD gets perturbative at large energies, it is non-perturbative at low energies

→ Quarks are confined

→ Spectroscopy is the method of choice to investigate the inner workings of QCD and the formation of matter

→ Although overall providing a good general understanding the quark model has certain short comings

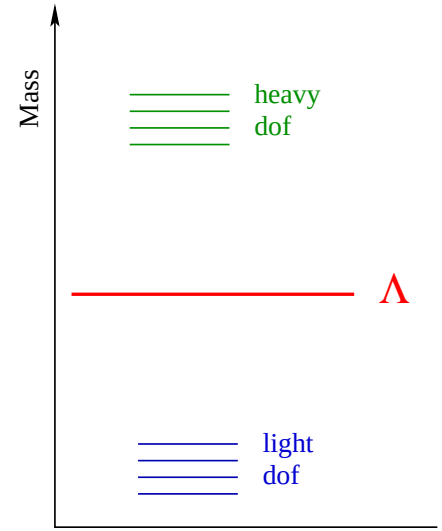
→ The physics of light and heavy quarks is rather different

Effective field theories

Weinberg 1979

→ Precondition: separation of scales
low vs. high energy dynamics

- ▷ low-energy dynamics in terms of relevant dof's: $E \sim \mathrm{p} \sim Q$
- ▷ high-energy dynamics not resolved → contact interactions



→ Small parameter(s) & power counting

- ▷ Standard QFT: trees + loops → renormalization
- ▷ Expansion in powers of Q over the large scale

$$M = \sum_\nu (Q/\Lambda)^\nu f(Q/\mu, C_i)$$

μ: regularization scale; C_i: low–energy constants
ν bounded from below → controlled expansion

Limiting cases of QCD

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{q}_f \left(\gamma_\mu D^\mu - m_f\right) q_f - \frac{1}{4T} \mathrm{Tr}\left(F^{\mu\nu} F_{\mu\nu}\right)$$

Limit of massless Quarks

Weinberg/ Gasser, Leutwyler

$$\mathcal{L}_{\rm QCD} = \bar{q}_L \left\{ i\partial\!\!\!/ + g A\!\!\!/^a t^a \right\} q_L + \bar{q}_R \left\{ i\partial\!\!\!/ + g A\!\!\!/^a t^a \right\} q_R + \mathcal{O}(m_f/\Lambda_{\rm QCD})$$

L and R Quarks decouple + spontaneous symmetry breaking

$\rightarrow$ Chiral Perturbation Theory (ChPT)

Limit of infinitely Heavy Quarks

Isgur, Wise, Manohar, Caswell/Lepage

$$\mathcal{L}_{\rm QCD} = \bar{q}_f \left\{ i v \cdot \partial + g v \cdot A^a t^a \right\} q_f + \mathcal{O}(\Lambda_{\rm QCD}/m_f)$$

Independent of Heavy Quark Spin and Flavour

$\rightarrow$ Heavy Quark Effective Field Theory (HQEFT)

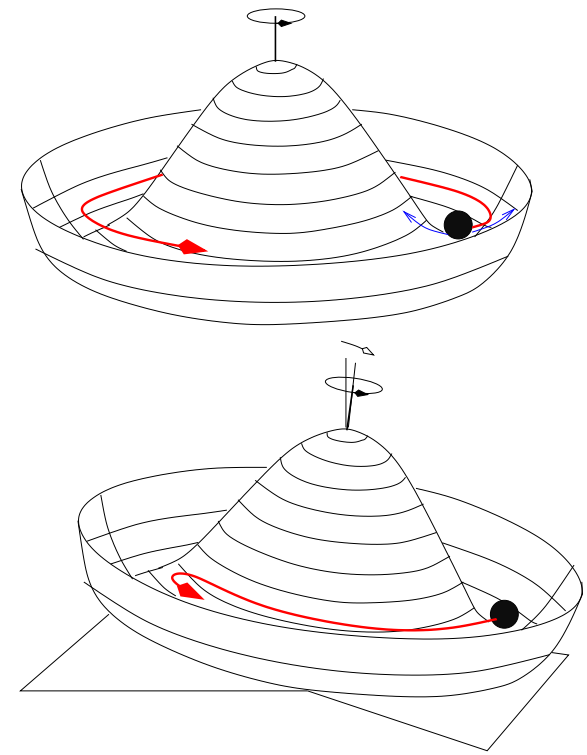
$\rightarrow$ Non-Relativistic QCD (NRQCD)

Chiral Perturbation Theory

→ Systematic expansion around
$m_f = 0 \; (f = u, d, (s))$

→ Mass gap provided by
spontaneous–symmetry breaking:
→ pions as Goldstonebosons

→ Finite quark masses as perturbations
→ $m_\pi^2 \propto (m_u + m_d) \;\rightarrow\; m_\pi / M_N \ll 1$



→ Example: $\pi\pi$-isoscalar s–wave scattering length:

$a_0 = (0.16 + 0.04 + 0.017) \pm 0.009$ at LO, NLO, N^{2}LO

Bijnens et al. PLB374(1996)210

Exp.: $a_0 = 0.221 \pm 0.006$ in excellent agreement

Batley et al. [NA48/2 Coll.] EPJC79(2010)635

Unitarization

Unitarity relation: $\mathrm{Im}(t) = \sigma |t|^2$ with $\sigma = \sqrt{1 - 4m_\pi^2/s}$

$\rightarrow$ Perturbative expansion consistent only to given order

$\rightarrow$ s–dependent terms quickly hit unitarity bound

Solution: Unitarization $\rightarrow$ can produce poles

Truong, Dorado, Pelaez, Kaiser, Weise, Oller, Oset, Lutz, Kolomeitsev, Guo, Meißner, C.H., ...

Different methods used (dep. needs to be clarified);

$\rightarrow$ universal picture emerges in many channels!

Example I: $\pi\pi$ scattering from the Inverse Amplitude Method

Idea: write unitarity as

$$\mathrm{Im}(t^{-1}) = -\sigma \qquad \longrightarrow t = \frac{1}{\mathrm{Re}(t^{-1}) - i\sigma}$$

use ChPT to fix $\mathrm{Re}(t^{-1})$ to the required accuracy

Results

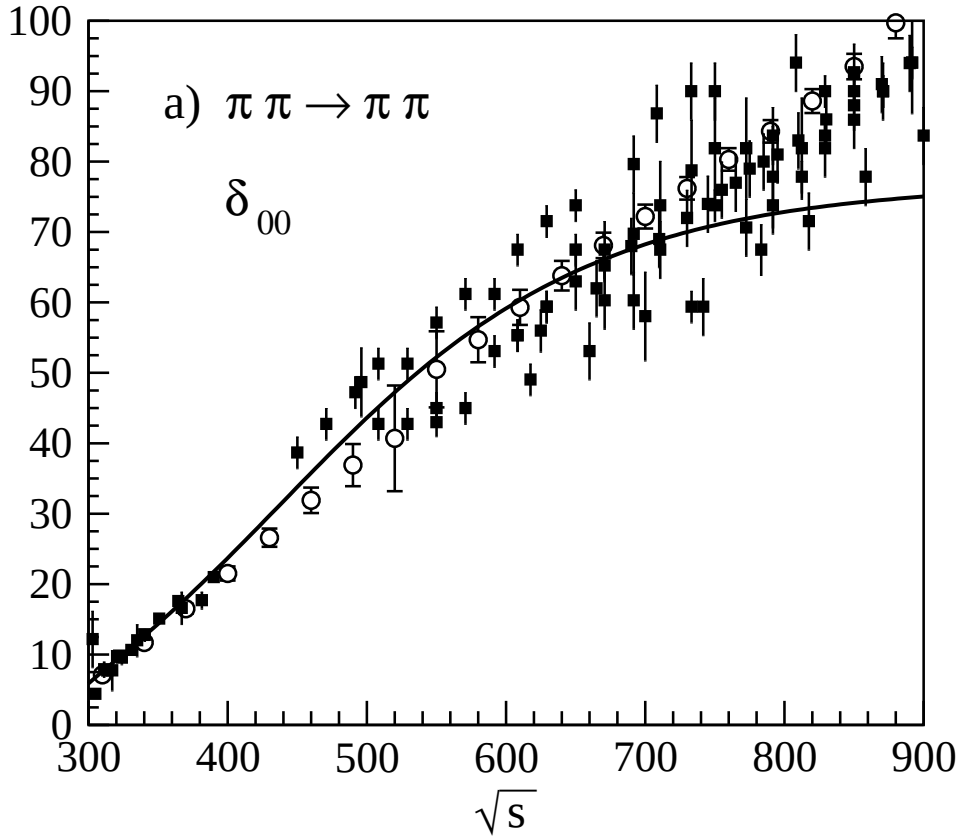
Using ChPT to NLO as input (parameters $\times 10^3$ at $\mu = m_\rho$)

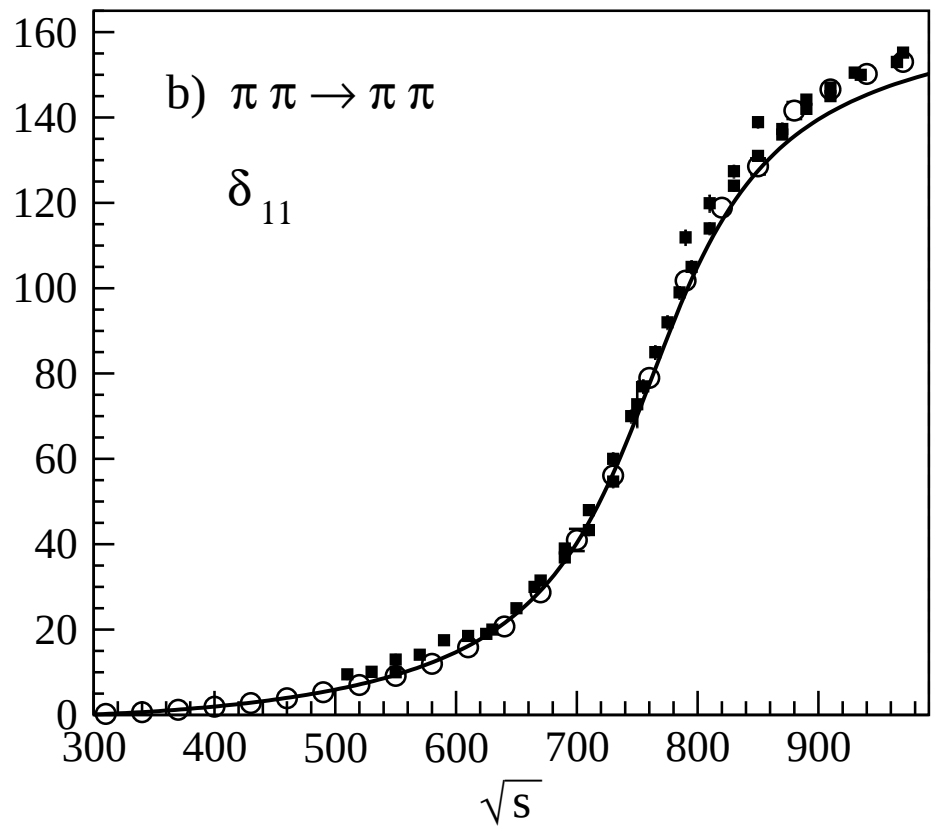
$l_3^r = 0.18 \pm 1.11$; $l_4^r = 6.17 \pm 1.39$ from the literature

Colangelo et al. (2001) & Colangelo et al. (2010)

$l_1^r = -3.7 \pm 0.2$; $l_2^r = 4.3 \pm 0.4$ fit to the data

Nebreda et al. (2011)





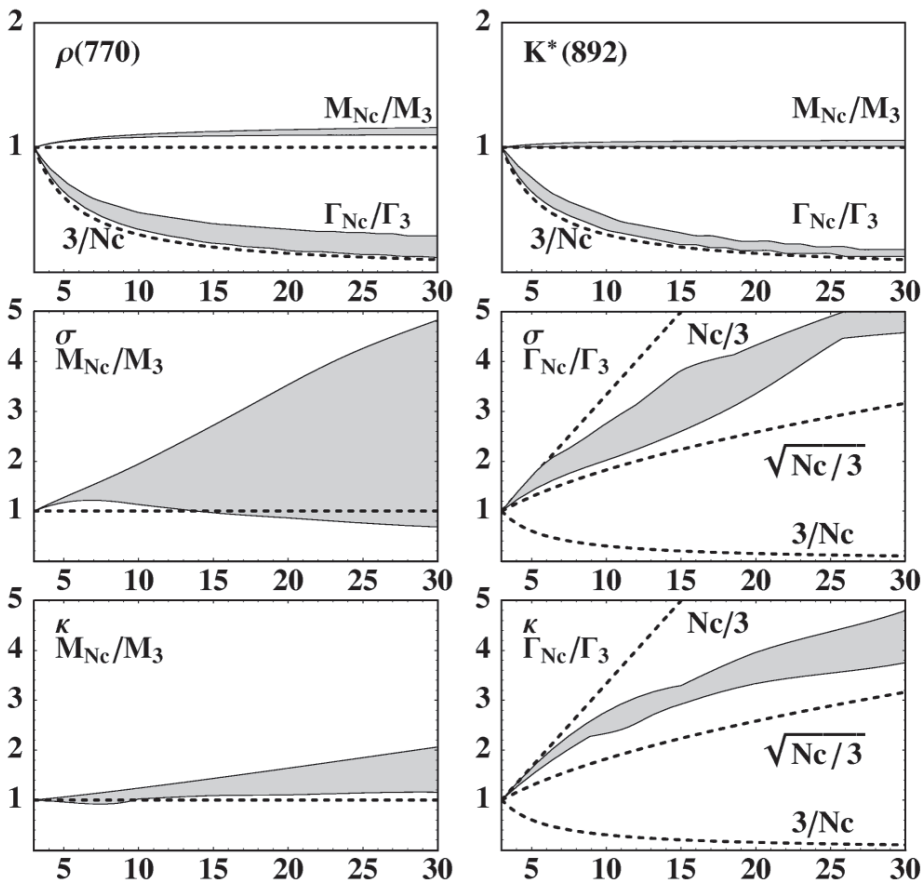
What does this tell about the nature of $f_0(500)$ and $\rho(770)$?

Large N_c

An interesting limit of QCD is $N_c \to \infty$, while $g_s^2 N_c = const.$

$\to$ N_c–scaling of low energy constants known

e.g. Peris & de Rafael. PLB348(1995)539



Pelaez PRL92(2004)102001

$\to$ N_C scaling for states:

Cohen et al. PRD90(2014)036003

- $\bar{q}q$:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c)$
- $\bar{q}\bar{q}qq$:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c)$
- gg:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c^2)$
- $(N_c - 1)\bar{q}q$:
  $M \sim \mathcal{O}(N_c);\ \Gamma \sim \mathcal{O}(1)$

ρ, K^* consistent with $\bar{q}q$; σ, κ do not match to any $\to$ rescattering?

Heavy Quark Symmetry

see, e.g., Neubert Phys. Rep. 245(1994)259

One may derive from the QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \bar{q}_f \left\{ iv \cdot \partial + gv \cdot A^a t^a \right\} q_f + \mathcal{O}(\Lambda_{\rm QCD}/m_f)$$

At leading order interaction spin and flavor independent!

heavy quark spin and $J_{\rm light}$ of light quarks conserved independently

Terms at $\mathcal{O}(\Lambda_{\rm QCD}/m_f)$ contain

$\rightarrow$ kinetic energy of heavy quark

$\rightarrow$ term breaking spin symmetry

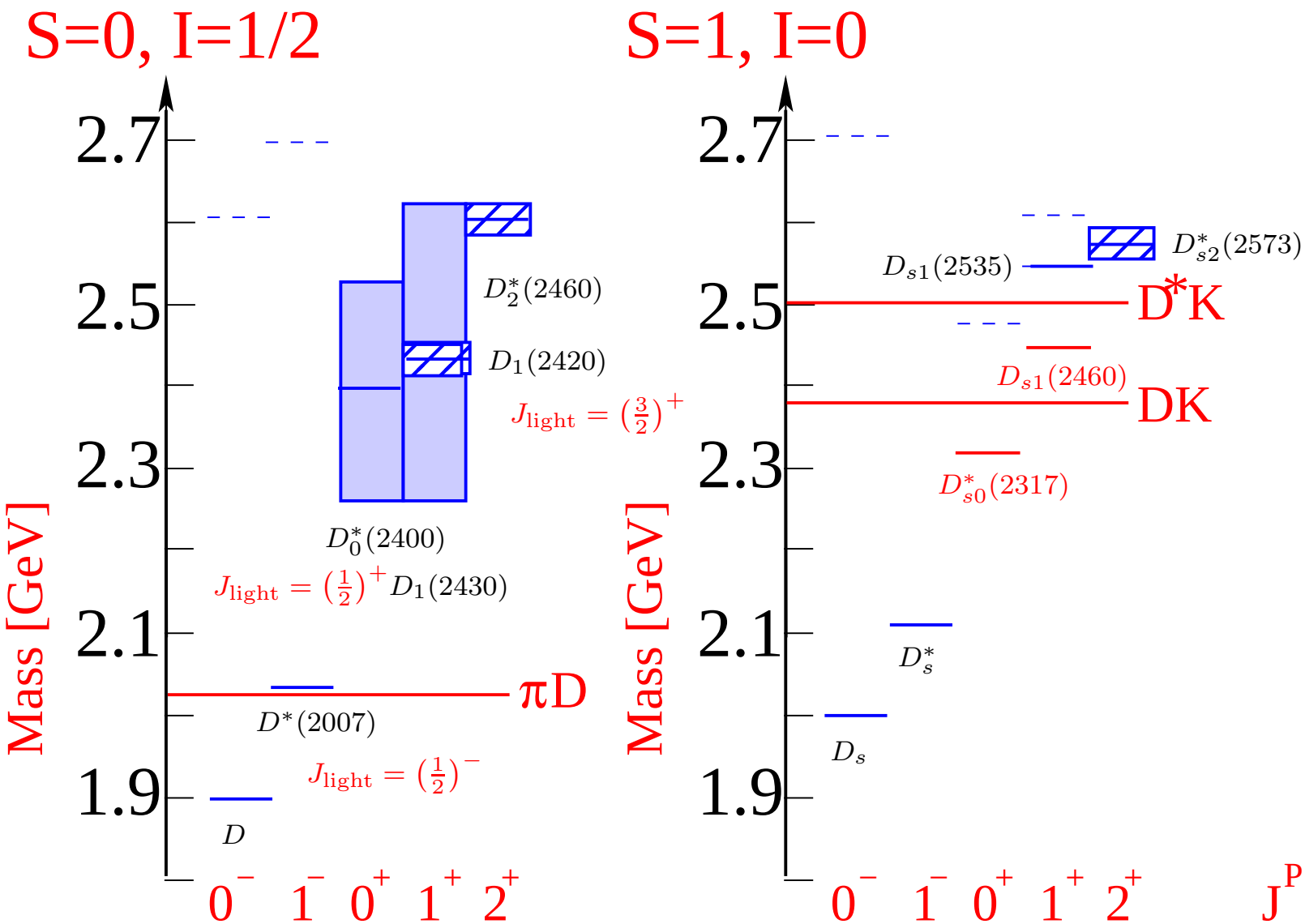
Consequence: mesons form spin multiplets with

$$m_{D^*} - m_D \sim \Lambda_{QCD} \ , \quad m_{B^*}^2 - m_B^2 \simeq m_{D^*}^2 - m_D^2$$

which works nicely - also for excited states

$\rightarrow$ Amount of spin symmetry violation important diagnostic tool!

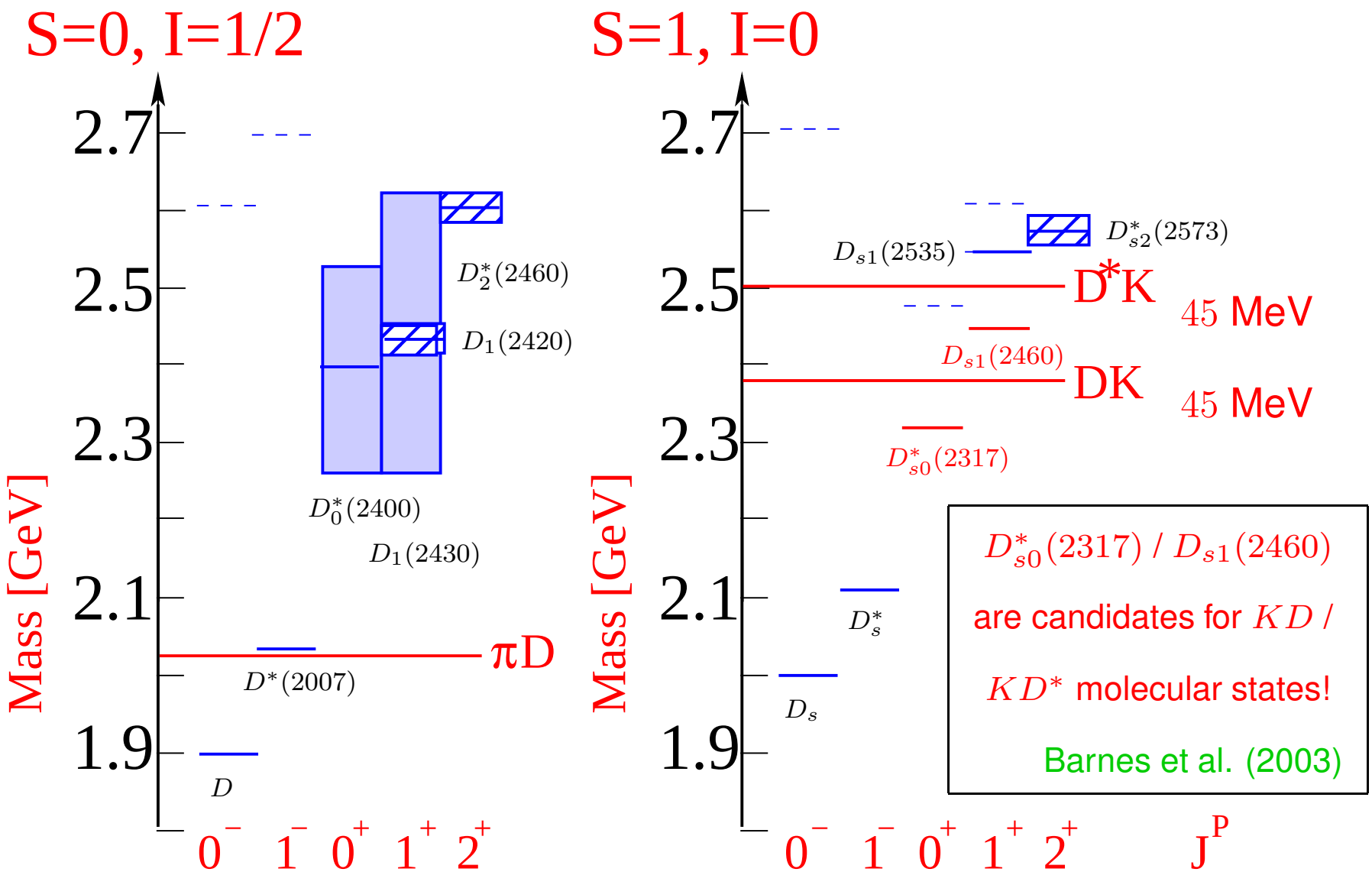
Example: Strange-Charm states



Quark Modell: M. Di Pierro and E. Eichten, PRD **64** (2001) 114004

Note: decay modes of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ either $D_s^{(*)}\pi$ or $D_s^{(*)}\gamma$ $\rightarrow$ narrow

Example: Strange-Charm states



$D_{s0}^*(2317)$ / $D_{s1}(2460)$ are candidates for KD / KD^* molecular states!

Barnes et al. (2003)

Quark Modell: M. Di Pierro and E. Eichten, PRD **64** (2001) 114004

Note: decay modes of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ either $D_s^{(*)}\pi$ or $D_s^{(*)}\gamma$ $\rightarrow$ narrow

Hadronic Molecules

→ are few-hadron states, bound by the strong force

→ do exist: light nuclei.
e.g. deuteron as pn & hypertriton as Λd bound state

→ are located typically close to relevant continuum threshold;
e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$

- ▷ $E_B^{\text{deuteron}} = 2.22$ MeV ($\gamma = 45$ MeV)
- ▷ $E_B^{\text{hypertriton}} = (0.13 \pm 0.05)$ MeV (to Λd) ($\gamma = 13$ MeV)

→ can be identified in observables (Weinberg compositeness):

$$\frac{g_{\text{eff}}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \rightarrow a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\,; \quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

where $(1-\lambda^2)$=probability to find molecular component in bound state wave function

Are there mesonic molecules?

Disclaimers

→ The formalism presented is 'diagnostic' — especially,

- ▷ it does not allow for conclusions on the binding force
- ▷ it allows one only to study individual states.

To go beyond that a dynamical model needs to be employed.

→ Quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim R\gamma$) ; we propose to stick to

'the larger the coupling the more molecular the state'

There are striking phenomenological implications from large couplings for they lead to

- ▷ relations between seemingly unrelated reactions
- ▷ rather specific, unusual line shapes.

Unitraized ChPT

Kolomeitsev/ Lutz PLBB582(2004)39; Guo et al. PLB641(2006)278
Gamermann et al. PRD76(2007)074016; Guo et al. PLB666(2008)251

$$\boxed{\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - m_D^2 D D^\dagger}$$

with $D = (D^0, D^+, D_s^+)$ denoting the D–mesons, and

$$\mathcal{D}_\mu = \partial_\mu + \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right) \text{ where } u = \exp\left(\frac{\sqrt{2} i \phi}{2 F_\pi}\right)$$

The Goldstone boson fields are collected in the matrix

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

LO potential parameter free; 1 regulator necessary

At NLO 6 low energy constants enter

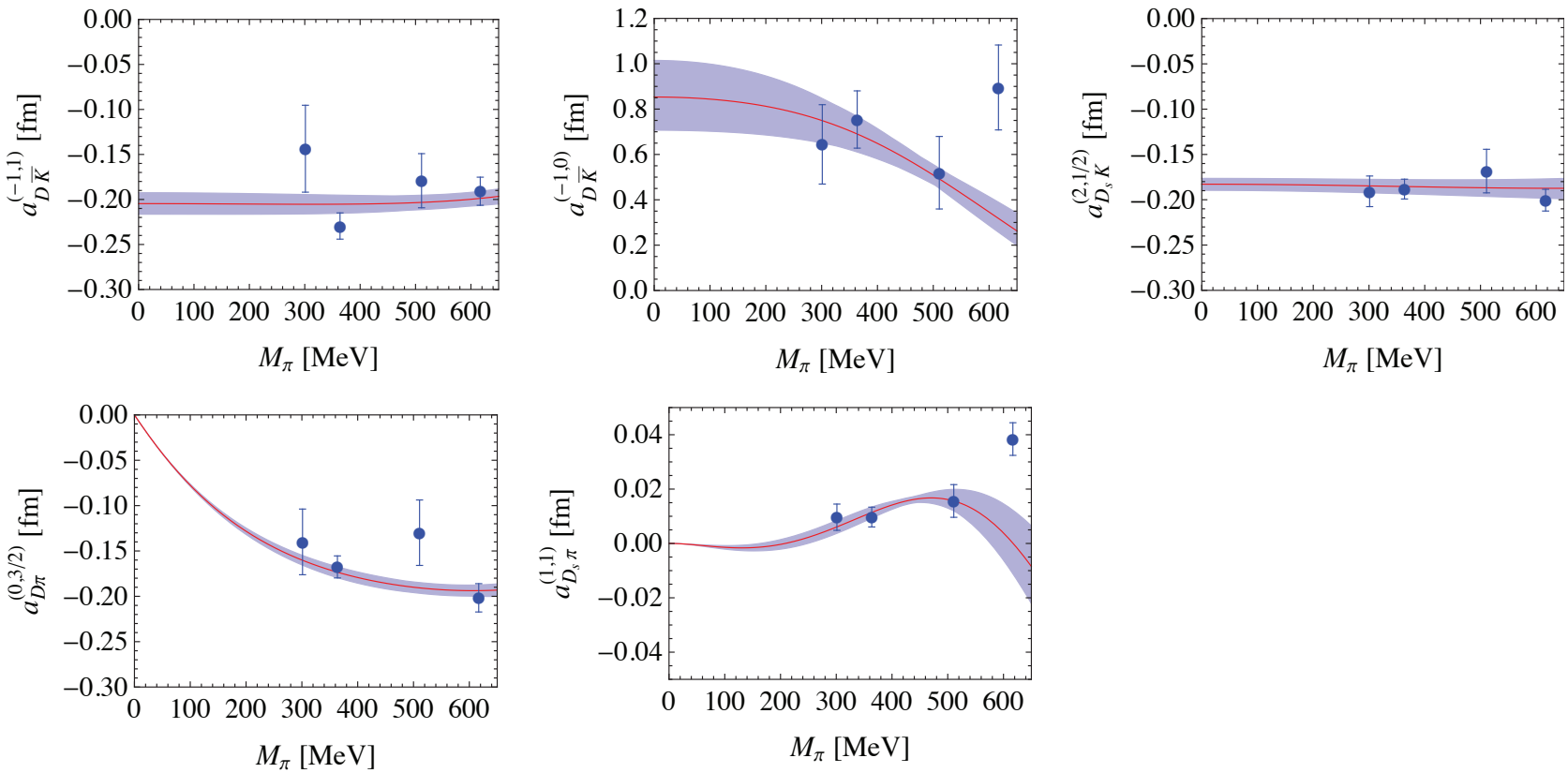
Heavy light Systems

→ $\pi/K/\eta$–D/D_s scattering in ChPT to NLO unitarized

→ controlled quark mass dependence

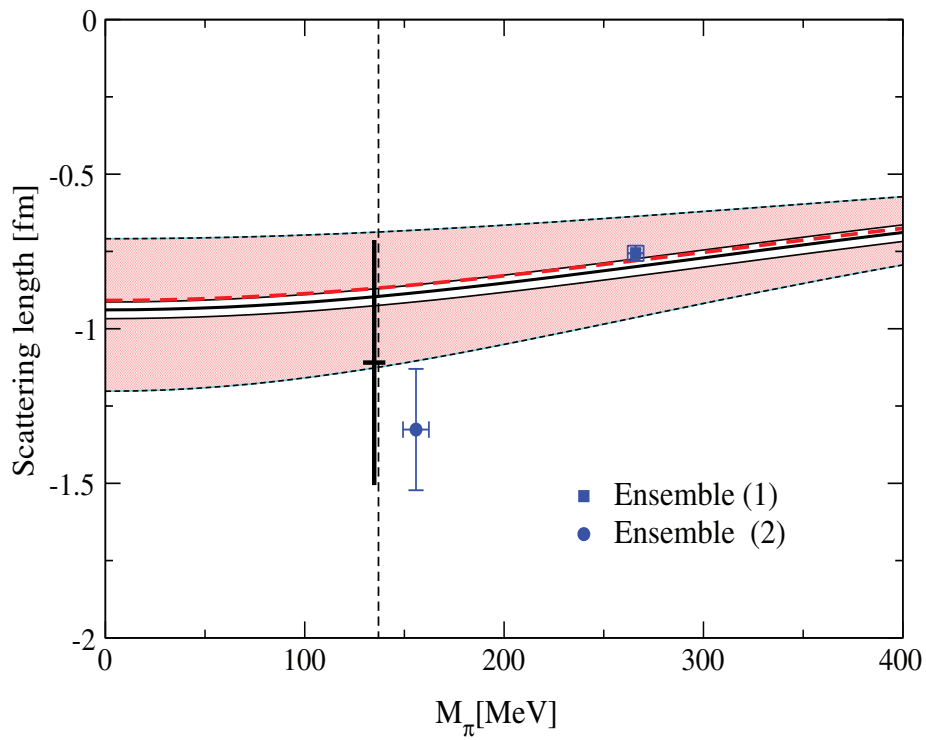
→ fit LECs to lattice data

Liu et al. PRD87(2013)014508



→ $D^*_{s0}(2317)$ emerges as a pole with $M_{D^*_{s0}} = 2315^{+18}_{-28}$ MeV.

Interpretation

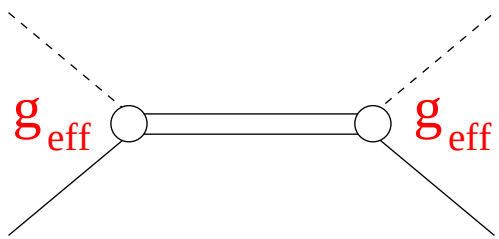


shaded band (dashed line):
full result (best fit)

white band (solid line):
$D^*_{s0}(2317)$ mass fixed to physical value

Liu et al. PRD87(2013)014508

Lattice: Mohler et al., PRL 11(2013)222001

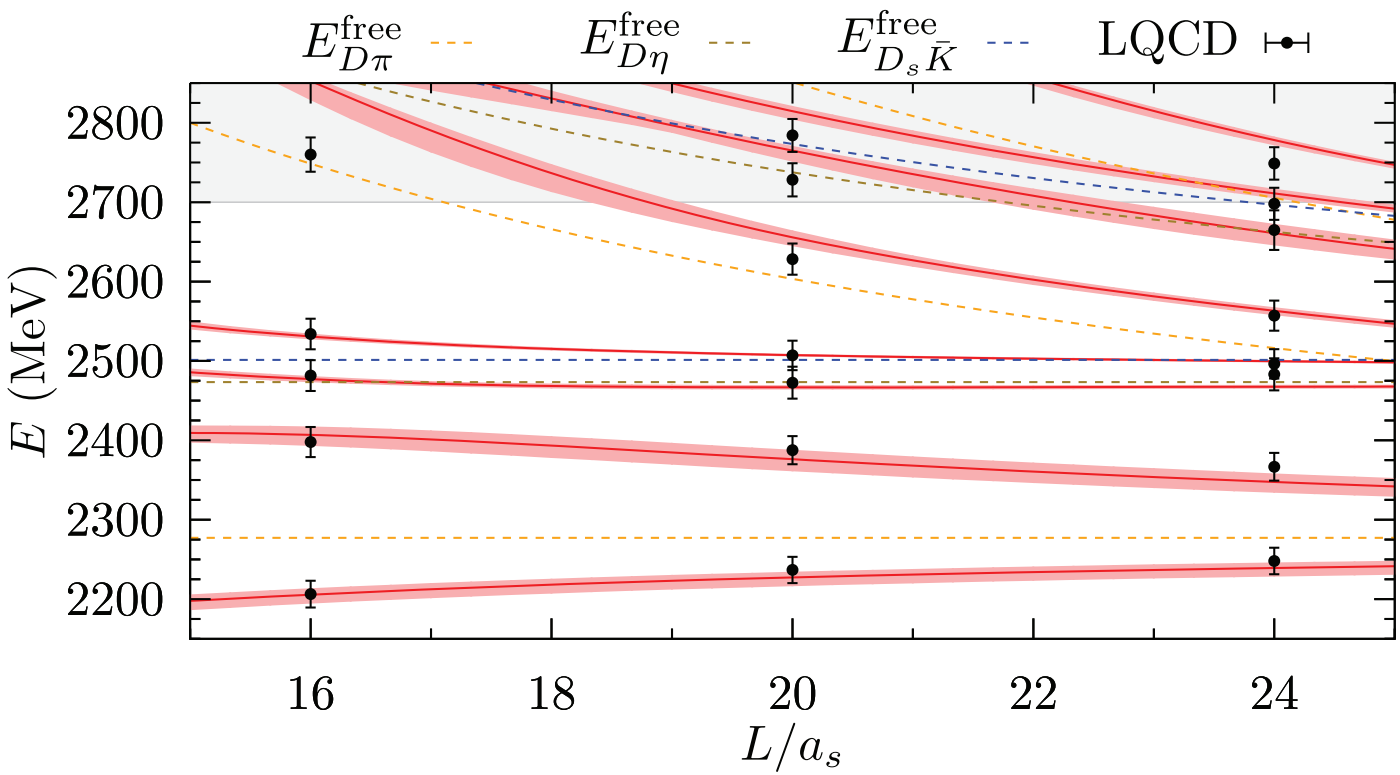
$D^*_{s0}(2317)$: $a =$ 

$+\mathcal{O}(1/\beta) \simeq \left(\frac{2\lambda^2}{1+\lambda^2}\right)\frac{-1}{\sqrt{2m_K E_B}}$

$a = -(1.05\pm0.36)$ fm for molecule $(\lambda^2 = 1)$; smaller otherwise

... and in the $s = 0$ sector

Keeping parameters fixed one gets:



Albaladejo et al., PLB767(2017)465; Lattice: Moir et al. JHEP1610(2016)011

→ Poles for $m_\pi \simeq 391$ MeV: $(2264, \quad 0)$ MeV [000] & $(2468, 113)$ MeV [110]

→ Poles for $m_\pi = 139$ MeV: $(2105, 102)$ MeV [100] & $(2451, 134)$ MeV [110]

Questions $c\bar{q}$ nature of lowest lying 0^+ D state, $D_0^*(2400)$

Exp. Test: Hadronic decays

Faessler et al. PRD76(2007)014005; Lutz, Soyeur NPA813(2008)14; Guo et al., PLB666 (2008)251

Isospin breaking (drives decay) via quark masses and charges

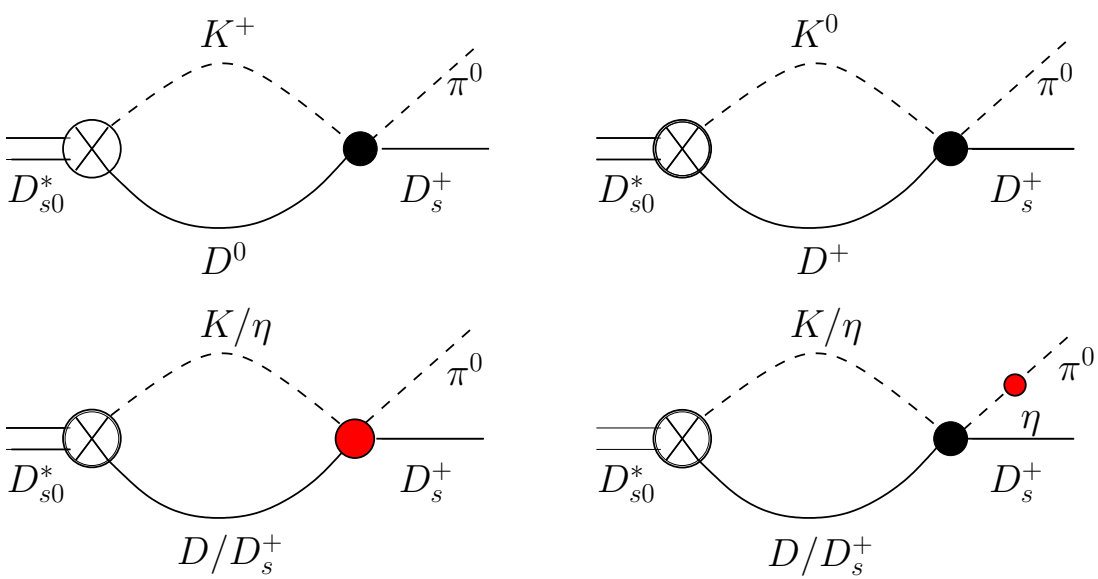
The same effective operators lead to

→ mass differences, e.g.

- ▷ $m_{D^+} - m_{D^0} = \Delta m^{\mathrm{q}} + \Delta m^{\mathrm{e.m.}} = ((2.5 \pm 0.2) + (2.3 \pm 0.6))$ MeV
- ▷ $\pi^0 - \eta$ mixing $\longrightarrow$ parameters fixed

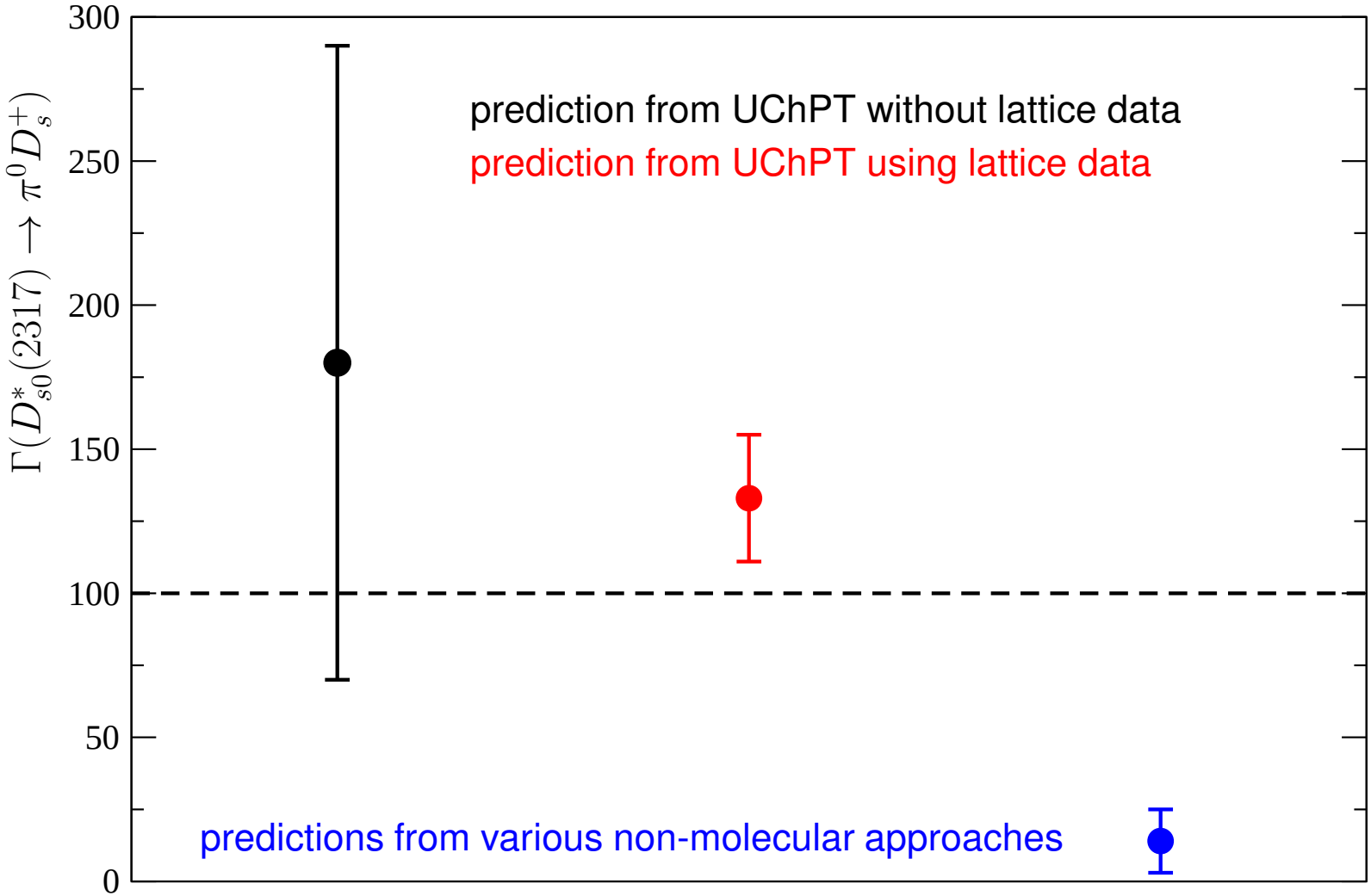
→ Isospin breaking scattering amplitude

- ▷ e.g. $KD \to \pi^0 D_s$ predicted



Specific for molecules!

Hadronic width



Measurement of width is decisive, if D^*_{s0} is molecular or not

Experiment needs very high resolution $\to$ PANDA

Summary lecture I

Various systematic approaches to QCD exist

→ lattice QCD

→ Chiral perturbation theory

→ Heavy quark effective field theory/ NRQCD

→ Large N_c

→ The Weinberg-analysis

Especially analyses combining the methods, supplemented with high quality experiments promise deep insights into the inner workings of QCD

Next lecture: The XYZ–story