# The Structure of the nucleon

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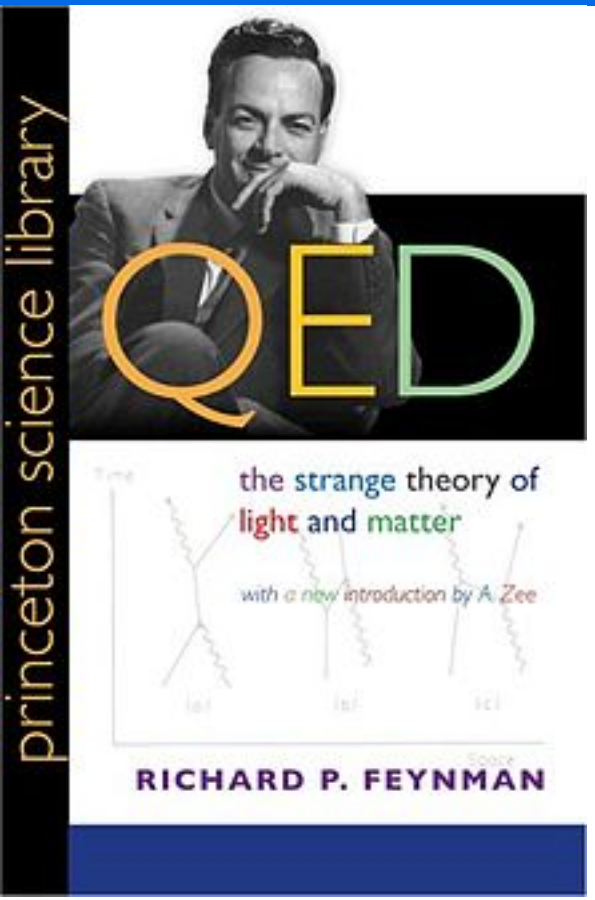
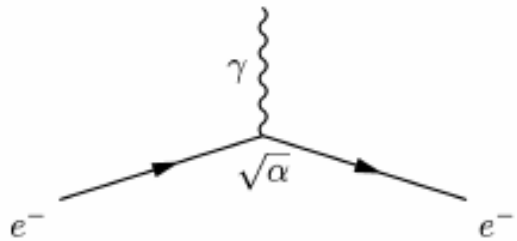
**PANDA Lecture Week 2017– GSI/Darmstadt**

# QED: the strange theory of light and matter




*Electromagnetic interaction via the exchange of virtual photons*




$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cong \frac{1}{137}$$

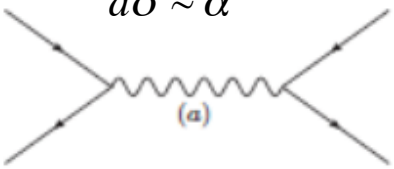
QED is a time dependent perturbation theory

One photon interaction: $\alpha=1/137$

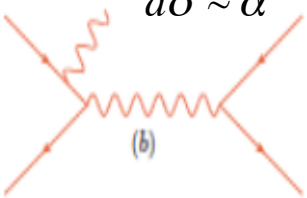
Perurbative corrections: $\alpha^n=(1/137)^n$

QED converges rapidly: accurate predictions

$d\sigma \sim \alpha^2$




$d\sigma \sim \alpha^3$

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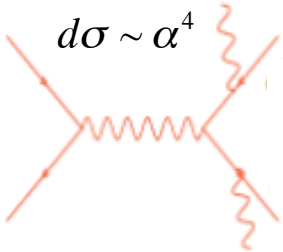
$d\sigma \sim \alpha^4$



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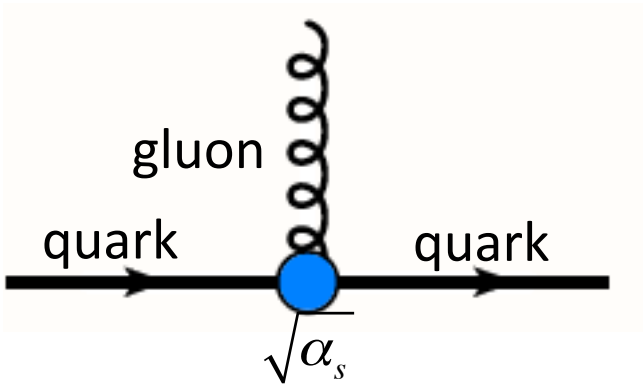
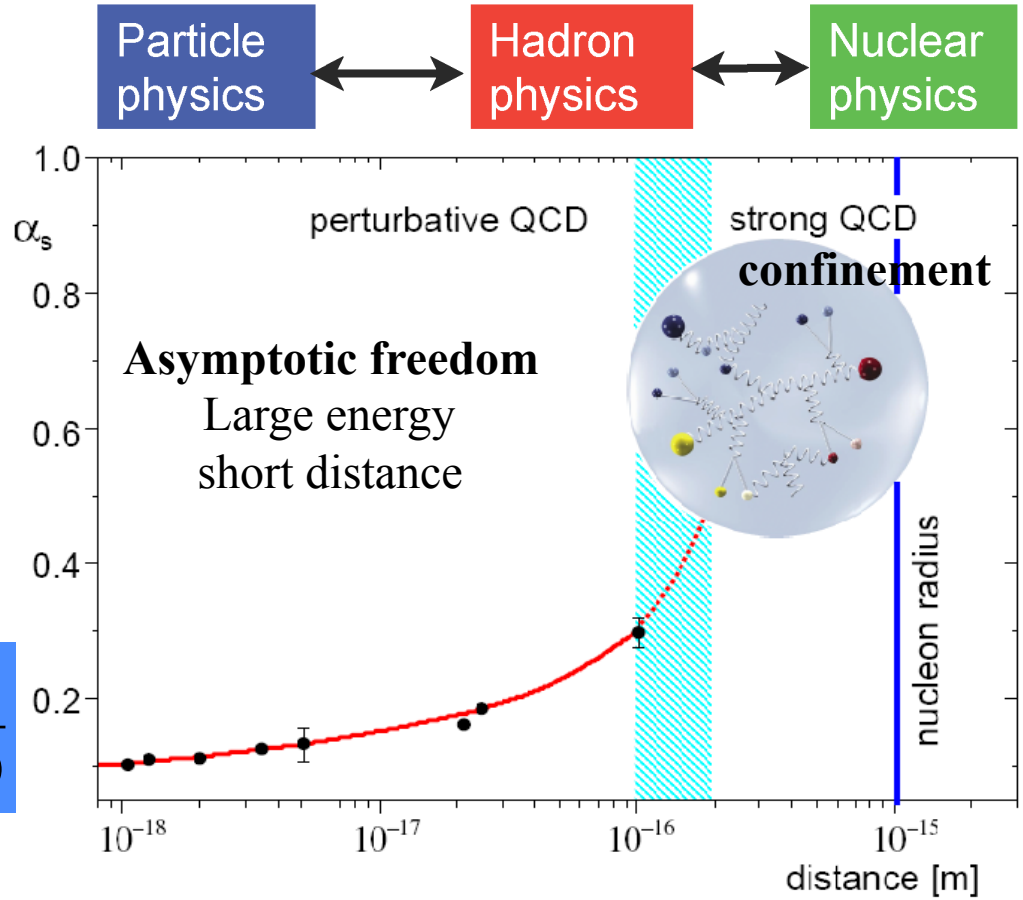
# From QED to the theory of the strong interaction QCD

**Quark model (Gell-Mann 1964) :**
hadrons are made of quarks which are held together by the strong interaction




$$\alpha_s(q^2) = \frac{\alpha_s(\Lambda_{QCD}^2)}{(1+\beta\alpha_s(\Lambda_{QCD}^2)\ln(q^2/\Lambda_{QCD}^2))}$$



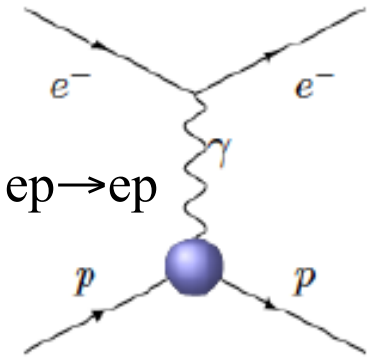

Hadronic scale ~ 1/fm (=1/139 MeV$^{-1}$) ~ $\Lambda_{QCD}$ is non-perturbative

Studying the nucleon structure is an investigation of the non perturbative QCD

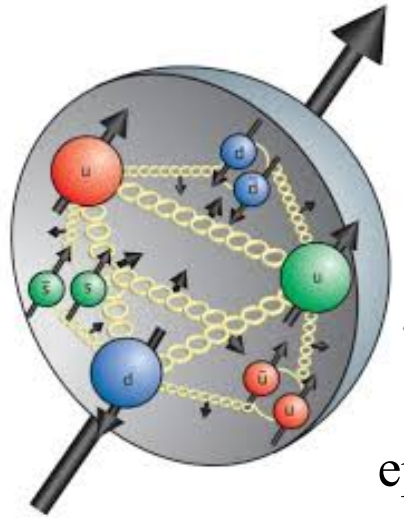
# Electromagnetic structure of hadrons

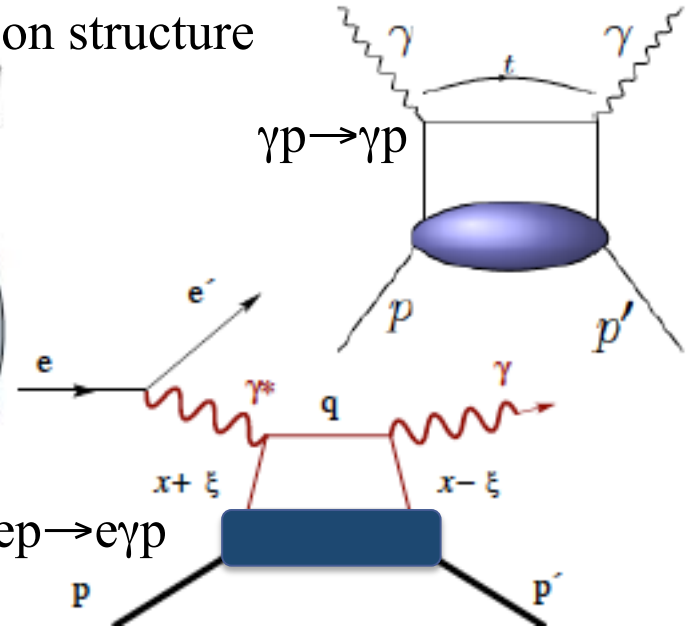
**QED interactions to probe the non perturbative QCD**

- Connect quarks and gluons to hadrons via non-perturbative but **universal distribution functions** (QCD factorization)
- Provide unified view of the nucleon structure

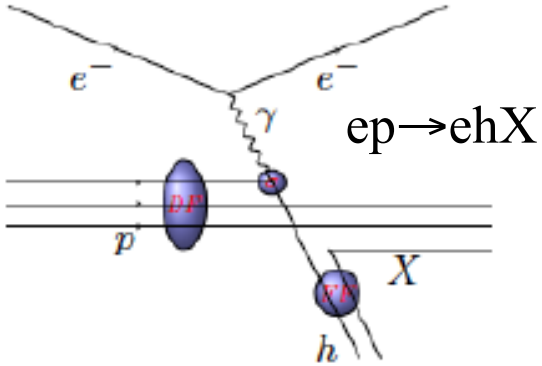



Elastic Electron Proton Scattering- FFs
- transverse spatial distributions






Deep Virtual Compton Scattering- Wide Angle Compton Scattering- GPDs – 1D momentum – 2D space distributions




Semi Inclusive Deep Inelastic Scattering- TMDs – 3D momentum distributions + Spin structure

and many other electromagnetic processes

# Outline

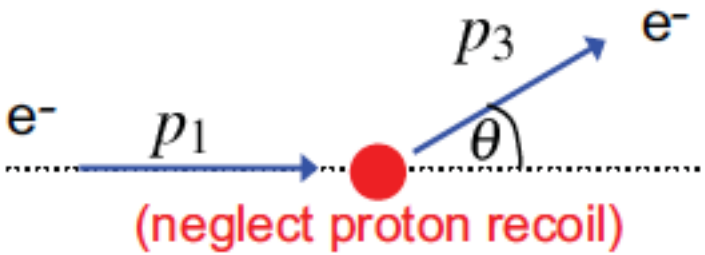
- Electromagnetic form factors of the proton
  - Space-like region
  - Time-like region
- Parton Distribution functions (PDF) in SIDIS and Drell-Yan
- Accessible nucleon structure functions at PANDA

In connection to the opportunities offered by the future **antiproton beams** of FAIR

# Elastic Electron-Proton Scattering

## Rutherford scattering cross section (~1911 )

- Non relativistic electron ($E_k << m_e$)
- No recoil of the target (neglected)
- Point-like target

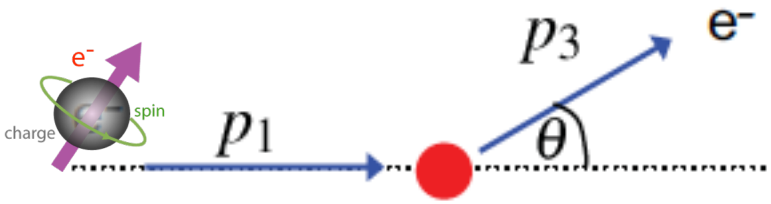



$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{\alpha^2}{16E_k^2}\frac{1}{\sin^4(\theta/2)}$$

Interaction between the electric charges of the particles matters.

## Mott scattering cross section (~1929)

- Relativistic electron ($E_k >> m_e$)
- Electron is carrying a spin
- Point-like target ( for a finite Mass M)




$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{1}{1+(2E/M)\sin^2(\theta/2)}$$

# Elastic Electron-Proton Scattering (Magnetic Scattering)

Elastic scattering of relativistic electrons by the magnetic moment of the proton has been predicted by Rosenbluth in 1950.

**Elastic scattering of relativistic electrons from a point-like Dirac spin ½ proton**

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left[1.00-\frac{q^2}{2M^2}\tan^2(\theta/2)\right]$$

**Elastic scattering of relativistic electrons from a point-like Dirac-pauli proton**

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left[1.00-\frac{q^2}{4M^2}\left(2(1.00+\kappa)^2\tan^2(\theta/2)+\kappa^2\right)\right]$$

NS: No Spin (Mott cross section)

# Experimental results

Theoretical curves:

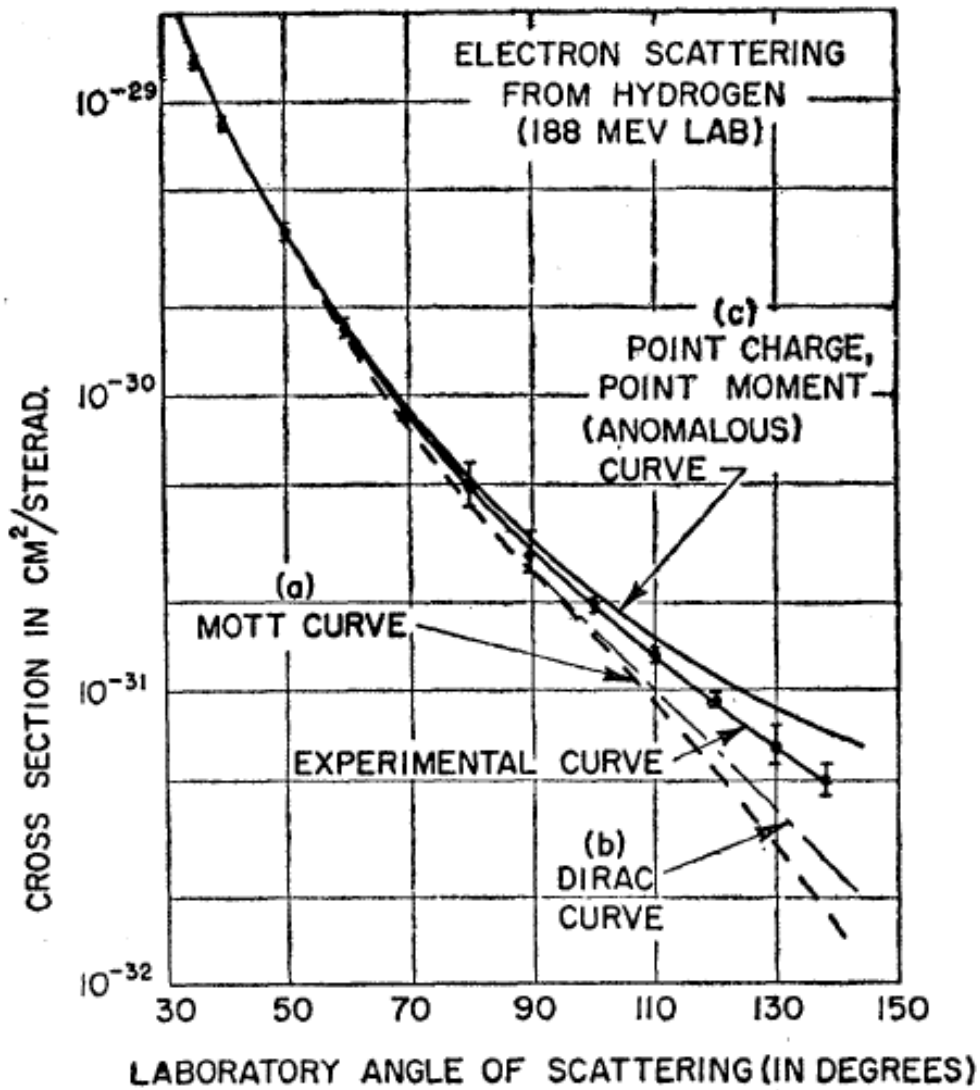
a) Mott cross section (lab frame): a spinless point charge proton

b) Dirac cross section: a point like proton and Dirac magnetic moment

c) Rosenbluth cross section: point-like proton with the corrected anomalous magnetic moment mu

Results deviate from theory due to a structure factor- **the proton is not a point-like but has finite size**.

**Form Factors introduce this deviation**




*R Hofstadter, Rev. Mod. Phys., 28:214–254, 1956*

# Elastic Electron-Proton Scattering (Rosenbluth fromula)

Dirac and Pauli form factors

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left[1.00-\frac{q^2}{4M^2}\left(2(1.00+\kappa)^2\tan^2(\theta/2)+\kappa^2\right)\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left[F_1^2-\frac{q^2}{4M^2}\left(2(F_1+F_2)^2\tan^2(\theta/2)+F_2^2\right)\right]$$

$F_1$ is introduced to take care of the **spread-out charge** and **spread-out Dirac magnetic moment**: Dirac Form Factor

$F_2$ is introduced to take care of the **spread-out Pauli magnetic moment**: Pauli Form Factor

- Electric $G_E$ and magnetic $G_M$ form factors:

$$G_E(q^2)=F_1(q^2)+\frac{q^2}{4M^2}F_2(q^2)$$

$$G_M(q^2)=F_1(q^2)+F_2(q^2)$$

$F_1$ and $F_2$ or ($G_E$ and $G_M$) are two independent phenomenological quantities

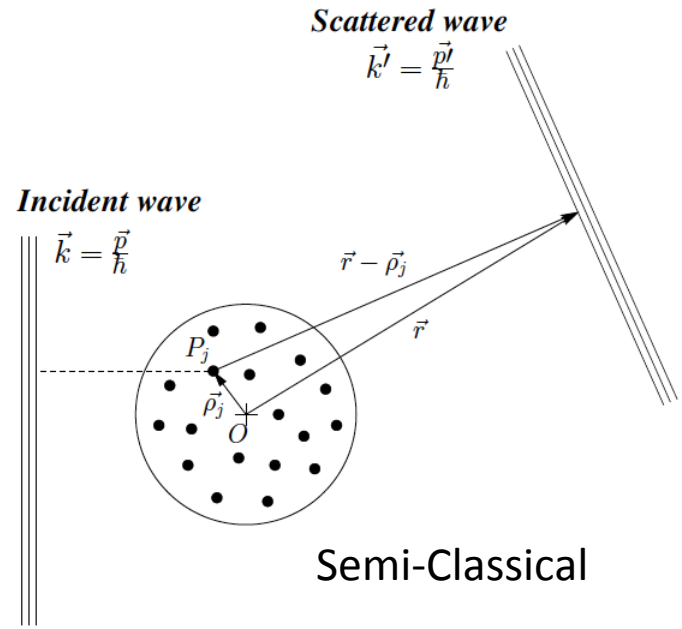
- Need to be measured experimentally

# Electron-Proton Elastic Scattering: the form factor

**Scattering cross section off finite size target (1943-1951)**

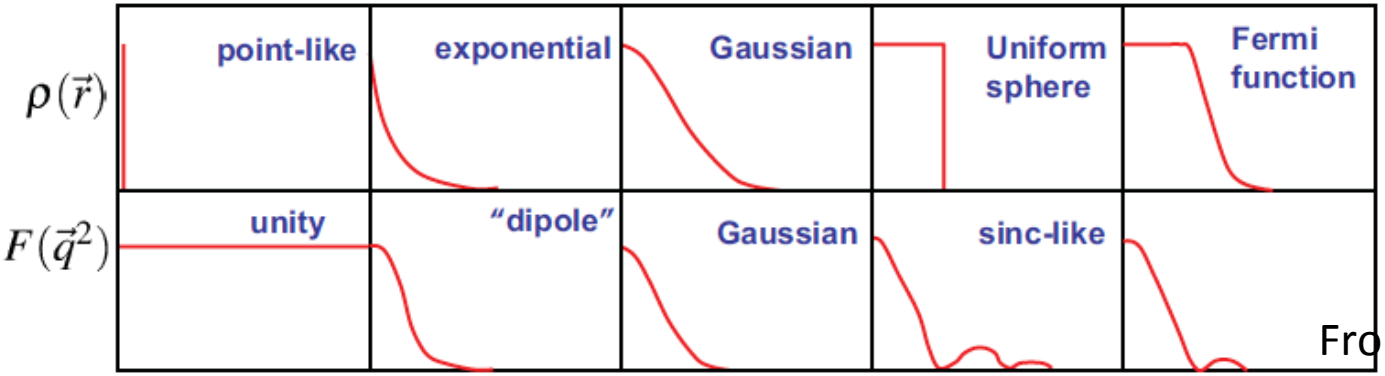
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{Mott}}{d\Omega}\left|\int \rho(\vec{x})e^{\vec{q}\cdot\vec{x}}d\vec{x}\right|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{Mott}}{d\Omega}|F(\vec{q}^{\,2})|^2$$



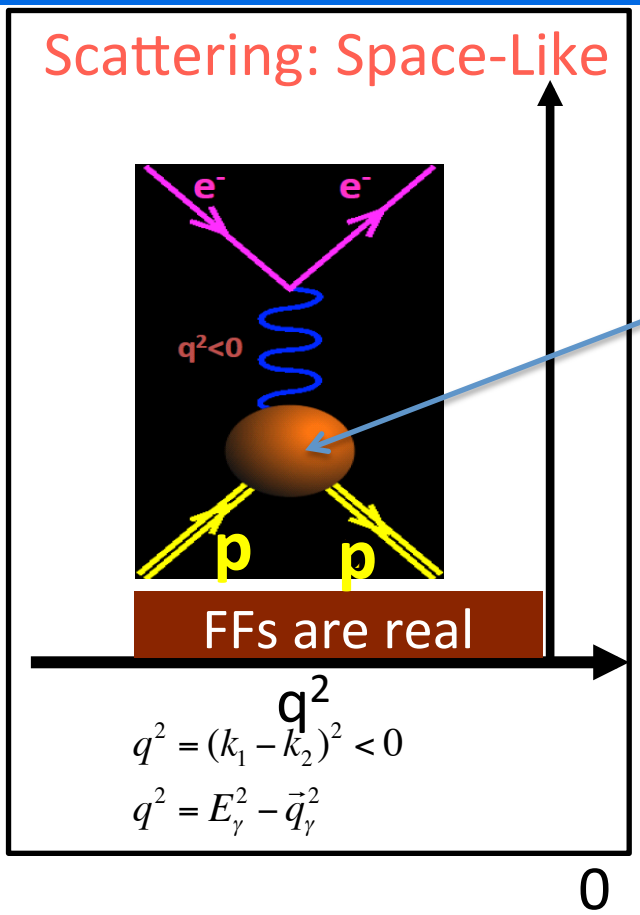

Semi-Classical

The resulting cross section is the cross section for scattering from a point source multiplied by the **form factor**




From E. Tomasi-Gustafsson

# Proton electromagnetic form factor




$$q^2 = (k_1 - k_2)^2 < 0$$

$$q^2 = E_\gamma^2 - \vec{q}_\gamma^{\,2}$$

Dirac and Pauli form factors:

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\gamma}{2M} F_2(q^2)$$

Sachs form factors:

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2), \quad G_E(0) = 1$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \quad G_M(0) = \mu_p$$

**(1951-1965) Rusenbluth formula**

- **Non point like proton**
- **Proton recoil is not neglected**
- **Magnetic moment of the proton is taking into account**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2} \frac{E_3}{E_1} \left( \frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

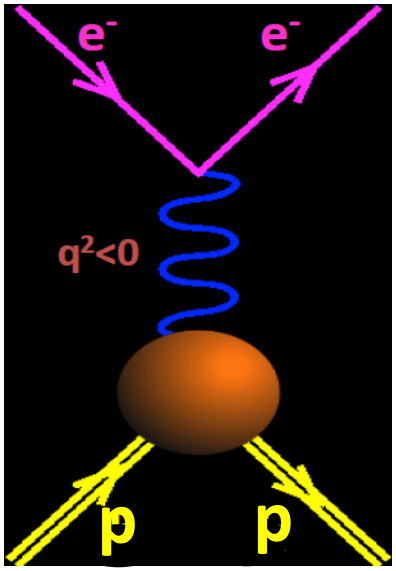
# Rosenbluth separation method

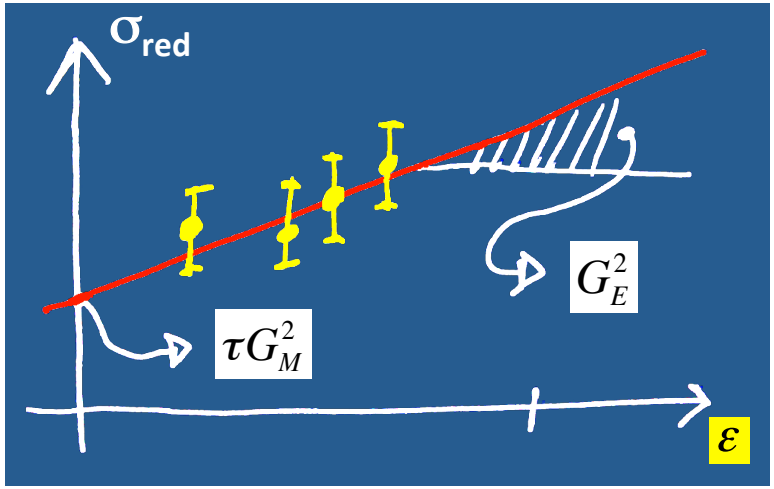
Unpolarized elastic ep scattering (Born approximation)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{Mott}}{d\Omega}\frac{1}{\varepsilon(1+\tau)}[\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)], \tau = Q^2/4M_p^2$$

$$\varepsilon = [1 + 2(1+\tau)\tan^2(\theta_e/2)]^{-1}$$

$$\sigma_{red} = \frac{d\sigma}{d\sigma_{Mott}}\varepsilon(1+\tau) = \varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)$$



# Rosenbluth separation method

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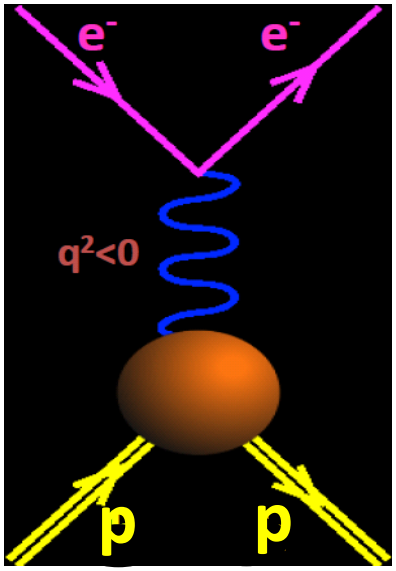
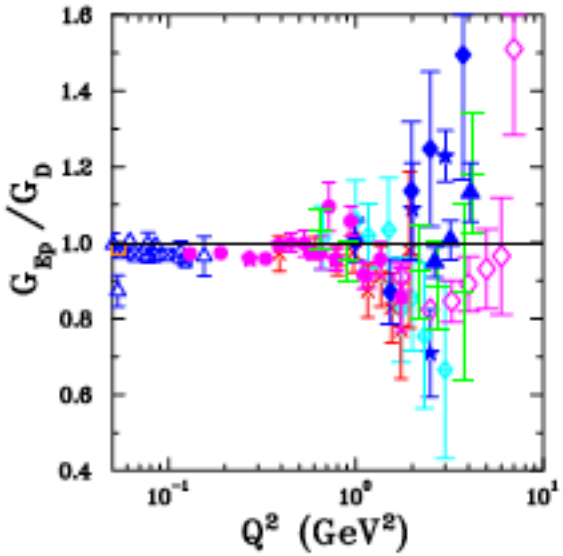
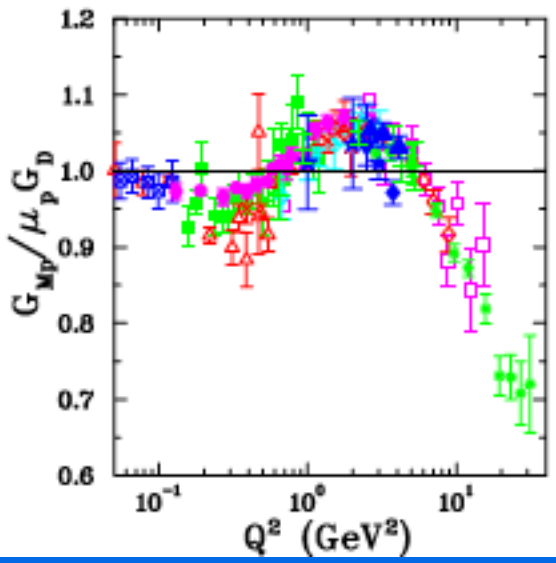



C. F. Perdrisat at al.
Prog. Part. Nucl. Phys. 59 (2007) 694

# Polarization method (1967)

SOVIET PHYSICS – DOKLADY VOL. 13, NO. 6 DECEMBER, 1968

PHYSICS

**POLARIZATION PHENOMENA IN ELECTRON SCATTERING BY PROTONS IN THE HIGH-ENERGY REGION**

Academician A. I. Akhiezer* and M. P. Rekalo

Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR

Translated from Doklady Akademii Nauk SSSR, Vol. 180, No. 5, pp. 1081–1083, June, 1968
Original article submitted February 26, 1967

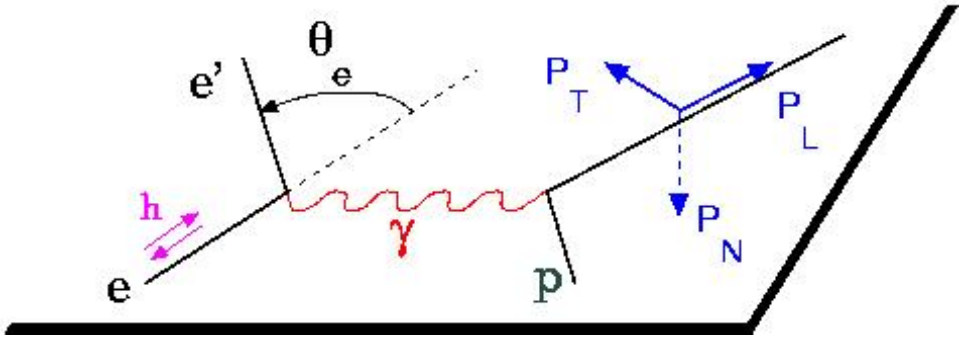
The polarization induces a term in the cross section proportional to $G_E G_M$

Polarized beam and target or

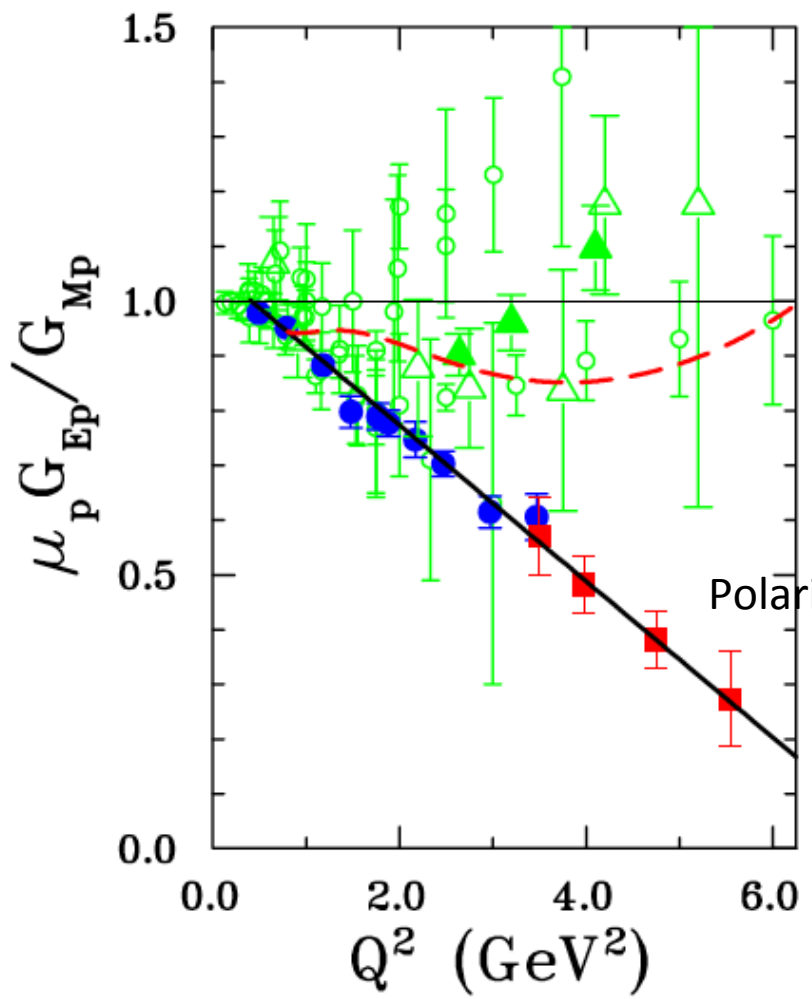
polarized beam and recoil proton polarization

GEp Collaboration at JLab

$$R = \frac{G_E}{G_M} = -\frac{P_t}{P_\ell}\frac{\epsilon_1 + \epsilon_2}{2M}\tan(\vartheta/2),$$



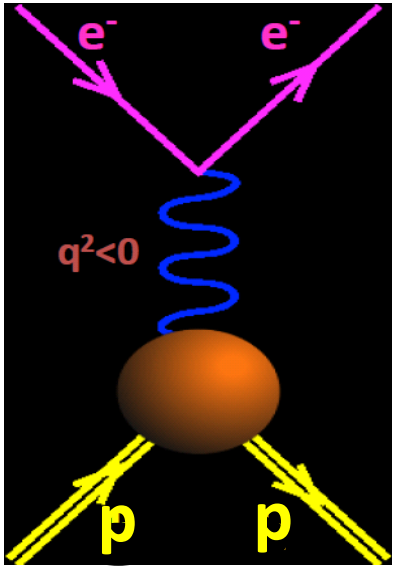
# JLab Polarization and Rosenbluth separation data




**Contradiction between polarized and unpolarized measurements**

Rosenbluth separation data

Polarization data




C. F. Perdrisat at al.
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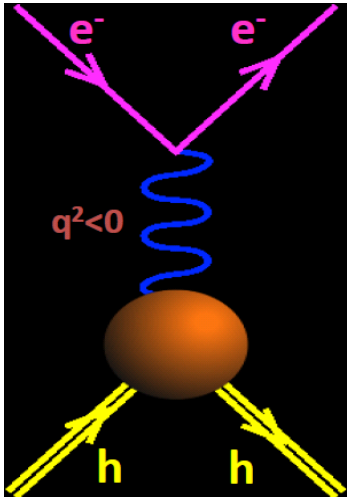
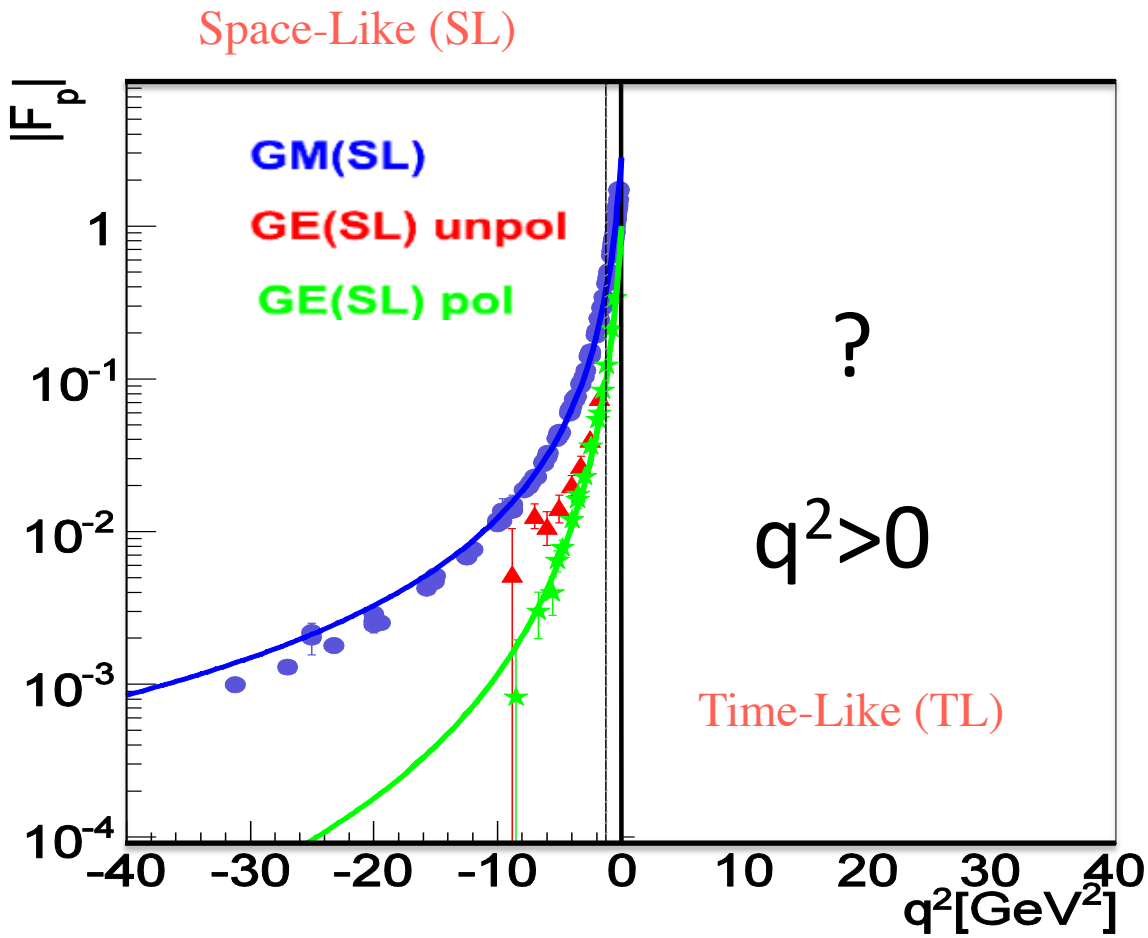
# Data on the proton electromagnetic FFs (SL)

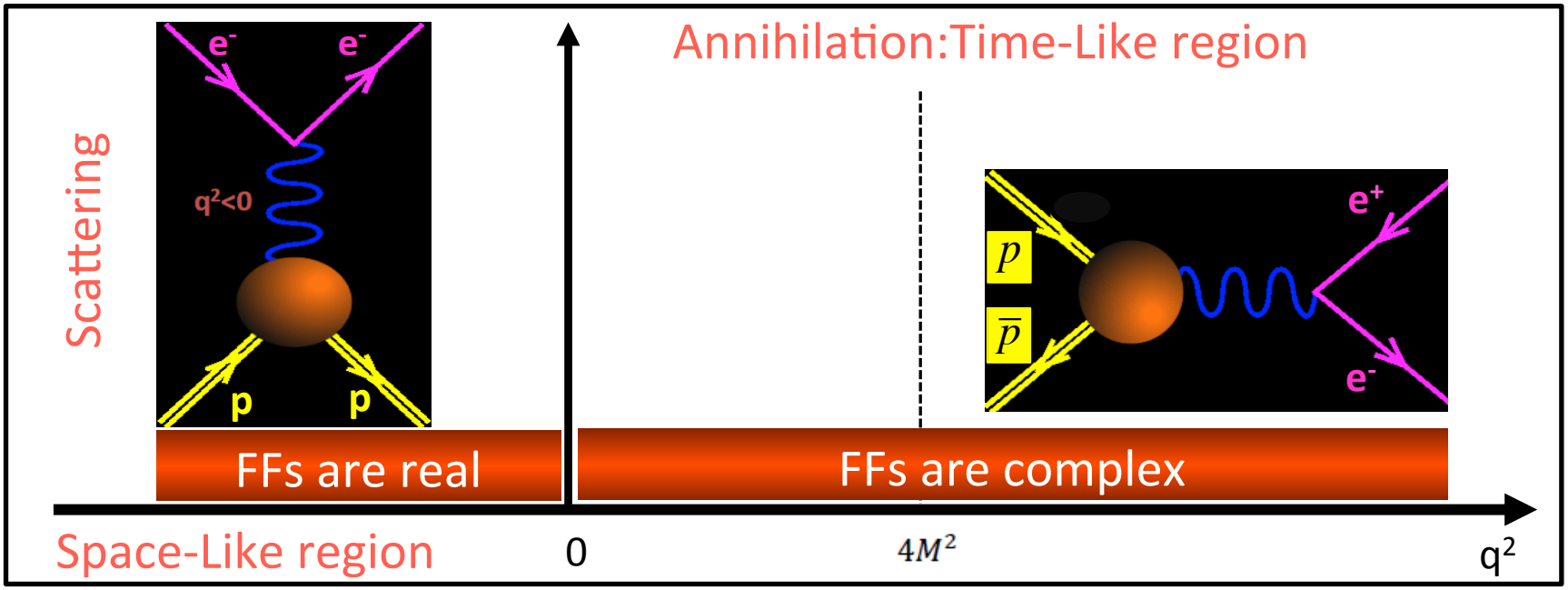
- **Electric $G_E$ and magnetic $G_M$** proton form factors are analytical functions of the momentum transfer squared $q^2$




$$q^2 = (k_1 - k_2)^2 < 0$$



# Electromagnetic Form Factors: the analyticity




At the threshold: $G_E(4M^2) = G_M(4M^2)$ (only s-wave)

Point-like proton: $G_E(4M^2) = G_M(4M^2) = 1$

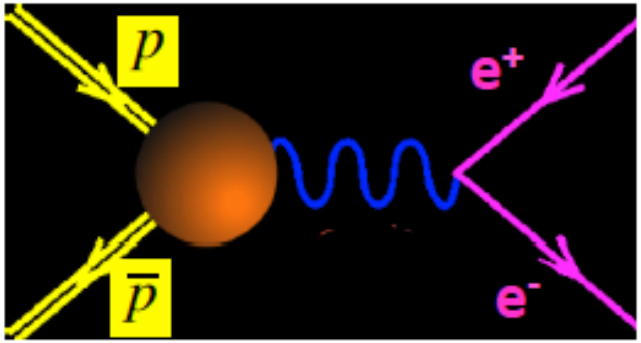
Unified frame for the description of FFs:

$$G(q^2) = \frac{1}{\pi}\left[\int_{4m_\pi^2}^{4m_p^2} \frac{\mathrm{Im}\,G(s)ds}{s-q^2} + \int_{4m_p^2}^{\infty} \frac{\mathrm{Im}\,G(s)ds}{s-q^2}\right]$$

$$\lim_{q^2\to-\infty} G_{E,M}^{SL}(q^2) = \lim_{q^2\to+\infty} G_{E,M}^{TL}(q^2)$$

The measurement of the From Factors at large $q^2$ and in all the kinematical region: test of the analytical nature of the FFs

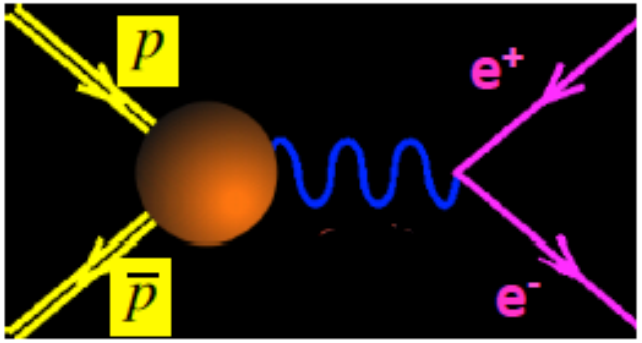
# Feasibility studies




- Identification of the signal and suppression of the background processes
- Determination of the statistical uncertainty on the proton form factors

# Measurements of proton form factors at PANDA

## Main Background processes




- Production of two charged particles

$$\bar{p}p \rightarrow \mu^+\mu^-$$

$$\bar{p}p \rightarrow \pi^+\pi^-$$

$$\bar{p}p \rightarrow K^+K^-$$

$$\bar{p}p \rightarrow \bar{p}p$$

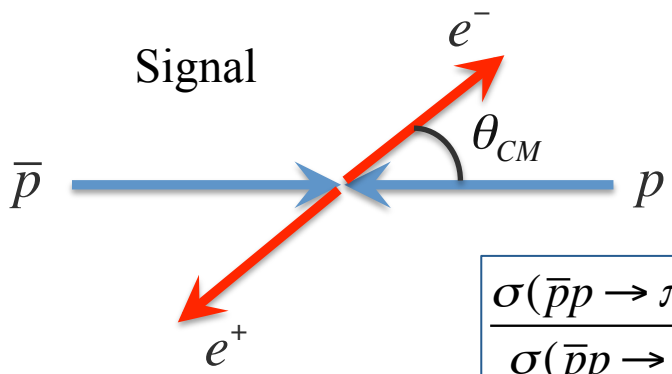
- Production of two charged particles + neutral/charged particle, i.e.:

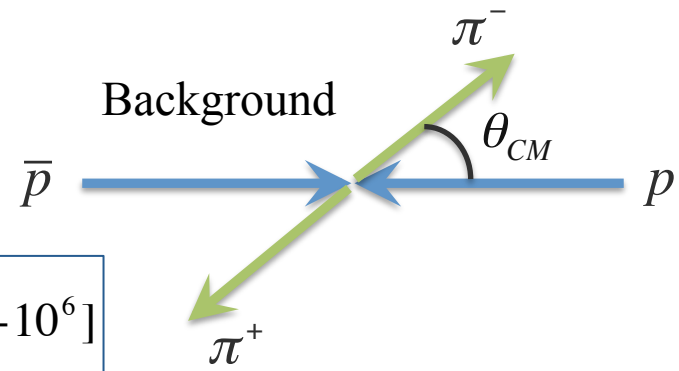
$$\bar{p}p \rightarrow \pi^+\pi^-\pi^0$$

$$\bar{p}p \rightarrow K^+K^-\pi^0$$

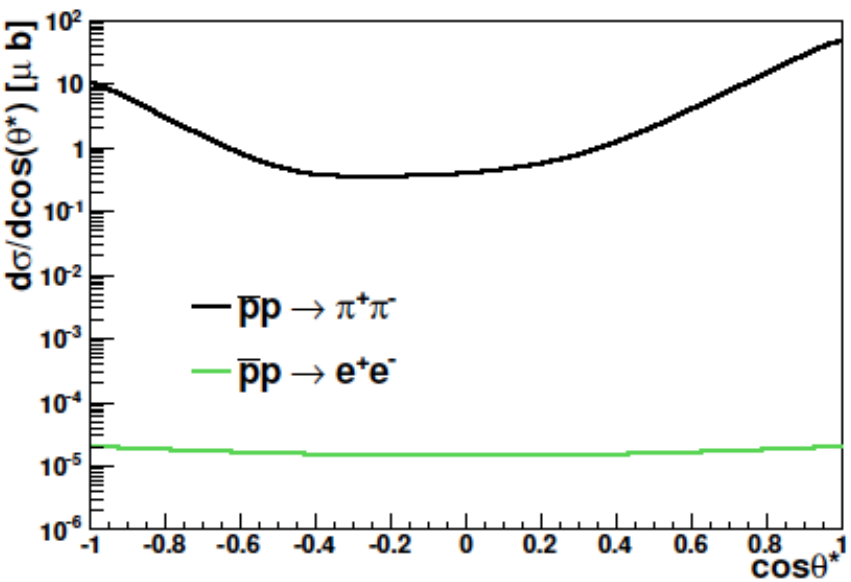
# Feasibility studies: TL proton FFs @ PANDA

- Main issue: signal identification from the huge hadronic background
- The main background is: $\bar{p}p \rightarrow \pi^+\pi^-$




$$\frac{\sigma(\bar{p}p \rightarrow \pi^+\pi^-)}{\sigma(\bar{p}p \rightarrow l^+l^-)} \propto [10^5 - 10^6]$$




- For one event of the signal we have $10^6$ events of the background
- Bakground Contamination:

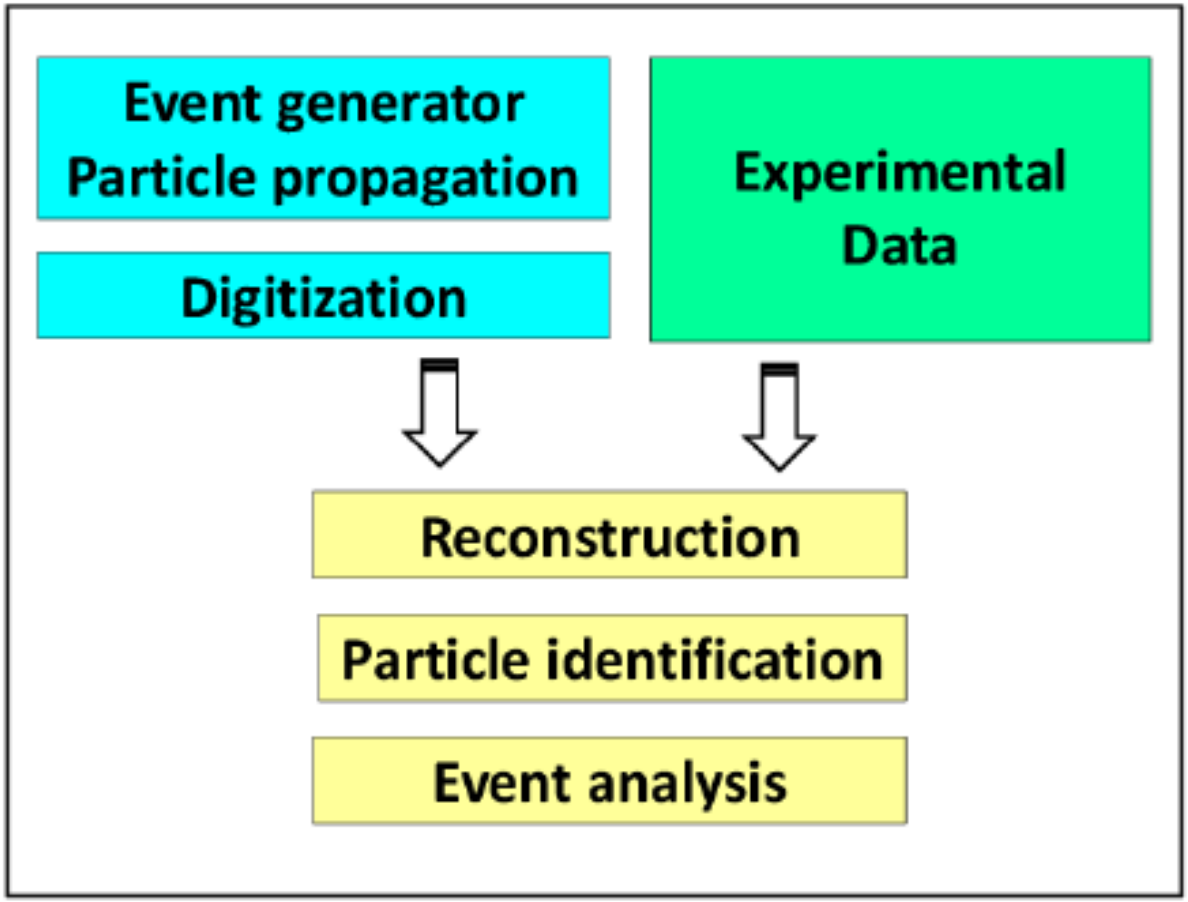
$$C = \frac{N_B \cdot \varepsilon_B}{N_S \cdot \varepsilon_S + N_B \cdot \varepsilon_B} \cong 10^6 \varepsilon_B = 0.01, \varepsilon_B = 10^{-8}$$

$10^8$ rejection factor of the background is needed

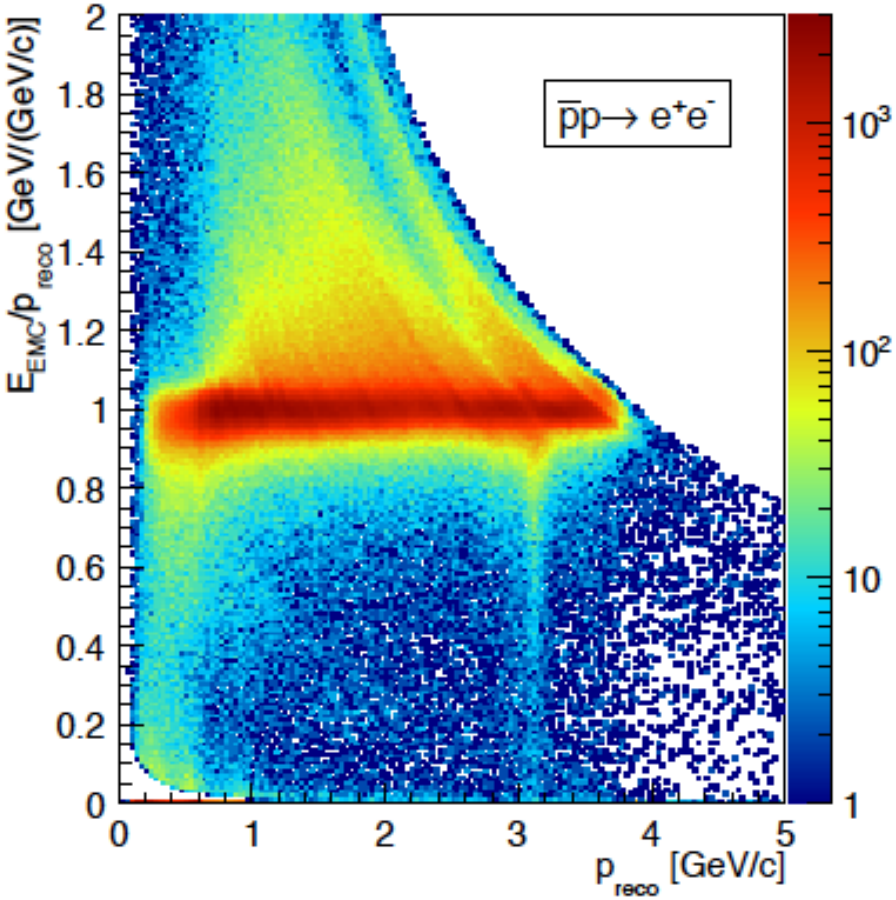
# Analysis chain in PANDARoot

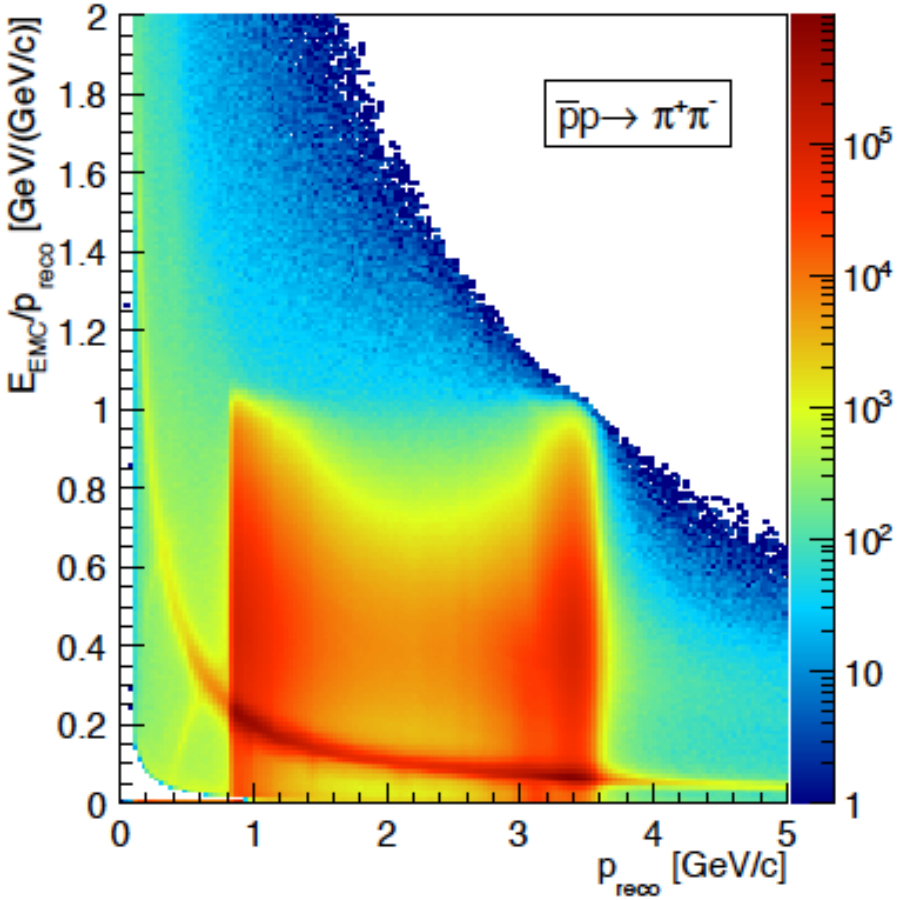
$$\bar{p}p \rightarrow e^{+}e^{-}$$

$$\bar{p}p \rightarrow \pi^{+}\pi^{-}$$



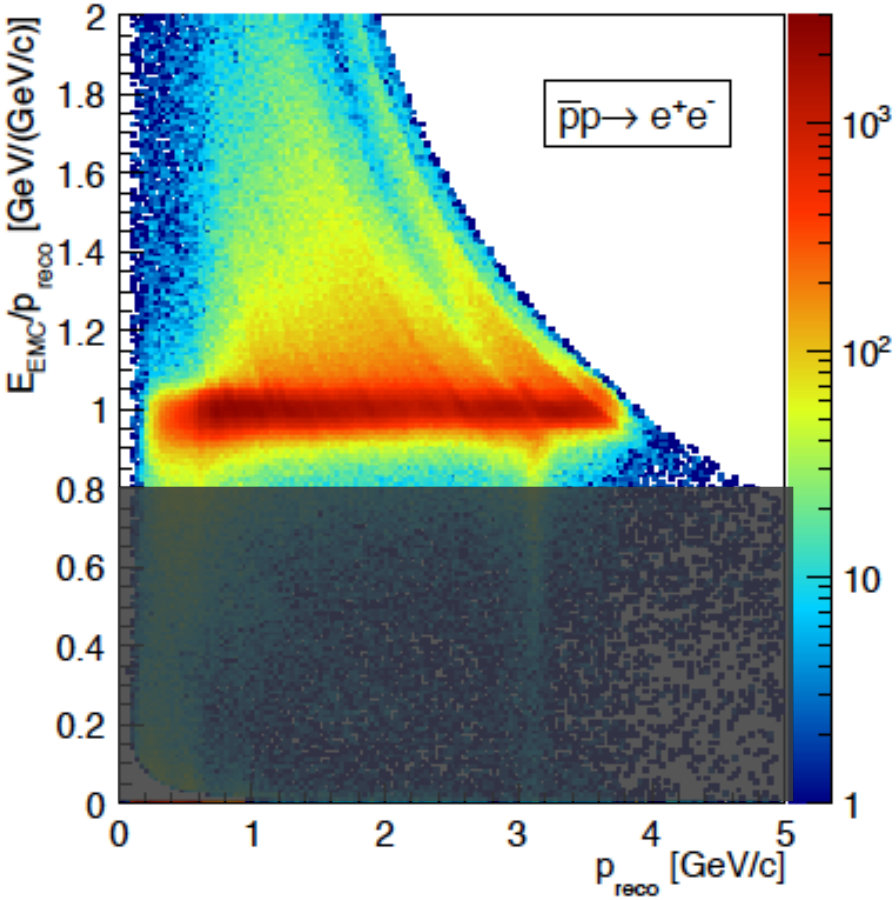
# How to identify the signal from the background?

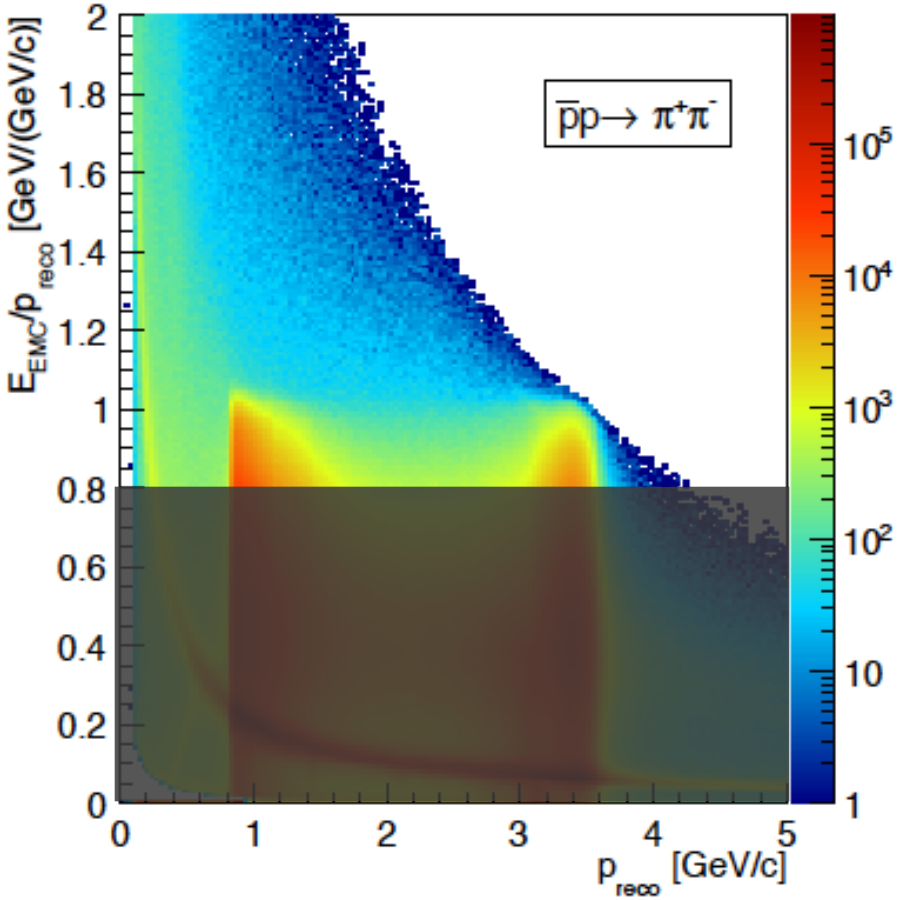



Energy deposited in the EMC over the tracking momentum: $E_{EMC}/p_{rec}$

# How to identify the signal from the background?

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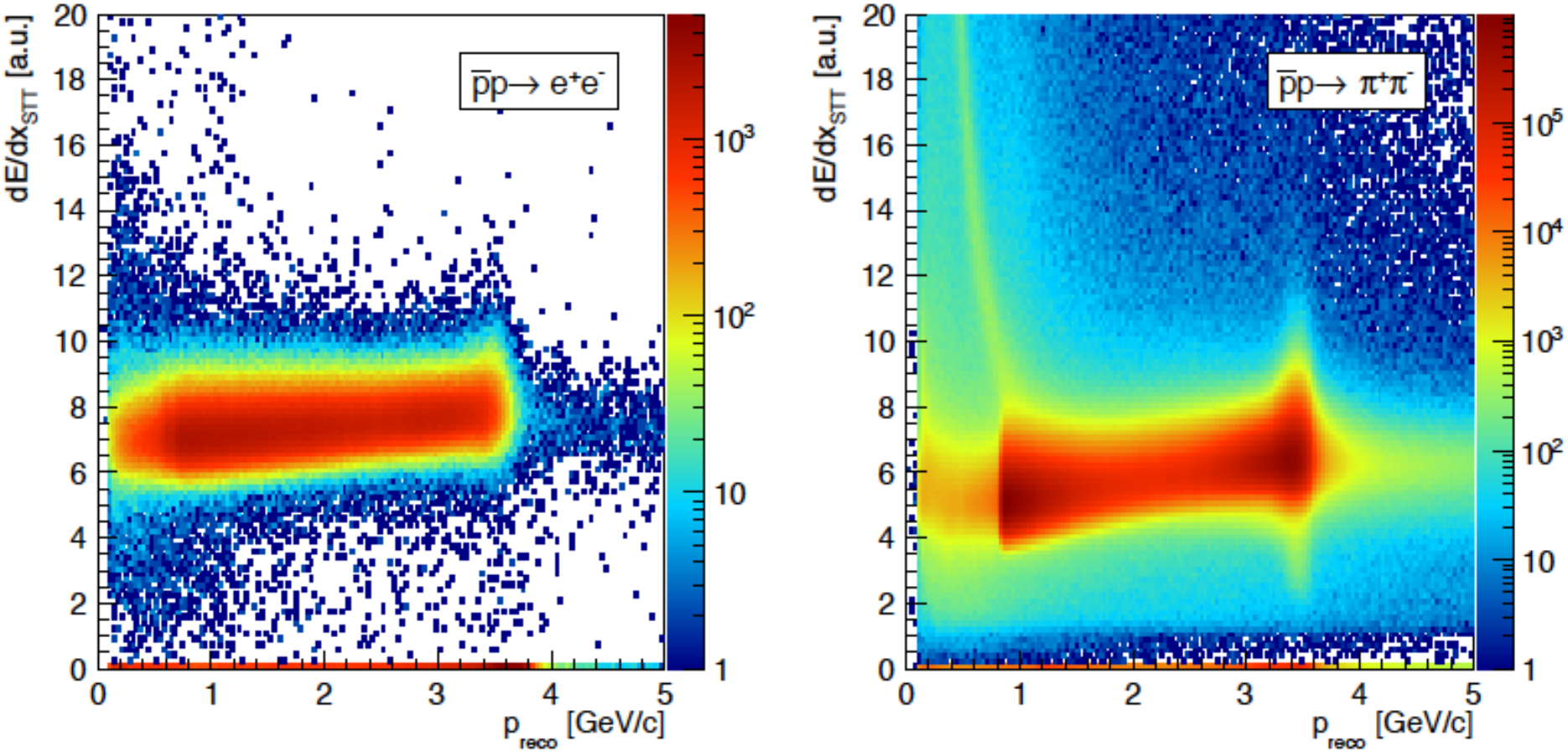
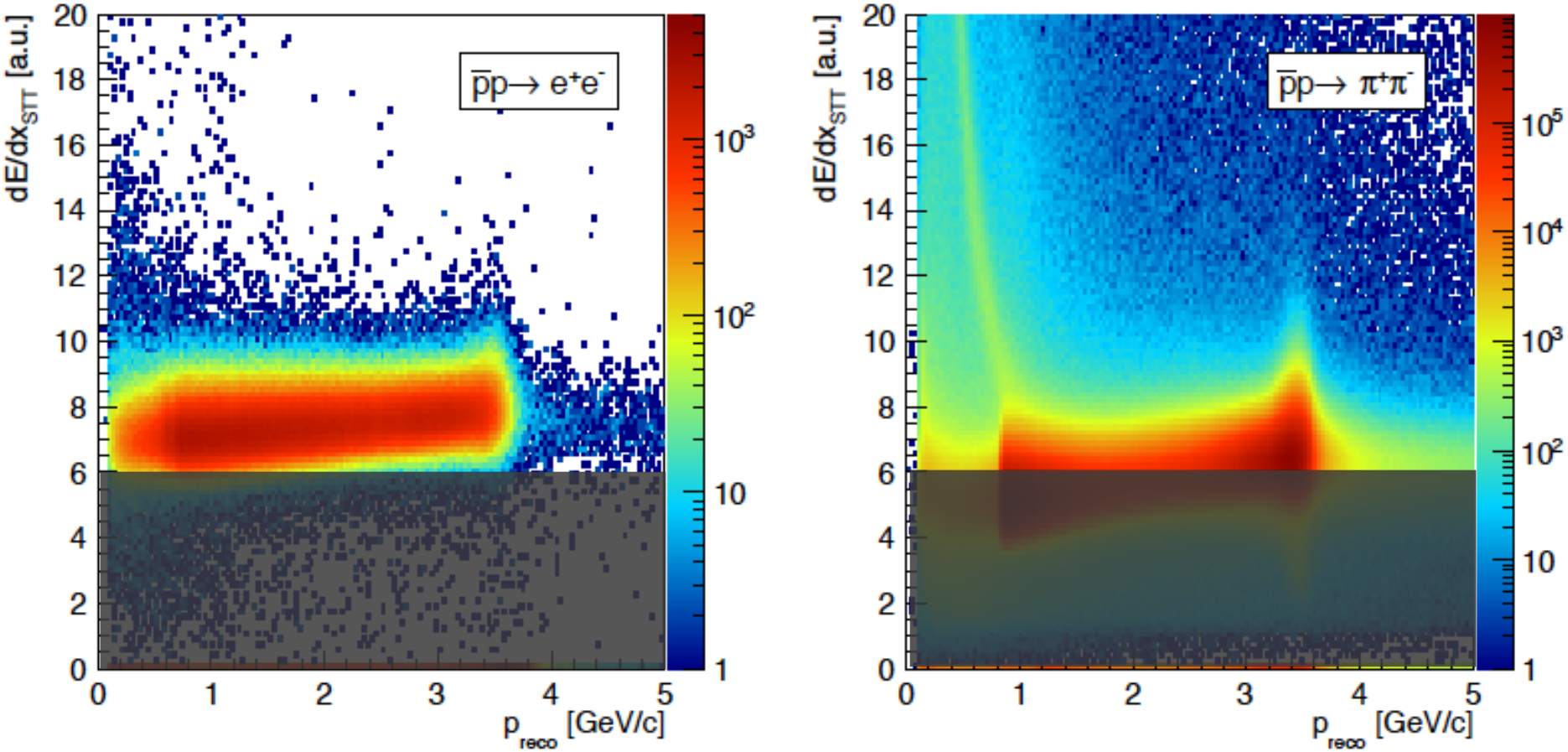


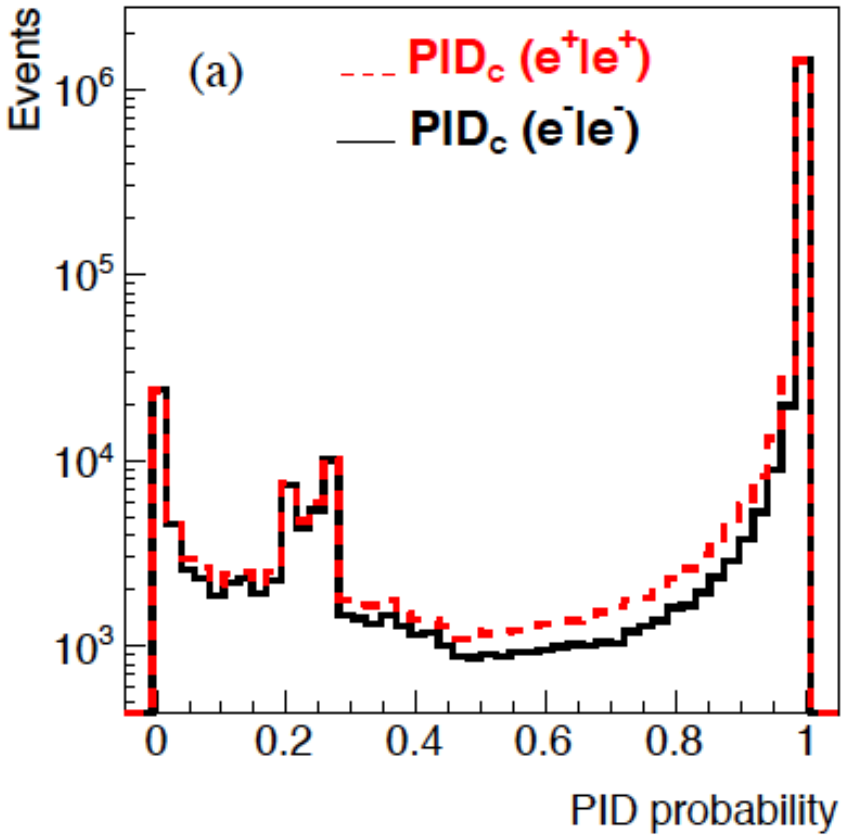

Energy loss dE/dx in the straw tube tracker STT

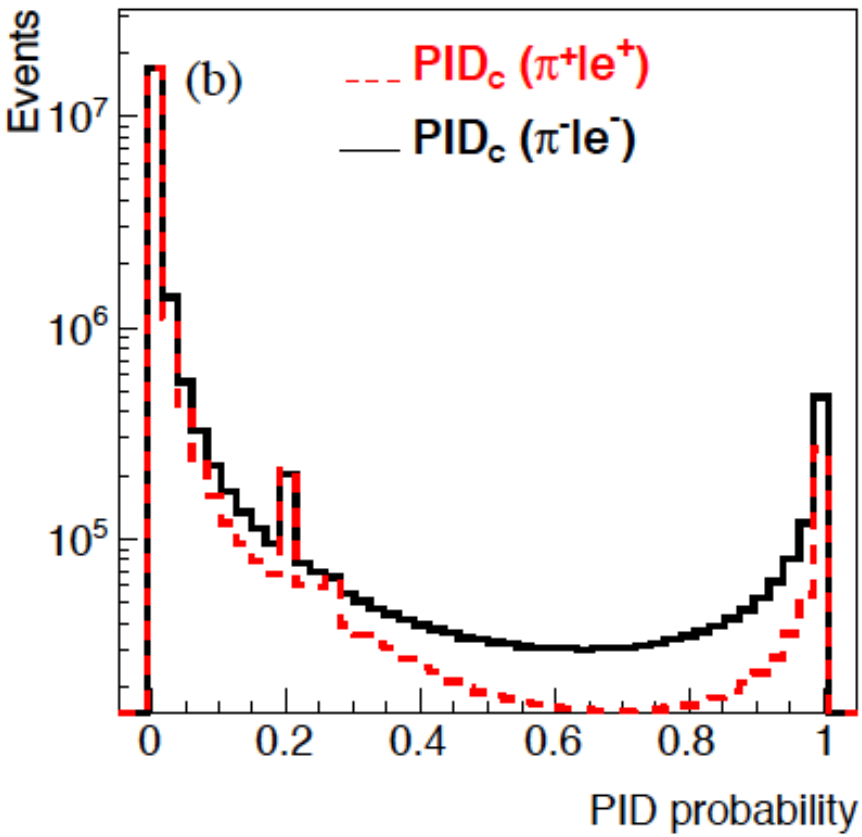
# How to identify the signal from the background?




Energy loss dE/dx in the straw tube tracker STT

# How to identify the signal from the background?

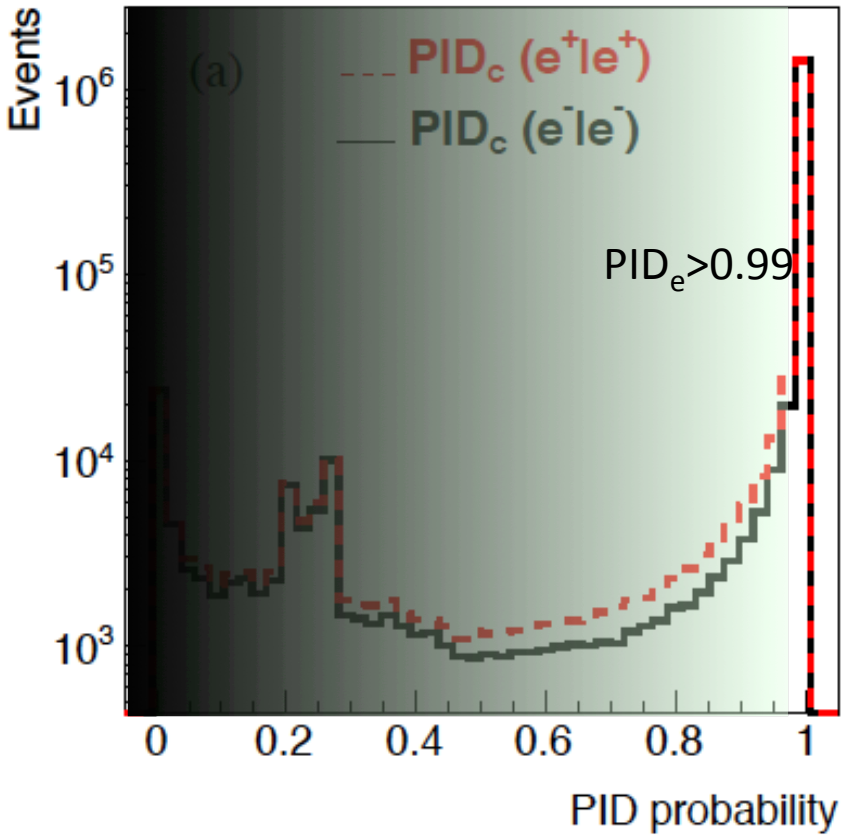


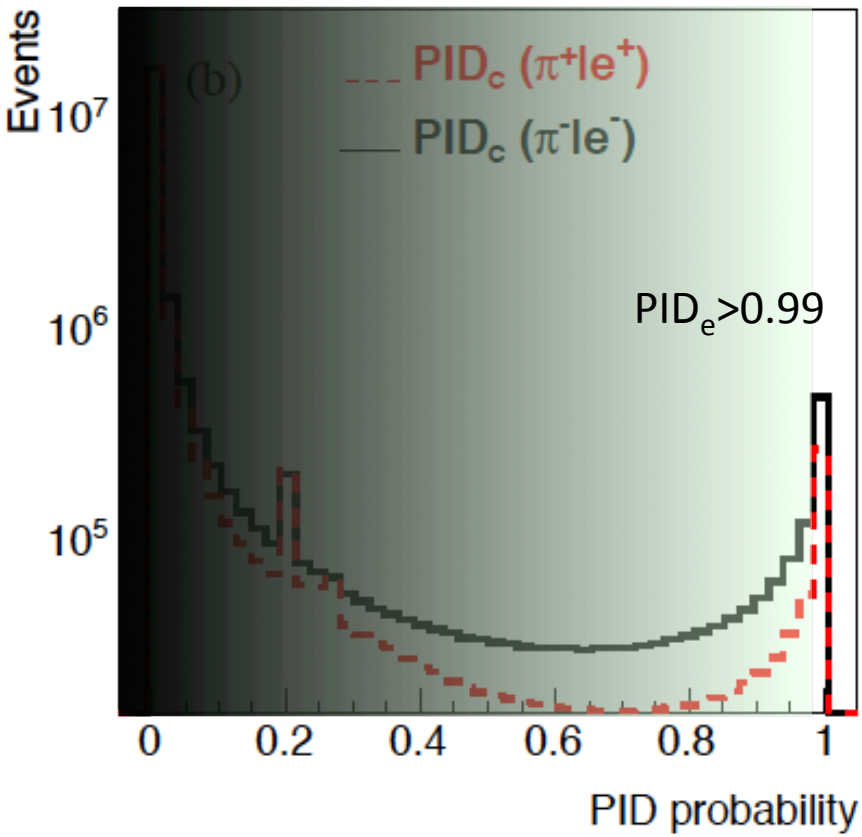




PID variables from the EMC, STT, DIRC, MVD → PID probability

What is the probability for the detected particle to be an electron/positron?

# How to identify the signal from the background?

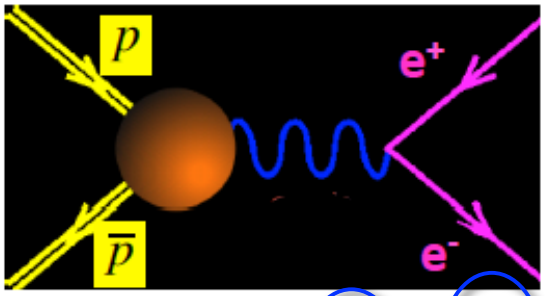



- **Background rejection ~ $10^{-8}$**
  **Signal pollution < 1%**

Clean signal → extraction of the proton form factors from the cross section

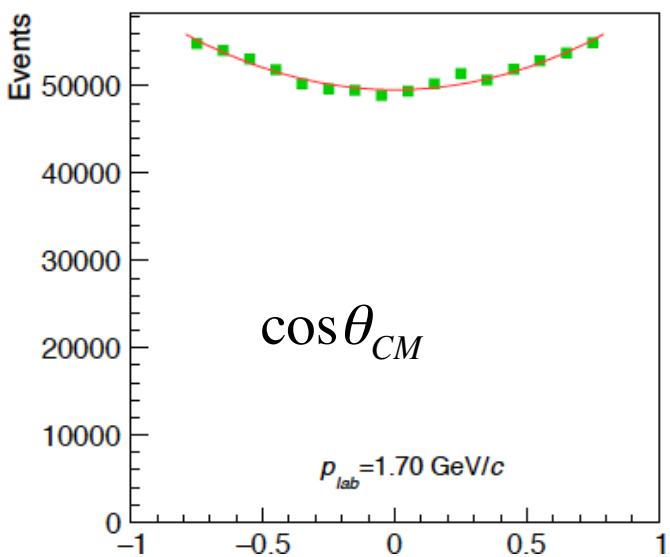
# Measurement of proton FFs in the TL region




$$\frac{d\sigma}{d\cos\theta_{CM}} \propto Norm \times \left[ (1+\cos^2\theta_{CM})|G_M|^2 + \frac{|G_E|^2}{\tau}(1-\cos^2\theta_{CM}) \right]$$

$$R = \frac{|G_E|}{|G_M|}$$

$$\frac{d\sigma}{d\cos\theta_{CM}} \propto Norm \times |G_M|^2 \left[ (1+\cos^2\theta_{CM}) + \frac{R^2}{\tau}(1-\cos^2\theta_{CM}) \right]$$




- Fit to the electron/positron angular distribution in CM

$$y = a\left[(1+\cos^2\theta_{CM}) + b(1-\cos^2\theta_{CM})\right]$$

b → R=|$G_E$|/|$G_M$|

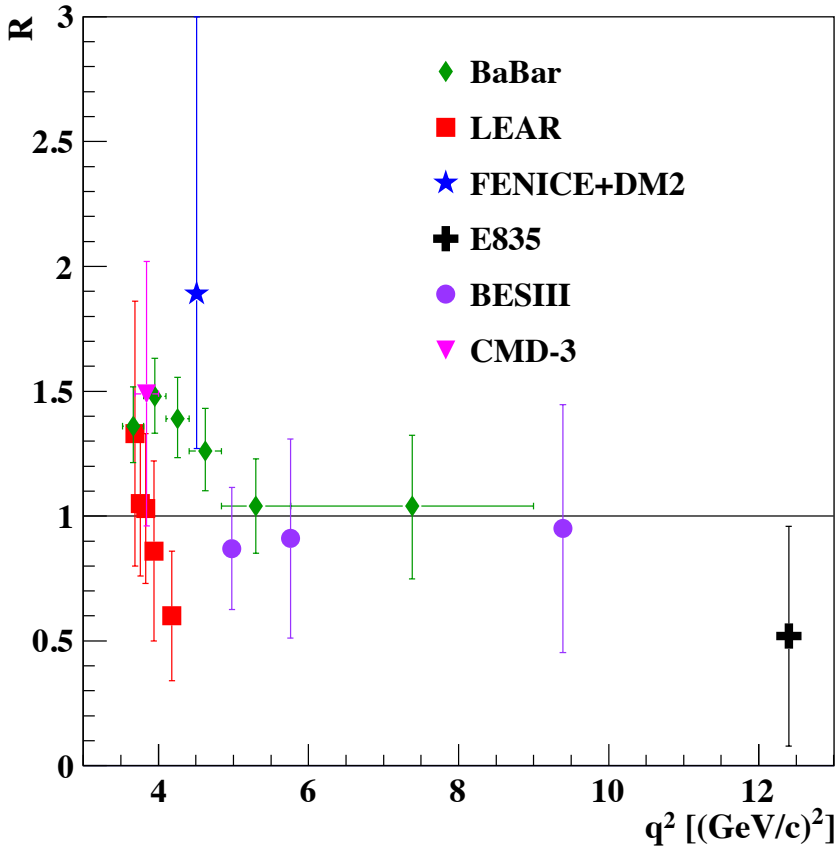
- Norm and luminosity are known:

$$a \rightarrow Norm \times |G_M|$$

|$G_E$| &|$G_M$|

- Low statistics → |$G_E$|=|$G_M$|: $|F_p|^2 \propto \sigma_{tot}$

# World data on TL proton form factor ratio




@ BaBar (SLAC): $e^+e^- \rightarrow \bar{p}p\gamma$

- data collection over wide energy range

@ PS 170 (LEAR): $\bar{p}p \rightarrow e^+e^-$

- data collection at low energies

Data from BaBar & LEAR show different trends

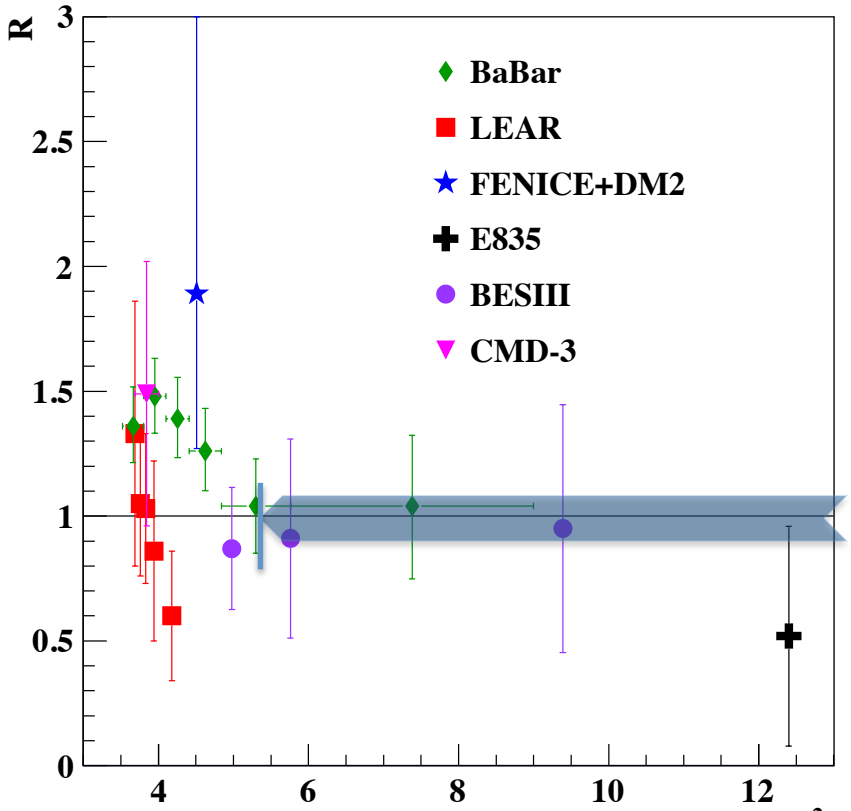
@ BESIII: $e^+e^- \rightarrow \bar{p}p$

- Measurement at different energies
- Uncertainties comparable to previous experiments

@ CMD-3 (VEPP2000 collider, BINP):

- Energy scan $\sqrt{s} = 1 - 2\ GeV$
- Uncertaincy comparable to the measurement by BaBar

# World data on TL proton form factor ratio




@ BaBar (SLAC): $e^+e^- \rightarrow \bar{p}p\gamma$

- data collection over wide energy range

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- Measurement at different energies
- Uncertainties comparable to previous experiments

@ CMD-3 (VEPP2000 collider, BINP):

Energy scan $\sqrt{s} = 1 - 2\ GeV$

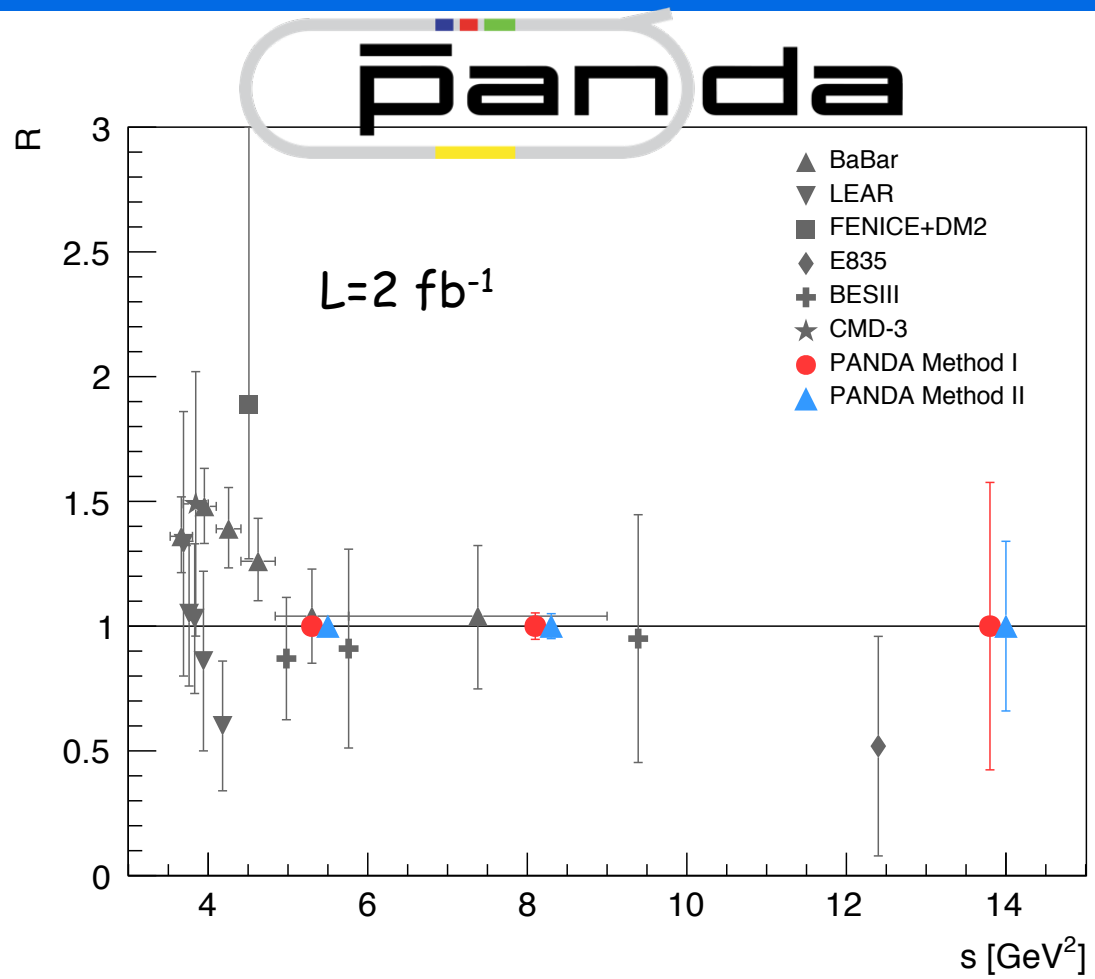
Uncertainc

measurem

**PANDA**: Measurement over wide range of $q^2$ with high precision

**Test of theoretical models & predictions!**

# Proton form factor measurements at PANDA




Measurement of proton FFs with unprecedented accuracy in $e^+e^-$ final state

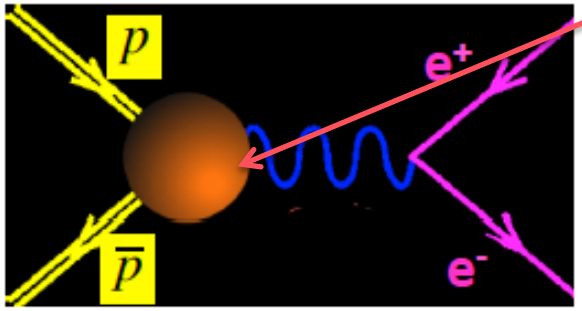
- E.W. Singh et al.: EPJA52, 325 (2016)

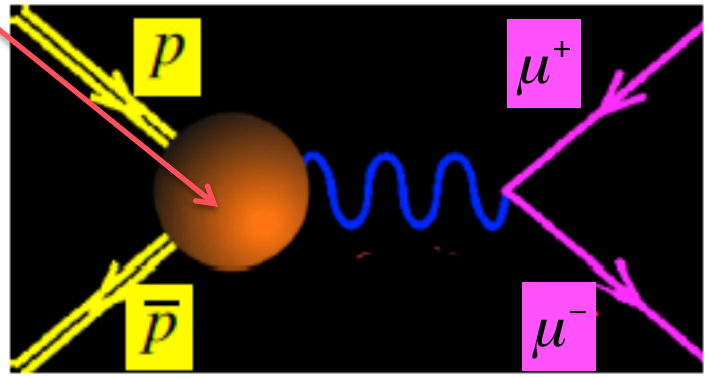
# Measurements of proton FFs with muons

$\bar{p}p \rightarrow e^+e^-$

**$G_E$ and $G_M$**

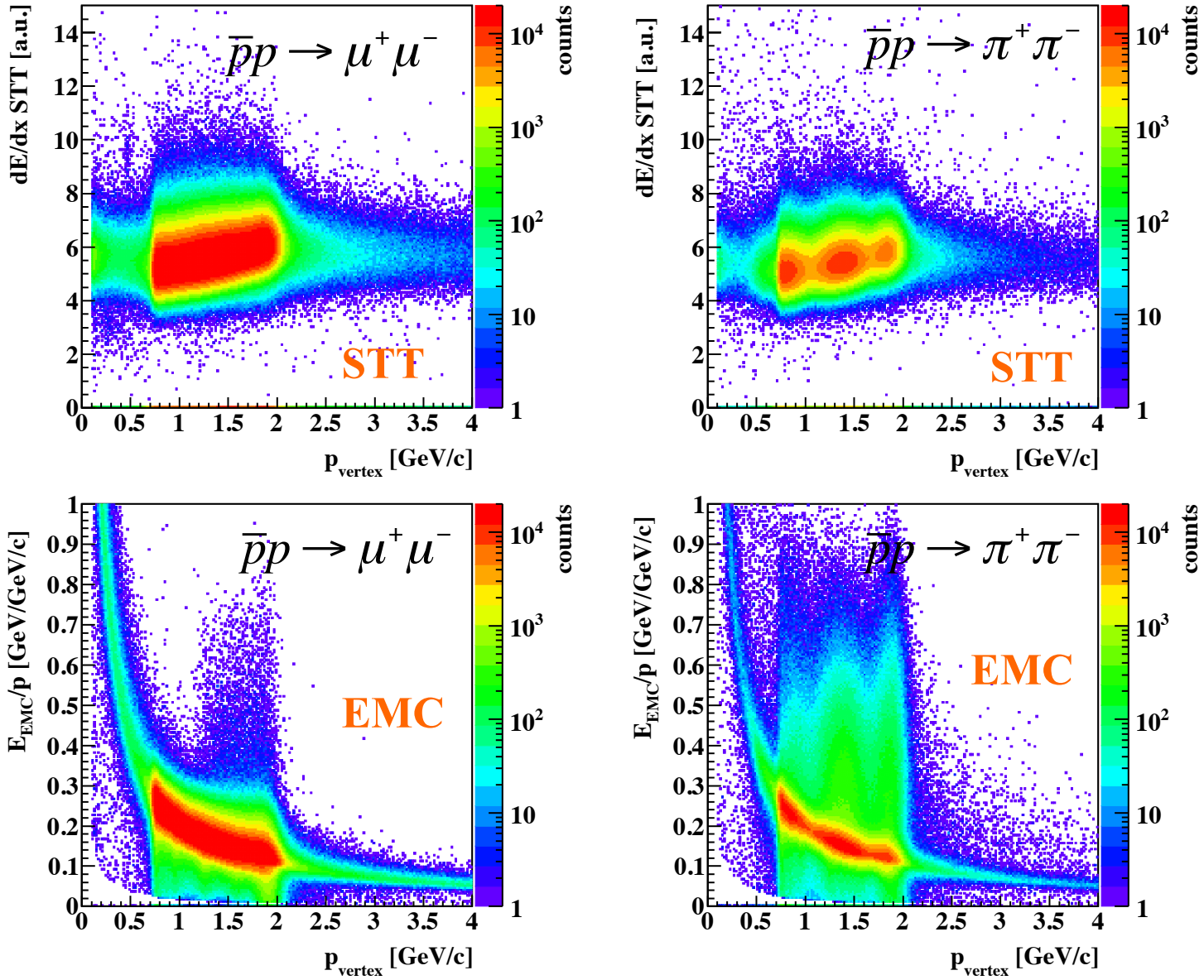
$\bar{p}p \rightarrow \mu^+\mu^-$



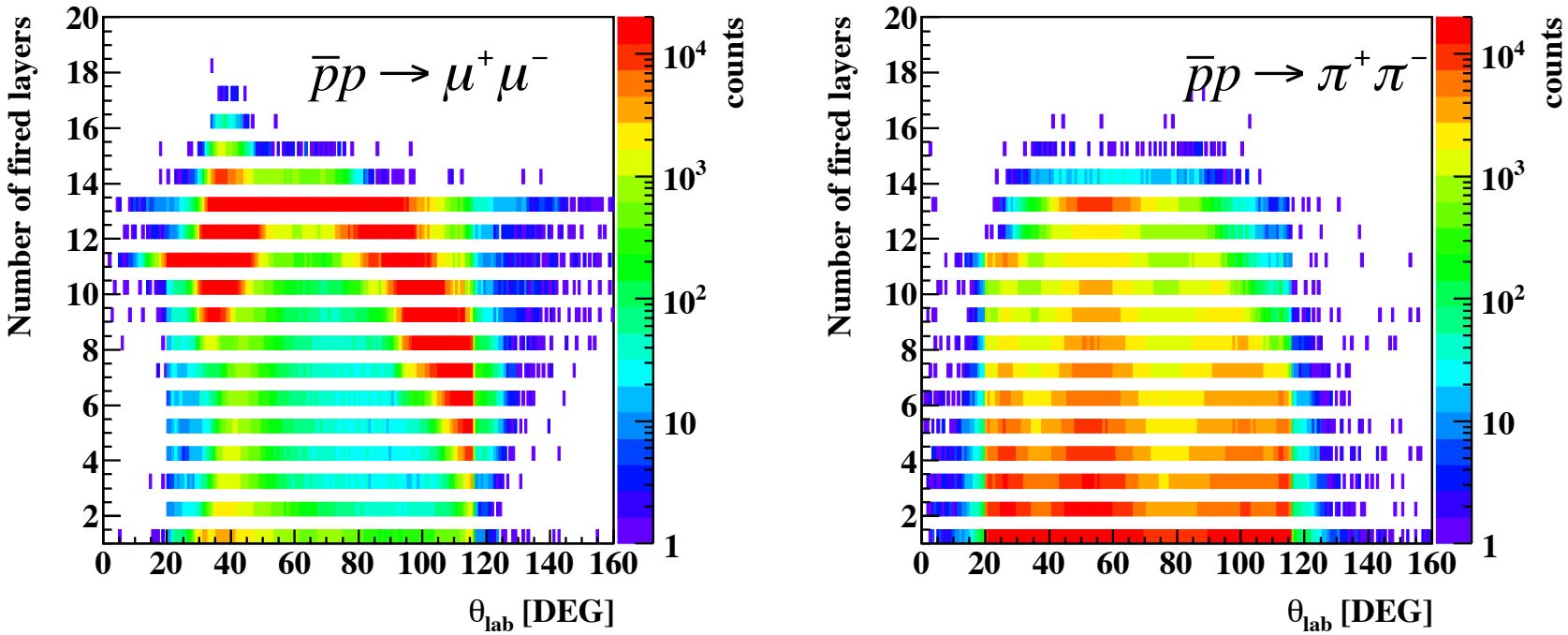

- **First time** measurement with **muons in final state**
- **Study of radiative corrections**
- **Consistency check** of proton form factor data
- **Test of lepton universality**

# Measurements of proton FFs at PANDA with muons



# Measurements of proton FFs at PANDA with muons

Number of fiered layers in the muon system



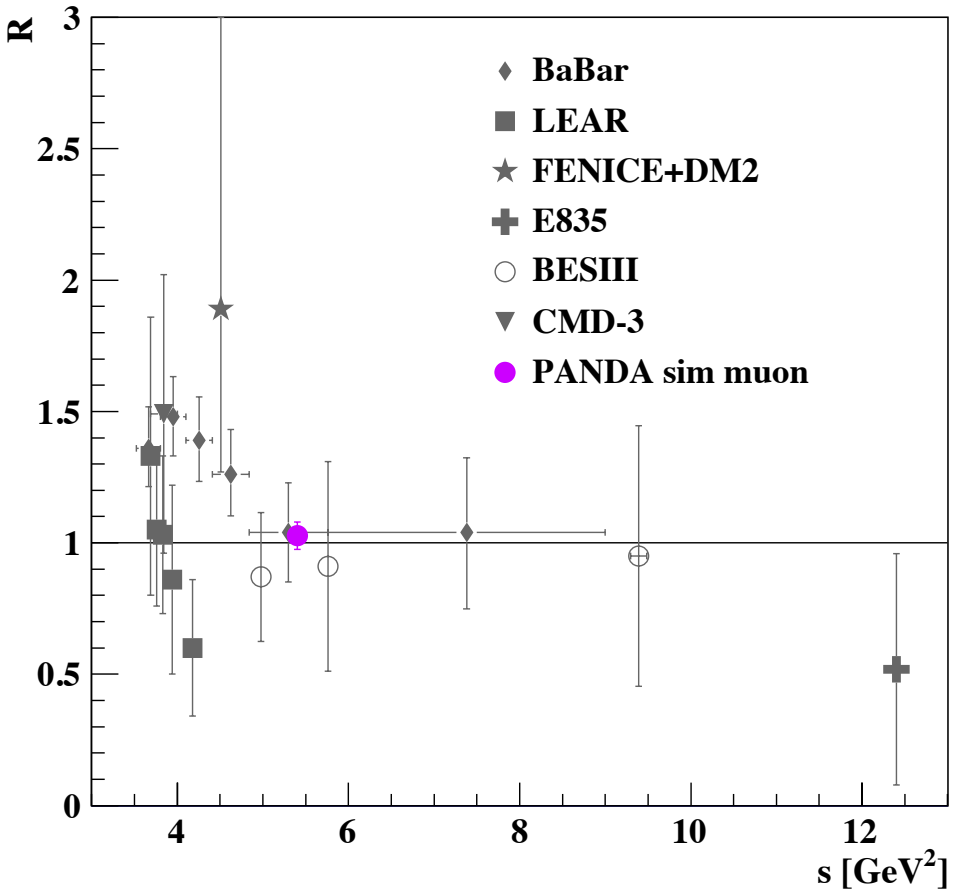

- **Background rejection factor ~ $10^{-6}$**

Background subtraction is needed:

$N_{signal}=N_{data}-N_{background}$, $(\Delta N_{signal})^2=(\Delta N_{data})^2+(\Delta N_{background})^2$

# Measurements of proton FFs at PANDA with muons

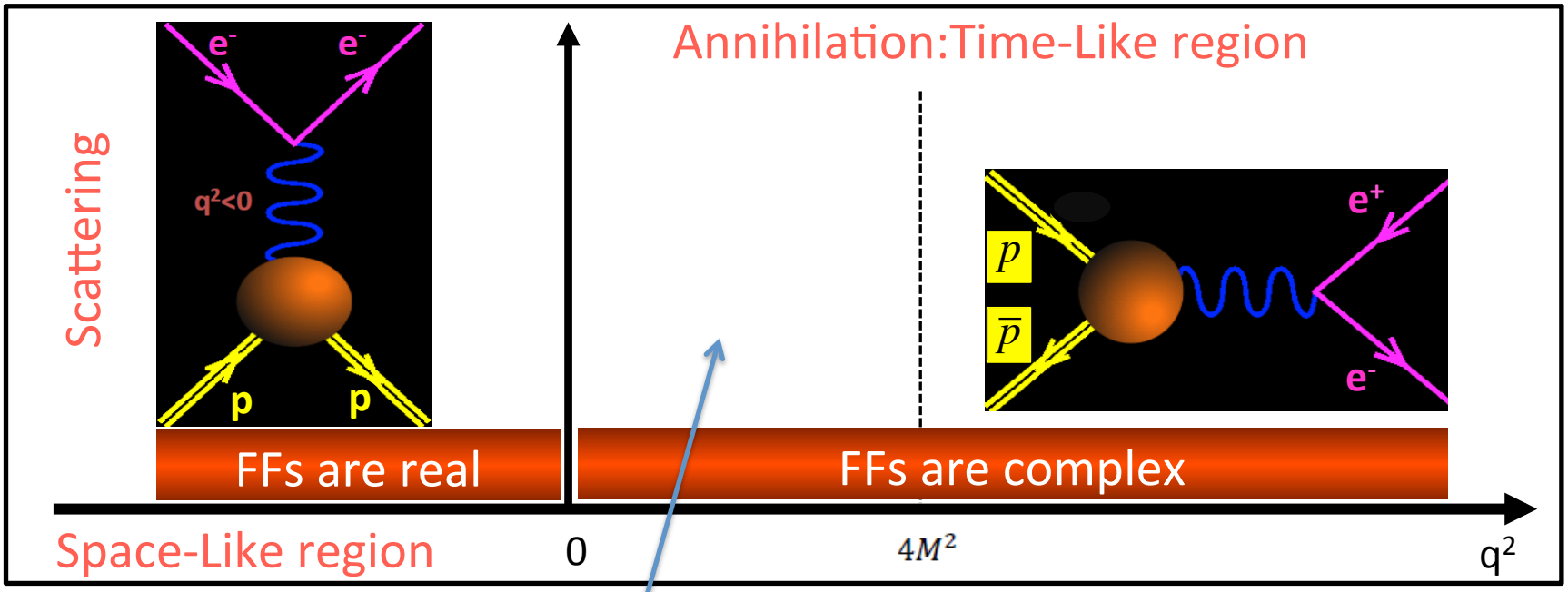
$$\bar{p}p \rightarrow \mu^+\mu^-$$




| Precision on R, $\lvert G_E \rvert$ & $\lvert G_M \rvert$ | |
|---|---|
| **p [GeV/c]** | **1.7** |
| $\Delta R/R$ | 5.1% |
| $\Delta\lvert G_E\rvert/\lvert G_E\rvert$ | 8.6% |
| $\Delta\lvert G_M\rvert/\lvert G_M\rvert$ | 4.1% |

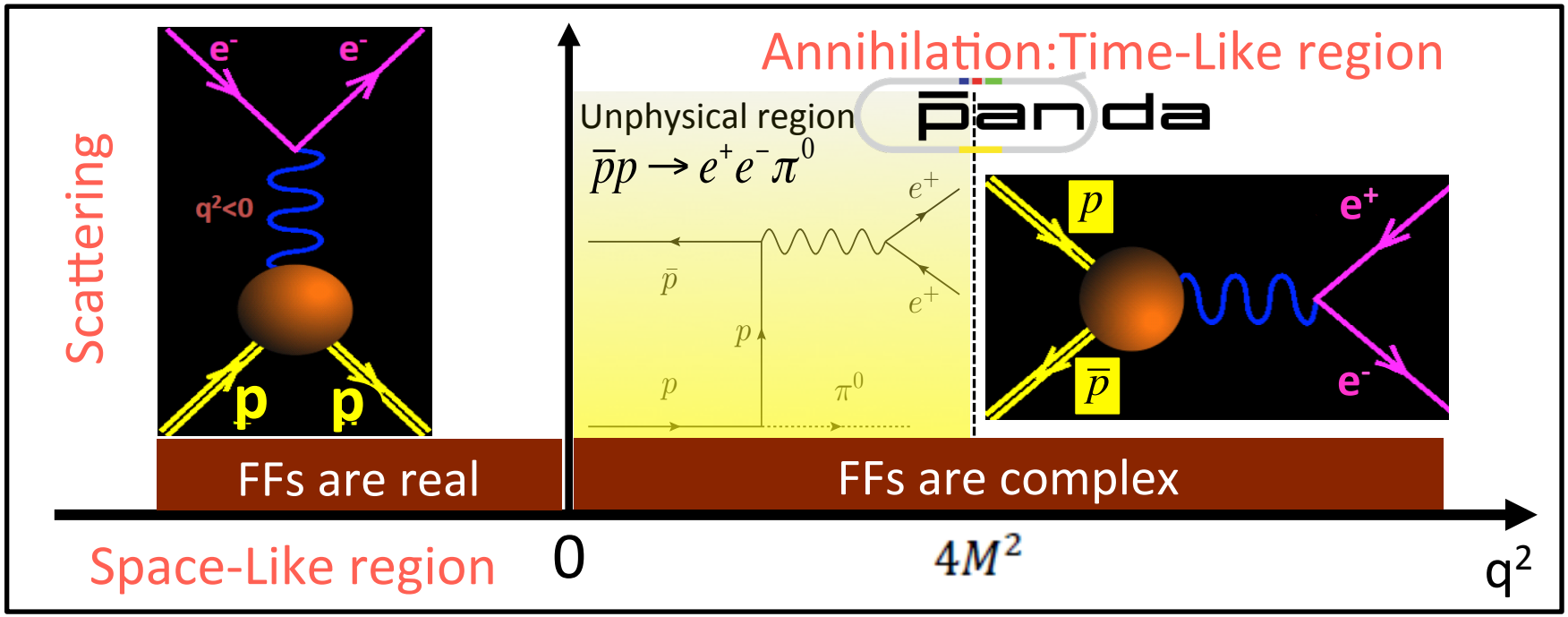
-PRELIMINARY-

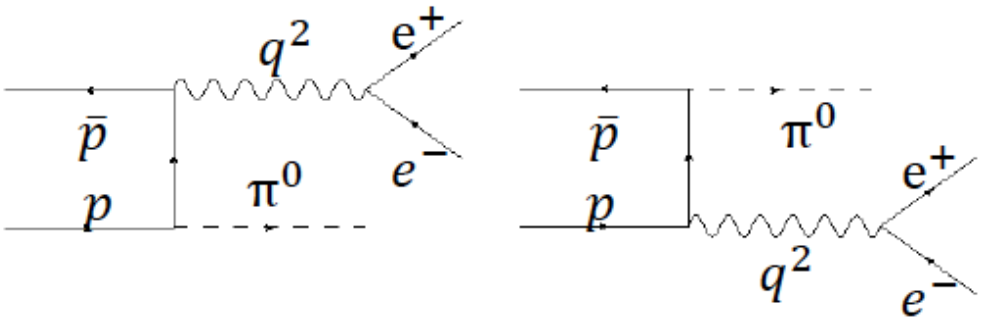
# Electromagnetic Form Factors: the analyticity




How to access the unphysical region?

# Electromagnetic Form Factors: the analyticity




- M.P. Rekalo. Sov. J. Nucl. Phys., 1:760, 1965
- Adamuscin, Kuraev, Tomasi-Gustafsson and F. Maas, Phys. Rev. C 75, 045205 (2007)
- C. Adamuscin, E.A. Kuraev, G. I. Gakh, …
- Feasibility studies (J. Boucher, M. C. Mora-Espi PhD)

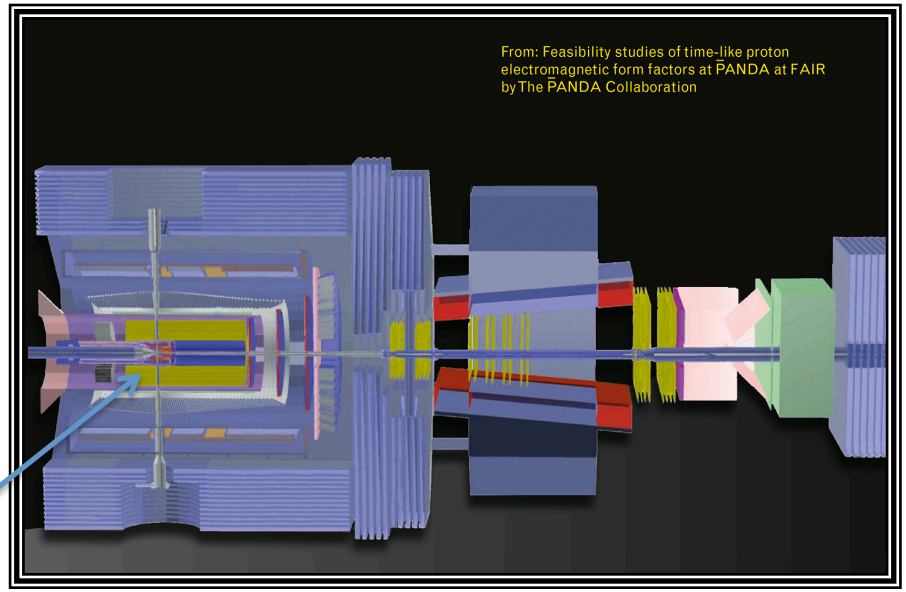
# Measurement of TL proton FFs at PANDA: Summary

- Measurements of the proton effective form factor in the TL region over a large kinematical region through: $\bar{p}p \rightarrow e^+e^-$ $\bar{p}p \rightarrow \mu^+\mu^-$
- Individual measurement of $|G_E|$ and $|G_M|$ and their ratio
- Possibility to access the relative phase of proton TL FFs
  - Polarization observables (**Born approximation**) give access to $G_E G_M{}^*$
  - Development of a transverse polarized proton target for PANDA in Mainz
- Measurement of proton FFs in the unphysical region: $\bar{p}p \rightarrow e^+e^-\pi^0$

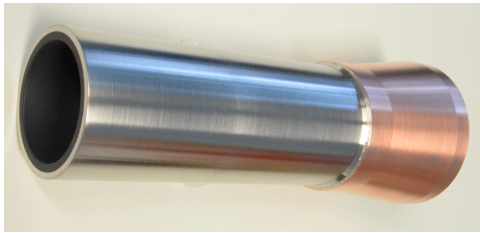
# Back-up

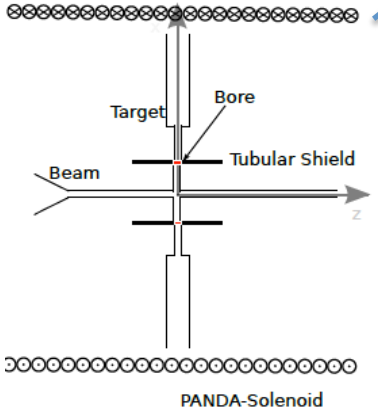
# Transverse Polarized target at PANDA

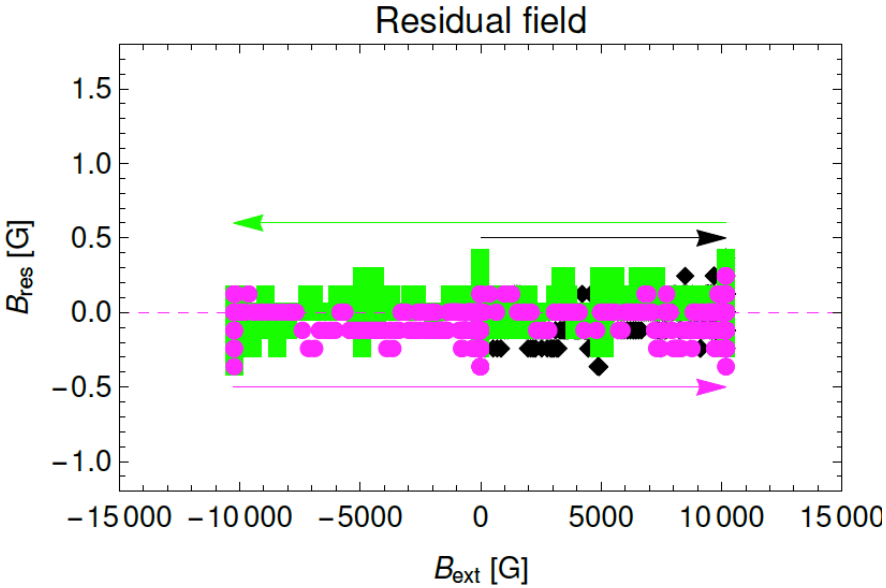
- To shield the target region from the longitudinal 2 T magnetic field induced by the PANDA solenoid one can use a superconducting tube
- The superconducting tube could induce a magnetic field opposite to the PANDA solenoid magnetic field




BSCCO-2212

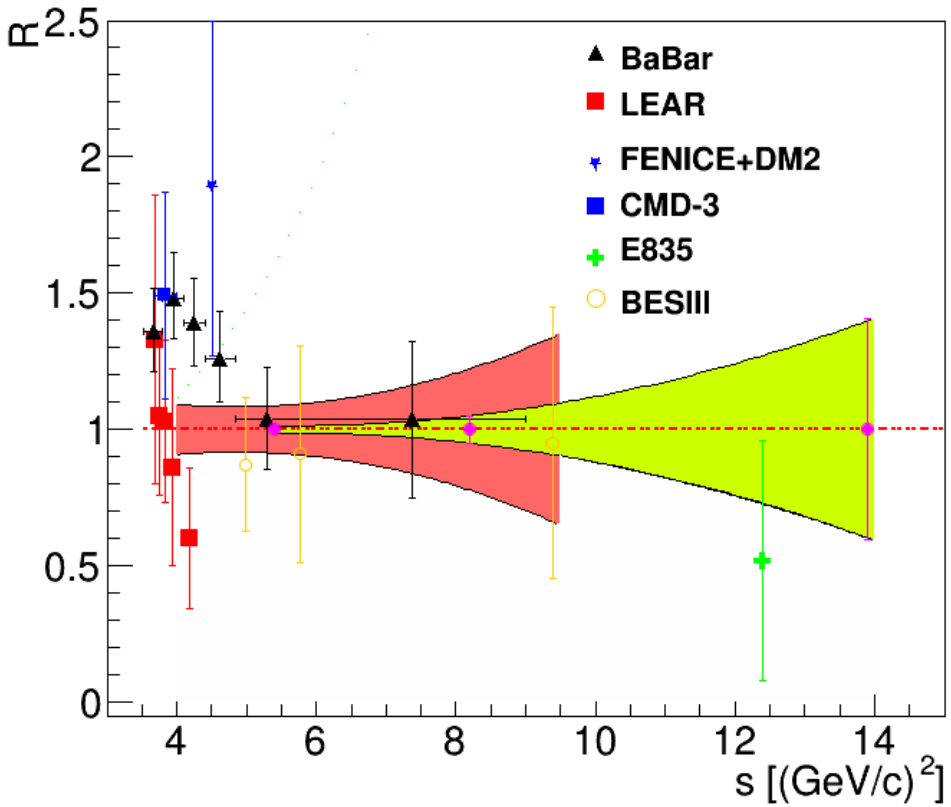





Current status (**Bertold Froehlich et al. (HIM)**): $B_{ext}$=1.0 T and **Residual field <1 Gauss (shielding factor >$10^4$)**

# Current/future experiments: BESII-PANDA




21 scan points 2015 (552 pb$^{-1}$)

Monte Carlo Sim., R=1 (C. Morales)

L=2 fb$^{-1}$
2.10$^{32}$ cm$^{-1}$ s$^{-1}$

| | BESIII | PANDA |
|---|---|---|
| s [(GeV/c)$^2$] | 4 - 9.5 | 5 - 14 |
| R=\|$G_E$\|/\|$G_M$\| | 9 % - 35 % | 1.4 % - 41 % |

# Proton form factors with a polarized proton target @ PANDA

**Access the relative phase between the proton form factors:**

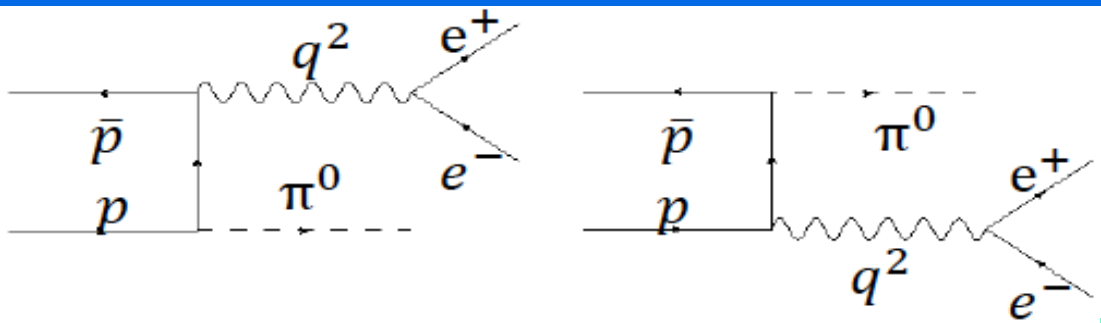
- Time-Like form factors are complex: $G_E = |G_E| e^{i\phi E}$ $G_M = |G_M| e^{i\phi M}$

- Differential cross section of unpolarized **signal reaction** $\bar{p}p \rightarrow e^+e^-$

$$\frac{d\sigma}{d\cos\theta_{CM}} \propto Norm \times \left[ (1+\cos^2\theta_{CM})|G_M|^2 + \frac{|G_E|^2}{\tau}(1-\cos^2\theta_{CM}) \right]$$

- with transverse polarized target: $\left(\frac{d\sigma}{d\Omega}\right)_0 A_{1,y} \propto \sin 2\Theta \, \mathrm{Im}\left(G_M G_E^*\right)$

# Proton FFs in the unphysical region




**J. Boucher, PhD Thesis 2011, IPNO**

One nucleon exchange model

**Feasibility** studies were performed @ **p=1.7 GeV/c** with:

- **$q^2$=0.605 ±0.005, 2.0±0.125 $(GeV/c^2)^2$, at each $q^2$:**
  - $10° < \theta_{\pi 0} < 30°$, $80° < \theta_{\pi 0} < 100°$ and $140° < \theta_{\pi 0} < 160°$ (Lab. System)

$\pi^0$ decay into $\gamma\gamma$ has to be taken into account