The Structure of the nucleon

Alaa Dbeyssi

Helmholtz-Institut Mainz

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Probing the structure of hadrons

The high-energy electron scattering is one of the most powerful tools to probe the hadron structure .

Historically, such experiments provided two crucial insights.

1) Elastic electron scattering established the extended nature of the proton, proton size ~ 10^{-13} cm.

R. Hofstadter, Nobel Prize 1961

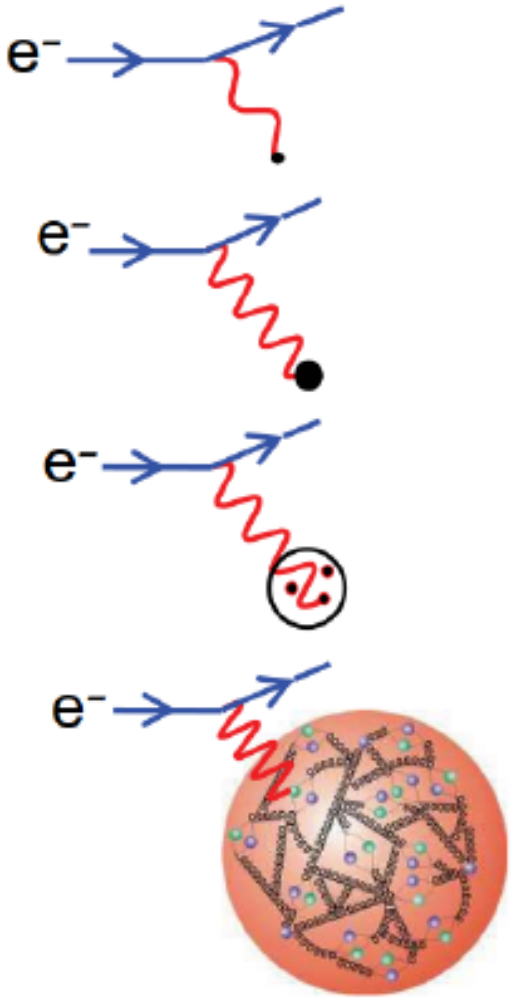
2) **Deep-Inelastic scattering (DIS)** discovered the existence of quasi-free point-like objects (quarks) inside the nucleon, which eventually paved the way to establish QCD.

Friedman, Kendall, Taylor, Nobel Prize 1990
Gross, Politzer, Wilczek, Nobel Prize 2004

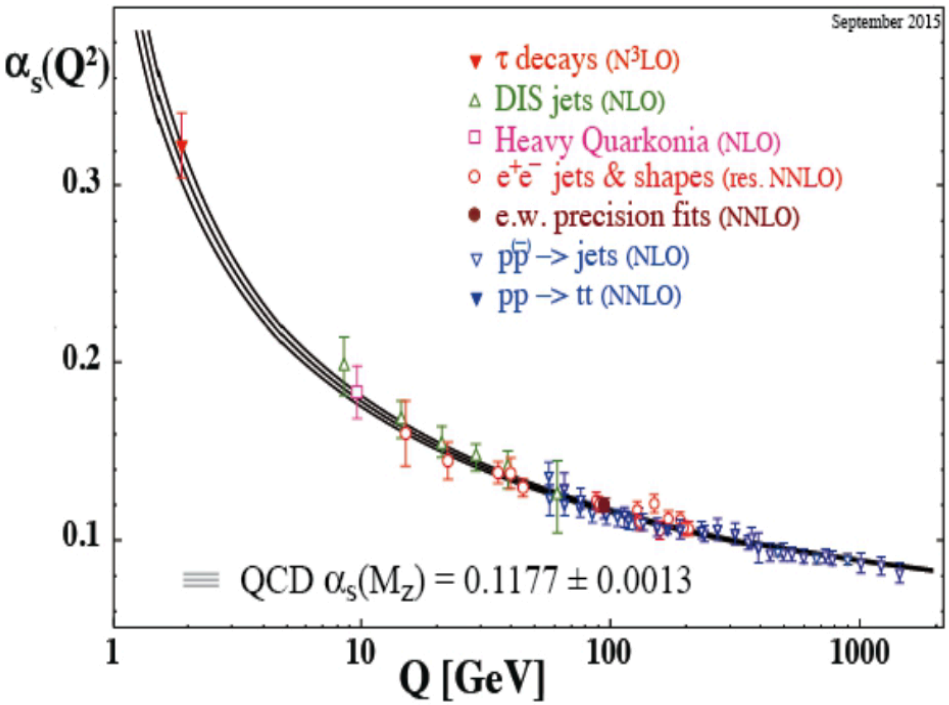
Probing the structure of hadrons

Wave length $\lambda = hc/E$

- At very Low electron beam ($\lambda >> r_p$): scattering from a **point-like spin-less** object
- Low electron beam energies ($\lambda \approx r_p$): scattering from an **extended charged** object
- High electron beam energies ($\lambda < r_p$): scattering from **constituent quarks**
- High electron beam energies ($\lambda << r_p$): proton appears as **sea of quarks and gluons**



Essential concept of QCD and Factorization theorems



1. Asymptotic freedom of quarks and gluons at large energy (short distance)

2. Confinement of quarks and gluons at small energy (large distance)

Factorization of high-energy and low-energy contributions

The power of the factorization theorem is that the same structure functions can be accessed in different processes as long as there is large scale, which guarantees validity of factorization.

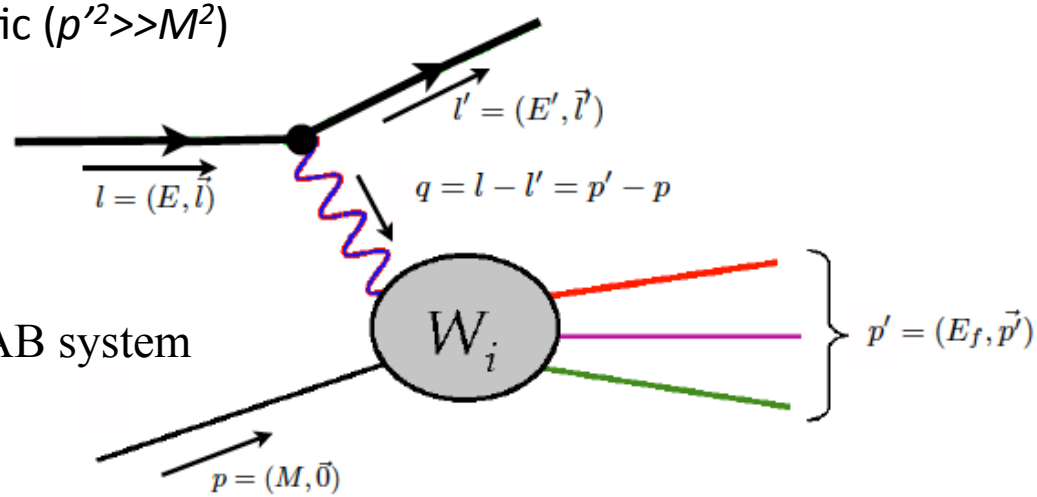
Inclusive Deep Inelastic Scattering

"Deep-inelastic" – probe the nucleon with high enough energies to break it up.

–"Inclusive" – measure *only* energy and angle of scattered lepton

relevant kinematics:

Deep ($Q^2 >> M^2 >> 1$ Gev^2)
Inelastic ($p'^2 >> M^2$)



$$Q^2 = -q^2 = -(p-p')^2 = -M^2 - p'^2 + 2pp'$$
$$Q^2 = -M^2 - p'^2 + 2p(p+q) = M^2 - p'^2 + 2p\cdot q$$
$$Q^2 = -M^2 - p'^2 + 2M(M+\nu), \nu = (E - E')$$

LAB system

Elastic scattering:

$$p'^2 = M^2$$
$$Q^2 = -(p-p')^2 = -M^2 - p'^2 + 2M(M+\nu) = 2M\nu = 2p\cdot q$$

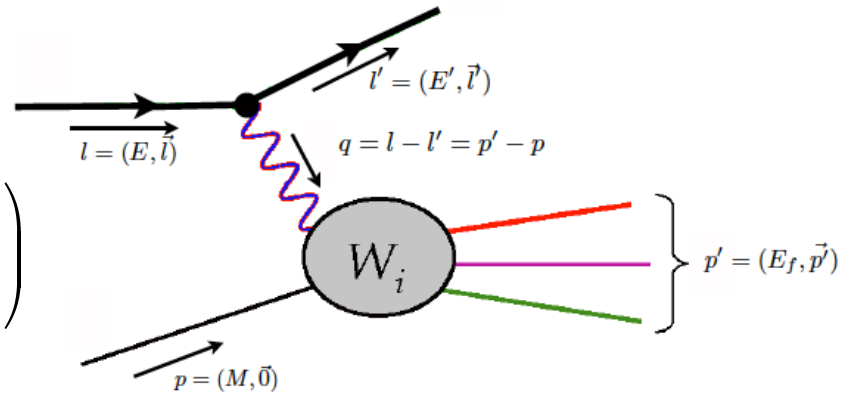
Q^2 and ν are independent

$$x_B = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2M\nu}, \nu = E - E'$$

Bjorken-x_B: deviation from elastic scattering (x_B=1)

Inclusive Deep Inelastic Scattering

Assuming elastic scattering with a point-like proton (Dirac particle of spin 1/2):



$$\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4} e_q^2 \frac{E}{E'}\left(\cos^2(\theta/2) + \frac{Q^2}{2M^2}\sin^2(\theta/2)\right)$$

Point-like objects have structure functions that are δ functions:

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi a^2}{Q^4} \boxed{e_q^2} \frac{E}{E'}\left(\cos^2(\theta/2) + \boxed{\frac{Q^2}{2M^2}}\sin^2(\theta/2)\right)\boxed{\delta(\nu - \frac{Q^2}{2M})}$$

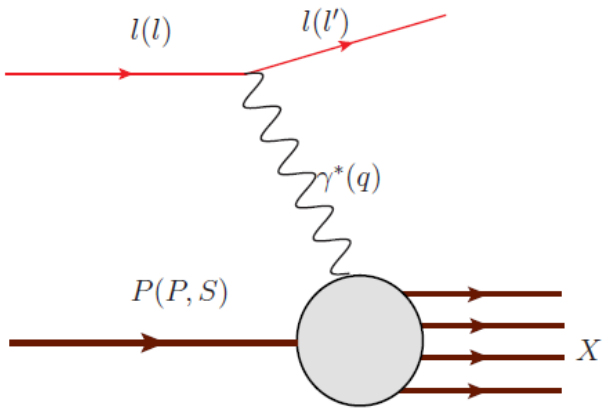
Define structure functions for Dirac point-like object W_1 and W_2

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi a^2}{Q^4}\frac{E}{E'}\left(W_2(Q^2,\nu)\cos^2(\theta/2) + 2W_1(Q^2,\nu)\sin^2(\theta/2)\right)$$

$$W_2(Q^2,\nu) = e_q^2\delta(\nu - \frac{Q^2}{2M}), W_1(Q^2,\nu) = e_q^2\frac{Q^2}{2M^2}\delta(\nu - \frac{Q^2}{2M})$$

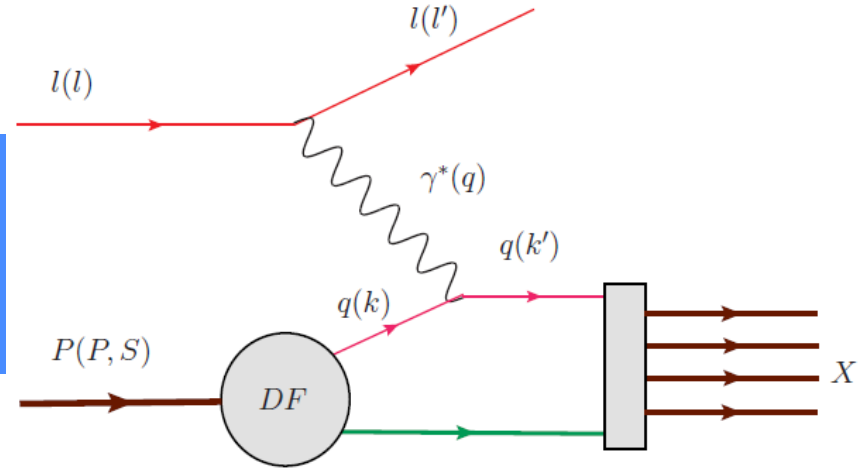
Inclusive Deep Inelastic Scattering

Deep inelastic scattering and naïve quark parton model: Effectively *elastic* scattering off of a point-like parton carrying a momentum fraction x_i of the proton



$$Q^2 \to \infty, \nu \to \infty$$

$$x_B = \frac{Q^2}{2M\nu} \to fixed$$



$$W_1(Q^2,\nu) = \sum_i \int dx f(x_i) e_i^2 \frac{Q^2}{4x_i M^2} \delta(\nu - \frac{Q^2}{2Mx_i}) = \sum_i e_i^2 f_i(x_B) \frac{1}{2M}$$

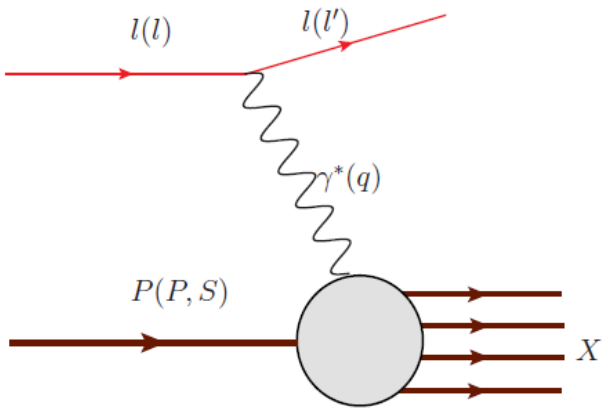
$$W_2(Q^2,\nu) = \sum_i \int dx f(x_i) e_i^2 \delta(\nu - \frac{Q^2}{2Mx_i}) = \sum_i e_i^2 f_i(x_B) \frac{x_B}{\nu}$$

Q^2 and ν are not independent

$f_i(x_B)$ describes the probability to find a parton of type i and having a fraction x_B of the proton momentum - **parton distribution function (pdf)**

Inclusive Deep Inelastic Scattering

Deep inelastic scattering and naïve quark parton model: Effectively *elastic* scattering off of a point-like quark or antiquark inside the nucleon



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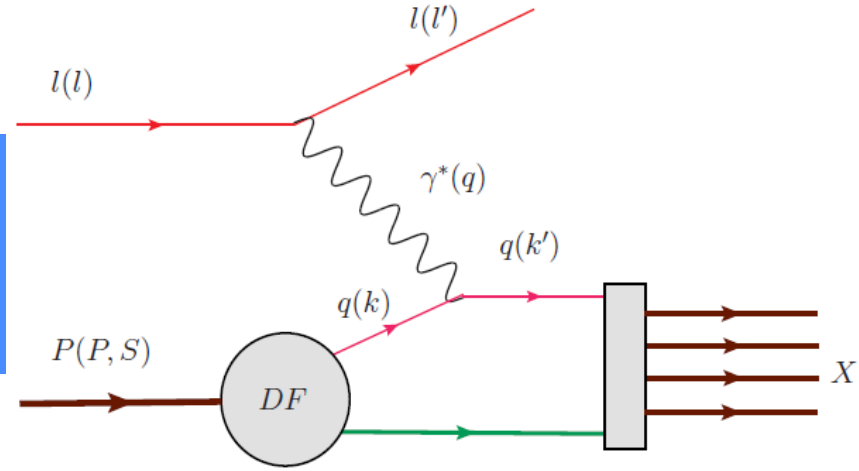


Bjorken Scaling

$$F_1(x_B) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x_B)$$

$$F_2(x_B) = \nu W_2(Q^2,\nu) = \sum_i e_i^2 x_B f_i(x_B)$$

Bjorken scaling: the Structure functions obey scaling law, independent of Q^2

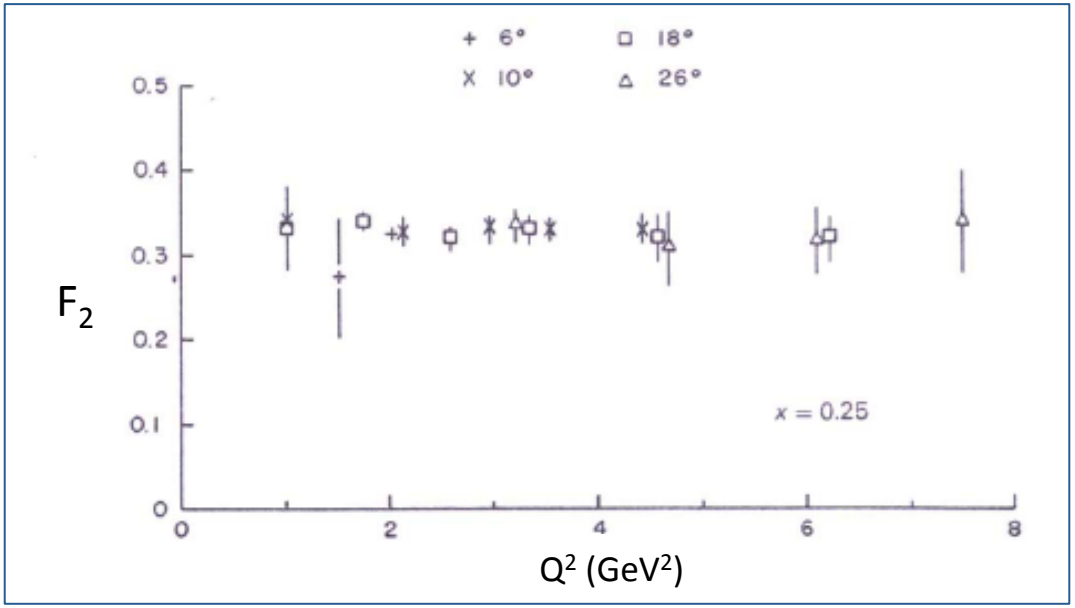
Parton Distribution Functions-Bjorken scaling

Structure functions

$$F_1(x_B) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x_B)$$

$$F_2(x_B) = \nu W_2(Q^2,\nu) = \sum_i e_i^2 x_B f_i(x_B)$$

$$x_B = \frac{Q^2}{2M\nu}$$ is the fraction of the nucleon's momentum carried by the struck quark.



No explicit dependence on Q^2: Indicates that the scattering takes place on Point-like constituents (partons)

Ann. Rev: Nucl. Sci. 22,203 (1972)

Data from SLAC experiments

Structure functions

$$F_1(x_B) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x_B)$$

$$F_2(x_B) = \nu W_2(Q^2,\nu) = \sum_i e_i^2 x_B f_i(x_B)$$

$$x_B = \frac{Q^2}{2M\nu}$$ is the fraction of the nucleon's momentum carried by the struck quark.

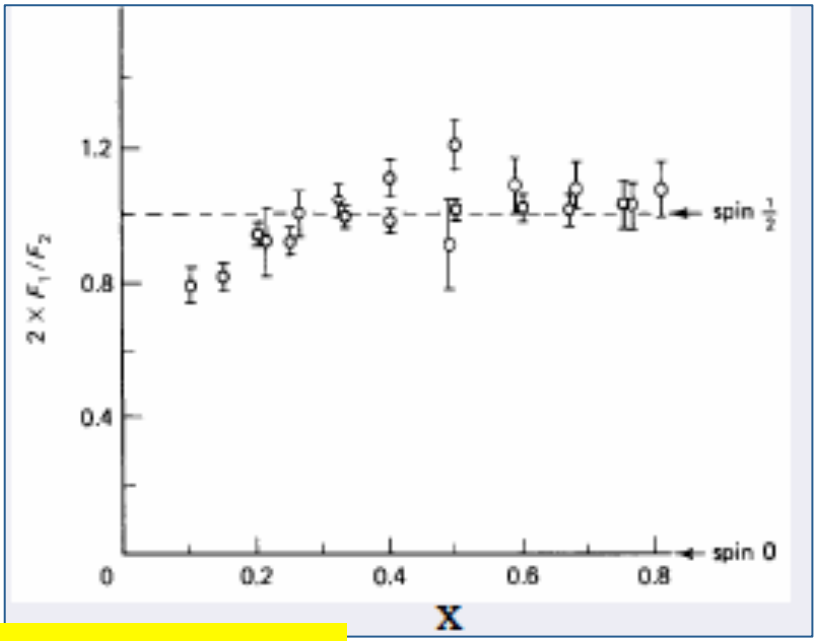
Spin ½ point like partons in the proton:

Callan-Gross relation

$$2x_B F_1(x_B) = F_2(x_B) \rightarrow 2xF_1(x_B)/F_2(x_B) = 1$$

If spin 0 point-like objects in the proton: the purely magnetic structure function F1=0

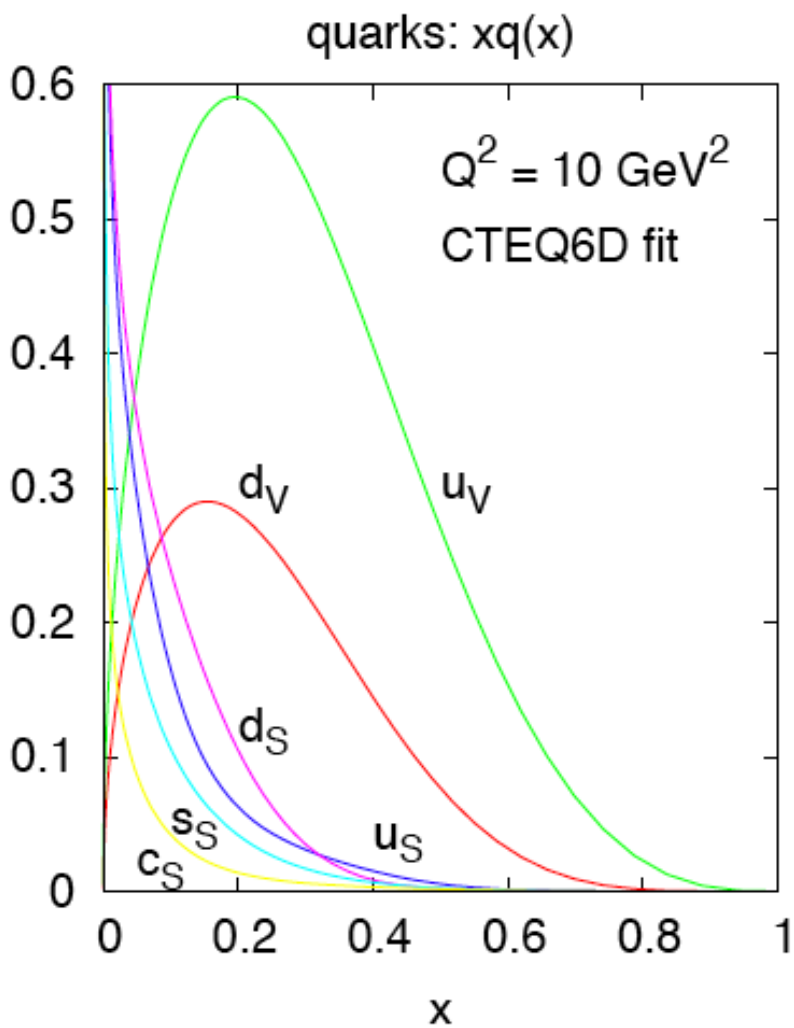
$$2xF_1(x_B)/F_2(x_B) = 0$$



Good agreement with spin 1/2 point-like objects inside proton: spin ½ nature of the quarks

Parton Distribution Functions

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \bar{s}^p(x)]$$



$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

This indicates a Large contribution from Gluons

Parton Distribution Functions

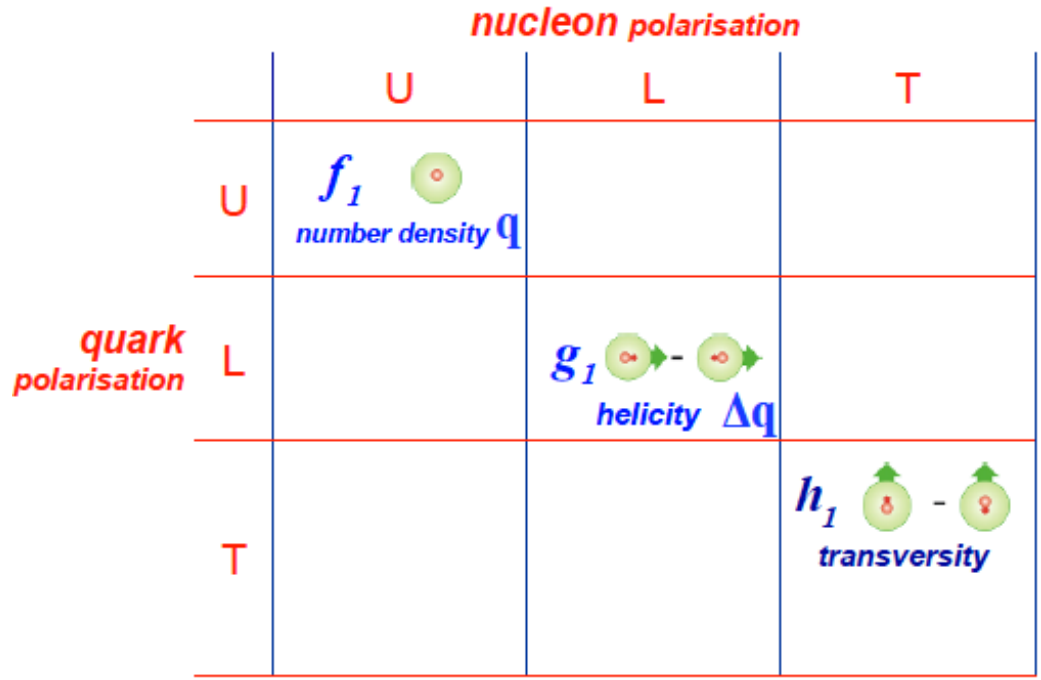
At leading order 3 PDFs are needed for a full description of the nucleon structure

- f_1 Probability to find quarks within the nucleon carrying a fraction x of the nucleon momentum

$$F(x) = \frac{1}{2}\sum_i e_i^2 f_i(x), f_i(x) = f_i^+(x) + f_i^-(x)$$

- g_1 the difference in probabilities to find quarks in a longitudinally polarized nucleon with their spin aligned or anti-aligned to the spin of the nucleon

$$\frac{1}{2}\sum_i e_i^2 g_{1i}(x), g_{1i}(x) = q_i^+(x) - q_i^-(x) = \Delta q(x)$$



- Transversity (h_1): correlation between the transverse spin of the nucleon and the transverse spin of the quark (not accessible in Inclusive Deep Inelastic scattering)

Parton Distribution Functions

At leading order 3 PDFs are needed for a full description of the nucleon structure

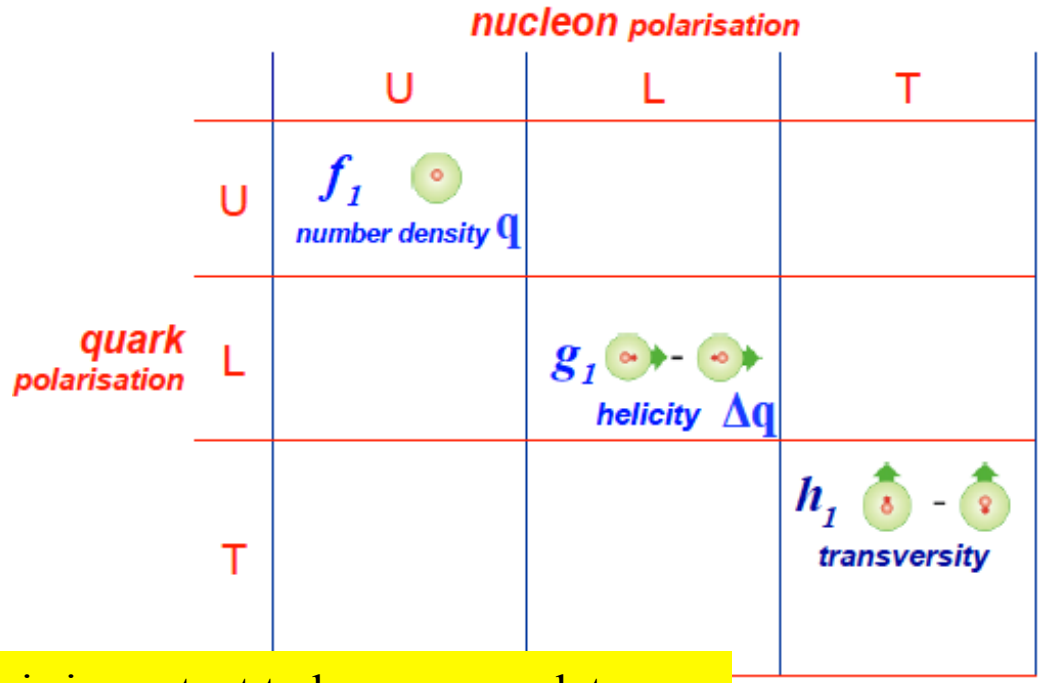
- f_1 Probability to find quarks within the nucleon carrying a fraction x of the nucleon momentum

$$F(x) = \frac{1}{2}\sum_i e_i^2 f_i(x), f_i(x) = f_i^+(x) + f_i^-(x)$$

- g_1 the difference in probabilities to find quarks in a longitudinally polarized nucleon with their spin aligned or anti-aligned

$$\frac{1}{2}\sum_i e_i^2$$

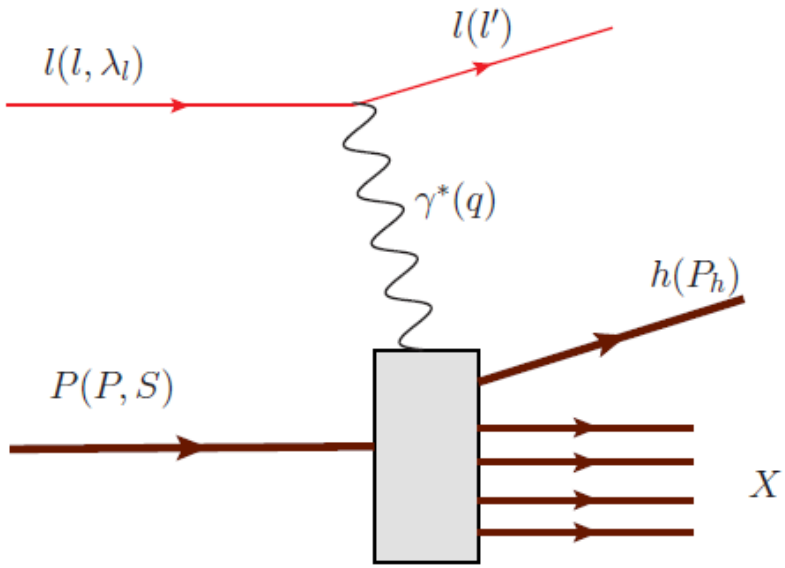
- Transv … the transverse spin of the quark (not accessible in Inclusive Deep Inelastic scattering)

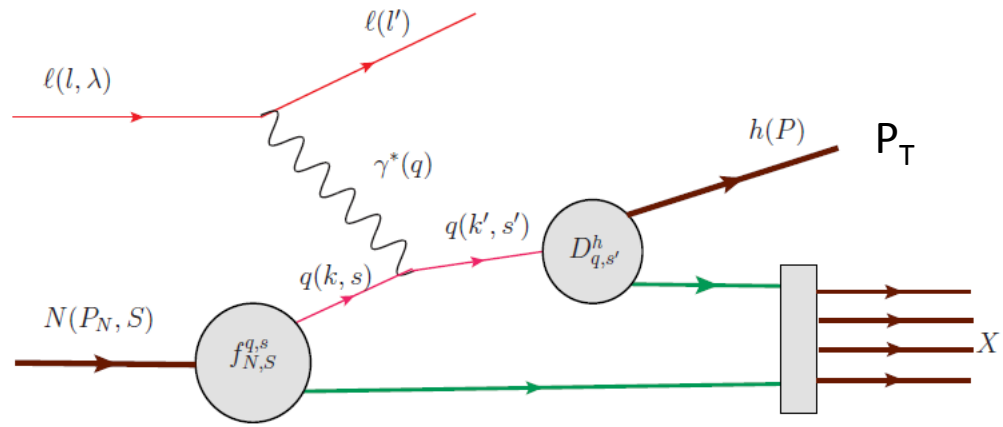


The measurement of the 3 PDFs is important to have a complete description of the nucleon structure in terms of:

- Spin distributions of the quarks
- **Longitudinal** momentum distributions (x_B) of the quarks

Semi-Inclusive Deep Inelastic Scattering (SIDIS)



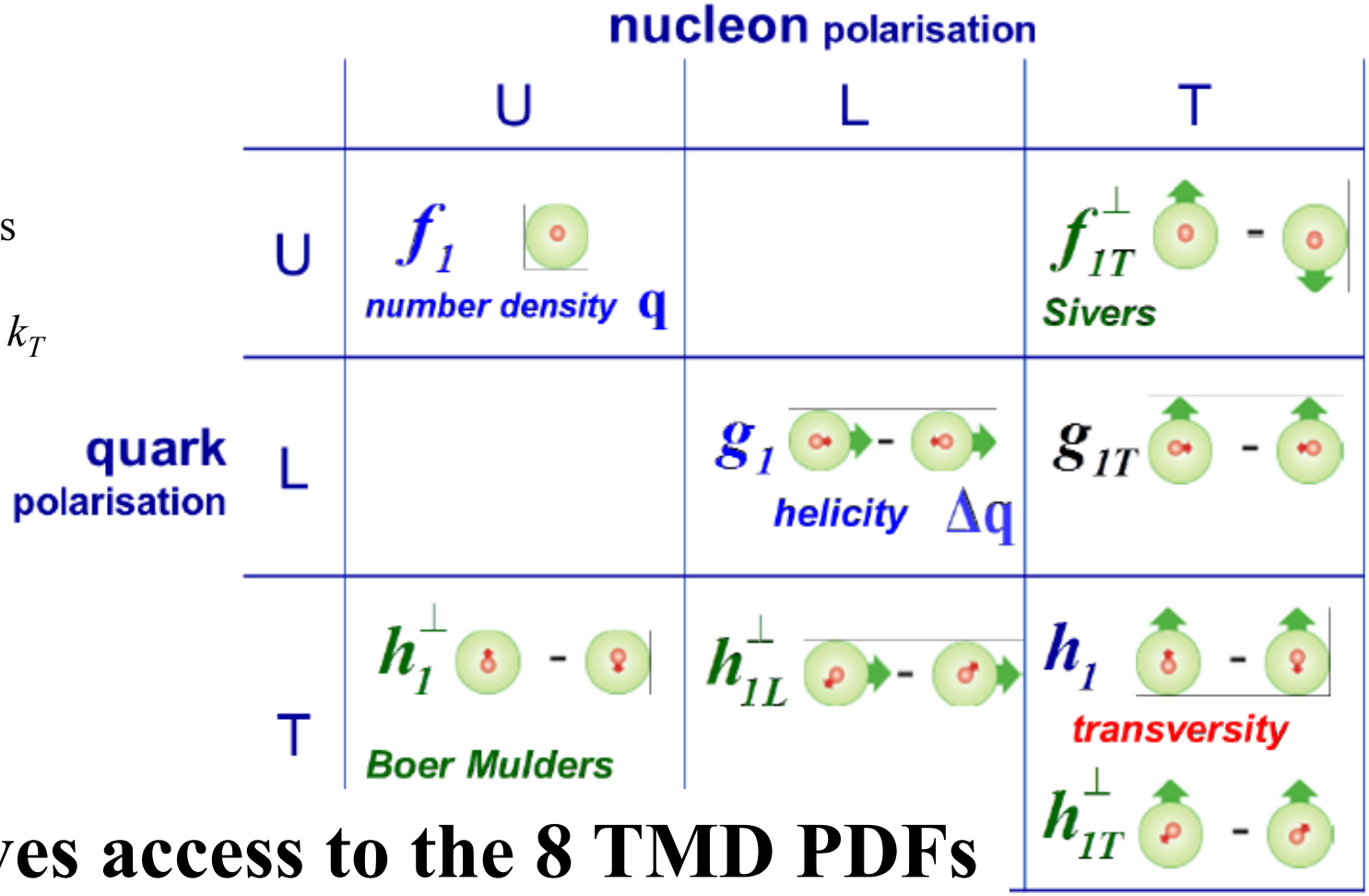


- Semi-inclusive deep-inelastic lepton-nucleon scattering (SIDIS): Measure energy and angle of scattered lepton and at least one final-state produced hadron
- Measuring scattered lepton and one hadron gives enough vectors **to probe transverse-momentum-dependent functions in the nucleon**
- **Description of the Nucleon parton structure in 3D Momentum**

The structure of the proton: TMD PDF

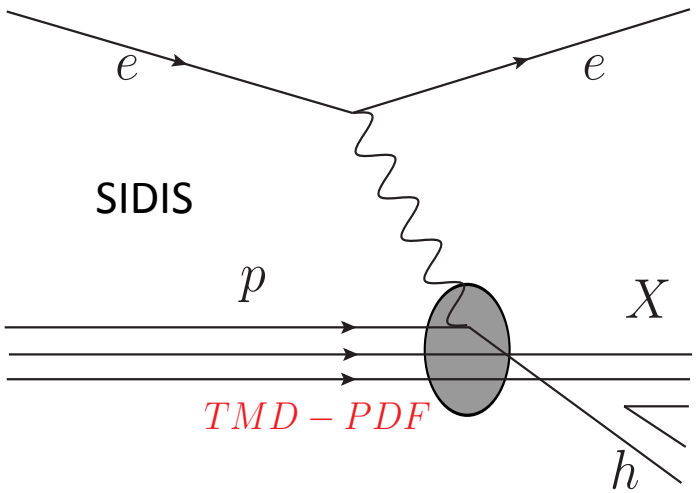
Taking into account the quark intrinsic transverse momentum k_T, at leading order 8 TMD PDFs are needed for a full description of the nucleon structure

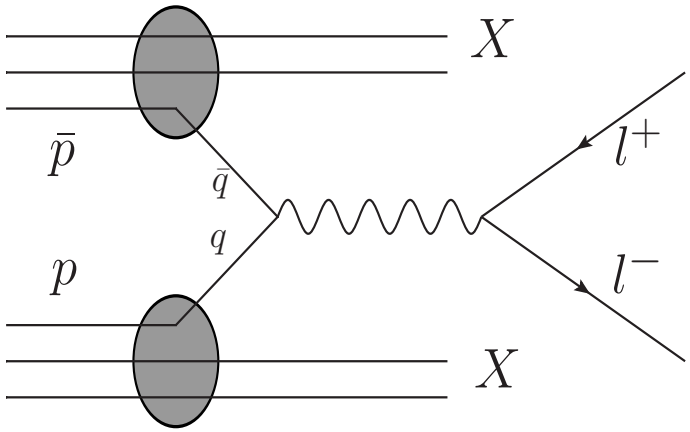
The 5 new TMDs vanishes when integrating over k_T



SIDIS gives access to the 8 TMD PDFs

Drell-Yan at PANDA





TMD-PDFs are convoluted with the fragmentation functions

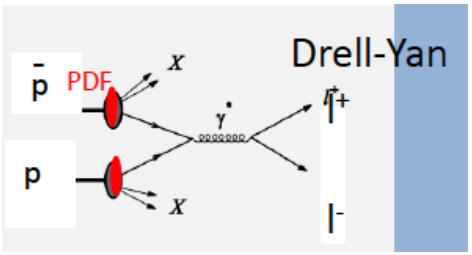
Direct access to TMD-PDFs

- Related to DIS (inclusive or semi-inclusive) by rotation of the Feynman diagram – Drell-Yan the *s*-channel process, SIDIS the *t*-channel process

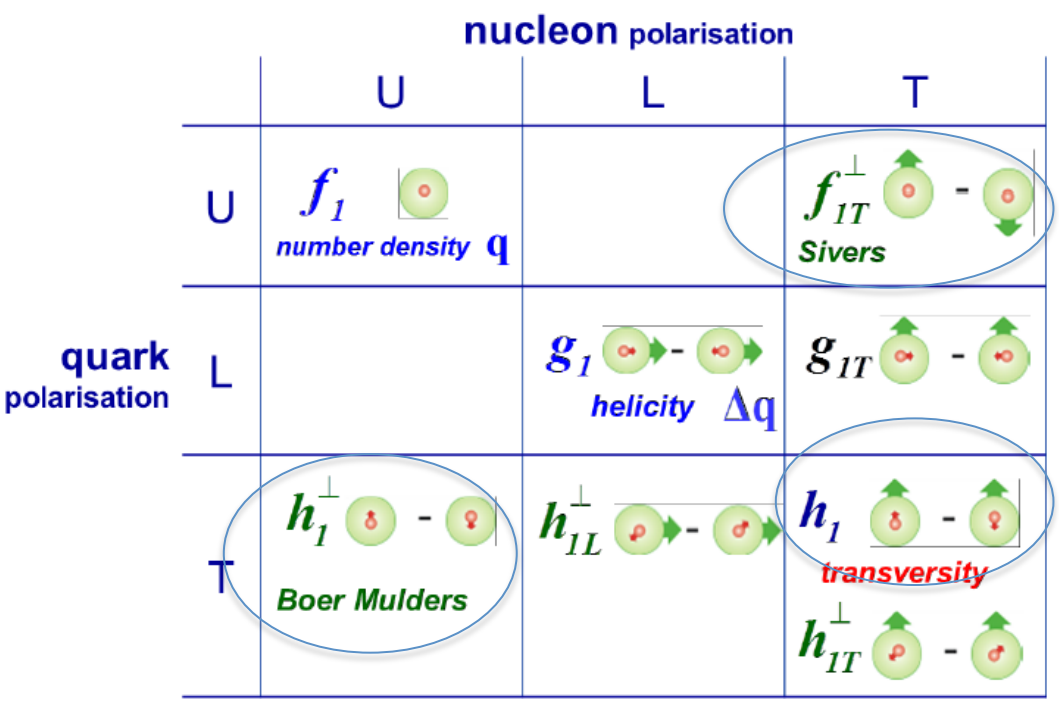
Test of Universality
and the QCD TMD factorization

"Drell-Yan" process – Drell and Yan, PRL 25, 316 (1970); Erratum PRL 25, 902 (1970)

Transverse momentum dependence PDFs @ PANDA



@ PANDA unique energy range up to s~30 GeV^2



Transversity

Sivers Boer-Mulders (BM)

Asymmetry measurements:

Unpolarized DY

$$A^{\cos 2\varphi} \rightarrow h_1^{\perp}$$

Single-polarized DY

$$A^{\sin(\varphi \pm \varphi s2)} \rightarrow h_1^{\perp}, h_1, f_{1T}^{\perp}$$

$$A = \frac{U - D}{U + D}$$

φ : angle between hadron and lepton planes

φ_{s2}: angle between hadron spin and lepton plane

$$U = N(\cos 2\varphi > 0) \qquad D = N(\cos 2\varphi < 0)$$

$$U = N(\sin(\varphi \pm \varphi_{s2}) > 0) \qquad D = N(\sin(\varphi \pm \varphi_{s2}) > 0)$$

Proton structure functions

Form Factors
$F_1(t)$, $F_2(t)$

$\int dx d^2 k_\perp$

2D Fourier Transform
$(t \leftrightarrow \vec{b}_\perp)$

Parton Distribution Functions
PDF f(x)

$\int d^2 k_\perp, t \to 0$

Wigner distributions

$(x, \vec{k}_\perp, \vec{b}_\perp)$

(x) Longitudinal momentum
$(\vec{k}_\perp)$ Transverse momentum
$(\vec{b}_\perp)$ Transverse spatial variable

$\int d^2 k_\perp$

Generalized Parton Distributions
GPDs, GDAs (t↔s)
F(x,ζ,t)

$t \to 0$

Transverse Momentum
Dependance (TMD) PDF

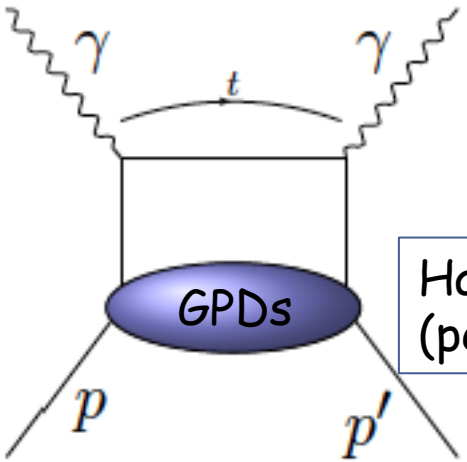
$f(x, \vec{k}_\perp)$

Hard exclusive processes at PANDA

(SL) $\gamma p \rightarrow \gamma p$

large transverse momentum
(hard scale)

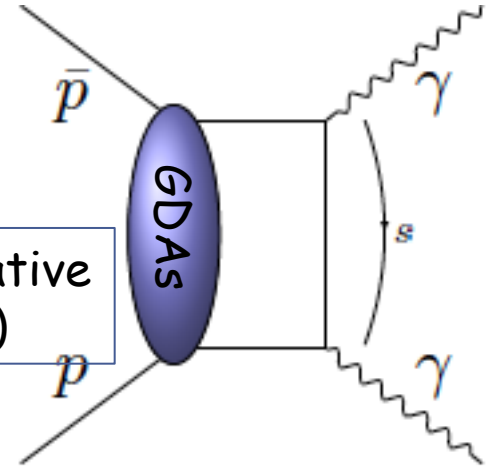
(TL) ppbar$\rightarrow \gamma\gamma$, $\pi^0\gamma$



QCD factorization

Hard scattering
(perturbative)

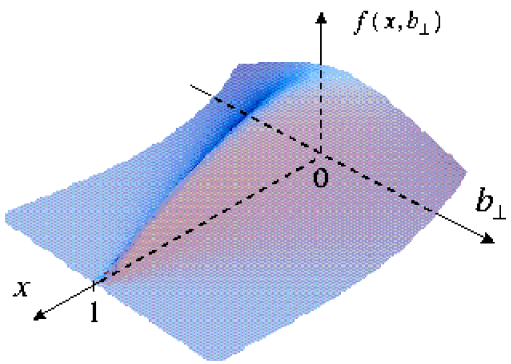
Non perturbative
(GPDs, GDAs)

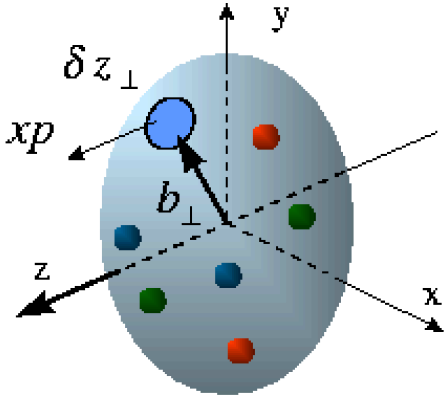


Wide Angle Compton Scattering
Generalized Parton Distributions GPDs

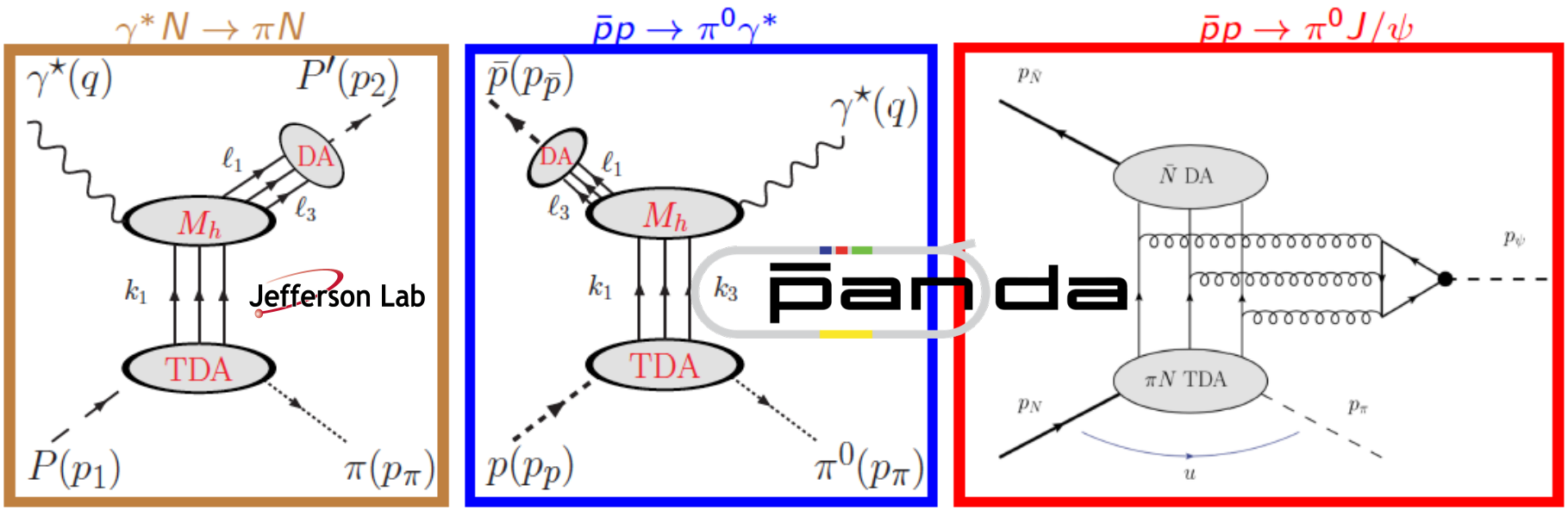
Time-Like Wide Angle Compton Scattering
Generalized Distribution Amplitudes GPDs

Correlated quark momentum and helicity distributions in transverse space - GPDs



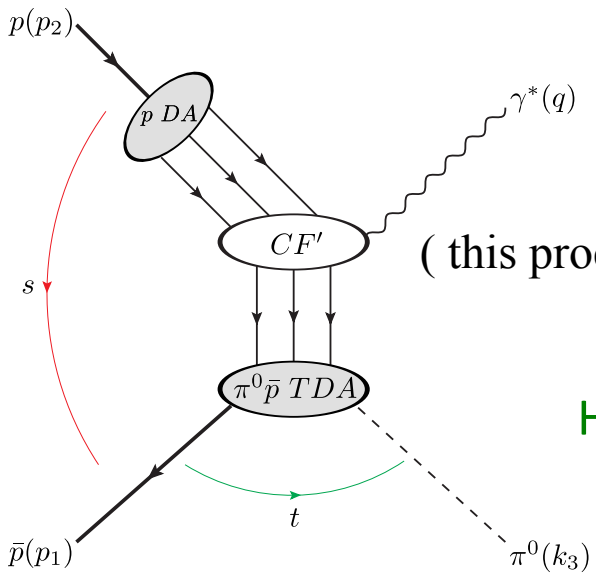


Nucleon to meson TDAs



- New class of non-perturbative structure functions
- Occur in collinear factorization description of various hard exclusive processes
- Are independent of reaction type, s and q^2
- Give information on pionic components of the nucleon wave-function

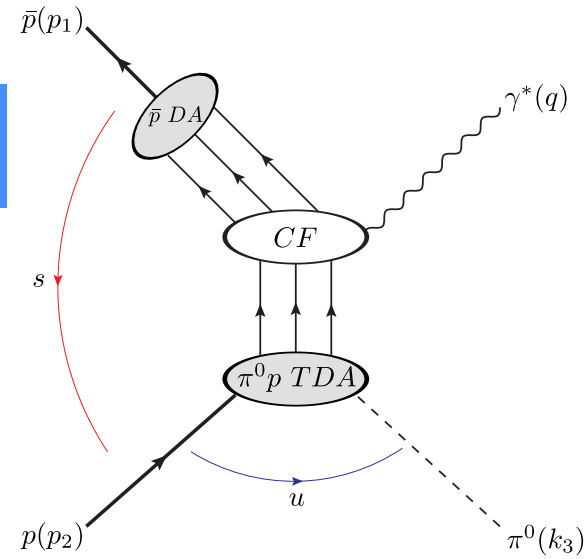
Nucleon to meson TDAs at PANDA



$$\bar{p}p \rightarrow \gamma^* \pi^0 -> e^+ e^- \pi^0$$

(this process never been measured)

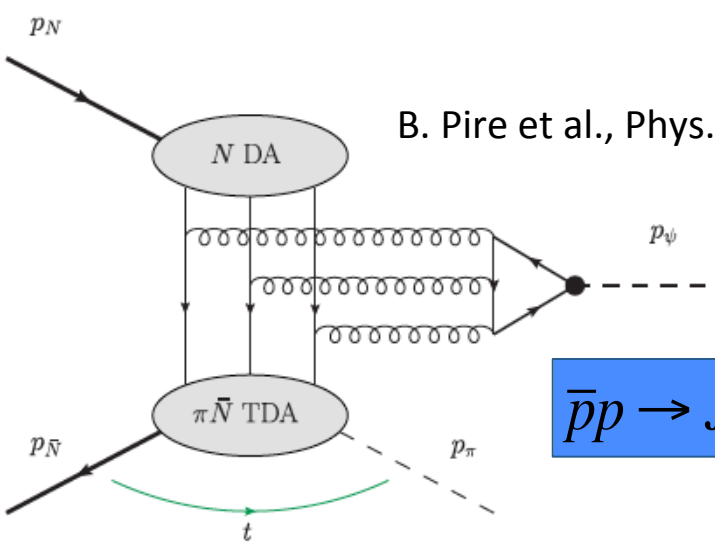
Hard scale: large q^2~s



t is small (forward kinematics)

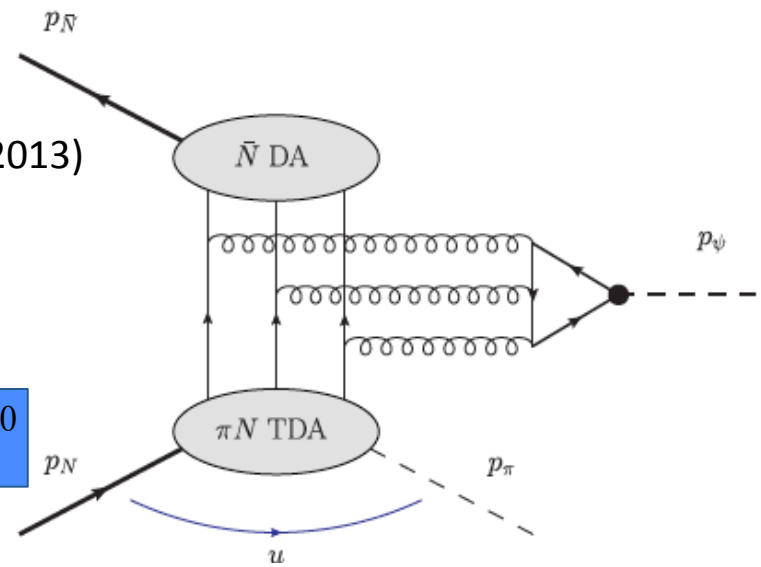
u is small (backward kinematics)

J. P. Lansberg et al., Phys Rev D 76, 111502(R) (2007)

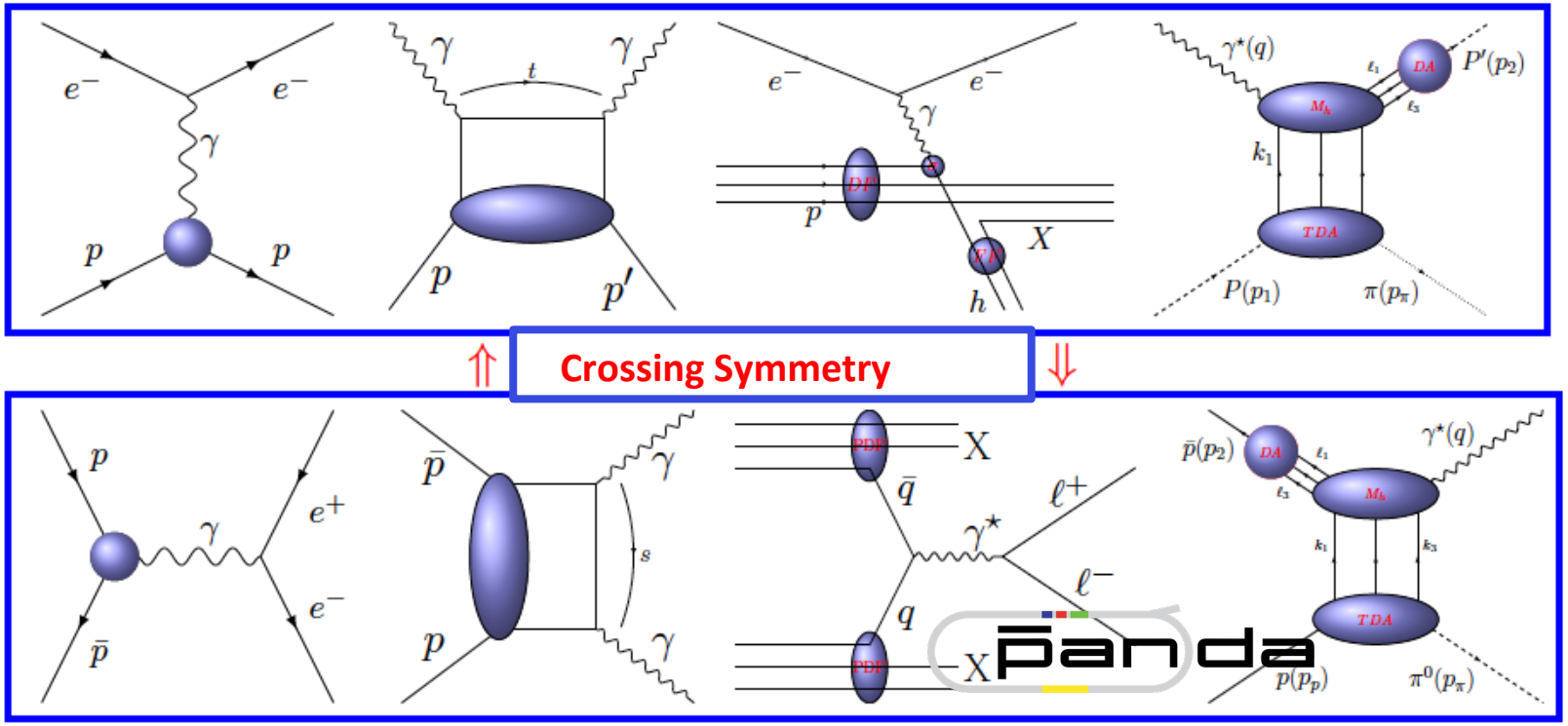


B. Pire et al., Phys. Lett. B. 724 99-107 (2013)

$$\bar{p}p \rightarrow J/\psi \pi^0 -> e^+ e^- \pi^0$$

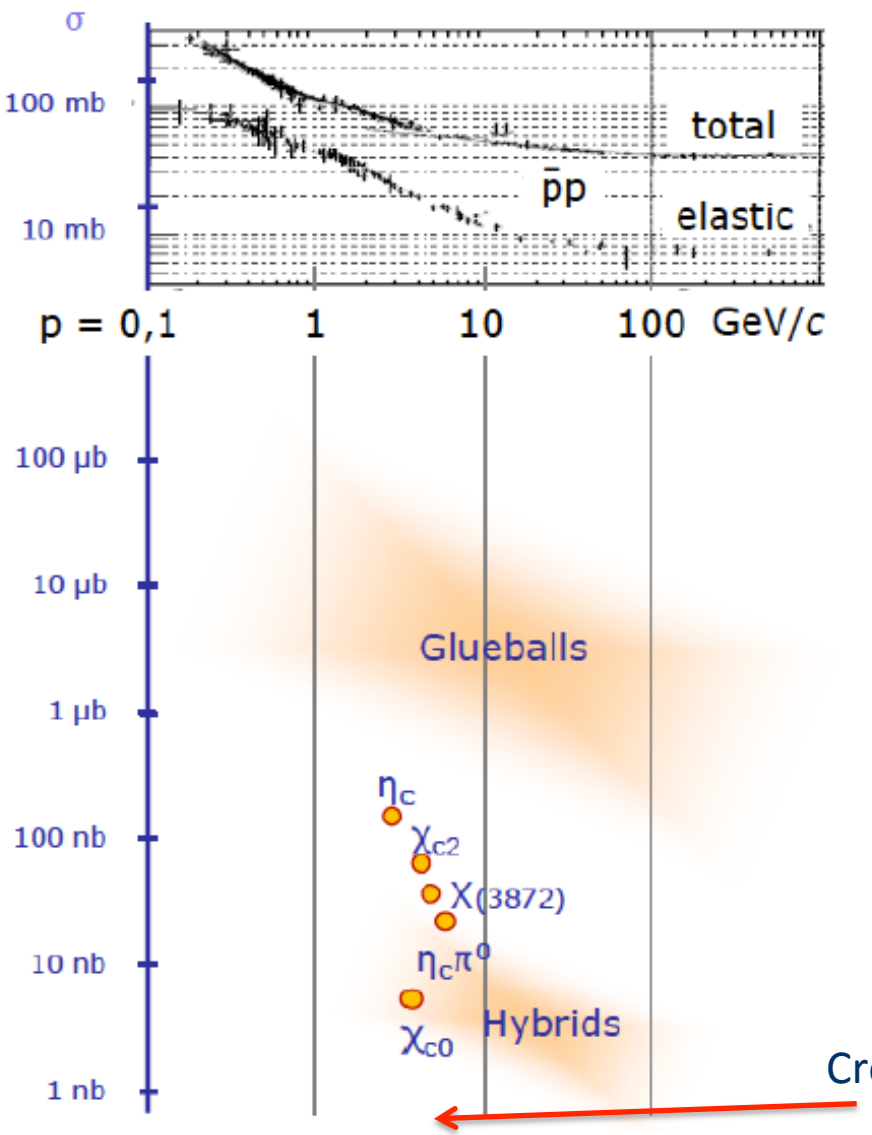


Study of the Nucleon Structure at PANDA



- Proton Electromagnetic Form Factors (FFs)
- Generalized Distribution Amplitudes (GDAs)
- Transverse Momentum Dependent Parton Distribution Functions (TMD-PDFs)
- Transition Distribution Amplitudes (TDAs)

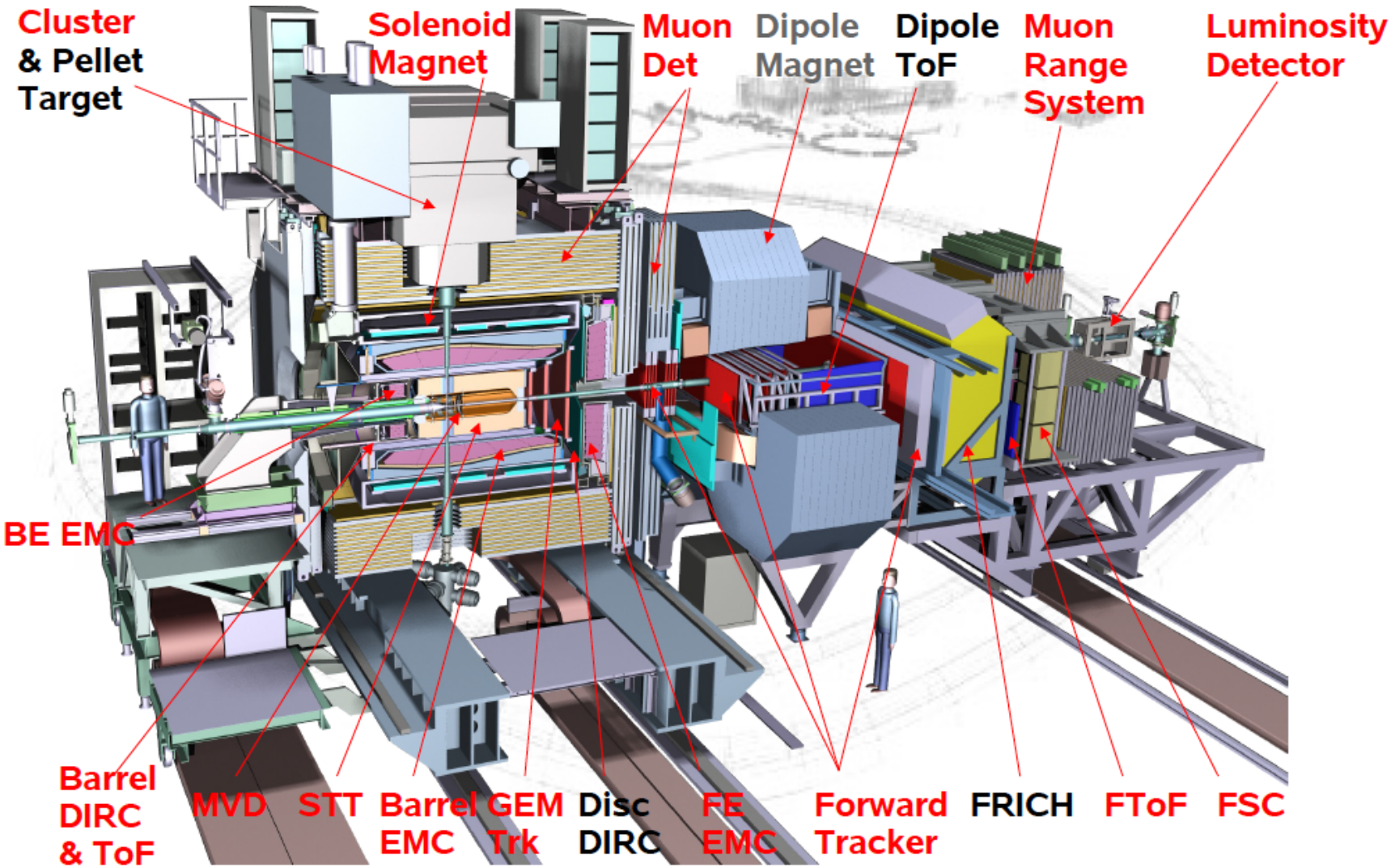
Detector requirements from physics case



Cross section for electromagnetic Processes

- 4π acceptance
- Momentum resolution: 1%
  central tracker in magnetic field
- Photon detection: 1 MeV - 10 GeV
  high dynamic range
  good energy resolution
- Particle identification: γ, e, μ, π, K, p
  Cherenkov detector
  time of flight, dE/dx, muon counter
- Displaced vertex info
  $c\tau = 317\ \mu m$ for $D\pm$
  $\gamma\beta \approx 2$

The PANDA detector (start/full setup)



Summary

- Hadron structure functions are universal frame to study various types of electromagnetic processes

- Measurements of the hadron structure functions in different channels and in different kinematical regions is required to test their universality/analyticity.

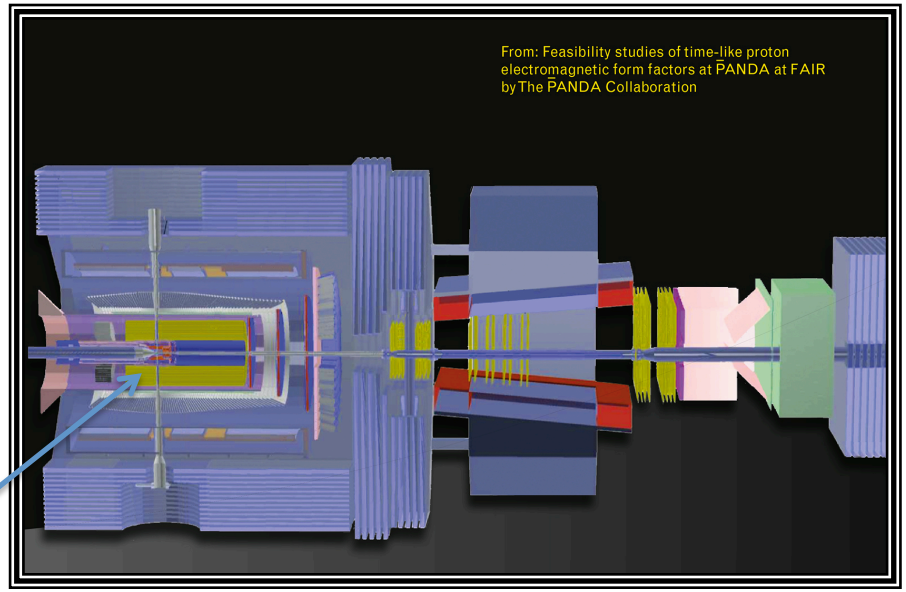
- The high quality antiproton beams of FAIR between 1.5 and 15 GeV/c allows the PANDA experimental to provide a **complementary** study of the nucleon structure with lepton or photon experiments

Thank you for your attention

Back-up

Transverse Polarized target at PANDA

- To shield the target region from the longitudinal 2 T magnetic field induced by the PANDA solenoid one can use a superconducting tube
- The superconducting tube could induce a magnetic field opposite to the PANDA solenoid magnetic field



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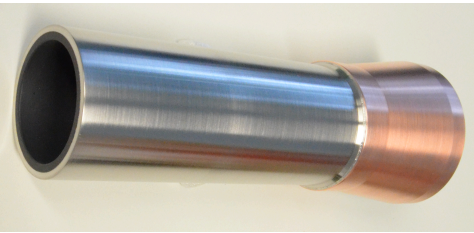
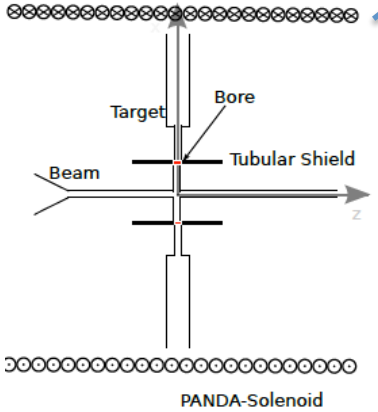
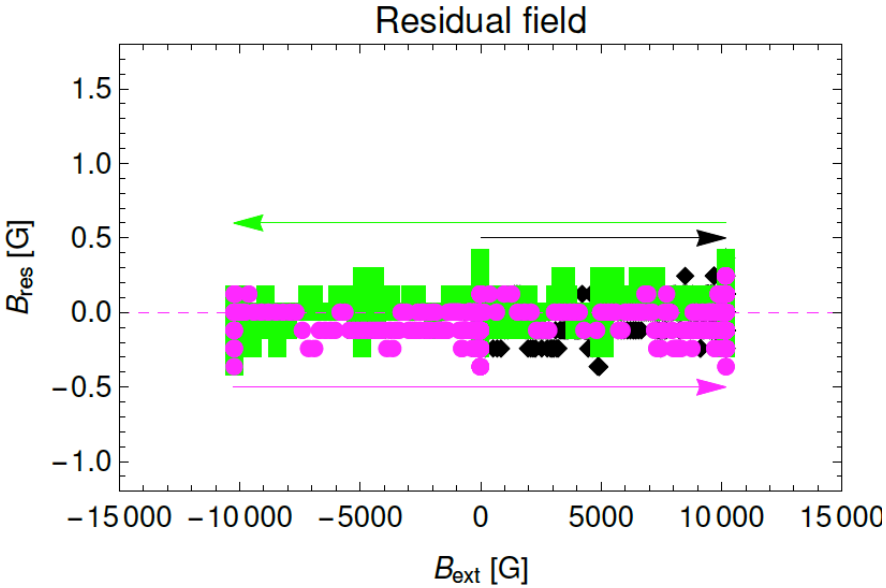






Current status (**Bertold Froehlich et al. (HIM)**): B_{ext}=1.0 T and **Residual field <1 Gauss (shielding factor >10^4)**

Proton form factors with a polarized proton target @ PANDA

Access the relative phase between the proton form factors:

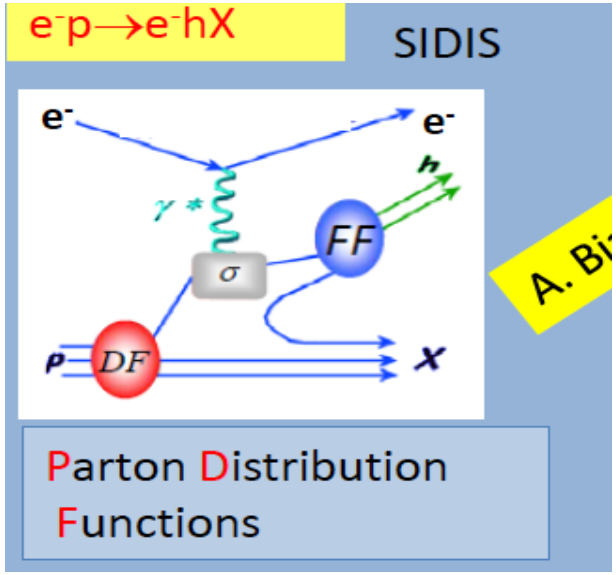
- Time-Like form factors are complex: $G_E = |G_E| e^{i\phi E}$ $G_M = |G_M| e^{i\phi M}$

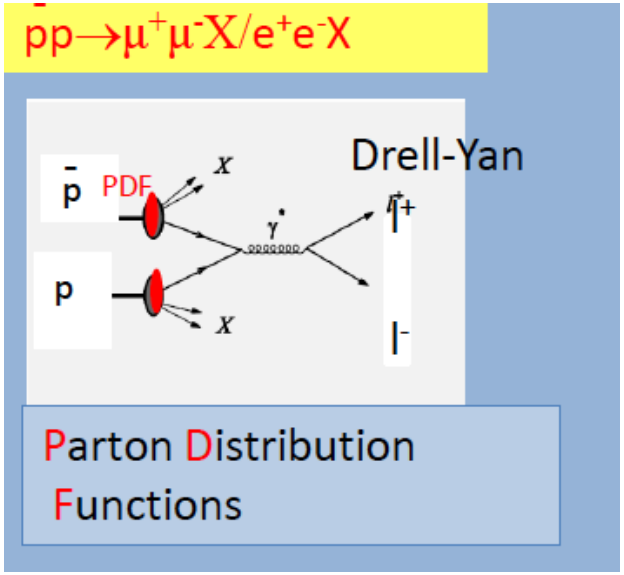
- Differential cross section of unpolarized **signal reaction** $\bar{p}p \rightarrow e^+e^-$

$$\frac{d\sigma}{d\cos\theta_{CM}} \propto Norm \times \left[(1+\cos^2\theta_{CM})|G_M|^2 + \frac{|G_E|^2}{\tau}(1-\cos^2\theta_{CM}) \right]$$

- with transverse polarized target: $\left(\frac{d\sigma}{d\Omega}\right)_0 A_{1,y} \propto \sin 2\Theta \, \mathrm{Im}\left(G_M G_E^*\right)$

Factorization and universality





PDFs are convoluted with the fragmentation functions

SIDIS@ hermes

SIDIS@ COMPASS

Jefferson Lab

Test of Universality
and the QCD TMD factorization

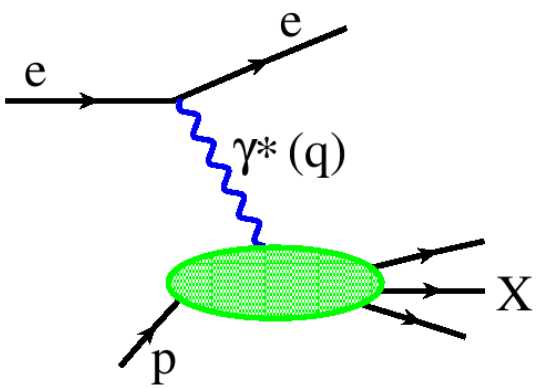
$$f_{1T}^{\perp}(DY) = -f_{1T}^{\perp}(SIDIS)$$

$$h_1^{\perp}(DY) = -h_1^{\perp}(SIDIS)$$

Drell-Yan@ Fermilab

Z production@ Fermilab

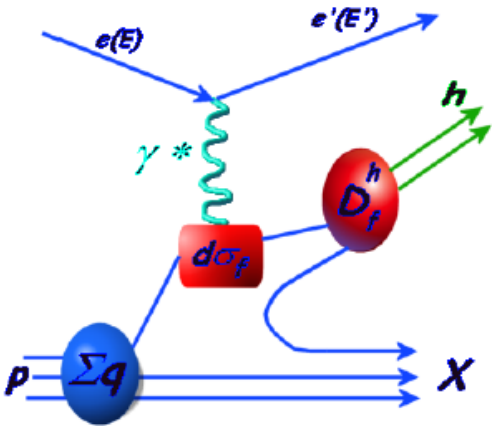
(Semi) Inclusive Deep Inelastic Scattering



- When we are scattering from individual point-like quarks within the target, we are in the regime of deep-inelastic scattering
- Scattering at high q^2 and $W^2=(p+q)^2$ (Bjorken limit)…. α_s is small → QCD factorization (perturbative and non perurbative parts)

$$\frac{d\sigma}{dx\,dQ^2} = \left(\frac{d\sigma}{dx\,dQ^2}\right)_{\text{point(eq}\to\text{eq)}} \cdot \sum_{q=u,d,s,\bar{u},\bar{d},\bar{s}} e_q^2\, q(x,Q^2)$$

PDFs: functions of the Bjorken x = fraction of nucleon momentum carried by struck quark



- In **SIDIS**, a hadron h is detected in coincidence with the scattered lepton:
- Scattering at high Q^2 and W^2 ... but create only one particle in final-state!

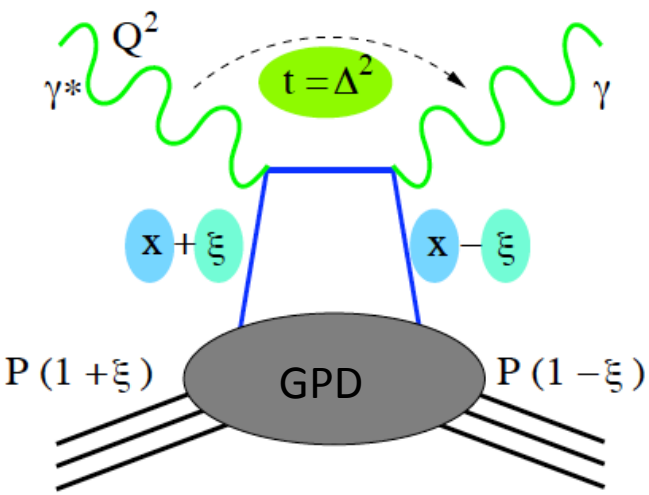
$$d\sigma^h \sim \sum_q e_q^2\, q(x) \cdot \hat{\sigma} \cdot D^{q\to h}(z)$$

Generalized Parton Distributions

Hard exclusive processes leads to a new class of parton distributions

Deep Virtual Compton Scattering

ep→eγp



- At twist-2 approximation there are four chiral-even functions for each parton, related to QCD operators by Fourier transform:

$$H^{g,q}(x,\xi,t), E^{g,q}(x,\xi,t)$$

$$\tilde{H}^{g,q}(x,\xi,t), \tilde{E}^{g,q}(x,\xi,t)$$

- Contain PDFs probed in DIS experiments:

$$H^q(x,\xi=0,t=0) = q(x), -\bar{q}(-x)$$

$$\tilde{H}^q(x,\xi=0,t=0) = \Delta q(x), \Delta\bar{q}(-x)$$

- They are related to the elastic Form Factors:

$$\int_{-1}^{+1} dx H^q(x,\xi,t) = F_1^q(t), \int_{-1}^{+1} dx E^q(x,\xi,t) = F_2^q(t)$$

- GPDs are 3D functions describing partonic structure of nucleons:

$$H^q(x,b_\perp) = \int \frac{d^2 b_\perp}{(2\pi)^2} e e^{-ib_\perp . b_\perp} H^q(x,\xi=0,t=-\Delta_\perp^2)$$