The muon $g - 2$: A sensitive probe for new physics

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PRISMA Cluster of Excellence, Institute for Nuclear Physics and Helmholtz Institute Mainz

PANDA Lecture Week
GSI Darmstadt
12 December 2017

HELMHOLTZ ASSOCIATION
Helmholtz Institute Mainz

PRISMA

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Magnetic moment of particles and nuclei

Particle with charge *e* and mass *m*:

$$\boldsymbol{\mu} = g\,\frac{e\hbar}{2m}\,\boldsymbol{S}, \qquad \boldsymbol{S} = \frac{\boldsymbol{\sigma}}{2}$$

Pauli equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = \left\{\frac{1}{2m}\left[\boldsymbol{\sigma}\cdot(\boldsymbol{p}-e\boldsymbol{A})\right]^2 + e\Phi\right\}\psi(\boldsymbol{x},t)$$

$$\Leftrightarrow$$

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = \left\{\frac{1}{2m}(\boldsymbol{p}-e\boldsymbol{A})^2 + e\Phi - \frac{e\hbar}{2m}\,\boldsymbol{\sigma}\cdot\boldsymbol{B}\right\}\psi(\boldsymbol{x},t)$$

* Non-relativistic limit of Dirac equation: $g = 2$
* Experimental measurement:

$$g_e = 2.0023193\ldots$$
$$g_\mu = 2.0023318\ldots$$

Anomalous magnetic moment

* Dirac value of $g = 2$ modified by quantum corrections

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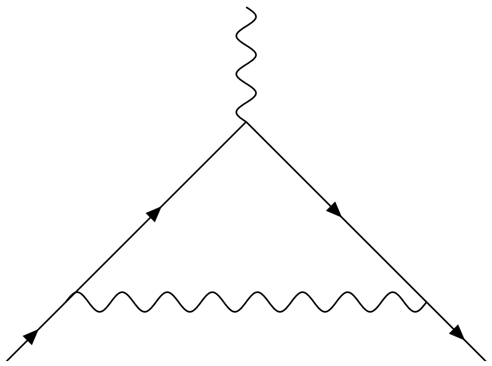
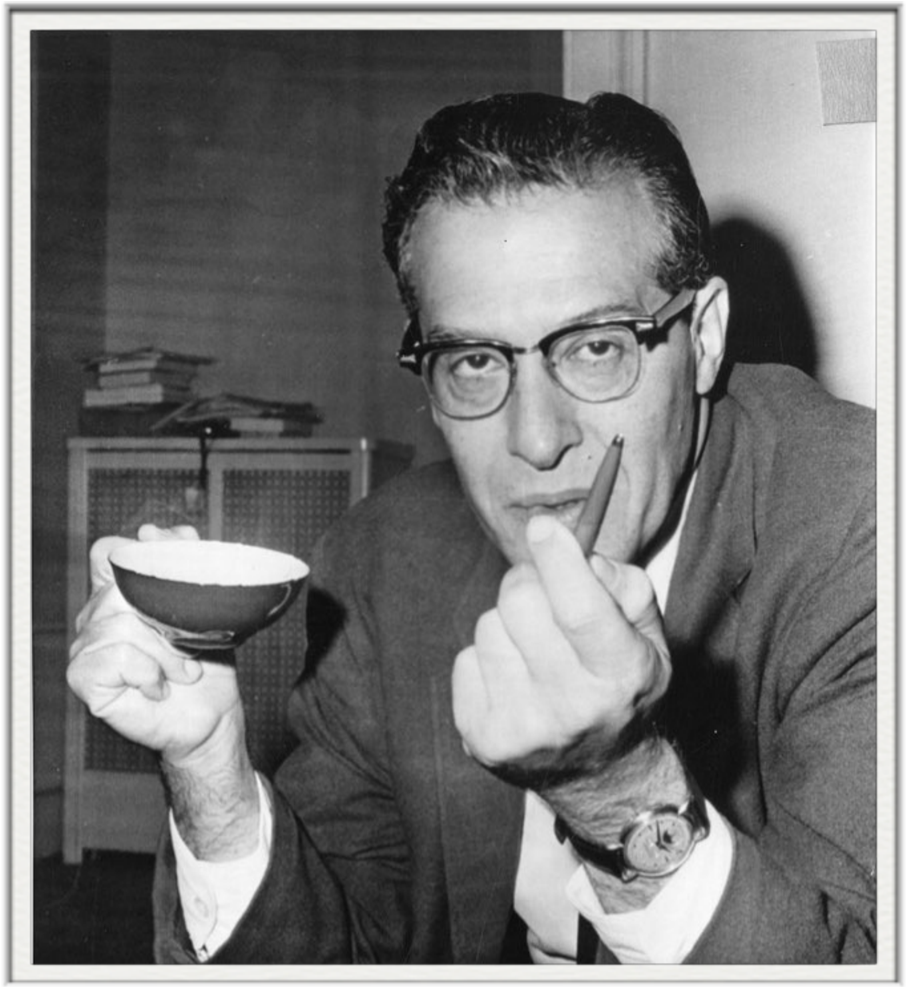


$$a^{(2)} = \frac{\alpha}{2\pi} = 0.001\,161\,40\ldots$$

[J. Schwinger, Phys Rev 73 (1948) 416]



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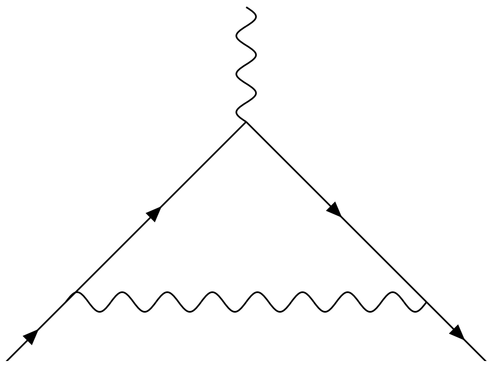

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$$a_\mu^{\exp} = 0.001\,165\,920\,9(6)$$

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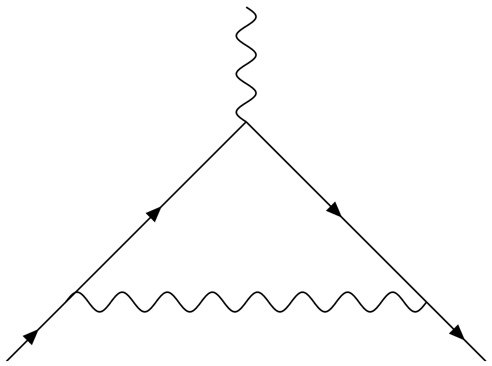

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Higher-order corrections

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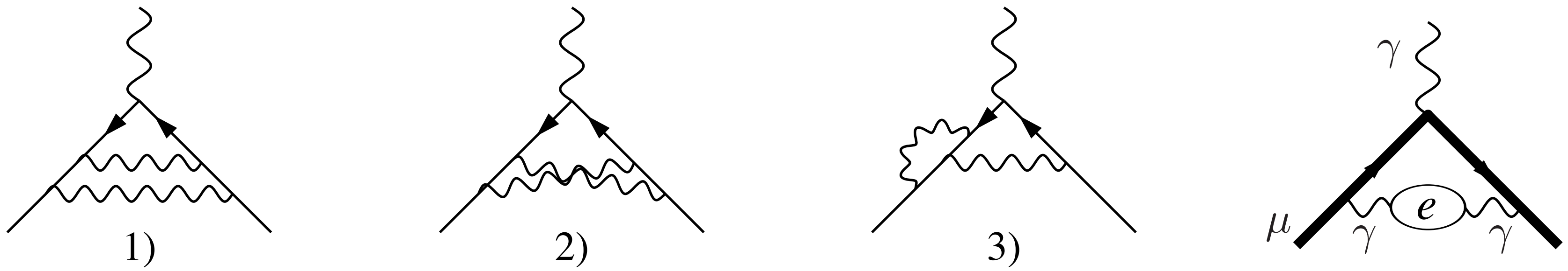

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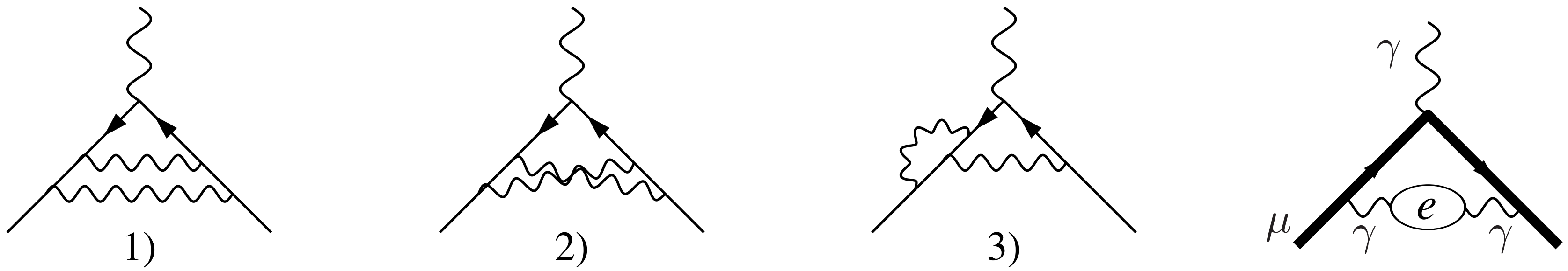

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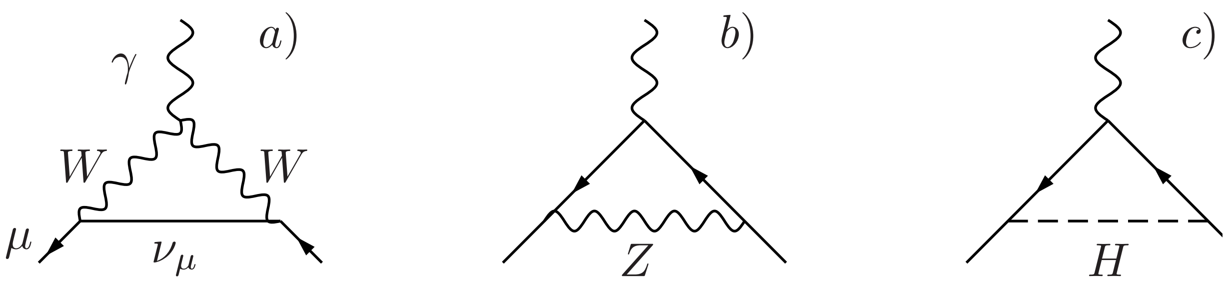

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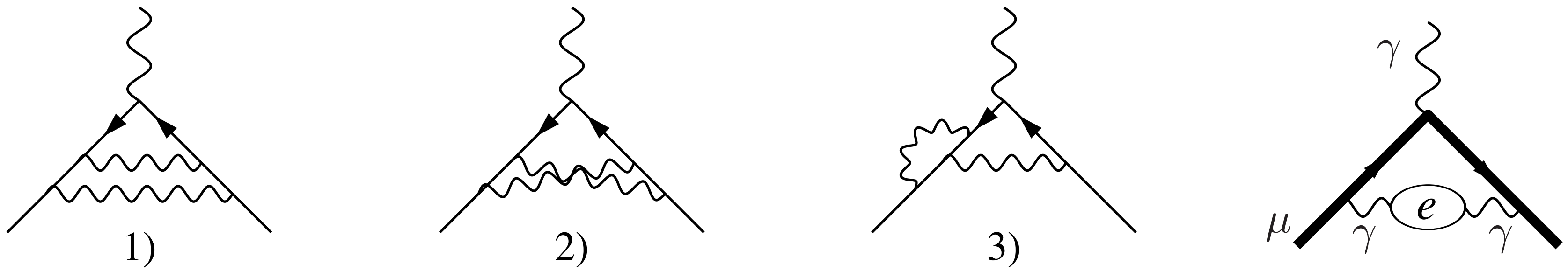

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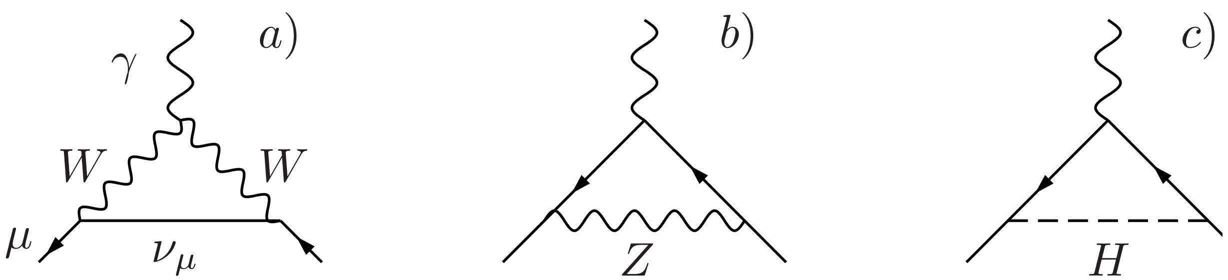

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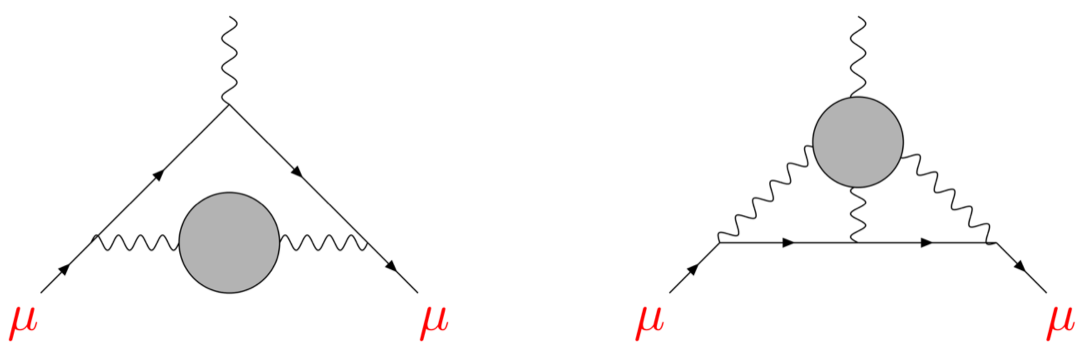
* Strong corrections:



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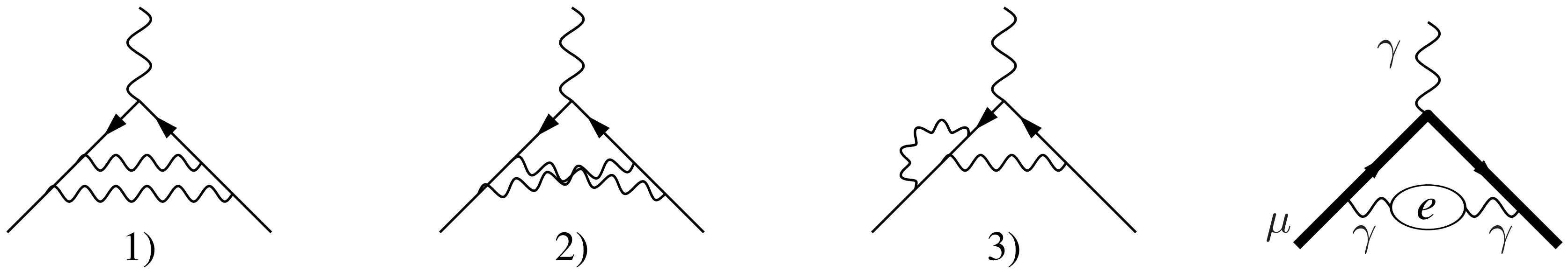

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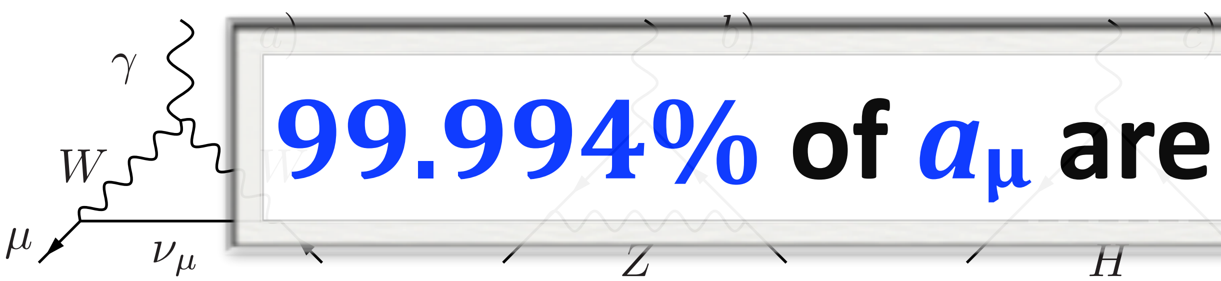
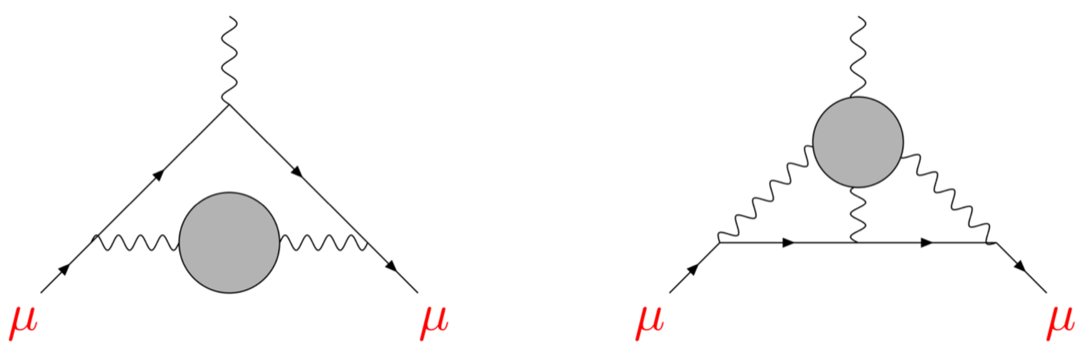


99.994% of a_μ are due to QED!

* Strong corrections:



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Why do we care?

* Standard Model estimate of a_μ deviates from experiment:

$$a_\mu^{\mathrm{exp}} = 116\,592\,089\,(54)_{\mathrm{stat}}\,(33)_{\mathrm{syst}} \cdot 10^{-11}$$ E821 @ BNL

$$a_\mu^{\mathrm{SM}} = 116\,591\,776\,(44) \cdot 10^{-11}$$ Jegerlehner 2017

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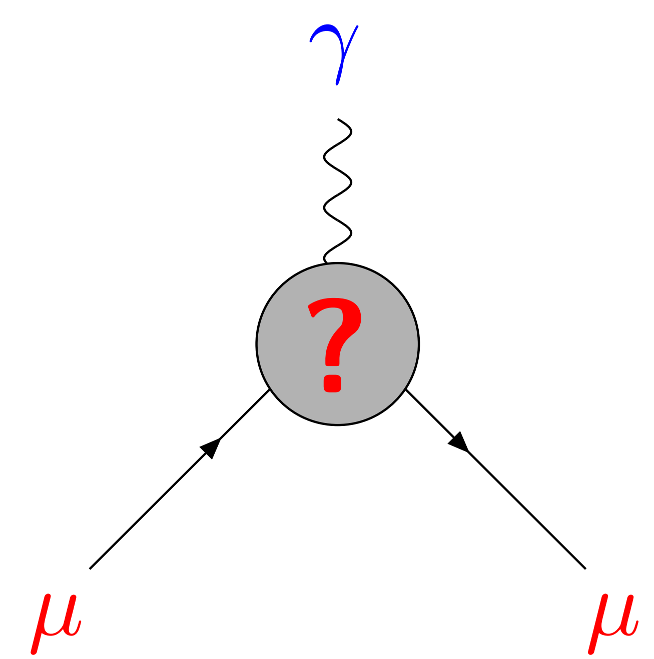
* Deviation may be signal for new physics

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm strong} + a_\mu^{\rm NP?}$$



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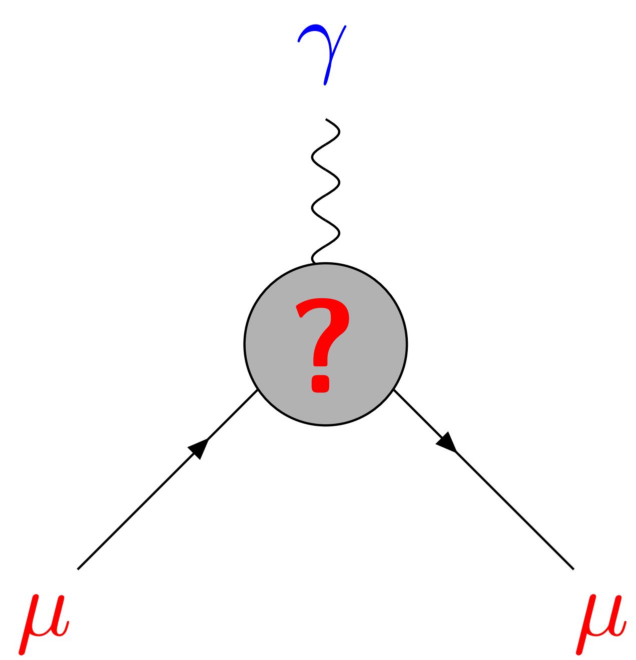

* New physics effects enhanced by

$$\delta a_\ell \propto m_\ell^2 / M_?^2$$

⇒ Muon is more sensitive by a factor

$$(m_\mu/m_e)^2 \approx 4.3 \cdot 10^4$$

Outline

Experimental determination of a_μ

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QED contribution to a_μ

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Hadronic contributions to a_μ from lattice QCD

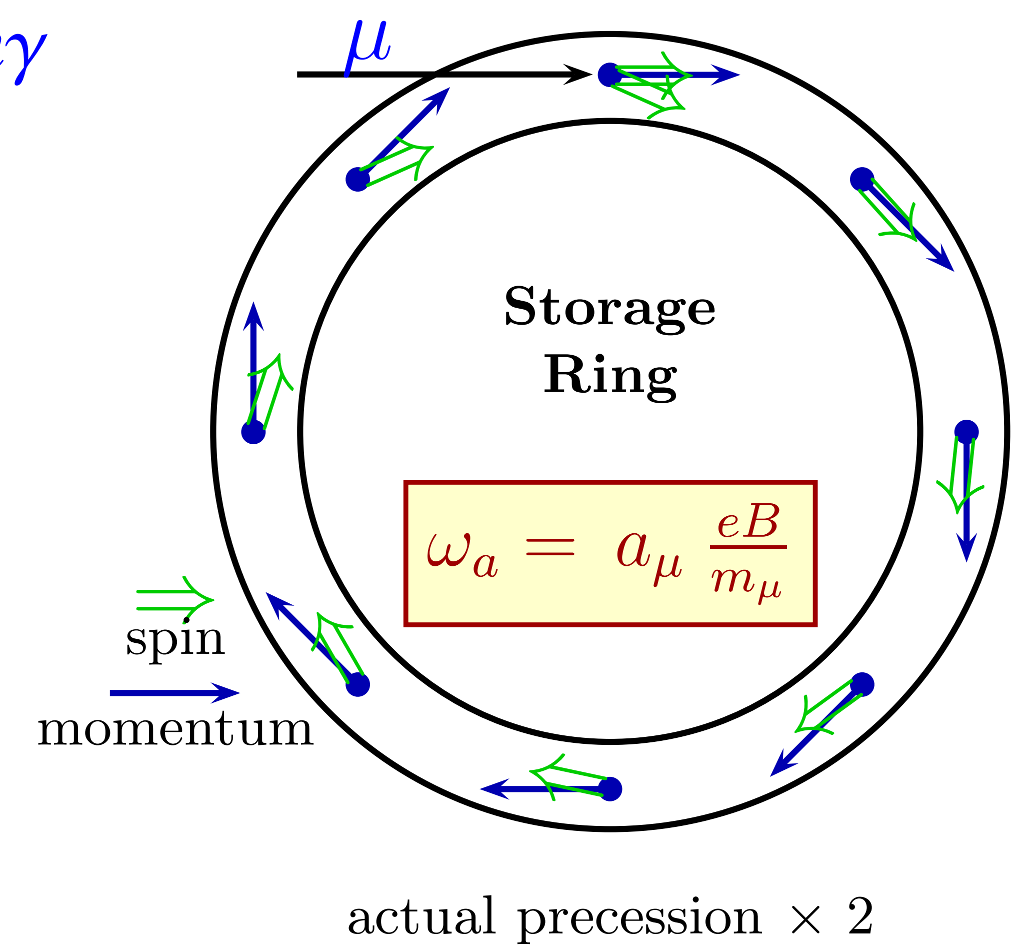
Measuring a_μ at storage rings

* Particle with charge e moving in a magnetic field:
    - Momentum turns with cyclotron frequency ω_C
    - Spin turns with ω_S

$$\omega_C = -\frac{eB}{m\gamma}, \quad \omega_S = -g\,\frac{eB}{2m} - (1-\gamma)\frac{eB}{m\gamma}$$

⇒ Spin turns relative to the momentum with frequency ω_a

$$\omega_a = \omega_S - \omega_C = -\underbrace{\tfrac{1}{2}(g-2)}_{a}\,\frac{eB}{m}$$



[Jegerlehner & Nyffeler, Phys Rep 477 (2009) 1]

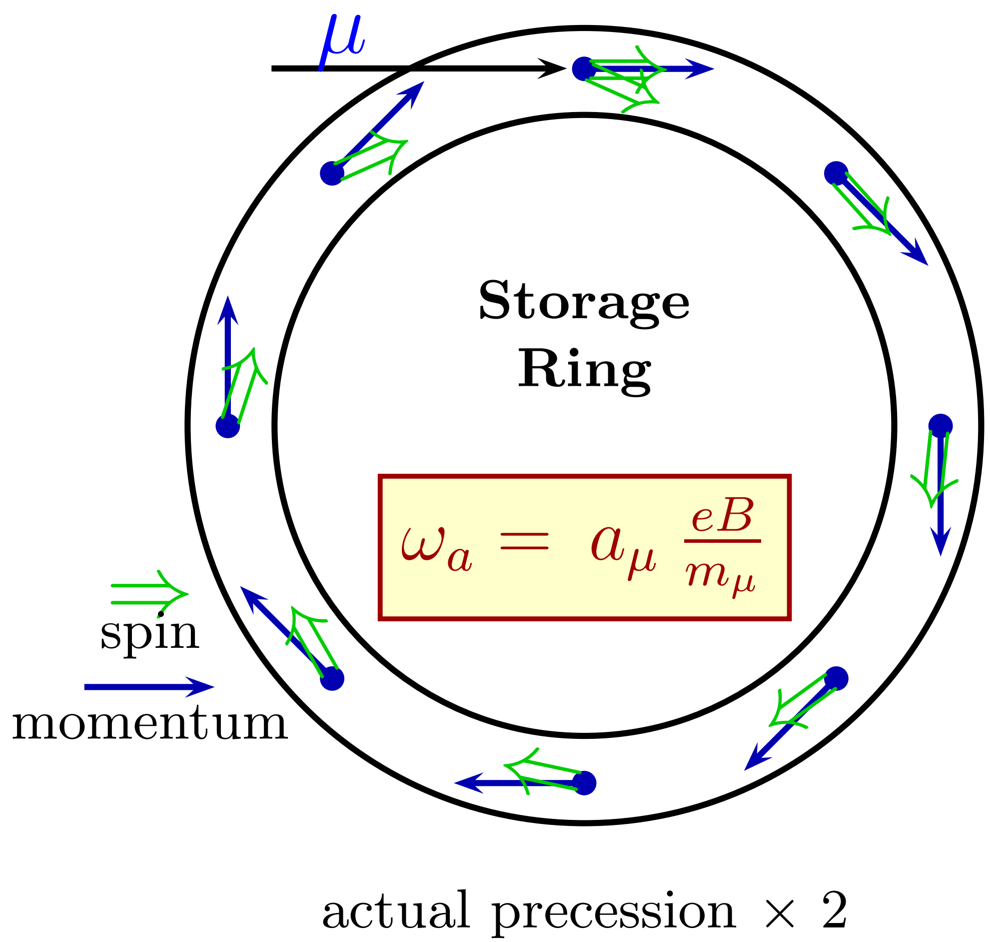
Measuring a_μ at storage rings

* Storage rings require vertical focussing — apply electric quadrupole field

$$\omega_a = -\frac{e}{m}\left\{a_\mu \boldsymbol{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\boldsymbol{\beta}\times\boldsymbol{E}}{c}\right\}$$

[Bargmann, Michel & Telegdi 1959]

* Tune γ such that term $\propto (\boldsymbol{\beta}\times\boldsymbol{E})$ vanishes



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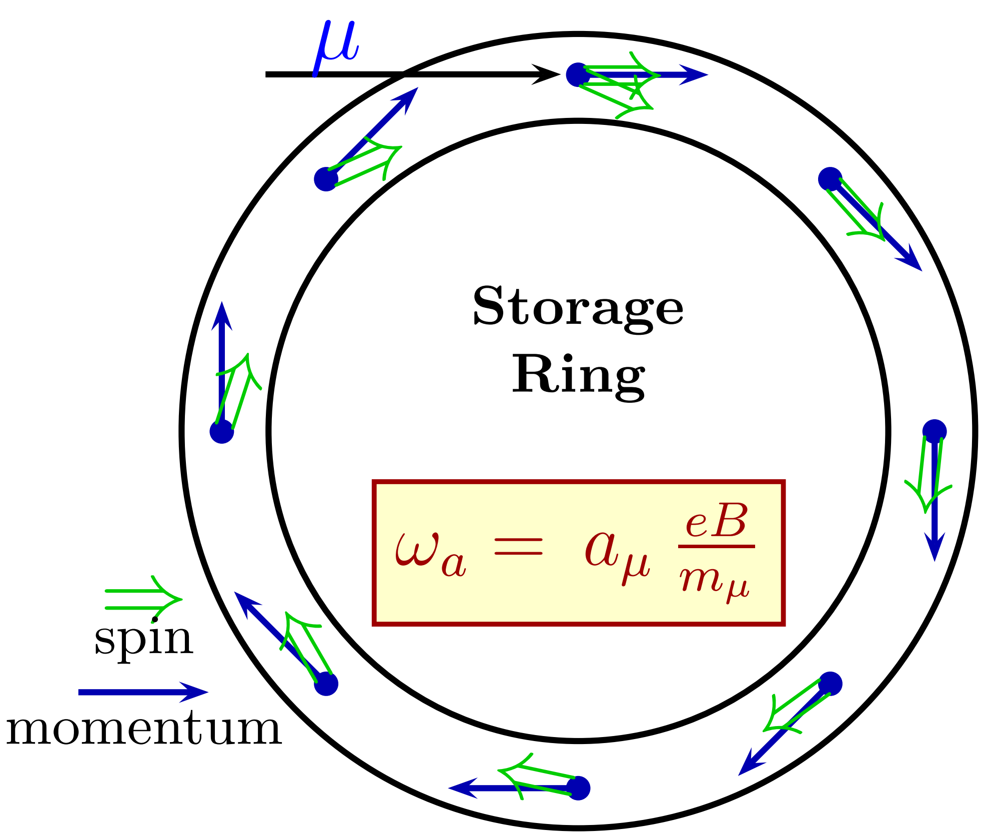
  "magic" γ :

$$\gamma_{\text{magic}} = 29.3 \quad \Leftrightarrow \quad p_{\text{magic}} = 3.09\,\text{GeV}/c$$



actual precession $\times$ 2

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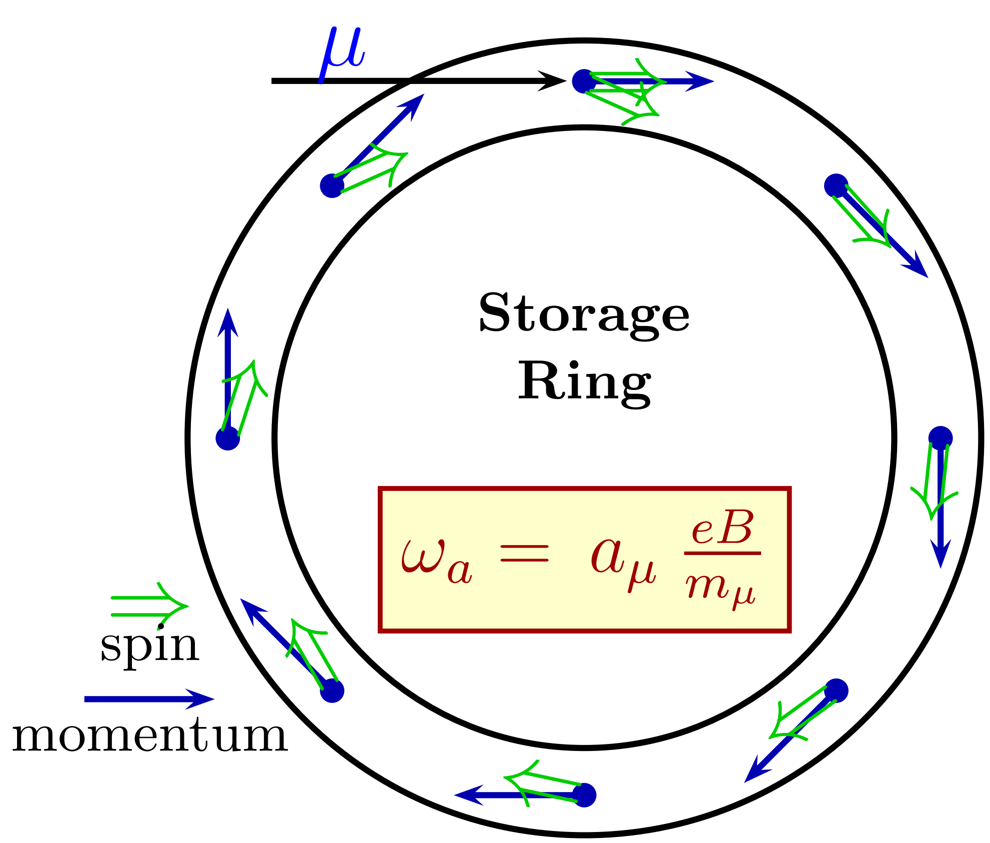

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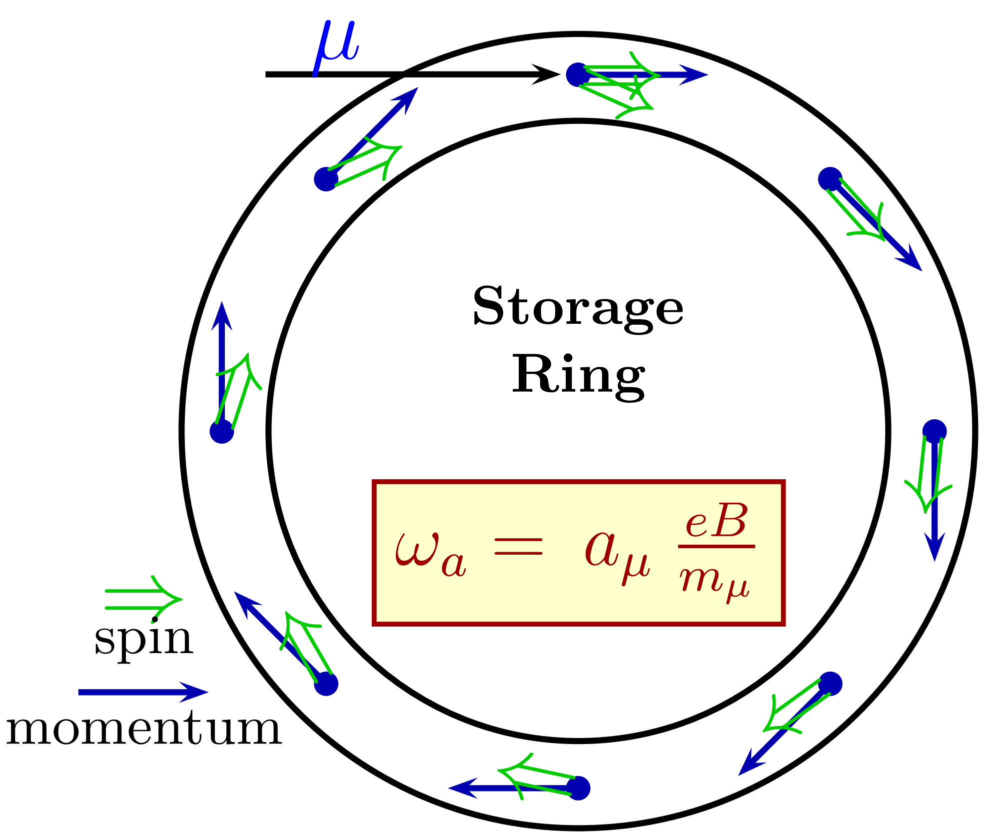
- Measure two quantities: $\quad \omega_a, \; \boldsymbol{B}$



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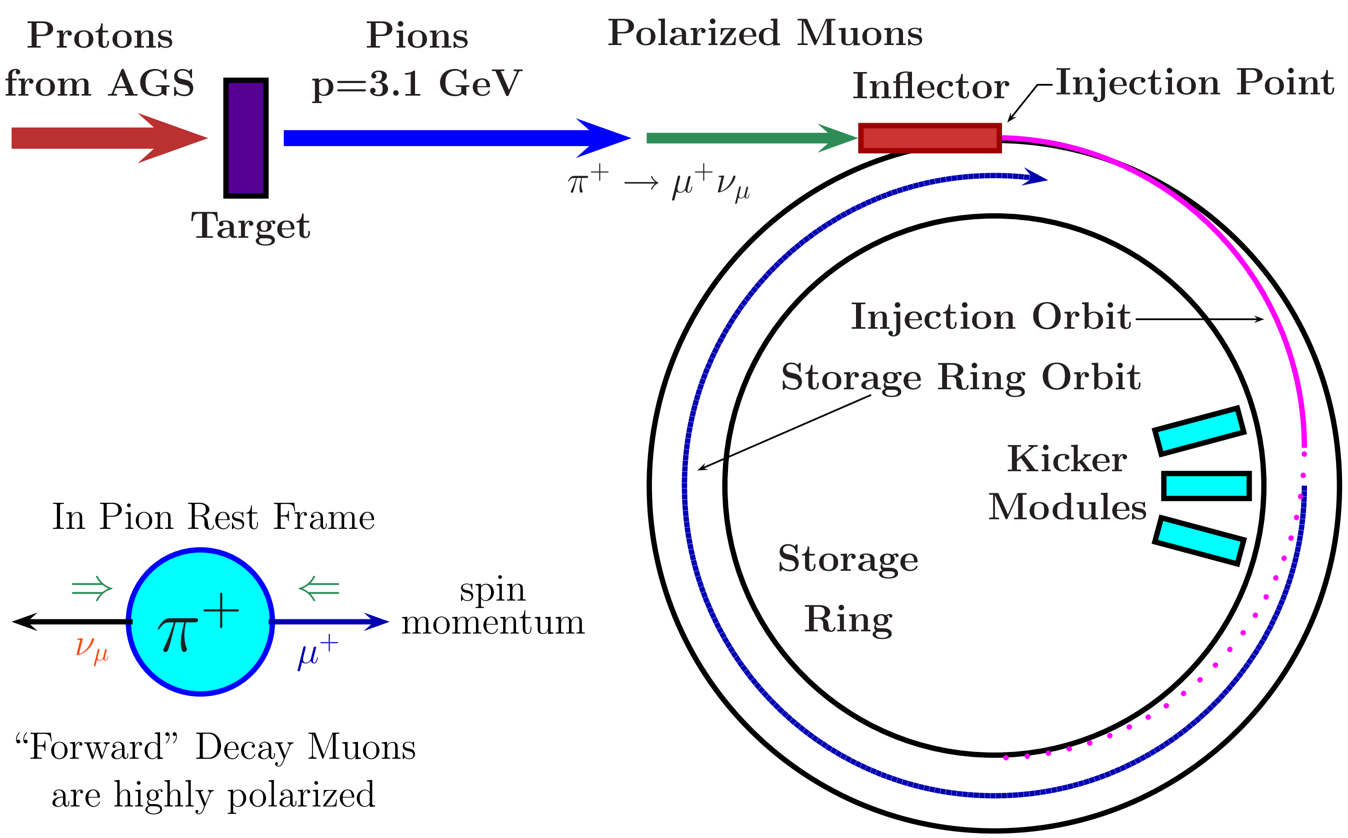
BNL experiment E821

Birth: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

Death: $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$



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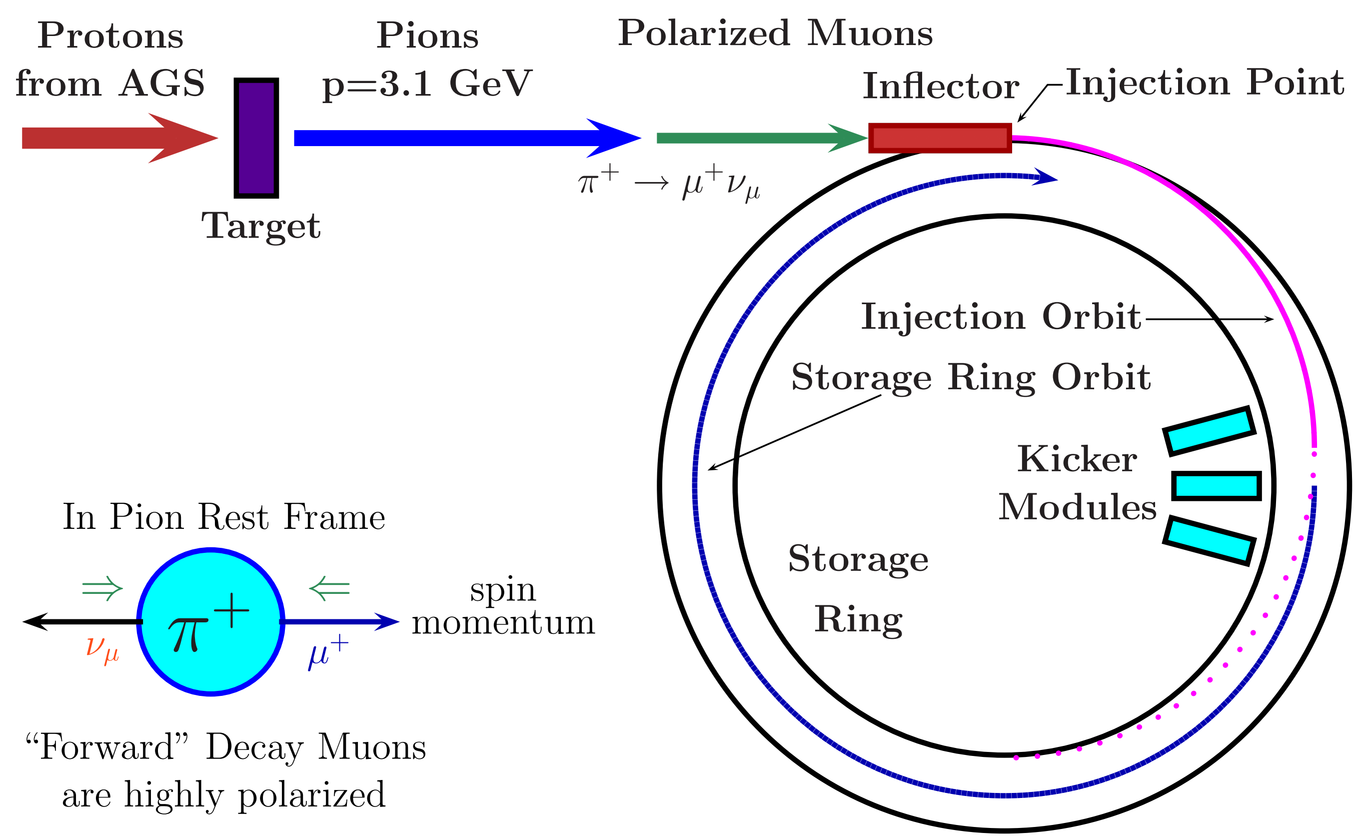
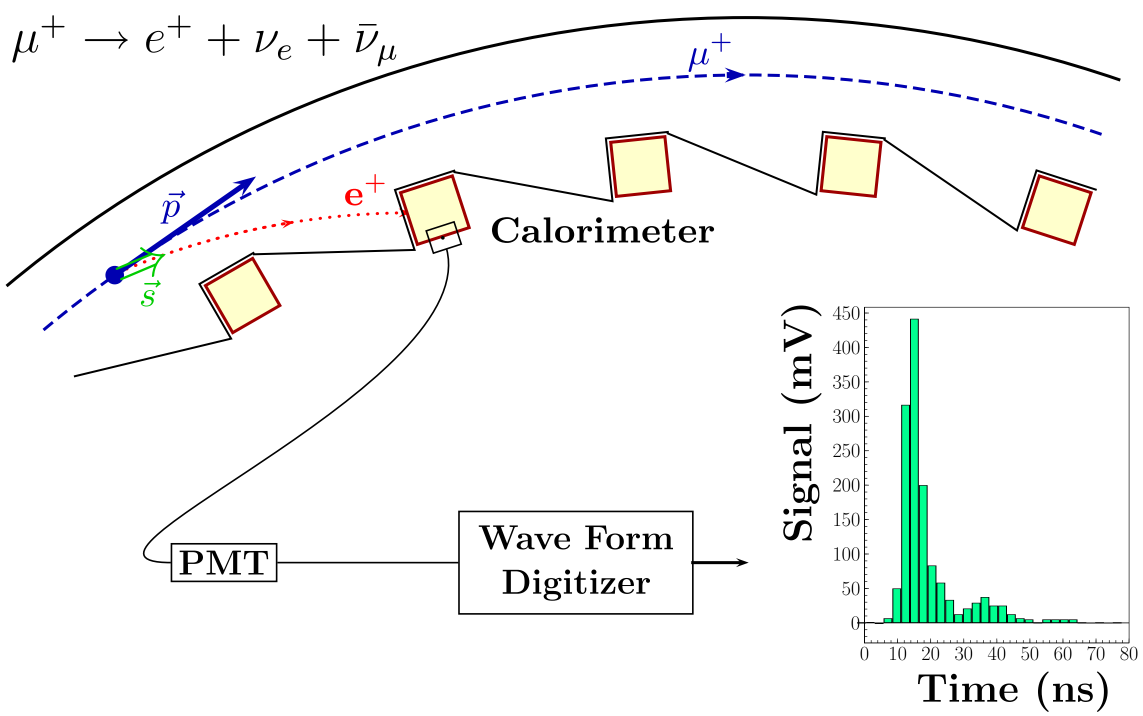




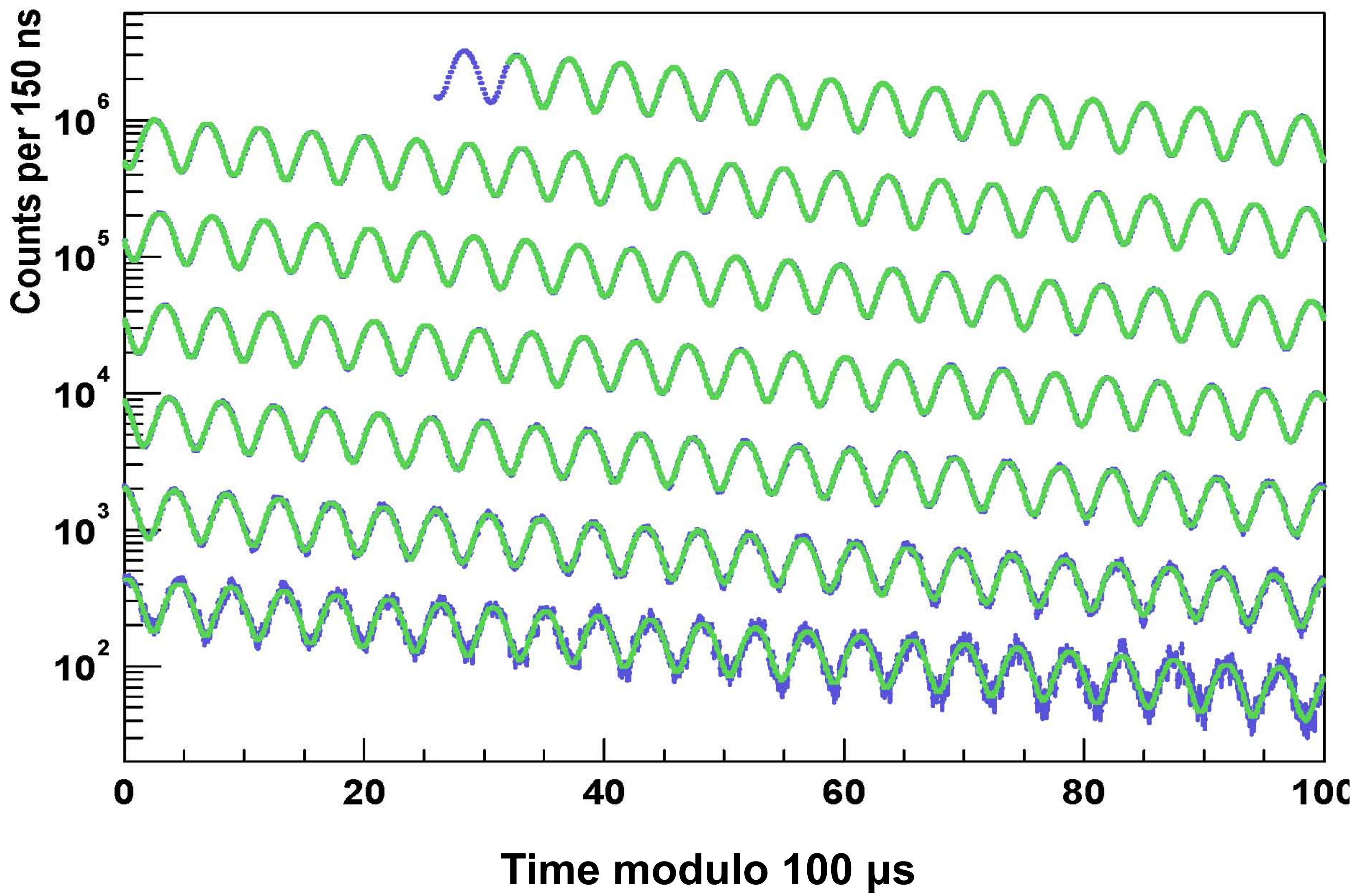
- Highest energy positrons emitted along muon spin axis
- Select positron above threshold energy

[Jegerlehner & Nyffeler, Phys Rep 477 (2009) 1]

BNL experiment E821

* Count rate and wiggle plot:

$$N(t) = N_0(E)\,\exp\left(-\frac{t}{\gamma\tau_\mu}\right)\left\{1 + A(E)\sin\left(\omega_a t + \phi(E)\right)\right\}$$



[B. Lee Roberts]

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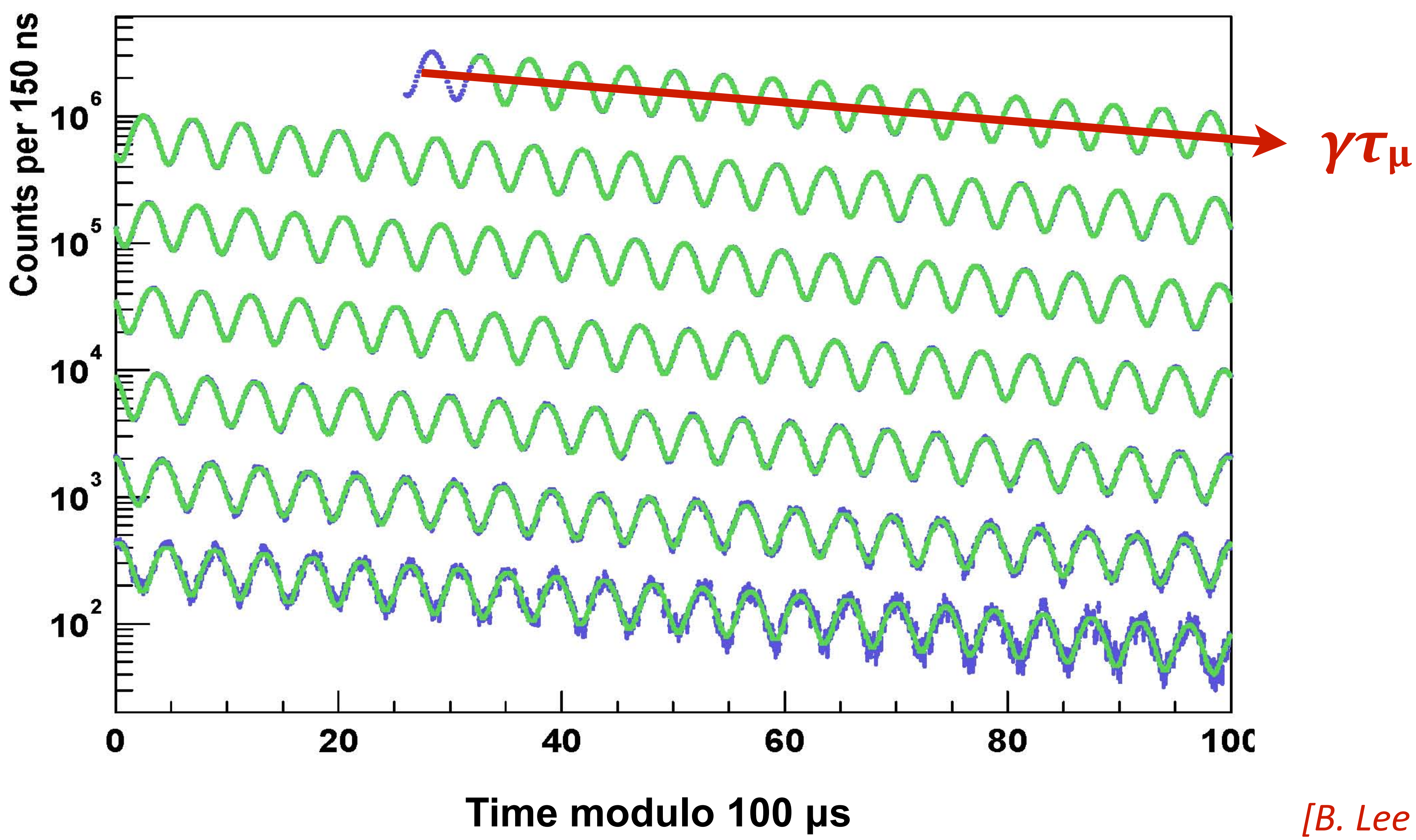

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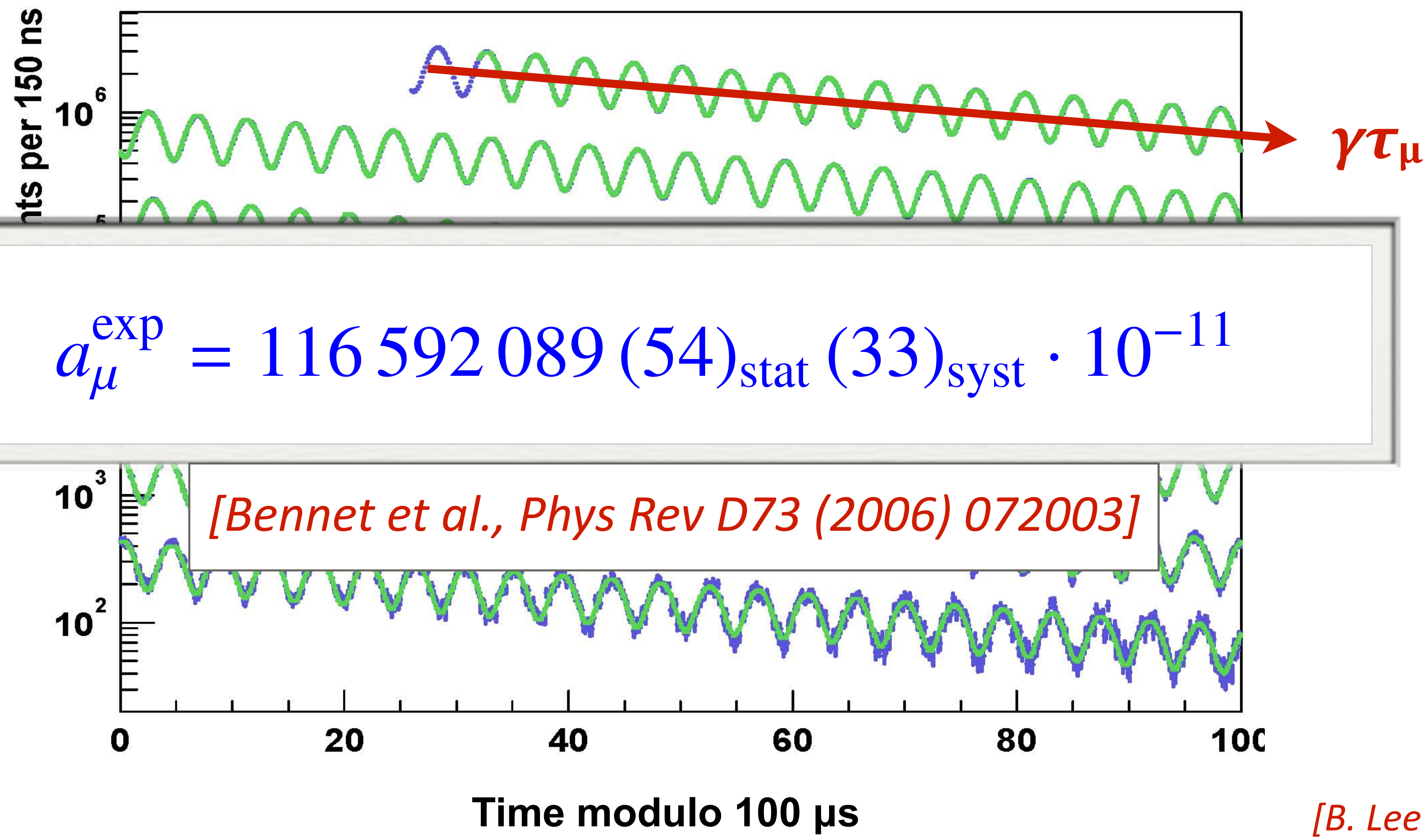

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[Bennet et al., Phys Rev D73 (2006) 072003]

[B. Lee Roberts]

From BNL E821 to Fermilab E989

$$a_\mu^{\rm exp} = 116\,592\,089\,(54)_{\rm stat}\,(33)_{\rm syst} \cdot 10^{-11}$$

- Total precision of 0.54 ppm, dominated by statistics
- Use hotter beam of Fermilab proton booster: 8 GeV/c
- Suppress pion background — longer pion decay channel

  BNL: 80 m —> Fermilab: 2 km

- Aim for 100 ppb statistical and 100 ppb systematic error

  —> 0.14 ppm total error

- Transport BNL storage ring to Fermilab


EMMERT

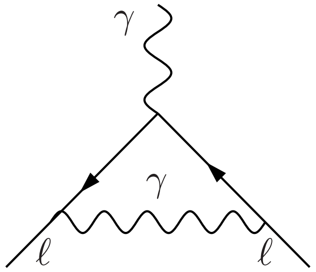
Re-assembly of the storage ring

[©B. Lee Roberts]

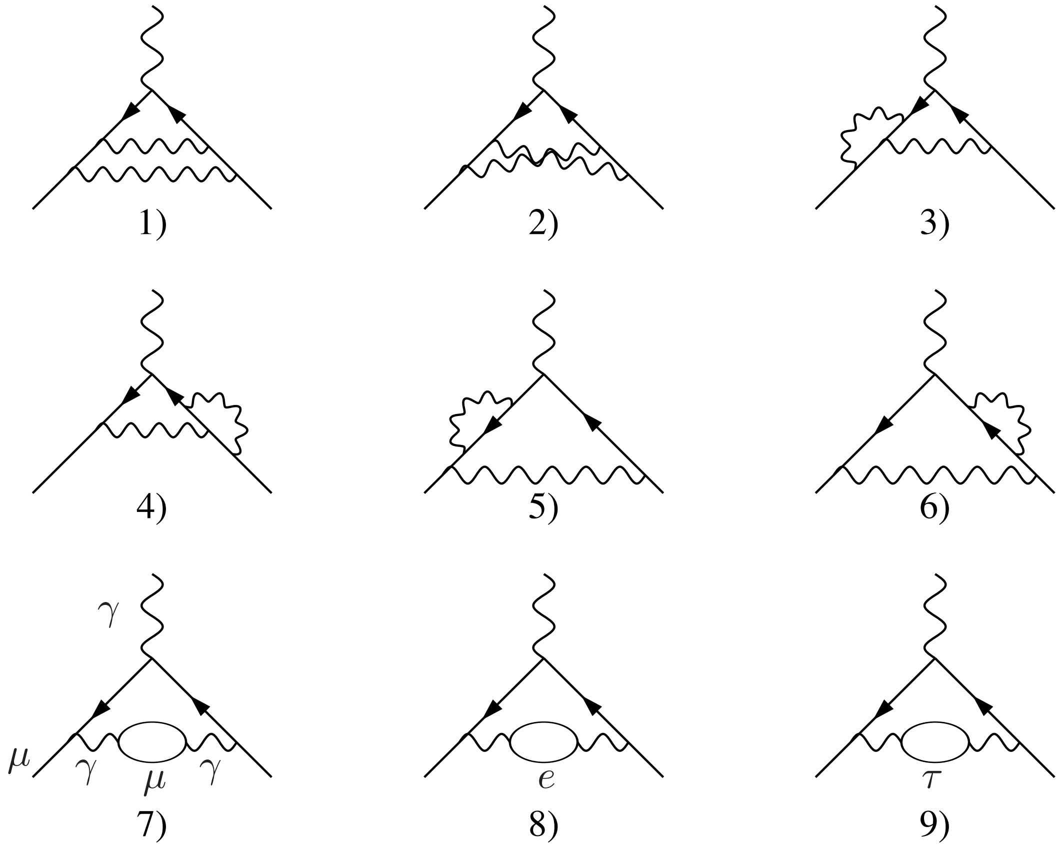
QED contribution to a_μ

- QED contribution has been worked out in perturbation theory to 5^{th} order in α

- Order α (1-loop)



- Order α^2 (2-loop)



QED contribution to a_μ

SM	116 591 776.000	100%	#diagrams
QED, tot	116 584 718.951	99,9939%	
2	116 140 973.318	99,6133%	1
4	413 217.629	0,3544%	9
6	30 141.902	0,0259%	72
8	381.008	0,0003%	891
10	5.094	$4 \cdot 10^{-6}$ %	12672

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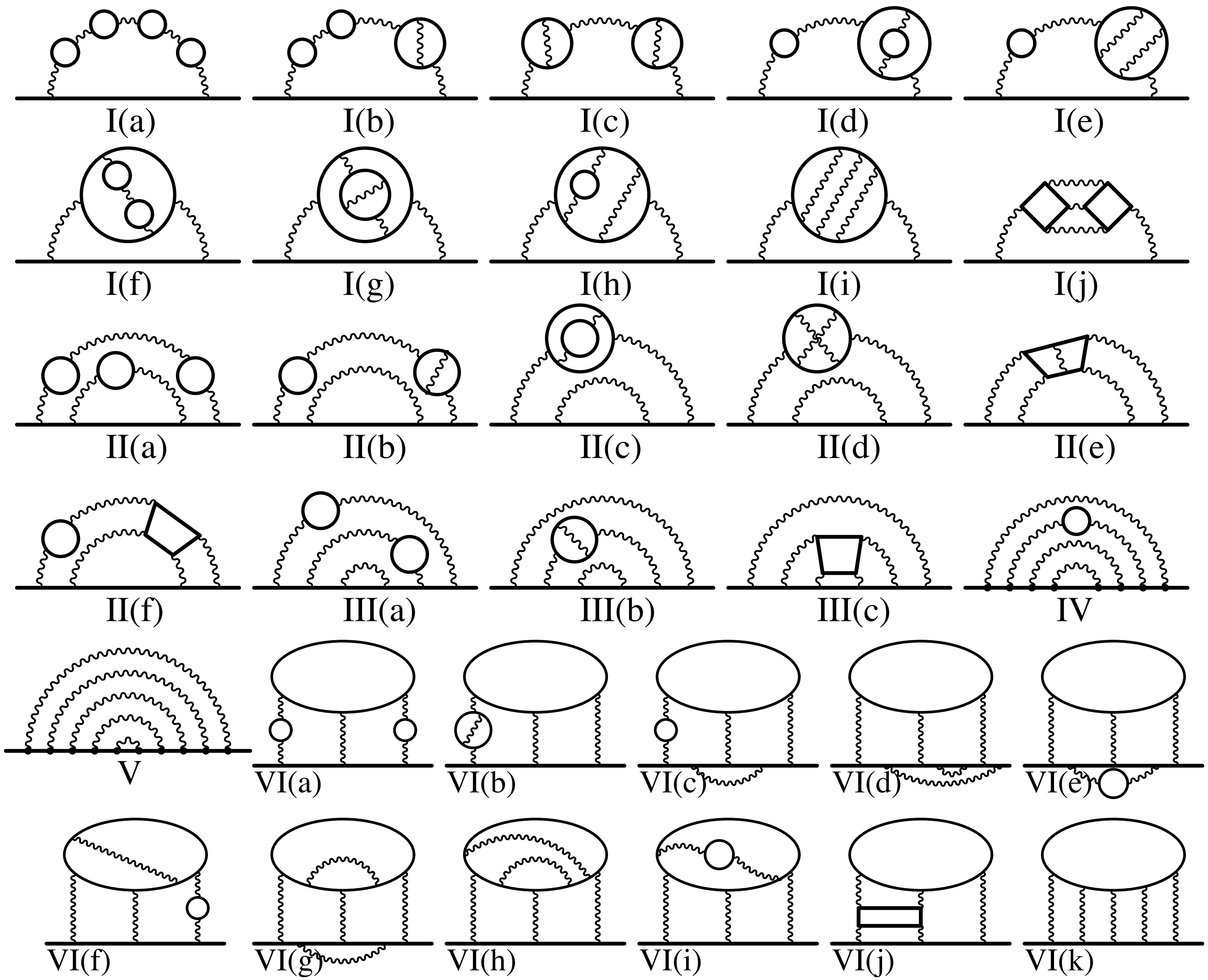
PRL **109,** 111808 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

Complete Tenth-Order QED Contribution to the Muon $g - 2$

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

[1]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[2]*Nishina Center, RIKEN, Wako, Japan 351-0198*
[3]*Department of Physics, Nagoya University, Nagoya, Japan 464-8602*
[4]*Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York 14853, USA*
(Received 24 May 2012; published 13 September 2012)

QED contribution to a_μ



QED contribution to a_μ

Physics Letters B 772 (2017) 232–238

High-precision calculation of the 4-loop contribution to the electron g-2 in QED

Stefano Laporta

Dipartimento di Fisica, Università di Bologna, Istituto Nazionale Fisica Nucleare, Sezione di Bologna, Via Irnerio 46, I-40126 Bologna, Italy

A B S T R A C T

I have evaluated up to 1100 digits of precision the contribution of the 891 4-loop Feynman diagrams contributing to the electron g-2 in QED. The total mass-independent 4-loop contribution is

$$a_e = -1.912245764926445574152647167439830054060873390658725345\ldots\left(\frac{\alpha}{\pi}\right)^4 .$$

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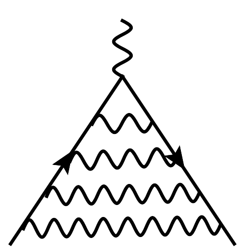

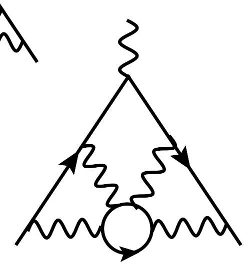

[Marquard,Smirnov,Smirnov, Steinhauser,Wellmann]	[Laporta'17]	[Aoyama,Hayakawa, [Kinoshita,Nio'12]
-2.161 ± 0.065		-2.1755 ± 0.0020
	$-2.176866027739540077443259355895893938670$	
0.077 ± 0.031		0.05596 ± 0.0001
	$0.056110899897828364831469274418908884223$	

Theory confronts experiment

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471 . 8851	0 . 036	Remiddi, Kinoshita et al.
Leading hadronic vac. pol.	688 . 77	3 . 38	data-driven $e^+e^- + \tau$
Subleading hadronic vac. pol.	-9 . 927	0 . 072	2016 update
NNLO hadronic vac. pol.	1 . 224	0 . 010	[31]
Hadronic light–by–light	10 . 34	2 . 88	[46, 69]
Weak incl. 2-loops	15 . 36	0 . 11	[11, 70]
Theory	11 659 177 . 6	4 . 4	–
Experiment	11 659 209 . 1	6 . 3	[2] updated
Exp.- The. 4.1 standard deviations	31 . 3	7 . 7	–

[Jegerlehner, arXiv:1705.00263]

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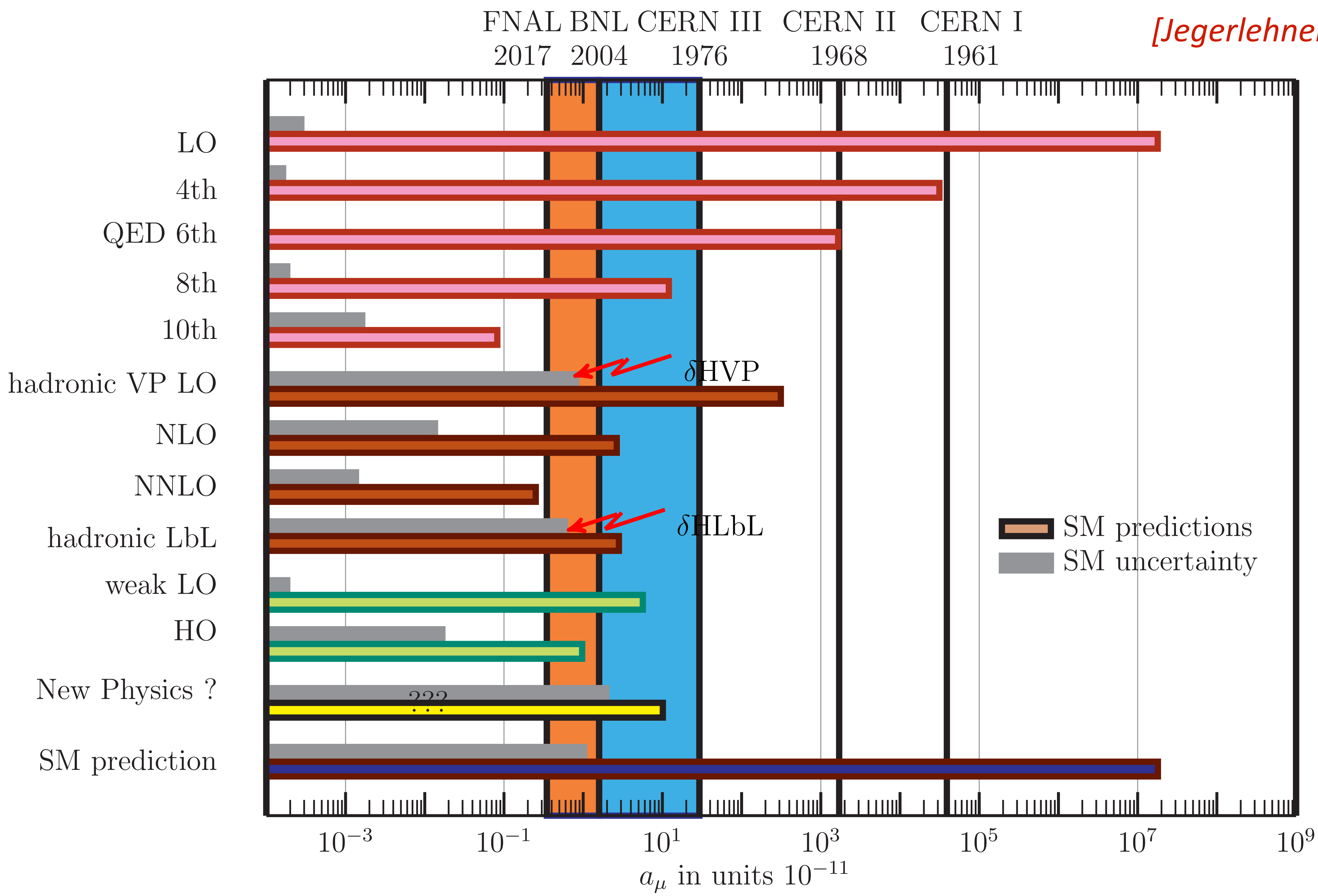
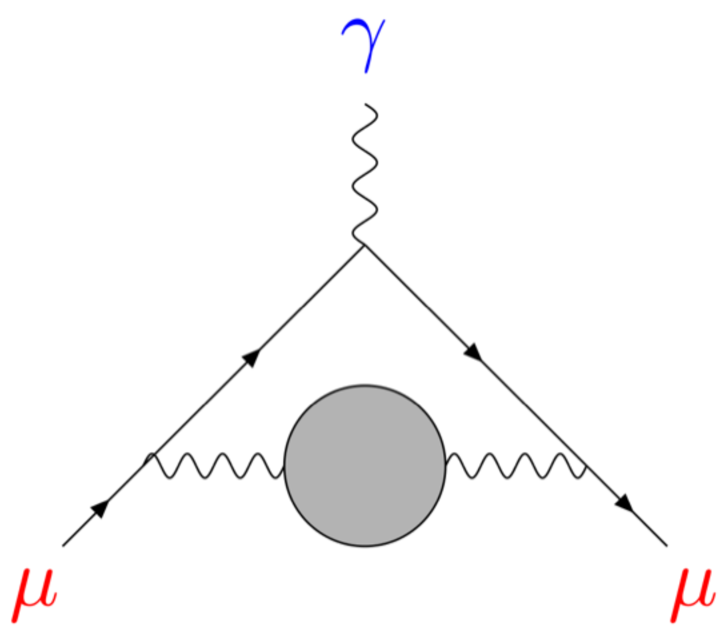


* Experimental sensitivity of E989 exceeds total theory uncertainty by far!

Hadronic contributions to a_μ

Hadronic vacuum polarisation:

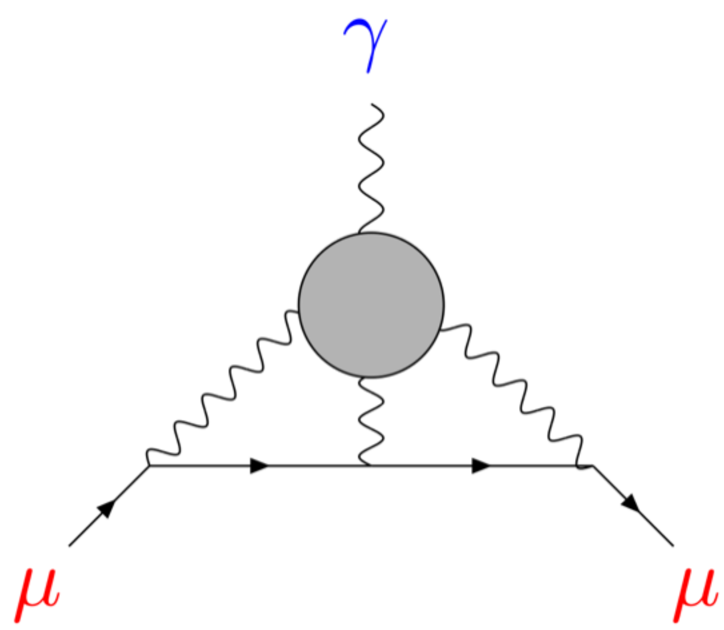


Dispersion theory:

$$a_\mu^{\mathrm{hvp}} = (6888 \pm 34) \cdot 10^{-11}$$

(combined e^+e^- and τ data)

Hadronic light-by-light scattering:



Model estimates:

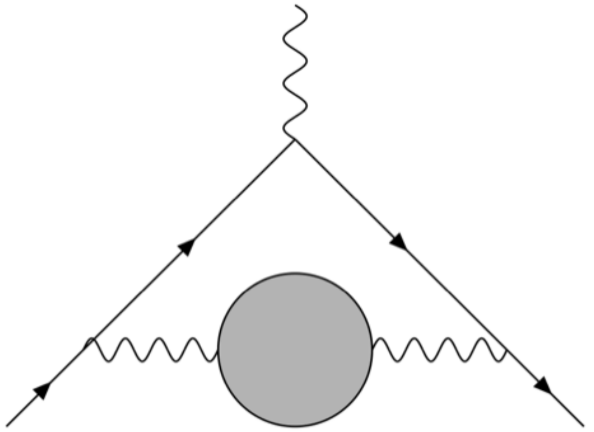
$$a_\mu^{\mathrm{hlbl}} = (105 \pm 26) \cdot 10^{-11}$$

"Glasgow consensus"

Hadronic vacuum polarisation

* Hadronic electromagnetic current:

$$J^\mu(x) = \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s + \tfrac{2}{3}\bar{c}\gamma^\mu c + \ldots$$



$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^\infty ds\, \frac{R_{\rm had}(s)\,\hat{K}(s)}{s^2}, \quad R_{\rm had}(s) = \sigma(e^+e^- \to \text{hadrons}) \Big/ \frac{4\pi\,\alpha(s)}{(3s)}$$

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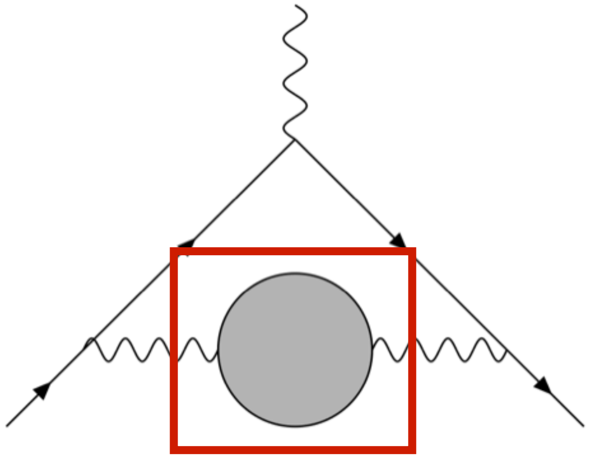

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\,\Pi(q^2) = ie^2 \int d^4x\, \mathrm{e}^{iq\cdot x}\, \langle 0|T\, J^\mu(x) J^\nu(0)|0\rangle$$

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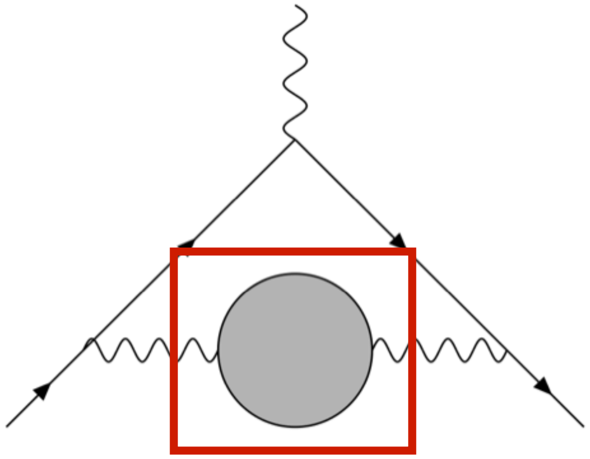
Hadronic vacuum polarisation



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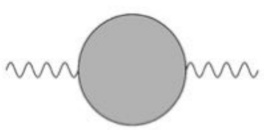
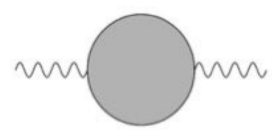
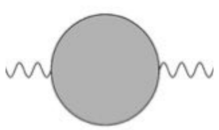
* **Optical theorem:**



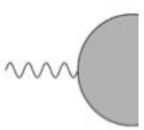
$$= \int \frac{ds}{\pi(s-q^2)}\,\mathrm{Im}$$



$$2\,\mathrm{Im}$$

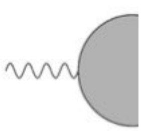


$$= \sum_{\text{had}} \int d\Phi \,\Bigg|$$



$$\Bigg|^2$$

$$\Bigg|$$



$$\Bigg|^2 \;\propto \sigma(e^+e^- \to \text{hadrons})$$

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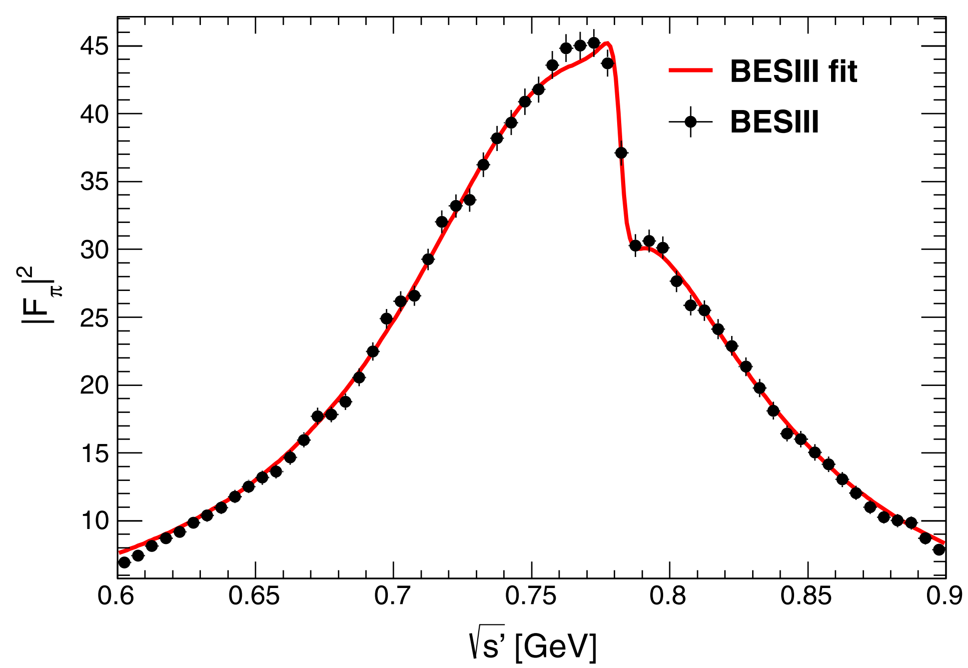
HVP contribution from dispersion relations

* Knowledge of $R_{\rm had}(s)$ required down to pion threshold

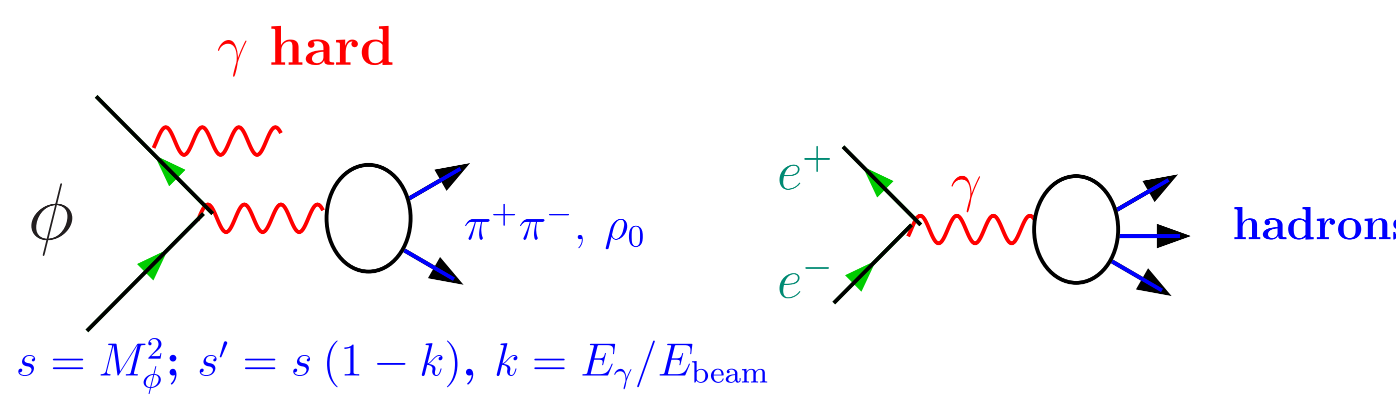
$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left\{ \int_{m_{\pi^0}^2}^{E_{\rm cut}^2} ds\, \frac{R_{\rm had}^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds\, \frac{R_{\rm had}^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right\}$$

⇒ Use experimental data for cross section ratio $R_{\rm had}(s)$



[BESIII Collaboration, 2016]

Initial state radiation (ISR) vs. beam energy tuning:



[Jegerlehner, arXiv:1705.00263]

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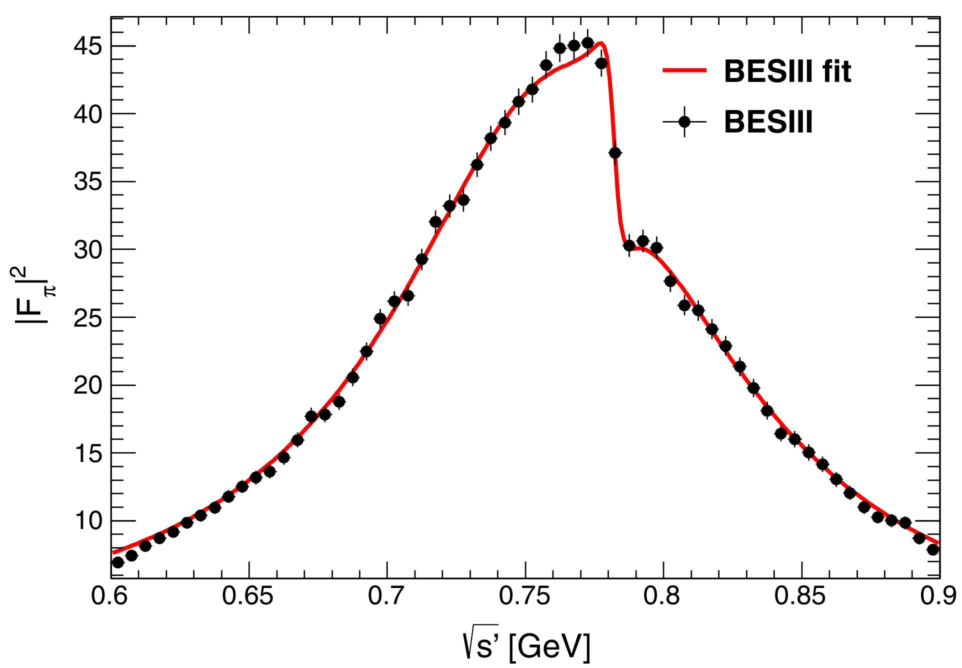
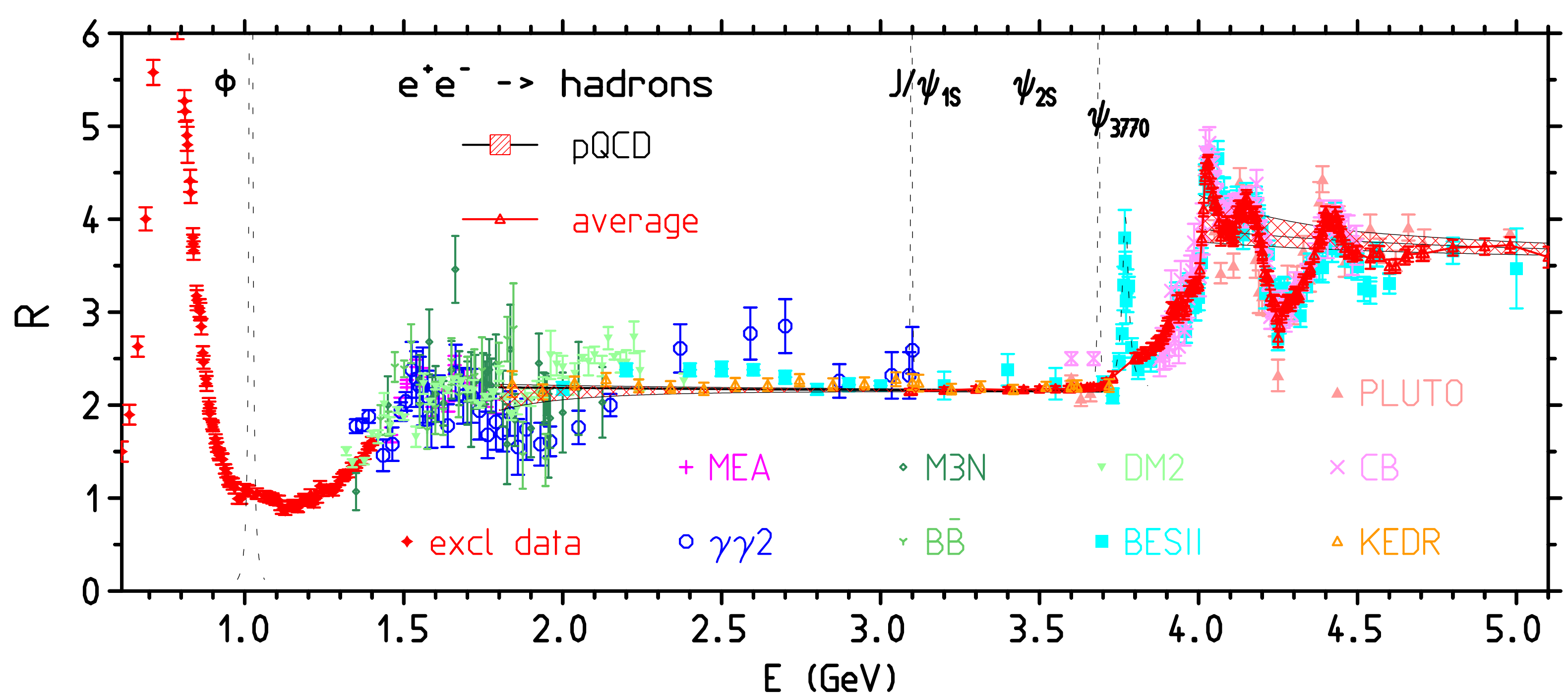
$\Rightarrow$ Use experimental data for cross section ratio $R_{\rm had}(s)$



[BESIII Collaboration, 2016]



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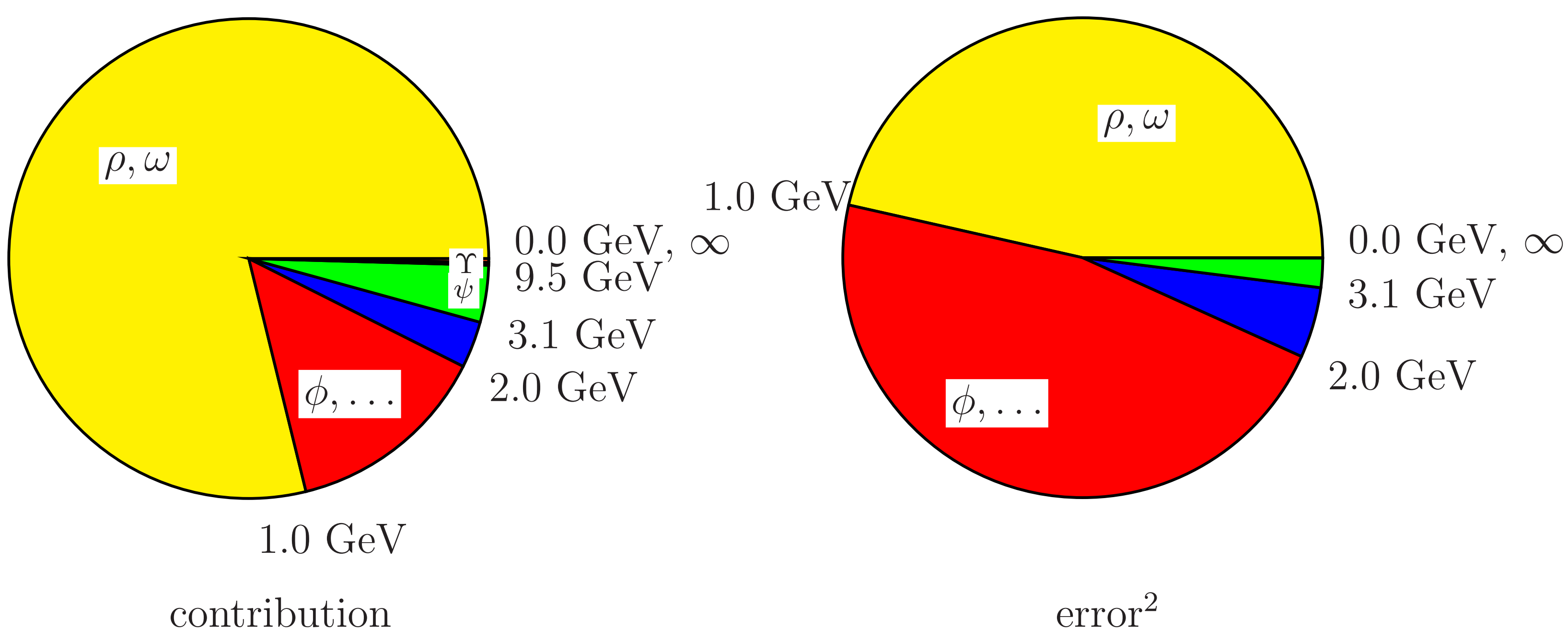
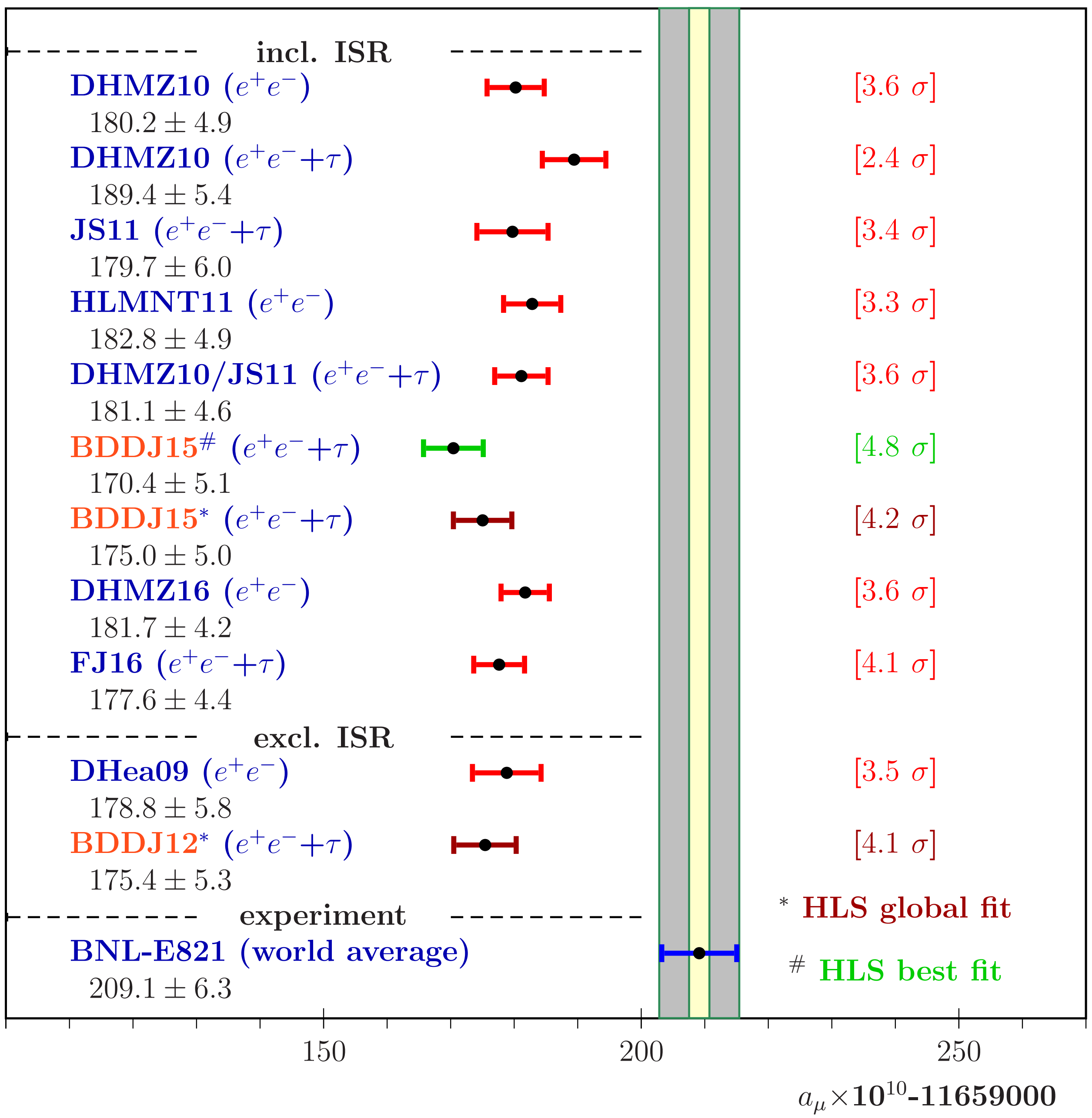
⇒ Use experimental data for hadronic cross section $R_{\rm had}(s)$



* Low-energy region dominates

[Jegerlehner, arXiv:1705.00263]

HVP contribution from dispersion relations



- ✱ Stable deviation of 3–4 standard deviations between SM and experiment
- ✱ Overall precision of HVP estimate: ≈ 0.5%
- ✱ Theory estimate subject to experimental uncertainties

$$a_\mu^{\rm hvp} = (6880.7 \pm 41.4) \cdot 10^{-11}$$

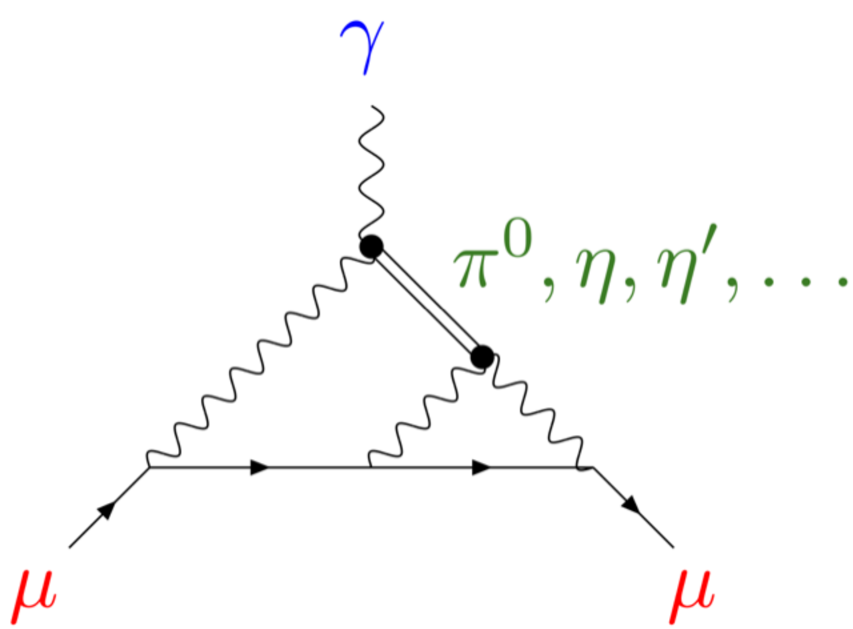
(combined $e^+ e^-$ data)

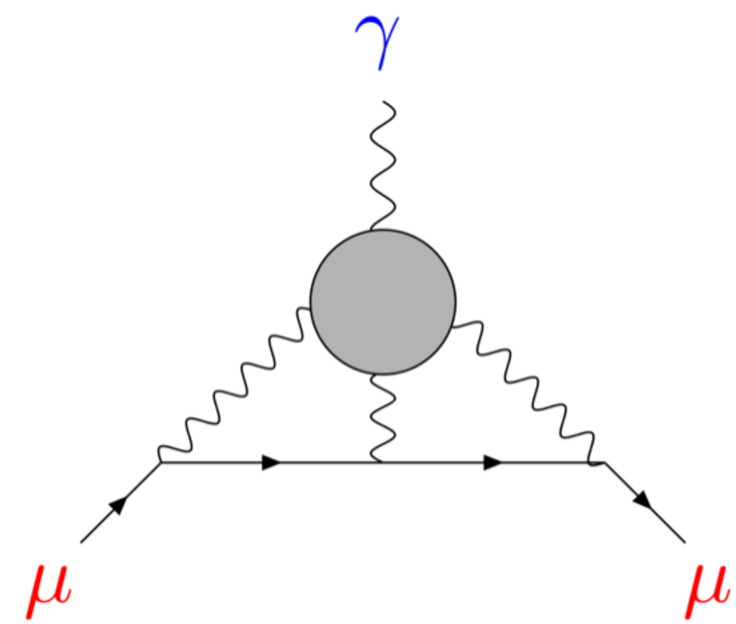
[Jegerlehner, arXiv:1705.00263]

Hadronic Light-by-Light scattering

- No simple dispersive framework

- Identify dominant sub-processes, e.g.





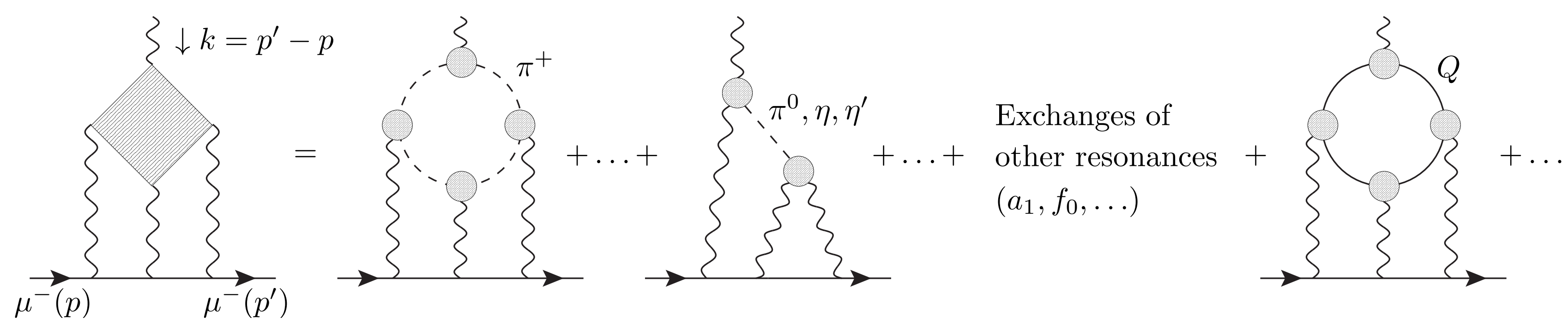
- Individual contributions estimated using model calculations

- Dispersive formalism set up for various subprocesses *[Colangelo et al., 2014 ff]*

- Lattice QCD calculations

Hadronic Light-by-Light scattering

* Dominant hadronic contributions to a_μ^{hlbl} *[Nyffeler, arXiv:1710.09742]*



Contribution:	pion-loop (dressed)	pseudoscalar exchanges		quark-loop (dressed)
Chiral counting:	p^4	p^6	p^8	p^8
N_c counting:	1	N_c	N_c	N_c

Hadronic Light-by-Light scattering

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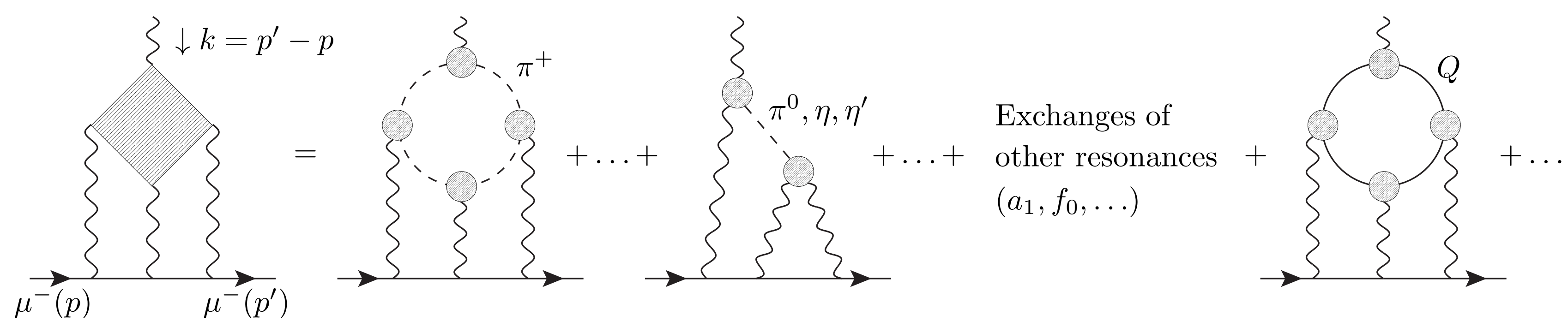

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Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99 ± 16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15 ± 10	22 ± 5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3 (c-quark)	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105 ± 26	116 ± 39

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

Hadronic Light-by-Light scattering

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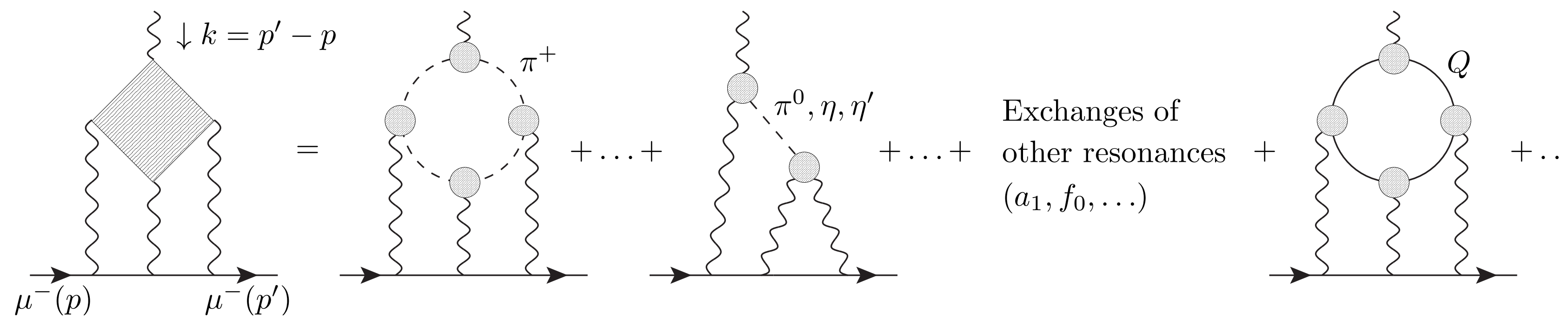

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"Glasgow consensus"

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The muon g – 2 in lattice QCD

Motivation for first-principles approach:

* No reliance on experimental data

  — except for simple hadronic quantities to fix bare parameters

* No model dependence

  — except for chiral extrapolation and constraining the IR regime

The muon g – 2 in lattice QCD

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* No model dependence
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* Can lattice QCD deliver estimates with sufficient accuracy in the coming years?

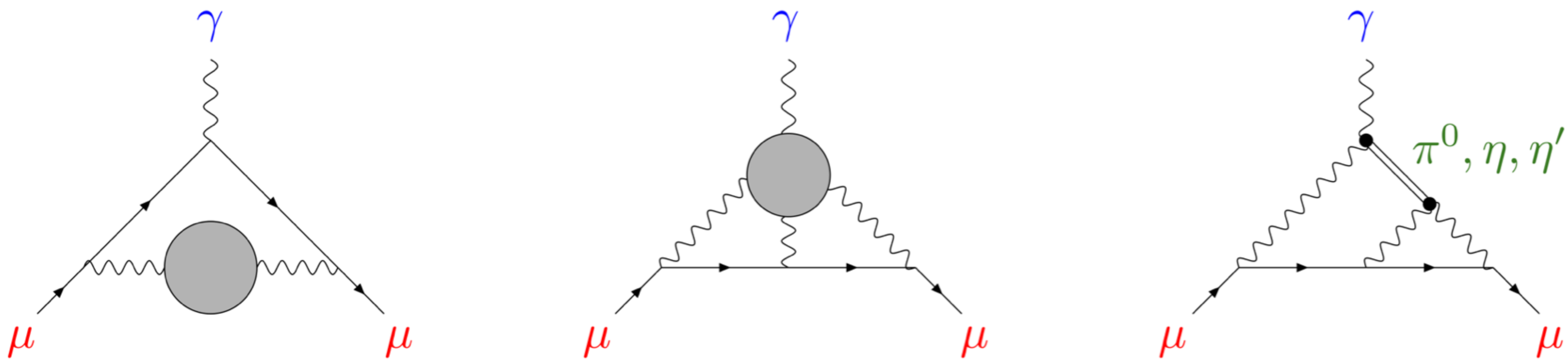
$$\delta a_\mu^{\rm hvp}/a_\mu^{\rm hvp} < 0.5\%, \qquad \delta a_\mu^{\rm hlbl}/a_\mu^{\rm hlbl} \lesssim 10\%$$

The Mainz $(g-2)_\mu$ project

Collaborators:

N. Asmussen, A. Gérardin, O. Gryniuk, G. von Hippel, B. Hörz, H. Horch, H.B. Meyer, A. Nyffeler, V. Pascalutsa, A. Risch, HW

M. Della Morte, A. Francis, J. Green, V. Gülpers, B. Jäger, G. Herdoíza



- Direct determinations of LO $a_\mu^{\rm hvp}$

- Exact QED kernel
- Forward scattering amplitude

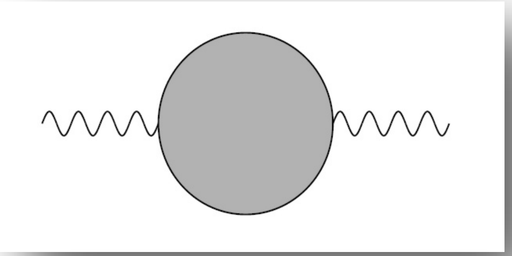
- Transition form factor for $\pi^0 \to \gamma^*\gamma^*$

Lattice QCD approach to HVP

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

$$a_\mu^{\rm hvp} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\left\{\Pi(Q^2) - \Pi(0)\right\}$$

$$\Pi_{\mu\nu}(Q) = \int e^{iQ\cdot(x-y)}\left\langle J_\mu(x) J_\nu(y)\right\rangle \equiv \left(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2\right)\Pi(Q^2)$$



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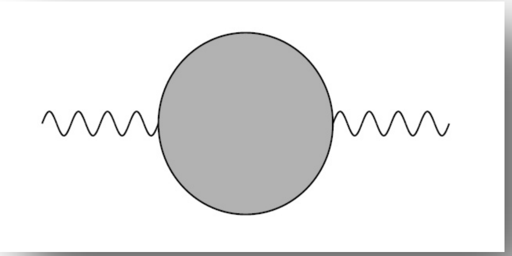

* Determine VPF $\Pi(Q^2)$ and additive renormalisation $\Pi(0)$
* Integrand peaked near $Q^2 \approx (\sqrt{5} - 2) m_\mu^2$

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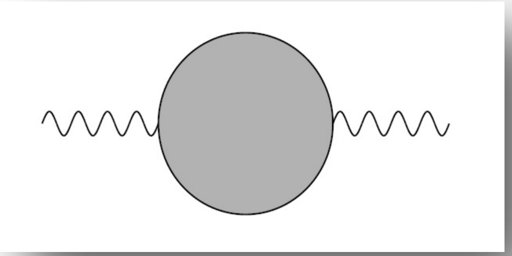

* Determine VPF $\Pi(Q^2)$ and additive renormalisation $\Pi(0)$
* Integrand peaked near $Q^2 \approx (\sqrt{5}-2)m_\mu^2$
* Lattice momenta are quantised: $Q_\mu = \dfrac{2\pi}{L_\mu}$
* Statistical accuracy of $\Pi(Q^2)$ deteriorates as $Q \longrightarrow 0$

Lattice QCD approach to HVP

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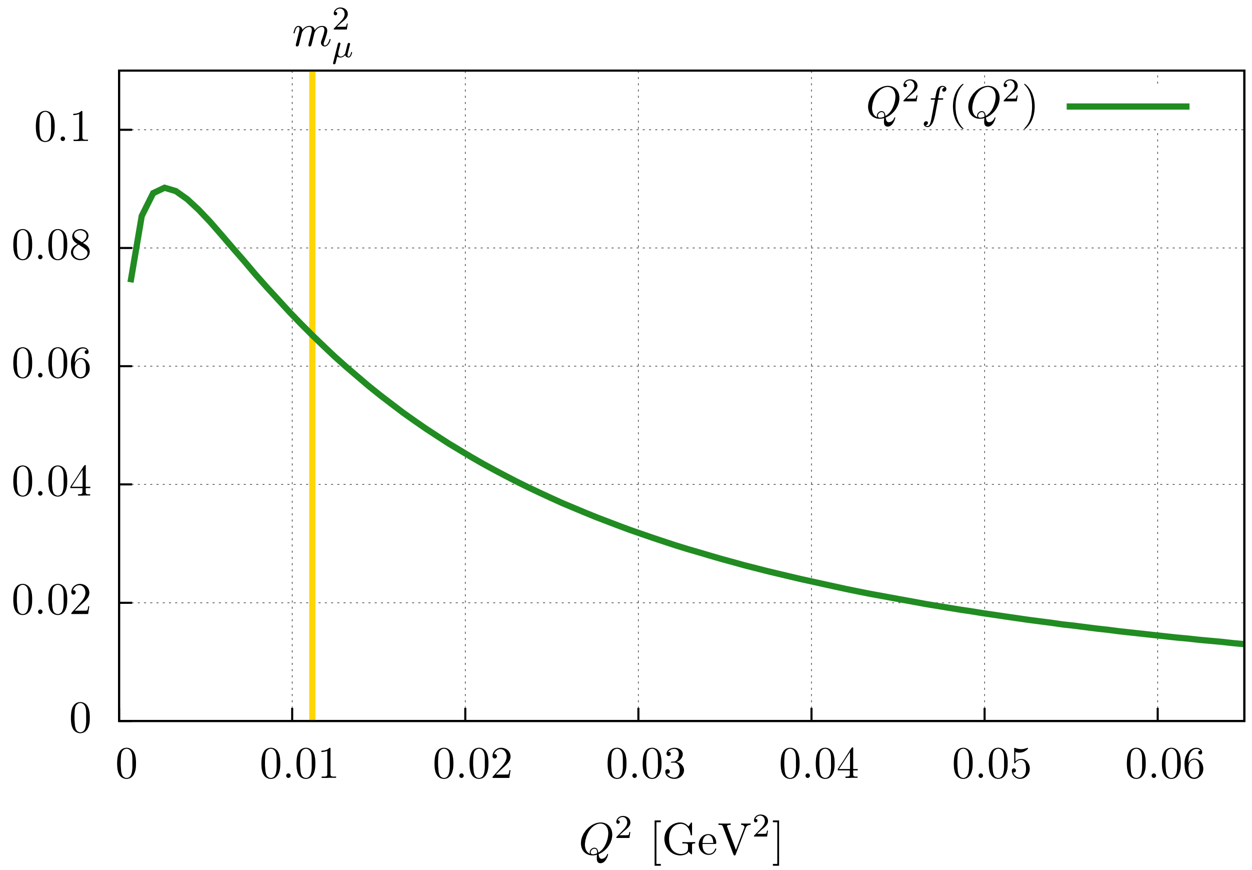

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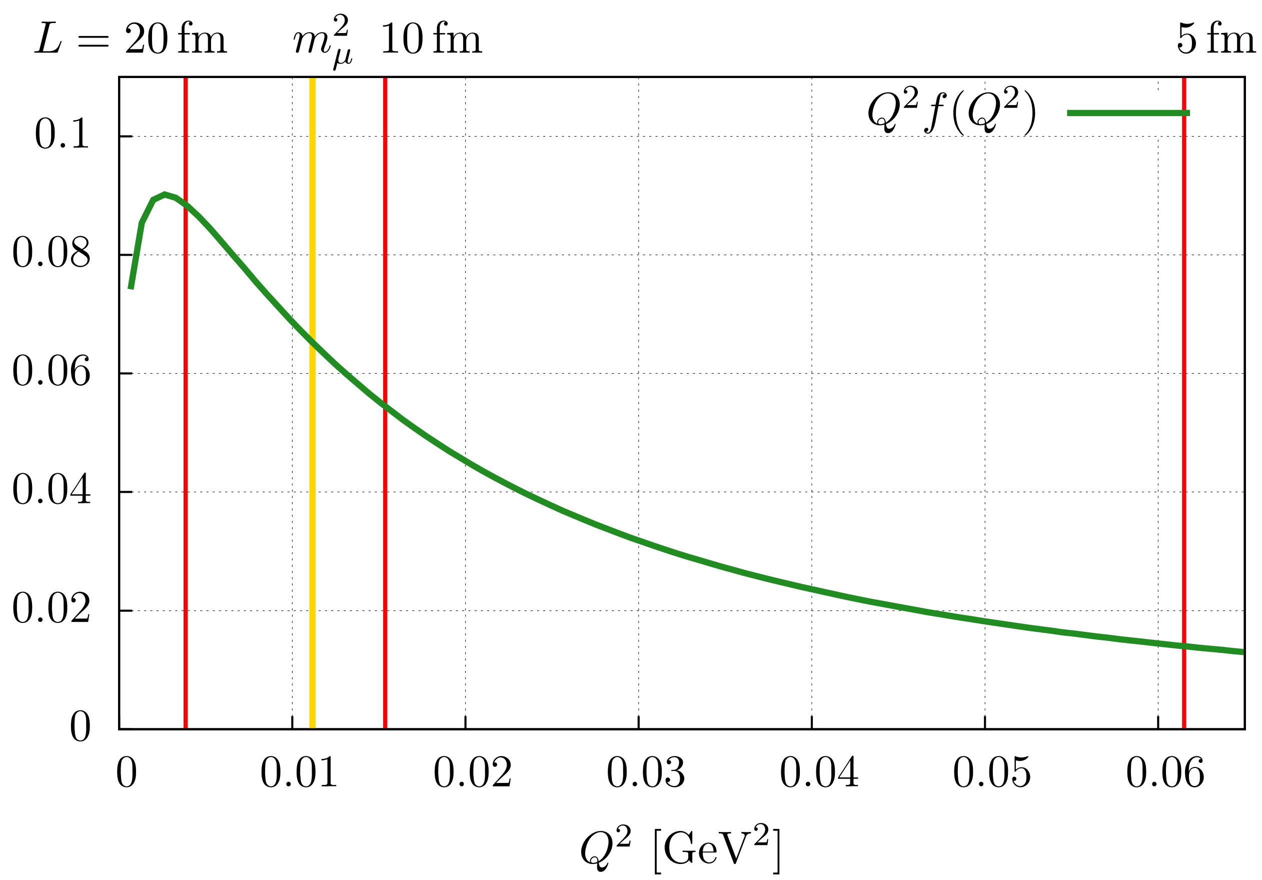

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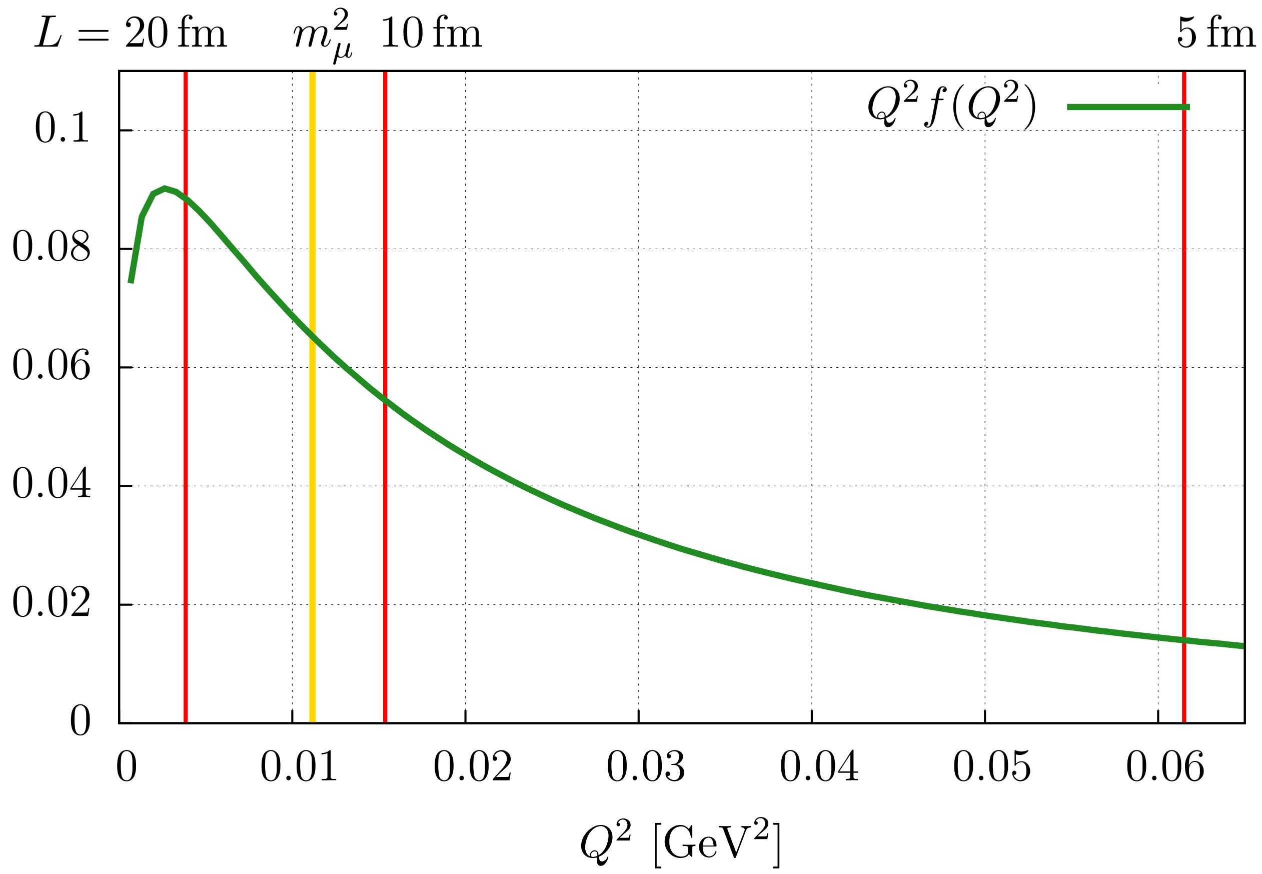


Accurate determination requires large statistics on large volumes!

Lattice QCD approach to HVP

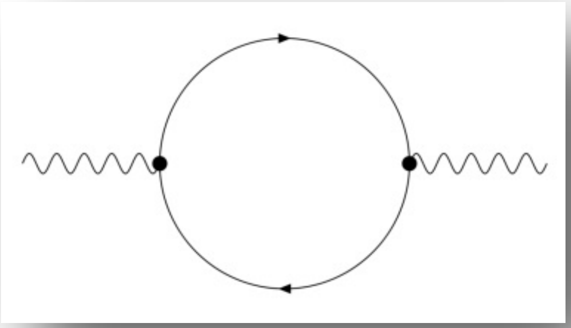
Main issues:

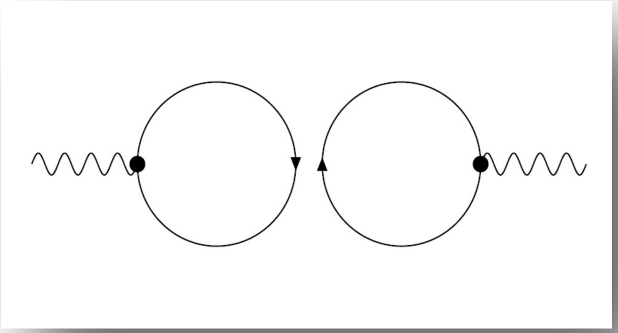
* Statistical accuracy at the sub-percent level required

* Reduce systematic uncertainty associated with region of small Q^2
  $\Leftrightarrow$ accurate determination of $\Pi(0)$

* Perform comprehensive study of finite-volume effects

* Include quark-disconnected diagrams

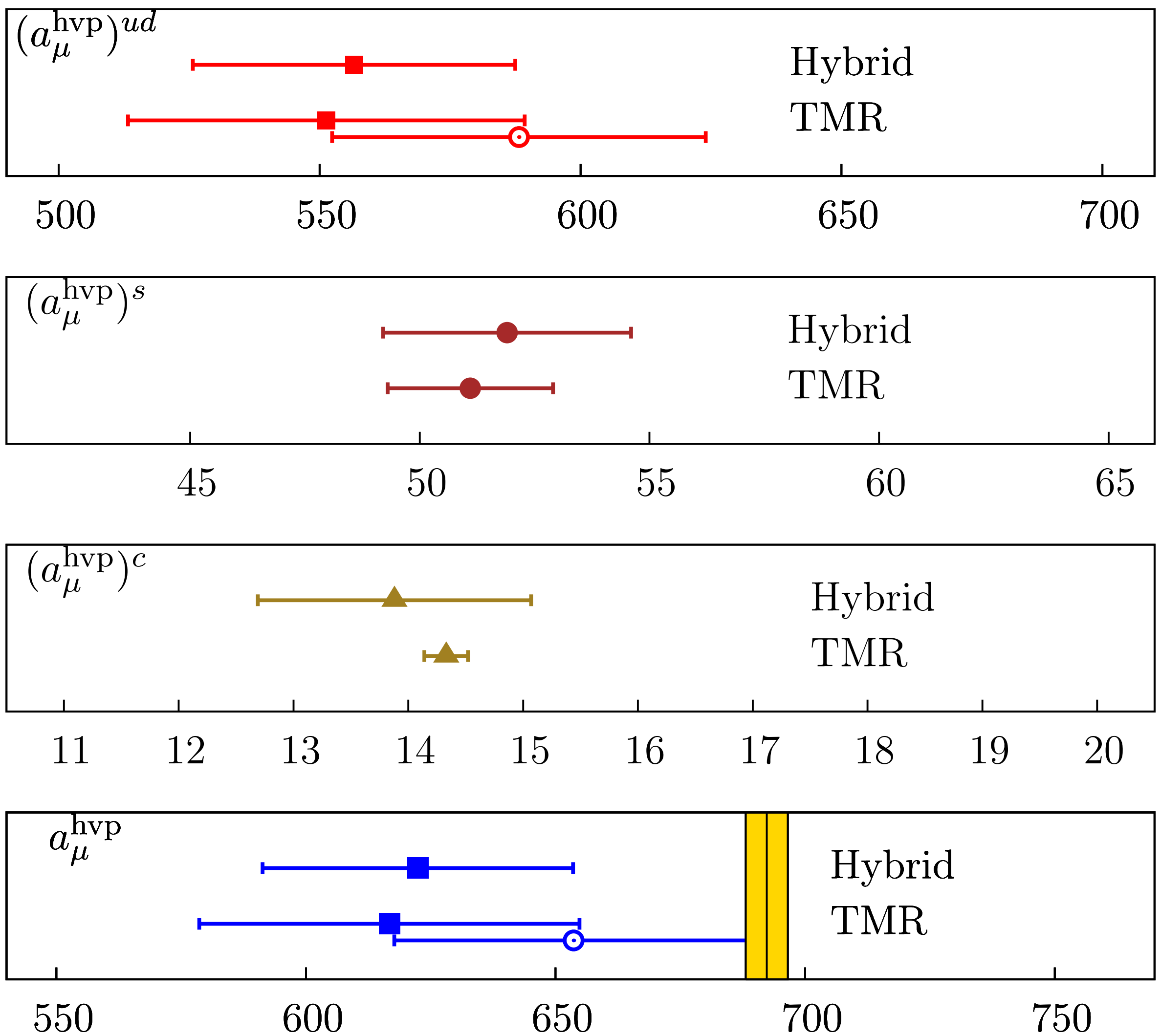




* Include isospin breaking: $m_u \neq m_d$, QED corrections

Results in two-flavour QCD

$$a_\mu^{\rm hvp} = (654 \pm 32_{\rm stat} \pm 17_{\rm syst} \pm 10_{\rm scale} \pm 7_{\rm FV}\,{}^{+\,0}_{-10}{}_{\rm disc}) \cdot 10^{-10}$$

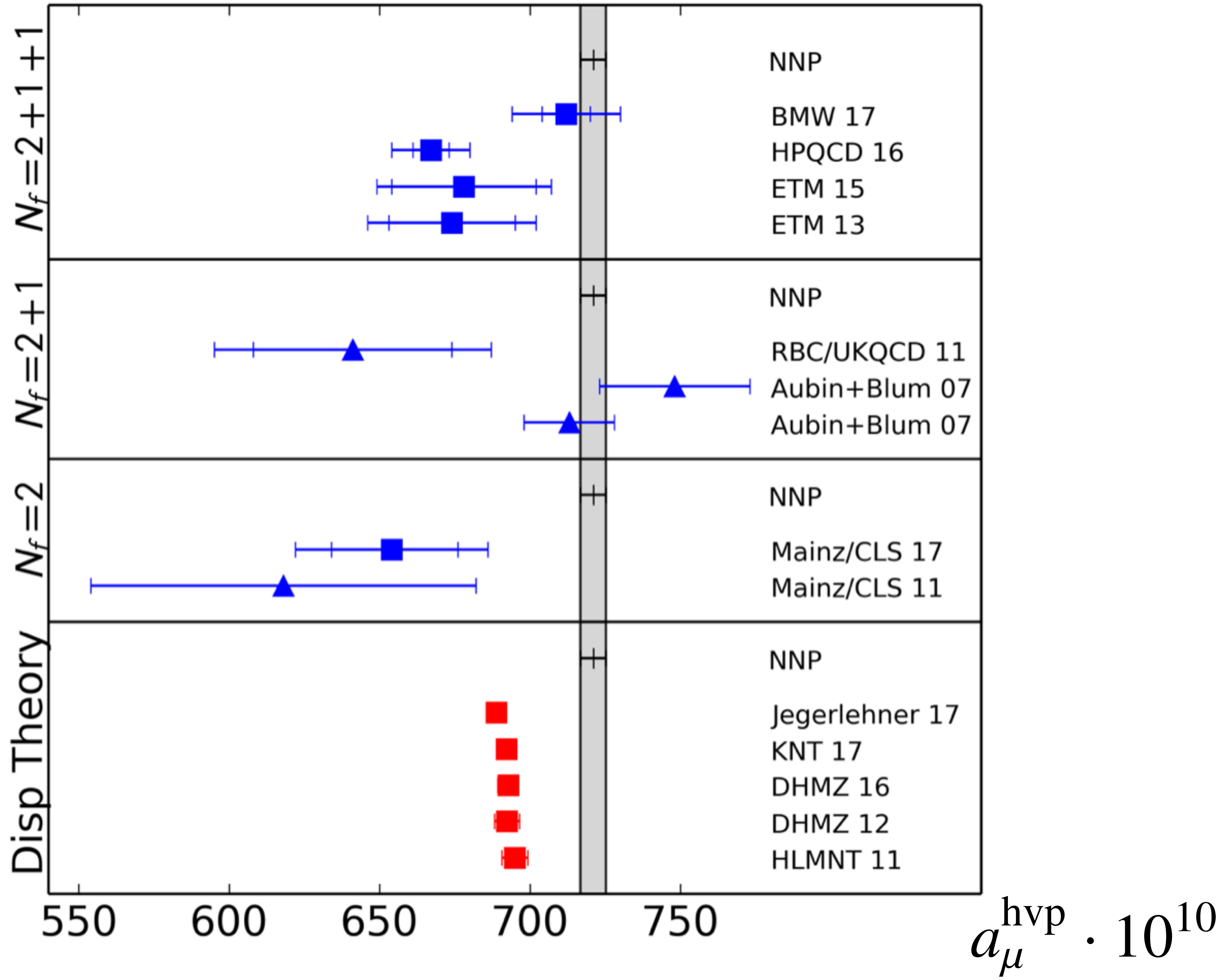


- Compare different methods to constrain infrared regime
- Finite-volume corrections sizeable
- Quark-disconnected diagrams contribute $< 2\%$

[Della Morte et al., JHEP 10 (2017) 020]

Compilation & comparison

* Lattice QCD vs. dispersion theory:



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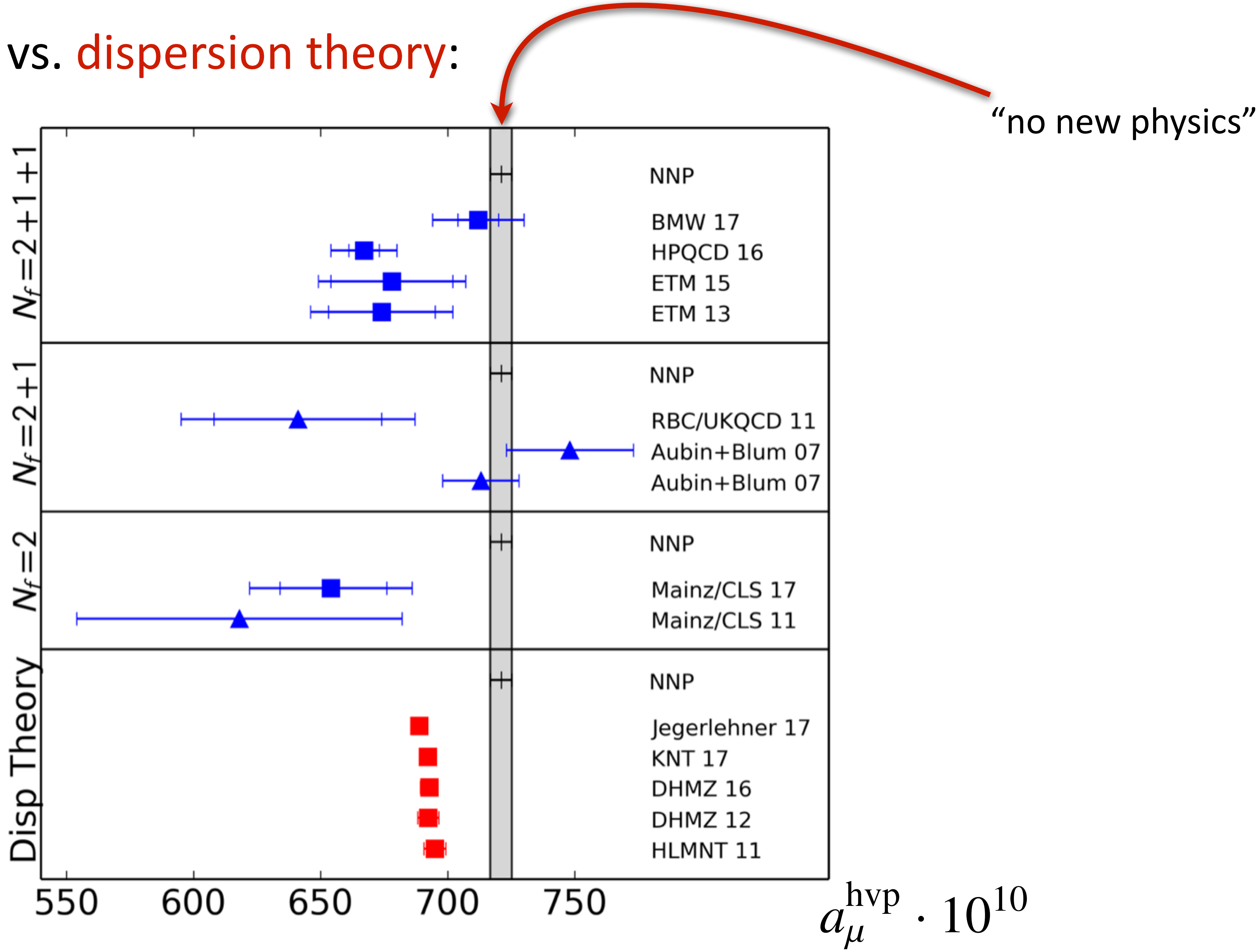

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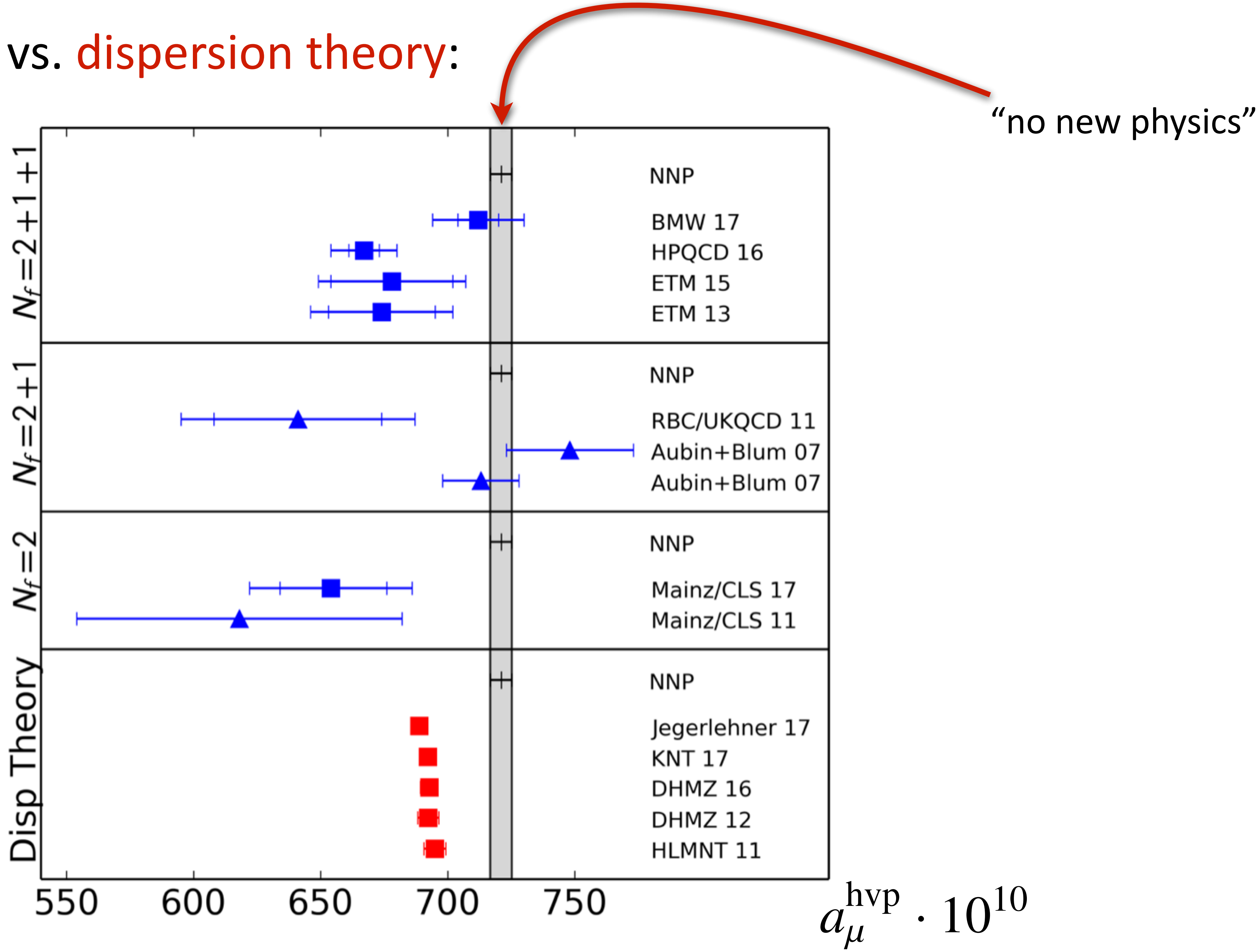
* Lattice QCD vs. dispersion theory:



* Increase overall precision of lattice QCD calculations

Lattice QCD approaches to HLbL

* Matrix element of e.m. current between muon initial and final states:

$$\left\langle \mu(\boldsymbol{p}', s') \left| J_\mu(0) \right| \mu(\boldsymbol{p}, s) \right\rangle = -e\, \overline{u}(\boldsymbol{p}', s') \left(F_1(Q^2)\gamma_\mu + \frac{F_2(Q^2)}{2m}\sigma_{\mu\nu}Q_\nu \right) u(\boldsymbol{p}, s)$$

$$a_\mu^{\rm hlbl} = F_2(0)$$

RBC/UKQCD:

* QCD + QED simulations *[Hayakawa et al. 2005; Blum et al. 2015]*
* QCD + stochastic QED *[Blum et al. 2016, 2017]*

Mainz group:

* Exact QED kernel in position space *[Asmussen et al. 2015, 2016, and in prep.]*
* Transition form factors of sub-processes *[Gérardin, Meyer, Nyffeler 2016]*
* Forward scattering amplitude *[Green et al. 2015, 2017]*

QCD + Stochastic QED

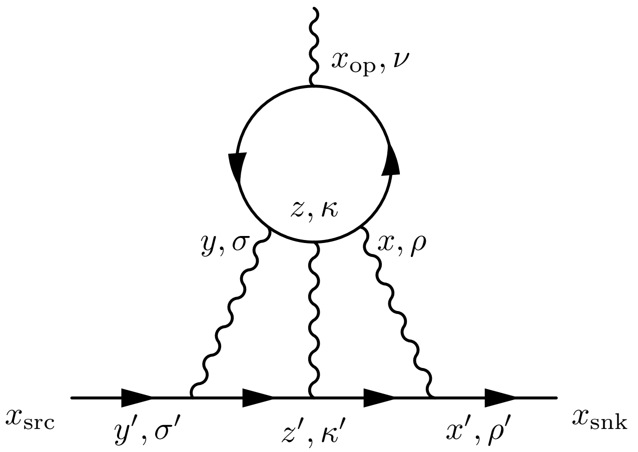
* Stochastic treatment of QED contribution:
  ⇒ insertion of three exact Feynman gauge photon propagators

$$G_{\mu\nu}(x,y) = \frac{1}{VT}\delta_{\mu\nu} \sum_{k,\,|\vec{k}|\neq 0} \frac{e^{ik\cdot(x-y)}}{\hat{k}^2}$$

[Blum et al., Phys Rev D93 (2016) 014503]

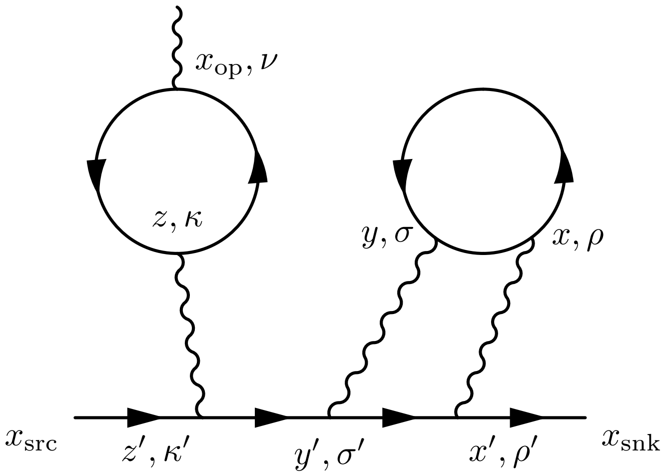
* Connected contribution:

$$(a_\mu^{\rm hlbl})_{\rm con} = (116.0 \pm 9.6)\cdot 10^{-11}$$



* Leading disconnected contribution:

$$(a_\mu^{\rm hlbl})_{\rm disc} = (-62.5 \pm 8.0)\cdot 10^{-11}$$



* Compute sub-leading disconnected diagrams

[Blum et al., Phys Rev Lett 118 (2017) 022005]

Exact QED kernel in position space

✱ Determine QED part perturbatively in the continuum in infinite volume

⇒ no power-law volume effects

$$a_\mu^{\mathrm{hlbl}} = F_2(0) = \frac{me^6}{3} \int d^4y \int d^4x \, \overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \, i\Pi_{\rho;\mu\nu\lambda\sigma}(x,y)$$

✱ QCD four-point function: $i\Pi_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \, z_\rho \left\langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \right\rangle$

✱ QED kernel function: $\overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ *[Asmussen, Green, Meyer, Nyffeler, in prep.]*

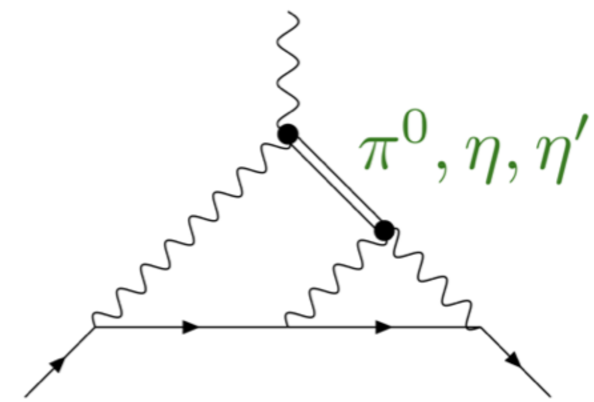
- Infra-red finite; can be computed semi-analytically
- Admits a tensor decomposition in terms of six weight functions which depend on $x^2, \quad y^2, \quad x \cdot y$

⇒ 3D integration instead of $\int d^4x \int d^4y$

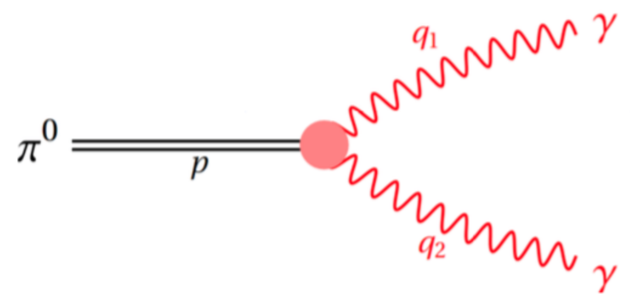
✱ Weight functions computed and stored on disk

Transition form factor $\pi^0 \longrightarrow \gamma^*\gamma^*$

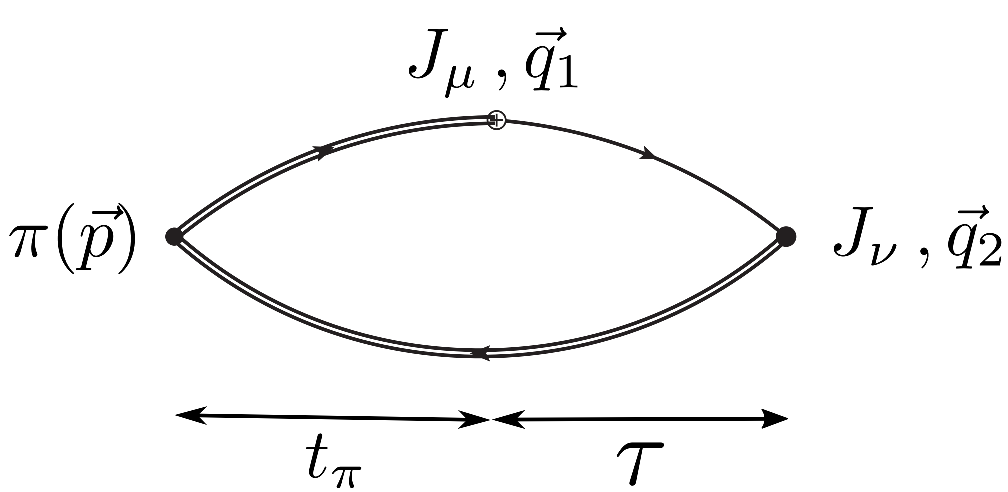
- Pseudoscalar meson pole expected to dominate LbL scattering



- Compute transition form factor between π^0 and two off-shell photons:



$$\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta\, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2; q_1^2, q_2^2) \equiv M_{\mu\nu}$$



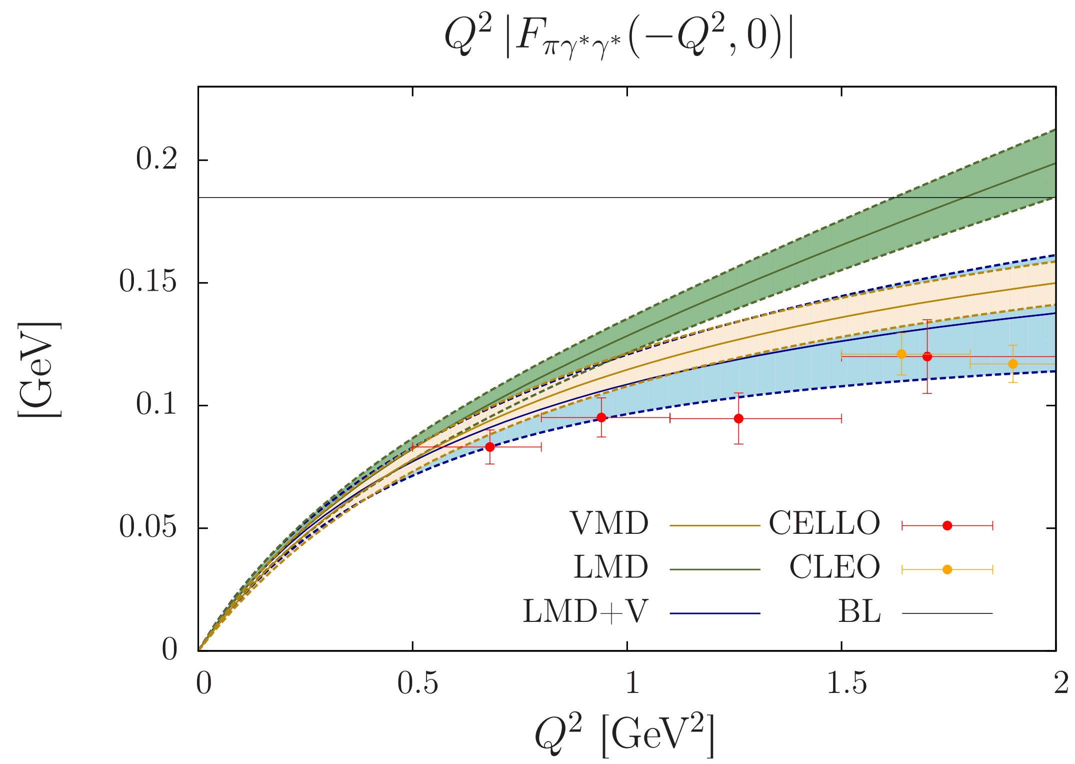
$$M_{\mu\nu} \sim C^{(3)}_{\mu\nu}(\tau, t_\pi; \vec{p}, \vec{q}_1, \vec{q}_2) = \sum_{\vec{x},\vec{z}} \left\langle T\left\{J_\nu(\vec{0}, \tau + t_\pi) J_\mu(\vec{z}, t_\pi) P(\vec{x}, 0)\right\}\right\rangle \mathrm{e}^{i\vec{p}\cdot\vec{x}} \mathrm{e}^{-i\vec{q}_1\cdot\vec{z}}$$

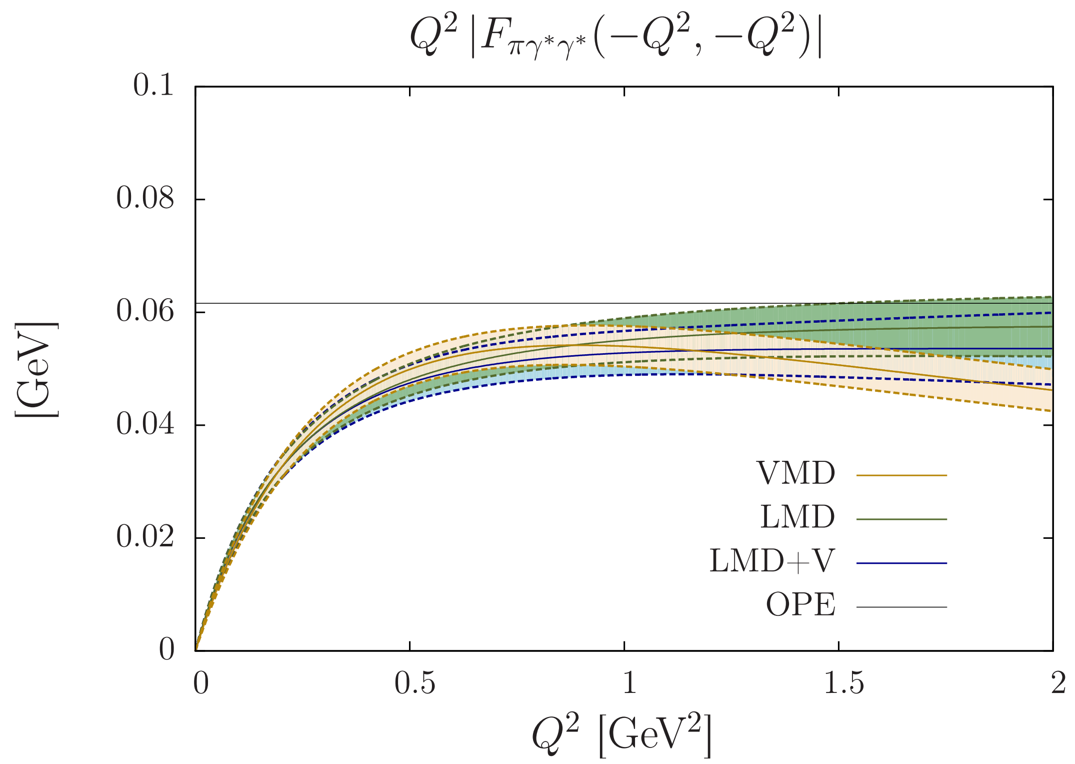
- Compute connected and disconnected contributions

Transition form factor $\pi^0 \longrightarrow \gamma^*\gamma^*$

* Fit VMD, LMD, LMD-V models, e.g.

$$\mathcal{F}^{\rm LMD}_{\pi^0\gamma^*\gamma^*} = \frac{\alpha M_V^4 + \beta(q_1^2+q_2^2)}{(M_V^2-q_1^2)(M_V^2-q_2^2)}$$





* Results for π^0 contribution to hadronic light-by-light scattering:

$$(a_\mu^{\rm hlbl})_{\pi^0} = (65.0 \pm 8.3)\cdot 10^{-11} \quad \text{(LMD+V)}$$ (stat. error only)

[Gérardin, Meyer, Nyffeler, Phys Rev D94 (2016) 074507]

Summary & Outlook

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Muon anomalous magnetic moment

- One of the most promising hints for new physics
- Beautiful interplay between theory and experiment
- Numerous technical and computational challenges
- New experiments will significantly increase sensitivity
- Theory must keep pace

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Lattice QCD

- Provides model-independent estimates for hadronic contributions
- HVP: difficult to reach sub-percent precision
- HLbL: $10–15\%$ calculation will have great impact

Summary & Outlook

First results from E989 expected in 2018

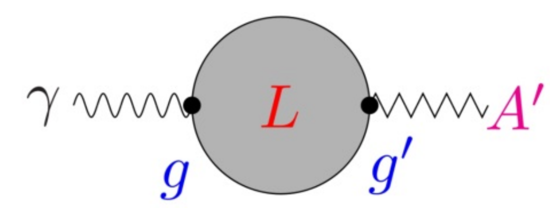
Stay tuned!

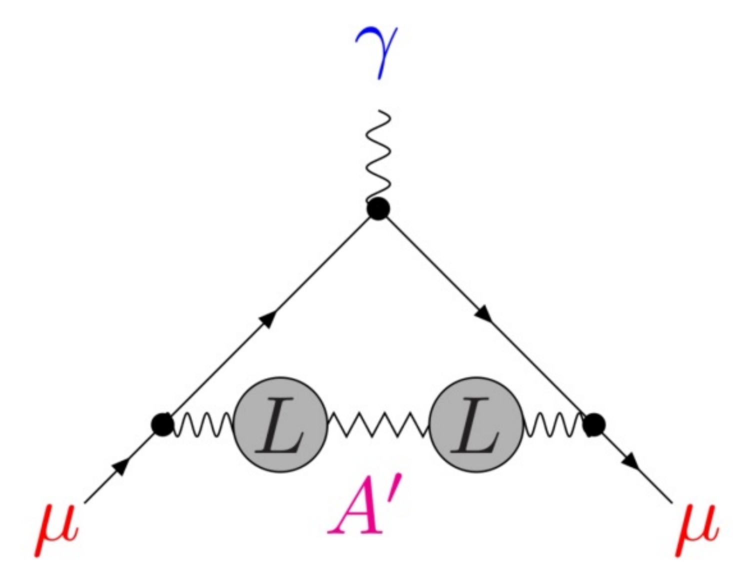
BSM manifestations in $(g-2)_\mu$

- SM extensions — larger symmetry groups: $G_{\mathrm{SM}} \longrightarrow G_{\mathrm{SM}} \times U(1)^n$

$\Rightarrow$ Additional $U(1)$ gauge bosons

- "Dark photons": messenger particles to dark sector





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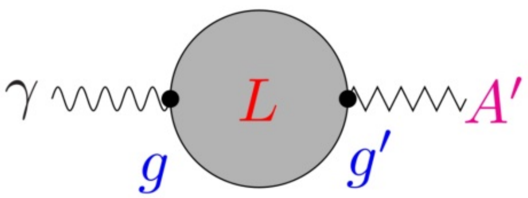
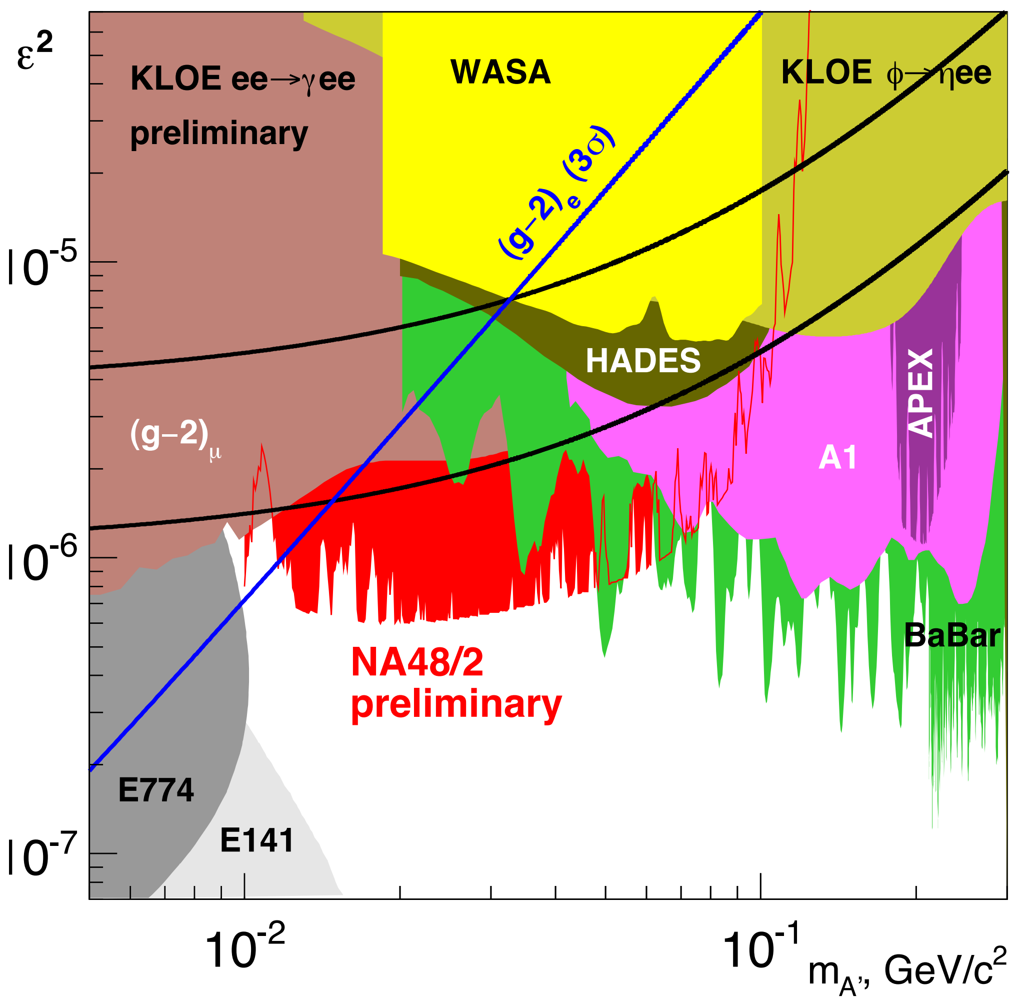
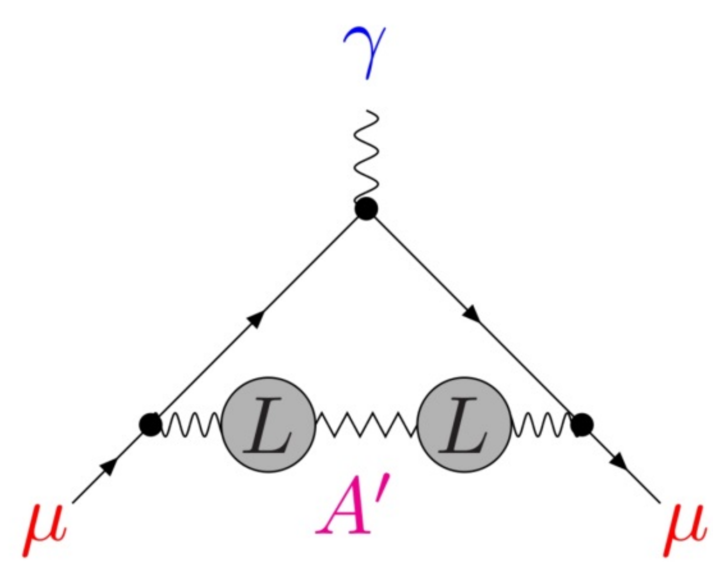




[M. Pospelov @ PhiPsi2017]



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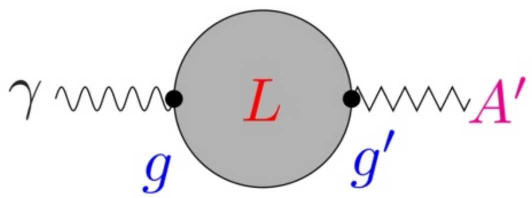
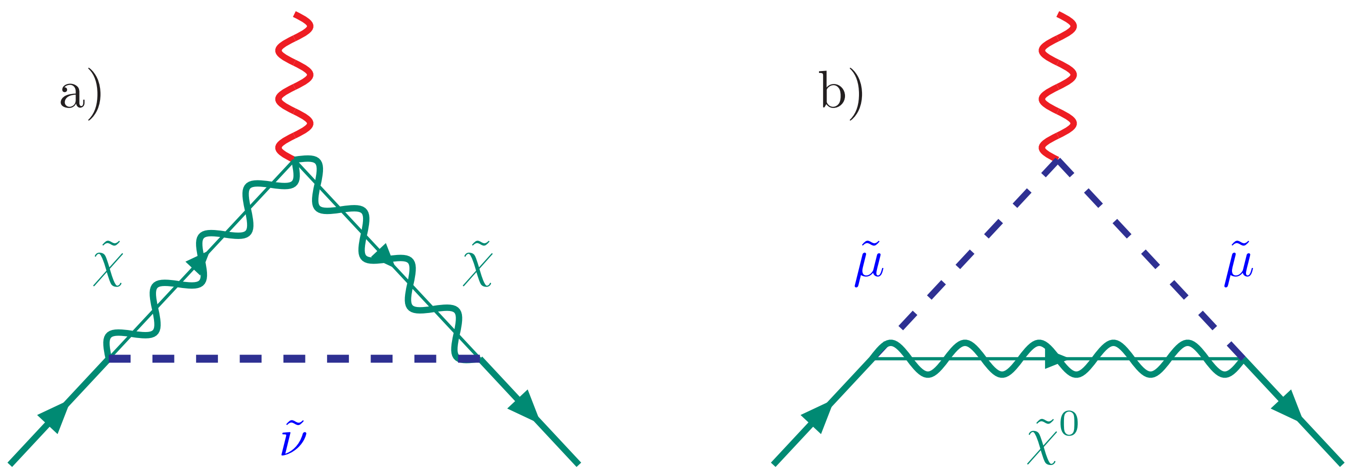


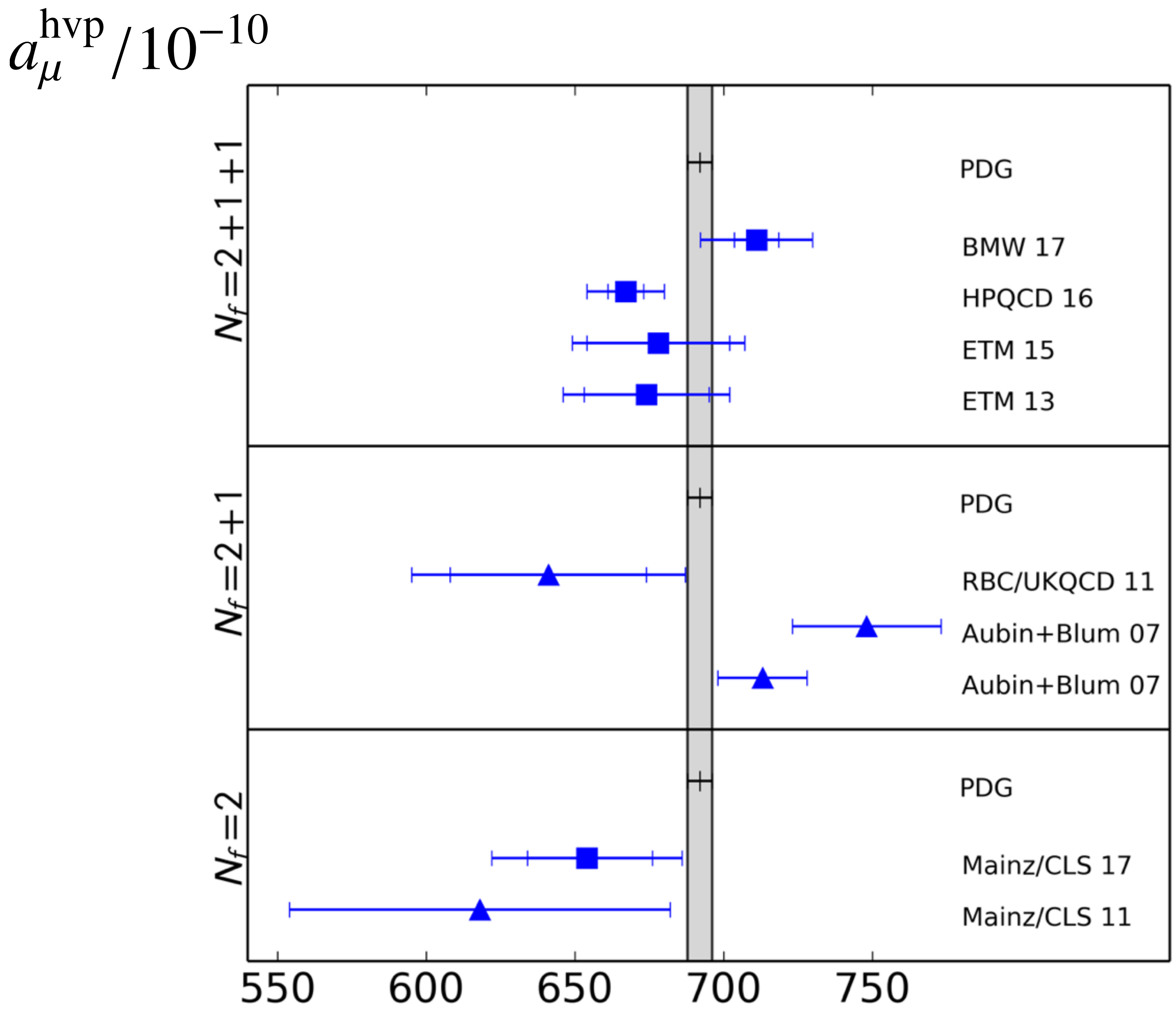
* Alternative: supersymmetric extensions



Leading SUSY contributions

[F. Jegerlehner, The Anomalous Magnetic Moment of the Muon, Springer Tracts Mod. Phys. 274 (2017) 1]

Compilation & comparison



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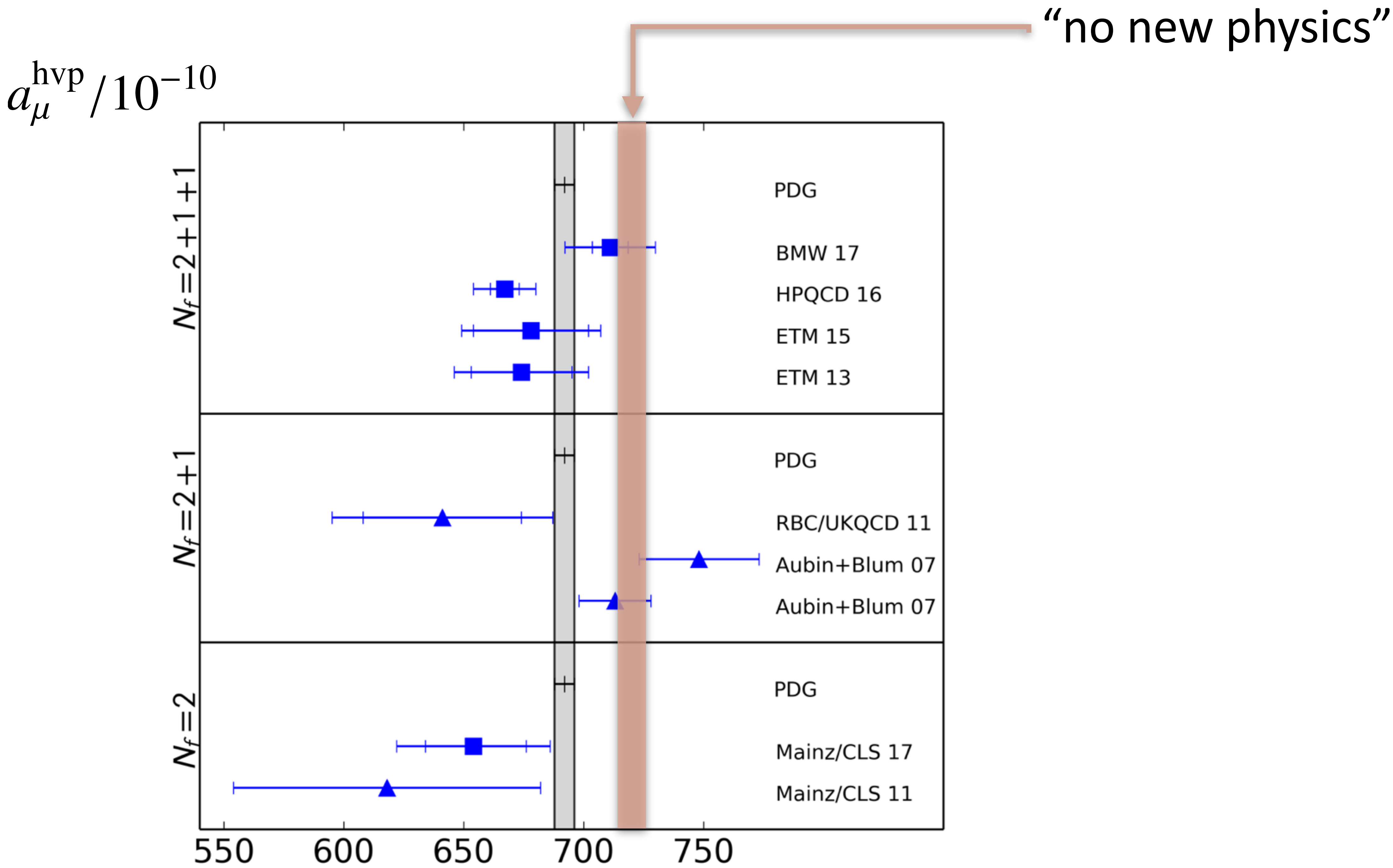
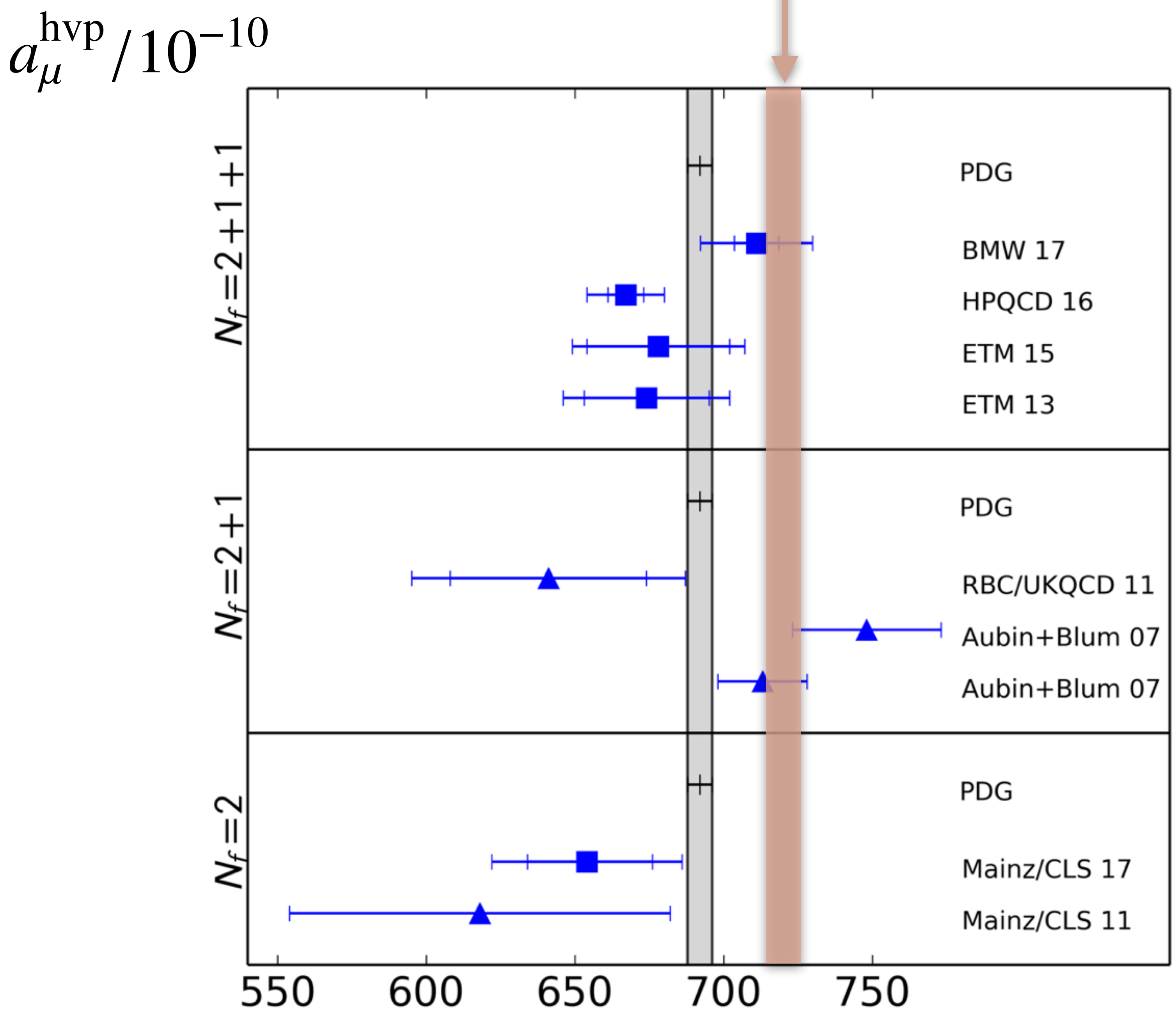


Compilation & comparison

“no new physics”



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