Nucleon formfactors in eVMD model

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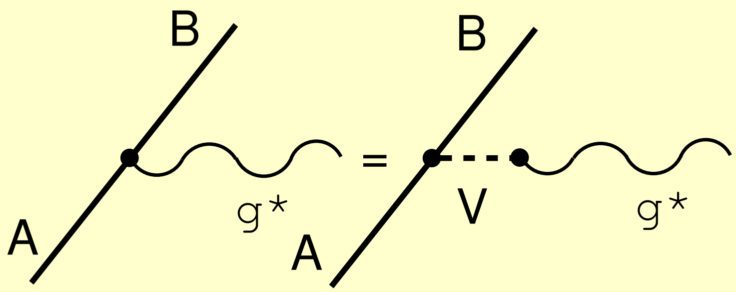
Motivations



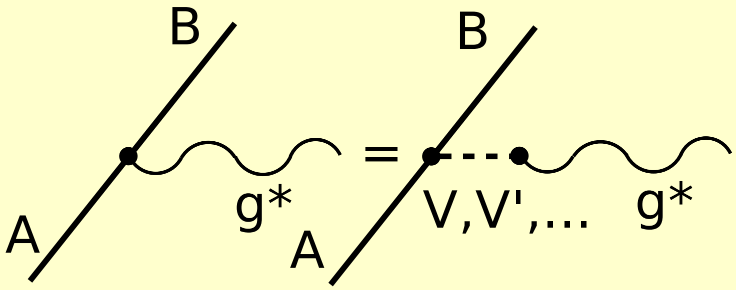
$$\mathbf{F(q^2) = F(0)\frac{m_V^2}{m_V^2 - q^2}}$$

Experiment: $\mathbf{F_{\omega\pi\gamma^*}(q^2) \sim 1/q^4}$, $\mathbf{F_1^N(Q^2) \sim 1/Q^4}$, $\mathbf{F_2^N(Q^2) \sim 1/Q^6}$

Quark counting rules: $\mathbf{F(Q^2) \sim 1/Q^{2n}}$, $\mathbf{Q^2 \to \infty}$, $\mathbf{n = 1, 2, ...}$ depending on the process.

Radiative versus vector meson decays of baryonic resonances: $\mathbf{\Gamma(R \to N\gamma)}$ is too large when calculated from $\mathbf{\Gamma(R \to NV)}$ using VDM.

Extension



$$\mathbf{F(q^2) = F(0)\frac{m_V^2 m_{V'}^2}{m_{V'}^2 - m_V^2}(\frac{1}{m_V^2 - q^2} - \frac{1}{m_{V'}^2 - q^2}) = F(0)\frac{m_V^2 m_{V'}^2}{(m_V^2 - q^2)(m_{V'}^2 - q^2)}}$$

The destructive interference of $\mathbf{V, V', ...}$ vector mesons can reduce $\mathbf{\Gamma(R \to N\gamma)}$ radiation width in comparison to contribution of only ground state vector meson $\mathbf{V}$.

Quark counting rules give constraints on the unknown couplings of excited vector mesons $\mathbf{V', ...}$ to baryons and photon.

Helicity amplitudes

The process of decay $R \to N\gamma^*$ of baryonic resonance with spin $J = l + 1/2$ to nucleon and virtual photon is described by helicity amplitudes

$$< \lambda\lambda_{\gamma^*}|\mathbf{S}|\mathbf{J}\lambda_* >; \quad \lambda_* = \lambda_{\gamma^*} - \lambda; \quad \lambda_{\gamma^*} = \mathbf{0}, \pm\mathbf{1}; \quad \lambda = \pm\frac{\mathbf{1}}{\mathbf{2}}$$

Due to P-invariance only 3 of them with $\lambda_* > \mathbf{0}$

$$\mathbf{A}_{\frac{3}{2}} \equiv < -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{1}|\mathbf{S}|\frac{\mathbf{3}}{\mathbf{2}} >, \quad \mathbf{A}_{\frac{1}{2}} \equiv < \frac{\mathbf{1}}{\mathbf{2}}\mathbf{1}|\mathbf{S}|\frac{\mathbf{1}}{\mathbf{2}} >, \quad \mathbf{S}_{\frac{1}{2}} \equiv < -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{0}|\mathbf{S}|\frac{\mathbf{1}}{\mathbf{2}} >$$

are independent and it means that there are 3 ($\mathbf{F_{1,2,3}(q^2)}$) independent invariant form factors

$$\langle\lambda\lambda_{\gamma^*}|\mathbf{S}|\mathbf{J}\lambda_*\rangle = \mathbf{e\bar{u}(p,\lambda)}\left\{\mathbf{q}_{\beta_1}...\mathbf{q}_{\beta_{l-1}}\sum_{\mathbf{k=1}}^{\mathbf{3}}\mathbf{\Gamma}^{\mathbf{k}}_{\beta_l\mu}\mathbf{F_k(q^2)}\right\}\mathbf{u}_{\beta_1...\beta_l}(\mathbf{p}_*,\lambda_*)\epsilon_\mu^{*(\lambda_{\gamma^*})}(\mathbf{q})$$

$$\mathbf{\Gamma}^{\mathbf{1}}_{\beta\mu} = \mathbf{m}_*(\mathbf{q}_\beta\gamma_\mu - \mathbf{\hat{q}g}_{\beta\mu})\gamma_5, \quad \mathbf{\Gamma}^{\mathbf{2}}_{\beta\mu} = (\mathbf{q}_\beta\mathbf{P}_\mu - \mathbf{q}\cdot\mathbf{Pg}_{\beta\mu})\gamma_5, \quad \mathbf{\Gamma}^{\mathbf{3}}_{\beta\mu} = (\mathbf{q}_\beta\mathbf{q}_\mu - \mathbf{q^2g}_{\beta\mu})\gamma_5$$

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Magnetic, electric and Coulomb form factors

The width

$$\Gamma(N^* \to N\gamma^*) = \frac{k}{32\pi^2 m_*^2} \frac{8\pi}{2J+1} (|A_{\frac{3}{2}}|^2 + |A_{\frac{1}{2}}|^2 + |S_{\frac{1}{2}}|^2)$$

is not diagonal in invariant form factors $F_{1,2,3}(q^2)$.

Another set of magnetic, electric and Coulomb amplitudes

$$A_M = \sqrt{\frac{l+2}{2(l+1)}} A_{\frac{3}{2}} + \sqrt{\frac{l}{2(l+1)}} A_{\frac{1}{2}}, \quad A_E = \sqrt{\frac{l}{2(l+1)}} A_{\frac{3}{2}} - \sqrt{\frac{l+2}{2(l+1)}} A_{\frac{1}{2}},$$

$$A_C = S_{\frac{1}{2}}$$

with corresponding magnetic, electric and Coulomb form factors

$$A_M = \lambda_l \sqrt{\frac{l+1}{l}} G_M(q^2), \quad A_E = \lambda_l \sqrt{(l+1)(l+2)} G_E(q^2), \quad A_C = \lambda_l \frac{\sqrt{q^2}}{m_*} G_C(q^2)$$

is introduced ($\lambda_l = e^{\frac{3(m^*+m)}{4m}} \sqrt{(m^*-m)^2 - q^2}\, k^{l-1} \sqrt{\frac{2^l (l!)^2 (l+1)}{(2l+1)!}}$).

Quark counting rules

Electroproduction of baryonic resonances in deep inelastic region $q^2 = -Q^2, \quad Q^2 \to \infty$ is described by quark model that gives in asymptotics

$$A_{\frac{3}{2}} \sim \frac{1}{(Q^2)^{\frac{5}{2}}}, \quad A_{\frac{1}{2}} \sim \frac{1}{(Q^2)^{\frac{3}{2}}}, \quad S_{\frac{1}{2}} \sim \frac{1}{(Q^2)^2}$$

This requires the following behaviour of invariant form factors

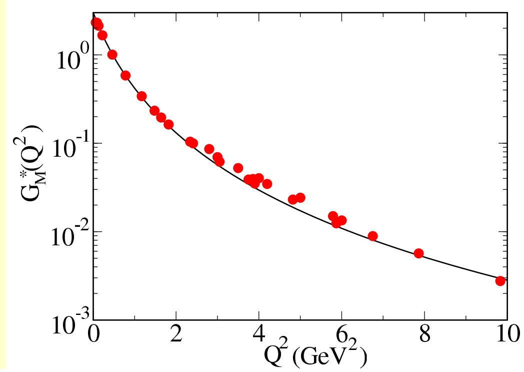
$$F_1(Q^2) \sim \frac{1}{(Q^2)^{l+2}}, \quad F_2(Q^2) \sim \frac{1}{(Q^2)^{l+3}}, \quad F_3(Q^2) \sim \frac{1}{(Q^2)^{l+3}} \ .$$

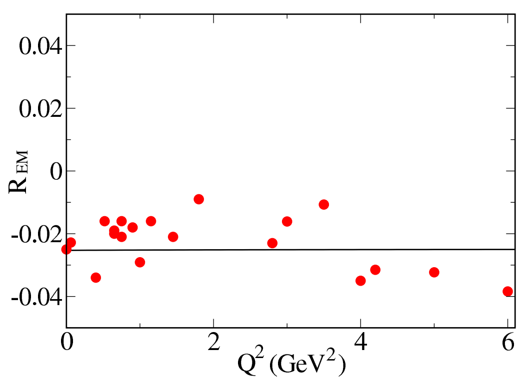
In extended VMD model it can be produced by destructive interference of the ground vector meson and $l+2$ its radial excitations

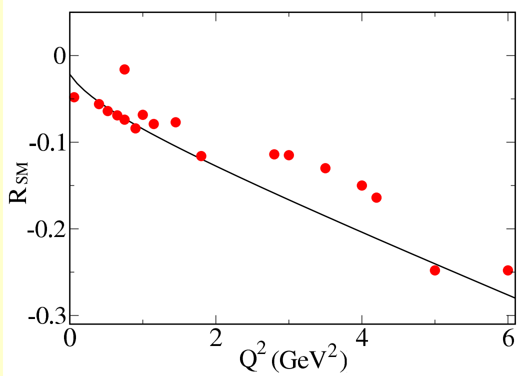
$$F_i(Q^2) = \frac{C_i - C_i' Q^2}{(1+\frac{Q^2}{m_V^2})(1+\frac{Q^2}{m_{V_1}^2})\ldots(1+\frac{Q^2}{m_{V_{l+2}}^2})}, \quad C_2' = C_3' = 0 \ .$$

So, we have 4 fitting parameters C_1, C_1', C_2, C_3 for the description of $RN\gamma^*$ transition.

Fit of $\Delta N\gamma^*$ *data*







$$\mathbf{C_1 = 1.77,\ C_1' = 0.025,\ C_2 = -1.1,\ C_3 = -0.93}$$

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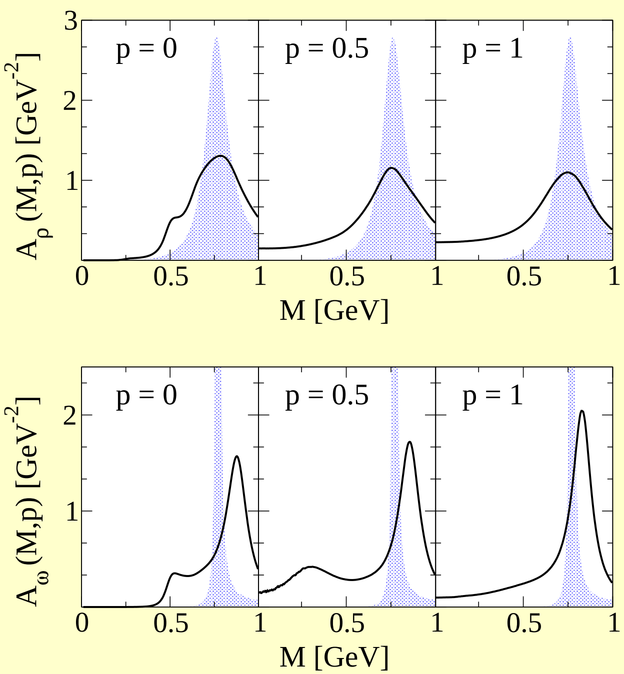
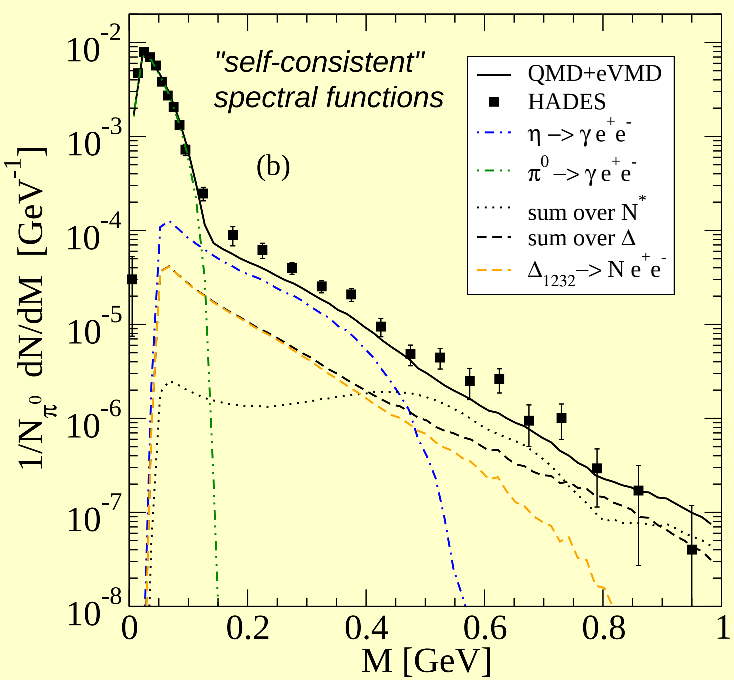
HIC

Using the coupling constants of nucleon resonances to the nucleons and vector mesons the spectral functions of vector mesons in nuclear medium can be determined.

They are used in calculation of dilepton production in HIC.





Nucleon form factors

Dirac and Pauli form factors

$$< N(p')|J_\mu|N(p) >= \bar{u}(p')(F_1^N(q^2)\gamma_\mu + \sigma_{\mu\nu}\frac{q_\nu}{2m}F_2^N(q^2))u(p)$$

Charges and anomalos magnetic moments

$$F_1^p(0) = 1, \quad F_2^p(0) = 1.79, \quad F_1^n(0) = 0, \quad F_2^n(0) = -1.92$$

Quark counting rules

$$F_1^N(Q^2) \sim \frac{1}{Q^4}, \quad F_2^N(Q^2) \sim \frac{1}{Q^6}$$

eVDM model

$$F_1^N(Q^2) = \frac{F_1^N(0) + c^N Q^2}{(1+\frac{Q^2}{m_V^2})(1+\frac{Q^2}{m_{V'}^2})(1+\frac{Q^2}{m_{V''}^2})}, \; F_2^N(Q^2) = \frac{F_2^N(0)}{(1+\frac{Q^2}{m_V^2})(1+\frac{Q^2}{m_V^2{}'})(1+\frac{Q^2}{m_{V''}^2})}$$

Isovector and isoscalar form factors

$$F_{1,2}^\rho(Q^2) = \frac{F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2)}{2}, \quad F_{1,2}^\omega(Q^2) = \frac{F_{1,2}^p(Q^2) + F_{1,2}^n(Q^2)}{2}$$

are linear superpositions of contributions from degenerate ρ and ω family mesons

Nucleon form factors

$$F^{\rho}_{1,2}(Q^2) = \frac{f^{\rho NN}_{1,2}}{g_\rho}\frac{m^2_\rho}{m^2_\rho+Q^2} + \frac{f^{\rho' NN}_{1,2}}{g_{\rho'}}\frac{m^2_{\rho'}}{m^2_{\rho'}+Q^2} + \frac{f^{\rho'' NN}_{1,2}}{g_{\rho''}}\frac{m^2_{\rho''}}{m^2_{\rho''}+Q^2}$$

$$F^{\omega}_{1,2}(Q^2) = \frac{f^{\omega NN}_{1,2}}{g_\omega}\frac{m^2_\omega}{m^2_\omega+Q^2} + \frac{f^{\omega' NN}_{1,2}}{g_{\omega'}}\frac{m^2_{\omega'}}{m^2_{\omega'}+Q^2} + \frac{f^{\omega'' NN}_{1,2}}{g_{\omega''}}\frac{m^2_{\omega''}}{m^2_{\omega''}+Q^2}$$

The fit of electric and magnetic form factors of nucleons

$$G^N_E = F^N_1 + \frac{q^2}{4m^2}F^N_2, \qquad G^N_M = F^N_1 + F^N_2$$

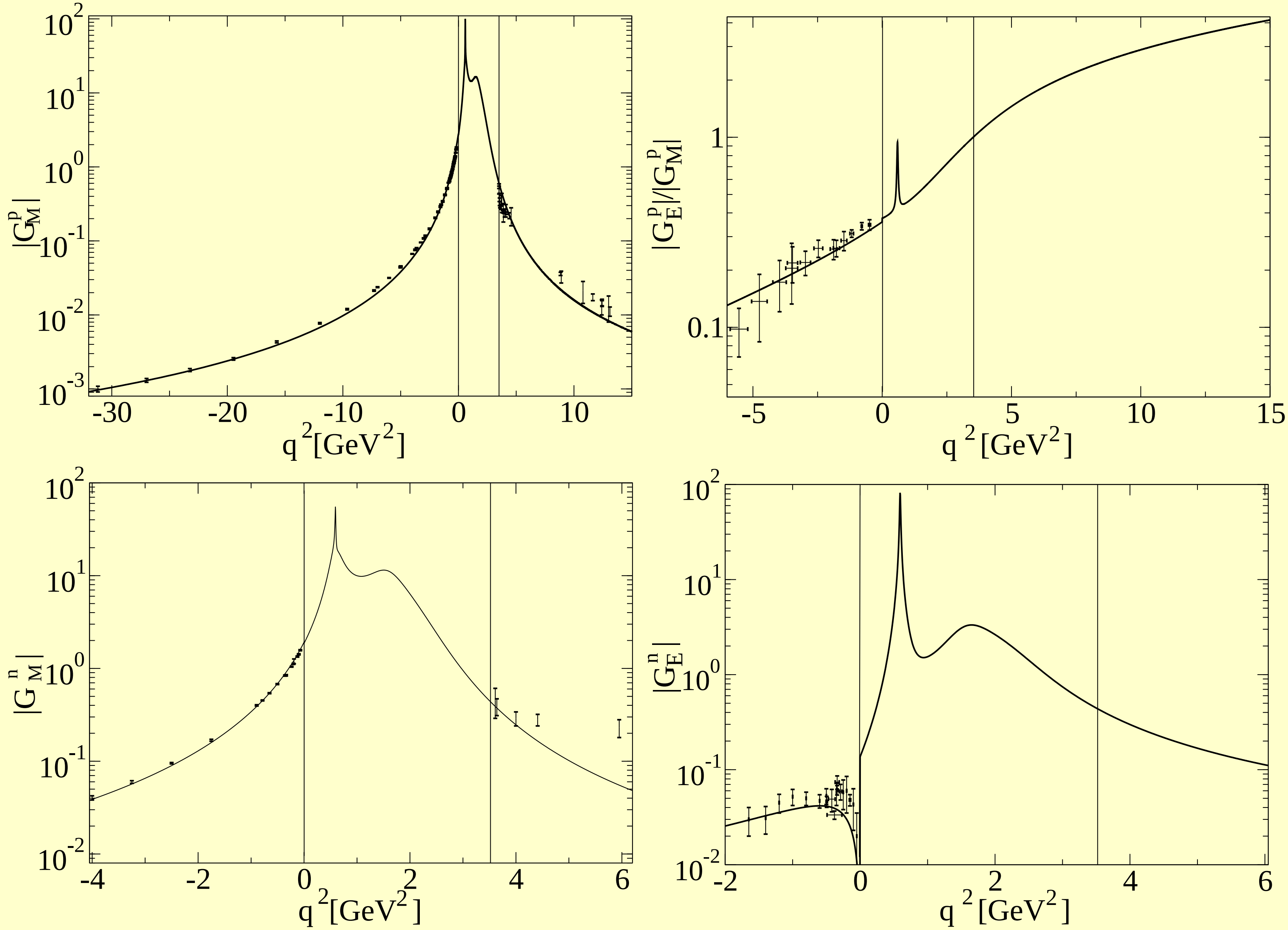
defines $c^p = 0.463\mathrm{GeV}^{-2}, \quad c^n = -0.297\mathrm{GeV}^{-2}$ At small Q^2 the decomposition $G^N_E \approx F^N_1(0) - \frac{1}{6}Q^2 < r^2_N >$ defines the charge radii of proton $\sqrt{< r^2_p >} = 0.83$ fm (exp: 0.875 fm) and neutron $< r^2_n >= -0.06$ fm^2 (exp: -0.113 fm^2). For known coupling constants of the photon to ρ and ω mesons $g_\rho = 5.03$ and $g_\omega = 17.1$ their coupling constants to the nucleon are equal to

$$f^{\rho NN}_1 = 3.02, \quad f^{\rho NN}_2 = 20.8, \quad f^{\omega NN}_1 = 17.2, \quad f^{\omega NN}_2 = -2.47$$

what is close to corresponding coupling constants used to describe Bonn potential of nucleon-nucleon interaction:

$$f^{\rho NN}_1 = 3.2, \quad f^{\rho NN}_2 = 19.8, \quad f^{\omega NN}_1 = 15.9, \quad f^{\omega NN}_2 = 0$$

Fit



Time-like region

In time-like region the finite widths of vector mesons should be taken into account

$$F^{\rho}_{1,2}(q^2) = \frac{f^{\rho NN}_{1,2}}{g_{\rho}} \frac{m^2_{\rho}}{m^2_{\rho} - i m_{\rho}\Gamma_{\rho} - q^2} + \frac{f^{\rho' NN}_{1,2}}{g_{\rho'}} \frac{m^2_{\rho'}}{m^2_{\rho'} - i m_{\rho'}\Gamma_{\rho'} - q^2}$$

$$+ \frac{f^{\rho'' NN}_{1,2}}{g_{\rho''}} \frac{m^2_{\rho''}}{m^2_{\rho''} - i m_{\rho''}\Gamma_{\rho''} - q^2}$$

$$F^{\omega}_{1,2}(q^2) = \frac{f^{\omega NN}_{1,2}}{g_{\omega}} \frac{m^2_{\omega}}{m^2_{\omega} - i m_{\omega}\Gamma_{\omega} - q^2} + \frac{f^{\omega' NN}_{1,2}}{g_{\omega'}} \frac{m^2_{\omega'}}{m^2_{\omega'} - i m_{\omega'}\Gamma_{\omega'} - q^2}$$

$$+ \frac{f^{\omega'' NN}_{1,2}}{g_{\omega''}} \frac{m^2_{\omega''}}{m^2_{\omega''} - i m_{\omega''}\Gamma_{\omega''} - q^2}$$

Conclusion

- An extended vector meson dominance model with a minimal number of free parameters is applied to the description of electromagnetic form factors of nucleons.
- The couplings of ground state ρ and ω mesons to the nucleons are calculated and appear to be close to those of Bonn potential model of nucleon interaction.
- In the time-like region the absolute values of electric form factors are considerably larger than those of magnetic form factors and this can be used in the reanalysis of experimental data obtained with the assumption $|G^p_E| = |G^p_M|$ in the proton case and $|G^n_E| = 0$ in the neutron case.