

Recent results from NPLQCD



Assumpta Parreño (U Barcelona)

for the NPLQCD Collaboration

nplqcd.ub.edu



EMMI Workshop: Anti-matter, hyper-matter and exotica production at the LHC
Nov. 6-10, 2017@Torino

Beane

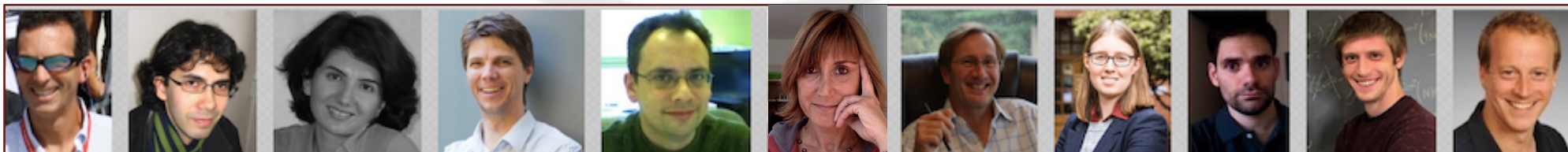
Davoudi

Orginos

Savage

Tiburzi

Winter



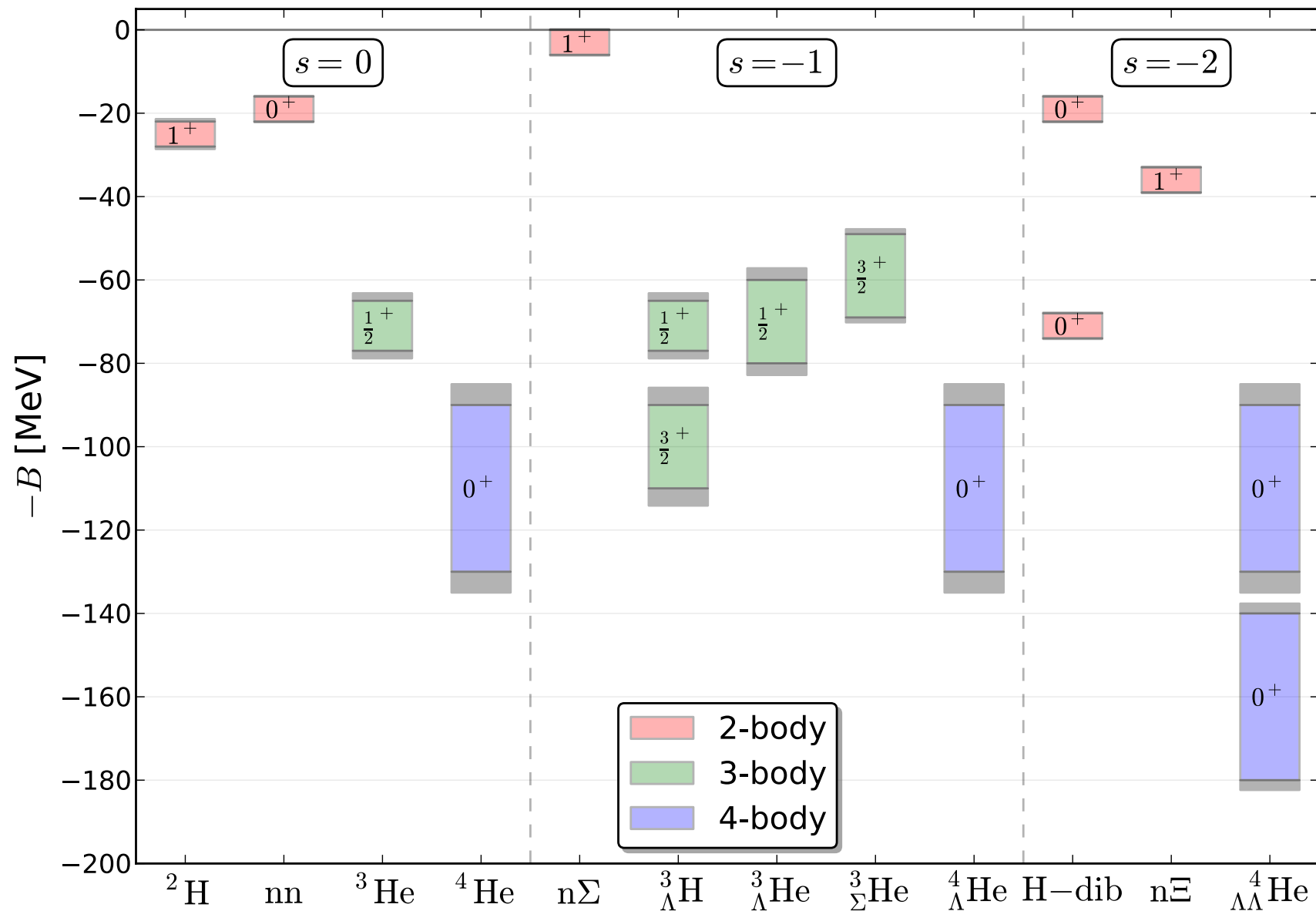
Chang

Detmold

Parreño

Shanahan

Wagman

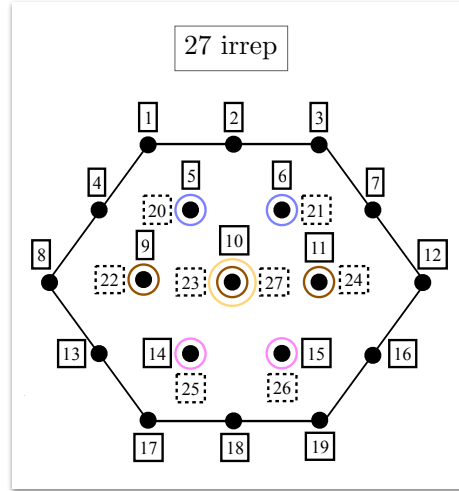


no e.m. interactions

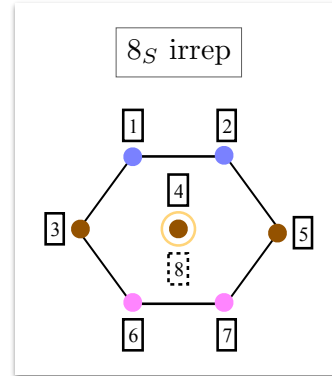
(hadronic labels for (J^π, I, s, A) states)

Two-baryon states

SU(3) decomposition

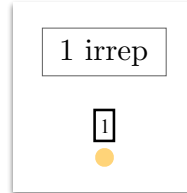


	Flavor channel		Flavor channel
1	nn	14	$-\sqrt{\frac{2}{3}}\Sigma^0\Xi^- + \sqrt{\frac{1}{3}}\Sigma^-\Xi^0$
2	$\frac{1}{\sqrt{2}}(np + pn)$	15	$\sqrt{\frac{1}{3}}\Sigma^+\Xi^- + \sqrt{\frac{2}{3}}\Sigma^0\Xi^0$
3	pp	16	$\Sigma^+\Xi^0$
4	Σ^-n	17	$\Xi^-\Xi^-$
5	$\sqrt{\frac{2}{3}}\Sigma^0n + \sqrt{\frac{1}{3}}\Sigma^-p$	18	$\frac{1}{\sqrt{2}}(\Xi^-\Xi^0 + \Xi^0\Xi^-)$
6	$-\sqrt{\frac{1}{3}}\Sigma^+n + \sqrt{\frac{2}{3}}\Sigma^0p$	19	$\Xi^0\Xi^0$
7	Σ^+p	20	$\Lambda n / -\sqrt{\frac{1}{3}}\Sigma^0n + \sqrt{\frac{2}{3}}\Sigma^-p$
8	$\Sigma^-\Sigma^-$	21	$\Lambda p / \sqrt{\frac{2}{3}}\Sigma^+n + \sqrt{\frac{1}{3}}\Sigma^0p$
9	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 + \Sigma^0\Sigma^-)$	22	$\Lambda\Sigma^- / \Xi^-n$
10	$\frac{1}{\sqrt{6}}(\Sigma^-\Sigma^+ - 2\Sigma^0\Sigma^0 + \Sigma^+\Sigma^-)$	23	$\Lambda\Sigma^0 / \frac{1}{\sqrt{2}}(\Xi^-p - \Xi^0n)$
11	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ + \Sigma^+\Sigma^0)$	24	$\Lambda\Sigma^+ / \Xi^0p$
12	$\Sigma^+\Sigma^+$	25	$\Lambda\Xi^- / \sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0$
13	$\Sigma^-\Xi^-$	26	$\Lambda\Xi^0 / -\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0$
27	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0n + \Xi^-p) / \Lambda\Lambda$		



	Flavor channel
1	$\Lambda n / -\sqrt{\frac{1}{3}}\Sigma^0n + \sqrt{\frac{2}{3}}\Sigma^-p$
2	$\Lambda p / \sqrt{\frac{2}{3}}\Sigma^+n + \sqrt{\frac{1}{3}}\Sigma^0p$
3	$\Lambda\Sigma^- / \Xi^-n$
4	$\Lambda\Sigma^0 / \frac{1}{\sqrt{2}}(\Xi^-p - \Xi^0n)$
5	$\Lambda\Sigma^+ / \Xi^0p$
6	$\Lambda\Xi^- / \sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0$
7	$\Lambda\Xi^0 / -\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0$
8	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0n + \Xi^-p) / \Lambda\Lambda$

J=0



	Flavor channel
1	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0n + \Xi^-p) / \Lambda\Lambda$

↑ strangeness

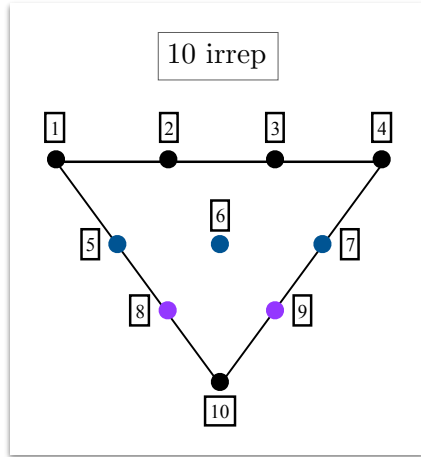
I_3

→

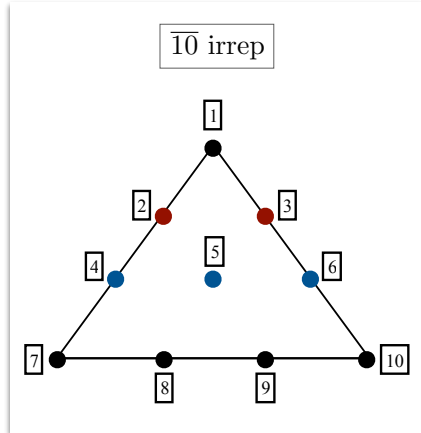
M.L. Wagman et al (NPLQCD)
ARXIV:1706.06550

SU(3) decomposition

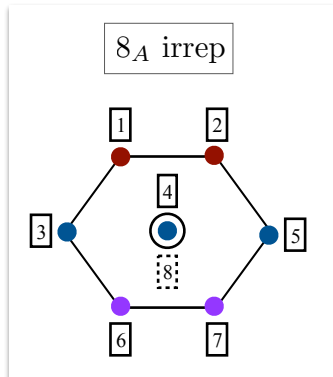
Two-baryon states



	Flavor channel
1	$\Sigma^- n$
2	$\sqrt{\frac{2}{3}}\Sigma^0 n + \sqrt{\frac{1}{3}}\Sigma^- p$
3	$-\sqrt{\frac{1}{3}}\Sigma^+ n + \sqrt{\frac{2}{3}}\Sigma^0 p$
4	$\Sigma^+ p$
5	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^0 - \Sigma^0 \Sigma^-)/\Xi^- n/\Lambda \Sigma^-$
6	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^+ - \Sigma^+ \Sigma^-)/\frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n)/\Lambda \Sigma^0$
7	$\frac{1}{\sqrt{2}}(\Sigma^0 \Sigma^+ - \Sigma^+ \Sigma^0)/\Xi^0 p/\Lambda \Sigma^+$
8	$\sqrt{\frac{1}{3}}\Sigma^0 \Xi^- + \sqrt{\frac{2}{3}}\Sigma^- \Xi^0/\Lambda \Xi^-$
9	$-\sqrt{\frac{2}{3}}\Sigma^+ \Xi^- + \sqrt{\frac{1}{3}}\Sigma^0 \Xi^0/\Lambda \Xi^0$
10	$\frac{1}{\sqrt{2}}(\Xi^0 \Xi^- - \Xi^- \Xi^0)$



	Flavor channel
1	$\frac{1}{\sqrt{2}}(pn - np)$
2	$-\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p/\Lambda n$
3	$\sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p/\Lambda p$
4	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^0 - \Sigma^0 \Sigma^-)/\Xi^- n/\Lambda \Sigma^-$
5	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^+ - \Sigma^+ \Sigma^-)/\frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n)/\Lambda \Sigma^0$
6	$\frac{1}{\sqrt{2}}(\Sigma^0 \Sigma^+ - \Sigma^+ \Sigma^0)/\Xi^0 p/\Lambda \Sigma^+$
7	$\Sigma^- \Xi^-$
8	$-\sqrt{\frac{2}{3}}\Sigma^0 \Xi^- + \sqrt{\frac{1}{3}}\Sigma^- \Xi^0$
9	$\sqrt{\frac{1}{3}}\Sigma^+ \Xi^- + \sqrt{\frac{2}{3}}\Sigma^0 \Xi^0$
10	$\Sigma^+ \Xi^0$



	Flavor channel
1	$-\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p/\Lambda n$
2	$\sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p/\Lambda p$
3	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^0 - \Sigma^0 \Sigma^-)/\Xi^- n/\Lambda \Sigma^-$
4	$\frac{1}{\sqrt{2}}(\Sigma^- \Sigma^+ - \Sigma^+ \Sigma^-)/\frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n)/\Lambda \Sigma^0$
5	$\frac{1}{\sqrt{2}}(\Sigma^0 \Sigma^+ - \Sigma^+ \Sigma^0)/\Xi^0 p/\Lambda \Sigma^+$
6	$\sqrt{\frac{1}{3}}\Sigma^0 \Xi^- + \sqrt{\frac{2}{3}}\Sigma^- \Xi^0/\Lambda \Xi^-$
7	$-\sqrt{\frac{2}{3}}\Sigma^+ \Xi^- + \sqrt{\frac{1}{3}}\Sigma^0 \Xi^0/\Lambda \Xi^0$
8	$\frac{1}{\sqrt{2}}(\Xi^0 n + \Xi^- p)$

J=1

↑
strangeness

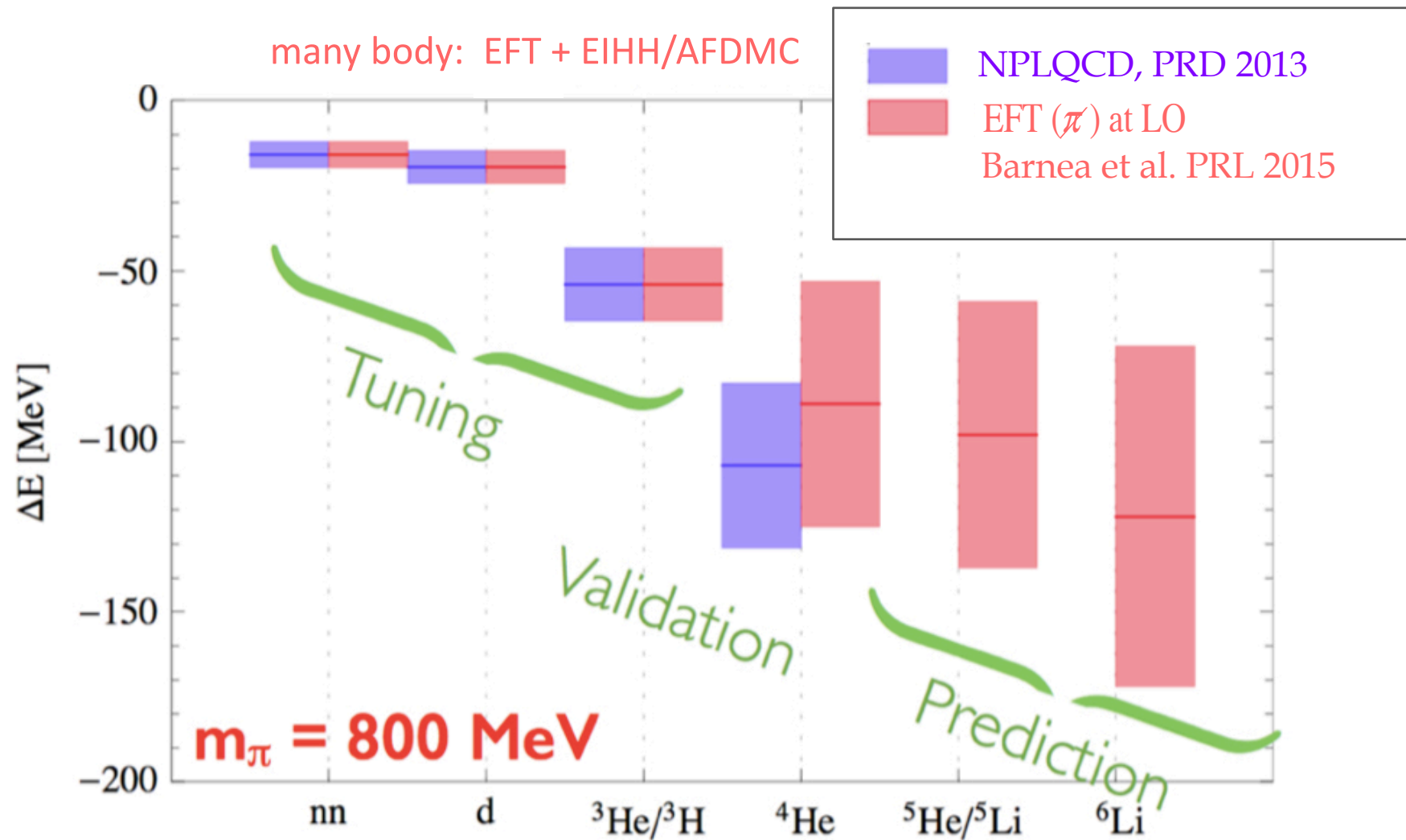
→
 I_3

M.L. Wagman et al (NPLQCD)
ARXIV:1706.06550

TWO-BARYON STATES AT THE SU(3) FLAVOR-SYMMETRIC POINT



LQCD \rightarrow FEW-BODY CALCS \rightarrow NUCLEAR MANY-BODY CALCS

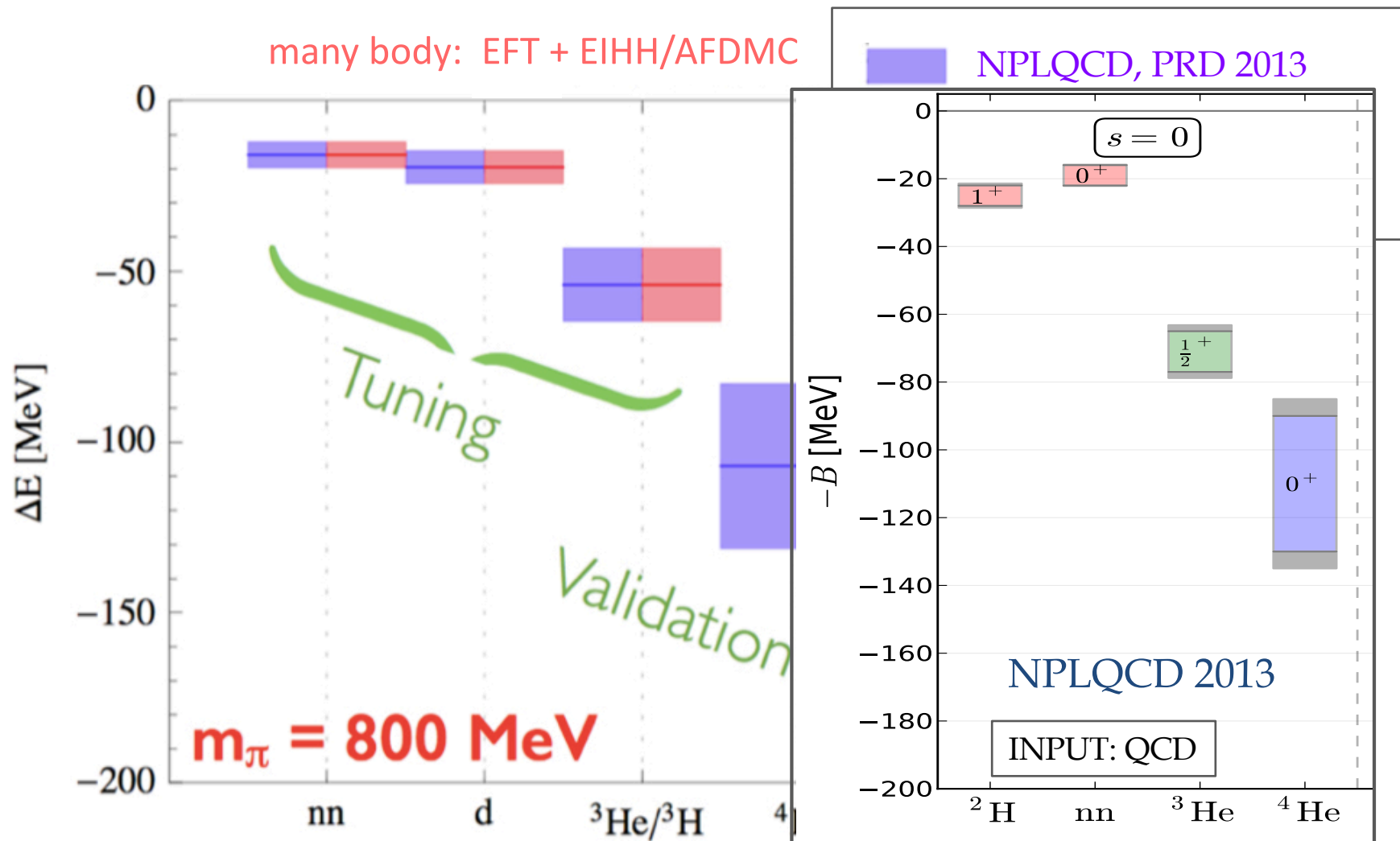


N. Barnea, L. Contessi, D. Gazit, F. Pederiva, and U. van Kolck, PRL **114** (2015) 052501

L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher and U. van Kolck, PLB **772** (2017) 839
Ground-state properties of ^4He and ^{16}O extrapolated from lattice QCD with pionless EFT



LQCD - QMC symbiosis

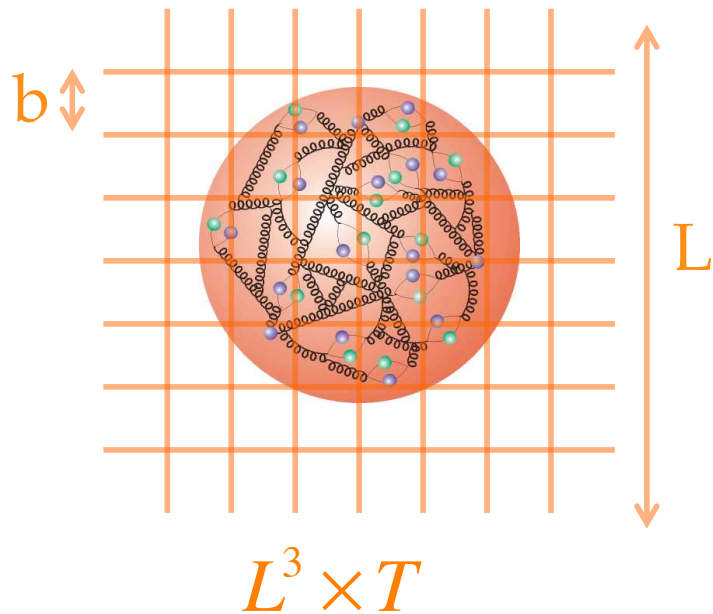
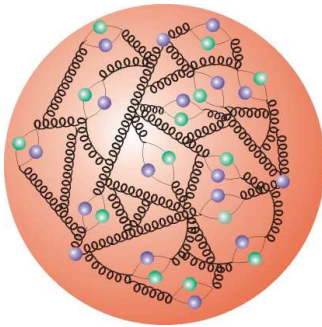


L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher and U. van Kolck, PLB **772** (2017) 839
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For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice

((anti) periodic (time) spatial boundary conditions) $N_s \times N_s \times N_s \times N_t$

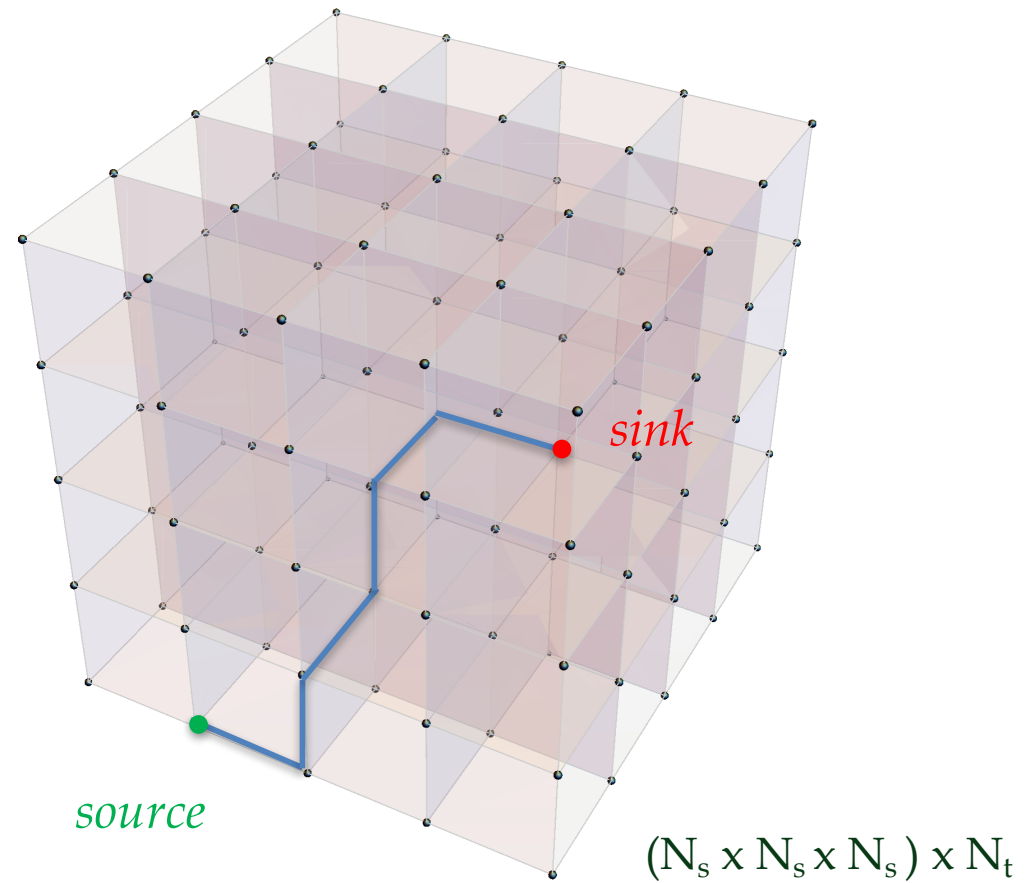
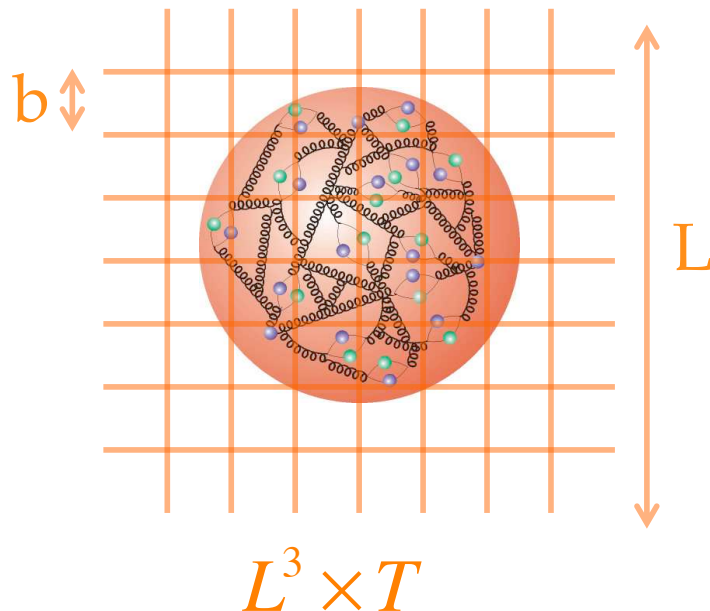
continuum



LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman **path-integral approach** to evaluate transition matrix elements

Wilson, 1970

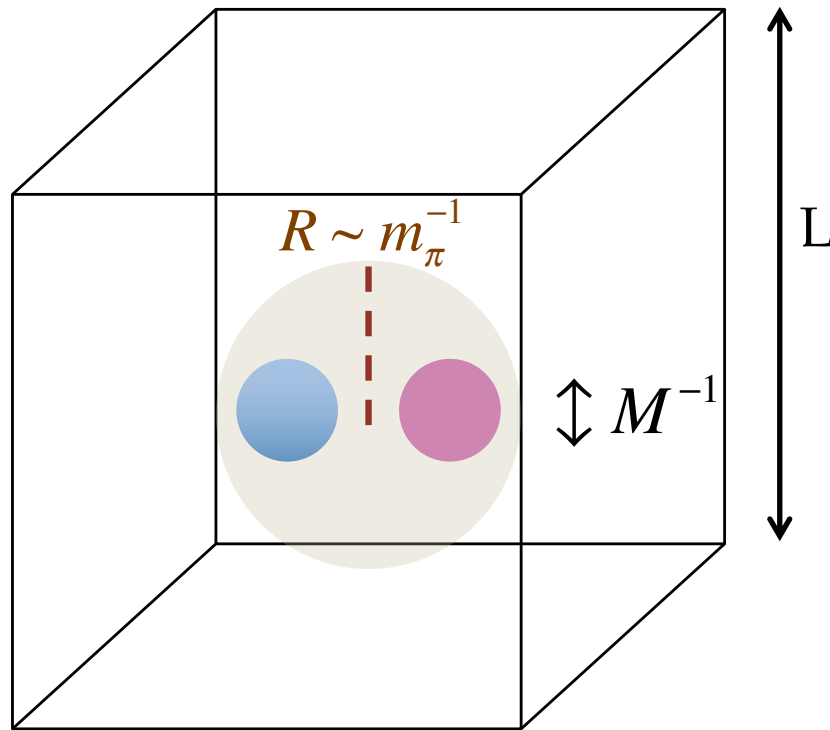
Monte-Carlo (probabilistic) evaluation of the Euclidean space path integral



(anti) periodic (time) spatial boundary conditions

$$L \gg \text{relevant scales} \gg b \quad \left(\frac{1}{L} \ll m_\pi \ll \Lambda_\chi \ll \frac{1}{b} \right)$$

→ i.e. finite number of d.o.f. (finite volume)



Lüscher, 1990

$$R < \frac{L}{2}$$

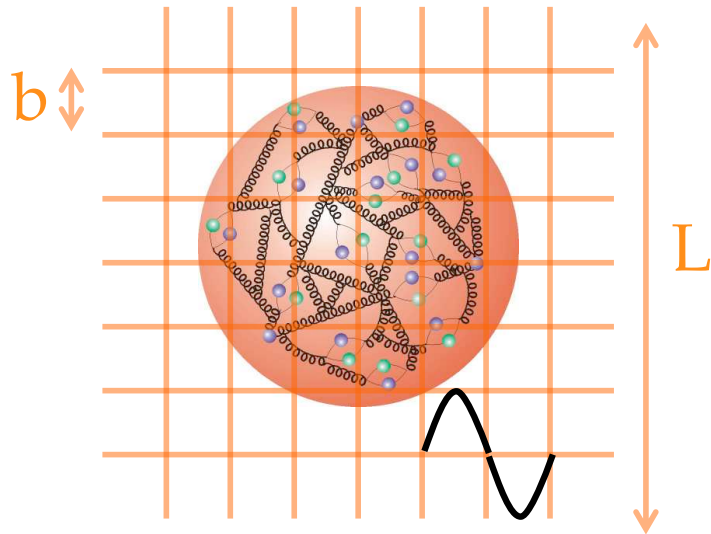
nucleon-nucleon scattering

$$\Rightarrow m_\pi L \gg 1 \quad (\text{infrared cutoff})$$

$$b \ll \frac{1}{M_N} \quad (\text{ultraviolet cutoff})$$

$$L \gg \text{relevant scales} \gg b \quad \left(\frac{1}{L} \ll m_\pi \ll \Lambda_\chi \ll \frac{1}{b} \right)$$

\rightarrow *i.e.* finite number of d.o.f. (finite volume)



$$x = b (n_1, n_2, n_3, n_4) \quad n_j \in \mathbb{Z}$$

$$L^3 \times T$$

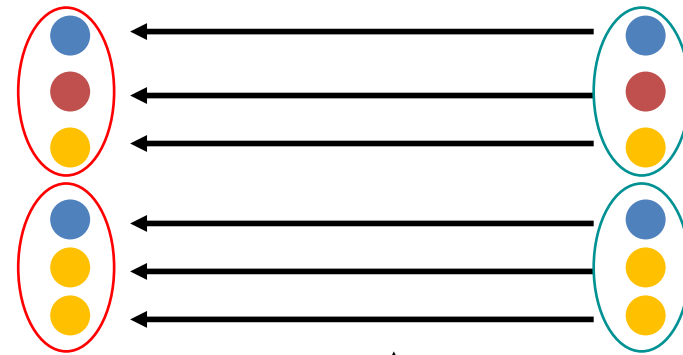
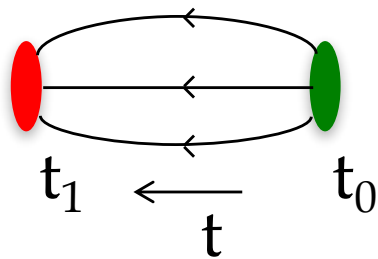
$$\lambda_{\min} = 2b \quad \text{shortest wave length}$$

$$\left. \begin{array}{l} \vec{p} = \frac{2\pi}{L} \vec{n}, \quad n_\mu \in \mathbb{Z} \\ x_\mu = m_\mu b, \quad \text{with } m_\mu = 0, 1, 2, \dots, N-1, \quad \text{and } L = Nb \end{array} \right\} \Rightarrow p_{\max} = n_{\max} \frac{2\pi}{L} = \frac{N}{2} \frac{2\pi}{Nb} = \frac{\pi}{b} \quad \text{cut-off}$$

(larger wave vector)

Computational cost increases as light quark masses decrease:
work with unphysical values of the quark masses and perform the correspondig
extrapolation

Direct Lattice QCD extraction \longleftrightarrow Compute correlation functions



smeared/point sink

$$\sum_n |n\rangle \langle n|$$

smeared source

construction of **SS/SP** correlation functions

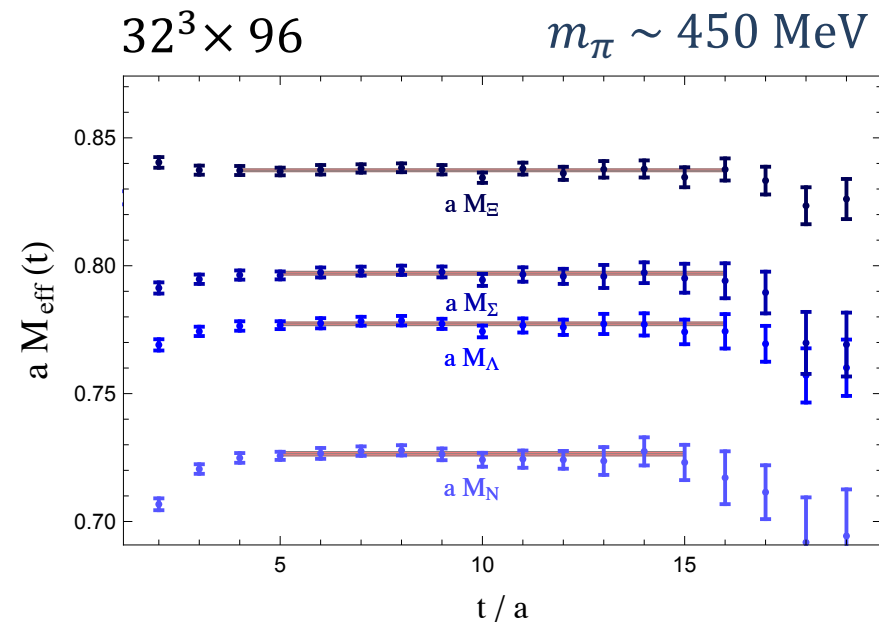
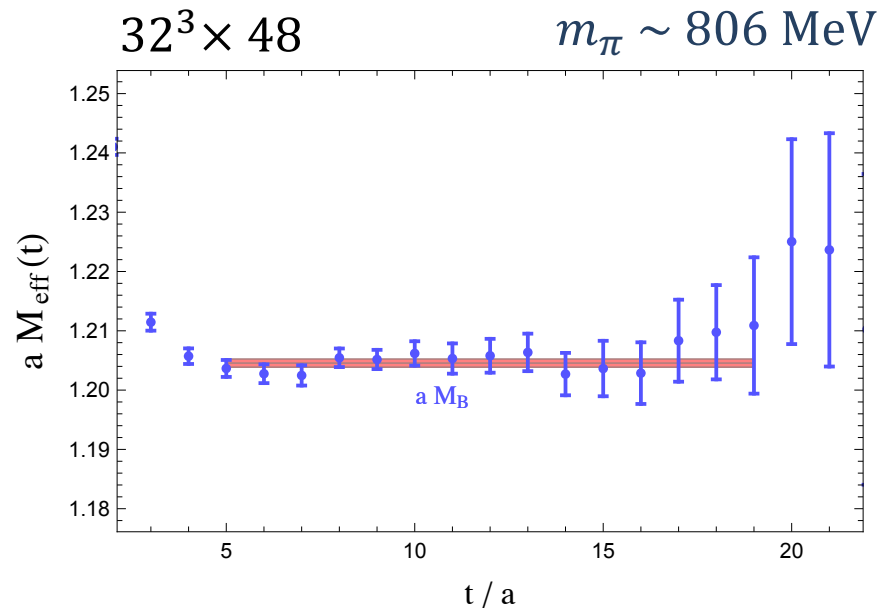
$$\begin{aligned}
 C_{\hat{O}, \hat{O}'}(t, \vec{d}) &= \sum_{\vec{x}} e^{2\pi i \vec{d} \vec{x} / L} \langle 0 | \hat{O}'(\vec{x}, t) \hat{O}^\dagger(\vec{0}, 0) | 0 \rangle \\
 &= \underbrace{Z_0^{snk} Z_0^{\dagger src}}_{\text{dominates at large } t} e^{-E^{(0)}t} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \dots
 \end{aligned}$$

projected into total momentum $\frac{2\pi \vec{d}}{L}$

Determination of the energy levels:

- Use a linear combination of SS/SP correlation functions to remove the excited-state contamination of the lowest lying state at earlier times (Matrix-Prony method/GPoF method)
- Perform a correlated χ^2 fit to single or two-exponential forms
- Work with an effective mass(energy) function:

$$C_{\hat{O},\hat{O}'}(\tau; \vec{d}, \tau_J) = \frac{1}{\tau_J} \log \left[\frac{C_{\hat{O},\hat{O}'}(\tau; \vec{d})}{C_{\hat{O},\hat{O}'}(\tau+\tau_J; \vec{d})} \right] \rightarrow E_0 \quad \text{at large times}$$



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Using the single and two-body correlation functions, one can compute the shift in the energy of the system resulting from two-body interactions:

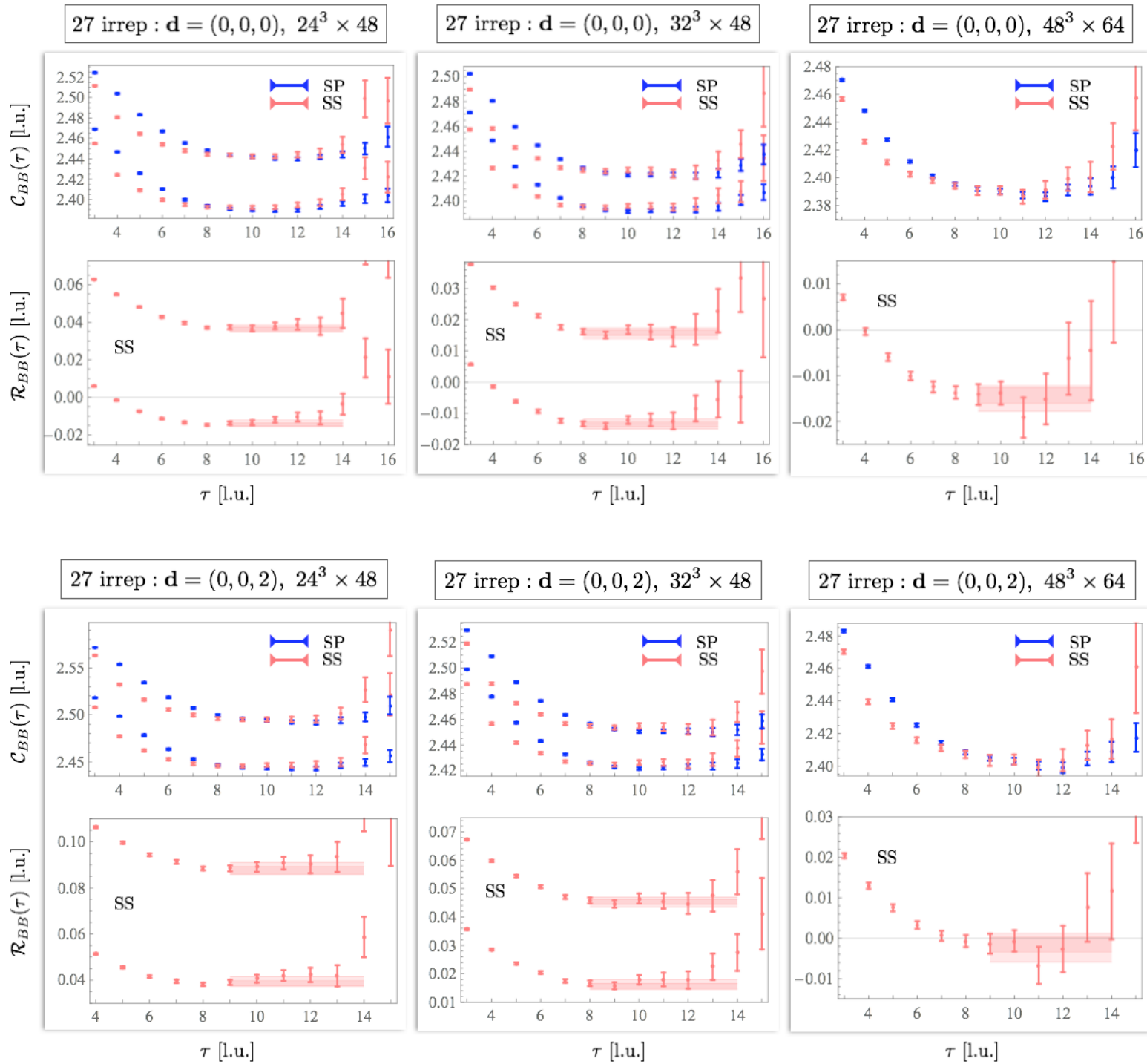
$$R(\tau; \mathbf{d}) = \frac{C_{\hat{O}_{BB},\hat{O}'_{BB}}(\tau; \mathbf{d})}{\left[C_{\hat{O}_B,\hat{O}'_B}(\tau; \mathbf{0}) \right]^2} = \mathcal{A}_1 e^{-(E_{BB}^{(0)} - 2M_B)\tau} \times \frac{1 + \mathcal{A}_2 e^{-(E_{BB}^{(1)} - E_{BB}^{(0)})\tau} + \dots}{\left[1 + \mathcal{A}_3 e^{-(E_B^{(1)} - M_B)\tau} + \dots \right]^2}$$

$$R^{\text{eff}}(\tau; \vec{d}, \tau_J) = \frac{1}{\tau_J} \log \left[\frac{R(\tau; \vec{d})}{R(\tau+\tau_J; \vec{d})} \right] \rightarrow \overline{\Delta E} \quad \text{at large times}$$

$$m_\pi \sim 806 \text{ MeV}$$

$$J=0$$

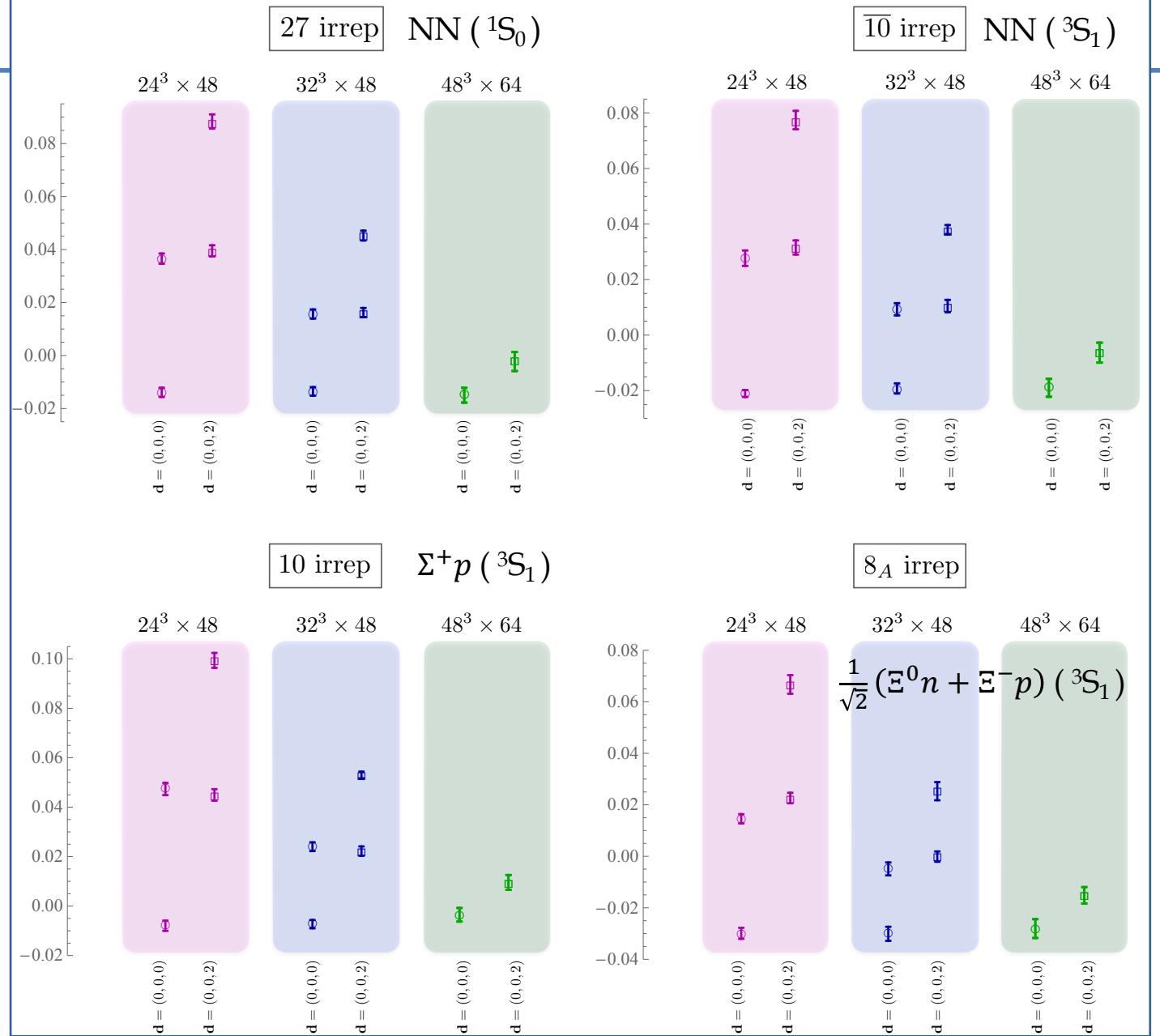
$$(nn)$$



Energy shifts

$$\overline{\Delta E} = E_{BB'} - 2M_B$$

10 kinematical points
for each irrep



M.L. Wagman et al (NPLQCD), PRD, ARXIV:1706.06550

$m_\pi \sim 806$ MeV

Energy shifts – Location of bound states

$$\overline{\Delta E} = E_{BB'} - 2M_B$$

10 kinematical points
for each irrep

input to constrain

— L=24 l.u. (3.4 fm)
— L=32 l.u. (4.5 fm)
— L=48 l.u. (6.7 fm)

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n}} \frac{1}{\vec{n}^2 - q^2}$$

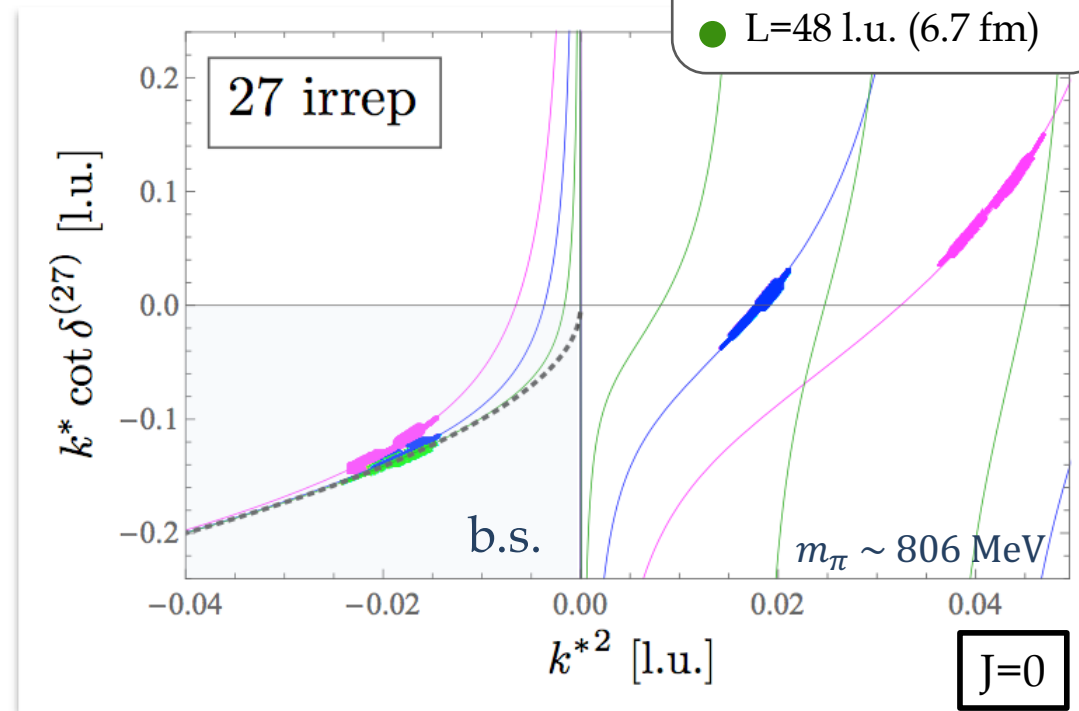
$$\mathcal{A} \sim \frac{4\pi}{M} \frac{1}{p \cot \delta(p) - ip}$$

$$\vec{n} \in Z^3 \quad q = \frac{Lk}{2\pi}$$

infinite volume

$$\text{b.s.} \quad p^2 = -\gamma^2$$

$$\cot \delta(i\gamma) = i$$



finite volume:

$$\cot \delta(i\gamma) \Big|_{k=i\gamma} = i - i \sum_{\vec{m} \neq 0} \frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

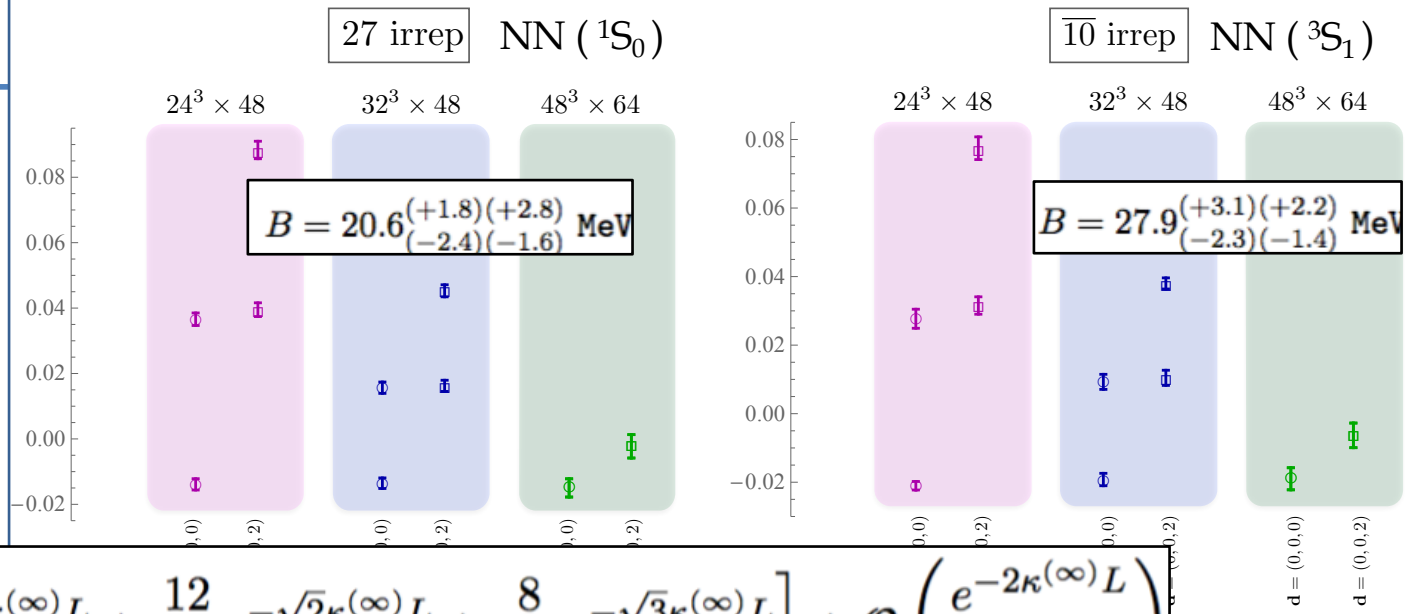


$$k^* \cot \delta(k^*) = -\sqrt{-k^{*2}} \dots\dots\dots$$

Binding energies

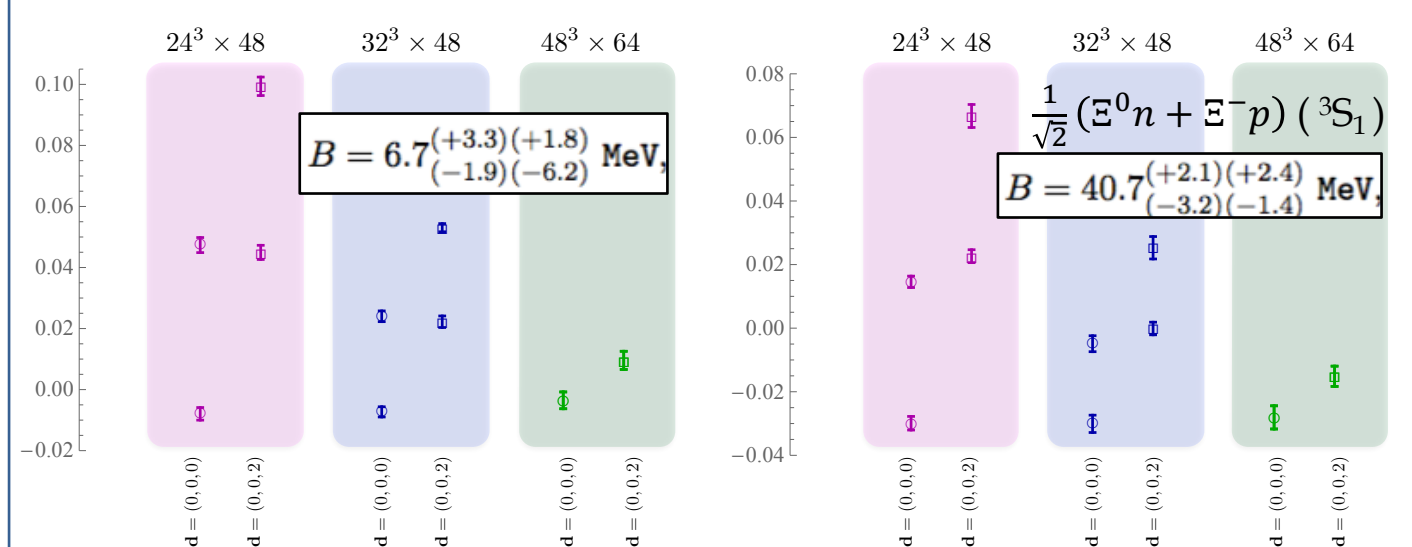
$$\overline{\Delta E} = E_{BB'} - 2M_B$$

10 kinematical points
for each irrep



$$|k^*| = \kappa^{(\infty)} + \frac{Z^2}{L} \left[6e^{-\kappa^{(\infty)}L} + \frac{12}{\sqrt{2}}e^{-\sqrt{2}\kappa^{(\infty)}L} + \frac{8}{\sqrt{3}}e^{-\sqrt{3}\kappa^{(\infty)}L} \right] + \mathcal{O}\left(\frac{e^{-2\kappa^{(\infty)}L}}{L}\right)$$

Lüscher's quantization condition

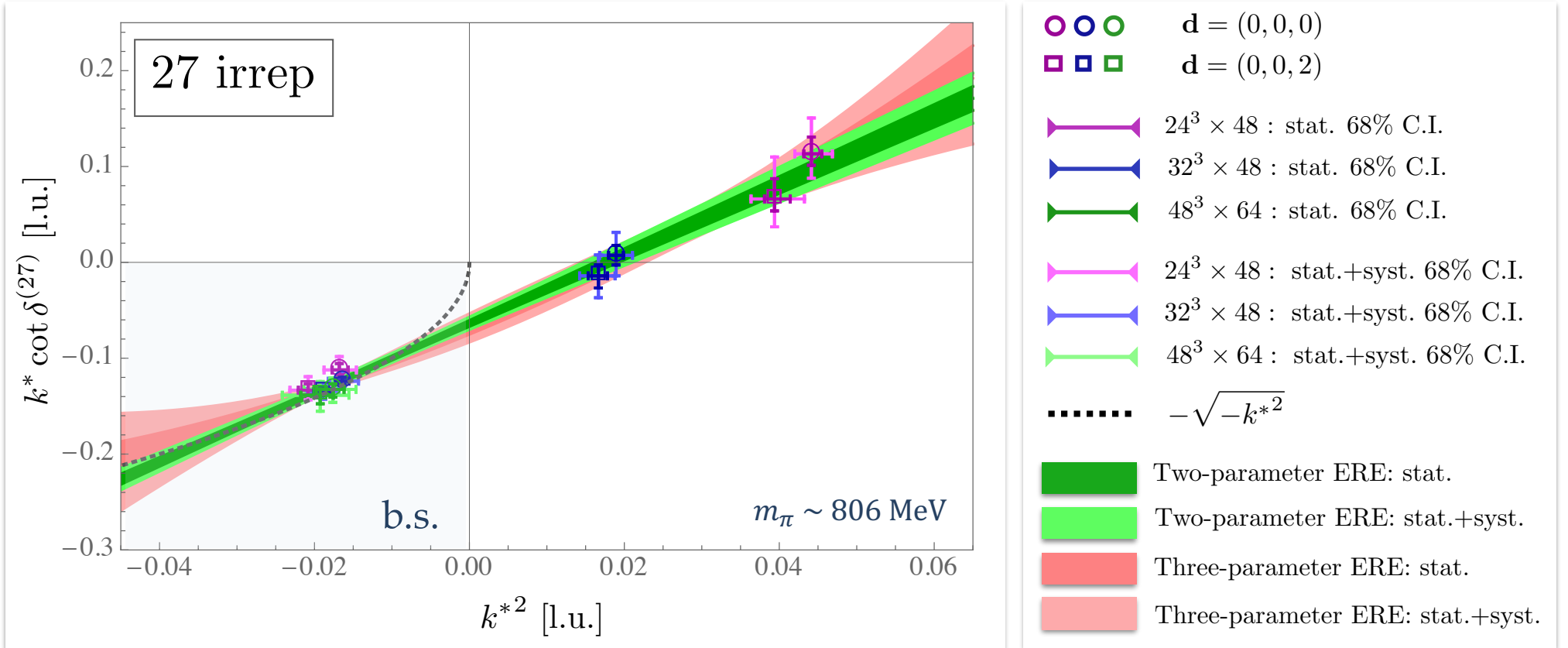


$$B = -2\sqrt{-\kappa^{(\infty)^2} + M_B^2} + 2M_B$$

M.L. Wagman et al (NPLQCD), PRD, ARXIV:1706.06550

$m_\pi \sim 806 \text{ MeV}$

Energy shifts – Location of bound states – Effective Range Expansion



$$\text{ERE } k^* \cot \delta(k^*) = -\frac{1}{a} + \frac{1}{2}rk^{*2} + Pk^{*4} + \dots$$

Energy shifts – Location of bound states – Effective Range Expansion

M.L. Wagman et al
(NPLQCD) PRD,
ARXIV:1706.06550

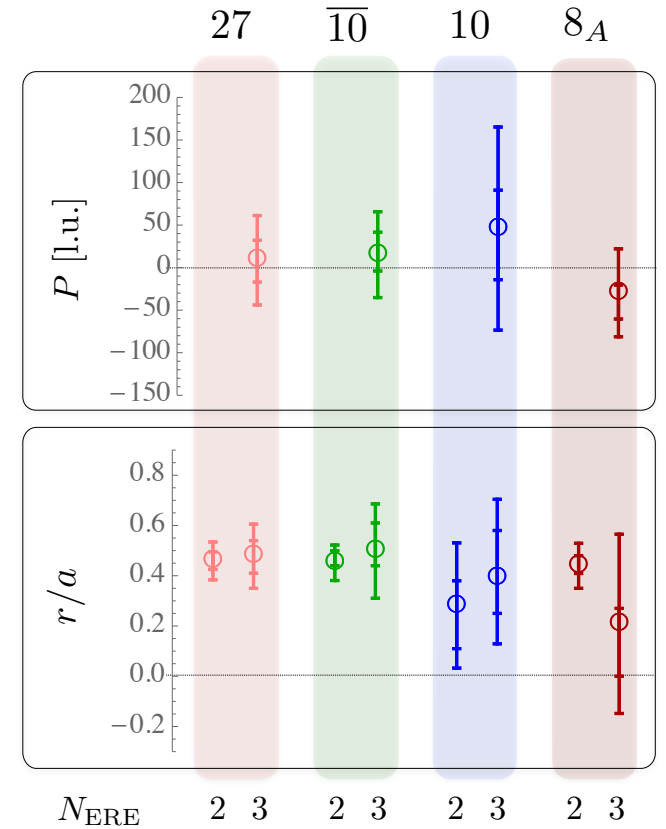
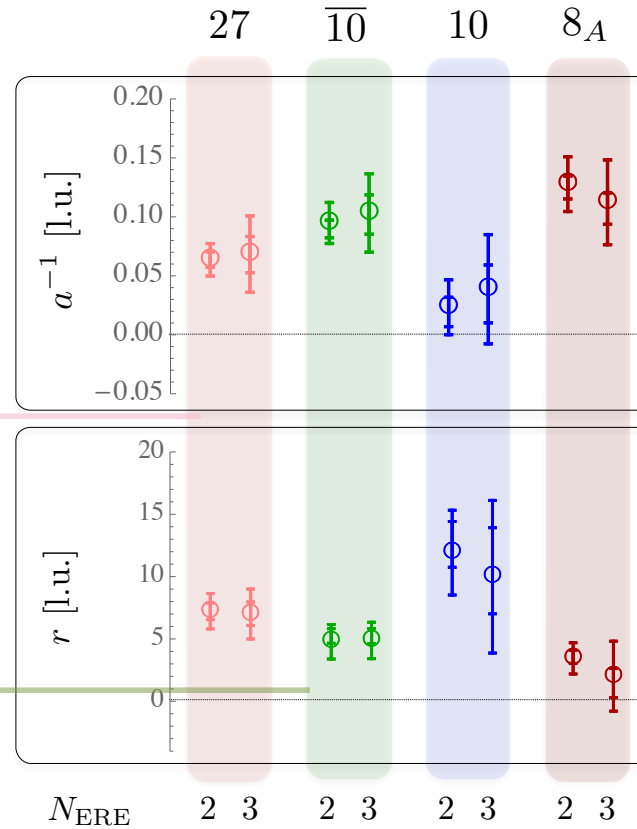
$m_\pi \sim 806$ MeV

$$a^{-1} = 0.44^{(+4)(+8)}_{(-5)(-8)} \text{ fm}^{-1}$$

$$r = 1.04^{(+10)(+18)}_{(-10)(-18)} \text{ fm}$$

$$a^{-1} = 0.63^{(+6)(+10)}_{(-5)(-11)} \text{ fm}^{-1}$$

$$r = 0.70^{(+16)(+12)}_{(-2)(-20)} \text{ fm}$$



Energy shifts – Location of bound states – Effective Range Expansion

M.L. Wagman et al
(NPLQCD) PRD,
ARXIV:1706.06550

$m_\pi \sim 806 \text{ MeV}$

$$a^{-1} = 0.44^{(+4)(+8)}_{(-5)(-8)} \text{ fm}^{-1}$$

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$$a^{-1} = 0.63^{(+6)(+10)}_{(-5)(-11)} \text{ fm}^{-1}$$

$$r = 0.70^{(+16)(+12)}_{(-2)(-20)} \text{ fm}$$

K. Orginos et al (NPLQCD)
PRD92,114512 (2015)

$m_\pi \sim 450 \text{ MeV}$

$$a^{-1}({}^1S_0) = 0.05^{(+0.06)(+0.08)}_{(-0.08)(-0.14)} \text{ fm}^{-1}$$

$$r({}^1S_0) = 2.96^{(+43)(+87)}_{(-34)(-055)} \text{ fm}$$

$$a^{-1}({}^3S_1) = -0.09^{(+0.15)(+0.19)}_{(-0.23)(-0.39)} \text{ fm}^{-1}$$

$$r({}^3S_1) = 3.4^{(+1.0)(+1.5)}_{(-0.7)(-0.8)} \text{ fm}$$

Nature

m_π physical

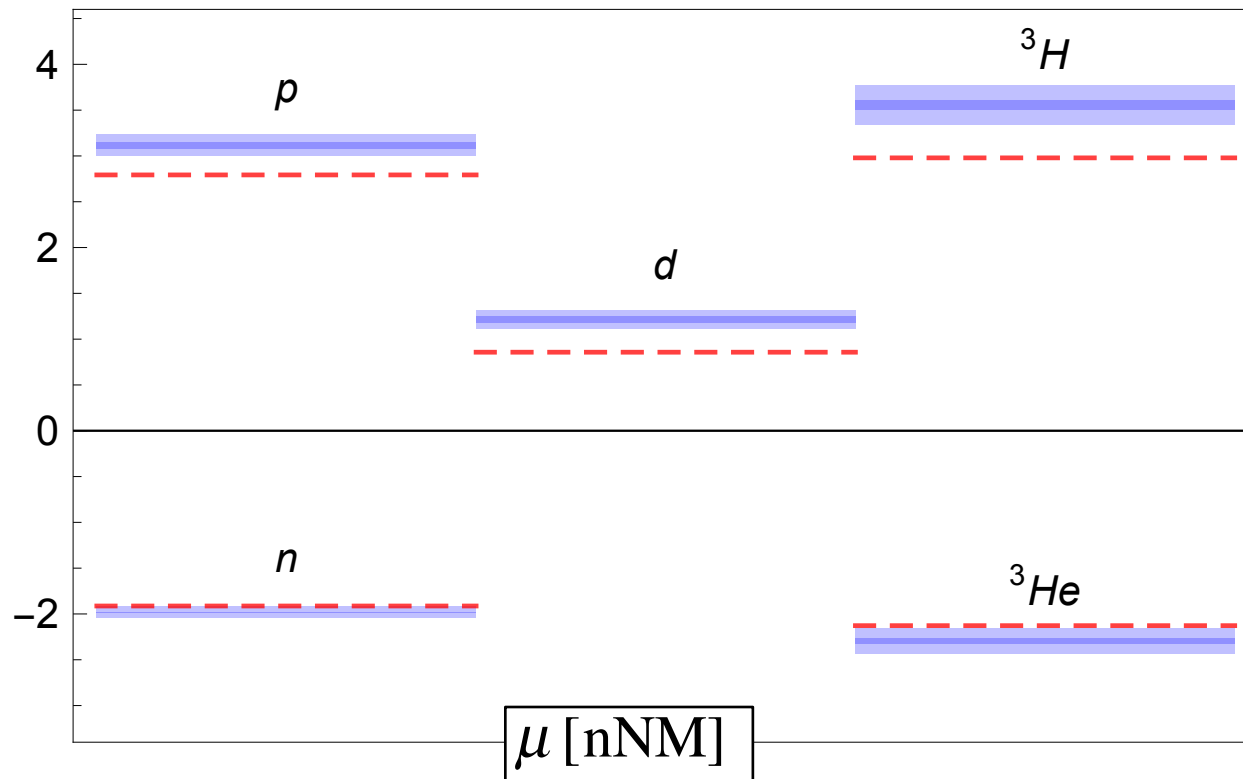
$$a^{-1}({}^1S_0) \approx -0.05 \text{ fm}^{-1}$$

$$r({}^1S_0) \approx 2.75 \text{ fm}$$

$$a^{-1}({}^3S_1) \approx 0.18 \text{ fm}^{-1}$$

$$r({}^3S_1) \approx 1.75 \text{ fm}$$

NPLQCD, *Phys. Rev. Lett.* 113 (2014) 25, 252001x



Octet baryon
magnetic moments

@ ~ 800 MeV

@ ~ 450 MeV

$$\text{nNM} = \frac{e}{2M_N^{\text{latt}}} = \frac{e}{2M_N(m_\pi^{\text{latt}})}$$

 LQCD @ $m_\pi \sim 800$ MeV

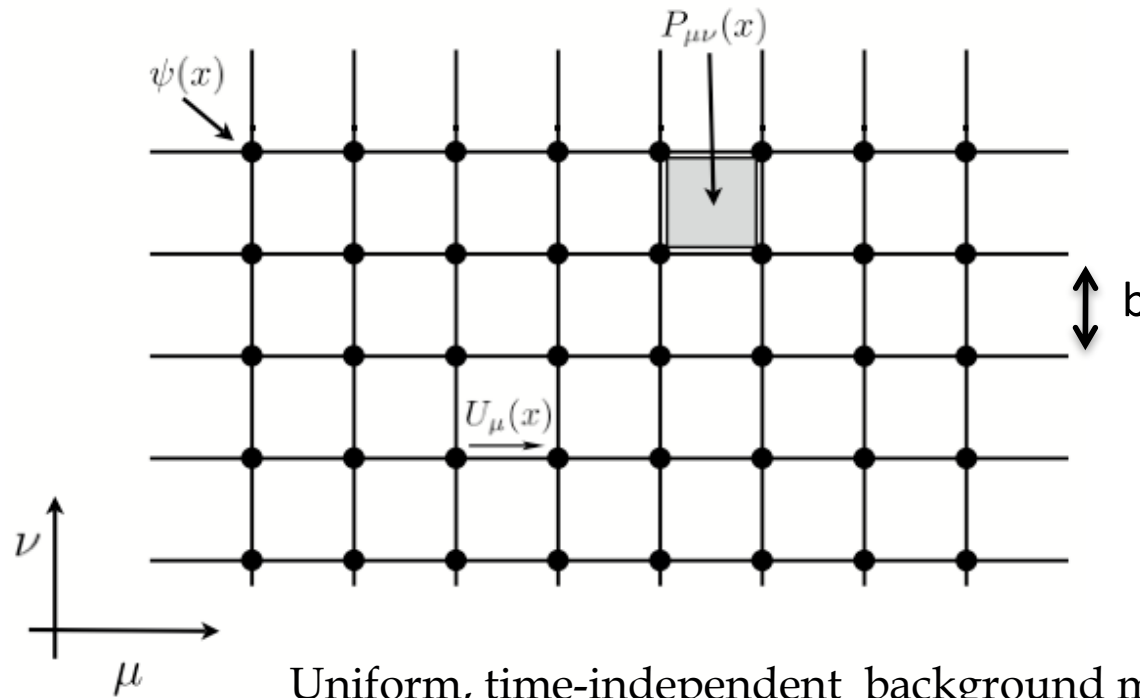
 experiment

Shell-model
predictions

$$\mu({}^3H) = \mu_p$$

$$\mu({}^3He) = \mu_n$$

$$\mu_d = \mu_n + \mu_p$$



Uniform, time-independent background magnetic field $\vec{B} = \hat{z} \cdot B$

$$U_\mu^b(x) \equiv e^{ig \int_x^{x+b\hat{\mu}} A_\mu dx^\mu} = e^{igbA_\mu^b(x)}$$

(SU(3)_c matrices, “links”)

$$U_\mu(x + L\hat{\nu}) = U_\mu(x)$$

$$U_\mu^b(x) \rightarrow U_\mu^b(x) U_\mu^{\text{ext}}(x)$$

$$U_0^{\text{ext}}(x) = U_3^{\text{ext}}(x) = 1$$

$$U_1^{\text{ext}}(x) = \begin{cases} 1 & \text{for } x_1 \neq L - b \\ \exp\left(-i Q n \frac{2\pi x_2}{L}\right) & \text{for } x_1 = L - b \end{cases}$$

$$U_2^{\text{ext}}(x) = \exp\left(i Q n \frac{2\pi b x_1}{L^2}\right)$$

G. t'Hooft, 1979

U(1) flux through each plaquette = $e^{iQeF_{\mu\nu}}$,
with $F_{12} = -F_{21} = B_z$

$$QeB_z = \frac{2\pi}{L^2} n$$

$$n = 3, -6, 12$$

$$\langle O(\underline{U}, \underline{q}, \bar{\underline{q}}) \rangle = \frac{1}{Z} \int [d\underline{U}] \underbrace{\prod_f}_{\text{propagators}} \underbrace{(D[\underline{U}] + m_f)^{-1} \det(D[\underline{U}] + m_f)}_{\text{configurations } (\sim P(\underline{U}))} e^{-S_g[\underline{U}]}$$

valence quarks sea quarks
 ↓ ↓

background electromagnetic field couples also to sea-quark degrees of freedom through the fermionic determinant.

$SU(3)_f$ sea quark contributions $\sim Q_u + Q_d + Q_s$

$$U_\mu^b(x) \rightarrow U_\mu^b(x) U_\mu^{\text{ext}}(x)$$

$$U_0^{\text{ext}}(x) = U_3^{\text{ext}}(x) = 1$$

$$U_1^{\text{ext}}(x) = \begin{cases} 1 & \text{for } x_1 \neq L - b \\ \exp\left(-i Q n \frac{2\pi x_2}{L}\right) & \text{for } x_1 = L - b \end{cases}$$

$$U_2^{\text{ext}}(x) = \exp\left(i Q n \frac{2\pi b x_1}{L^2}\right)$$

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$U(1)$ flux through each plaquette $= e^{iQeF_{\mu\nu}}$,
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$$QeB_z = \frac{2\pi}{L^2} n$$

$$n = 3, -6, 12$$

$$E_B^{(s)}(B_z) = M_B + \underbrace{\frac{|Q_B e B_z|}{M_B} \left(n_L + \frac{1}{2} \right)}_{\substack{n_L \text{ Landau level} \\ \text{occupied by the baryon} \\ \text{(charged particles)}}} - \underbrace{2 \mu_B s B_z}_{\text{circled}} - \underbrace{2\pi \beta_B^{(M0)} |B|^2 + 2\pi \beta_B^{(M2)} \langle \hat{T}_{ij} B_i B_j \rangle}_{\substack{O(B^2) \quad (\text{polarizabilities})}} + \dots$$

$\left(s = \pm \frac{1}{2}, p_z = 0 \right)$

$$\delta E \equiv E_B^{(+1/2)}(\vec{B}) - E_B^{(-1/2)}(\vec{B}) = -2 \mu_B B_z + \dots$$

Determination of the magnetic moments relies on the determination of the Zeeman splittings

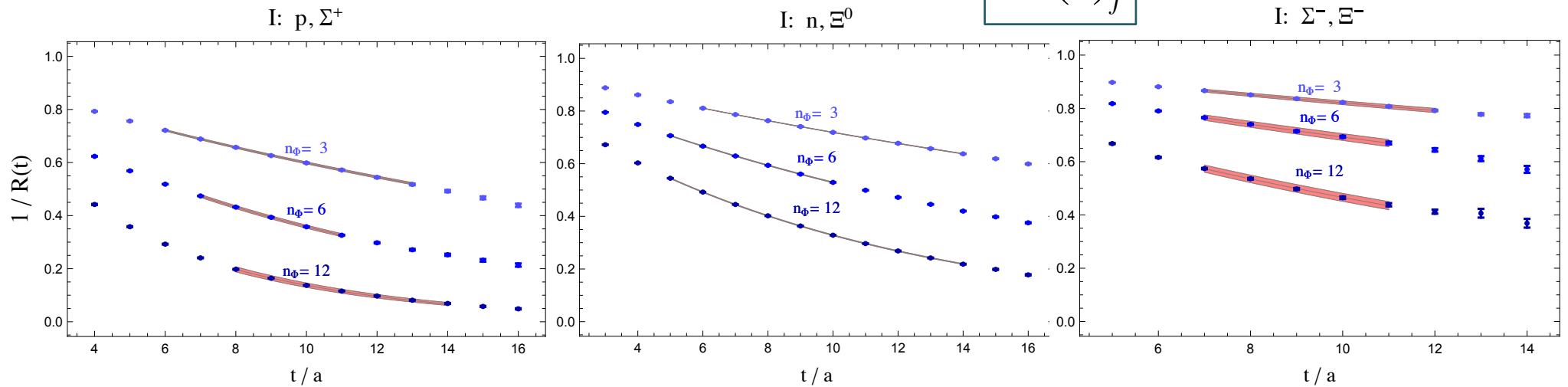
$$R_i(t) = \frac{G_i^{(+\frac{1}{2})}(t, n_\Phi)}{G_i^{(-\frac{1}{2})}(t, n_\Phi)} \bigg/ \frac{G_i^{(+\frac{1}{2})}(t, 0)}{G_i^{(-\frac{1}{2})}(t, 0)} \qquad \frac{1}{R_i(t)} \xrightarrow{t \rightarrow \infty} Z e^{-\delta E^B t}$$

$$QeB_z = \frac{2\pi}{L^2} n$$

$$n = 3, -6, 12$$

	L/a	T/a	β	am_l	am_s	$a[\text{fm}]$	$m_\pi[\text{MeV}]$	N_{cfg}
I	32	48	6.1	-0.2450	-0.2450	0.1453(16)	806.9(8.9)	1006

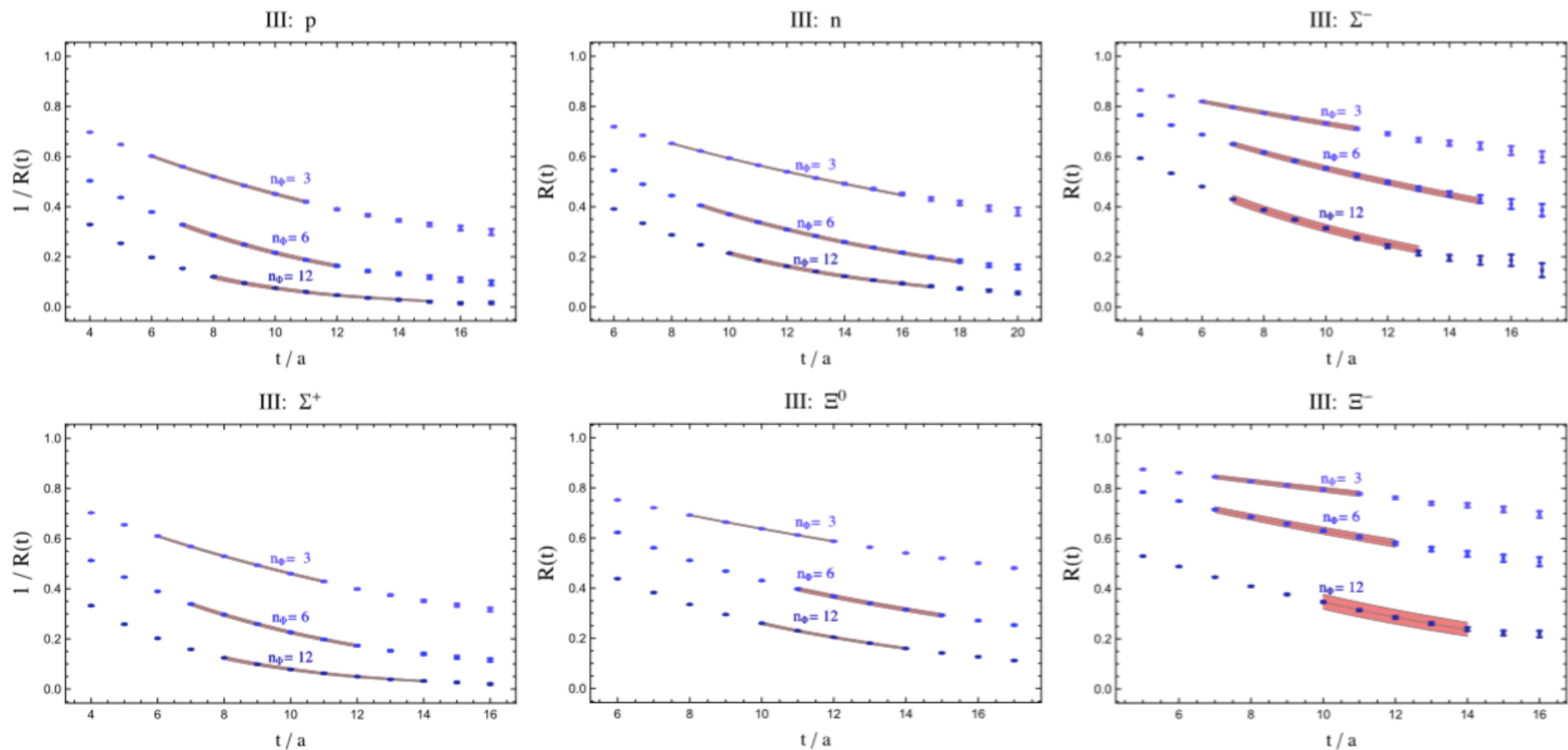
$SU(3)_f$



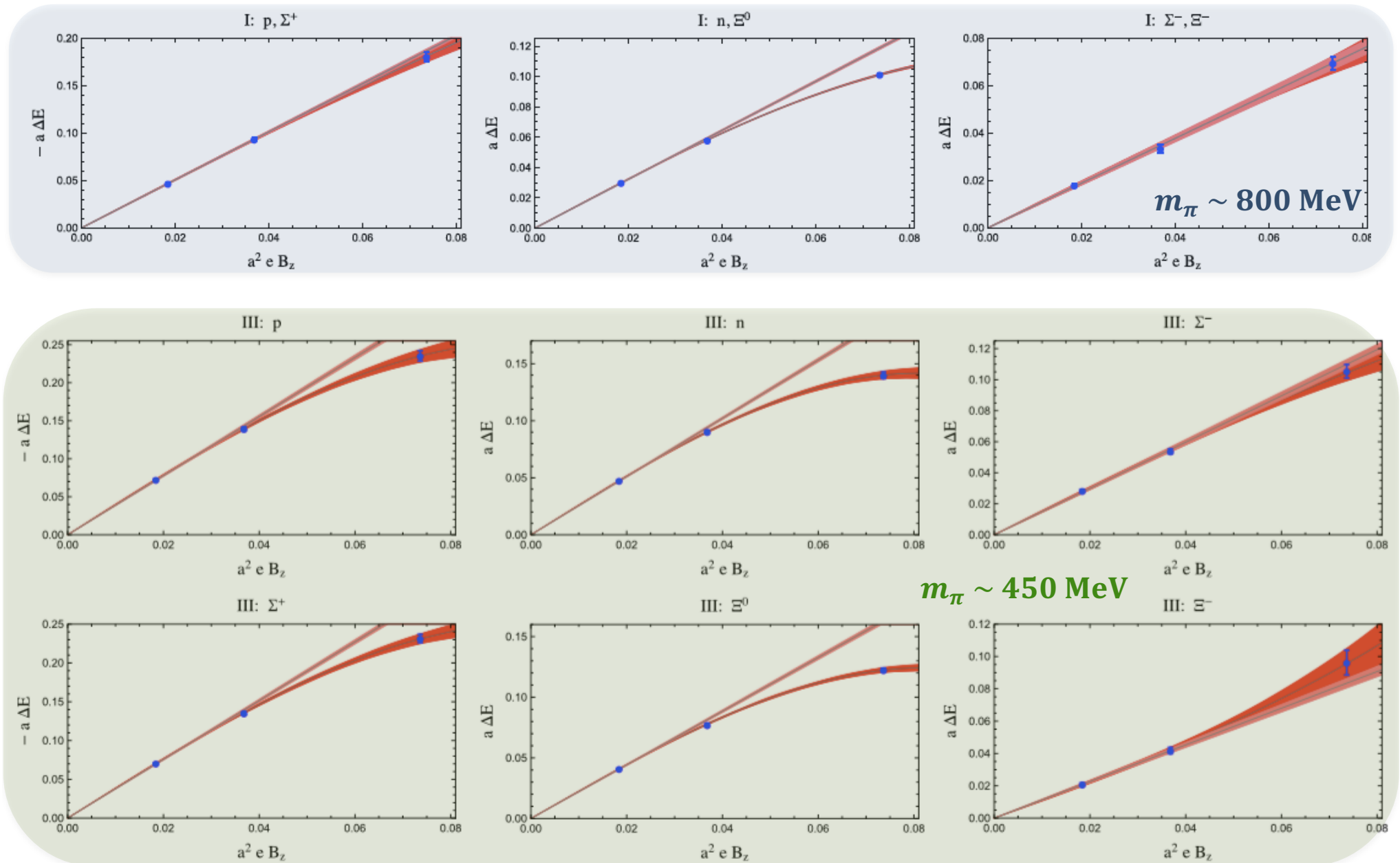
$$R_i(t) = \frac{G_i^{(+\frac{1}{2})}(t, n_\Phi)}{G_i^{(-\frac{1}{2})}(t, n_\Phi)} \bigg/ \frac{G_i^{(+\frac{1}{2})}(t, 0)}{G_i^{(-\frac{1}{2})}(t, 0)}$$

$$\frac{1}{R_i(t)} \xrightarrow{t \rightarrow \infty} Z e^{-\delta E^B t}$$

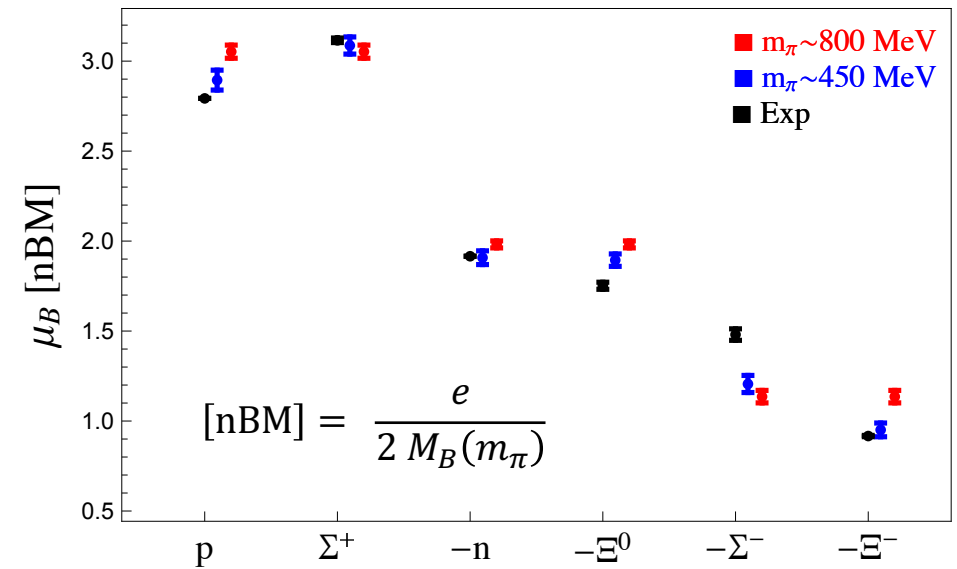
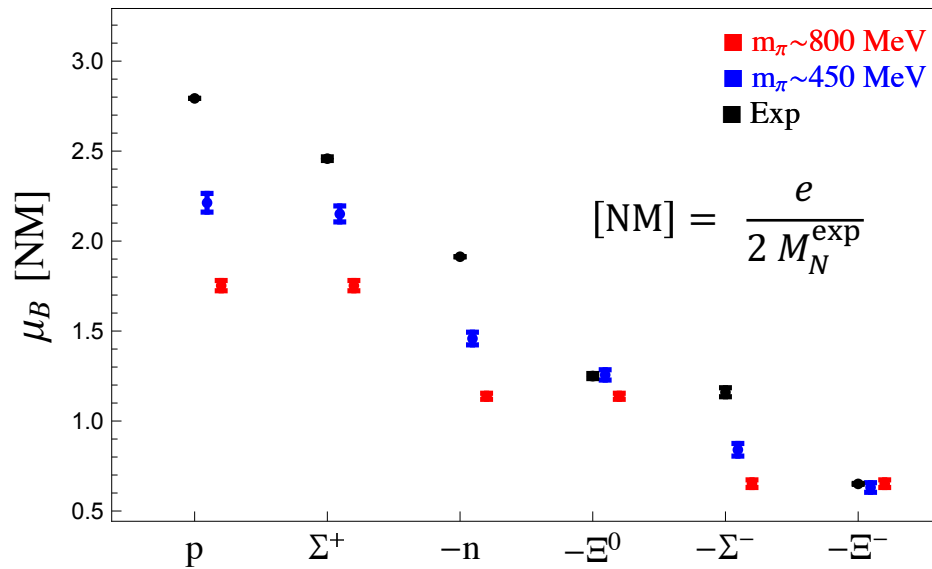
$$q_d = q_s \Rightarrow \begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & U & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad U \in SU(2)$$



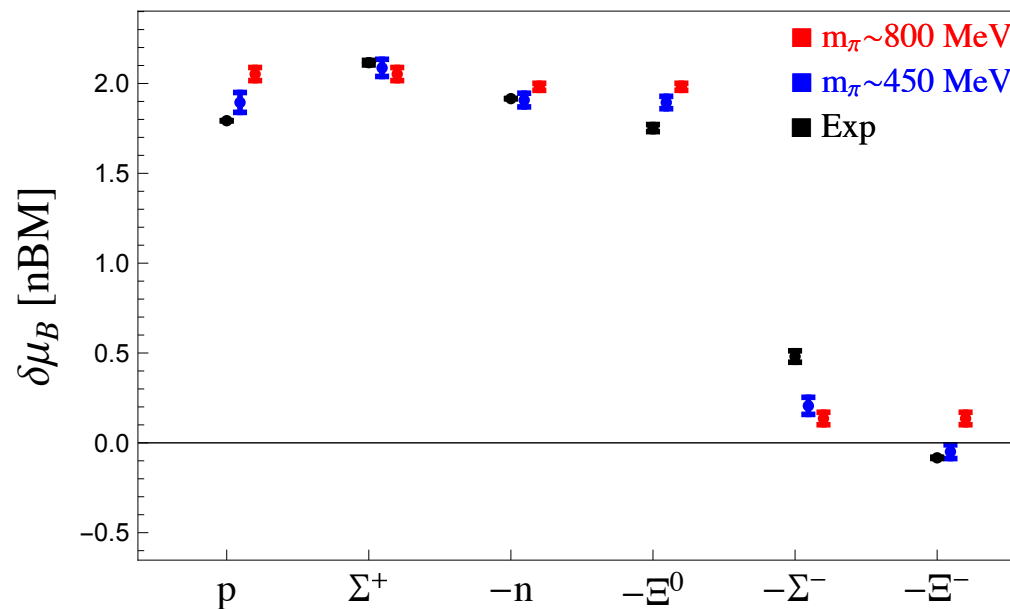
	L/a	T/a	β	am_l	am_s	$a[\text{fm}]$	$m_\pi[\text{MeV}]$	N_{cfg}
III	32	96	6.1	-0.2800	-0.2450	0.1167(16)	449.9(4.6)	544



$$\delta E \equiv E_B^{(+1/2)}(\vec{B}) - E_B^{(-1/2)}(\vec{B}) = -2 \mu_B B_z + \dots$$

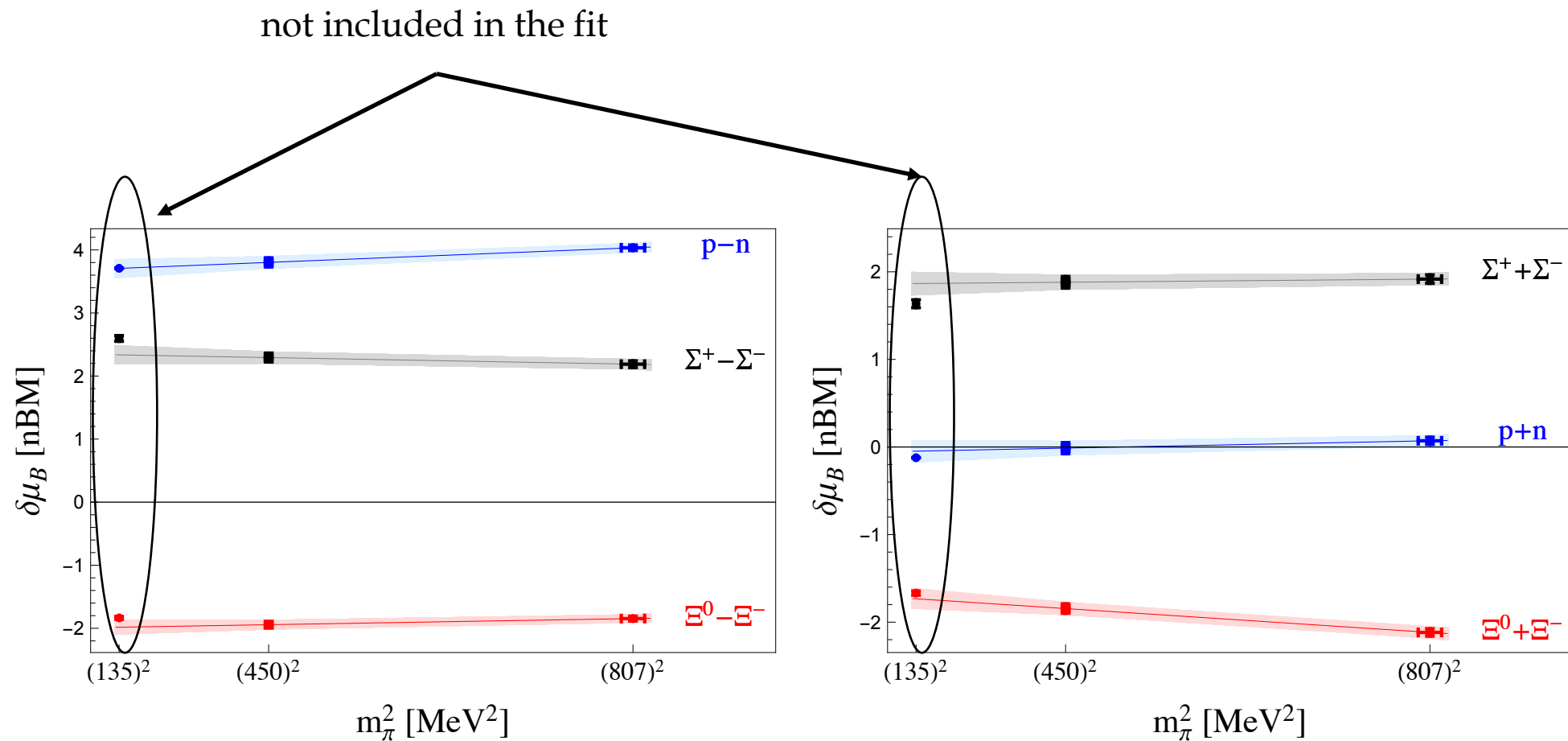


$$(\mu_{-B} \equiv -\mu_B)$$



Dirac operator

$$\delta\mu_B[nBM] = \mu_B[nBM] - Q_B$$



Quark mass extrapolation of the anomalous part of the isovector and isoscalar magnetic moments assuming

$$\delta\mu_B^I(m_\pi^2) = \delta\mu_B^I(0) + A_B^I m_\pi^2$$

NPLQCD, ongoing

1. *Computation of heavier nuclei. In collaboration with A. Lovato (INFN-TIFPA)*

Extend the domain of LQCD predictions to heavier nuclei in a model-independent way

*π EFT \rightarrow fit of LECs to LQCD calculations for few-baryon systems
+ QMC techniques*

2. *Analysis of YN and YY results from the 450 MeV pion mass calculation*

3. *Continue production at lighter pion mass (~ 300 MeV)*

