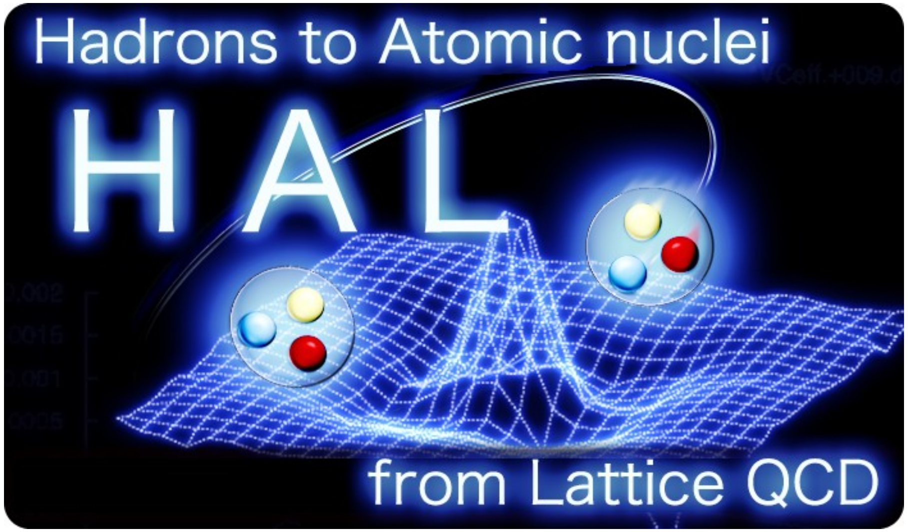
Hyperons in infinite nuclear matter based on the hyperon-baryon interactions from the HALQCD method

Takashi Inoue @Nihon Univ.
for
the HALQCD Collaboration

S. Aoki	YITP Kyoto Univ.
T. Doi	RIKEN Nishina
T. Hatsuda	RIKEN Nishina
Y. Ikeda	RCNP Osaka Univ
T. I.	Nihon Univ.
N. Ishii	RCNP Osaka Univ.
K. Murano	RCNP Osaka Univ.
H. Nemura	RCNP Osaka Univ.
K. Sasaki	YITP Kyoto Univ.
F. Etminan	Univ. Birjand
T. Miyamoto	YITP Kyoto Univ.
T. Iritani	RIKEN Nishina
S. Gongyo	RIKEN Nishina
D. Kawai	Kyoto Univ.
T. Doi	RIKEN Nishina
T. Aoyama	YITP Kyoto Univ



EMMI Workshop, Nov 9, 2017, Trino

Introduction

- Nuclear physics
  - Theories have been developed extensively from 1930's
    - mean field theory, shell model, few-body technique etc.
  - Properties of nuclei are explained and even predicted.

Introduction

- ## Nuclear physics
  - Theories have been developed extensively from 1930's
    - mean field theory, shell model, few-body technique etc.
  - Properties of nuclei are explained and even predicted.
  - But, we need input data for nuclear force from experiment.

Introduction

- Nuclear physics
  - Theories have been developed extensively from 1930's
    - mean field theory, shell model, few-body technique etc.
  - Properties of nuclei are explained and even predicted.
  - But, we need input data for nuclear force from experiment.

- Quantum Chromodynamics
  - is the fundamental theory of the strong interaction,
  - has no free parameter almost,
  - must explain everything, e.g. hadron spectrum, mass of nuclei.

Introduction

Nuclear physics

- Theories have been developed extensively from 1930's
  - mean field theory, shell model, few-body technique etc.
- Properties of nuclei are explained and even predicted.
- But, we need input data for nuclear force from experiment.

Quantum Chromodynamics

- is the fundamental theory of the strong interaction,
- has no free parameter almost,
- must explain everything, e.g. hadron spectrum, mass of nuclei.
- But, that is difficult due to a non-perturbative nature of QCD.
- One way to handle the non-perturbative nature of QCD is ….

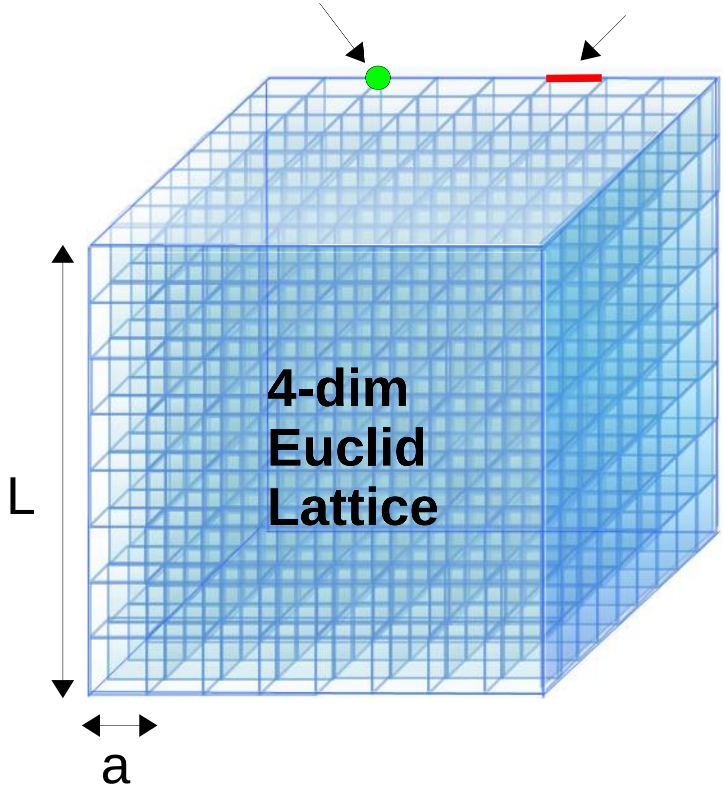
Lattice QCD

$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}\gamma^{\mu}\left(i\partial_{\mu} - g\, t^{a} A^{a}_{\mu}\right) q - m\bar{q}q$$

Lagrangian !

quarks q
on the sites

gluons $U = e^{iaA_\mu}$
on the links



Vacuum expectation value

path integral

$$\langle O(\bar{q}, q, U)\rangle$$

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

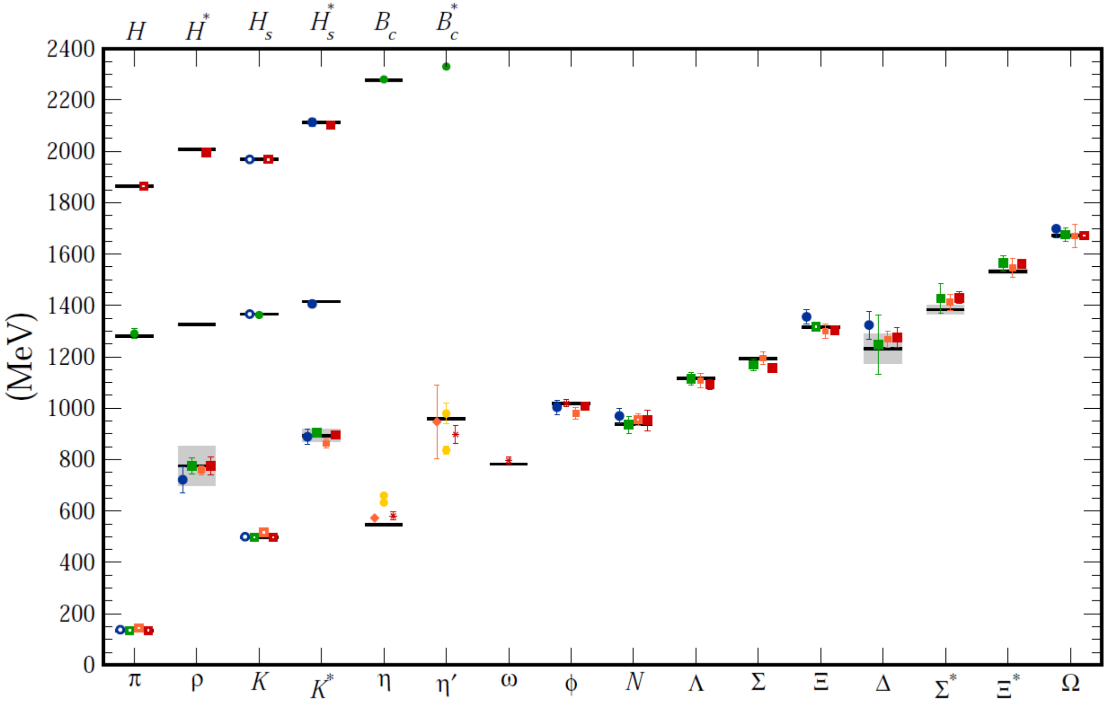
$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- ★ Well defined (regularized)
- ★ Manifest gauge invariance
- ★ Fully non-perturvative
- ★ Highly predictive

Lattice QCD

- LQCD simulations w/ the physical quark ware done.
  - PACS-CS, Phys. Rev. D81 (2010) 074503
  - BMW, JHEP 1108 (2011) 148

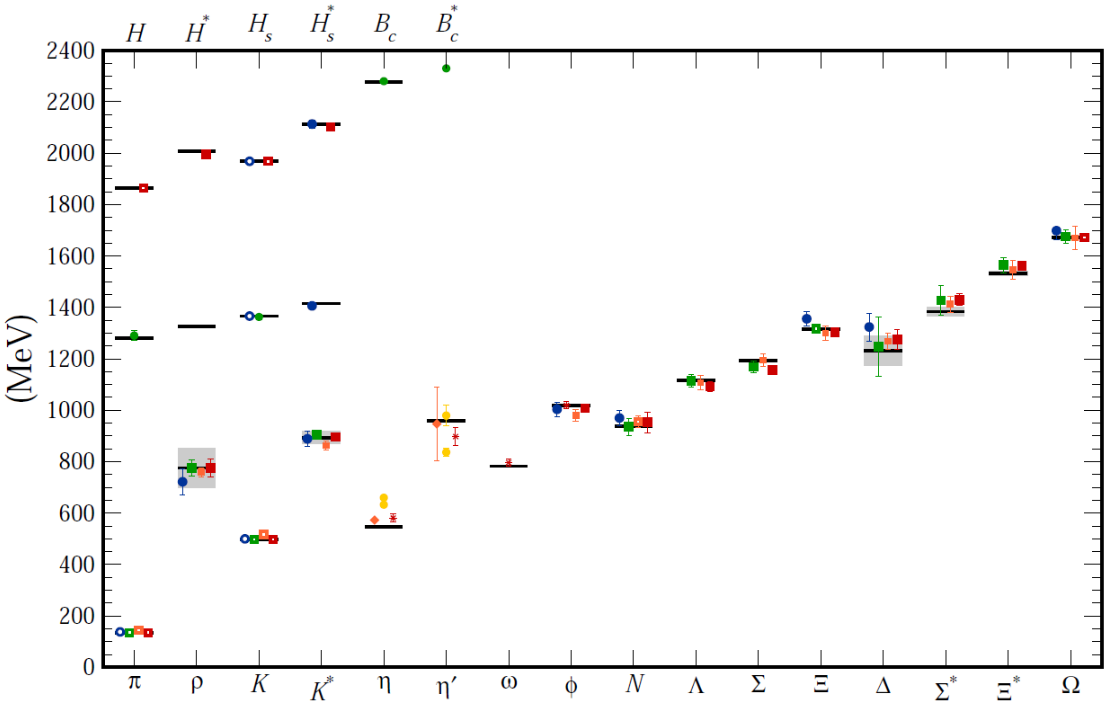


Summary by Kronfeld,
arXive 1203.1204

- Mass of (ground state) hadrons are well reproduced!

Lattice QCD

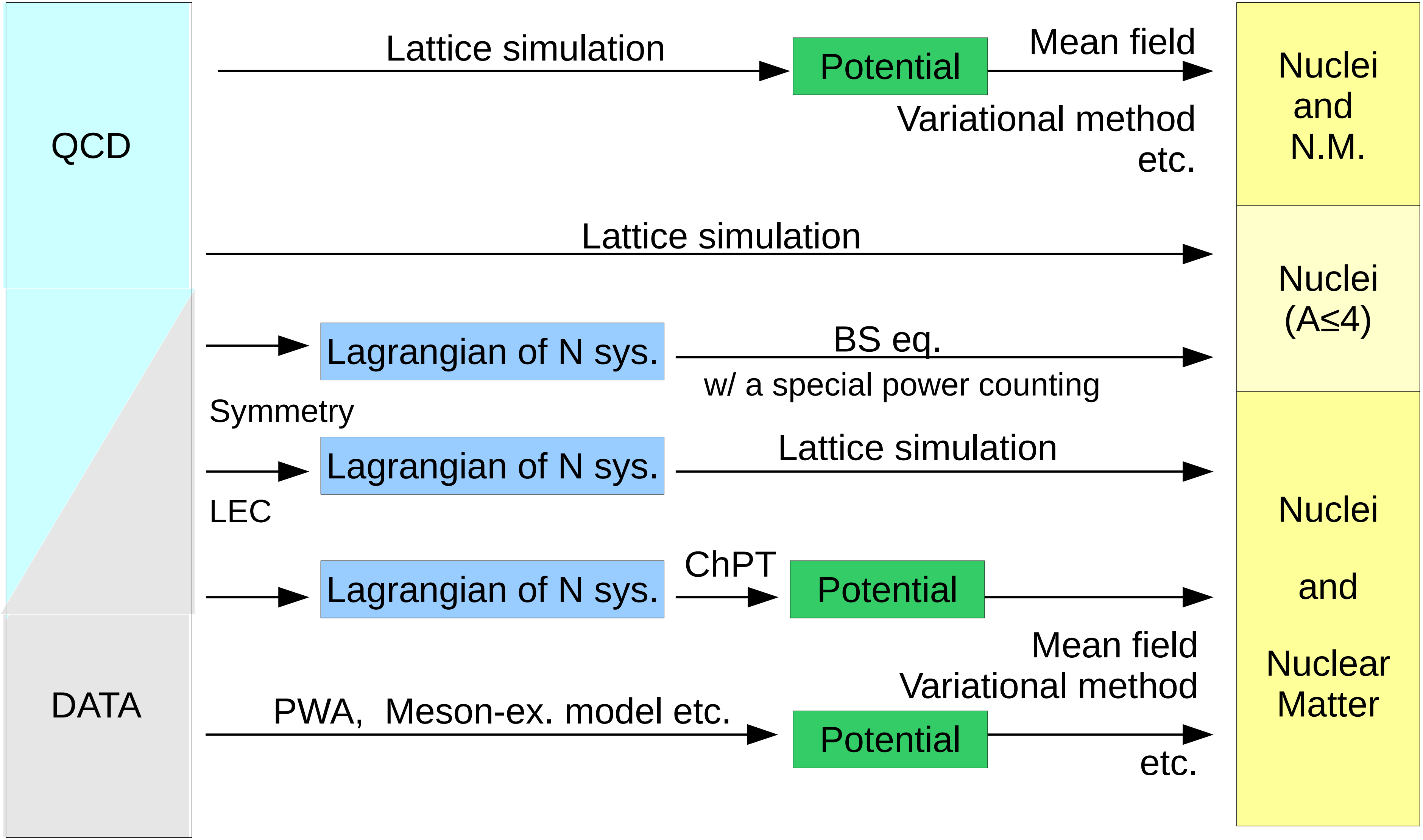
- LQCD simulations w/ the physical quark ware done.
    - PACS-CS, Phys. Rev. D81 (2010) 074503
    - BMW,   JHEP 1108 (2011) 148



Summary by Kronfeld,
arXive 1203.1204

- Mass of (ground state) hadrons are well reproduced!
- What about (hyper-)nuclei or matter from LQCD?

Various approaches in nuclear phys.



QCD

DATA

Lattice simulation

Potential

Mean field

Variational method

etc.

Nuclei
and
N.M.

Lattice simulation

Nuclei
(A≤4)

Lagrangian of N sys.

BS eq.

w/ a special power counting

Symmetry

Lagrangian of N sys.

Lattice simulation

LEC

Lagrangian of N sys.

ChPT

Potential

Nuclei

and

Nuclear
Matter

Mean field

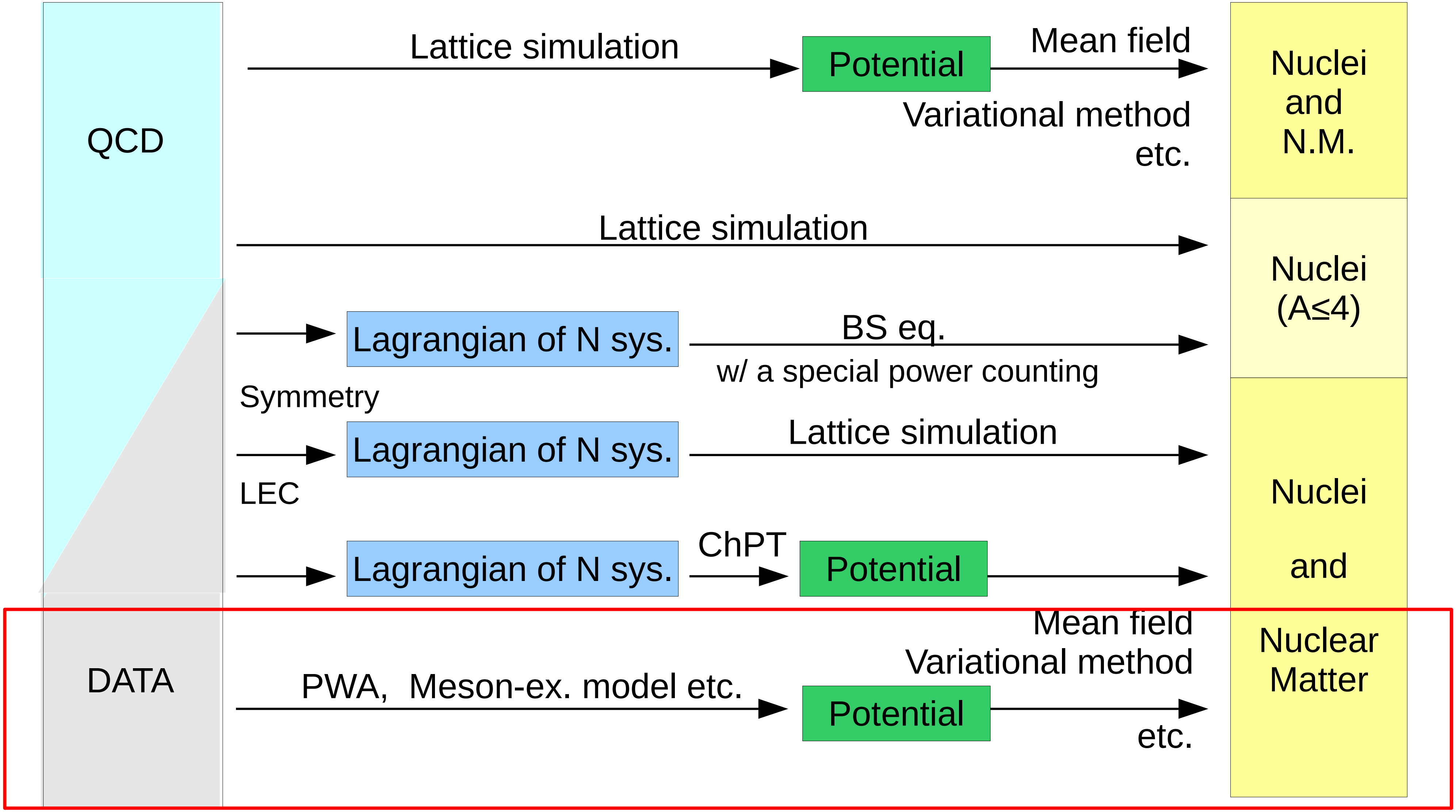
Variational method

PWA, Meson-ex. model etc.

Potential

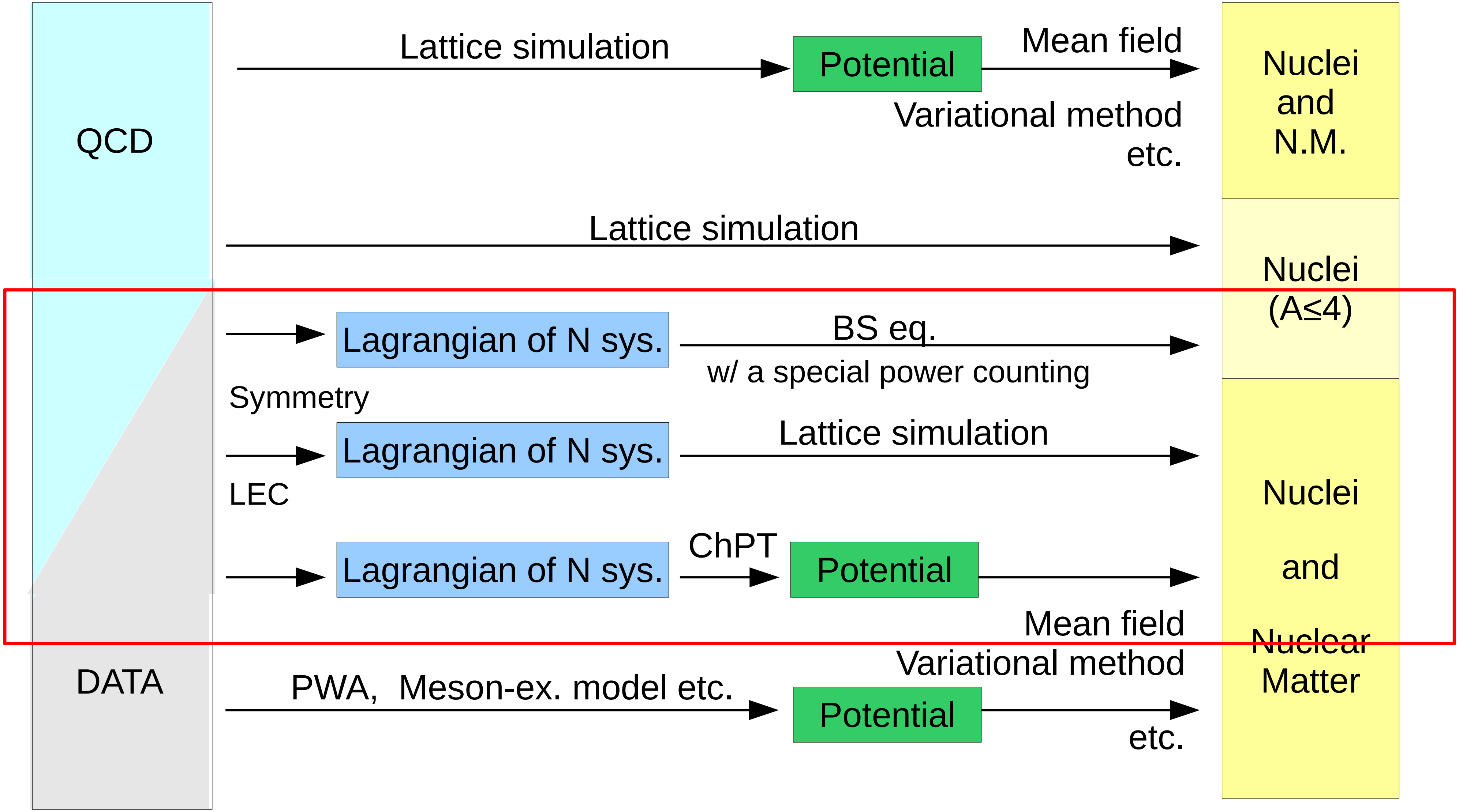
etc.

Various approaches in nuclear phys.



Most traditional. Many success.

Various approaches in nuclear phys.



QCD

Lattice simulation

Potential

Mean field

Variational method
etc.

Nuclei
and
N.M.

Lattice simulation

Nuclei
(A≤4)

Lagrangian of N sys.

BS eq.

w/ a special power counting

Symmetry

Lagrangian of N sys.

Lattice simulation

LEC

Lagrangian of N sys.

ChPT

Potential

Nuclei

and

Nuclear
Matter

DATA

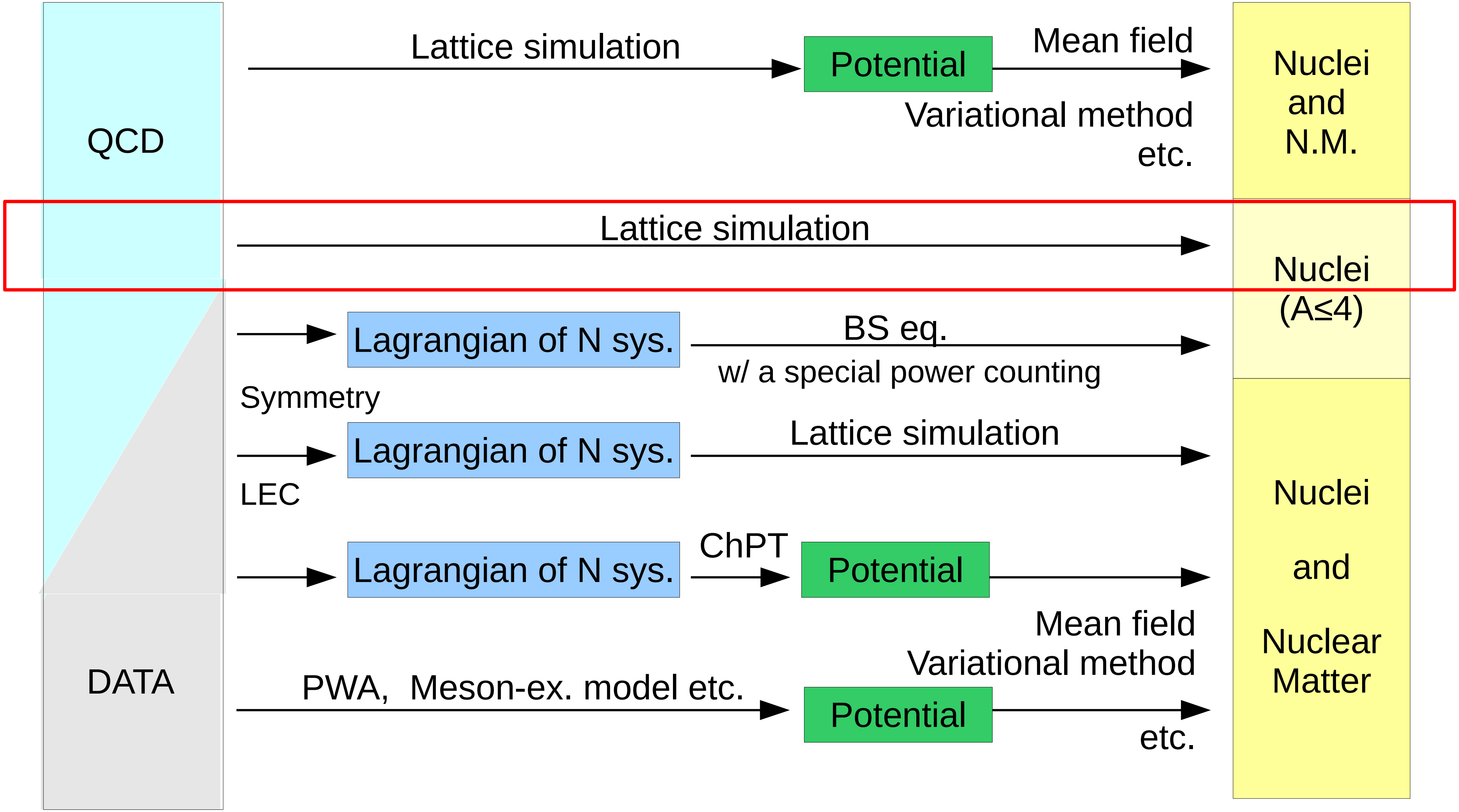
PWA, Meson-ex. model etc.

Potential

Mean field
Variational method
etc.

Very popular today. Let's say chiral approach.

Various approaches in nuclear phys.



QCD

Lattice simulation

Potential

Mean field

Variational method

etc.

Nuclei
and
N.M.

Lattice simulation

Nuclei
(A≤4)

Lagrangian of N sys.

BS eq.

w/ a special power counting

Symmetry

Lagrangian of N sys.

Lattice simulation

LEC

Lagrangian of N sys.

ChPT

Potential

Nuclei

and

Nuclear
Matter

DATA

Mean field

Variational method

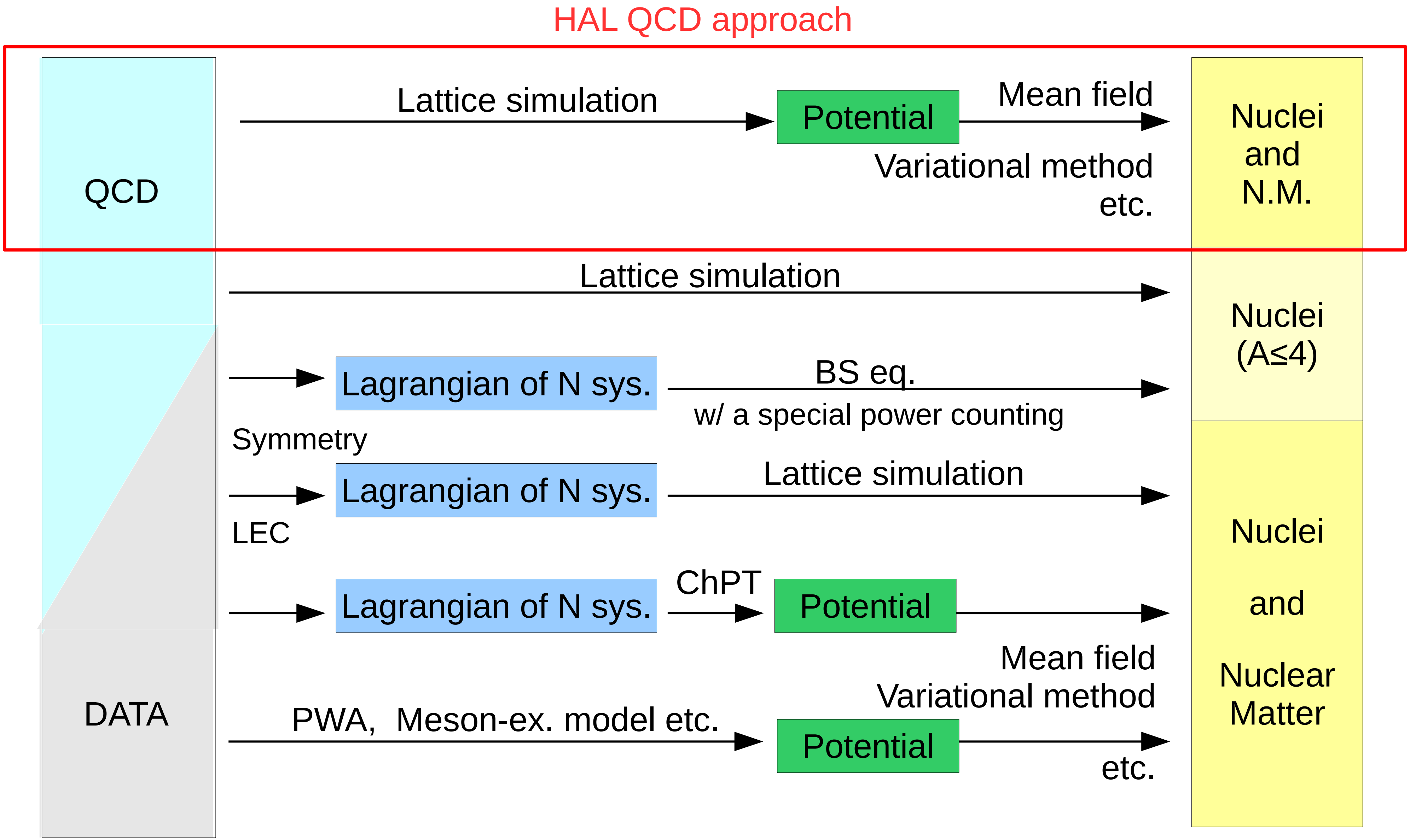
PWA, Meson-ex. model etc.

Potential

etc.

Very challenging. Let's call LQCD direct approach.

Various approaches in nuclear phys.


HAL QCD approach

QCD

DATA

Lattice simulation → Potential → Mean field / Variational method etc. → Nuclei and N.M.

Lattice simulation → Nuclei (A≤4)

Symmetry → Lagrangian of N sys. → BS eq. w/ a special power counting → Nuclei (A≤4)

LEC → Lagrangian of N sys. → Lattice simulation → Nuclei and Nuclear Matter

→ Lagrangian of N sys. → ChPT → Potential → Nuclei and Nuclear Matter

PWA, Meson-ex. model etc. → Potential → Mean field / Variational method / etc. → Nuclei and Nuclear Matter

Our approach. I focus on this one in this talk.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

- Disappointing points
  1. Demand long time and huge money at the 1st stage.
     - We had to deal with un-physical QCD world before.
     - Un-realistically heavy u,d quark, far from chiral symmetry.
  2. Depend on method/approximation used at the 2nd stage.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

- Disappointing points
  1. Demand long time and huge money at the 1st stage.
     - We had to deal with un-physical QCD world before. Getting solved.
     - Un-realistically heavy u,d quark, far from chiral symmetry.
  2. Depend on method/approximation used at the 2nd stage.

We can improve step by step.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

- Disappointing points
  1. Demand long time and huge money at the 1st stage.
     - We had to deal with un-physical QCD world before. Getting solved.
     - Un-realistically heavy u,d quark, far from chiral symmetry.
  2. Depend on method/approximation used at the 2nd stage.

     We can improve step by step.

- Today, in this talk, I want to show
  - results of HALQCD approach to strange nuclear physics and want to demonstrate that our approach is promising.

Outline

1. Our approach and method
   - Introduction
   - HAL QCD method
   - BB interactions from QCD

2. Application to strange nuclear physics
   - Hyperon single-particle potentials
   - Hyperon onset in high density matter

3. Summary and outlook

HAL QCD method

Multi-hadron in LQCD

- Direct：utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL：utilize spatial correlation and “potential” $V(r) + \dots$

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate.
    Just need to measure
  - Then, potential can be extracted.
  - Demand a minimal lattice volume.
    No need to extrapolate to V=∞.
  - Can output many observables.

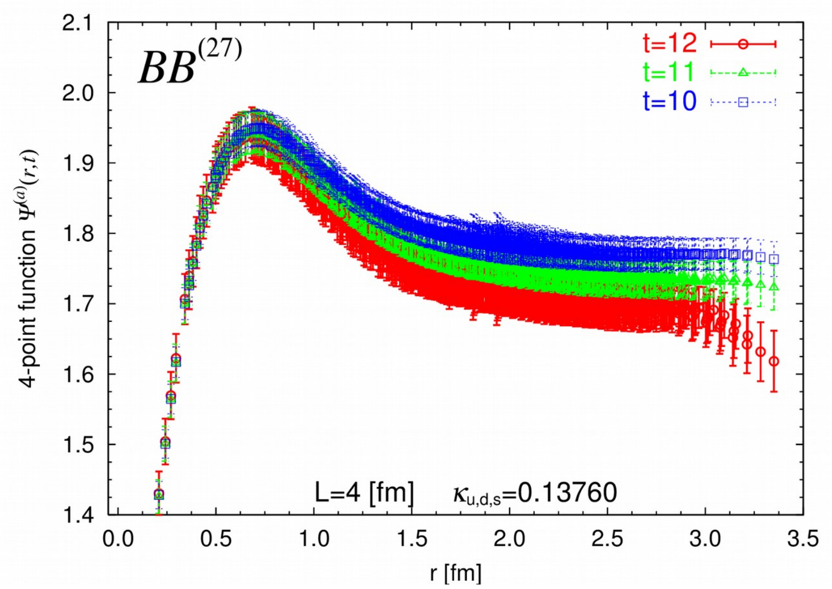
Multi-hadron in LQCD

- Direct：utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL：utilize spatial correlation and “potential” $V(r) + \ldots$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



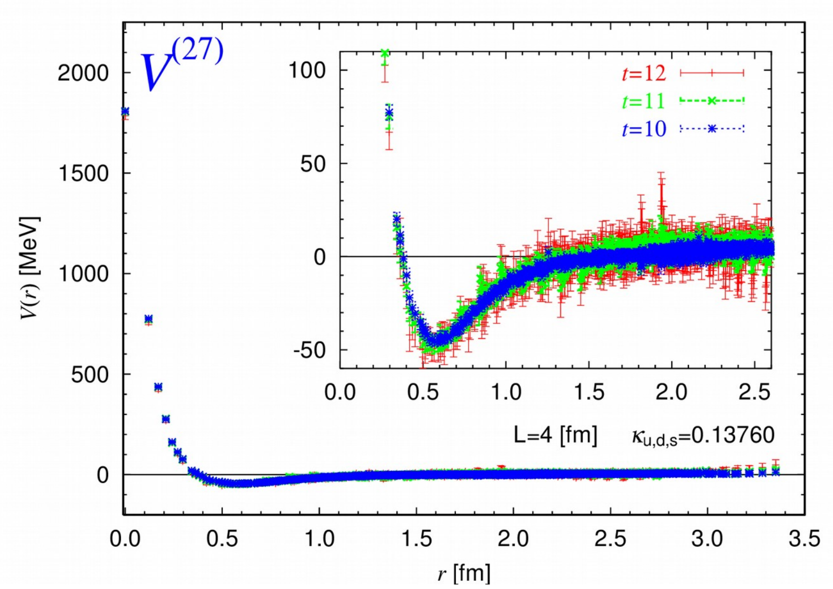
Multi-hadron in LQCD

- Direct：utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL：utilize spatial correlation and “potential” $V(r) + \ldots$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



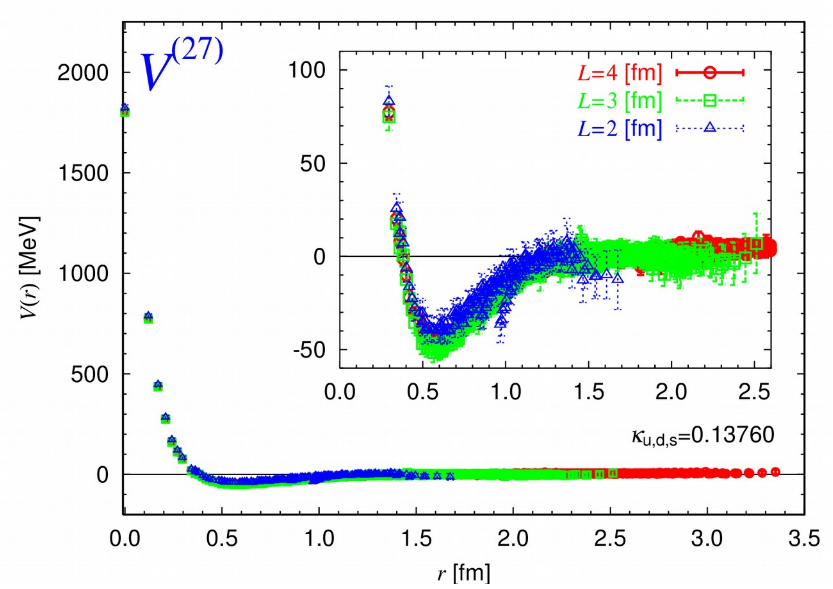
Multi-hadron in LQCD

- Direct：utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL：utilize spatial correlation and “potential” $V(r) + \ldots$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function
contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



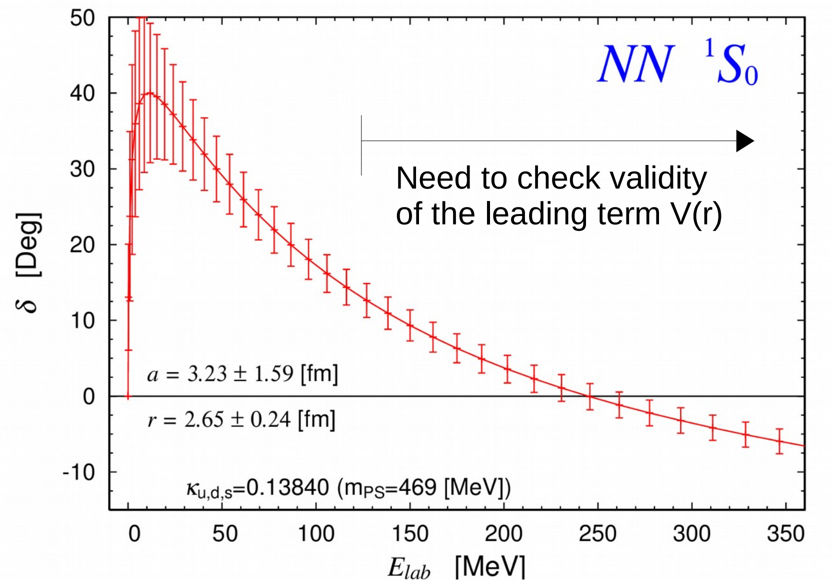
Multi-hadron in LQCD

- Direct : utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL : utilize spatial correlation and “potential” $V(r)+\ldots$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output more observables.



Need to check validity of the leading term V(r)

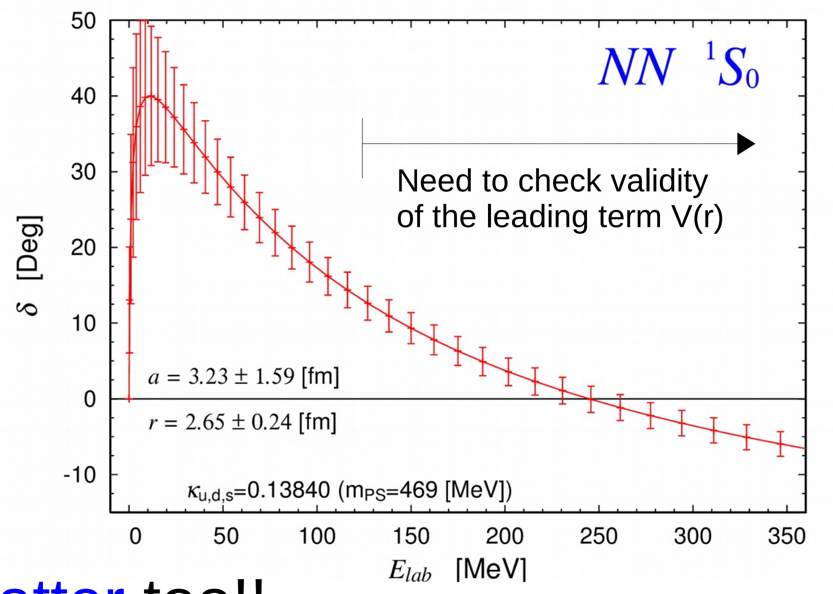
Multi-hadron in LQCD

- Direct：utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL：utilize spatial correlation and “potential” $V(r) + ...$

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function
contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output more observables.



★ We can attack nuclei and infinite matter too!!

HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

Define a common "potential" U for all E eigenstates via "Schrödinger" eq.

$$\left[-\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \phi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})$$

Non-local but
energy independent
below inelastic threshold

Measure 4-point function in LQCD

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) \; J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion
& truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$$

Therefor, in
the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

Source and sink operator

- NBS wave function and 4-point function

$$\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, \overbrace{t) B_j(\vec{x}, t}^{\text{equal}}) | B=2, \vec{k} \rangle \; \text{QCD eigenstate}$$

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | \underbrace{B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t)}_{\text{sink}} \; \underbrace{J(t_0)}_{\text{source}} | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

- Point type octet baryon field operator at sink

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} \, u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with} \quad \xi_i = \{c_i, \beta_i, \underline{x}\}$$

$$\Lambda_\alpha(x) = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} \sqrt{\frac{1}{6}} \left[d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

- Wall type source of two-baryon state

e.g. $\overline{BB}^{(1)} = -\sqrt{\frac{1}{8}} \, \overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}} \, \overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}} \, \overline{\mathrm{N}}\,\overline{\Xi}$ for flavor-singlet

Hyperon interactions from QCD

LQCD simulation setup

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \simeq (8\ \mathrm{fm})^4$ large volume
  - 1/a = 2333 MeV, a = 0.0845 fm
  - $M_\pi \simeq 146$, $M_K \simeq 525$ MeV almost physical point

    $M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV
  - Collaboration in HPCI Strategic Program Field 5 Project 1

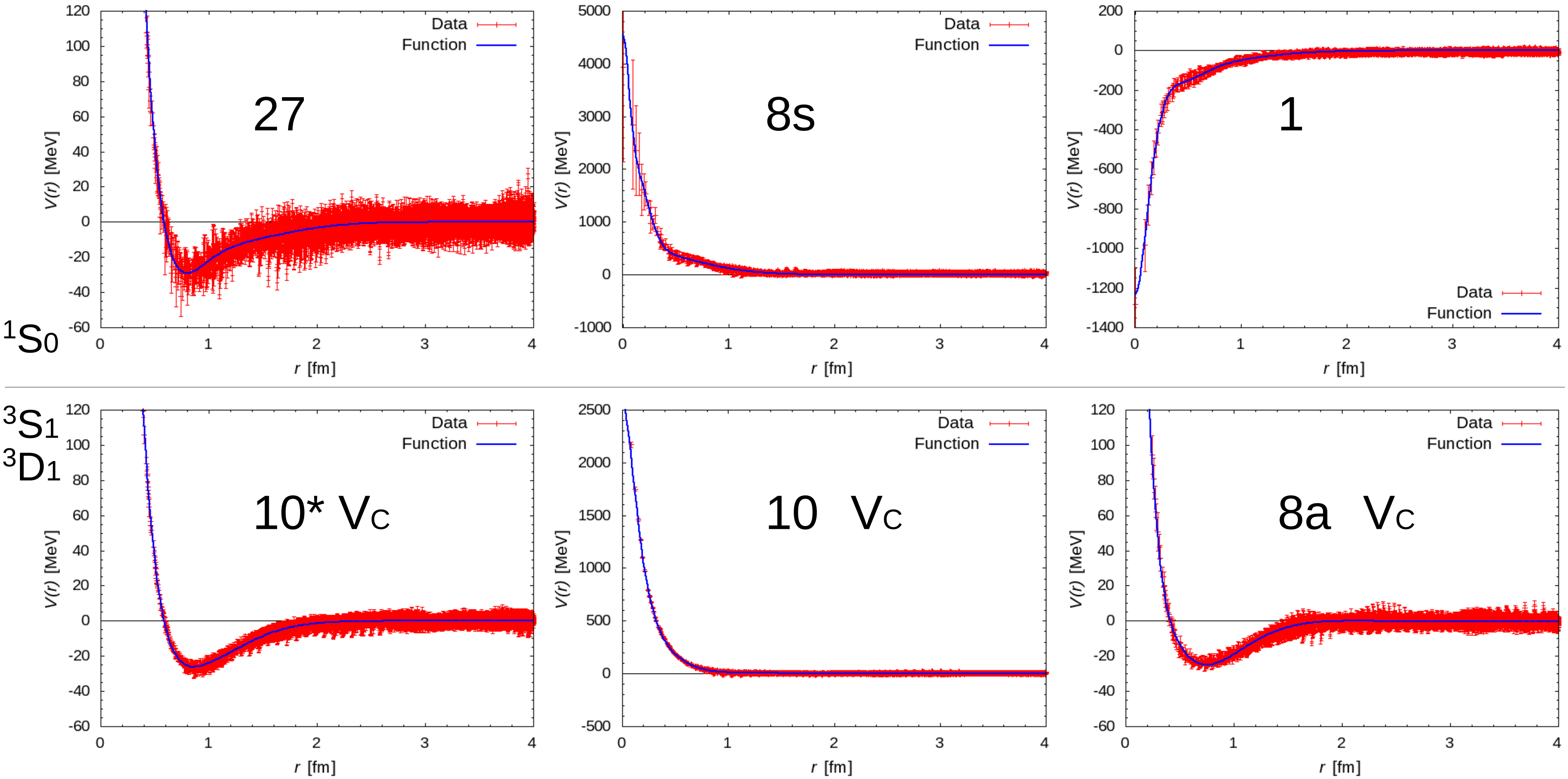
K-configuration

- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - #data = 414 confs × 4 rot × (72,96) src.

almost final

BB S-wave potentials

(72,96) src
$t-t_0 = 12$



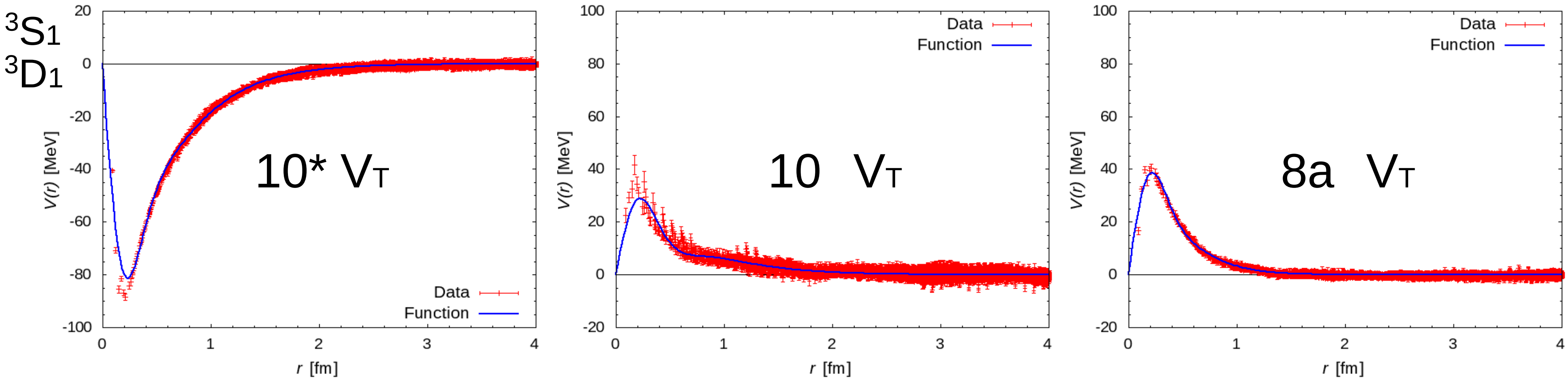
- Rotated into the flavor irreducible-representation basse

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1\ ,\ ^3D_1}$$

by using data
in S=−2 sector

BB S-wave potentials

You can see original $V_{\mathrm{BB,BB}}(r)$ in the next talk by K. Sasaki



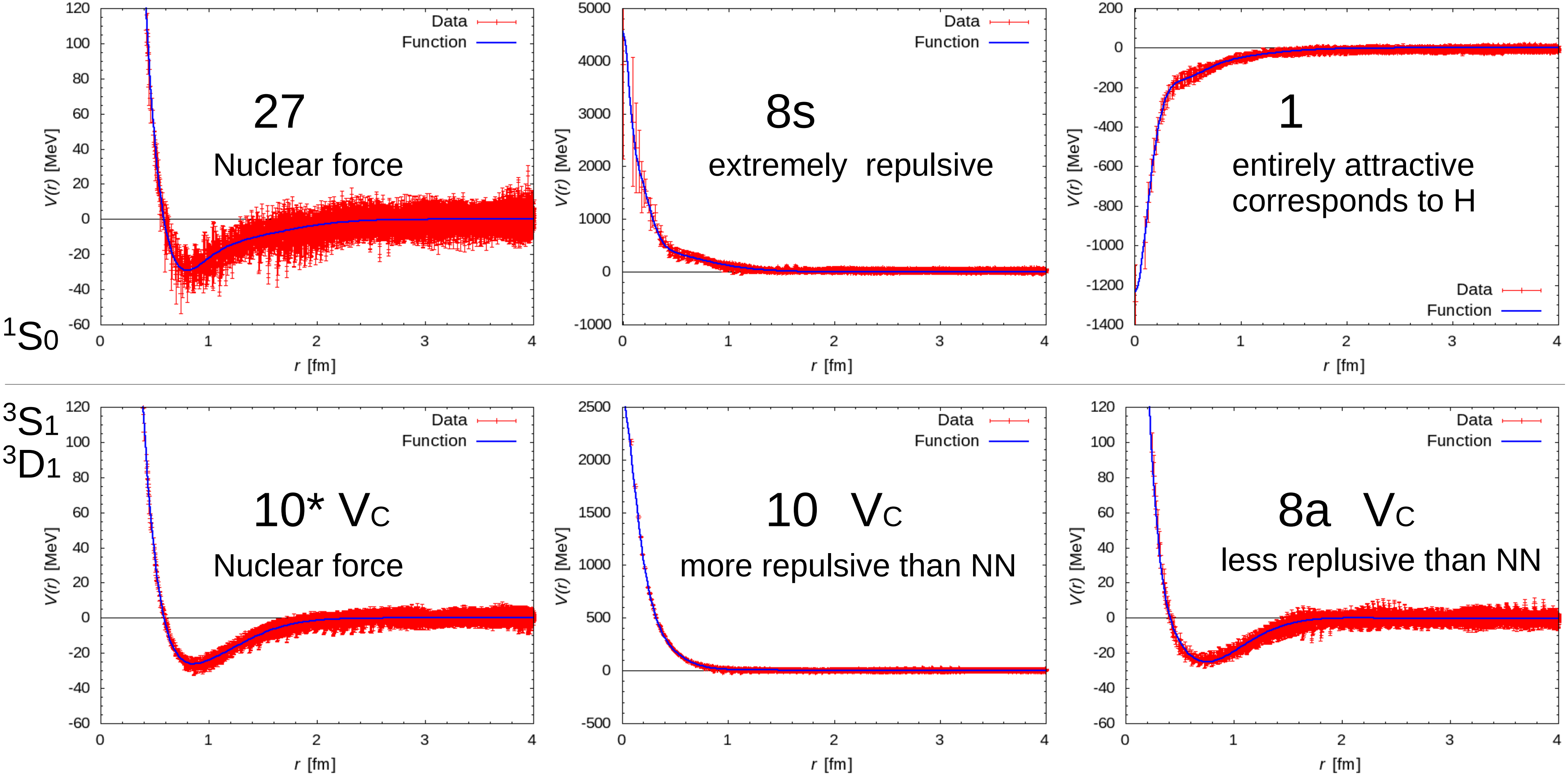
- Functions fitted to data

$$V_C(r) = a_1\, e^{-a_2 r^2} + a_3\, e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2$$

$$V_T(r) = a_1 \left(1 - e^{-a_2 r^2}\right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2}\right) \frac{e^{-a_3 r}}{r} + a_4 \left(1 - e^{-a_5 r^2}\right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2}\right) \frac{e^{-a_6 r}}{r}$$

- Since SU(3)$_F$ is broken at the physical point (K-conf.), there are irre.-rep. base off-diagonal potentials.
- But, I omit them and constract V_{YN}, V_{YY} with these irre.-rep. diagonal potentials and the C.G. coefficient.

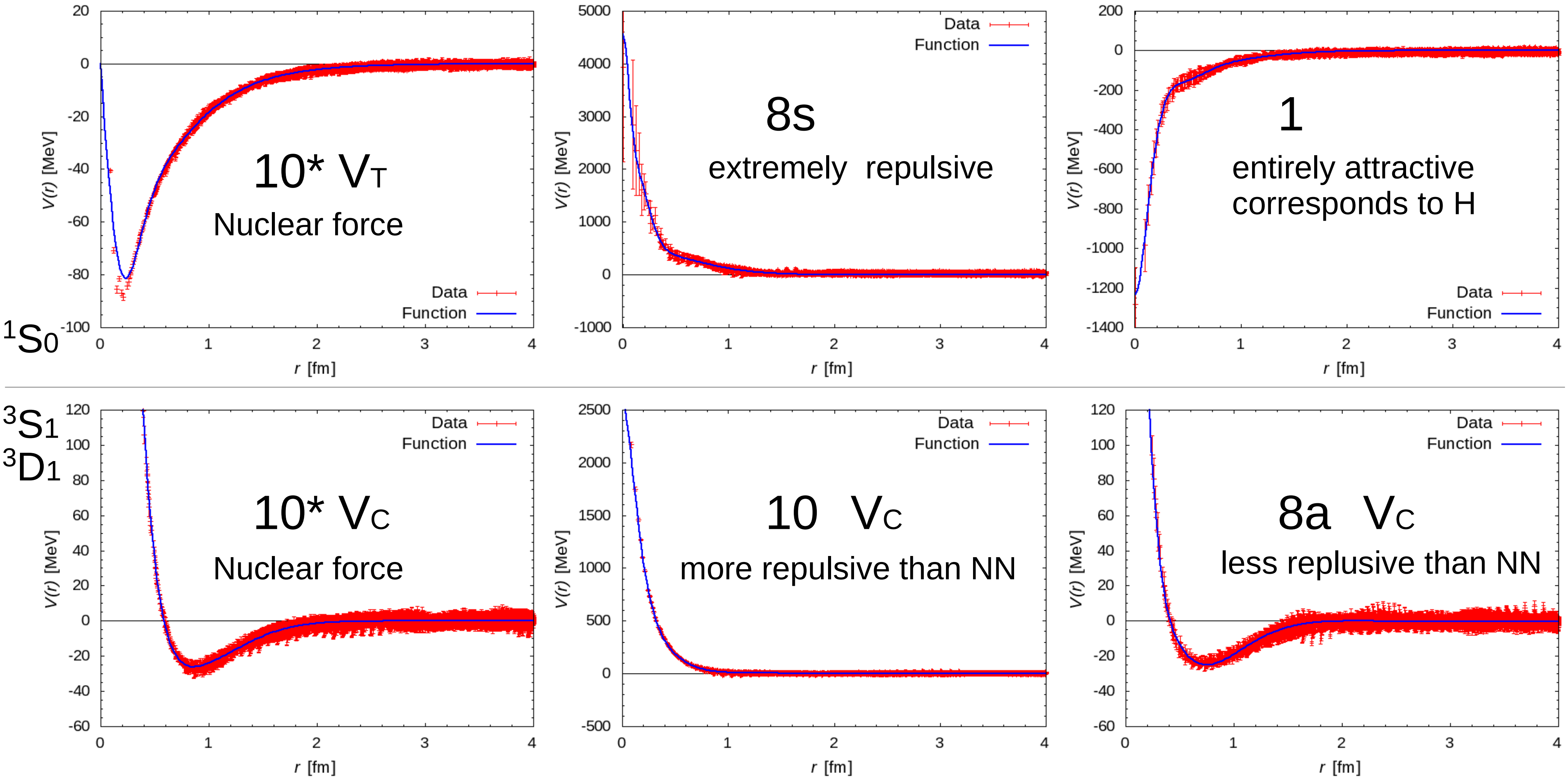
BB S-wave potentials



- Qualitatively reasonable NN forces are obtained from QCD.
- Features can be understood by the quark Pauli + OGE.

e.g. Oka, Shimizu, Yazaki, Nucl. Phys. A464 (1987)

BB S-wave potentials

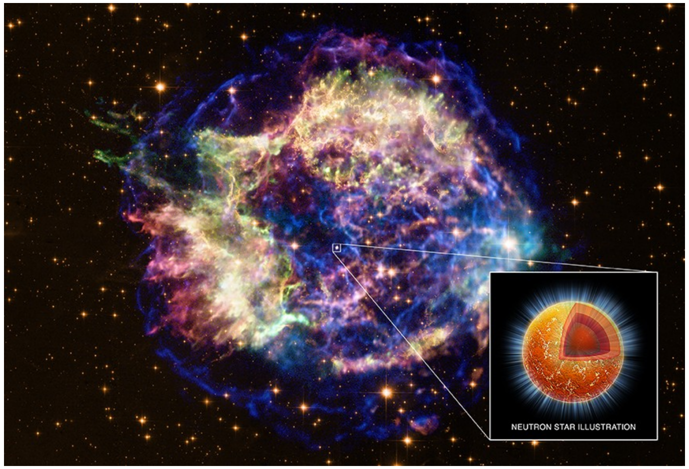


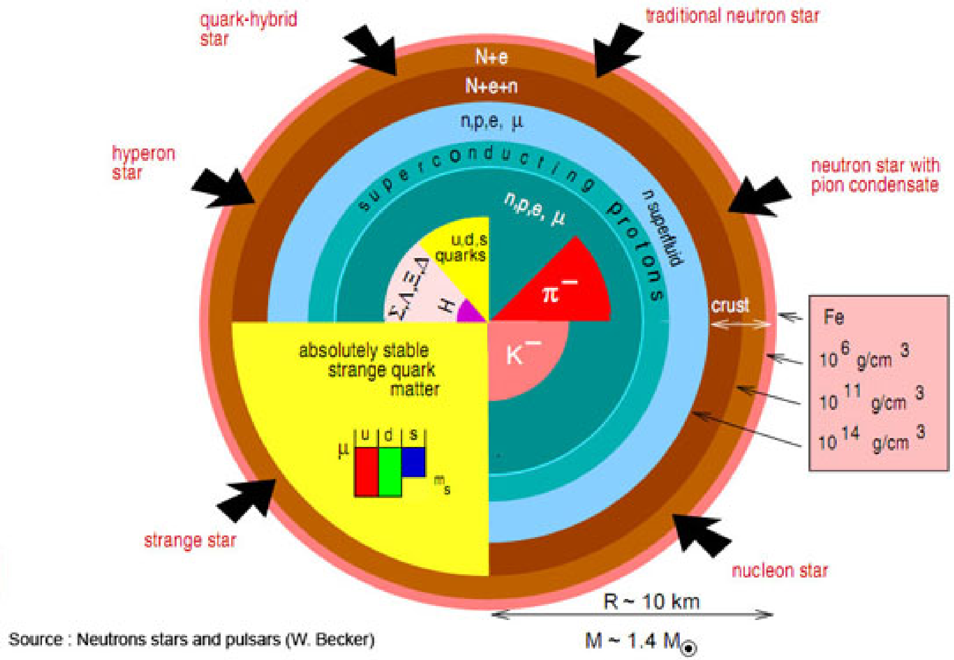
- Qualitatively reasonable NN forces are obtained from QCD.
- Features can be understood by the quark Pauli + OGE.

e.g. Oka, Shimizu, Yazaki, Nucl. Phys. A464 (1987)

Hyperons in infinite nuclear matter

Introduction





Source : Neutrons stars and pulsars (W. Becker)

- Hyperon is a serious subject in physics of NS.
  - Does hyperon appear inside neutron star core?
  - How EoS of NS mater can be so stiff with hyperon?

cf. PSR J1614-2230 1.97±0.04 $M_\odot$

- Tough problem due to ambiguity of hyperon forces
  - comes form difficulty of hyperon scattering experiment.

Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  - measure h-h NBS w.f. in lattice QCD simulation.
  - define & extract interaction “potential” from the w.f.

HALQCD
applapch

Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  - measure h-h NBS w.f. in lattice QCD simulation.
  - define & extract interaction “potential” from the w.f.

HALQCD applapch

- Today, we study hyperons in nuclear matter by basing on YN,YY interactions predicted from QCD.
  - We calculate hyperon single-particle potential $U_Y(k;\rho)$
  - defined by $e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$

$e_Y(k;\rho)$: sepectrum in medium

  - U_Y is crucial for hyperon chemical potential.

Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  - measure h-h NBS w.f. in lattice QCD simulation.
  - define & extract interaction “potential” from the w.f.

HALQCD
applapch

- Today, we study hyperons in nuclear matter by basing on YN,YY interactions predicted from QCD.
  - We calculate hyperon single-particle potential $U_Y(k;\rho)$
  - defined by $e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$

    $e_Y(k;\rho)$: sepectrum in medium
  - U_Y is crucial for hyperon chemical potential.
- Hypernuclear experiment suggest that

$@\rho = 0.17\,[\mathrm{fm}^{-3}]$
$x = 0.5$

$$U_{\underline{\Lambda}}^{\mathrm{Exp}}(0) \simeq -30, \quad U_{\underline{\Xi}}^{\mathrm{Exp}}(0) \simeq -10, \quad U_{\underline{\Sigma}}^{\mathrm{Exp}}(0) \geq +20 \quad [\mathrm{MeV}]$$

attraction attraction small repulsion

Nuclear matter

- Uniform matter consisting an infinite number of nucleon interacting each other via nuclear force
- Theories
  - Brueckner Hartree Fock
    - K.A. Brueckner and J.L.Gammel Phys. Rev. 109 (1958) 1023
  - Relativistic Mean Field
    - J. D. Walecka, Ann. Phys. 83 (1974) 491
  - Fermi Hyper-Netted Chain
    - A. Akmal, V.R. Phandharipande, D.G. Ravenhall Phys. Rev. C 58 (1998) 1804
  - Cupled Cluster
    - G.Baardsen, A. Ekstrom, G.Hagen, M.Hjorth-Jensen, Phys. Rev. C88(2013)
  - Self-consistent Green's function
    - W. H. Dickhoff, C. Barbieri, Prog. Part. Nucl. Phys. (2004),377
  - Quantum Monte Carlo
    - J. Carlson, J. Morales, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C68(2003) 025802

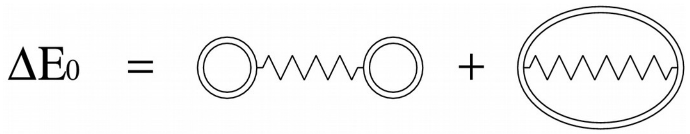
Nuclear matter

- Uniform matter consisting an infinite number of nucleon interacting each other via nuclear force
- Theories
  - Brueckner Hartree Fock
    - K.A. Brueckner and J.L.Gammel Phys. Rev. 109 (1958) 1023
  - Relativistic Mean Field
    - J. D. Walecka, Ann. Phys. 83 (1974) 491
  - Fermi Hyper-Netted Chain
    - A. Akmal, V.R. Phandharipande, D.G. Ravenhall Phys. Rev. C 58 (1998) 1804
  - Cupled Cluster
    - G.Baardsen, A. Ekstrom, G.Hagen, M.Hjorth-Jensen, Phys. Rev. C88(2013)
  - Self-consistent Green's function
    - W. H. Dickhoff, C. Barbieri, Prog. Part. Nucl. Phys. (2004),377
  - Quantum Monte Carlo
    - J. Carlson, J. Morales, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C68(2003) 025802

Brueckner-Hartree-Fock LOBT

M.I. Haftel and F. Tabakin, Nucl. Phys. A158(1970) 1-42

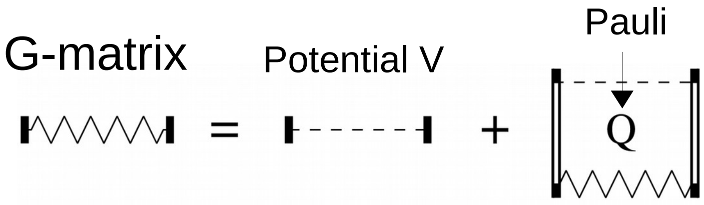
- Ground state energy in BHF framework



$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \mathrm{Re} \langle G_i \left(e(k) + e(k') \right) \rangle_A$$

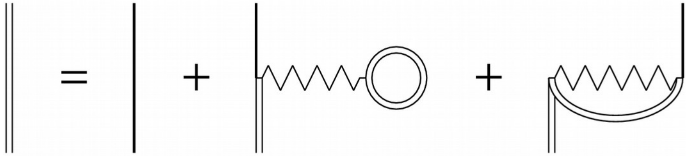
- G-matrix

AV18 + “UIX”



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$

- Single particle spectrum & potential



$$e(k) = \frac{k^2}{2M_{\mathrm{N}}} + U(k)$$

Physical

$$U(k) = \sum_{i} \sum_{k' \le k_F} \mathrm{Re} \langle k k' | G_i \left(e(k) + e(k') \right) | k k' \rangle_A$$

- Partial wave decomposition $^{2S+1}L_J = {}^1S_0\, ,\ {}^3S_1\, ,\ {}^3D_1\, ,\ {}^1P_1\, ,\ {}^3P_J \cdots$
- Continuous choice w/ effective mass approx. Angle averaged Q-operator

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze,
Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k k' | G^{SLJ}_{(YN)(YN)}\big(e_Y(k) + e_N(k')\big) | k k' \rangle$$

$$^{2S+1}L_J \;=\; {}^1S_0\,,\;\; {}^3S_1\,,\;\; {}^3D_1\,,\;\; {}^1P_1\,,\;\; {}^3P_J \;\;\cdots$$

in our study

limitation

- YN G-matrix using $M^{\text{Phys}}_{N,Y}$, $U^{\text{AV18+UIX}}_{n,p}$, $V^{\text{LQCD}}_{S=-1}$ and, U^{LQCD}_{Y}

Q=0
$$\begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$$

Q=+1
$$\begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

Q=−1 $G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)}$

Q = +2 $G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$

Brueckner-Hartree-Fock

- Hyperon single-particle potential

$$U_{\Xi}(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k'\leq k_F} \langle k\, k' |\, G^{SLJ}_{(\Xi N)(\Xi N)}\left(e_{\Xi}(k)+e_N(k')\right) | k\, k' \rangle$$

- ΞN G-matrix using $M^{\text{Phys}}_{N,Y}$, $U^{\text{AV18+UIX}}_{n,p}$, $U^{\text{LQCD}}_{\Lambda,\Sigma}$, $V^{\text{LQCD}}_{S=-2}$, U^{LQCD}_{Ξ}

Flavor symmetric $^1S_0$ sectors

Q=0

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Sigma^0)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} & G_{(\Xi^0 n)(\Lambda\Lambda)} \\
G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Sigma^0)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} & G_{(\Xi^- p)(\Lambda\Lambda)} \\
G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Sigma^0)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} & G_{(\Sigma^+\Sigma^-)(\Lambda\Lambda)} \\
G_{(\Sigma^0\Sigma^0)(\Xi^0 n)} & G_{(\Sigma^0\Sigma^0)(\Xi^- p)} & G_{(\Sigma^0\Sigma^0)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Sigma^0)(\Lambda\Lambda)} \\
G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Lambda)(\Lambda\Lambda)} \\
G_{(\Lambda\Lambda)(\Xi^0 n)} & G_{(\Lambda\Lambda)(\Xi^- p)} & G_{(\Lambda\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Lambda\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Lambda\Lambda)(\Sigma^0\Lambda)} & G_{(\Lambda\Lambda)(\Lambda\Lambda)}
\end{pmatrix}$$

Q=+1

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\
G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)}
\end{pmatrix}$$

Q=−1

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\
G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)}
\end{pmatrix}$$

Brueckner-Hartree-Fock

- ΞN G-matrix using $M_{N,Y}^{\mathrm{Phys}}$, $U_{n,p}^{\mathrm{AV18+UIX}}$, $U_{\Lambda,\Sigma}^{\mathrm{LQCD}}$, $V_{S=-2}^{\mathrm{LQCD}}$, U_{Ξ}^{LQCD}

Flavor anti-symmetric $^3S_1$, $^3D_1$ sectors

Q=0

$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} \\ G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} \\ G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} \end{pmatrix}$$

Q=+1

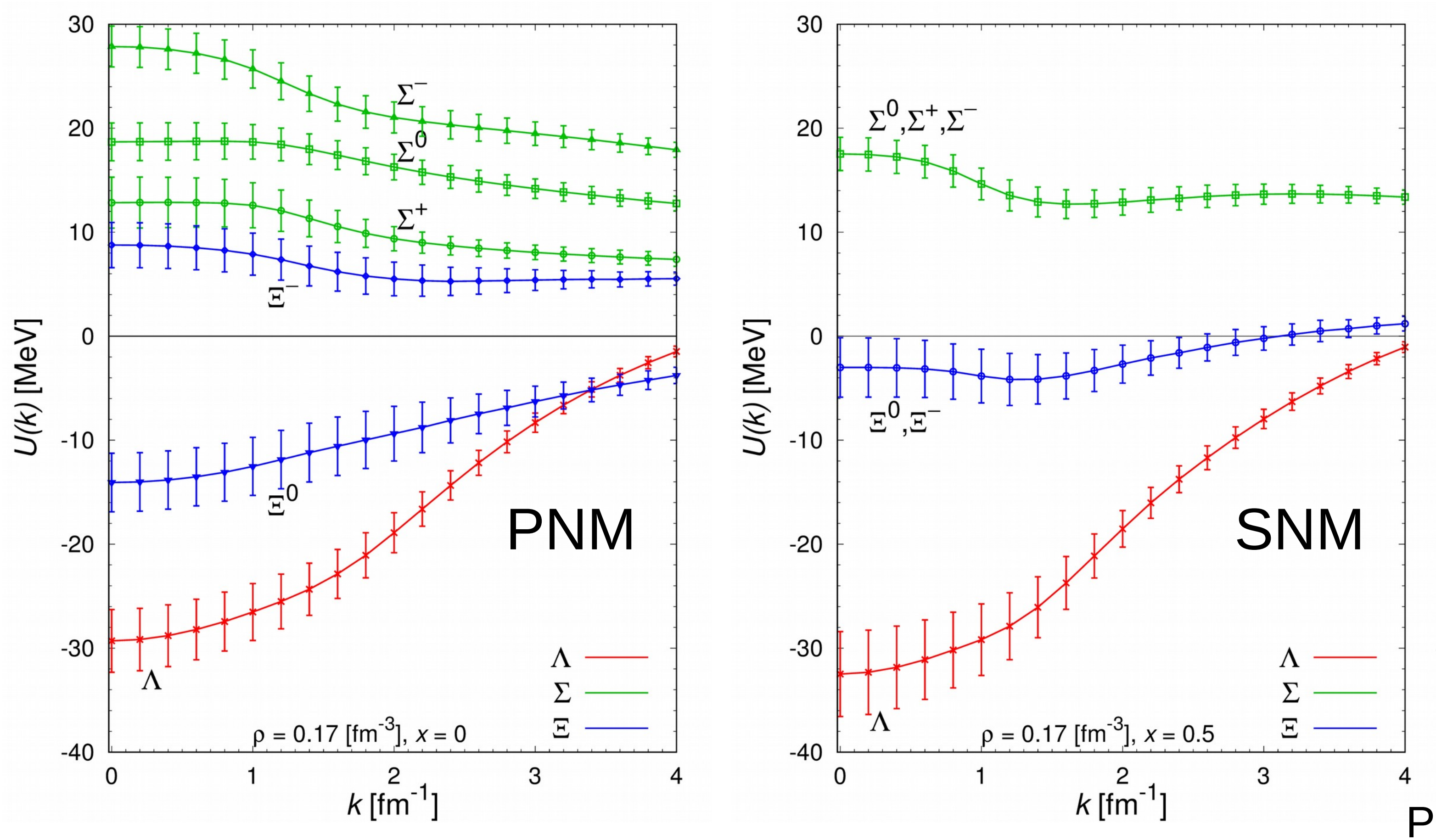
$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Sigma^0)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Sigma^0)(\Xi^0 p)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)} \end{pmatrix}$$

Q=−1

$$\begin{pmatrix} G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Sigma^0)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Sigma^0)(\Xi^- n)} & G_{(\Sigma^-\Sigma^0(\Sigma^-\Sigma^0))} & G_{(\Sigma^-\Sigma^0)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Sigma^0)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)} \end{pmatrix}$$

Results

Hyperon single-particle potentials



@ρ=0.17 [fm^{-3}]

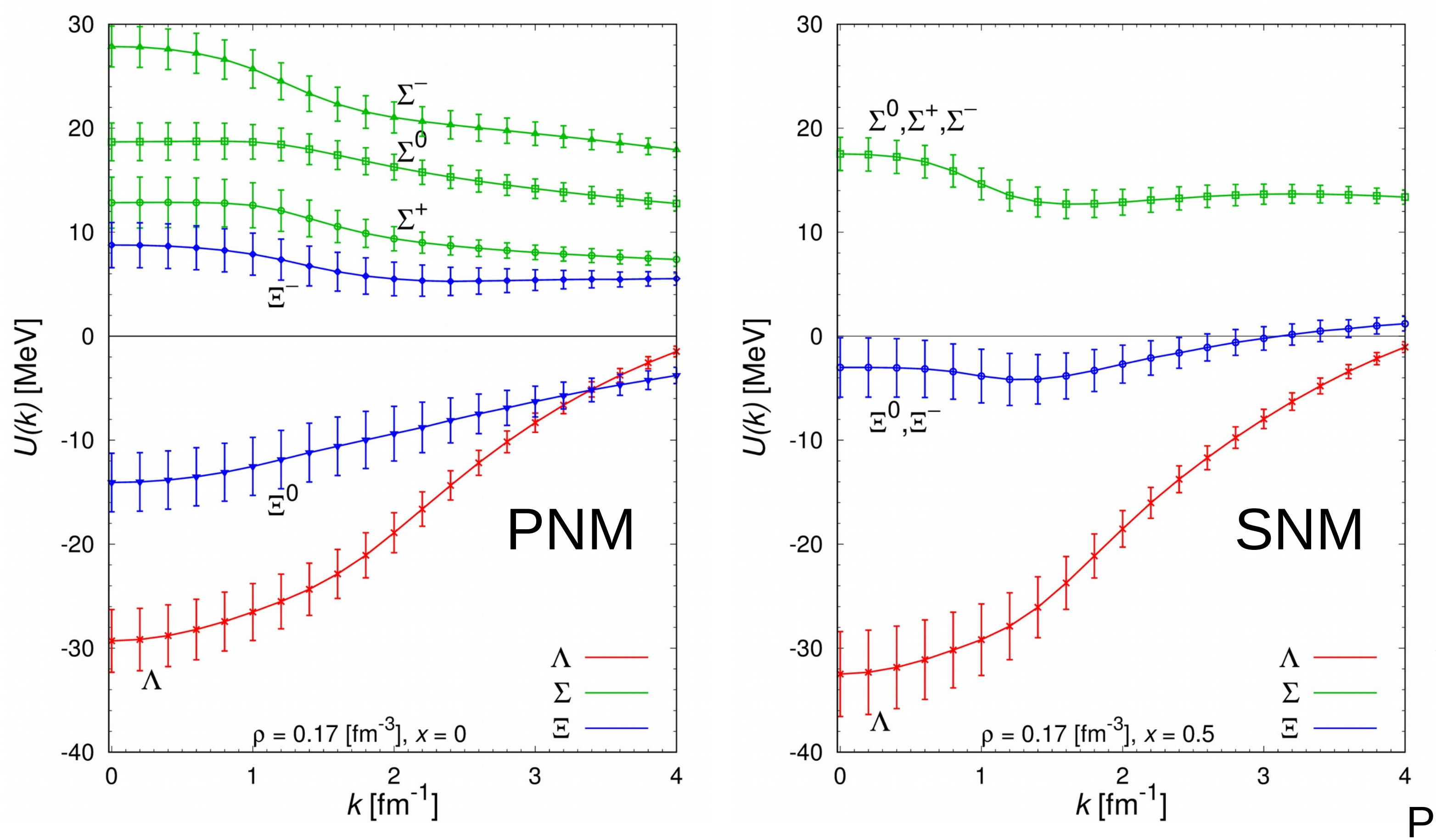
Preliminary

Vertical vars show statistical error only

PoS INPC2016 277

- obtained by using YN,YY S-wave forces form QCD.

Hyperon single-particle potentials



$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

Preliminary

Vertical vars show statistical error only

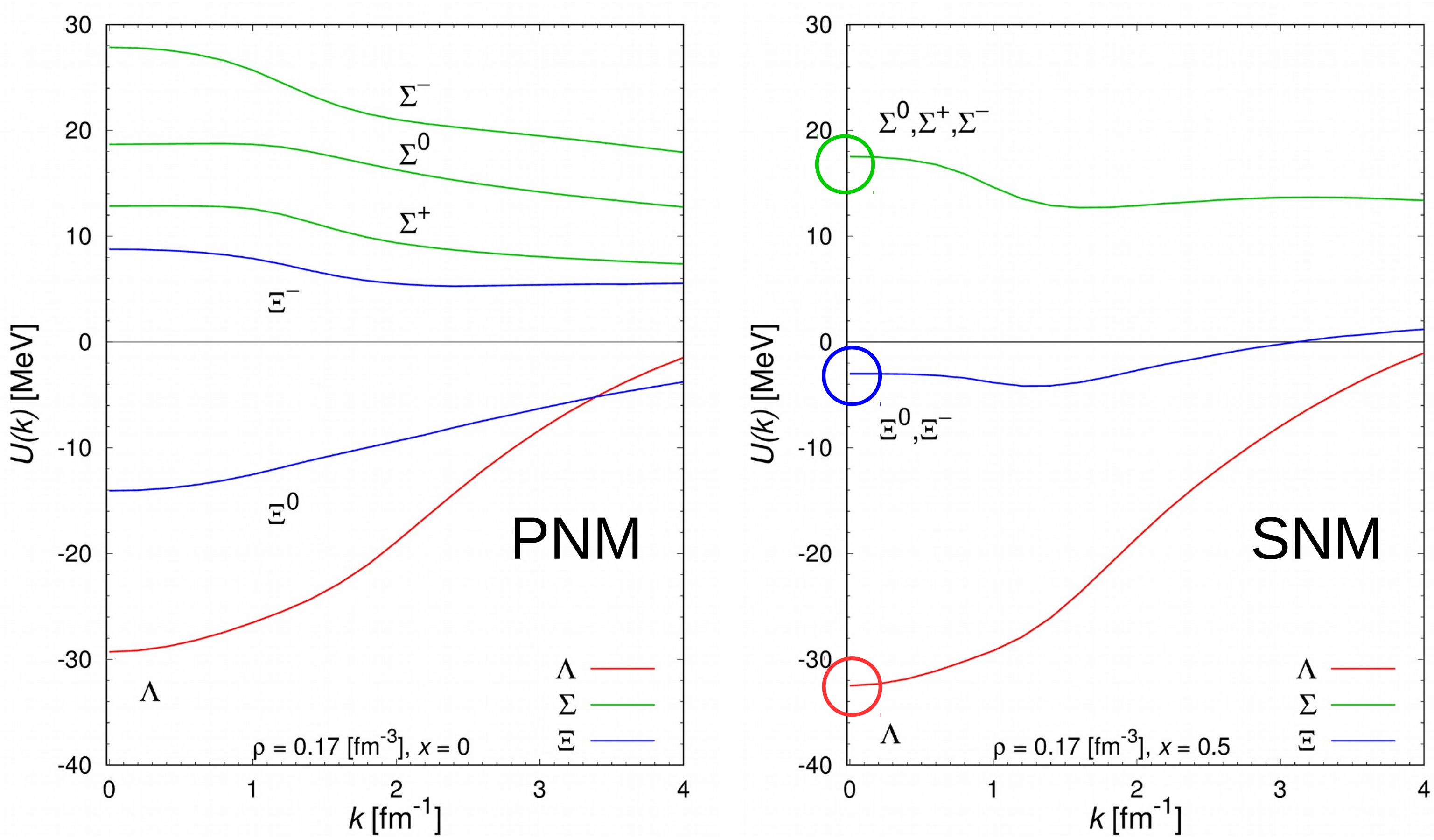
PoS INPC2016 277

- obtained by using YN,YY S-wave forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\text{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\text{Exp}} \simeq -10, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction attraction small repulsion

Hyperon single-particle potentials



$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

Preliminary

Remarkable.
Encouraging.

- obtained by using YN,YY S-wave forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\text{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\text{Exp}} \simeq -10, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction attraction small repulsion

Hyperon single-particle potentials

Preliminary

- $G^{SLJ}_{YN,YN}$ contributions to $U_Y(0;\rho_0)$ in SNM

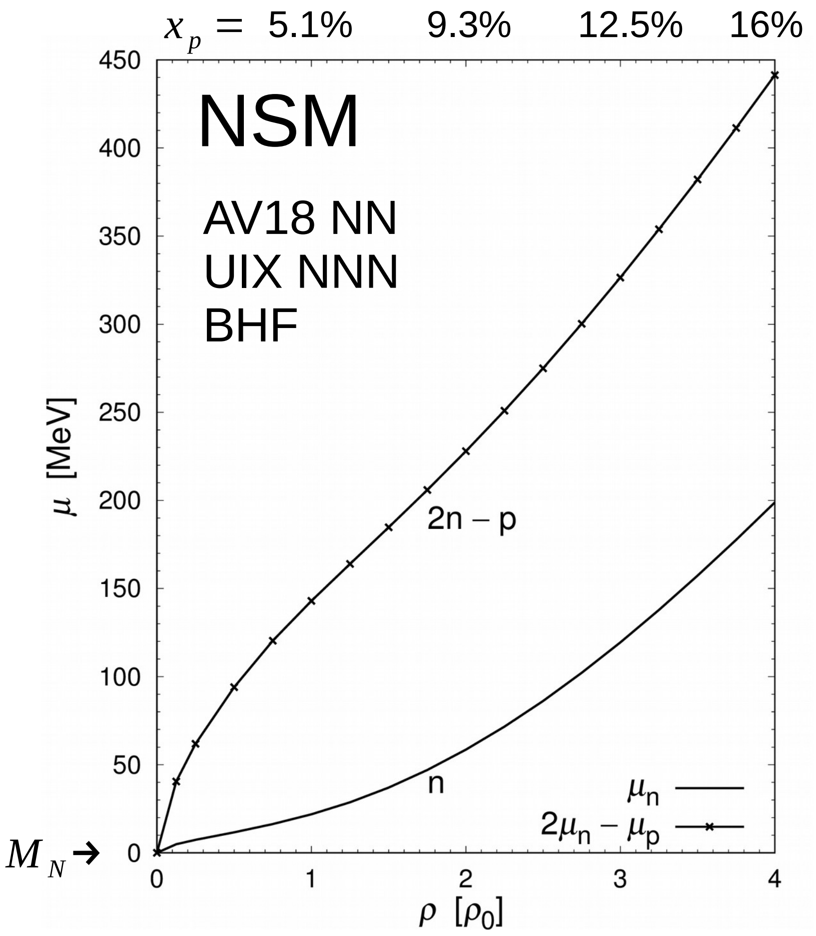
	$I=1/2$						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	-3.84	-28.70	0.06				-32.49

	$I=1/2$			$I=3/2$			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	10.22	-10.76	0.03	-6.16	24.34	-0.13	17.52

	$I=0$			$I=1$			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	-4.80	-5.83	-0.10	12.35	-4.60	-0.02	-3.01

Note: including spin and iso-spin multiplicity

Chemical potentials in NSM



- Neutron Star Matter : ANM + e^-, μ^- @Q=0, β-eq.
- Parabola approx. for ANM

$$\mu_p(\rho;\beta) = \mu_N^{SNM}(\rho) + \beta^2 \frac{d E^{sym}(\rho)}{d\rho} - \beta(\beta+2)\, E^{sym}(\rho)$$

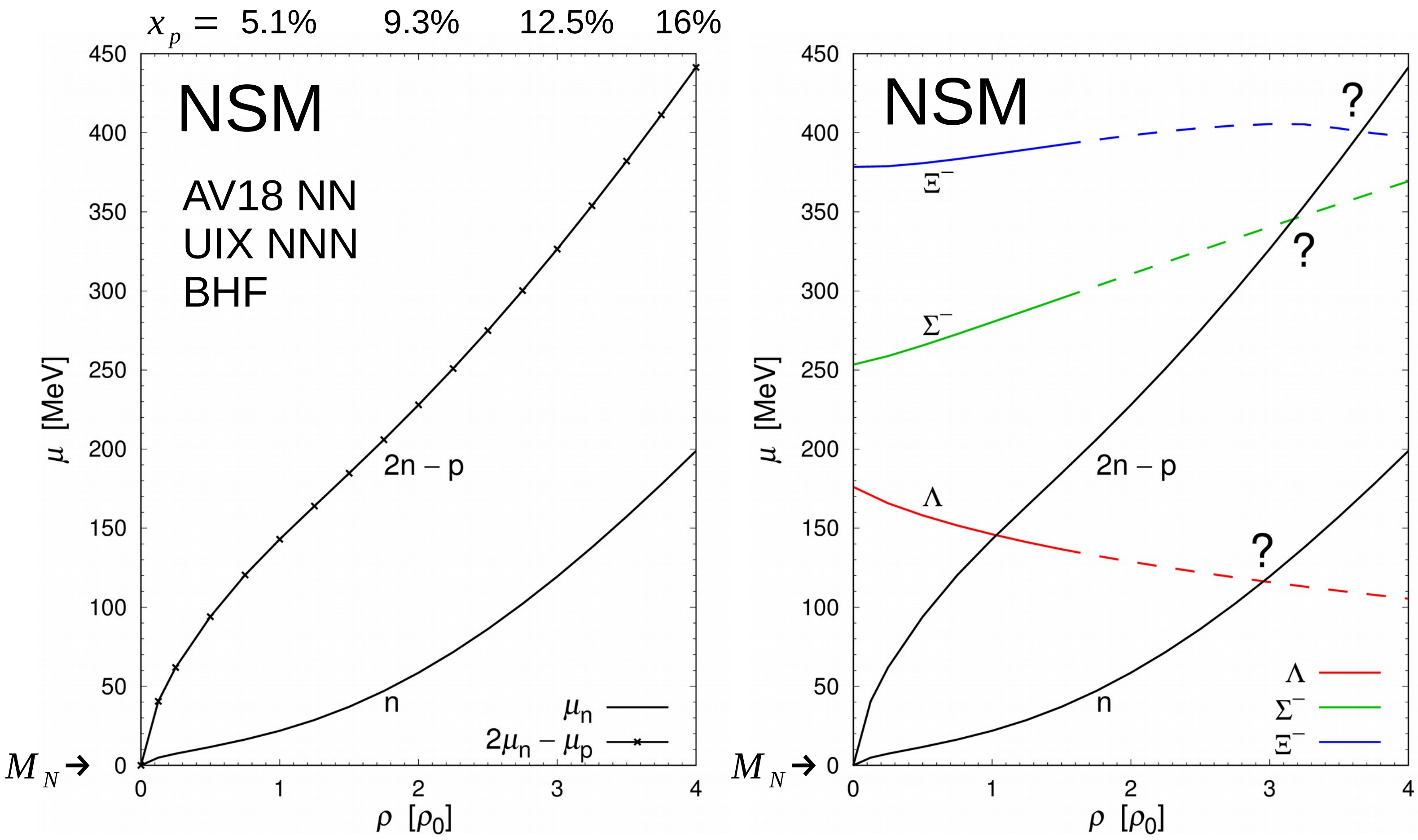
$$\mu_n(\rho;\beta) = \mu_N^{SNM}(\rho) + \beta^2 \frac{d E^{sym}(\rho)}{d\rho} - \beta(\beta-2)\, E^{sym}(\rho)$$

$$4E^{sym}(\rho) = \mu_n^{PNM}(\rho) - \mu_p^{PNM}(\rho),\ \ \beta = 1-2x_p$$

- Hyperon chemical in NSM $\mu_Y(\rho) \simeq M_Y - M_N + U_Y^{ANM}(0;\rho)$
- Hyperons appear as $n \rightarrow Y^0$ when $\mu_n > \mu_{Y^0}$

$nn \rightarrow pY^-$ when $2\mu_n > \mu_p + \mu_{Y^-}$

Hyperon onset in NSM (just for fun)



S-wave YN only

Preliminary

- Result indicate Λ, Σ^-, Ξ^- appear around ρ = 3.0 - 4.0 ρ_0
- However,
  - $YN^{L=1,2...}$ and YNN force could be important at high density.
  - We may need more sophisticated μ_n, μ_p than BHF.

Summary and Outlook

Summary and Outlook

- We've explained our goal and approach
  - Want to do (strange) nuclear physics starting from QCD.
  - Extract BB interaction potentials in lattice QCD simulation.
  - Then, apply potentials to many-body theories and so on.
- We've introduced HALQCD method
  - Utilize spatial correlation containing information of interaction.
  - This method avoid difficulty in a temporal plateau approach to multi-hadron system in lattice QCD.
- We've shown HALQCD BB potentials
  - We obtain QCD prediction of hyperon interactions.
  - We obtain (qualitatively) reasonable two-nucleon force.
  - We reveal nature of general BB S-wave interactions.

Summary and Outlook

✯ Resuls of application

- We studied hyperon s.p. potentials w/ the YN,YY forces.
  - This time, I used rotated data diagonal in the irre.-rep. base.
- We obtained U_Y compatible with experiment!
  - In SNM, Λ and Ξ feel attracsion, while Σ feels repulsion.
- This is remarkable success, at least encouraging.
  - Recall that we've never used any experimental data about hepron interactions, but we used only QCD.

✯ Outlook

- We'll use original data to take the physical $SU(3)_F$ breaking.
- We'll try to extract hyperon forces in higher partial waves, higher order terms of ∇-expansion, and BBB forces so that we can attack high density matter like NS.

Next generation super-computer

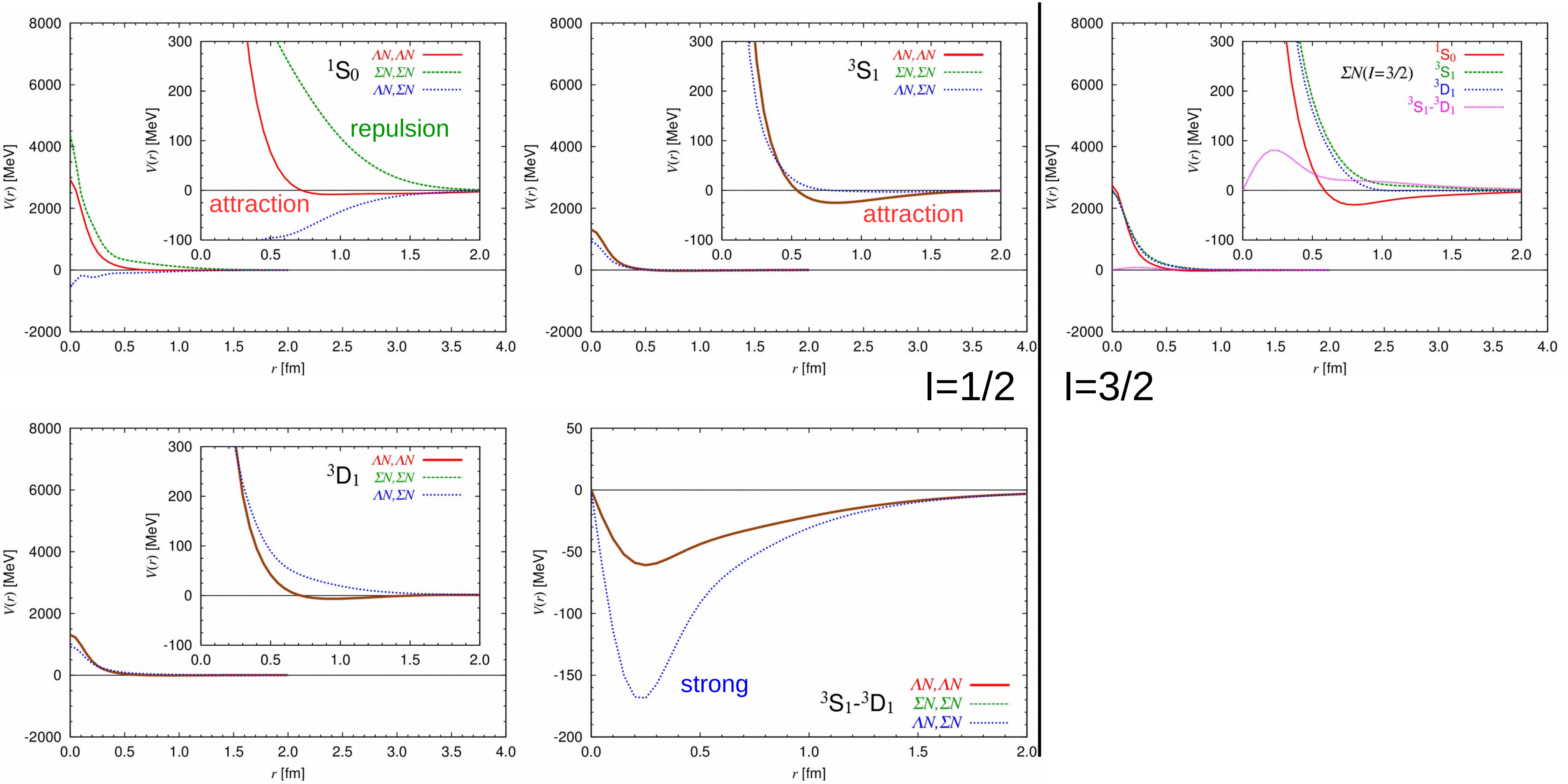
- I hope we can explain hypernuclei from QCD and we can solve hyperon puzzle of NS, in near future.

Thank you !!

Backup

LQCD ΛN-ΣN

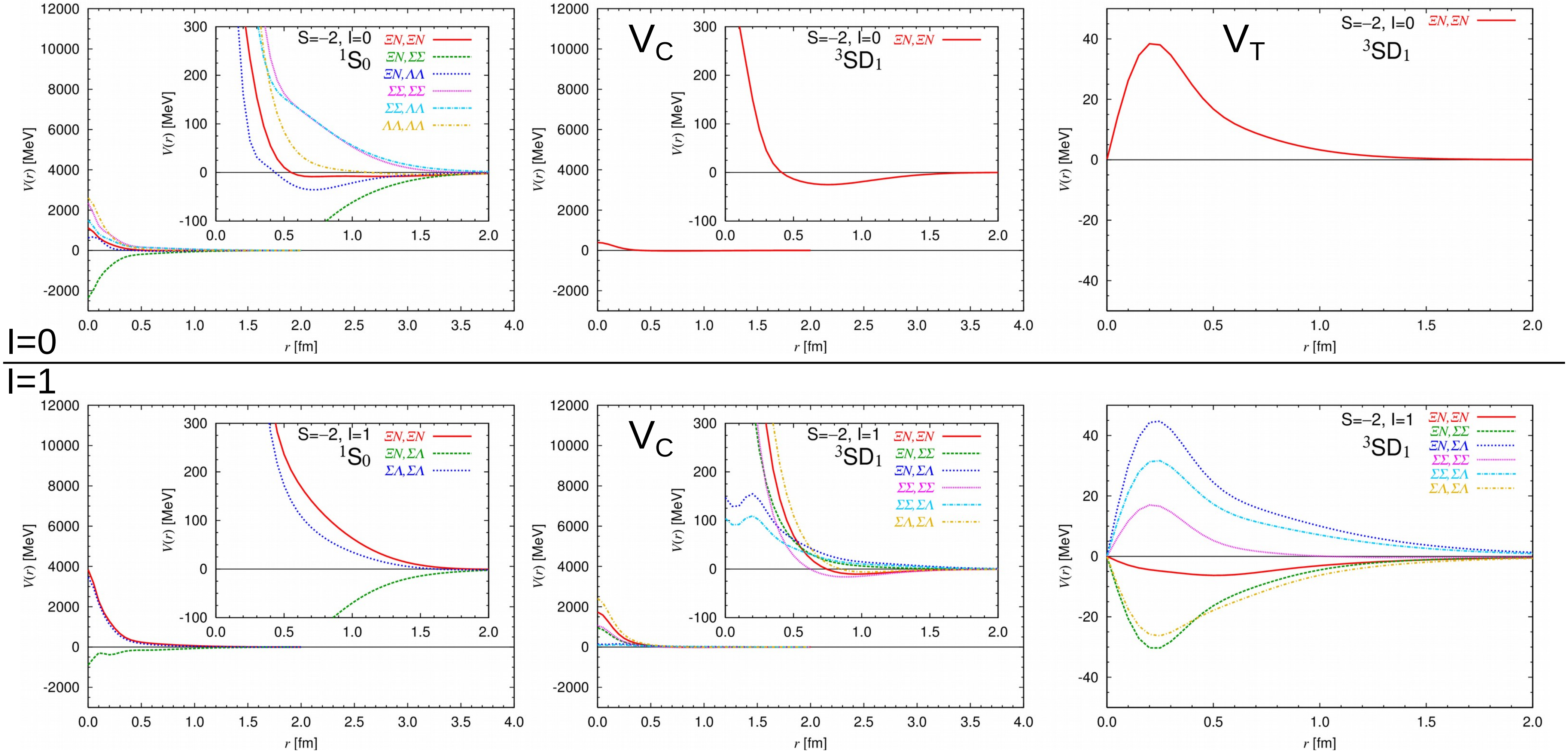
From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- In I=1/2, $^1S_0$ channel, ΛN has an attraction, while ΣN is repulsive.
- In I=1/2, $^3S_1$ channel, both ΛN and ΣN have an attraction.
- In I=1/2, strong tensor coupling in flavor off-diagonal.

LQCD ΞN-YY

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- Many experimentally unknown coupled-channel potentials.
- One can see predictive power of the HALQCD method.

Hyperon single-particle potentials

Preliminary

- $G^{SLJ}_{YN,YN}$ contributions to $U_Y(0;\rho_0)$ in SNM

	Yn			Yp			total
	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
Λ	-1.92	-14.35	0.03	-1.92	-14.35	0.03	-32.49
Σ^0	2.03	6.79	-0.06	2.03	6.79	-0.06	17.52
Σ^+	8.68	-4.68	-0.01	-4.62	18.26	-0.10	17.52
Σ^-	-4.62	18.26	-0.10	8.68	-4.68	-0.01	17.52
Ξ^0	-0.68	-7.37	-0.10	8.23	-3.07	-0.02	-3.01
Ξ^-	8.23	-3.07	-0.02	-0.68	-7.37	-0.10	-3.01

Note: including spin multiplicity

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

FAQ

1. Does your potential depend on the choice of source?

➔ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?

➔ Yes. It can be regarded as the "scheme" to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.

FAQ

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

- ➔ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

  However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in our study.

  If we find some dependence, we will determine the next leading term of the expansion from the dependence.

FAQ

in SU(3)$_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

FAQ

in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

➔ Both.
There is no distinct difference between two in QCD.
Note that baryon is made of three quarks in QCD.
Imagine a compact 6-quark object in $(0S)^6$ configuration.
This configuration can be re-written in a form of
$(0S)^3 \times (0S)^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r.
This demonstrate that a compact six-quark object,
at the same time, has a BB configuration.
In LQCD simulation at $SU(3)_F$ limits, we've established
existence of a B=2, S=−2, I=0 stable QCD eigenstate.

Nijmegen

Partial wave contributions to $U_\Lambda(\rho_0)^{(a)}$

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	sum
ESC08c1	–14.3	–29.9	2.7	0.2	1.6	–3.1	–1.6	–44.3
ESC08c1$^+$	–13.2	–26.8	2.9	0.3	1.8	–2.6	–1.5	–39.1
ESC08c2	–13.9	–34.1	2.8	0.2	1.6	–3.2	–1.6	–48.4
ESC08c2$^+$	–12.0	–28.9	3.2	0.3	1.9	–2.4	–1.5	–39.3

Partial wave contributions to $U_\Sigma(\rho_0)$

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ	Γ_Σ
ESC08c1	$1/2$	10.5	–22.6	2.2	1.9	–5.5	–1.1	–0.7		
	$3/2$	–14.1	29.9	–4.6	–1.8	5.6	–1.8	–0.3	–2.3	
ESC08c1$^+$	$1/2$	10.7	–21.5	2.3	1.9	–5.4	–1.0	–0.6		
	$3/2$	–13.3	31.4	–4.4	–1.7	5.8	–1.5	–0.2	+2.4	
ESC08c2	$1/2$	14.6	–22.0	3.1	1.9	–5.5	–1.1	–0.6		
	$3/2$	–15.5	35.2	–4.7	–1.7	5.9	–1.0	–0.2	+8.3	
ESC08c2$^+$	$1/2$	14.8	–20.8	3.2	1.9	–5.3	–0.8	–0.5		
	$3/2$	–14.1	37.6	–4.3	–1.6	6.1	–0.5	–0.1	+15.4	

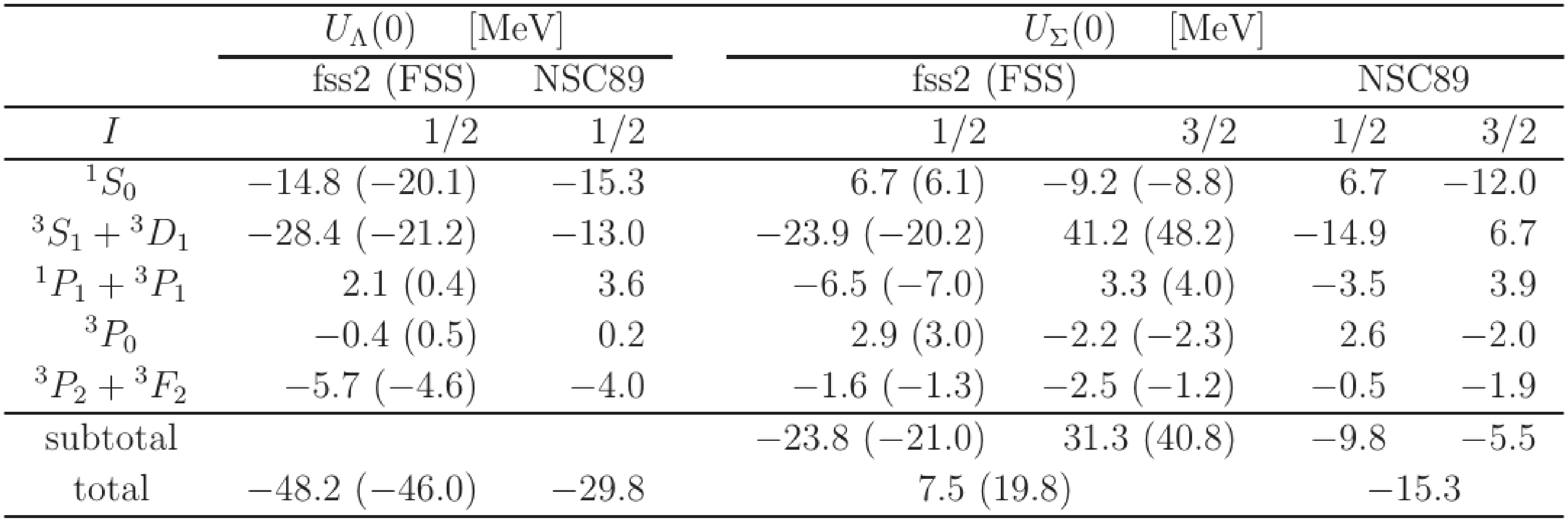
Nijmegen

Partial wave contributions to $U_\Xi(\rho_0)$

model		$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	U_Ξ	Γ^c_Ξ
ESC08c1	$T = 0$	3.1	–9.8	–0.1	0.5	1.7	–1.5		
	$T = 1$	9.1	–7.6	1.3	1.0	–2.4	0.0	–4.7	6.4
ESC08c1$^+$	$T = 0$	2.9	–8.8	–0.1	0.5	1.8	–1.4		
	$T = 1$	9.7	–5.3	1.5	1.0	–2.2	0.4	+0.1	6.3
ESC08c2	$T = 0$	3.6	–11.1	–0.1	0.2	1.8	–1.4		
	$T = 1$	8.7	–10.1	1.2	0.9	–2.7	–0.5	–9.6	5.1
ESC08c2$^{+\prime\prime}$	$T = 0$	3.4	–9.5	–0.0	0.2	1.9	–1.1		
	$T = 1$	9.8	–6.2	1.6	1.0	–2.4	0.1	–1.3	4.8

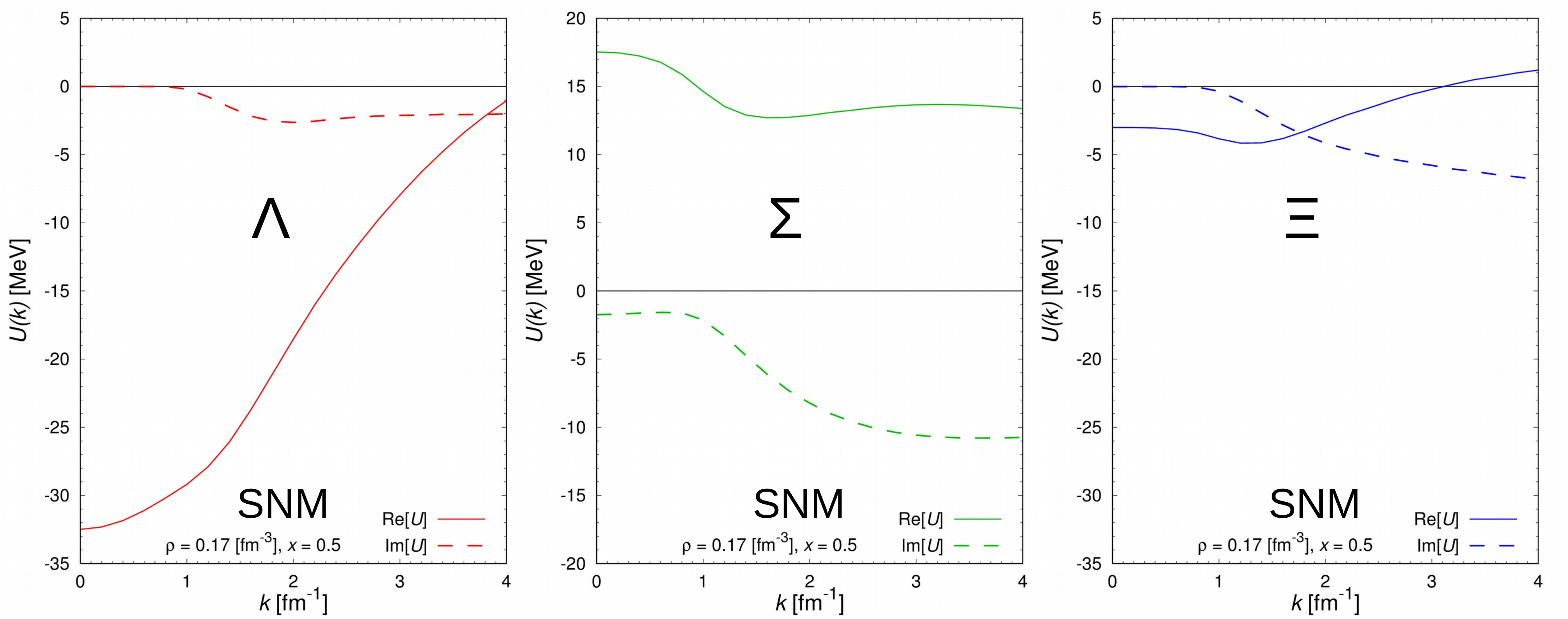
Quark model

Taken from M. Kohno etal.
Prog. Part. Nucl. Phys. 58, 439-520 (2007)



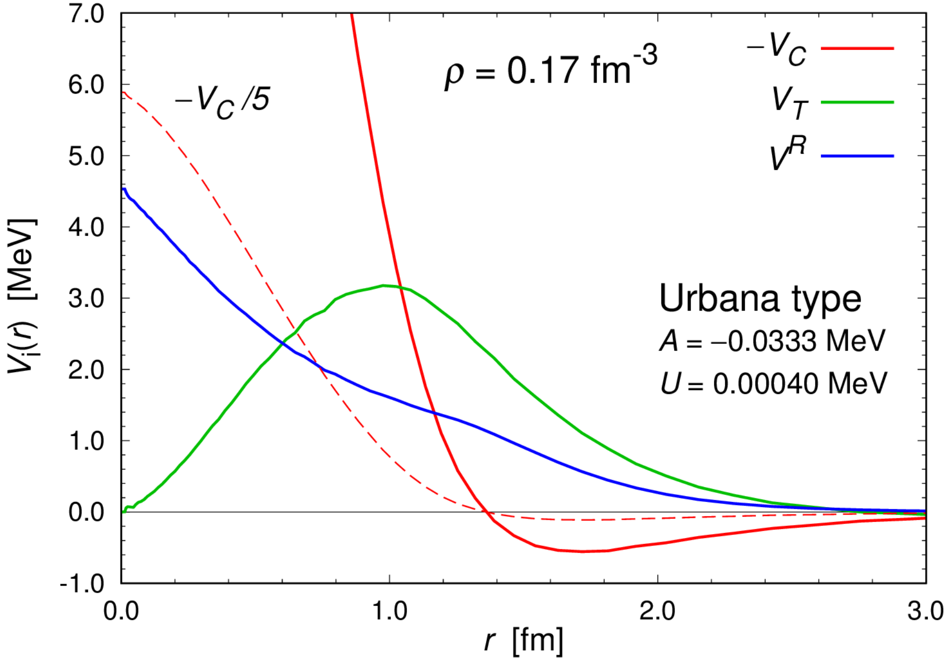
	$U_\Lambda(0)$ [MeV]		$U_\Sigma(0)$ [MeV]			
	fss2 (FSS)	NSC89	fss2 (FSS)		NSC89	
I	$1/2$	$1/2$	$1/2$	$3/2$	$1/2$	$3/2$
$^1S_0$	-14.8 (-20.1)	-15.3	6.7 (6.1)	-9.2 (-8.8)	6.7	-12.0
$^3S_1 + {}^3D_1$	-28.4 (-21.2)	-13.0	-23.9 (-20.2)	41.2 (48.2)	-14.9	6.7
$^1P_1 + {}^3P_1$	2.1 (0.4)	3.6	-6.5 (-7.0)	3.3 (4.0)	-3.5	3.9
$^3P_0$	-0.4 (0.5)	0.2	2.9 (3.0)	-2.2 (-2.3)	2.6	-2.0
$^3P_2 + {}^3F_2$	-5.7 (-4.6)	-4.0	-1.6 (-1.3)	-2.5 (-1.2)	-0.5	-1.9
subtotal			-23.8 (-21.0)	31.3 (40.8)	-9.8	-5.5
total	-48.2 (-46.0)	-29.8	7.5 (19.8)		-15.3	

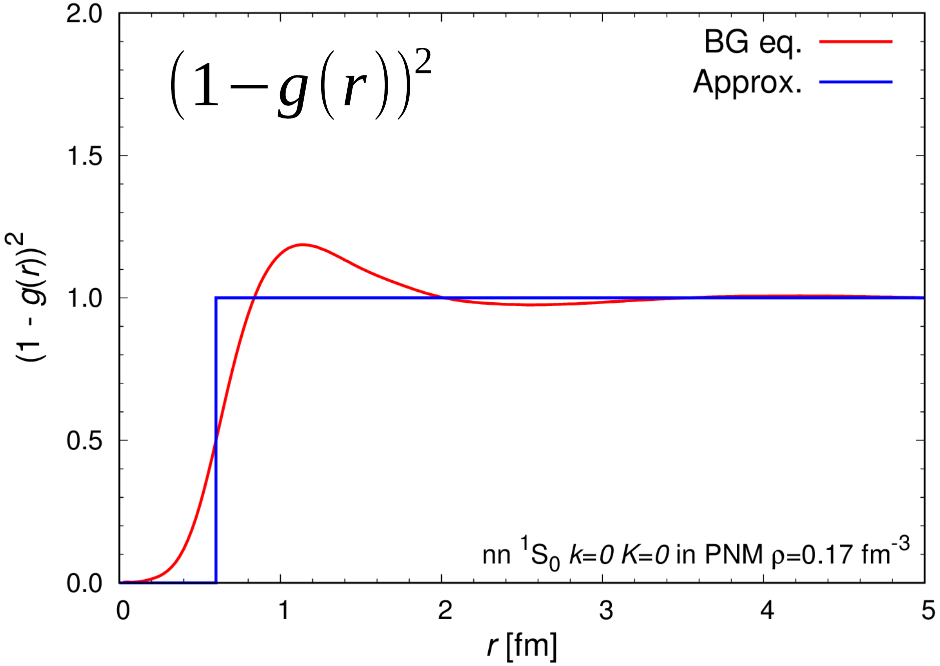
Hyperon single-particle potentials



- $\mathrm{Im}[U_Y]$ are obtained by summing up $\mathrm{Im}[G_{YN,YN}]$.
- But, $\mathrm{Im}[U]$ are not considered in the B.G. equation.

NNN force



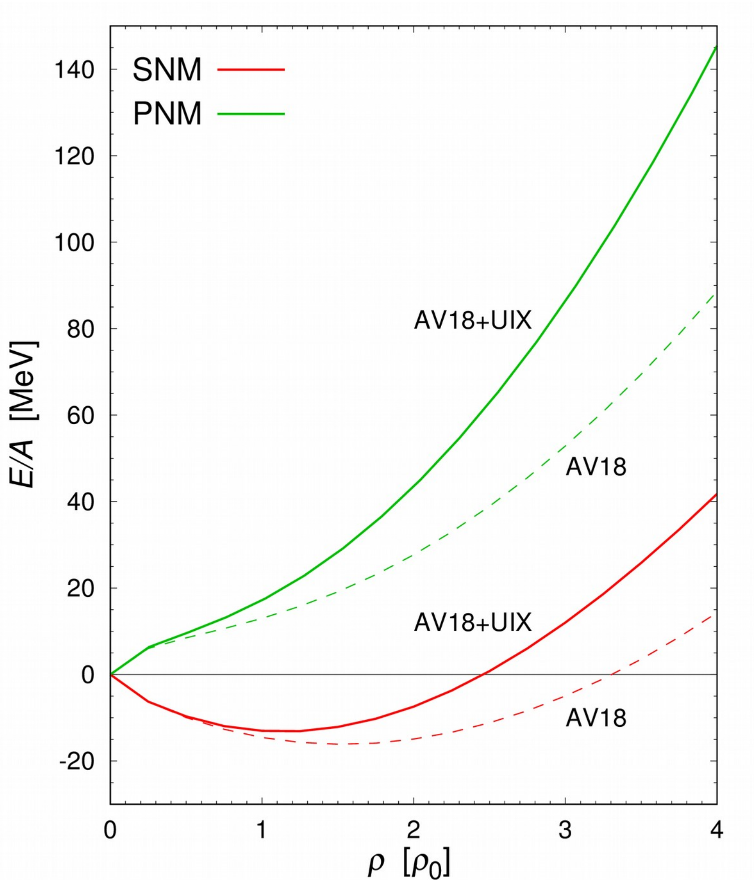


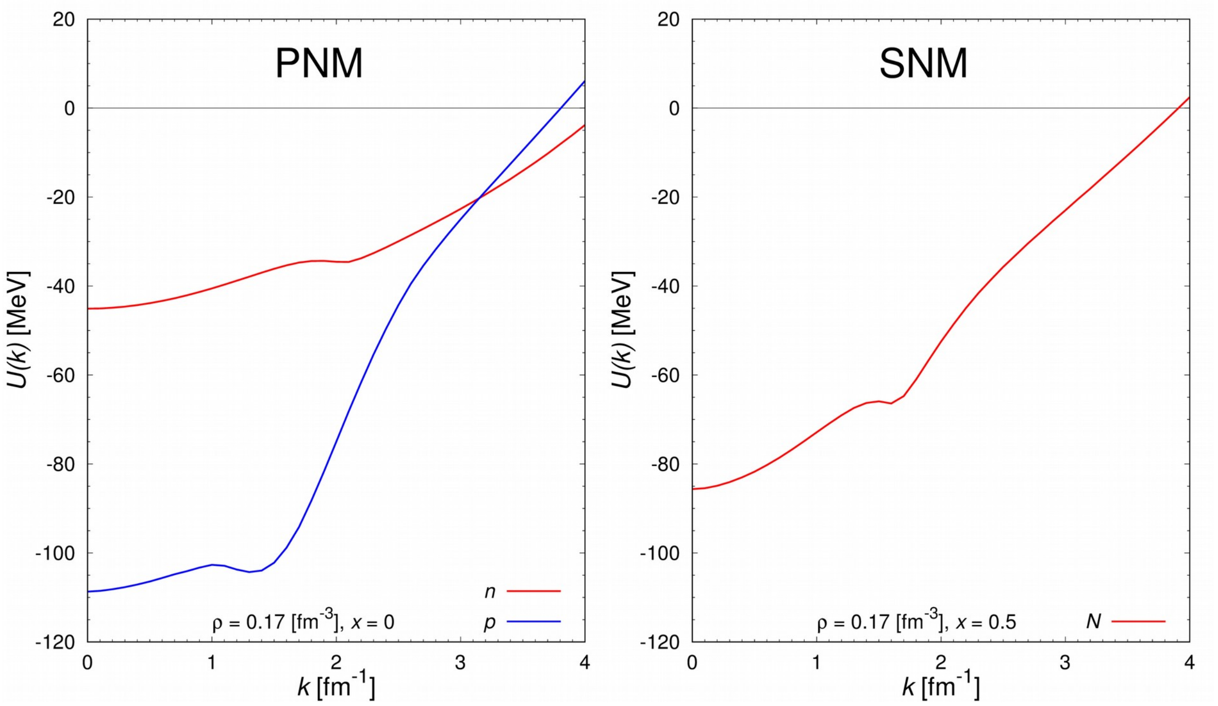
- Effective two-body potential

$$\bar{V}(\rho;\boldsymbol{r}) = \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\underline{V_C}(\rho;r) + S_{12}(\hat{\boldsymbol{r}})\underline{V_T}(\rho;r)\right] + \underline{V^R}(\rho;r)$$

- obtained by integrating out positon of 3rd nucleon.
- Here, $\bar{V}(\rho,\boldsymbol{r})$ is ρ-propotional due to a fixed defect.

Nuclear matter





- Urbana NNN force is adjusted so that AV18 + Urbana reproduce the “emprical” saturation property of SNM.

roughly