Exotica structure in a Bethe-Salpeter approach

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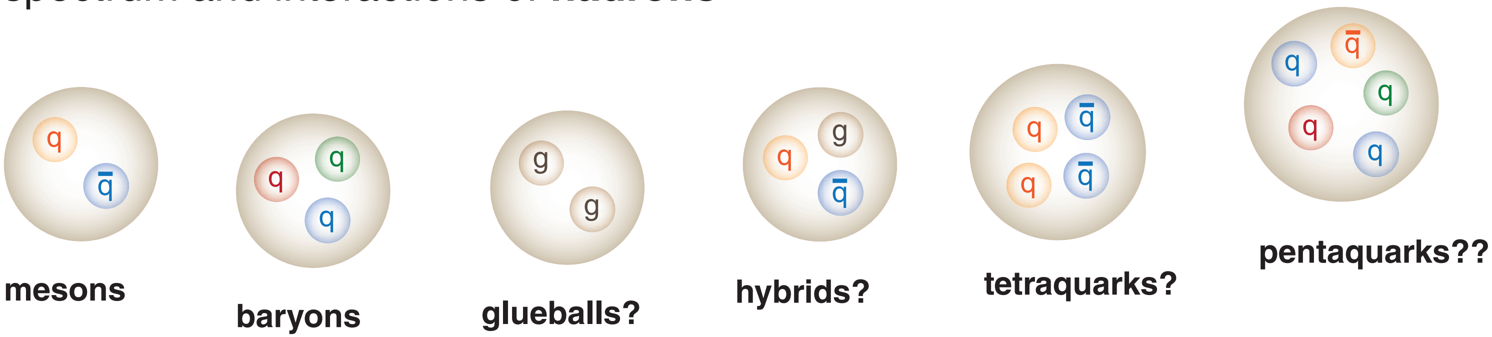
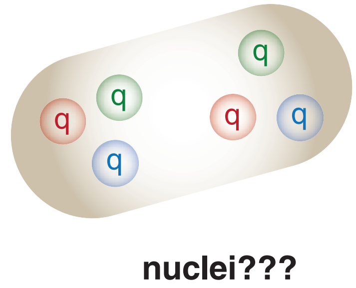
EMMI Workshop: Anti-matter, hyper-matter and exotica production at the LHC
Torino, Italy
November 9, 2017

Exotica

- To understand QCD, we need to understand spectrum and interactions of **hadrons**





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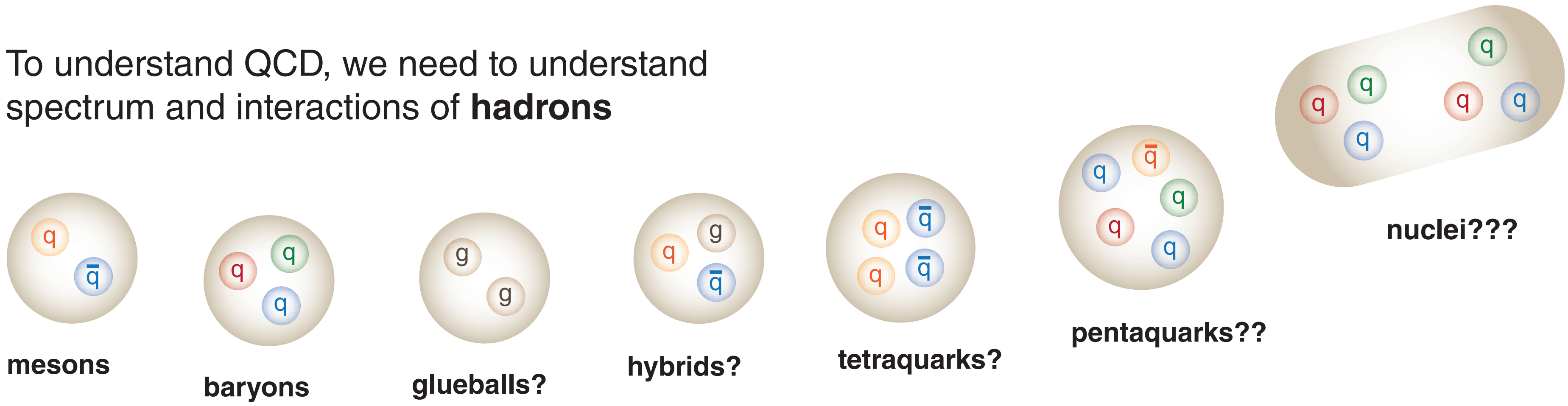
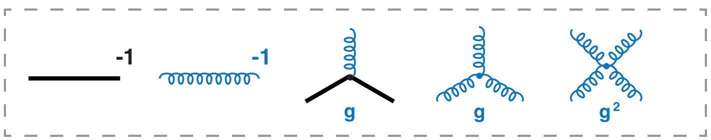


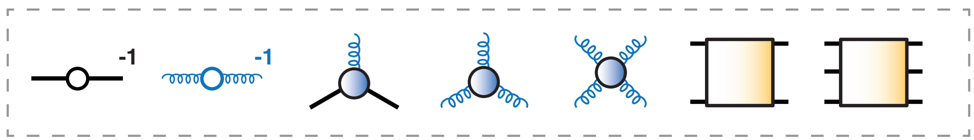
QCD's classical action:

$$S = \int d^4x \, \left[\bar{\psi} \, (\not\partial + ig\not{A} + m) \, \psi + \tfrac{1}{4} \, F^a_{\mu\nu} \, F^{\mu\nu}_a \right]$$

$=$ 

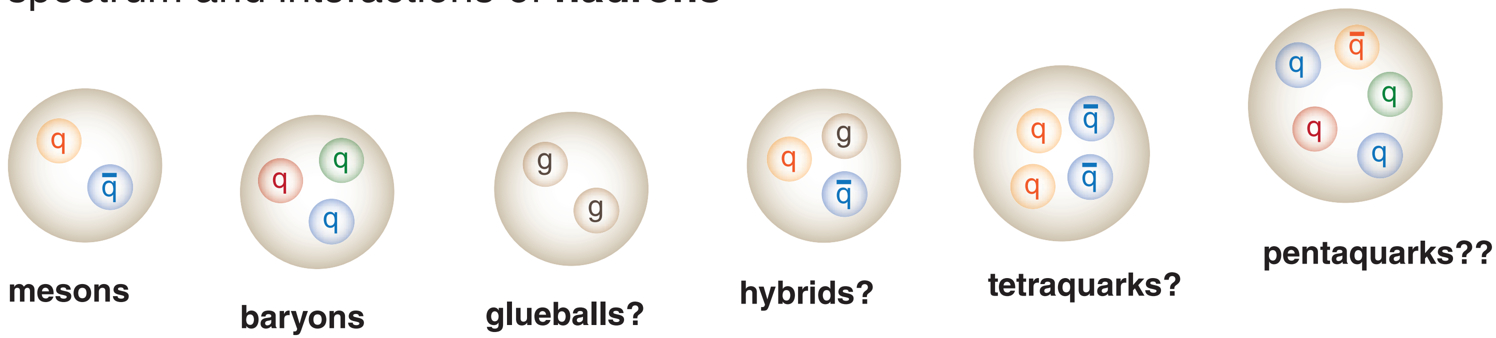
Quantum "effective action":

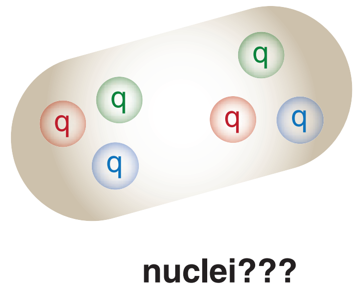
$$\int \mathcal{D}[\psi, \bar{\psi}, A] \, e^{-S} = e^{-\Gamma}$$



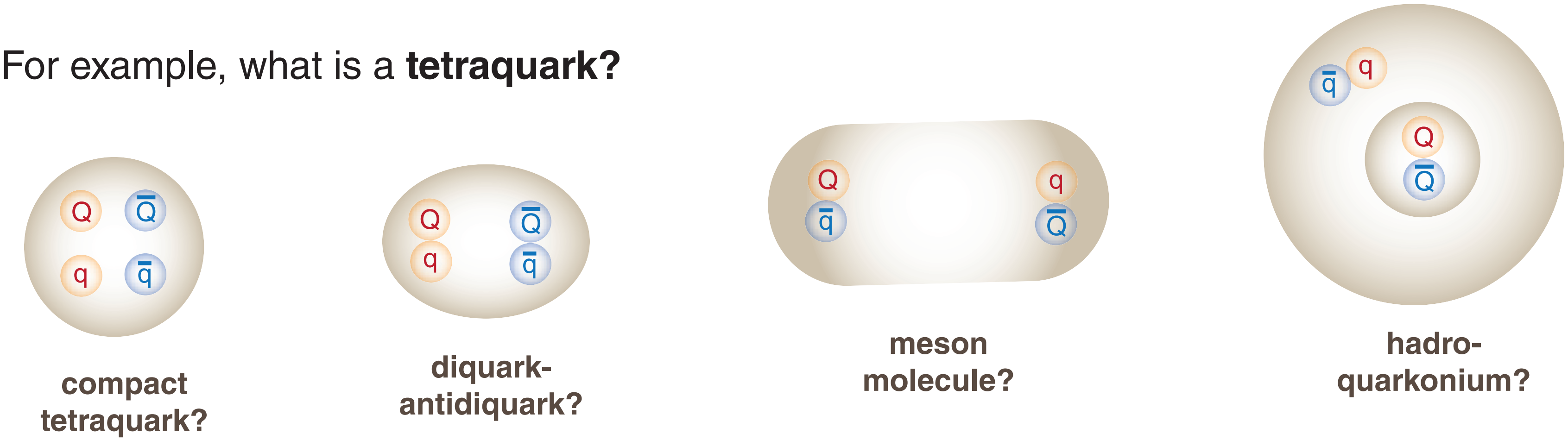
Exotica

- To understand QCD, we need to understand spectrum and interactions of **hadrons**





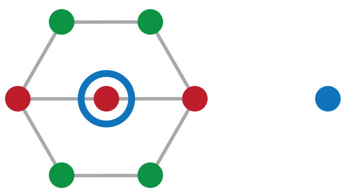
- For example, what is a **tetraquark?**



The hadron zoo

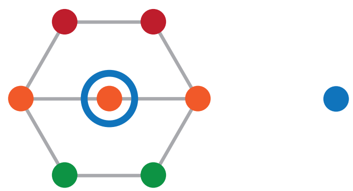
Mesons

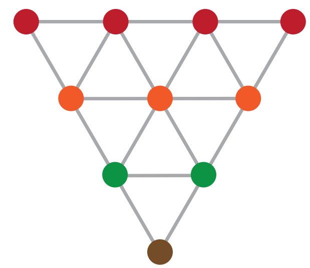
0^{-+}	0^{++}	1^{-+}	1^{--}	1^{++}	1^{+-}	2^{-+}	2^{++}	3^{--}
$\pi(140)$ $\pi(1300)$ $\pi(1800)$	$a_0(980)$ $a_0(1450)$ $a_0(1950)$	$\pi_1(1400)$ $\pi_1(1600)$	$\rho(770)$ $\rho(1450)$ $\rho(1570)$ $\rho(1700)$ $\rho(1900)$	$a_1(1260)$ $a_1(1420)$ $a_1(1640)$	$b_1(1235)$	$\pi_2(1670)$ $\pi_2(1880)$	$a_2(1320)$ $a_2(1700)$	$\rho_3(1690)$ $\rho_3(1990)$
$K(494)$ $K(1460)$ $K(1830)$	$K_0^*(800)$ $K_0^*(1430)$ $K_0^*(1950)$		$K^*(892)$ $K^*(1410)$ $K^*(1680)$	$K_1(1400)$ $K_1(1650)$	$K_1(1270)$	$K_2(1580)$ $K_2(1770)$ $K_2(1820)$	$K_2^*(1430)$ $K_2^*(1980)$	$K_3^*(1780)$
$\eta(548)$ $\eta'(958)$ $\eta(1295)$ $\eta(1405)$ $\eta(1475)$ $\eta(1760)$	$f_0(500)$ $f_0(980)$ $f_0(1370)$ $f_0(1500)$ $f_0(1710)$		$\omega(782)$ $\phi(1020)$ $\omega(1420)$ $\omega(1650)$ $\phi(1680)$	$f_1(1285)$ $f_1(1420)$ $f_1(1510)$	$h_1(1170)$ $h_1(1380)$ $h_1(1595)$	$\eta_2(1645)$ $\eta_2(1870)$	$f_2(1270)$ $f_2(1430)$ $f_2'(1525)$ $f_2(1565)$ $f_2(1640)$ $f_2(1810)$ $f_2(1910)$ $f_2(1950)$	$\omega_3(1670)$ $\phi_3(1850)$



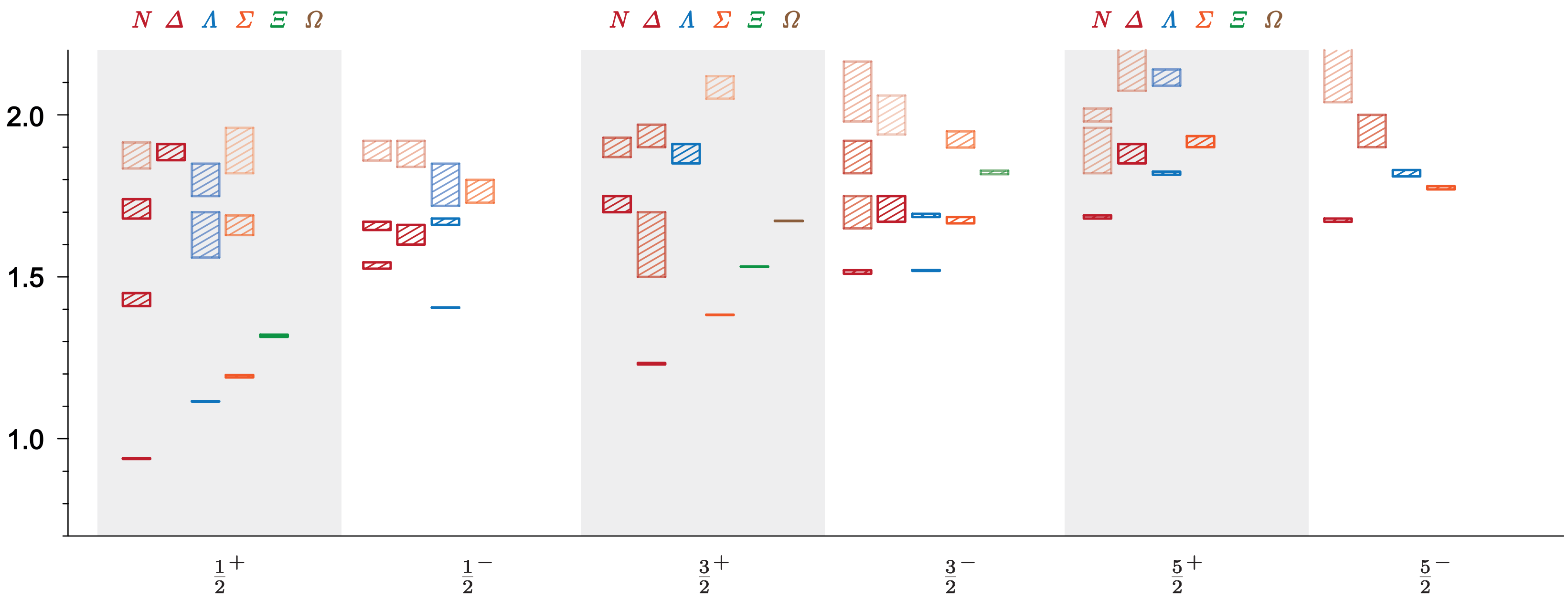
Baryons

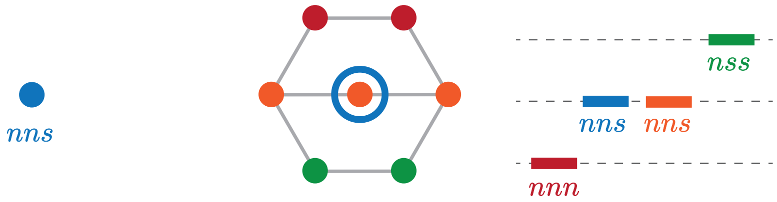
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				

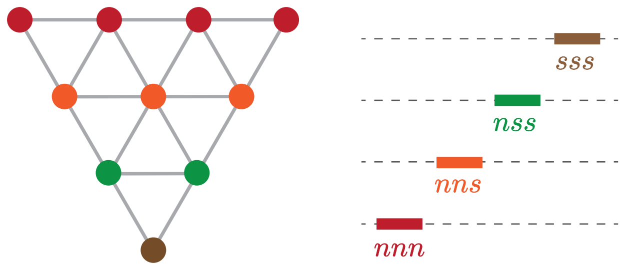




The hadron zoo







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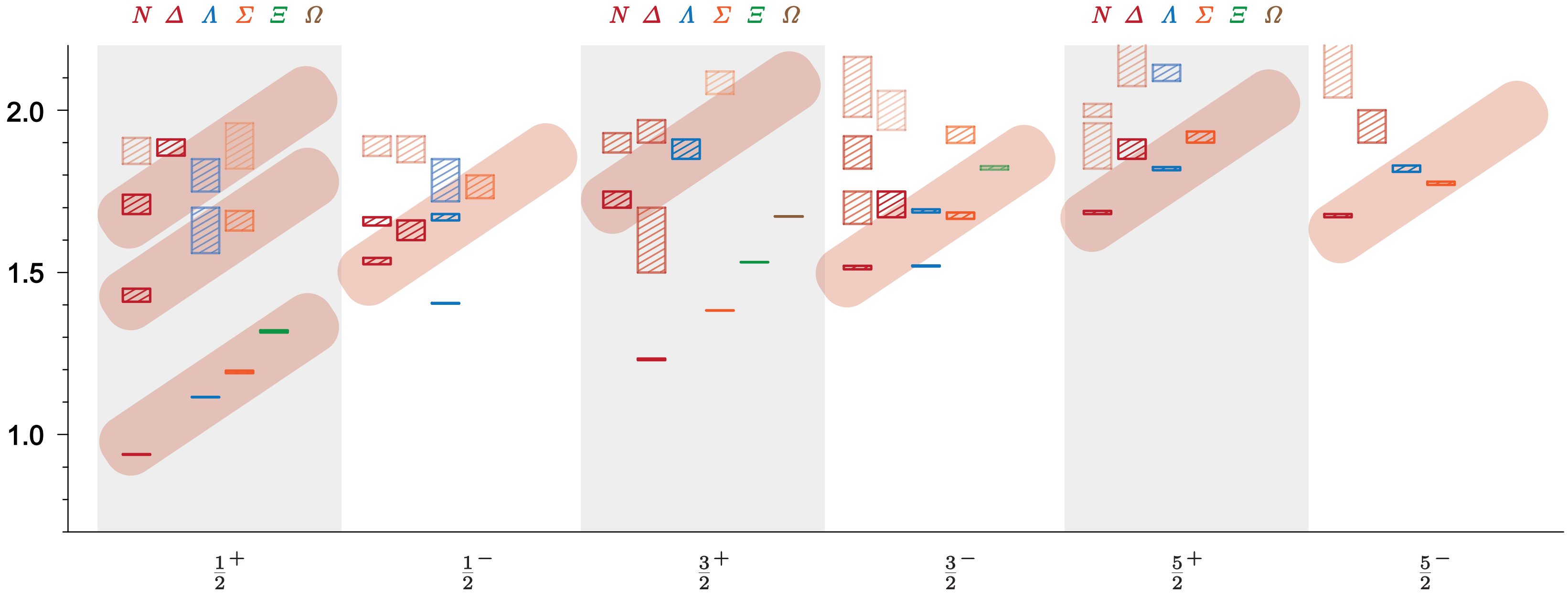

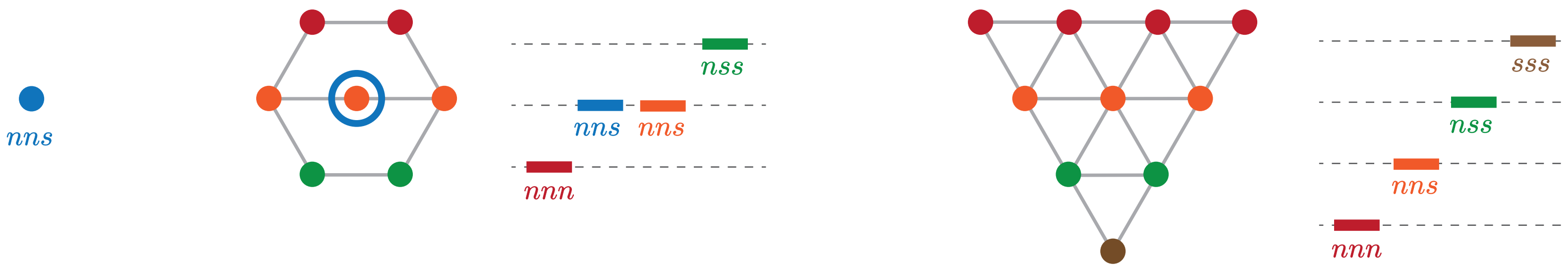

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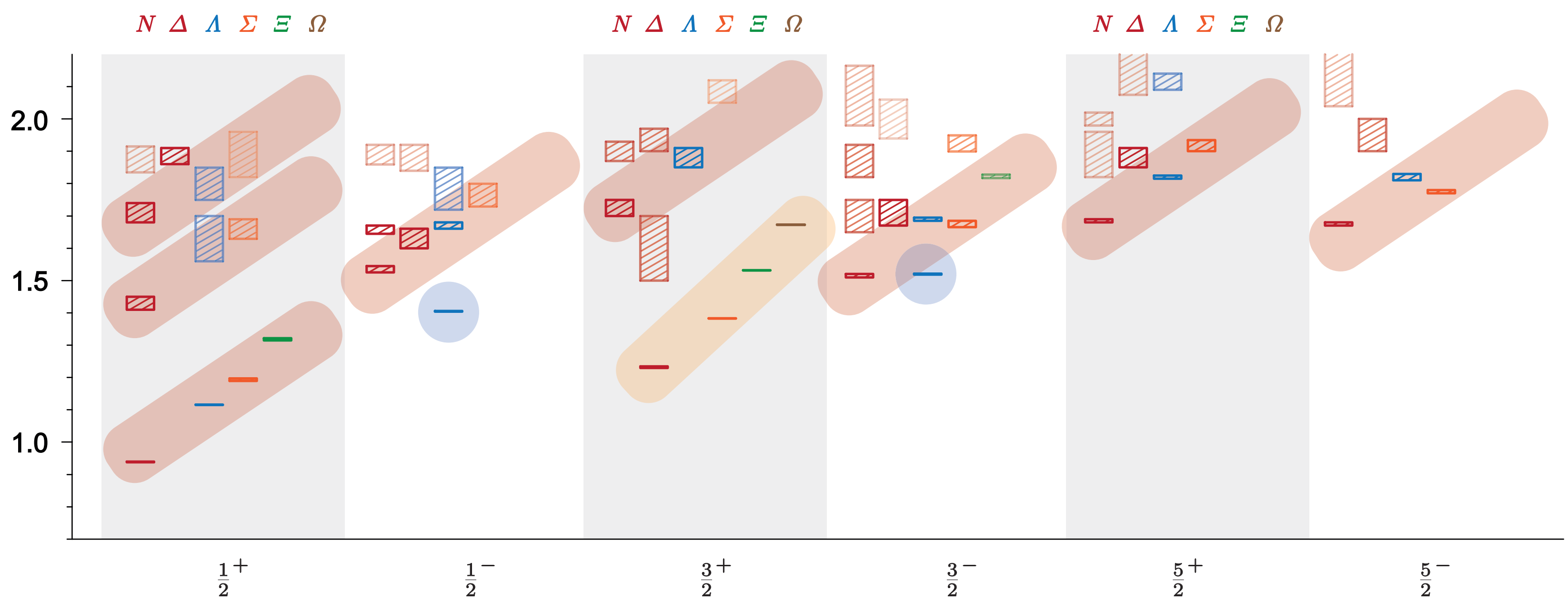
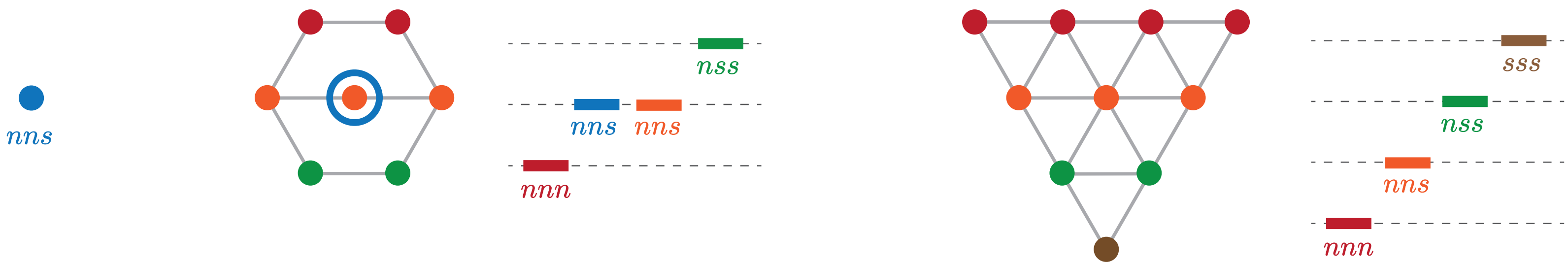
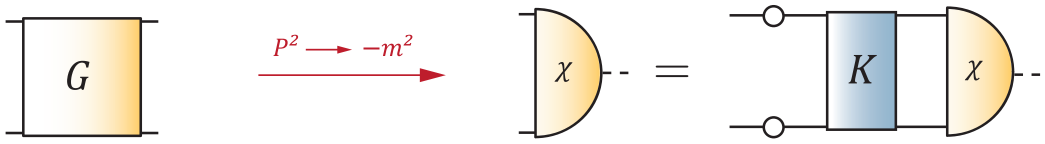


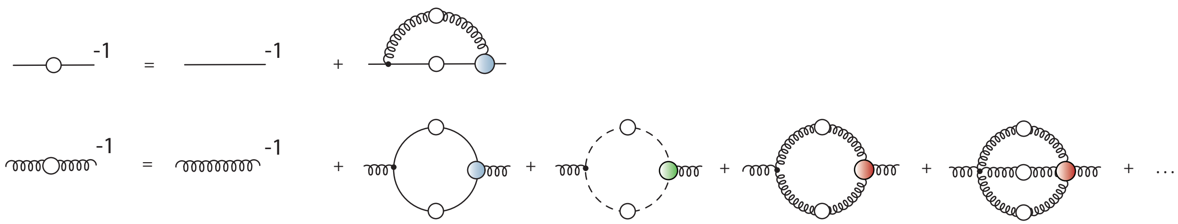


Mesons

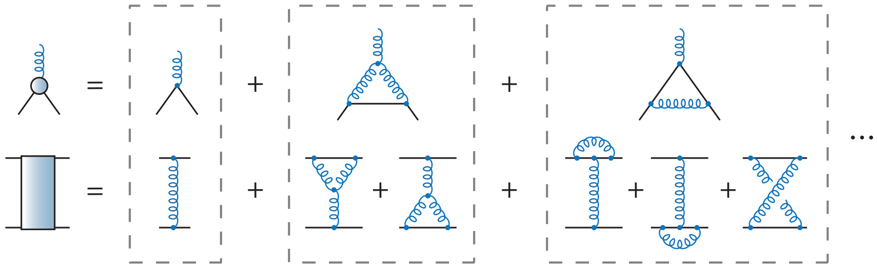
- Homogeneous **Bethe-Salpeter equation** for BS wave function:



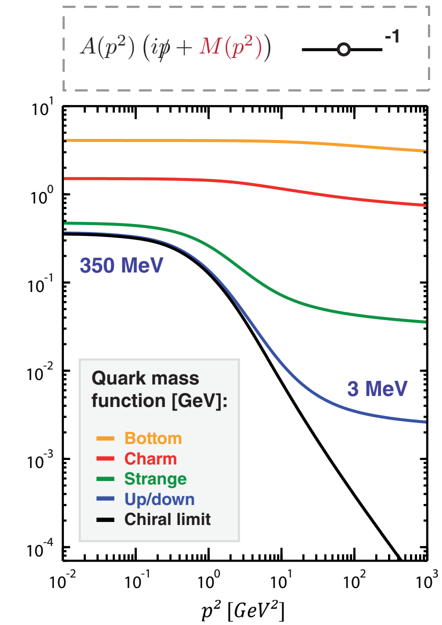
- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**



- Kernel can be derived in accordance with **chiral symmetry:**



Agreement between lattice, DSE & FRG within reach!



Quark propagator:
DCSB generates 'constituent-quark masses'

Mesons

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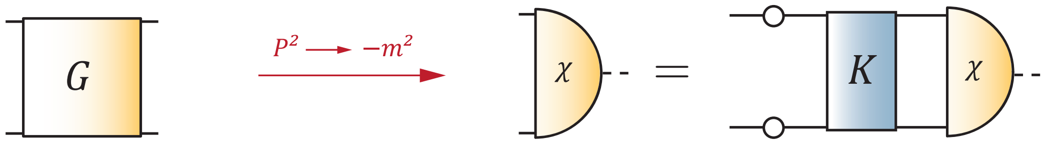

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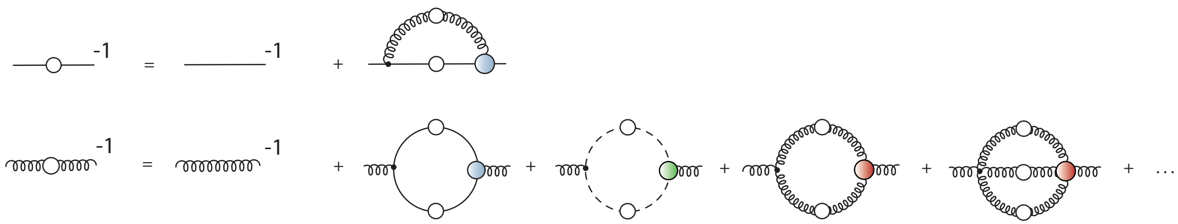

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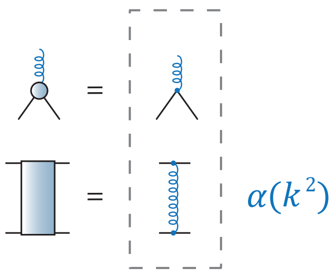
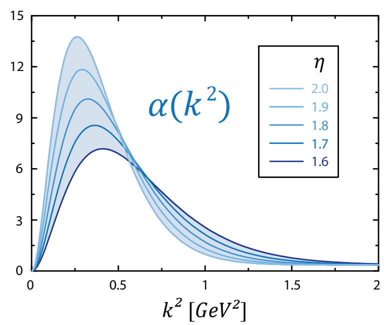




Rainbow-ladder:
effective gluon exchange

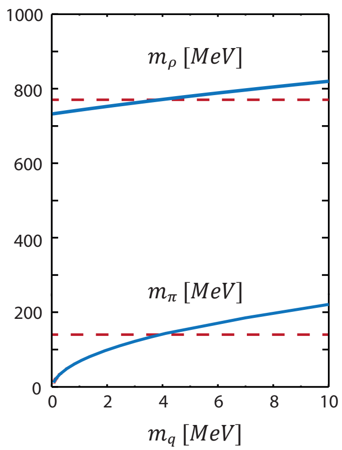
$$\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

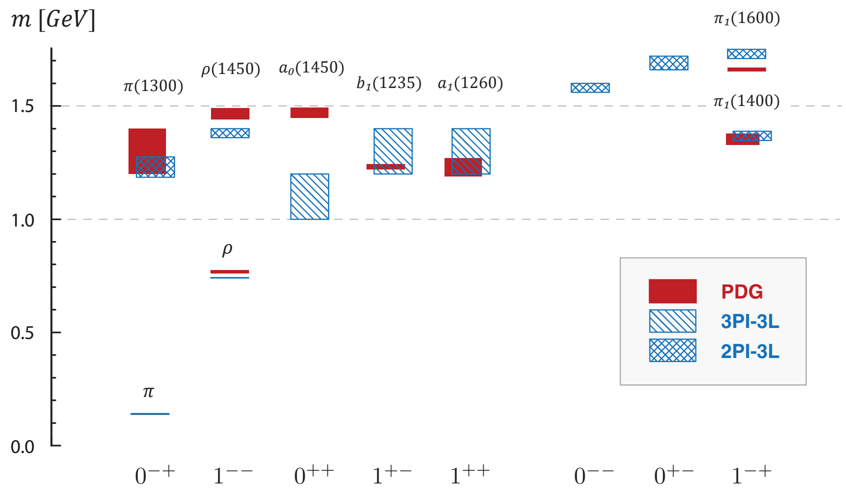
Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

Mesons

- Pion is **Goldstone boson:** $m_\pi^{\,2} \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

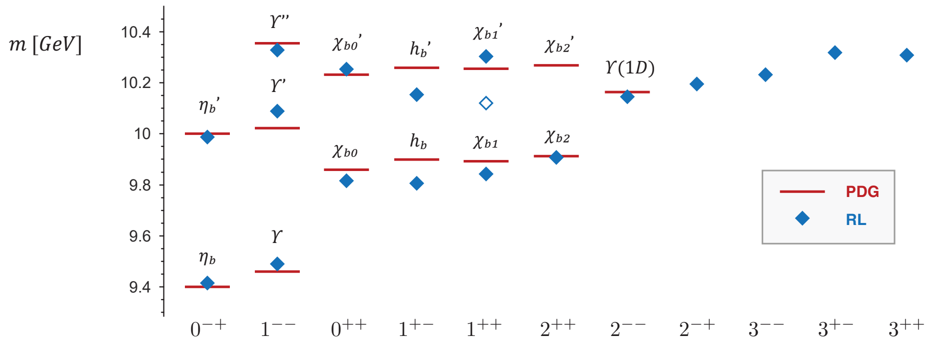


Williams, Fischer, Heupel, PRD 93 (2016)

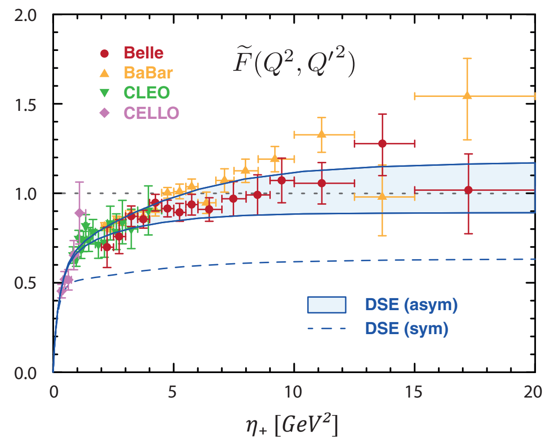
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Charmonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



- **Pion transition form factor**

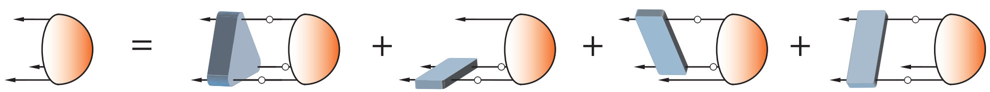


GE, Fischer, Weil, Williams, PLB 774 (2017)

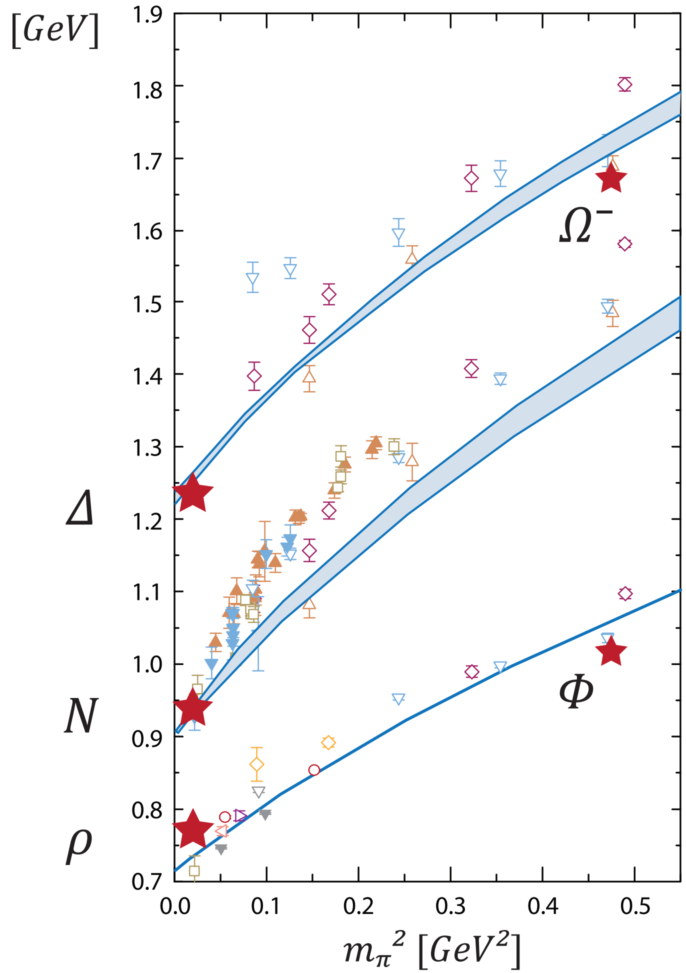
Baryons

Covariant Faddeev equation for baryons:

GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

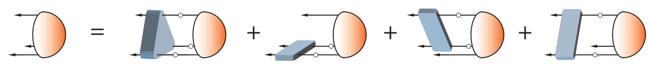


- 3-gluon diagram vanishes ⇒ **3-body effects small?**
- 2-body kernels same as for mesons, no further approximations: $M_N = 0.94\,\text{GeV}$
- **Relativistic bound states** carry OAM: 64 (128) tensors for nucleon (Δ)
- Octet & decuplet baryons, pion cloud effects, first steps beyond rainbow-ladder
- **Baryon form factors:** nucleon and Δ FFs, $N \to \Delta\gamma$ transition, ...

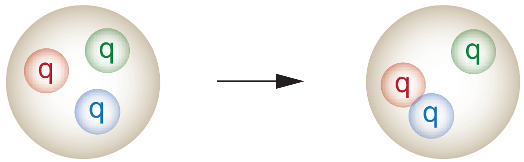


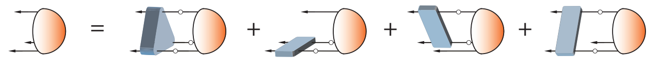
Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

DSE / Faddeev landscape $N \to N^*\gamma$

	Three-quark		
			
	RL	bRL	bRL + 3q
N, Δ masses	√	√	$\dots$
N, Δ em. FFs	√		
$N \to \Delta\gamma$	√		
Roper	√	$\dots$	
$N \to N^*\gamma$	$\dots$		
$N^*(1535), \dots$	√	$\dots$	
$N \to N^*\gamma$	$\dots$		

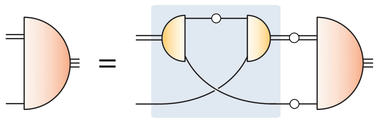
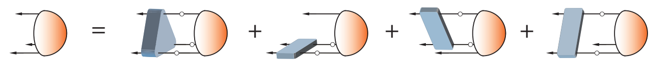
DSE / Faddeev landscape $N \to N^*\gamma$





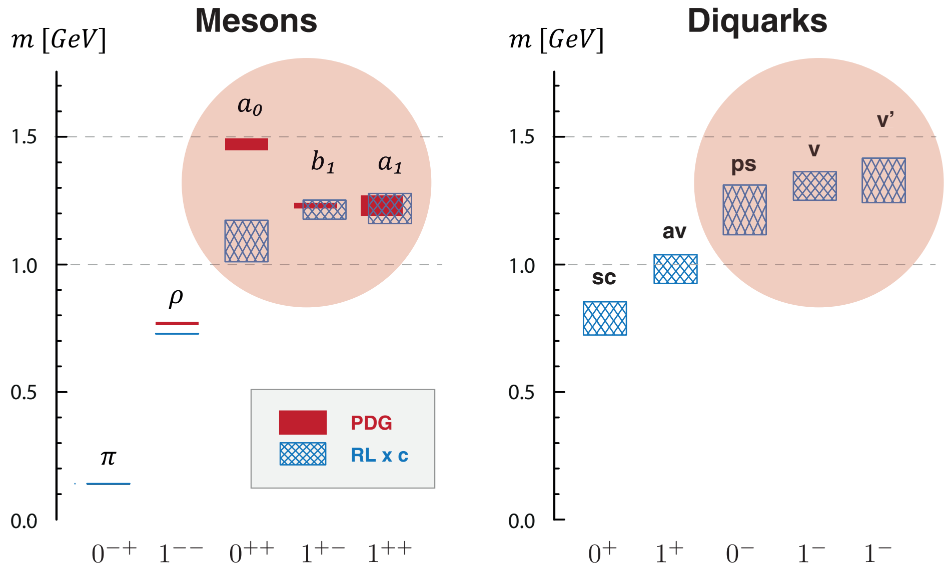
	Quark-diquark	Three-quark		
		RL	**bRL**	**bRL + 3q**
N, Δ masses		√	√	...
N, Δ em. FFs		√		
$N \to \Delta\gamma$		√		
Roper		√	...	
$N \to N^*\gamma$		...		
$N^*(1535), \ldots$		√	...	
$N \to N^*\gamma$		...		

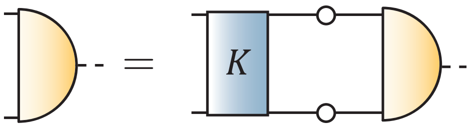
DSE / Faddeev landscape $N \to N^*\gamma$

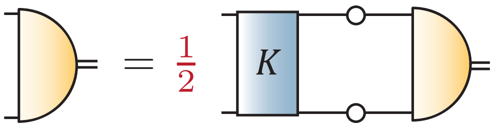
	Quark-diquark			Three-quark		
						
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	√		
Roper	√	√	√	√	...	
$N \to N^*\gamma$	√	√	...	...		
$N^*(1535), \dots$	...	...	√	√	...	
$N \to N^*\gamma$	...	...	...	...		
	Roberts, Bashir, Segovia, Chen, Wilson, Lu, ...	Oettel, Alkofer, Roberts, Cloet, Segovia, ...	GE, Alkofer, Nicmorus, ...	GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, ...		

The role of diquarks

Mesons and 'diquarks' closely related:
after taking traces, only factor 1/2 remains
⇒ **diquarks 'less bound' than mesons**





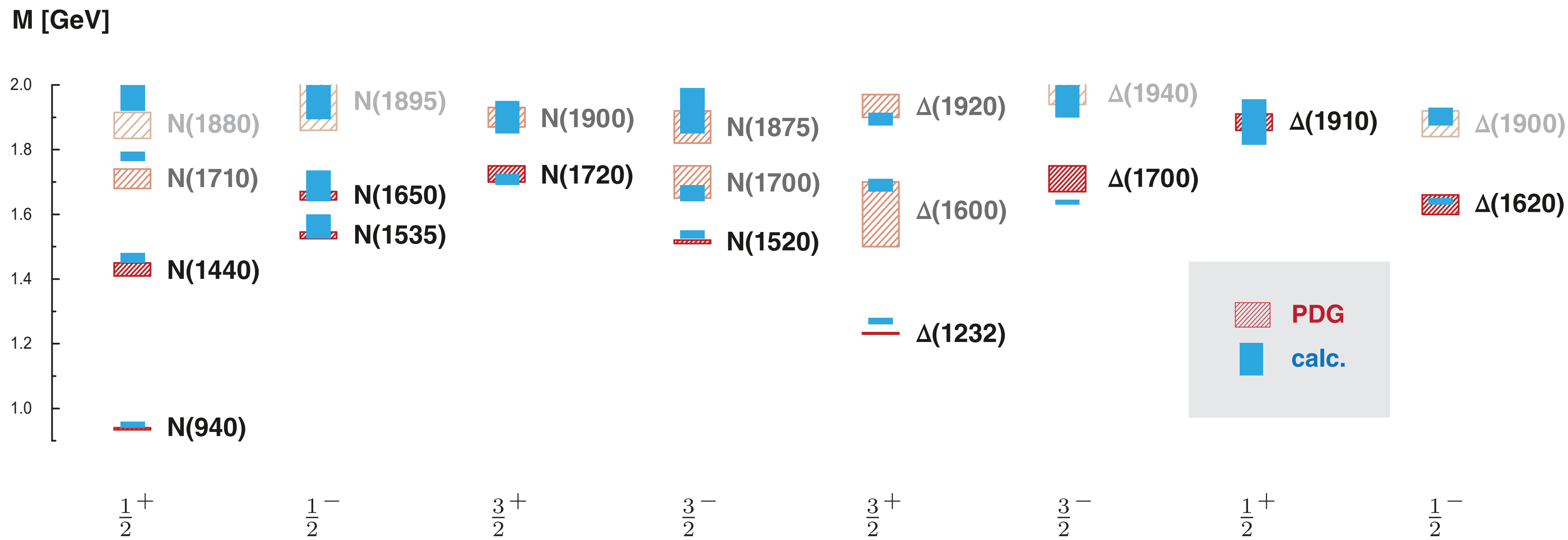


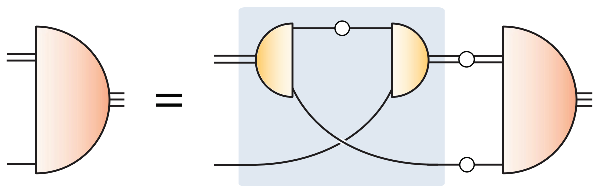
Pseudoscalar & vector mesons ⇔ **Scalar & axialvector diquarks**
already good in rainbow-ladder — sufficient for nucleon and Δ

Scalar & axialvector mesons ⇔ **Pseudoscalar & vector diquarks**
too light, repulsion beyond RL — important for remaining channels

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

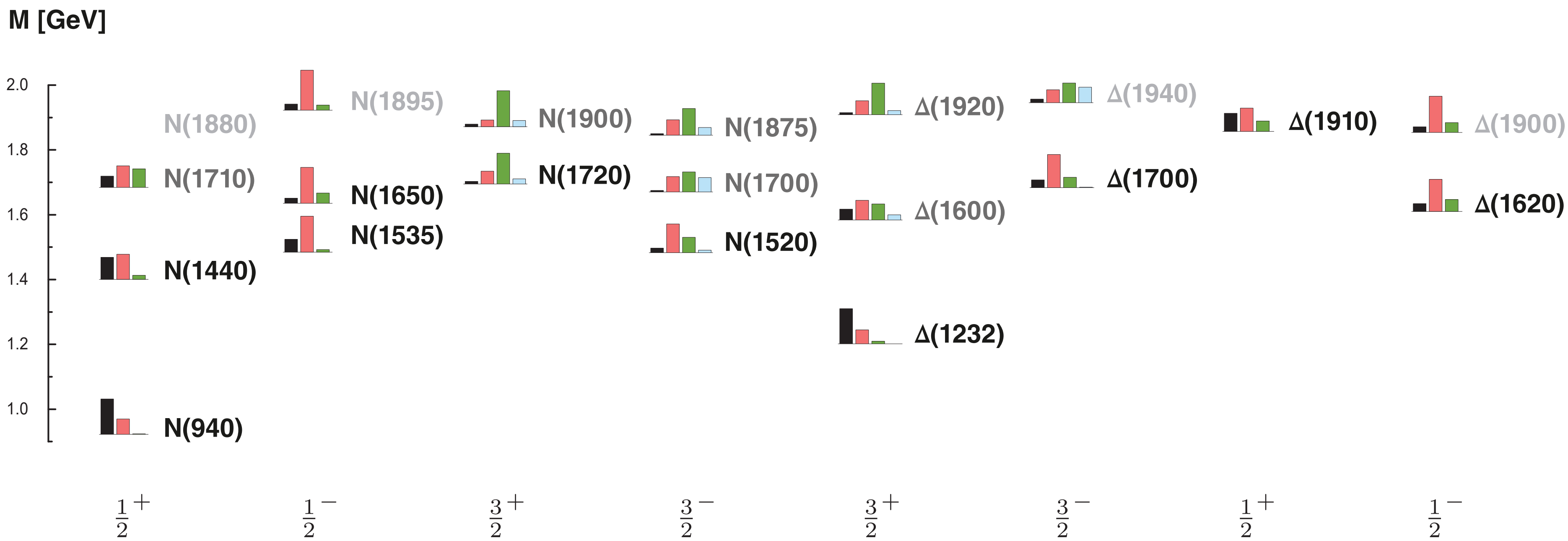




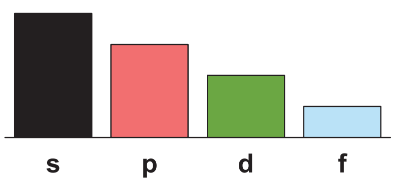
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, FBS 58 (2017)

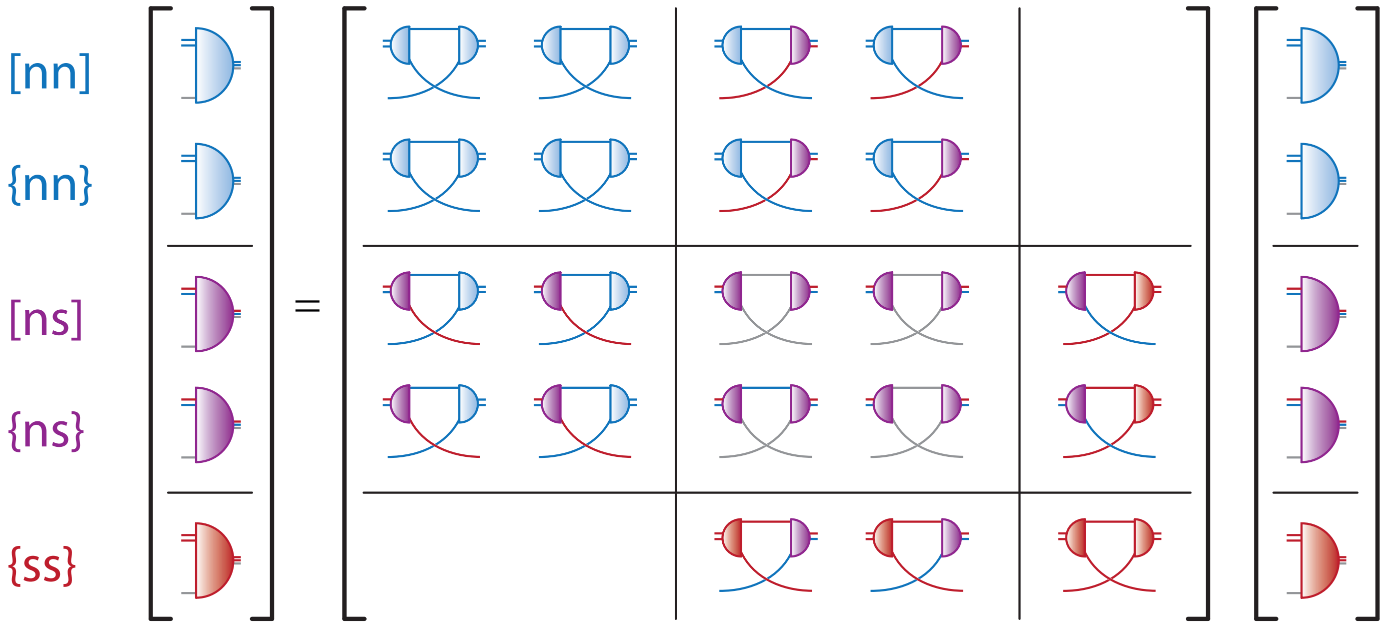


Orbital angular momentum content:

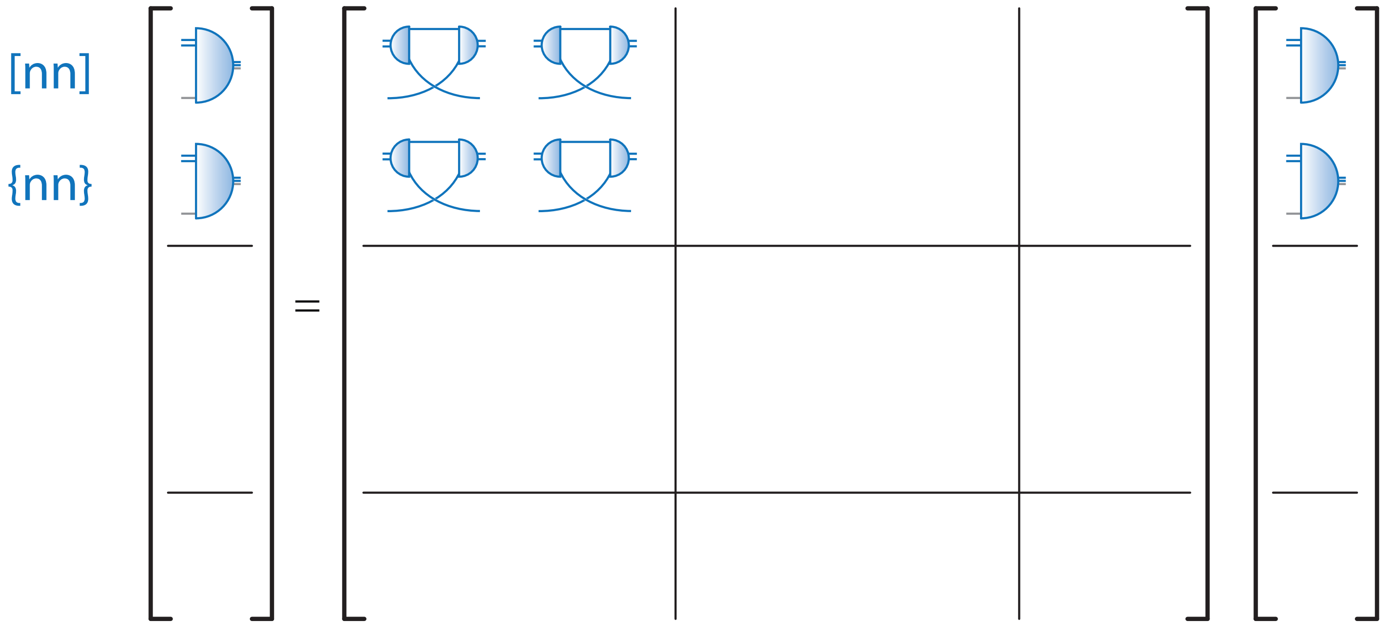


- in nonrelativistic quark model:
  N, **Δ** ~ **s waves,** negative-parity states ~ **p waves,** etc.
- Here: 'quark-model forbidden' contributions are always present, e.g. **Roper: dominated by p waves ⇒ relativity is important!**

Strange baryons

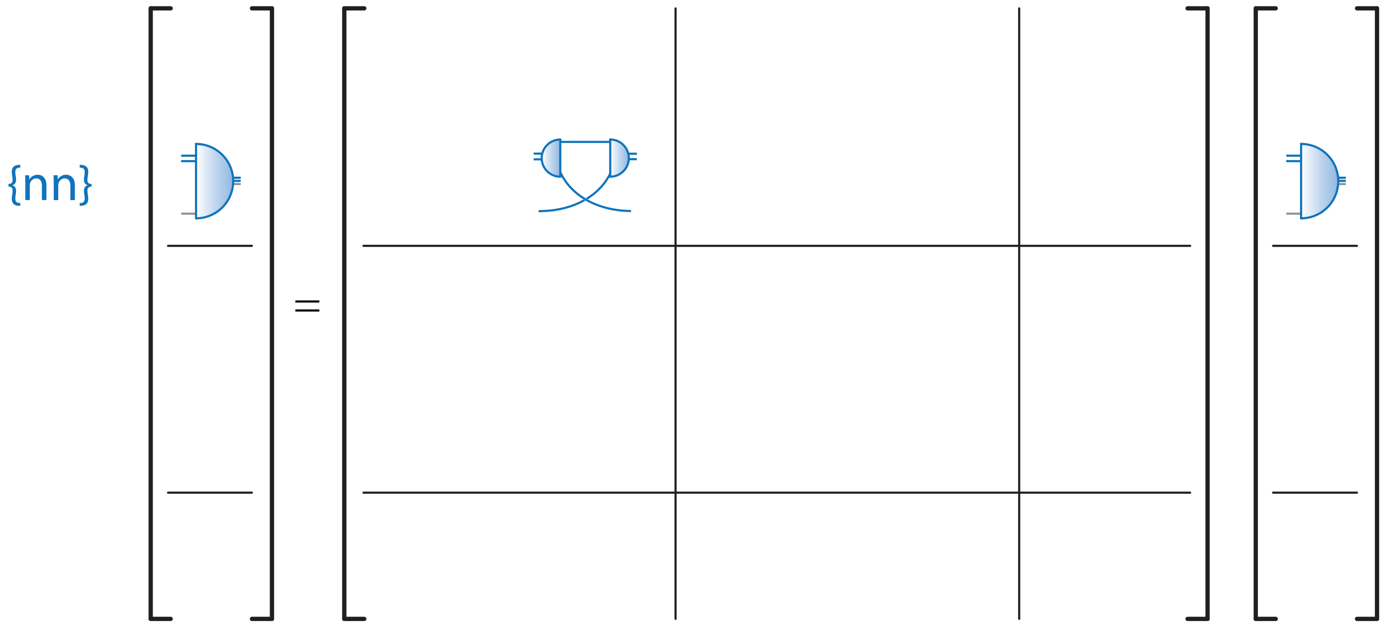


Strange baryons



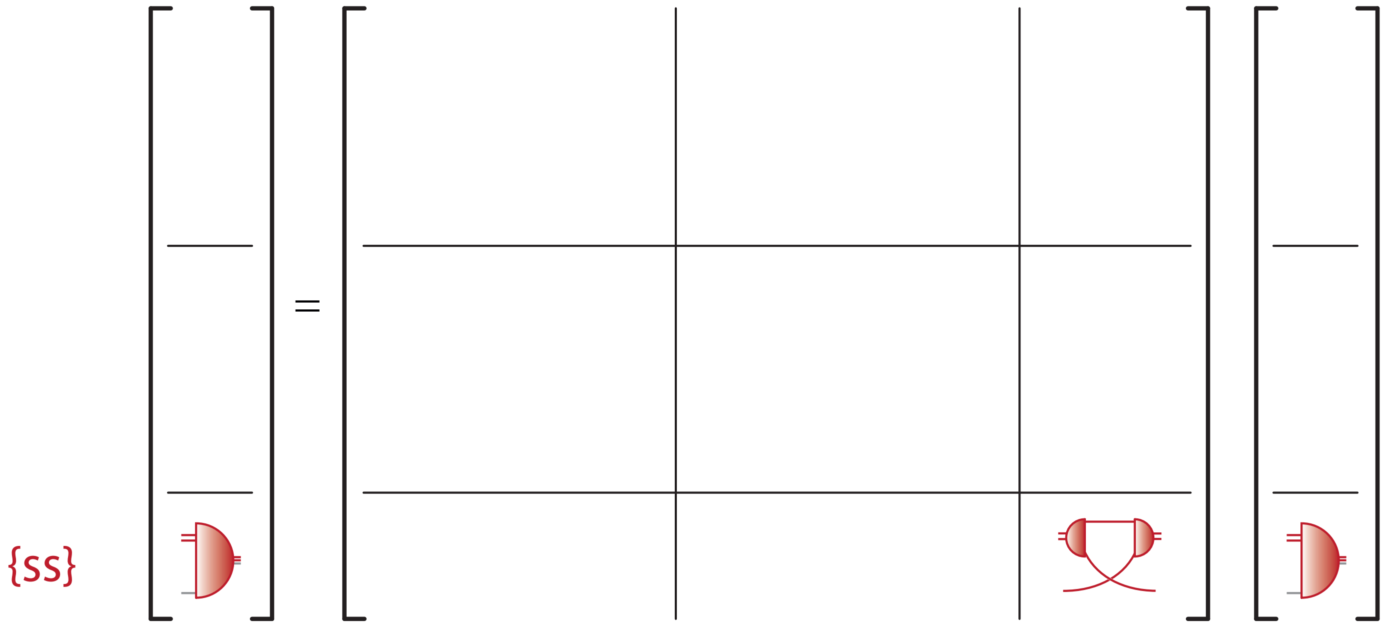
Nucleon

Strange baryons



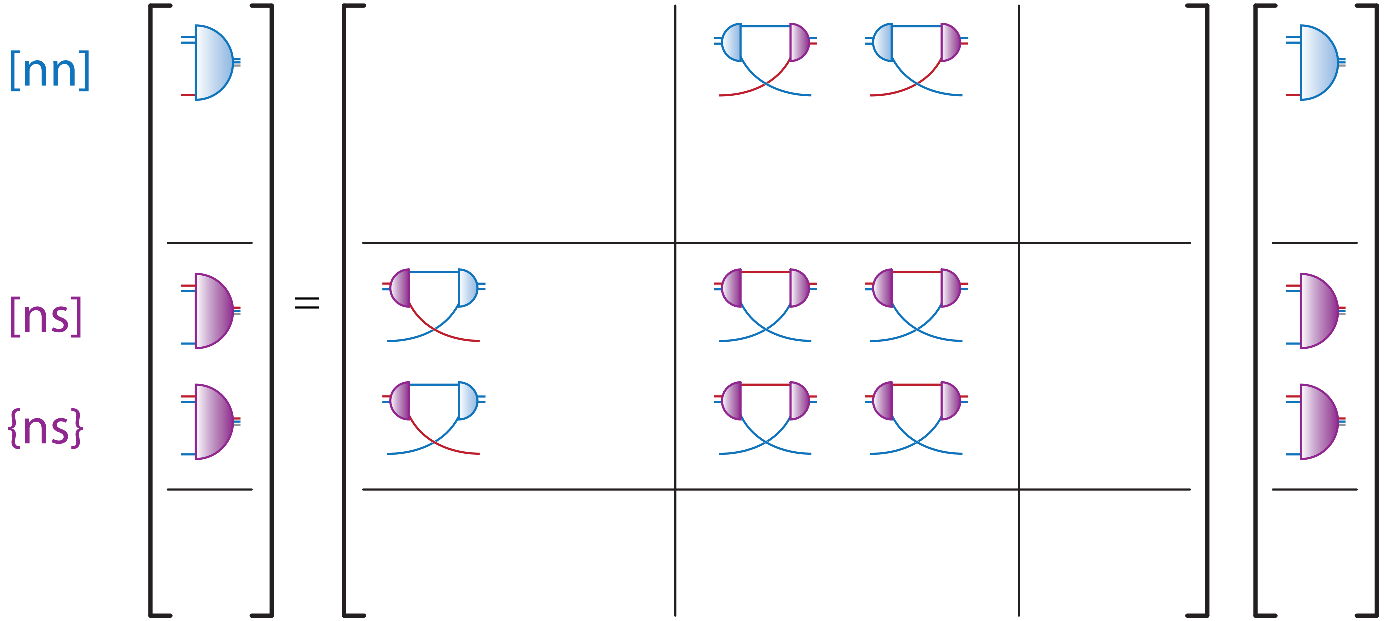
Delta

Strange baryons



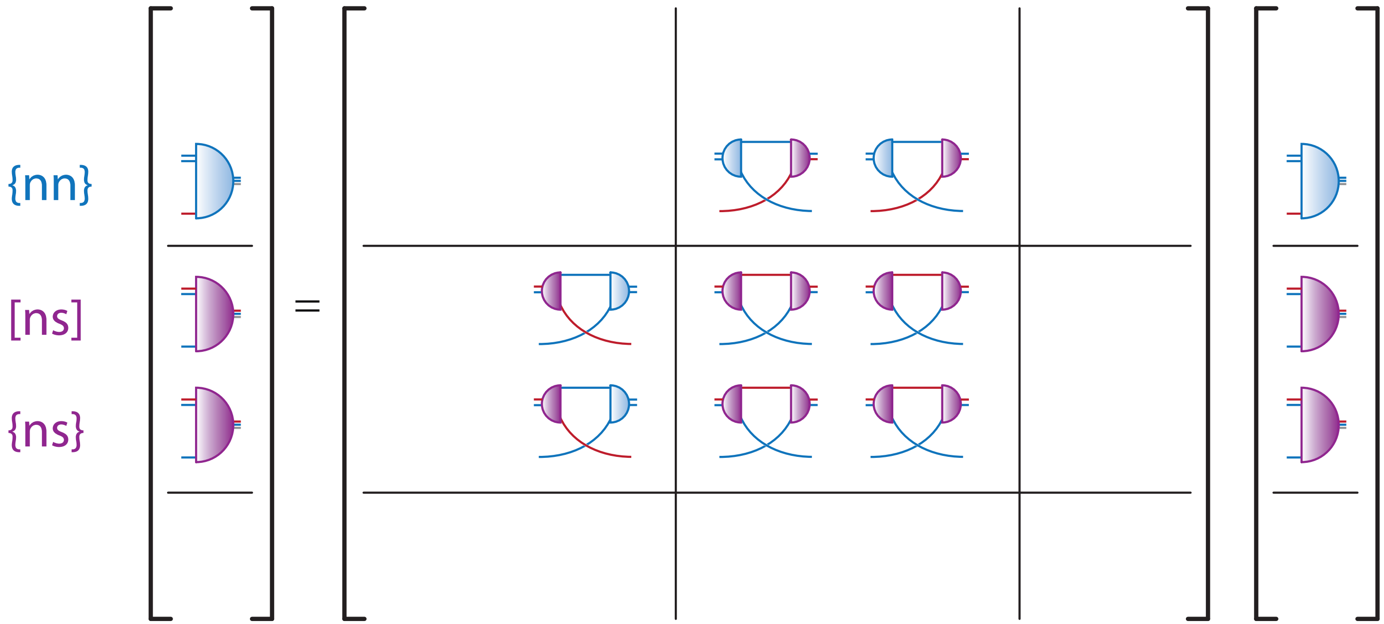
Omega

Strange baryons



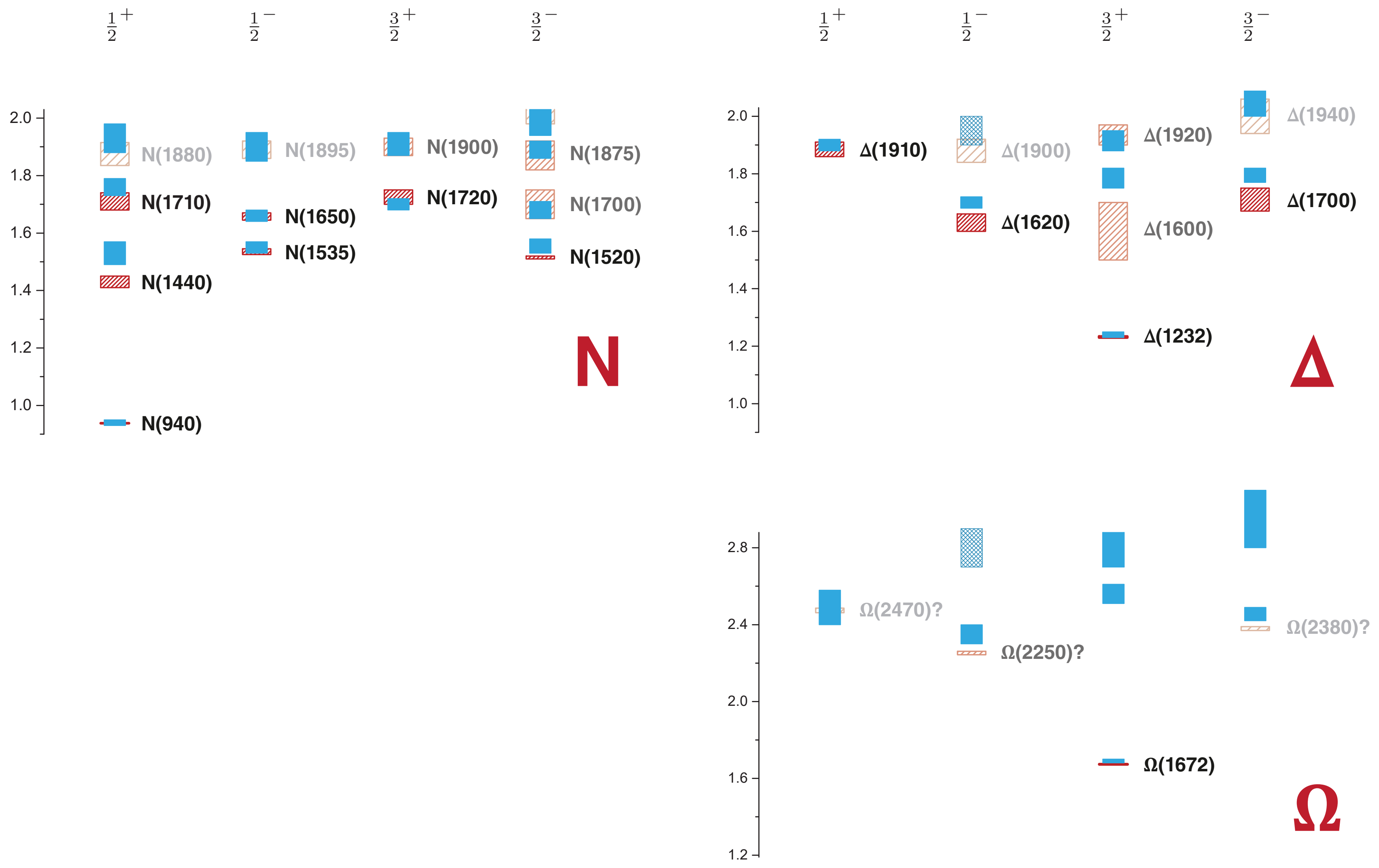
Lambda

Strange baryons

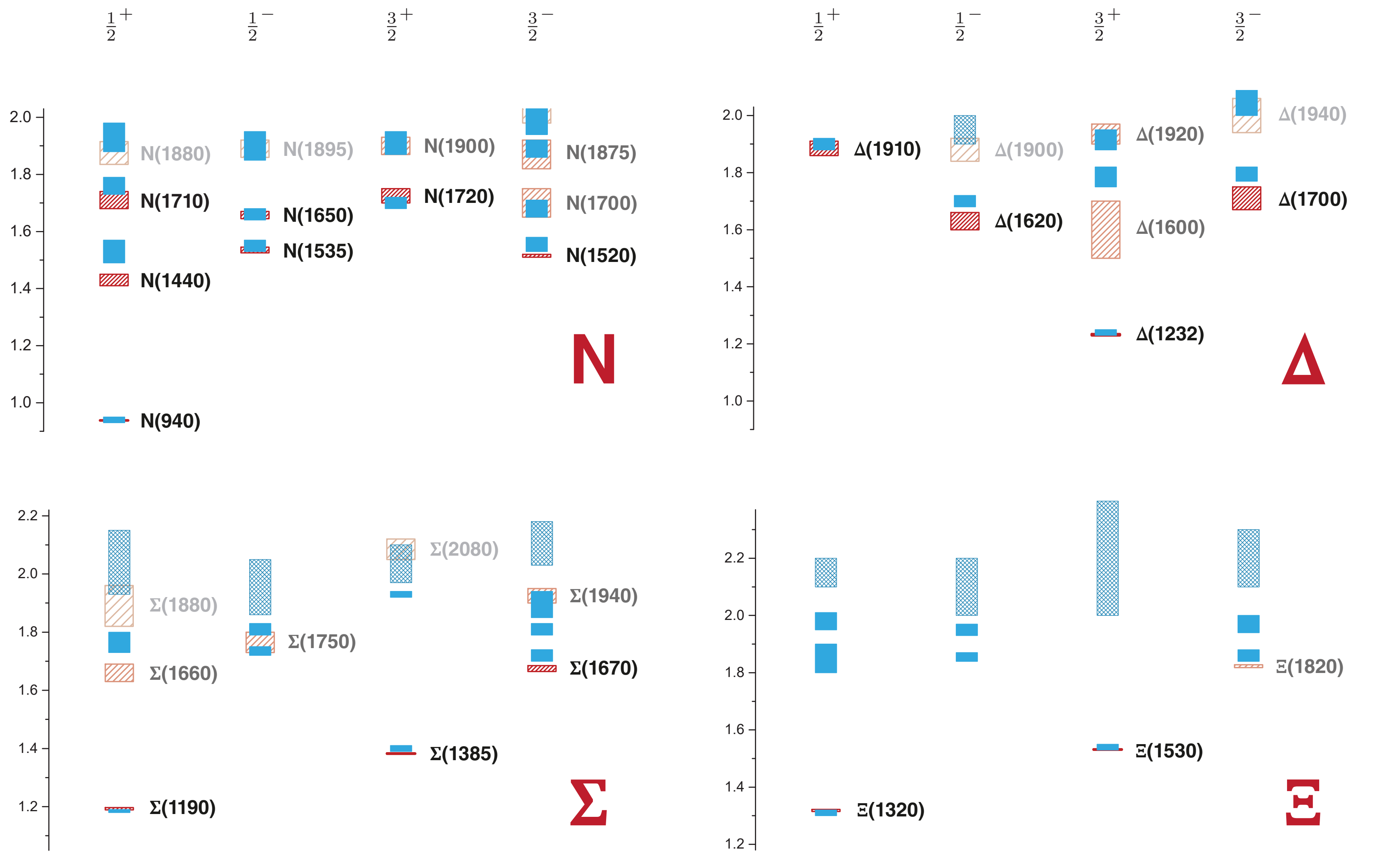


Sigma

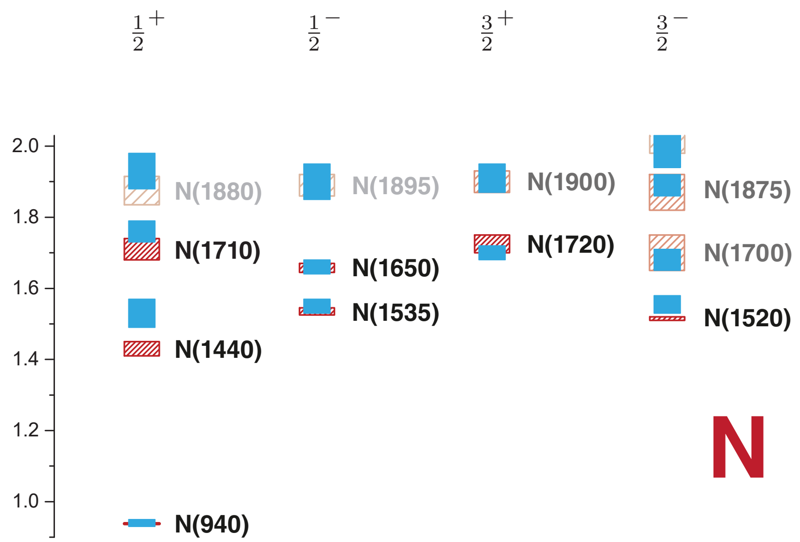
Strange baryons

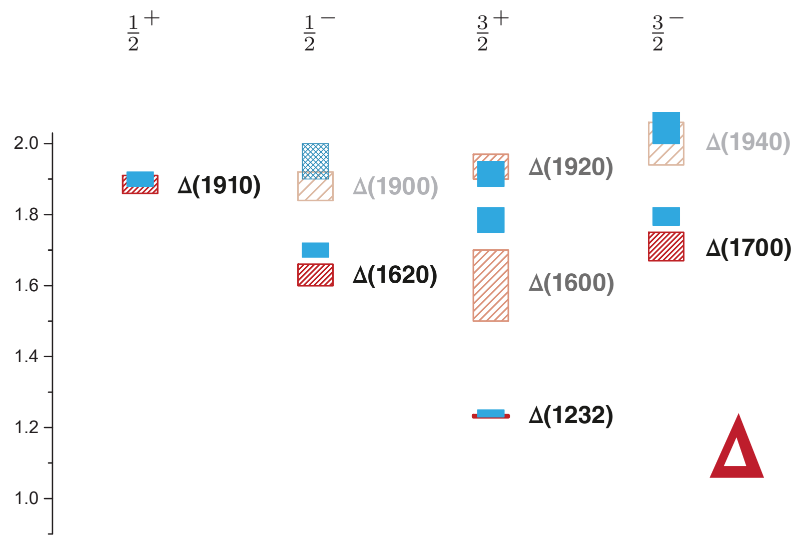


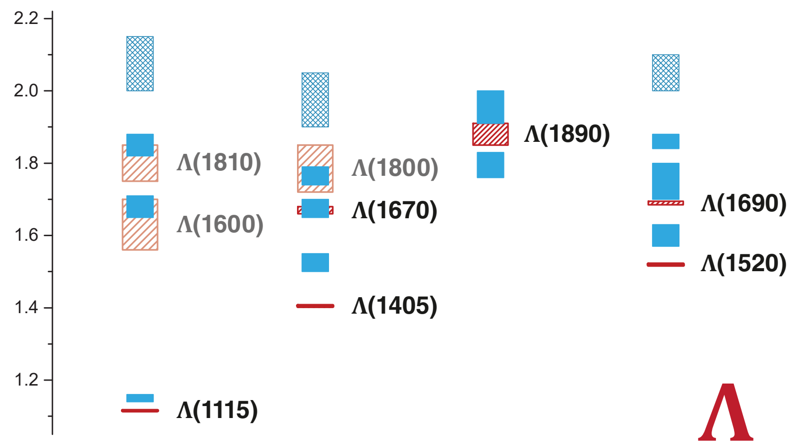
Strange baryons



Strange baryons







- Strange baryons similar to **light baryons:**

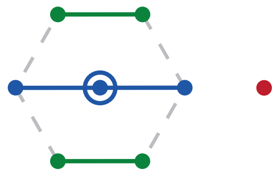
  $\mathbf{\Omega} \rightarrow \mathbf{\Delta}$
  $\mathbf{\Sigma}, \mathbf{\Xi} \rightarrow \mathbf{N} + \mathbf{\Delta}$ $\quad\rightarrow$ rich spectrum!
  $\mathbf{\Lambda} \rightarrow \mathbf{N}$ + singlets

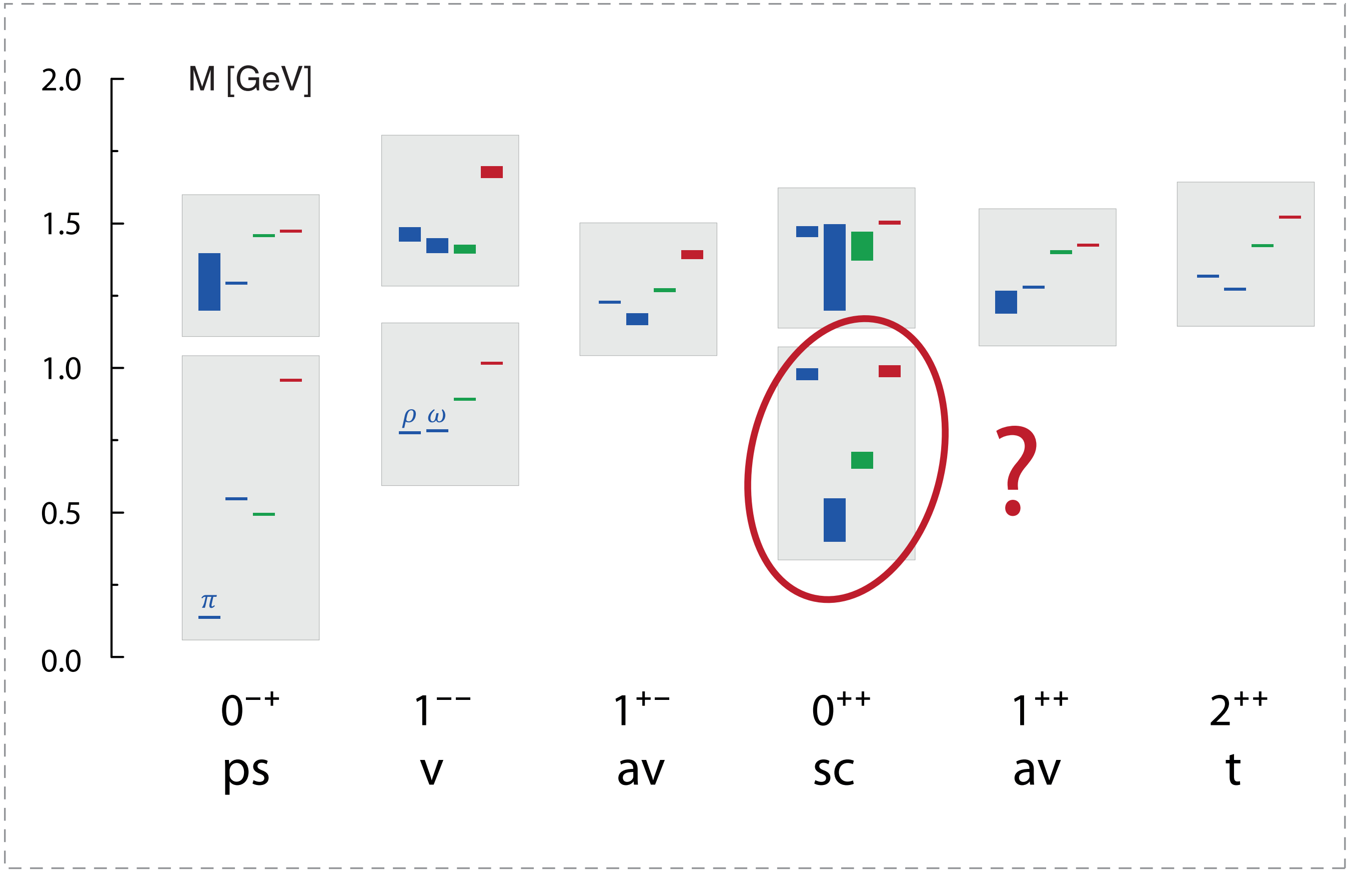
- Roper, $\mathbf{\Delta}$(1600), $\mathbf{\Lambda}$(1405), $\mathbf{\Lambda}$(1520):
  levels are there, but additional dynamics?

- **Structure information?**
  OAM, decays, form factors!

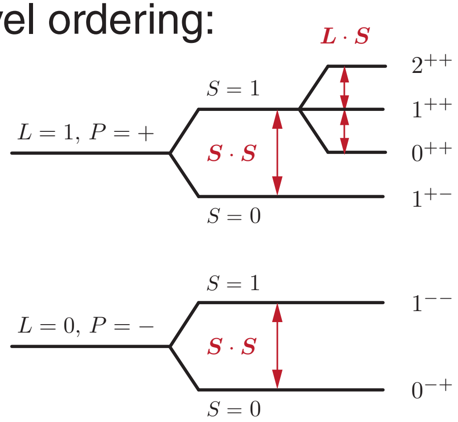
Tetraquarks?

Light meson spectrum (PDG):
grouped with J^{PC} and flavor content

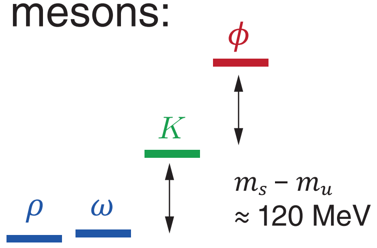




- Nonrelativistic level ordering:



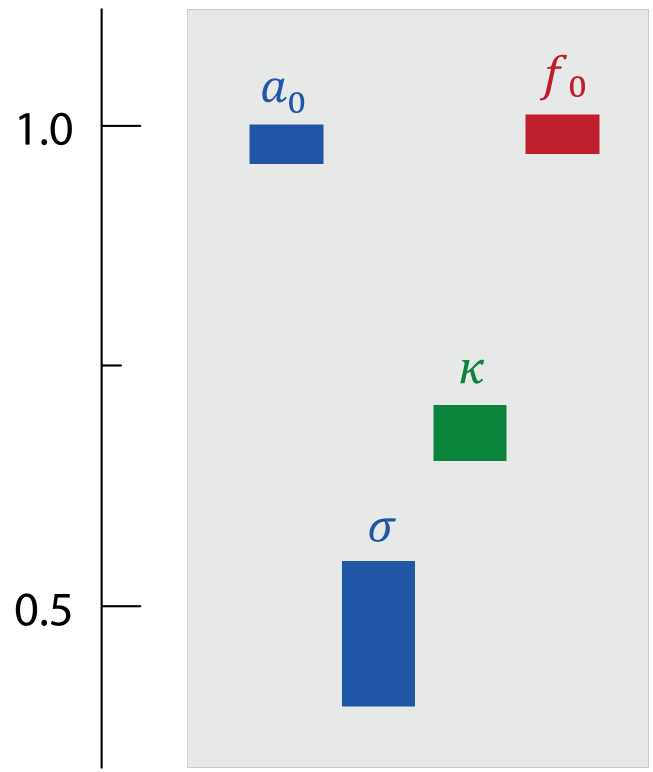
- Vector mesons:

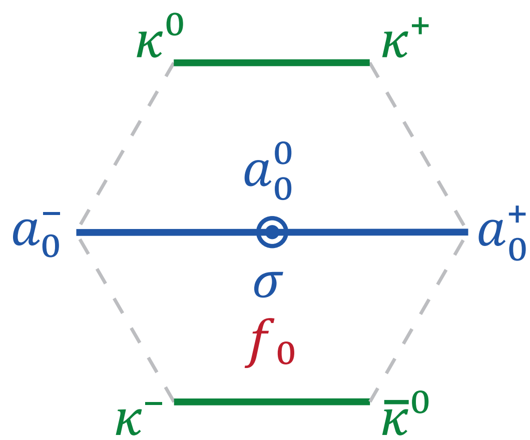


- **Pseudoscalar mesons?** spontaneous chiral symmetry breaking & axial anomaly
- **Scalar mesons?!**

Tetraquarks?

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





f_0 (980 MeV) $s\bar{s}$
κ (680 MeV) $u\bar{s}, d\bar{s}$
a_0 (980 MeV), σ (500 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$

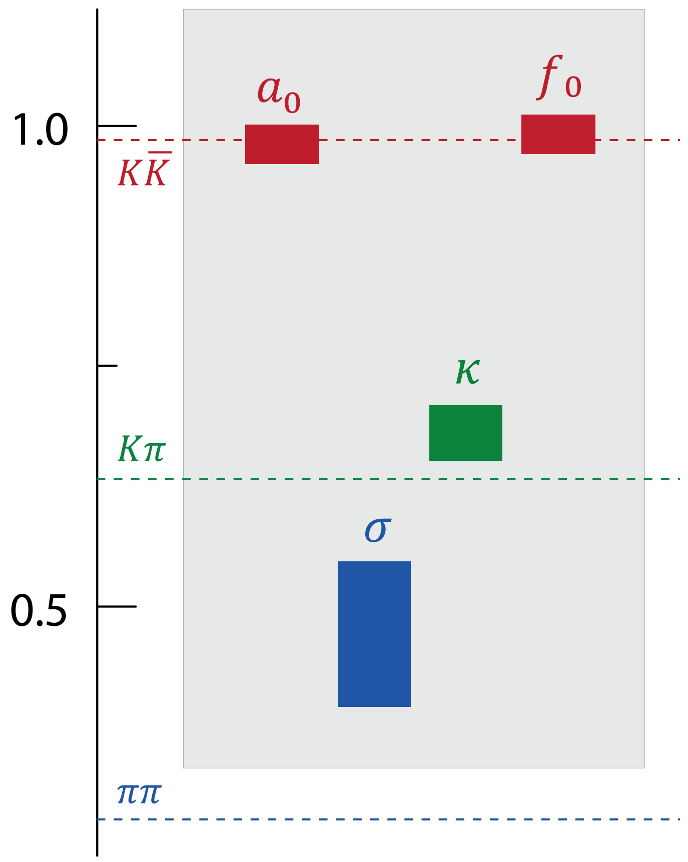
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

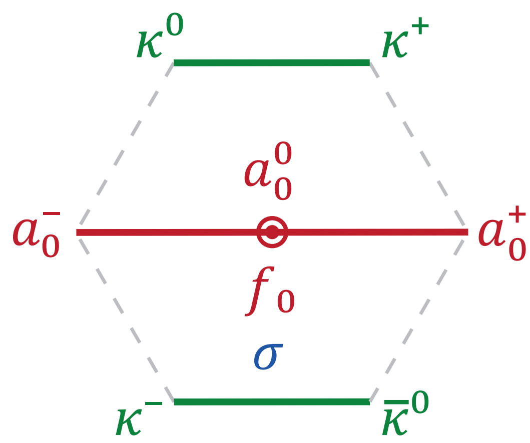
  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV

- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have
  masses similar to axialvector & tensor mesons ~ 1.3 GeV

Tetraquarks?

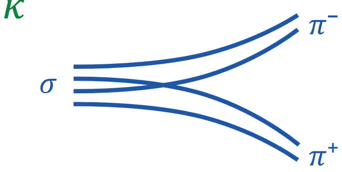
What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004





f_0 (980 MeV) } $us\bar{u}\bar{s}$, ...
a_0 (980 MeV)
κ (800 MeV) $us\bar{u}\bar{d}$, ...
σ (500 MeV) $ud\bar{u}\bar{d}$

- Explains **mass ordering** & **decay widths**:
  f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ

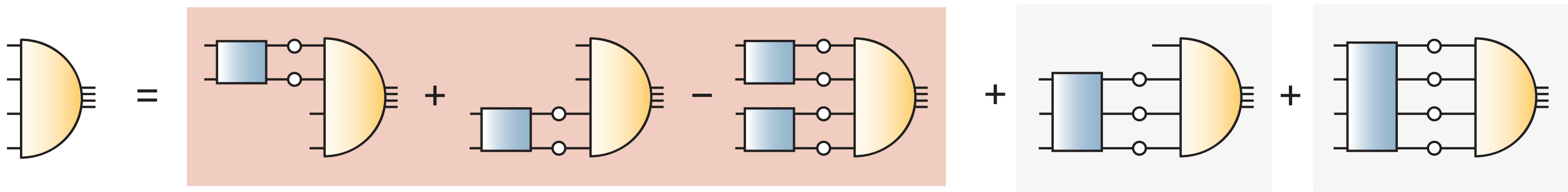


- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- Support for **non-q$\bar{\text{q}}$ nature** of σ from dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models
  Pelaez 2004, Weinberg 2013, Cohen, Llanes-Estrada, Pelaez, Ruiz de Elvira 2014, Londergan, Nebreda, Pelaez, Szczepaniak 2014, Parganlija, Giacosa, Rischke 2010, . . .

Tetraquarks: four-body equation

Four-quark bound-state equation:



Two-body interactions | **3-body** (+ permutations) | **4-body**

- plus permutations:

  $(qq)(\color{blue}{\bar{q}\bar{q}}\color{black}), \ (q\color{blue}{\bar{q}}\color{black})(q\color{blue}{\bar{q}}\color{black}), \ (q\color{blue}{\bar{q}}\color{black})(q\color{blue}{\bar{q}}\color{black})$

  (12) (34) (23) (14) (13) (24)

- $K \otimes I + I \otimes K - K \otimes K$ prevents overcounting in T-matrix $T = K + K\,G_0\,T$, avoids Van-der-Waals forces

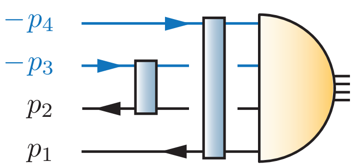
  Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Keep **two-body interactions** with **rainbow-ladder kernel:** well motivated by meson & baryon studies

Structure of the amplitude

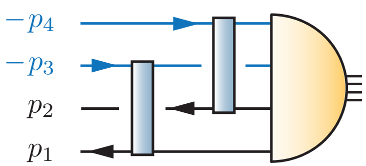
4-quark Bethe-Salpeter amplitude $\Gamma(p,q,k,P)$:
one total momentum & three relative momenta:

$$P = p_1 + p_2 + p_3 + p_4$$



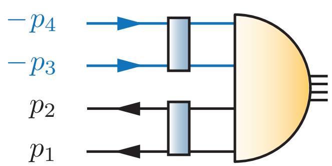
$$p = \tfrac{1}{2}\,(p_2 + p_3 - p_1 - p_4)$$

's channel'



$$q = \tfrac{1}{2}\,(p_3 + p_1 - p_2 - p_4)$$

'u channel'



$$k = \tfrac{1}{2}\,(p_1 + p_2 - p_3 - p_4)$$

't channel'

General structure:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2$$

$$\begin{aligned} \omega_1 &= q\cdot k & \eta_1 &= p\cdot P \\ \omega_2 &= p\cdot k & \eta_2 &= q\cdot P \\ \omega_3 &= p\cdot q & \eta_3 &= k\cdot P \end{aligned}$$

$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\overline{3}, \quad 6\otimes\overline{6}$ or

$1\otimes 1, \quad 8\otimes 8$

(Fierz-equivalent)

Structure of the amplitude

4-quark Bethe-Salpeter amplitude $\Gamma(p,q,k,P)$:
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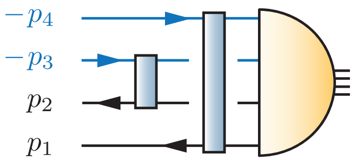
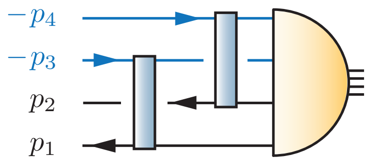
$$P = p_1 + p_2 + p_3 + p_4$$



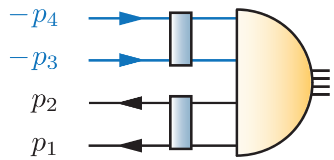
$$p = \frac{1}{2}\,(p_2 + p_3 - p_1 - p_4)$$

‘s channel’



$$q = \frac{1}{2}\,(p_3 + p_1 - p_2 - p_4)$$

‘u channel’



$$k = \frac{1}{2}\,(p_1 + p_2 - p_3 - p_4)$$

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$\otimes$ Color $\otimes$ Flavor

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(Fierz-equivalent)

Kaeding, nucl-th/9502037

Structure of the amplitude

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$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

Keep **s waves** only:
16 Dirac-Lorentz tensors,
Fierz-complete

e.g. $\left\{\begin{array}{c} C^T\gamma_5\otimes\gamma_5 C \\ C^T\gamma^\mu\otimes\gamma^\mu C \\ \dots \end{array}\right\}$ in (12)(34)

automatically includes also
$\gamma_5\otimes\gamma_5$ in (23)(14), (31)(24)

Structure of the amplitude

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P)$$

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$$p^2, \quad q^2, \quad k^2$$

$$\begin{aligned} \omega_1 &= q\cdot k & \eta_1 &= p\cdot P \\ \omega_2 &= p\cdot k & \eta_2 &= q\cdot P \\ \omega_3 &= p\cdot q & \eta_3 &= k\cdot P \end{aligned}$$

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automatically includes also
$\gamma_5 \otimes \gamma_5$ in (23)(14), (31)(24)

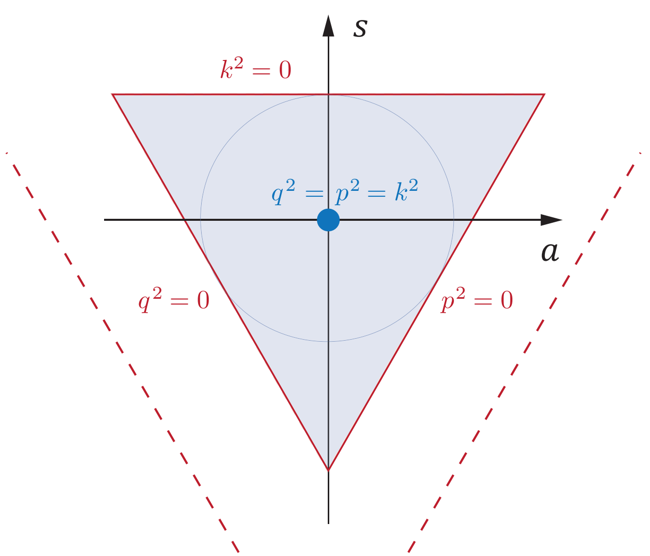
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \tfrac{1}{4}\,(p^2+q^2+k^2)$$

- **Doublet:** $\mathcal{D}_0 = \dfrac{1}{4\mathcal{S}_0}\begin{bmatrix} \sqrt{3}\,(q^2-p^2) \\ p^2+q^2-2k^2 \end{bmatrix}$

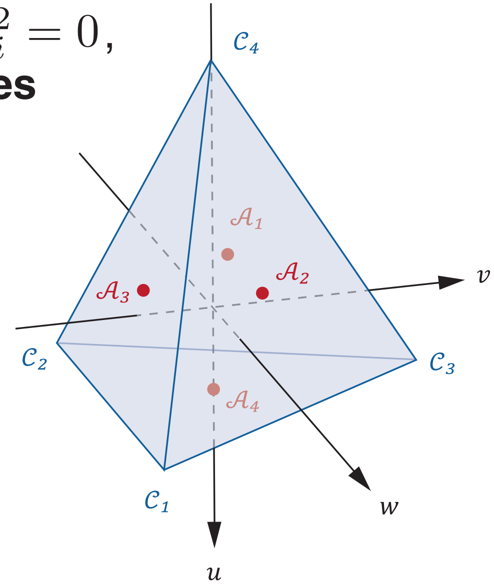
  Mandelstam triangle,
  outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \dfrac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\omega_1+\omega_2+\omega_3) \\ \sqrt{2}\,(\omega_1+\omega_2-2\omega_3) \\ \sqrt{6}\,(\omega_2-\omega_1) \end{bmatrix}$

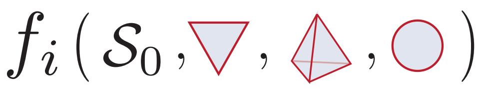
  tetrahedron bounded by $p_i^2=0$,
  outside: **quark singularities**



- Second **triplet:**
  3dim. sphere

$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\eta_1+\eta_2+\eta_3) \\ \sqrt{2}\,(\eta_1+\eta_2-2\eta_3) \\ \sqrt{6}\,(\eta_2-\eta_1) \end{bmatrix}$$

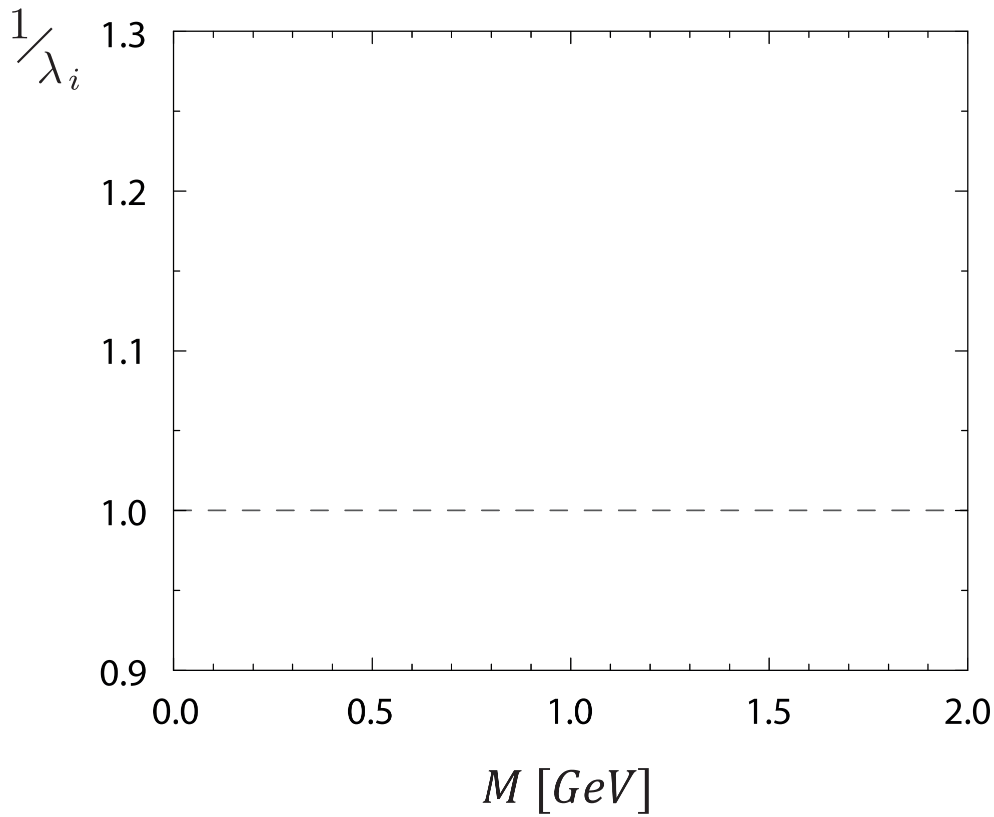
Tetraquark mass



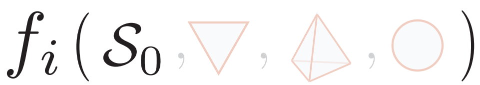
Idea: use symmetries to figure out **relevant** momentum dependence

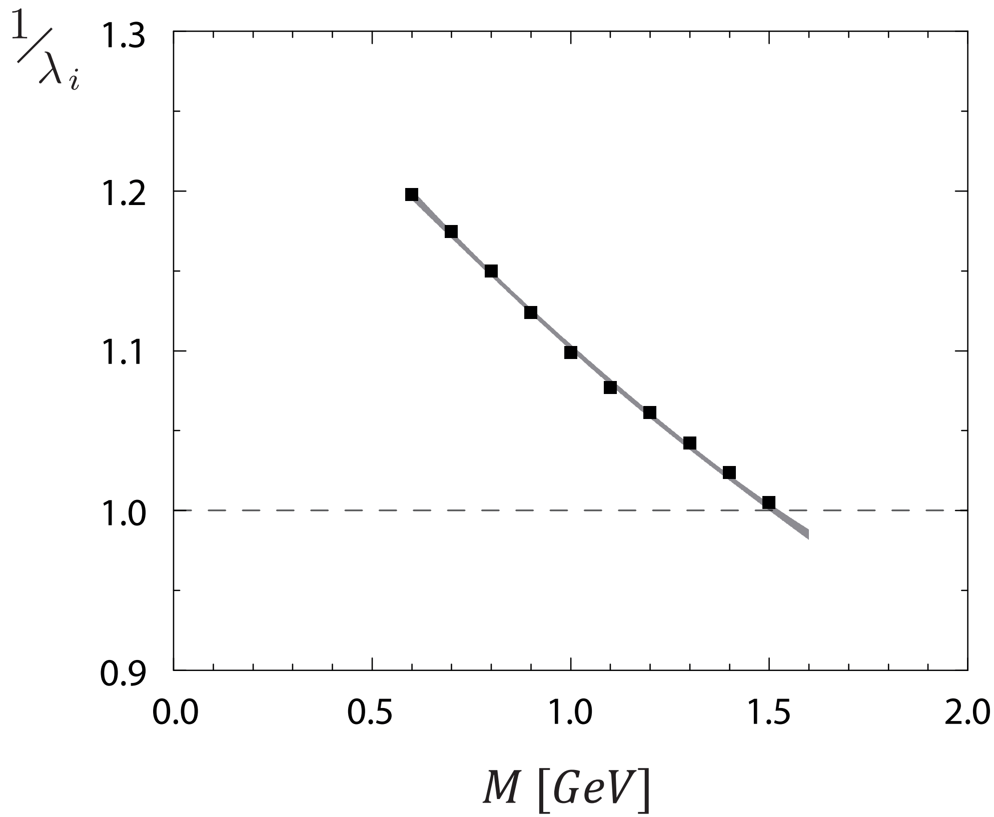
similar:

- **Three-gluon vertex**
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)
- HLbL scattering for **muon g-2**
  GE, Fischer, Heupel, PRD 92 (2015)



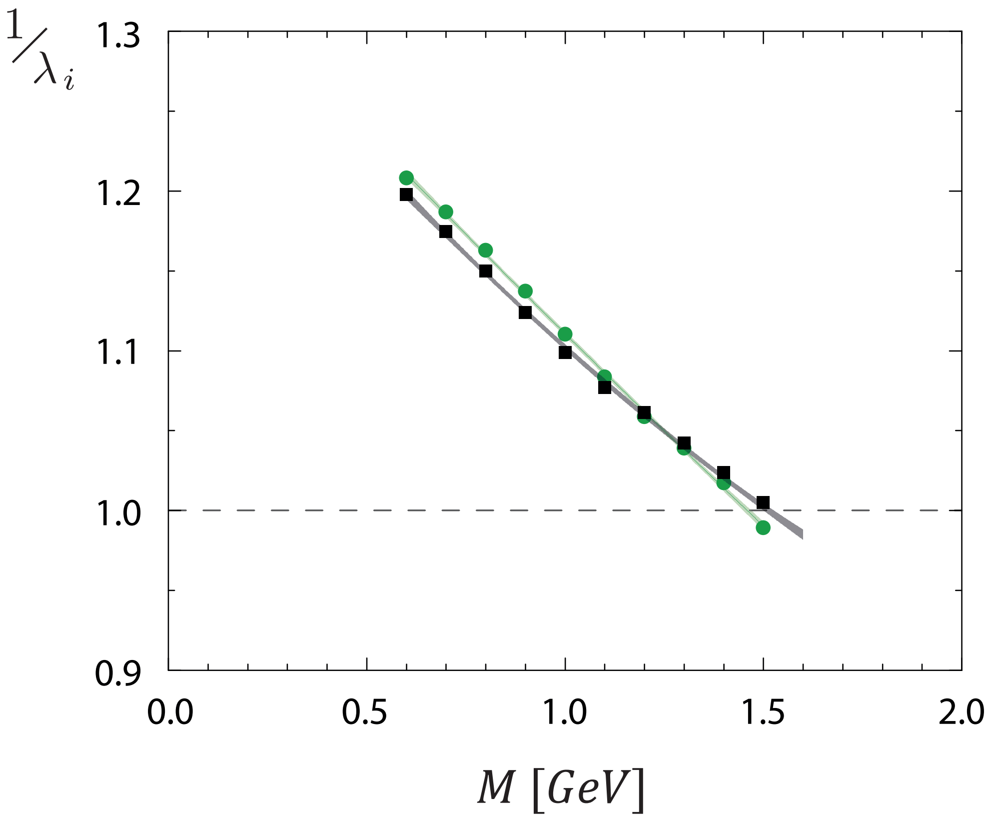
Tetraquark mass





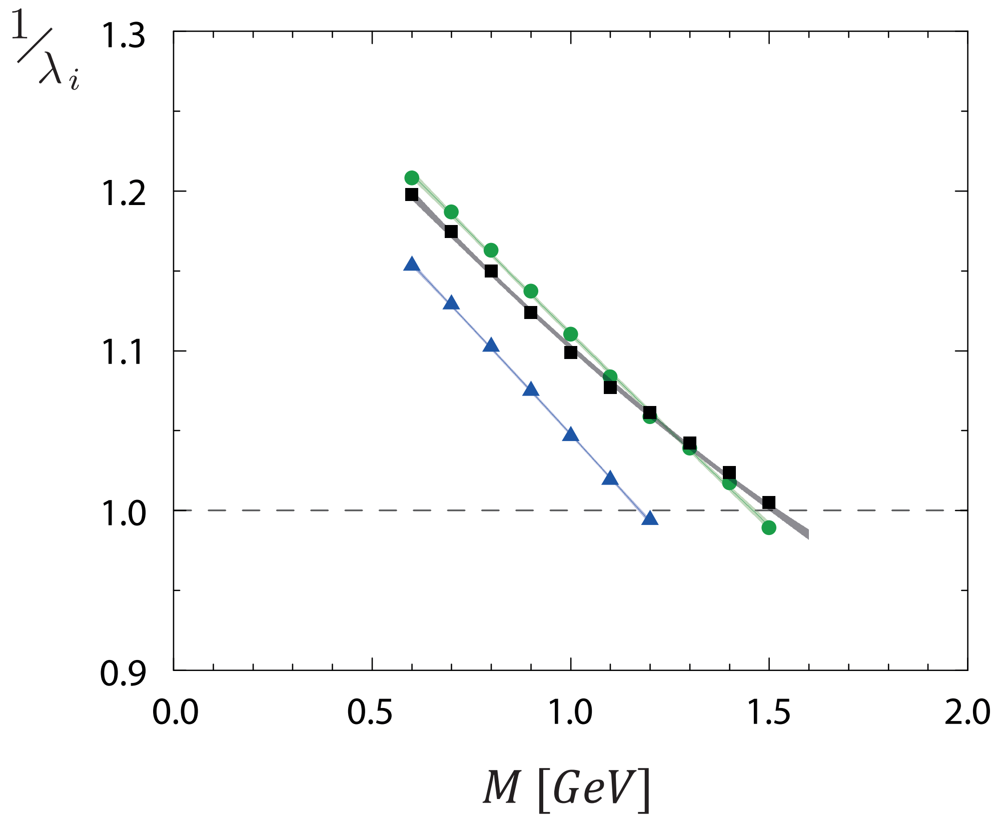
$$K_{ij}\, f_j^{(n)} = \lambda_n(P^2)\, f_i^{(n)} \qquad \lambda_n \xrightarrow{P^2 \to -M_n^2} 1$$

Tetraquark mass



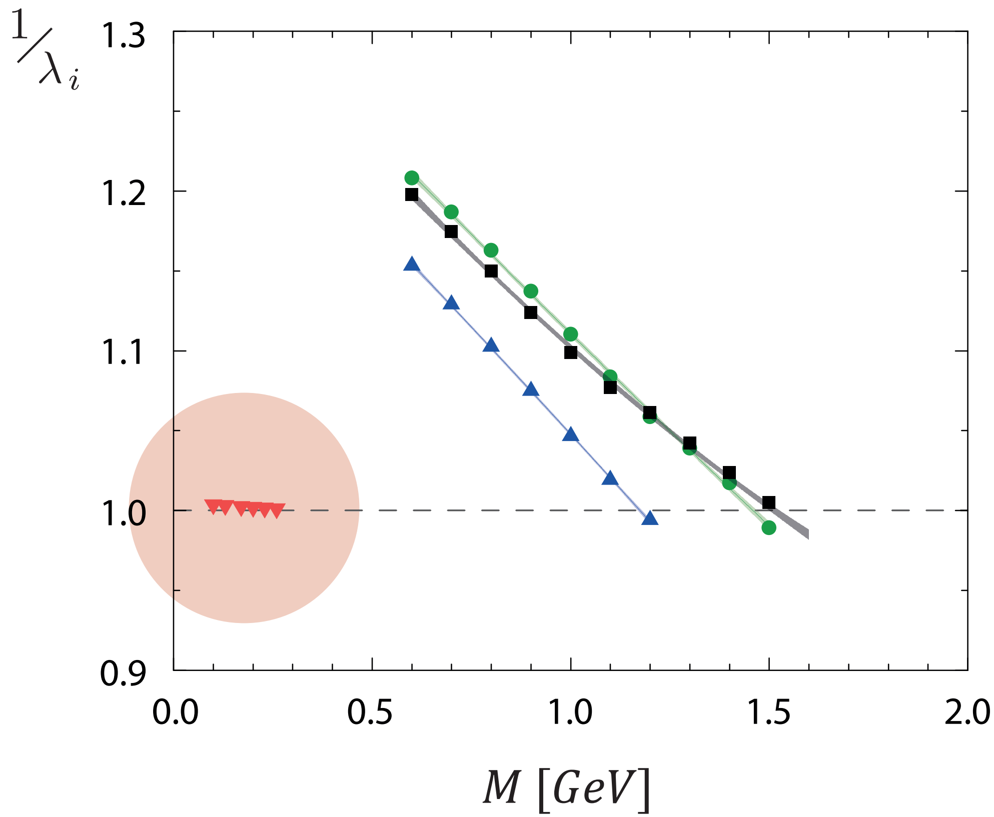
Tetraquark mass

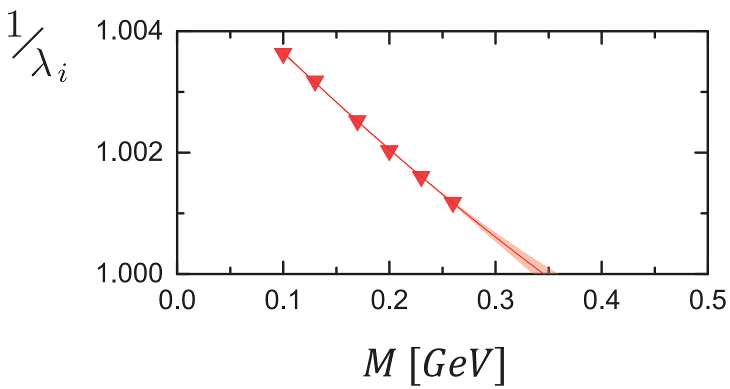
$$f_i\,(\,\mathcal{S}_0\,, \triangledown\,, \triangle\,, \circ\,)$$



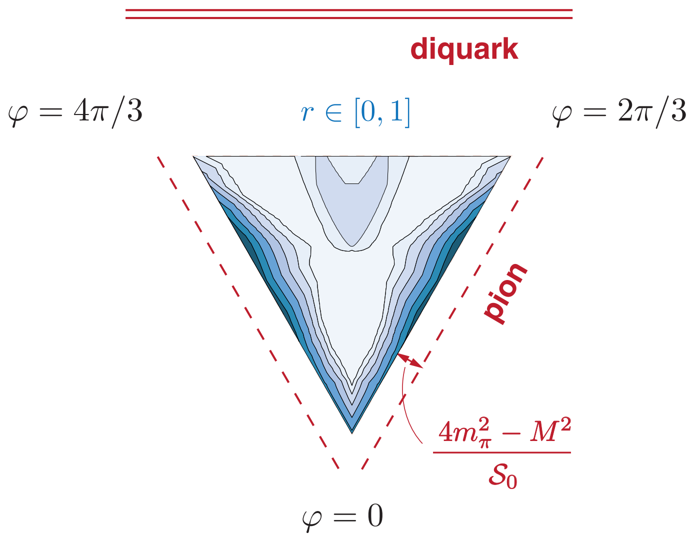
Tetraquark mass

$$f_i\,(\,\mathcal{S}_0\,, \bigtriangledown\,, \triangle\,, \bigcirc\,)$$

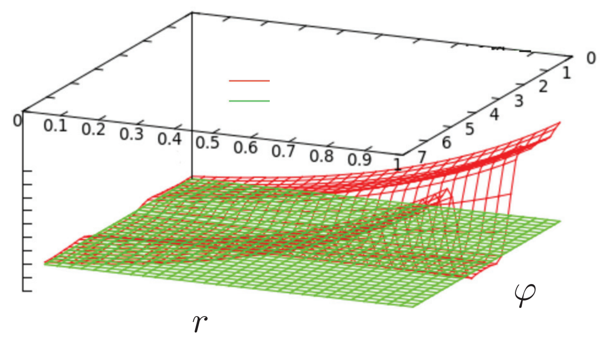




Tetraquark mass

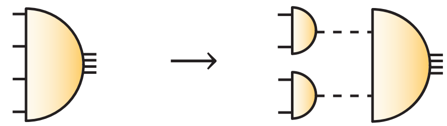


Gap in Mandelstam triangle due to **pion poles!**



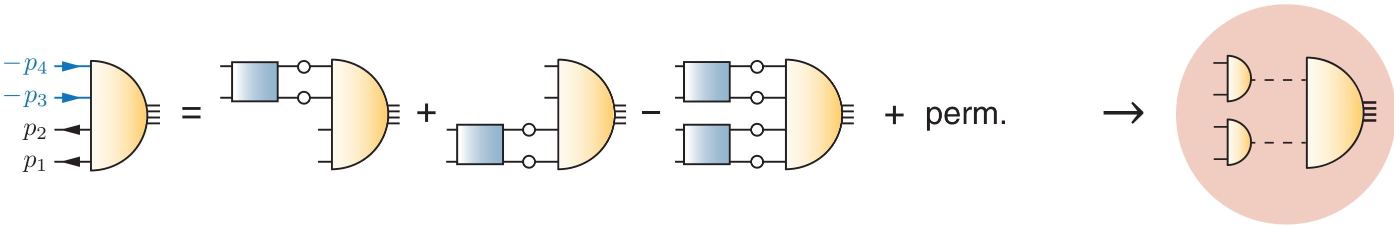
- Four-quark BSE dynamically generates pion poles in BS amplitude, although **equation knows nothing about pions!**
- drive σ mass from 1.5 GeV to ~350 MeV
  $\Rightarrow$ light tetraquarks are indirect consequence of SχSB
- **Poles enter integration domain** above threshold $M > 2m_\pi$: the tetraquark becomes a **resonance**

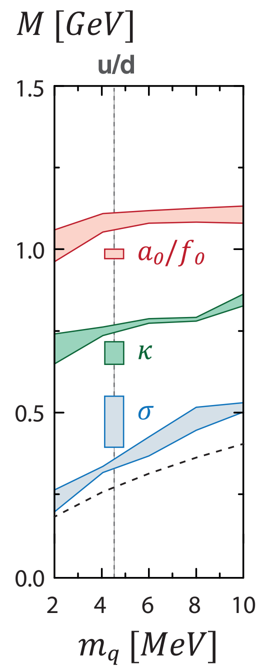
- Four quarks rearrange to **"meson molecule"**



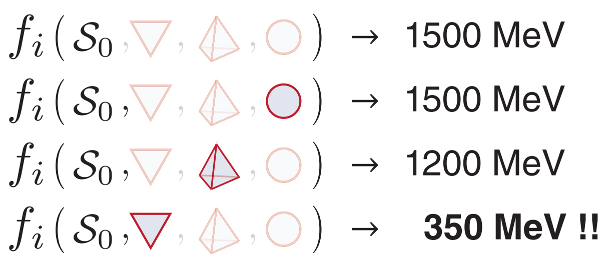
Tetraquarks

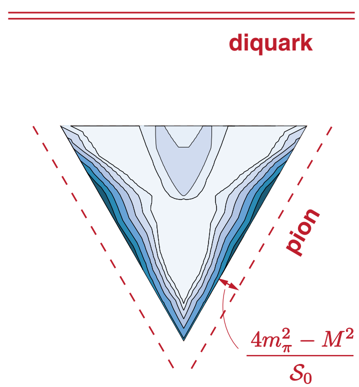
- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)





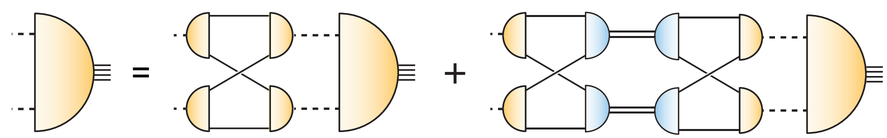
BSE dynamically generates **meson poles** in wave function:



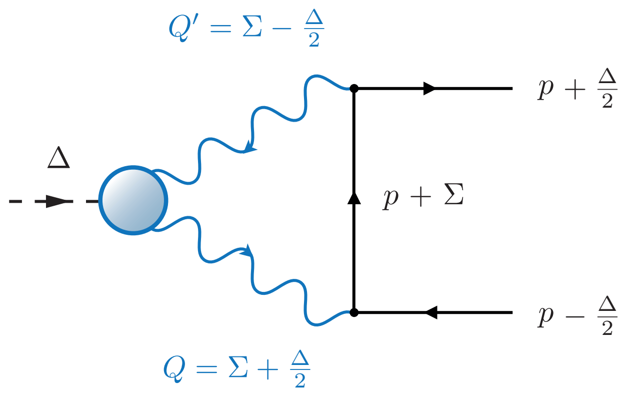


Four quarks rearrange to "**meson molecule**"

- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)



Rare pion decay $\pi^0 \to e^+e^-$



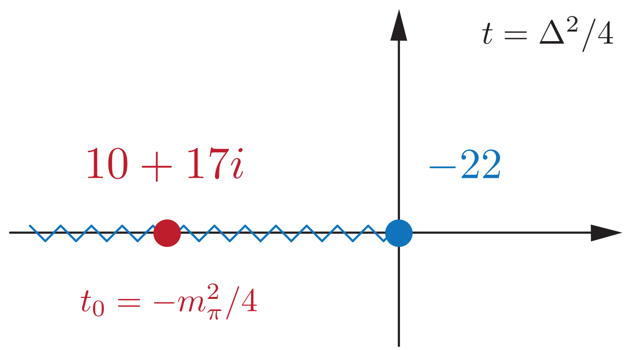
- After reanalysis of radiative corrections still 2σ discrepancy in branching ratio between exp and theory:

$6.87(36) \times 10^{-8}$ — **KTeV Collab.:** Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)

$6.23(09) \times 10^{-8}$ — Dorokhov, JETP Lett. 91 (2010), Masjuan, Sanchez-Puertas, 1504.07001

- Depends on **pion transition FF** as input:

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2}$$

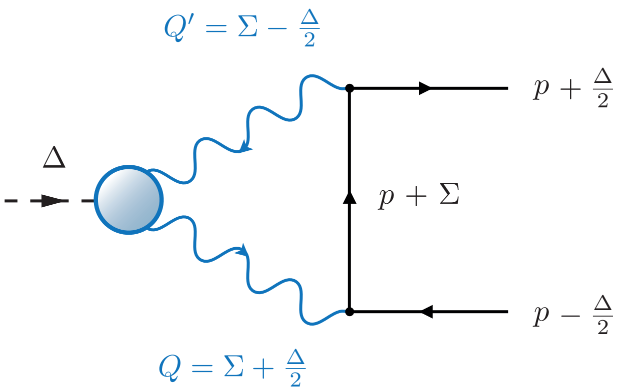


- cannot be calculated directly in Euclidean kinematics because of **photon and lepton poles**

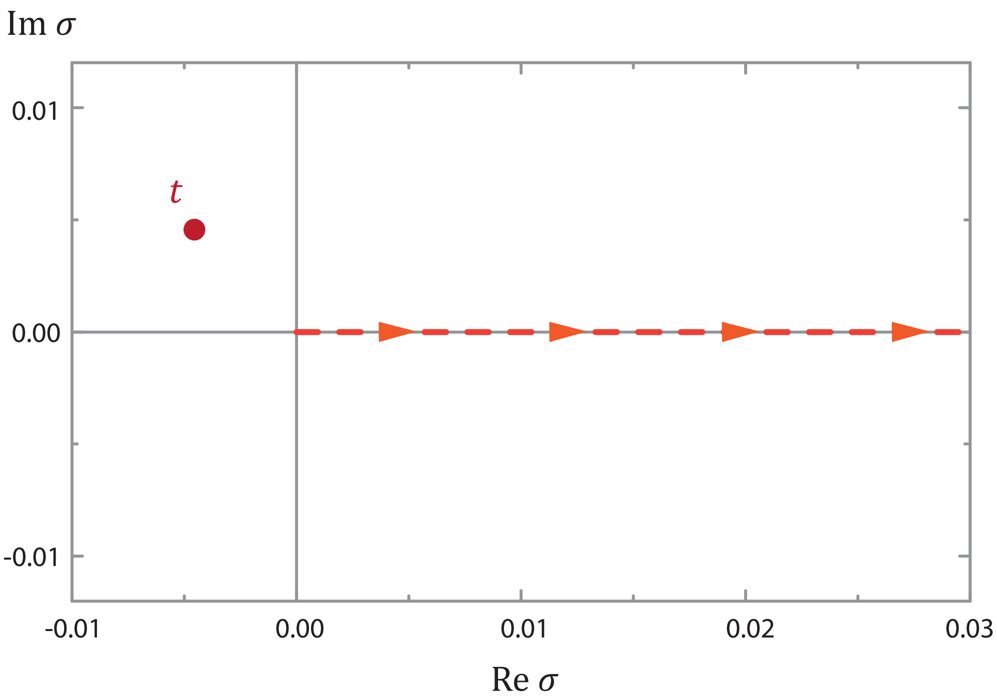
- workaround with dispersion relations:

$$\text{Im}\, \mathcal{A}^{\text{LO}}(t) = \frac{\pi \, \ln \gamma(t)}{2\beta(t)} F(0,0) \quad \Rightarrow \quad \text{Re}\, \mathcal{A}(t) = \mathcal{A}(0) + \frac{\ln^2 \gamma(t) + \frac{1}{3}\pi^2 + 4\, \text{Li}_2(-\gamma(t))}{4\beta(t)}$$

Rare pion decay $\pi^0 \to e^+ e^-$



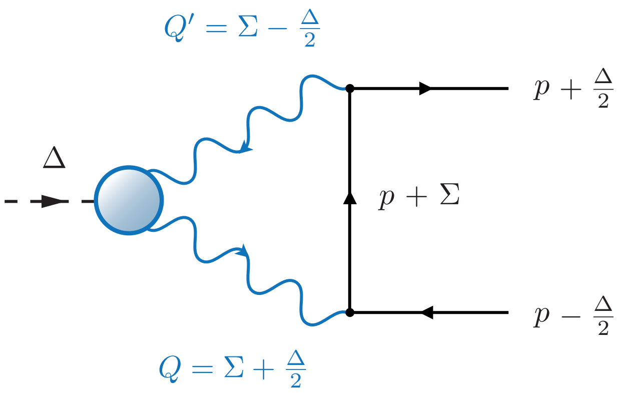
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2}$$



Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$

Rare pion decay $\pi^0 \to e^+e^-$



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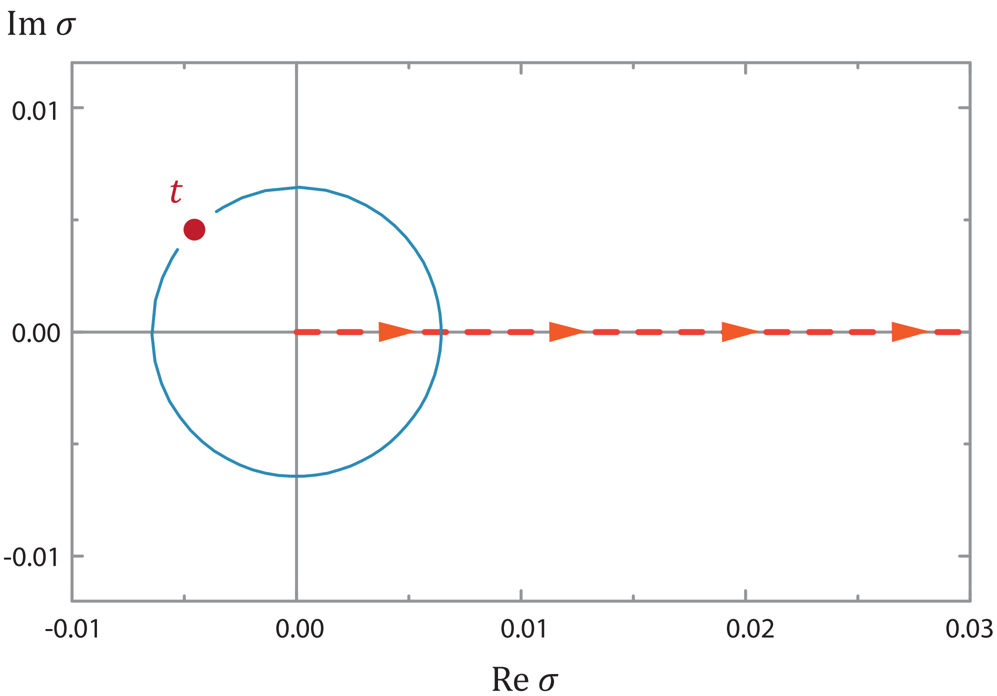

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

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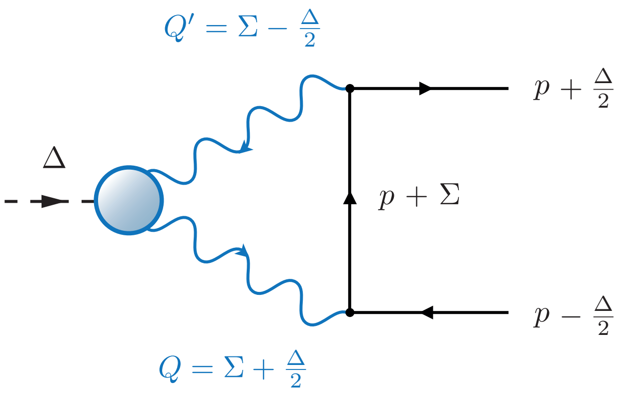

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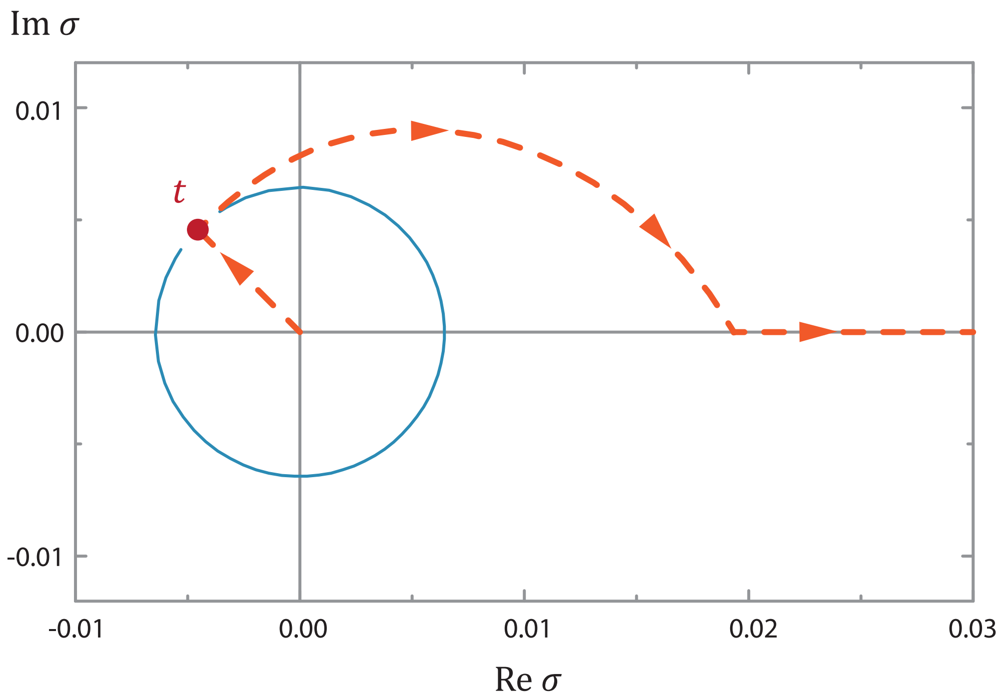

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

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- deform integration contour: cut opens at t

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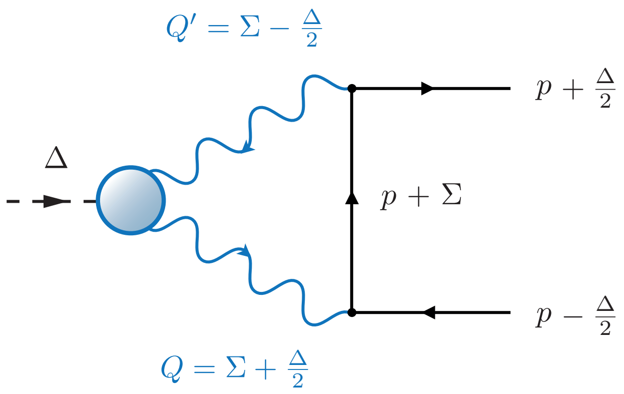

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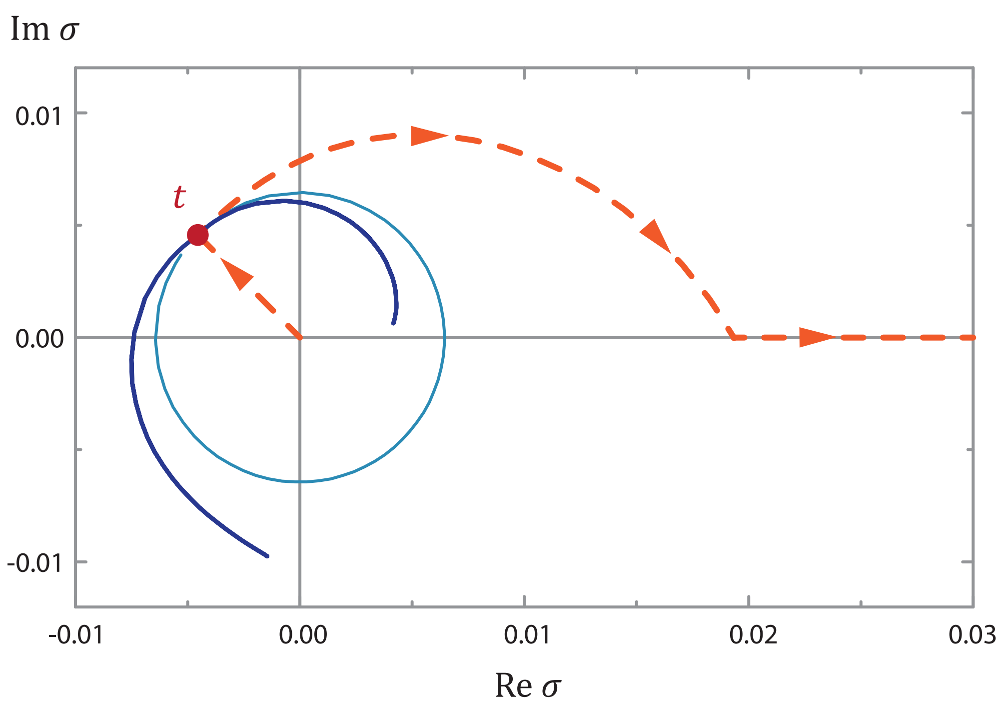

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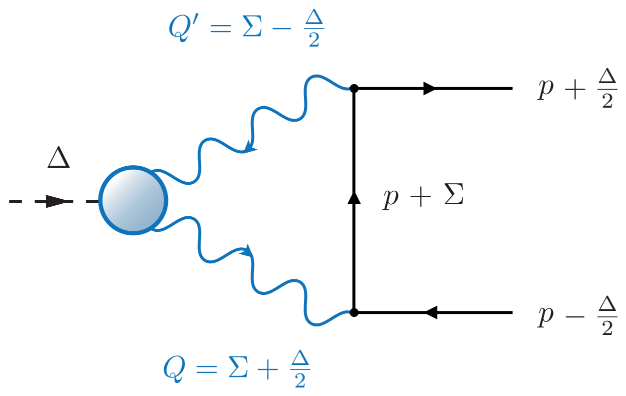

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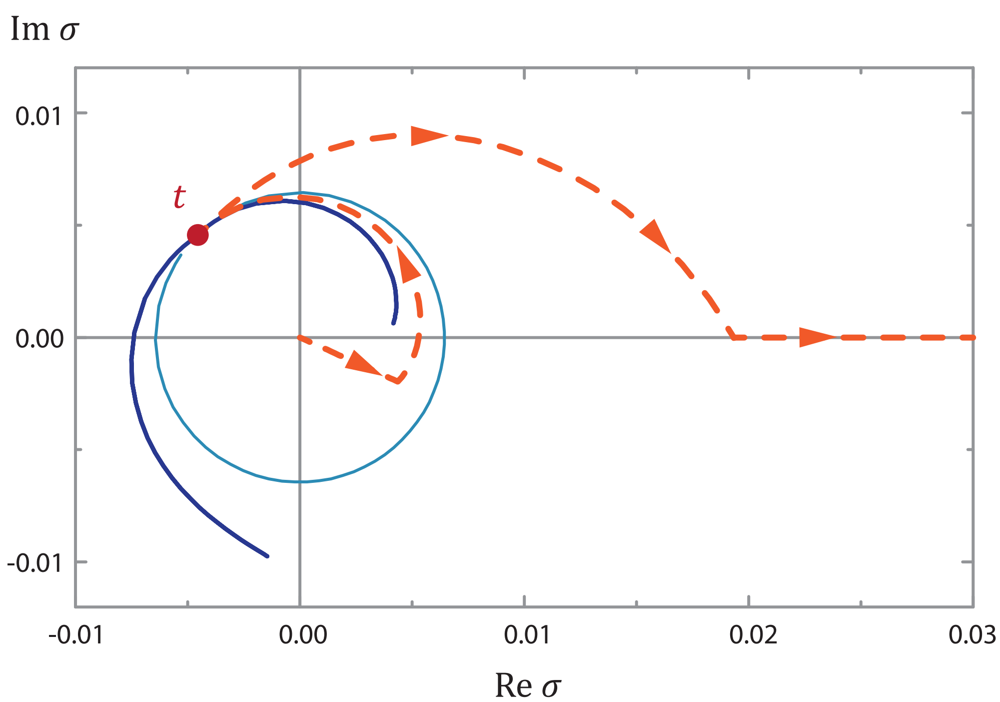
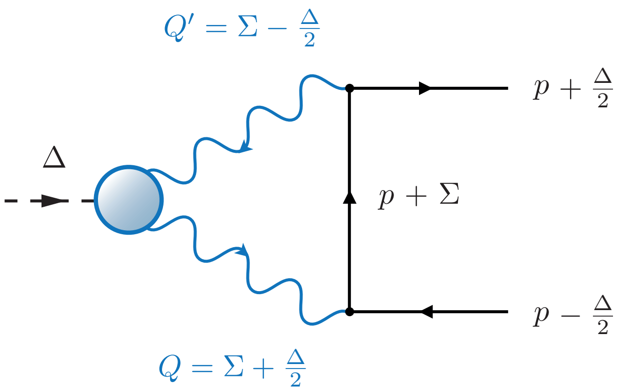


Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at t
- but lepton cut does **not** open at t!
- **deform contour** such that it never crosses any cut!

Rare pion decay $\pi^0 \to e^+e^-$



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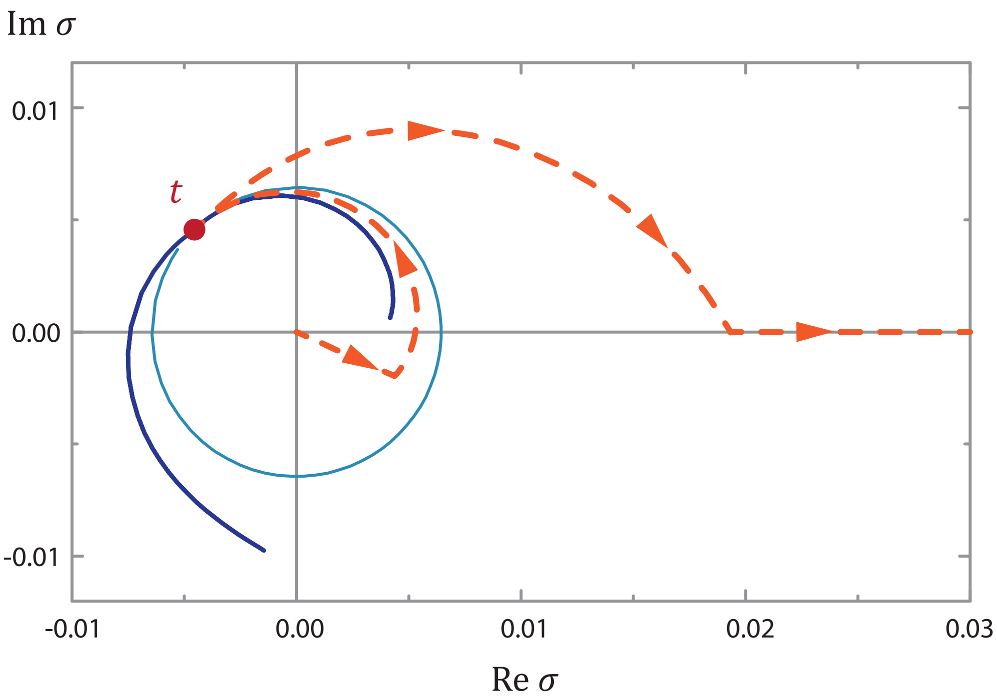

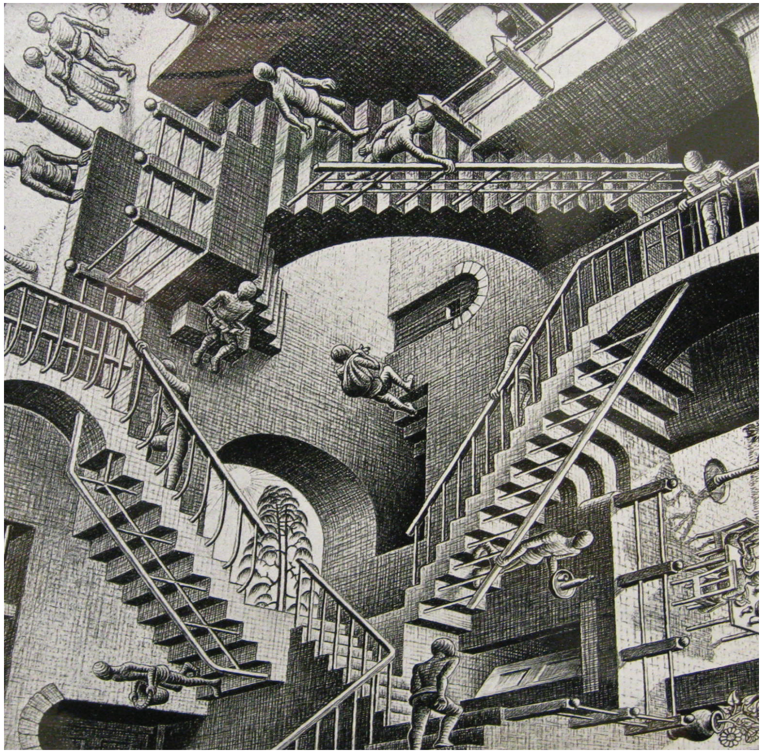

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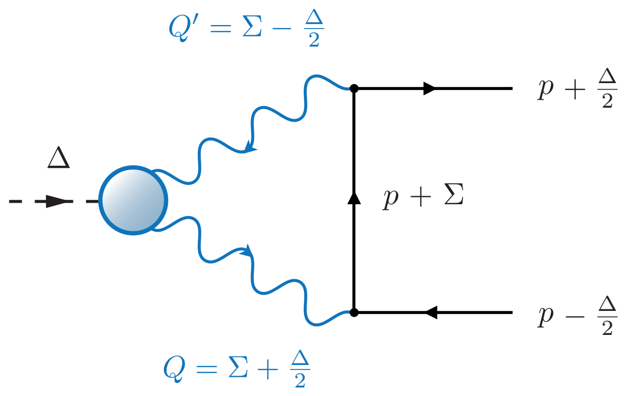

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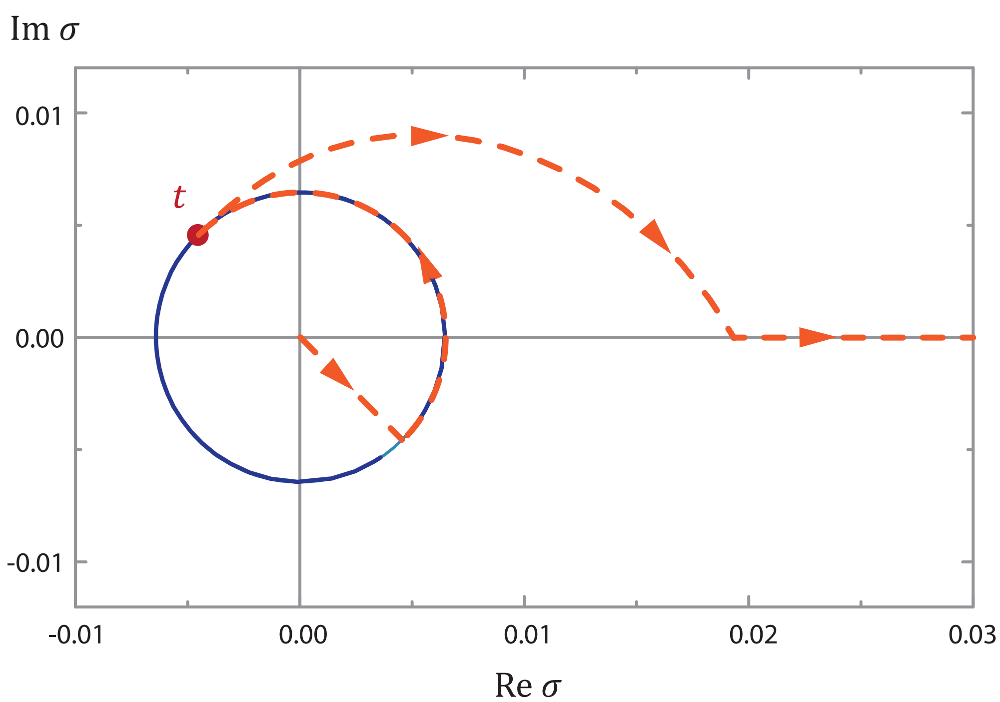

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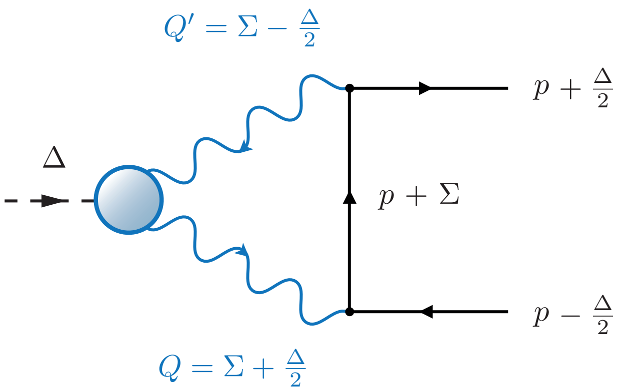

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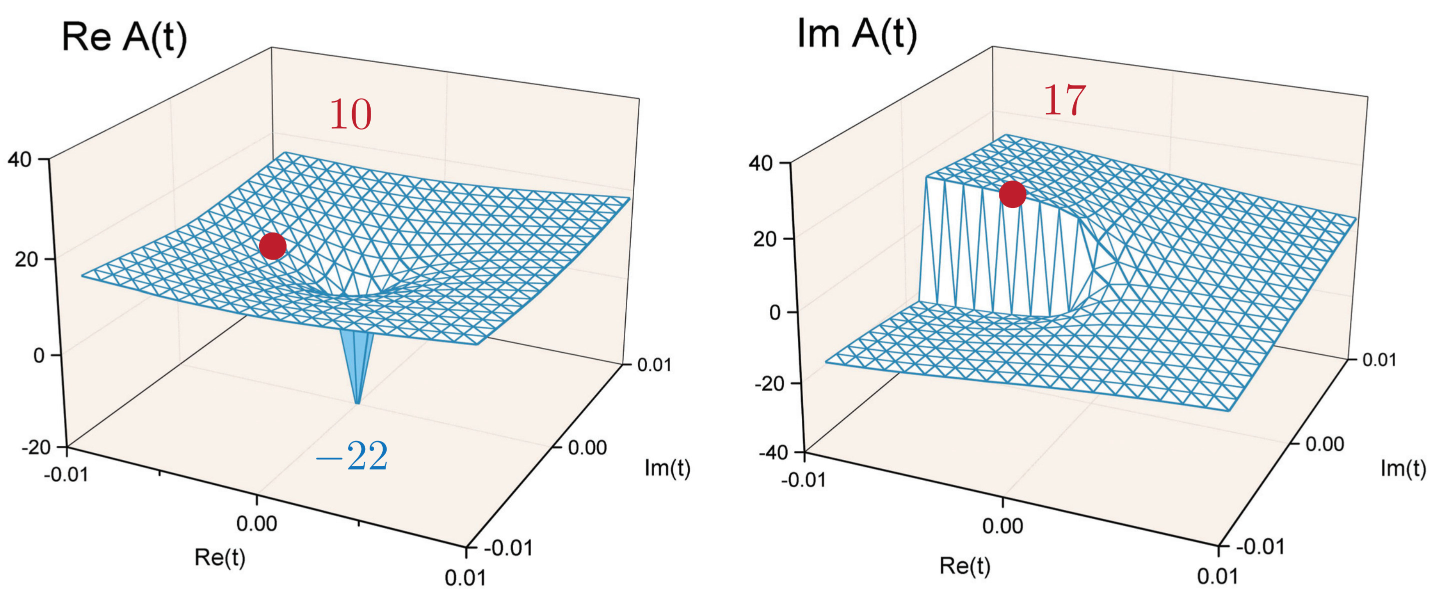
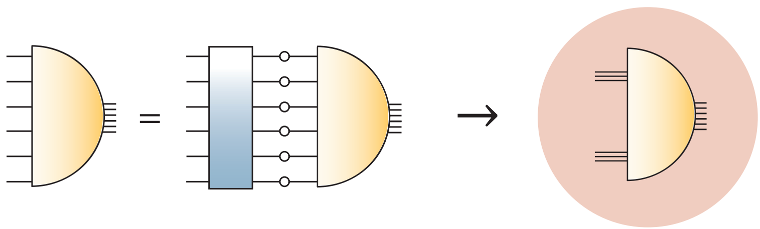


- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known
- Useful for treating **resonances!**

Weil, GE, Fischer, Williams, PRD 96 (2017)

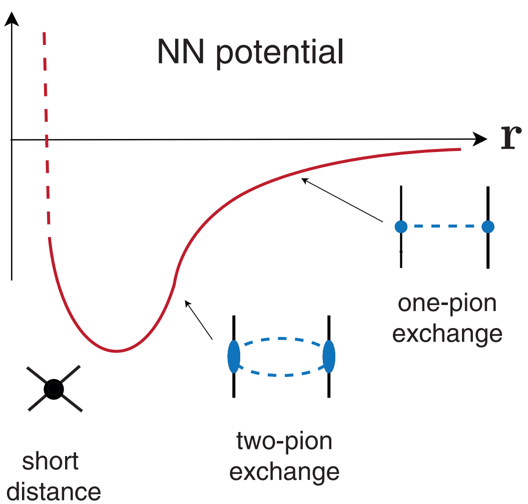
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

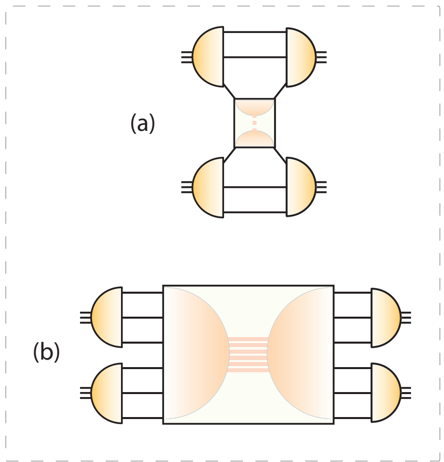


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



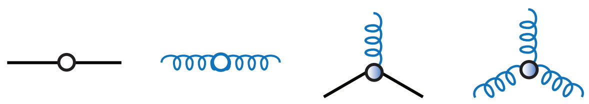
Weise, Nucl. Phys. A805 (2008)

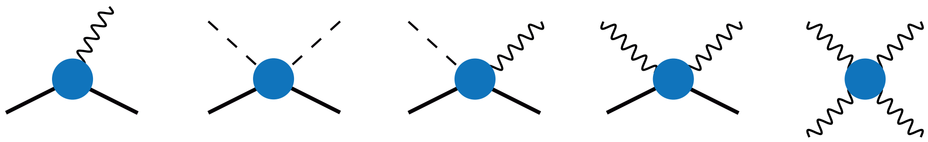


- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Hadron physics with functional methods

Understand properties of **elementary n-point functions** ↔ Calculate hadronic **observables**: mass spectra, form factors, scattering amplitudes, . . .





- **QCD**
- **symmetries** intact (Poincare invariance & chiral symmetry important)
- access to all momentum scales & all quark masses
- compute mesons, baryons, tetraquarks, . . . **from same dynamics**

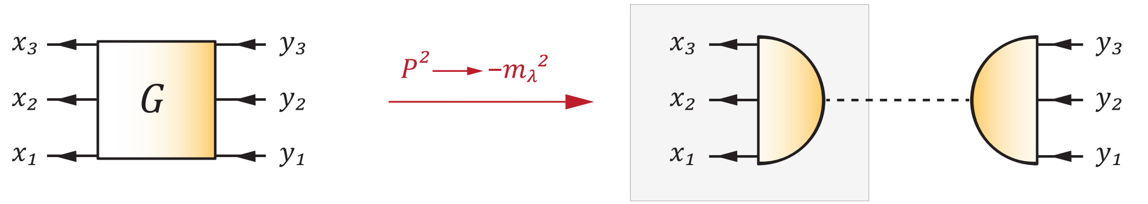
- **systematic** construction of truncations
- technical challenges: coupled integral equations, complex analysis, structure of 3-, 4-, ... point functions, **need lots of computational power!**

access to underlying nonperturbative dynamics!

Backup slides

Hadrons?

- Simplest n-point function that encodes information on **baryons:** quark 6-point correlator
  $\langle\, \psi_\alpha(x_1)\, \psi_\beta(x_2)\, \psi_\gamma(x_3)\, \bar\psi_\rho(y_1)\, \bar\psi_\sigma(y_2)\, \bar\psi_\tau(y_3)\, \rangle$



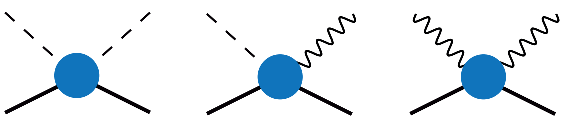
⇒ extract **gauge-invariant** baryon poles from gauge-fixed 6-quark function

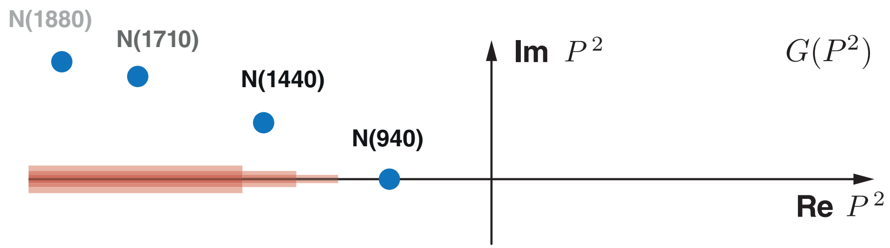
Bethe-Salpeter wave function:
residue at pole, contains all information about baryon

- Spectral decomposition:

$$\sum_\lambda |\lambda\rangle\langle\lambda| \quad \to \quad \sum_\lambda \frac{\cdots}{P^2+m_i^2}$$

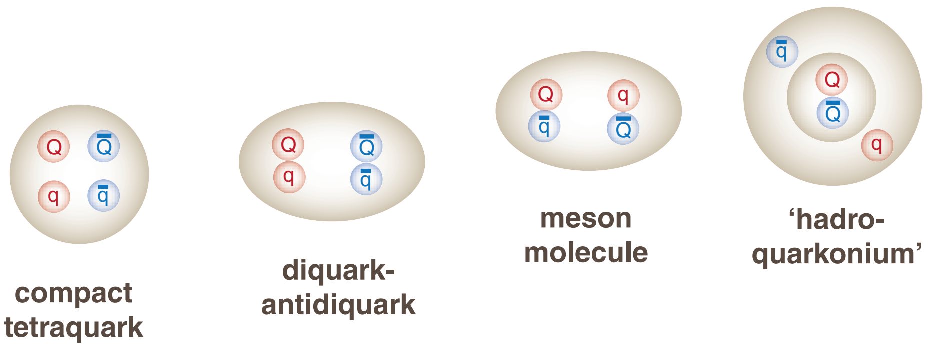
⇒ Same singularity structure as in



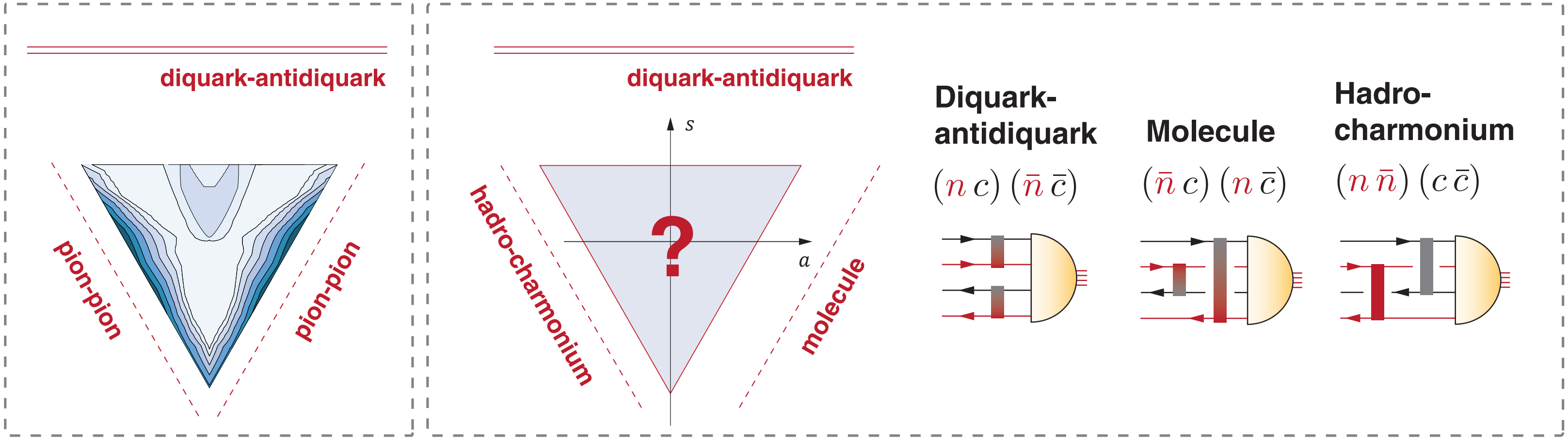


Tetraquarks in charm region?

- Can we **distinguish** different tetraquark configurations?

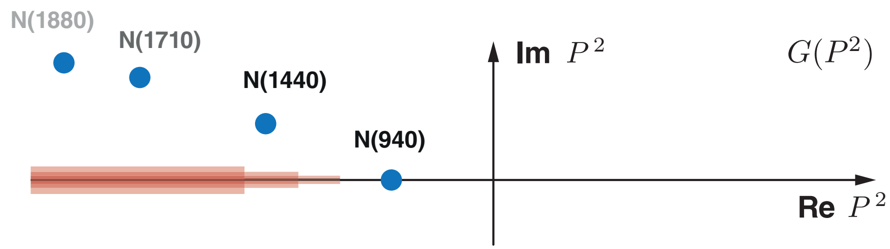


- **Four quarks** dynamically rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium; strengths determined by four-body BSE:

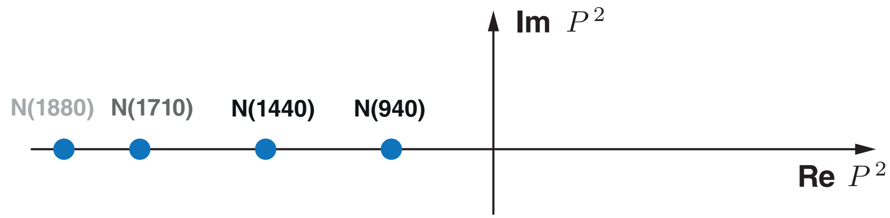


Resonances?

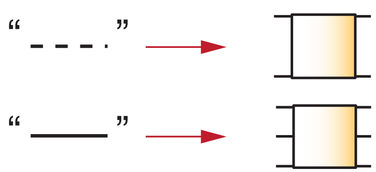
Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



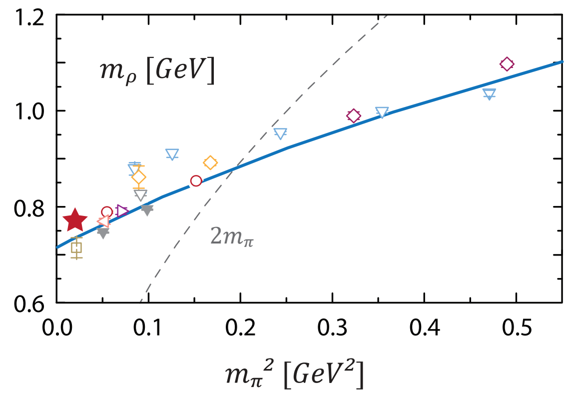
Without them: **bound states without widths**



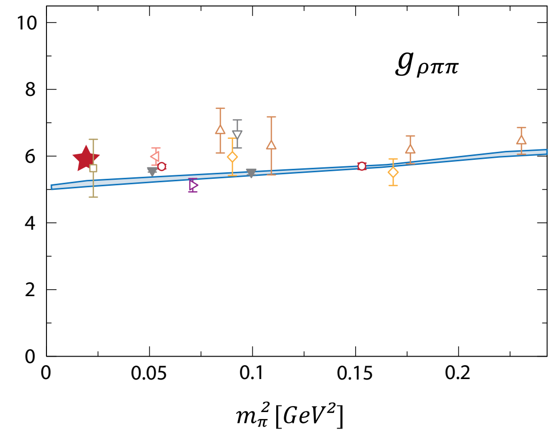
To generate resonances dynamically at **quark level:** complicated topologies beyond rainbow-ladder



cf. ρ **meson:** bound state vs. resonance below / above $\pi\pi$ threshold



resonance dynamics shifts pole into complex plane, effect on real part small?

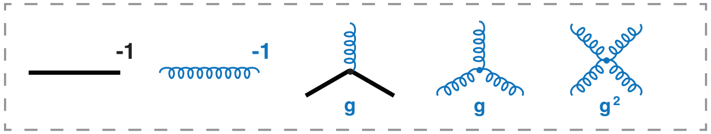


References:
see GE et al.,
PPNP 91 (2016)
1606.09602

QCD

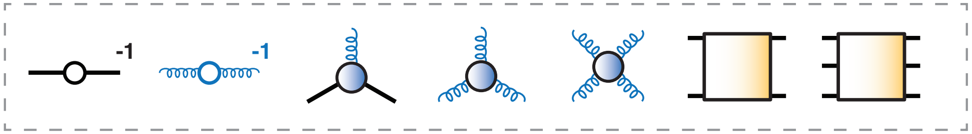
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + ig A\!\!\!/ + m \right) \psi + \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$



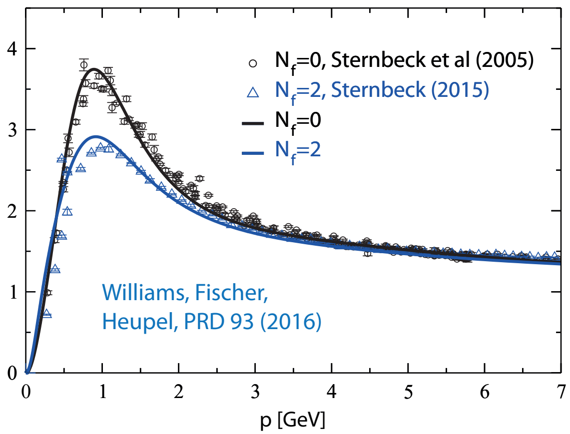
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



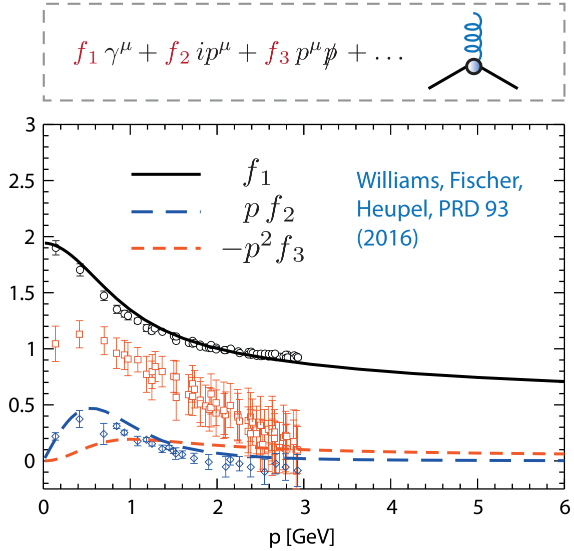
- **Gluon propagator**

$$\frac{D(p^2)}{p^2} \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)$$



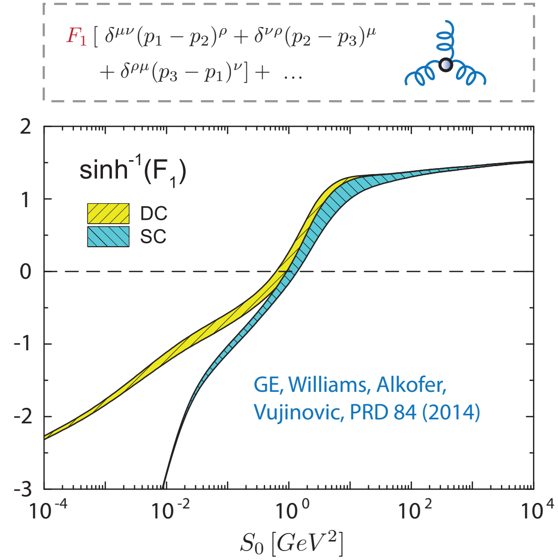
- **Quark-gluon vertex**

$$f_1\, \gamma^\mu + f_2\, i p^\mu + f_3\, p^\mu p\!\!\!/ + \dots$$



- **Three-gluon vertex**

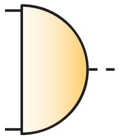
$$F_1 \left[\delta^{\mu\nu}(p_1 - p_2)^\rho + \delta^{\nu\rho}(p_2 - p_3)^\mu + \delta^{\rho\mu}(p_3 - p_1)^\nu \right] + \dots$$



Agreement between lattice, DSE & FRG within reach!

Bethe-Salpeter equations

- Example **pion:** quark-antiquark **bound state** ⇔ Goldstone boson of **DCSB**



$$= \gamma_5 \left(f_1 + f_2\, \not{P} + f_3\, q\cdot P\, \not{q} + f_4\, [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q\cdot P, P^2 = -m^2)$$

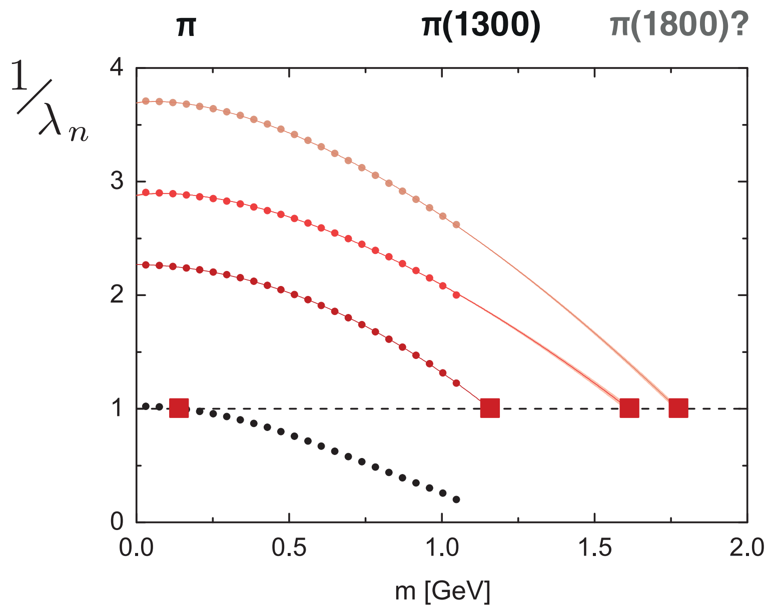
⟹ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Homogeneous BSE becomes

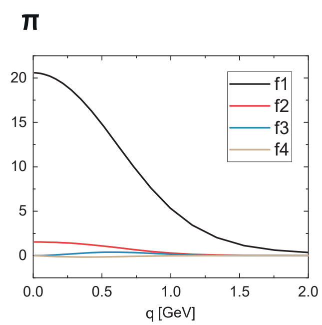
$$f_i(q^2, z) = \int d^4q'\, K_{ij}(q^2, q'^2, z, z', q\cdot q')\, f_j(q'^2, z')$$

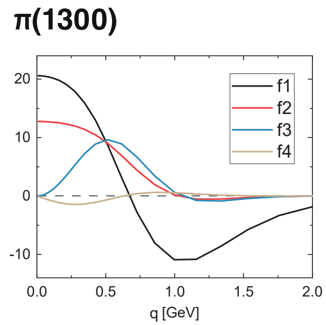
Eigenvalue spectrum of BS kernel:

$$K_{ij\,qq'\,zz'}\, f^{(n)}_{jq'z'} = \lambda_n(P^2)\, f^{(n)}_{iqz} \qquad \lambda_n \xrightarrow{P^2 \to -m_n^2} 1$$



- Eigenvectors = BS amplitudes

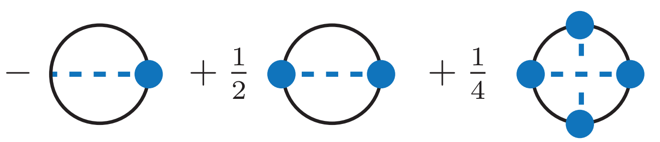




nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.

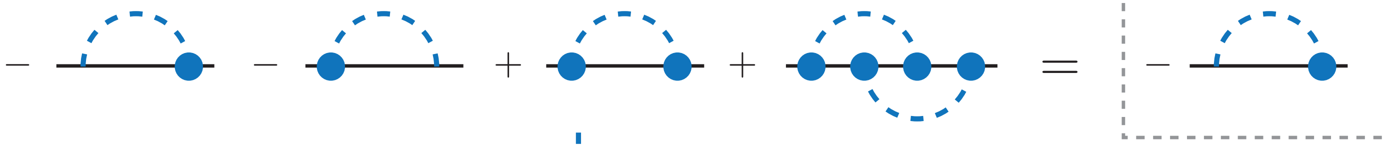
$\Gamma_2 =$



see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

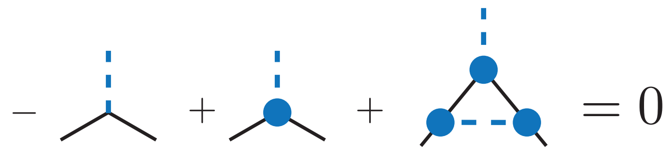
Self-energy:

$$\Sigma = \frac{\delta\Gamma_2}{\delta D} =$$



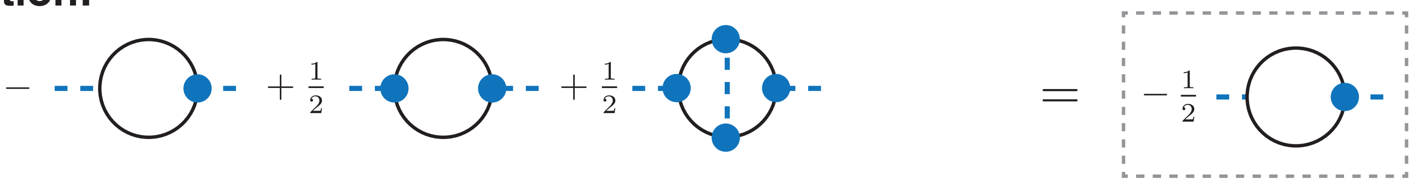
Vertex:

$$\frac{\delta\Gamma_2}{\delta V} = 0 \quad \Rightarrow$$



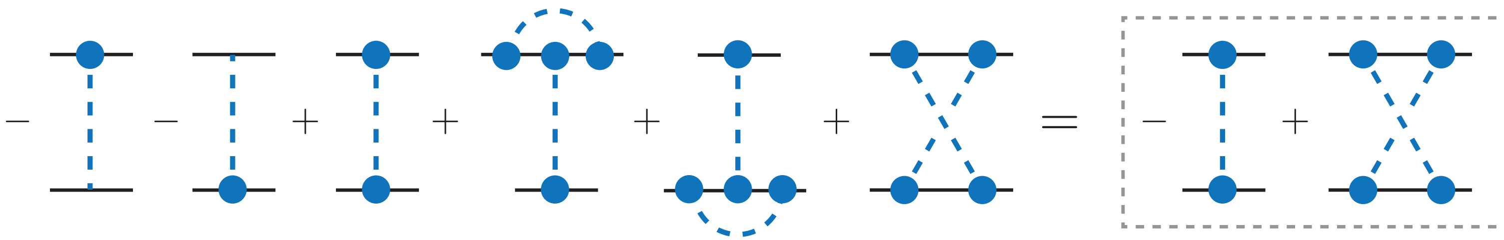
Vacuum polarization:

$$\Sigma' = \frac{\delta\Gamma_2}{\delta D'} =$$



BSE kernel:

$$-K = \frac{\delta\Sigma}{\delta D} =$$



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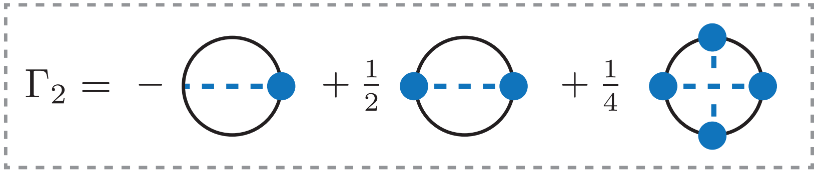

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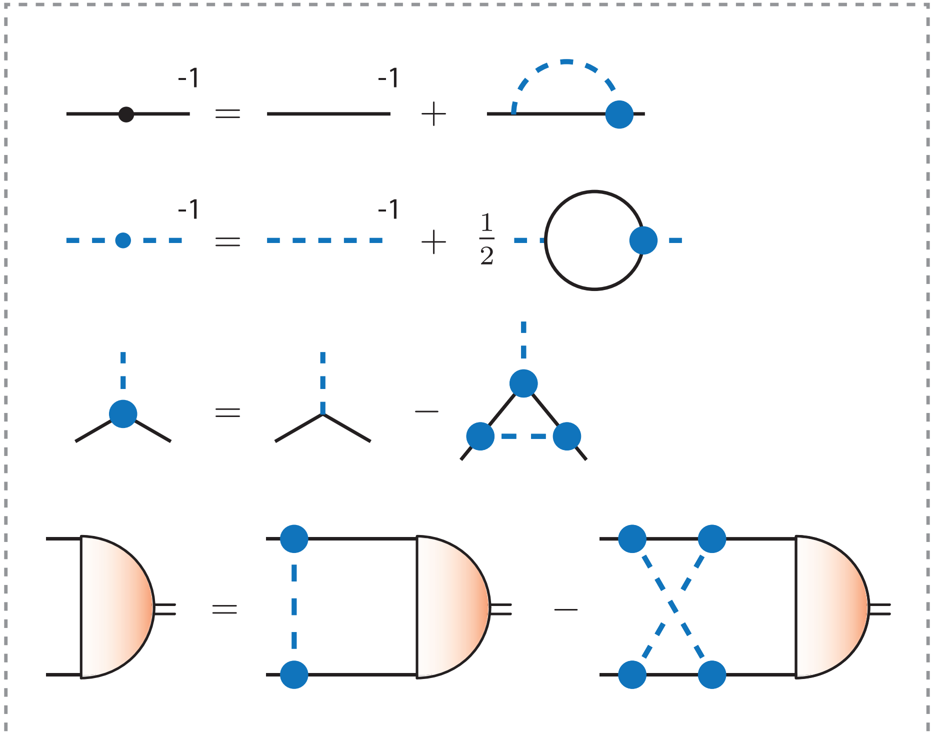

- Crossed ladder
  cannot be added by hand,
  requires **vertex correction!**

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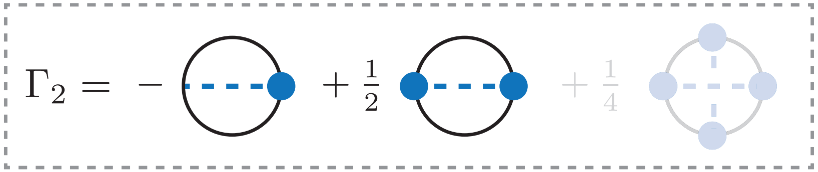

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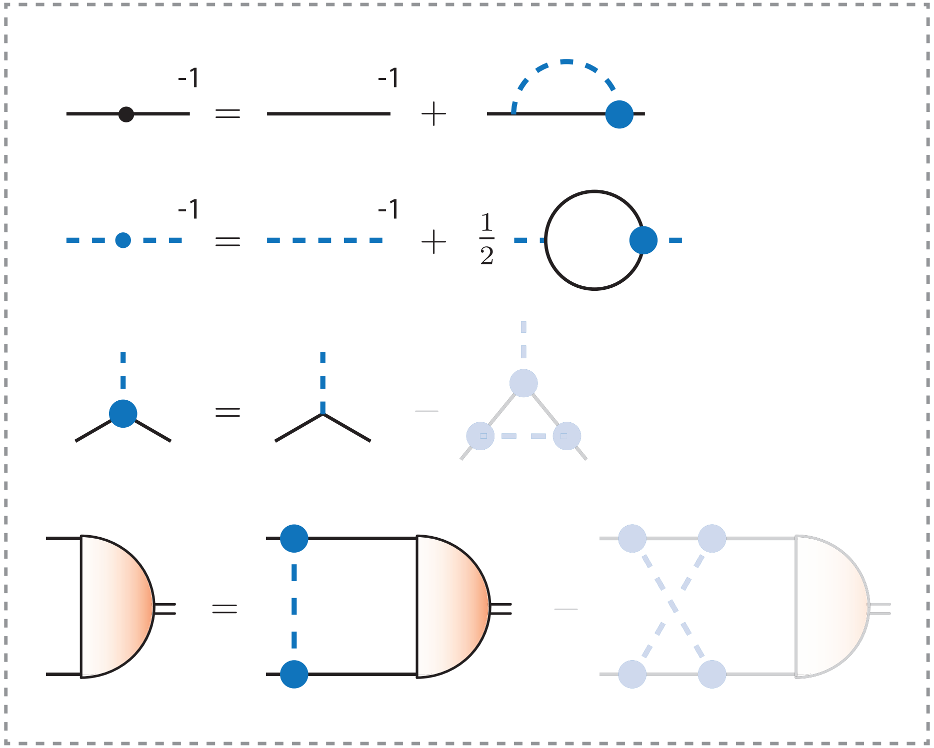
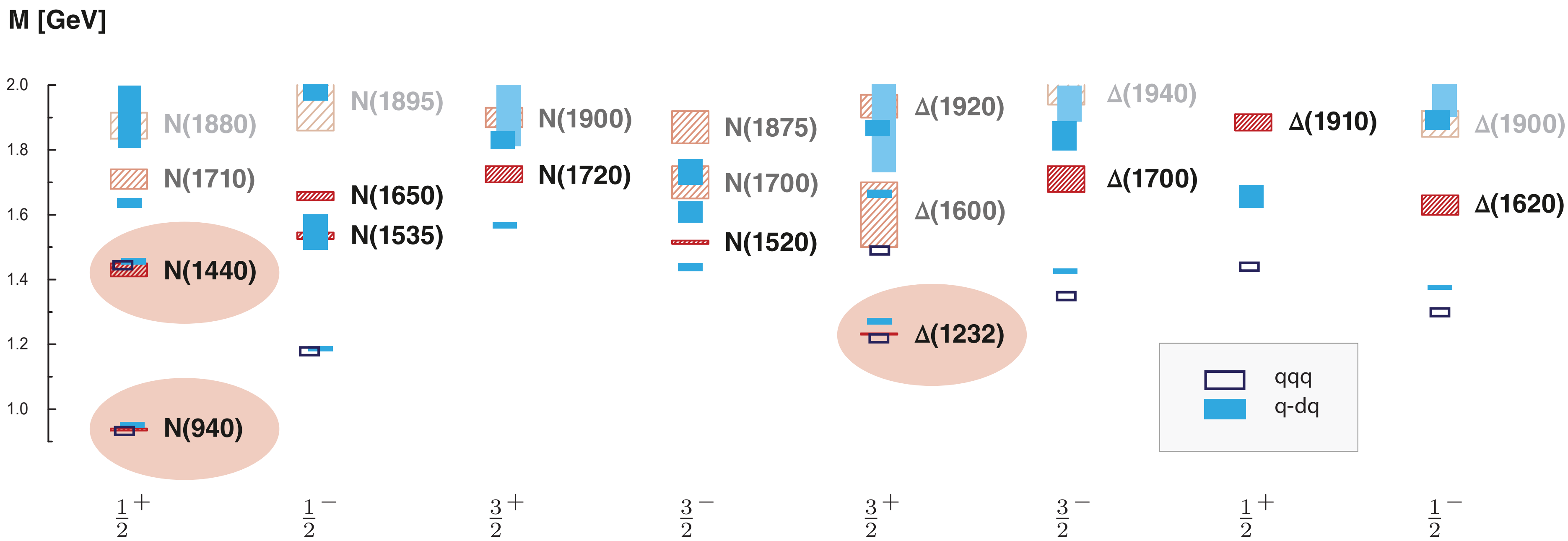


- Crossed ladder cannot be added by hand, requires **vertex correction!**
- without 3-loop term: **rainbow-ladder** with tree-level vertex ⇒ 2PI
- but still requires **DSE solutions** for propagators!
- Similar in QCD. nPI truncation guarantees chiral symmetry, massless pion in chiral limit, etc.

Baryon spectrum I

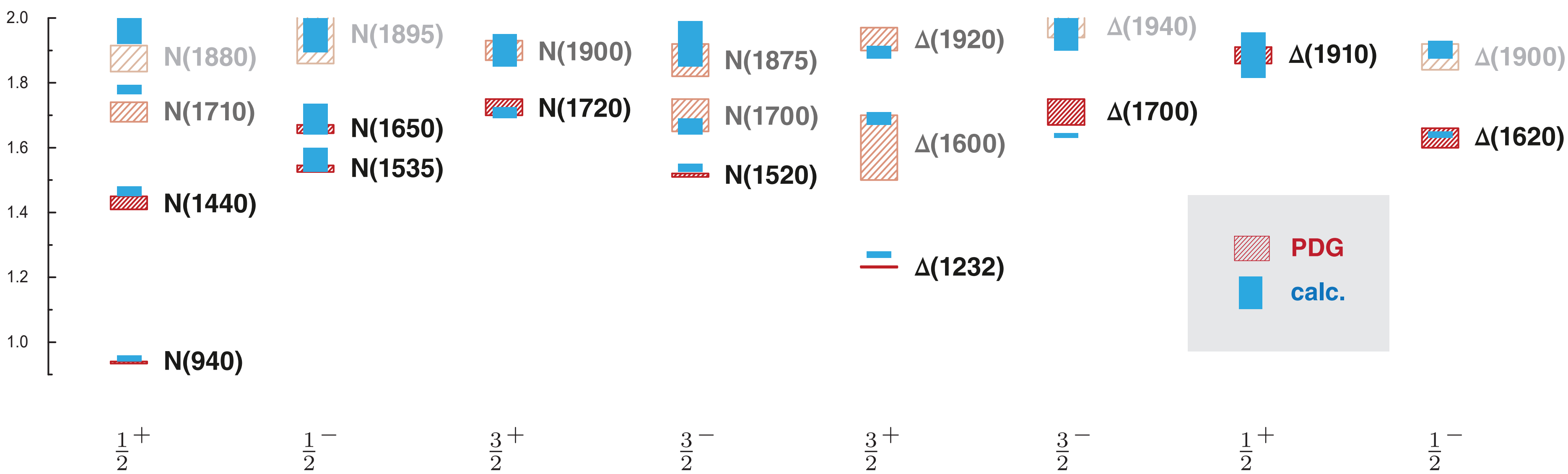
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

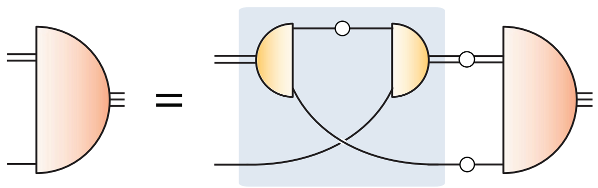


- **qqq** and **q-dq** agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)

Baryon spectrum

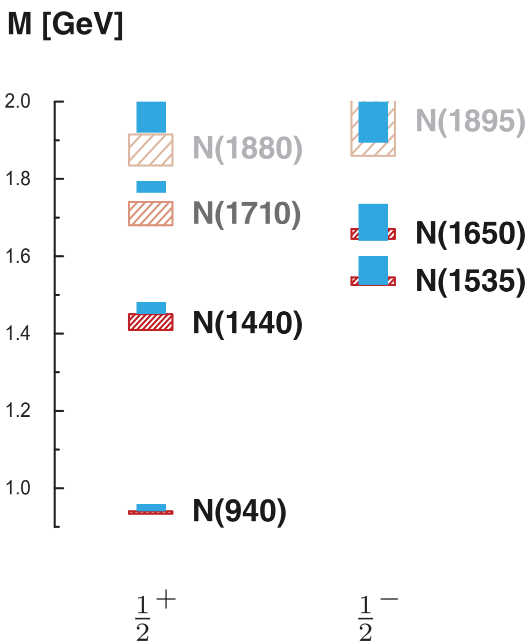
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)





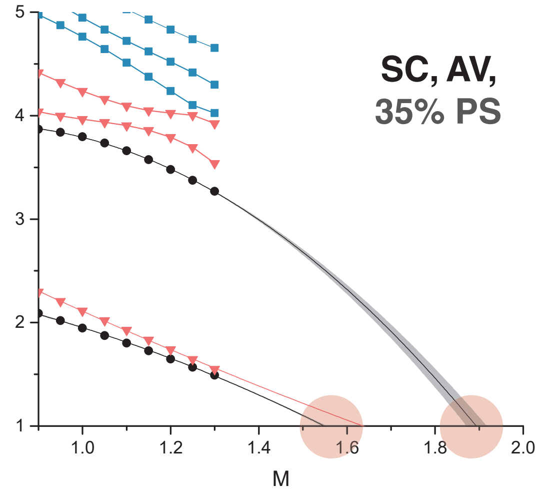
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

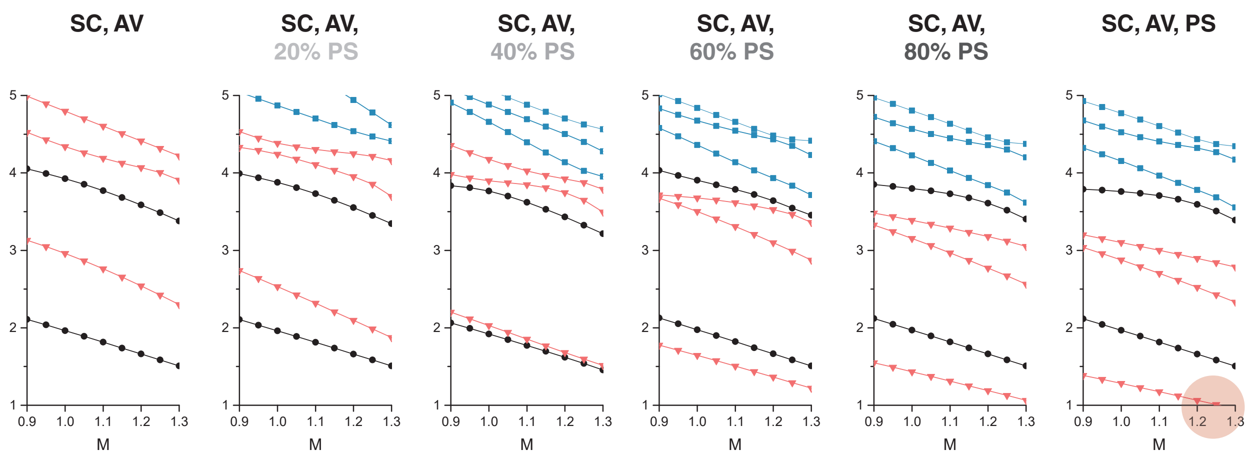
Baryon spectrum



Level ordering between
Roper and N(1535):

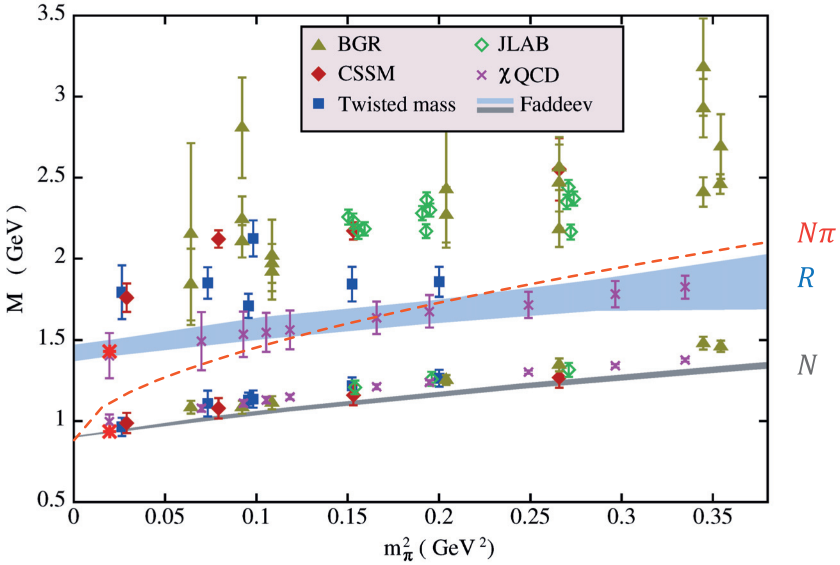
dynamics of ps diquark produces
2 nearby states: **N(1535), N(1650)**



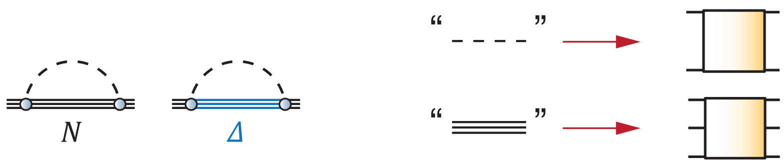


Resonances

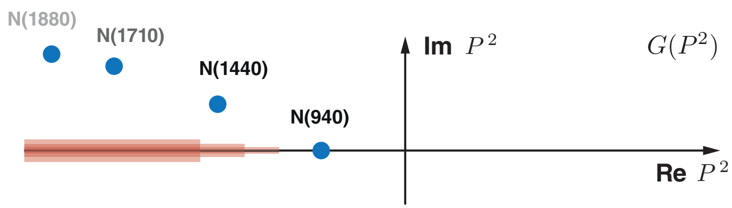
- **Current-mass evolution** of Roper:
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



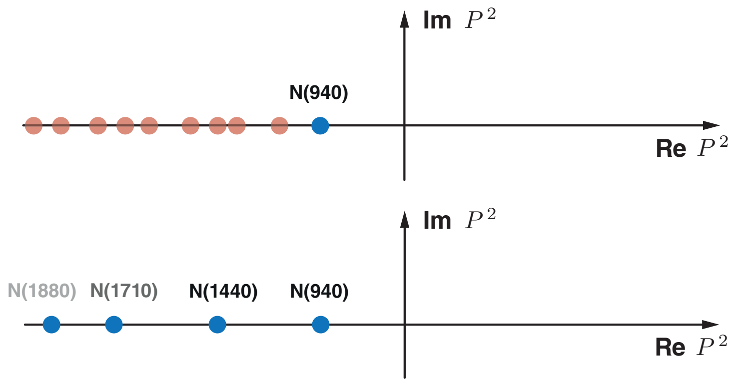
- **'Pion cloud'** effects difficult to implement at **quark-gluon level:**



- Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



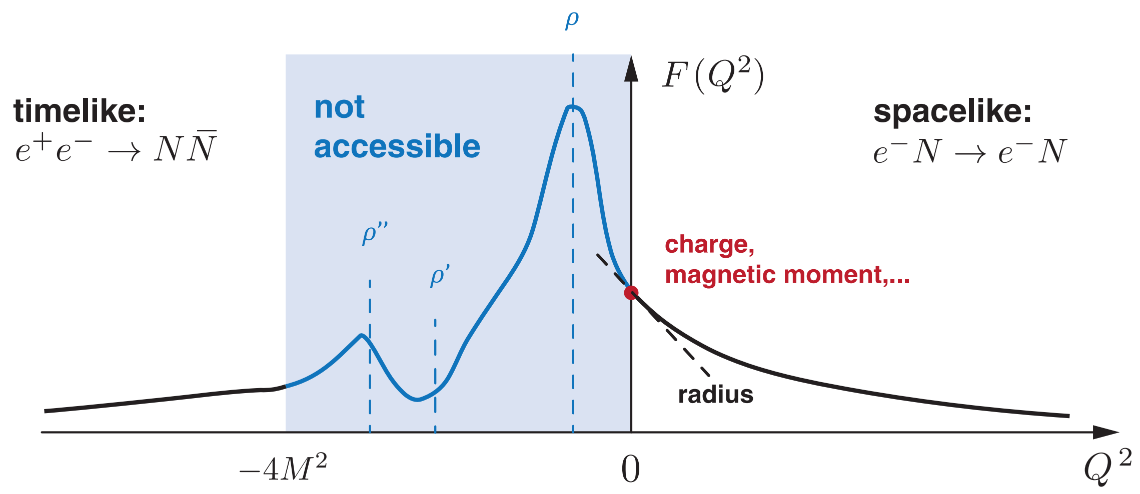
- **Lattice:** finite volume, **DSE** (so far): bound states



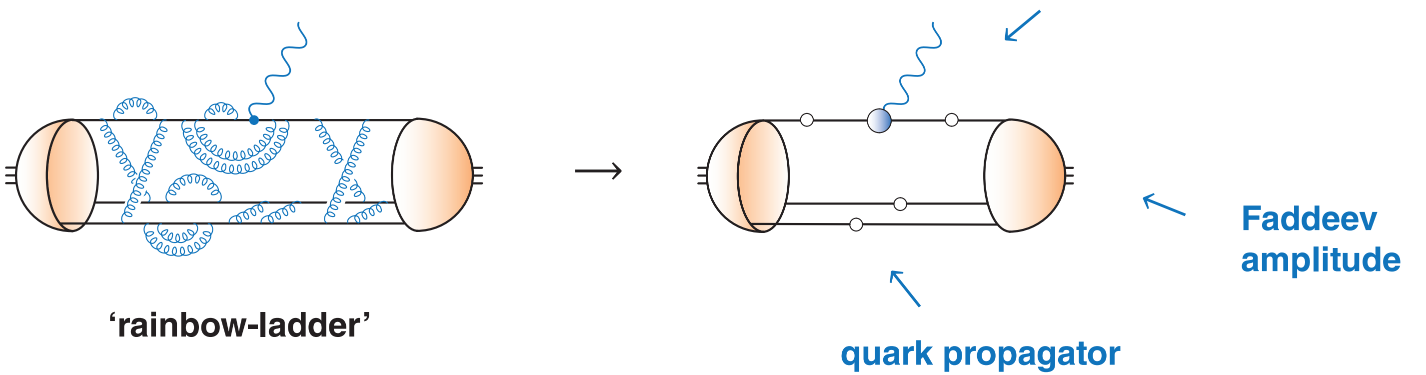
Resonance dynamics
shifts poles into complex plane,
but effects on real parts small?

Form factors

Sketch of a generic electromagnetic form factor:

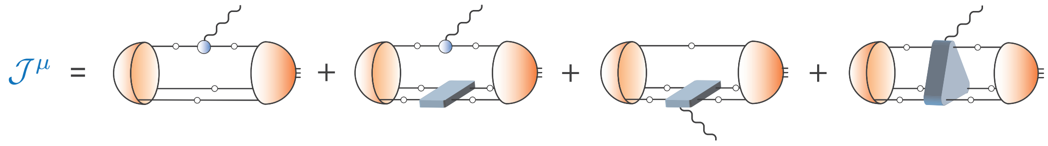


How can we calculate this from the **quark level?**



Form factors

Nucleon em. form factors
from three-quark equation
GE, PRD 84 (2011)

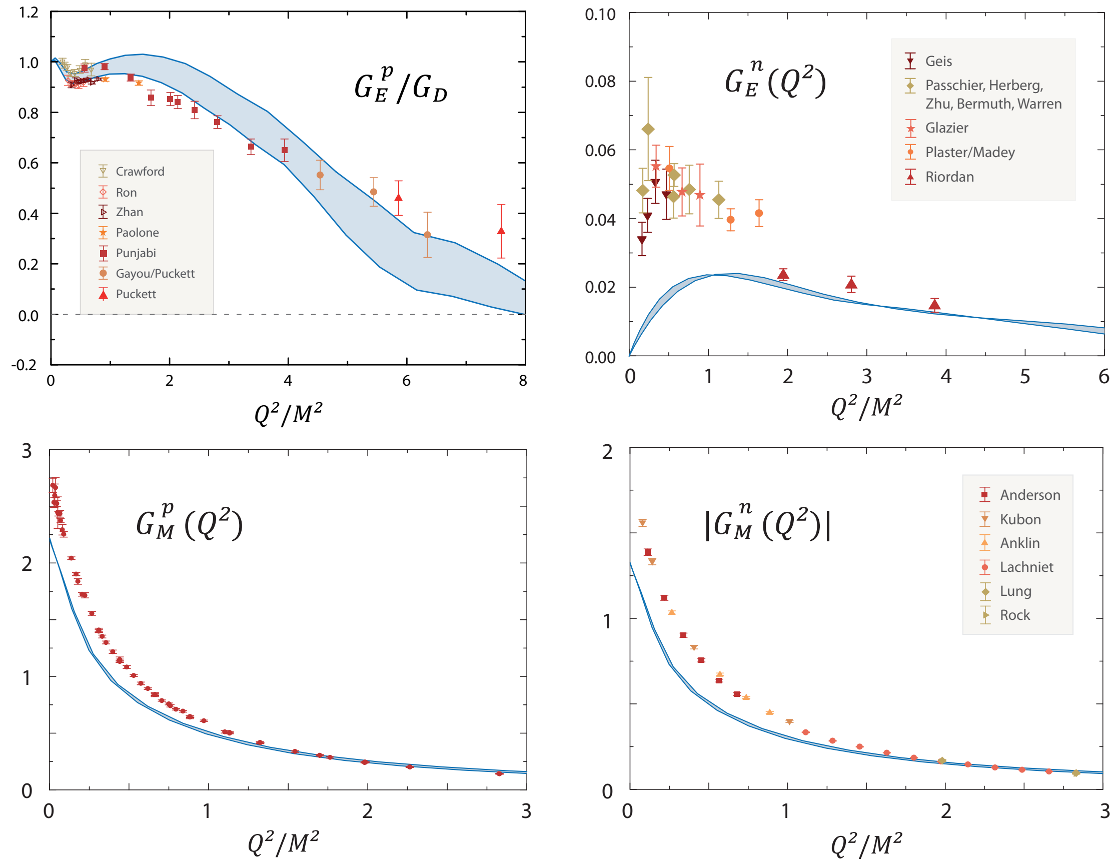


- **Timelike vector-meson poles** generated in quark-photon vertex
- **"Quark core without pion-cloud"**
- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

  Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

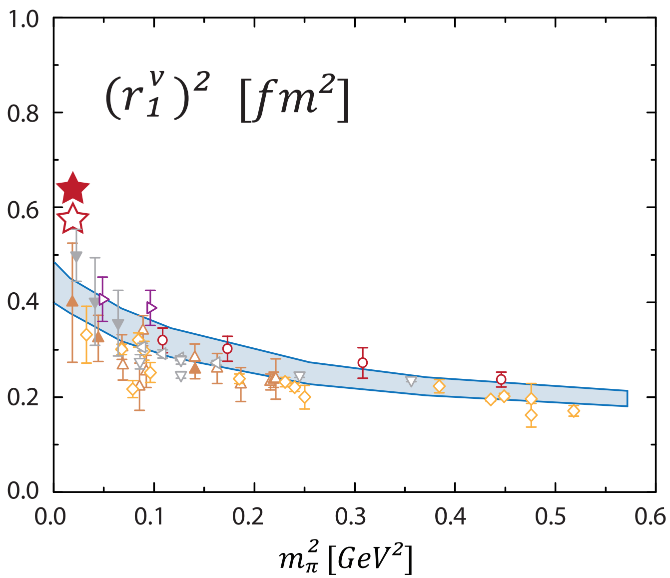
- $\pi \to \gamma\gamma^*$ transition: vm. poles modify asymptotic scaling!

  GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]

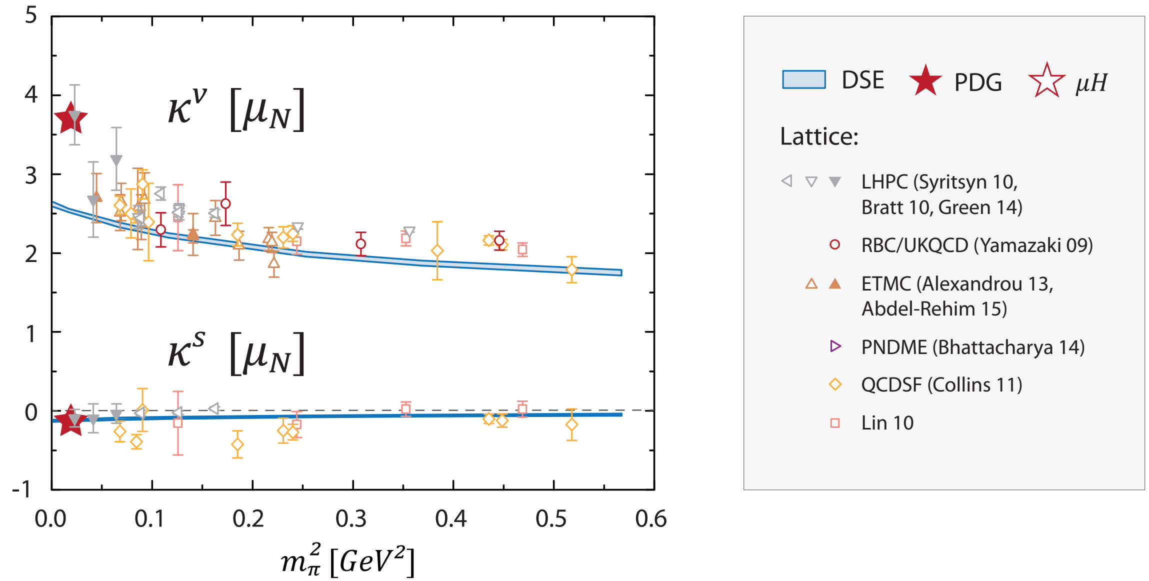


Nucleon em. form factors

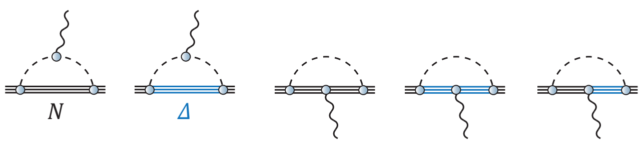
Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



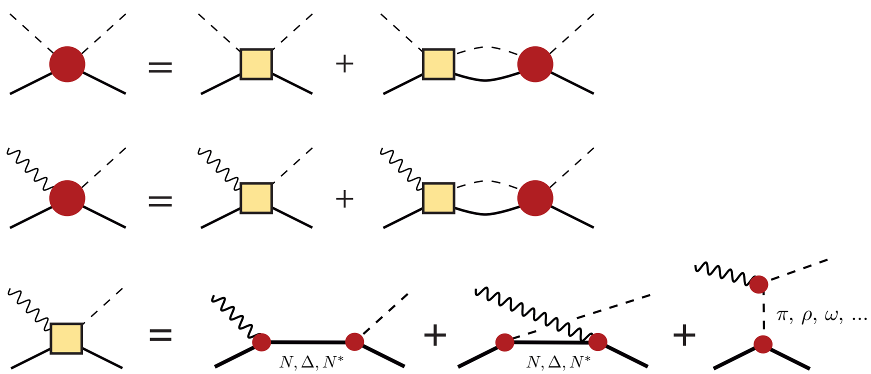
- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

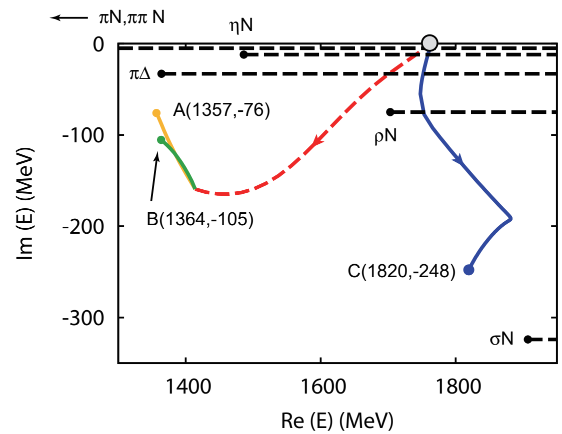
GE, PRD 84 (2011)

Extracting resonances

Hadronic coupled-channel equations:



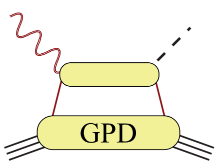
Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI, JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina, JPAC, . . .



Suzuki et al., PRL 104 (2010)

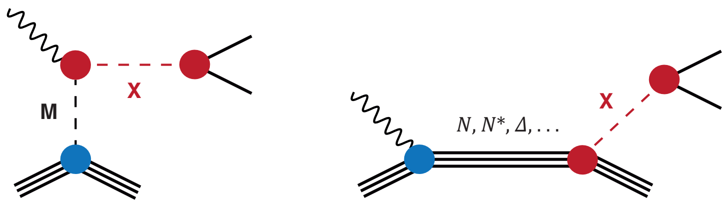
Microscopic effects?

What is an “offshell hadron”?



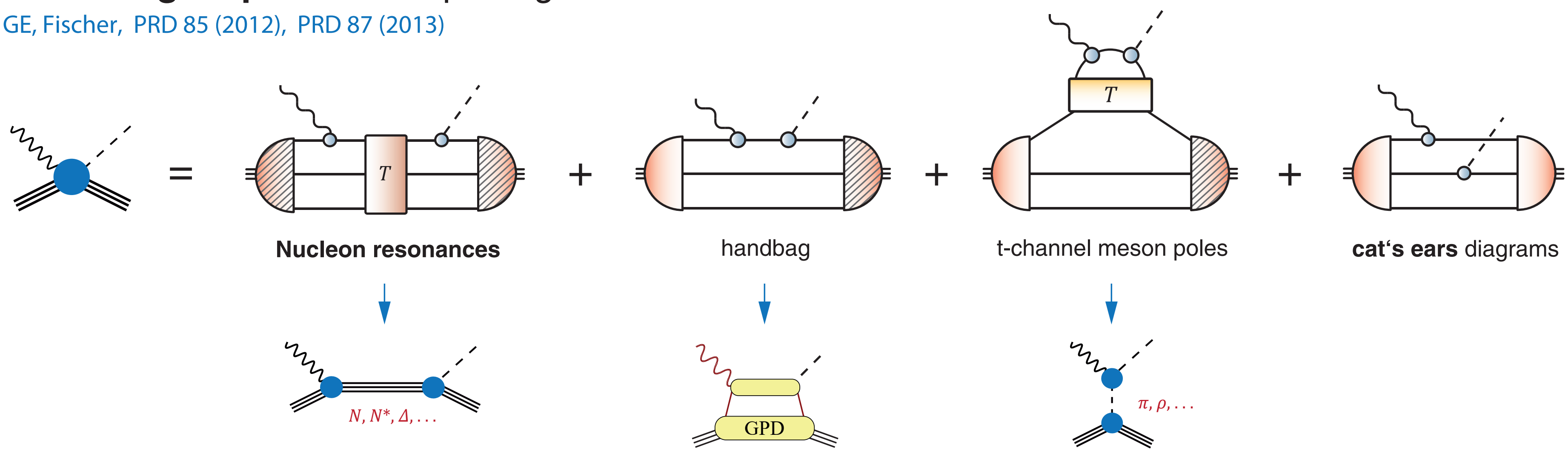
Extracting resonances

Photoproduction of **exotic mesons** at JLab/GlueX:



What if exotic mesons are **relativistic $q\bar{q}$ states?**
⇒ study with DSE/BSE!

Scattering amplitudes at quark-gluon level:
GE, Fischer, PRD 85 (2012), PRD 87 (2013)



Scattering amplitudes

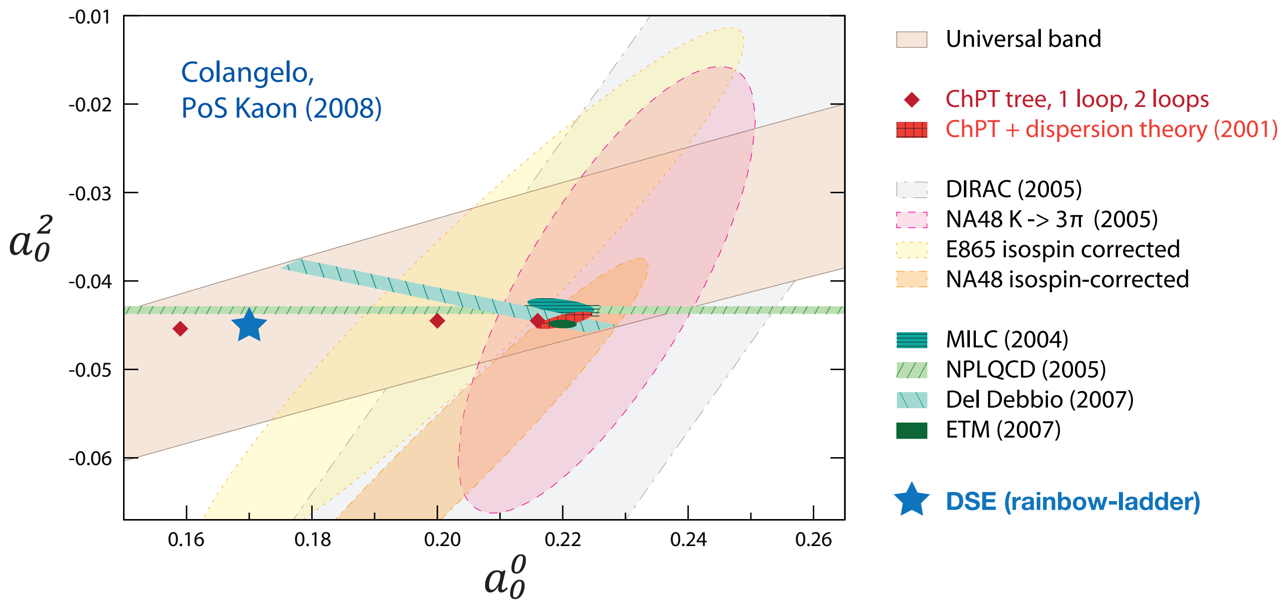
Scattering amplitudes from quark level:

- $\pi\pi$ **scattering**

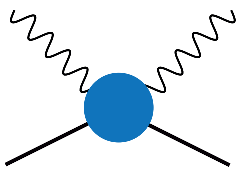
  DSE: Bicudo, Cotanch, Llanes-Estrada, Maris, Ribeiro, Szczepaniak, PRD 65 (2002),

  Cotanch, Maris, PRD 66 (2002)

  CST: Biernat, Pena, Ribeiro, Stadler, Gross, PRD 90 (2014)



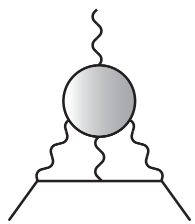
- **Nucleon Compton scattering**



  GE, Fischer, PRD 85 (2012) & PRD 87 (2013), GE, FBS 57 (2016)

- **Hadronic light-by-light scattering**

  Goecke, Fischer, Williams, PLB 704 (2011), GE, Fischer, Heupel, PRD 92 (2015)

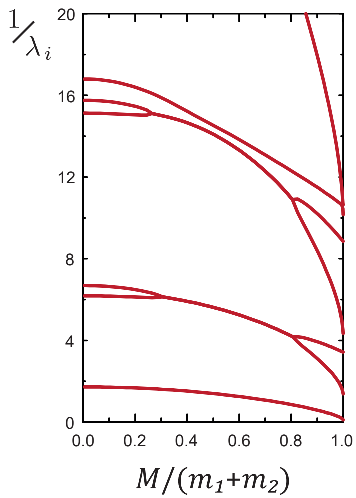


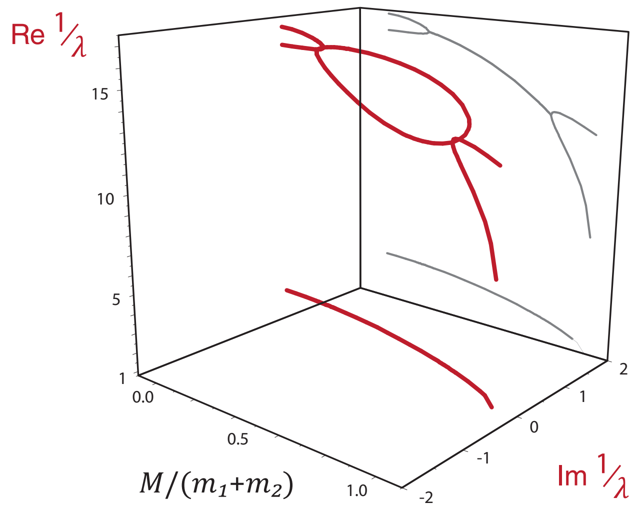
Complex eigenvalues?

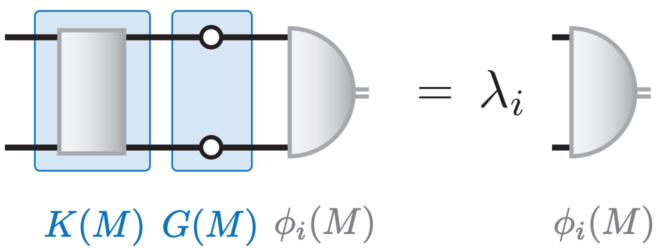
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

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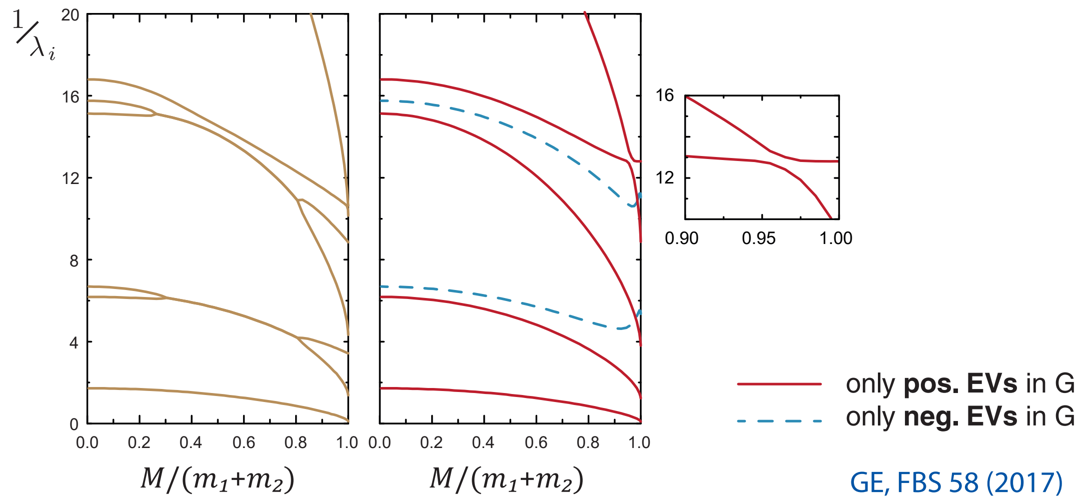
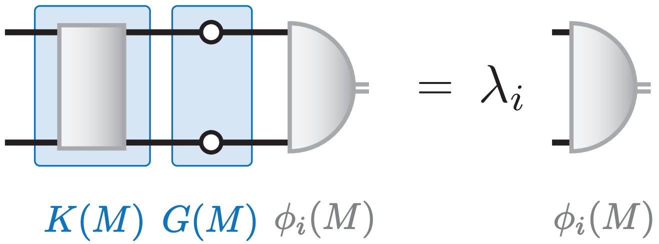


GE, FBS 58 (2017)



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⇒ Hermitian problem with same EVs!

⇒ all EVs strictly **real**
⇒ level repulsion
⇒ “anomalous states” removed?

Complex eigenvalues?

Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous**" **states?**
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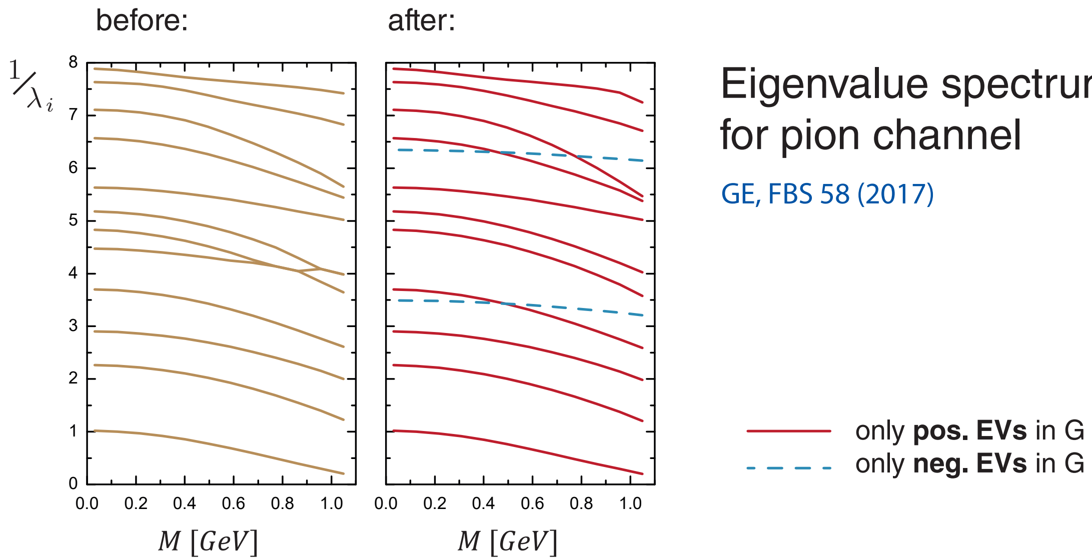
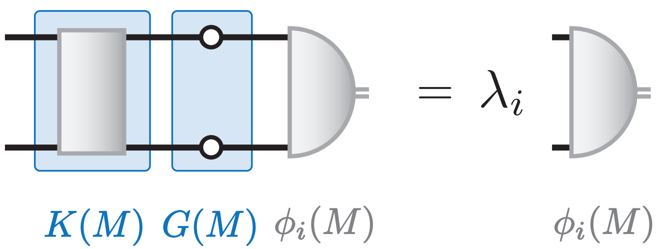


Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)



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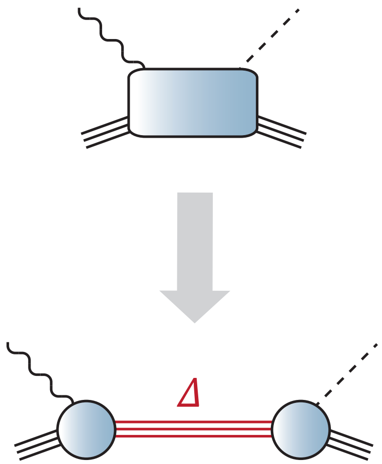
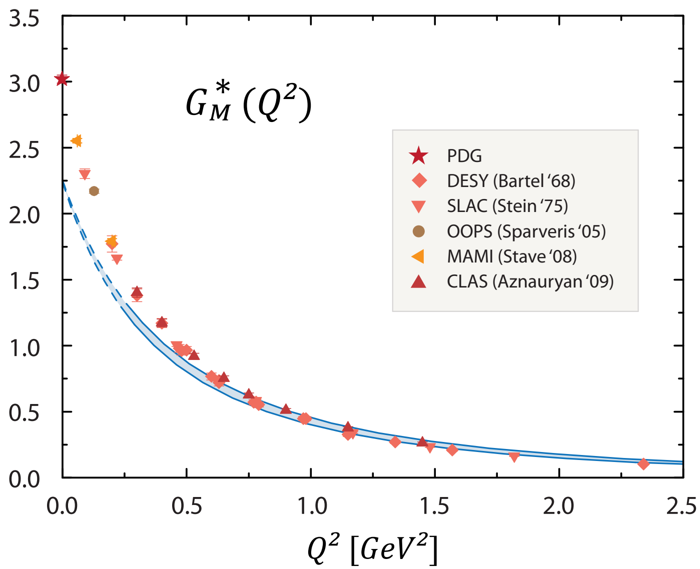
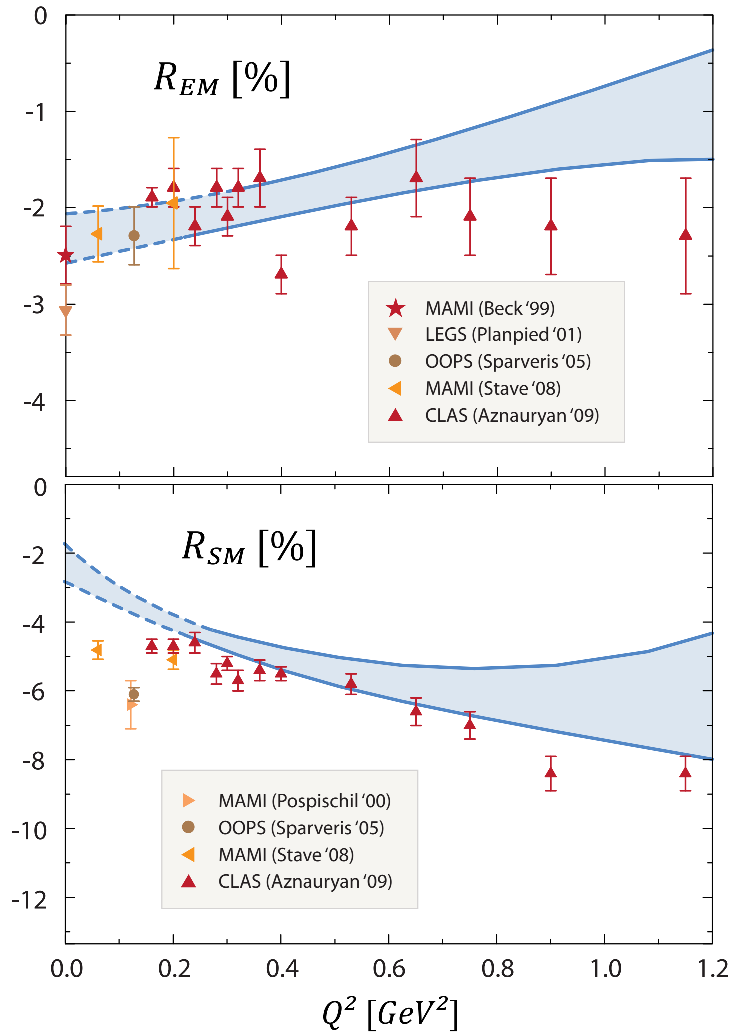
$\Rightarrow$ Hermitian problem with same EVs!

$\Rightarrow$ all EVs strictly **real**
$\Rightarrow$ level repulsion
$\Rightarrow$ "anomalous states" removed?

Nucleon-Δ-γ transition







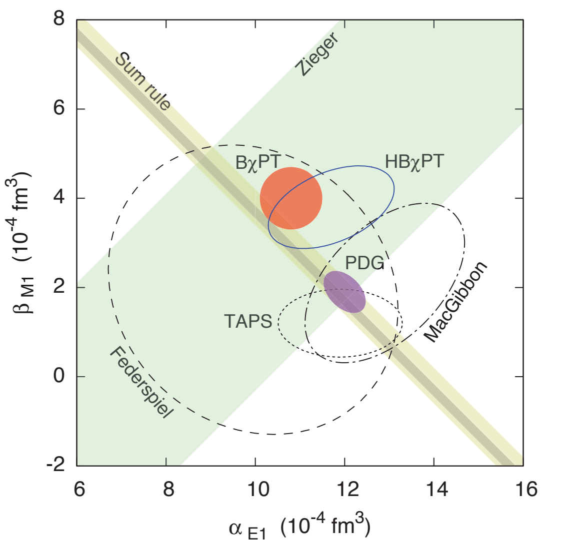
- **Magnetic dipole transition (G_M^*)** dominant:
  quark spin flip (s wave). "Core + 25% pion cloud"

- **Electric & Coulomb quadrupole ratios**
  small & negative, encode deformation.
  Reproduced without pion cloud: **OAM** from **p waves!**

  GE, Nicmorus, PRD 85 (2012)

Compton scattering

Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)



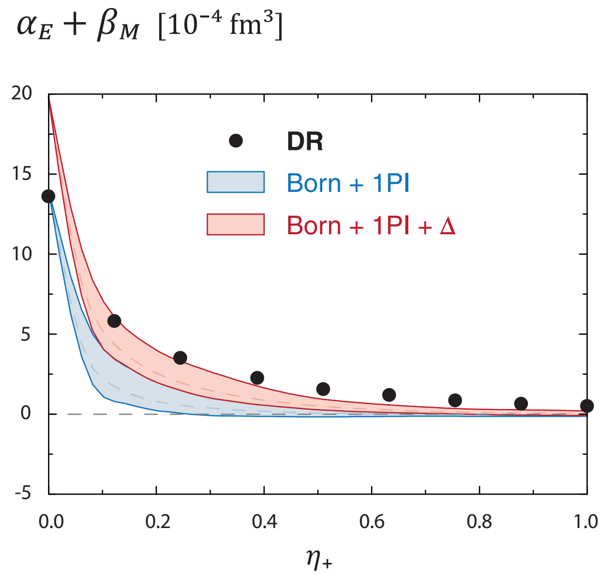
First DSE results:
GE, FBS 57 (2016)

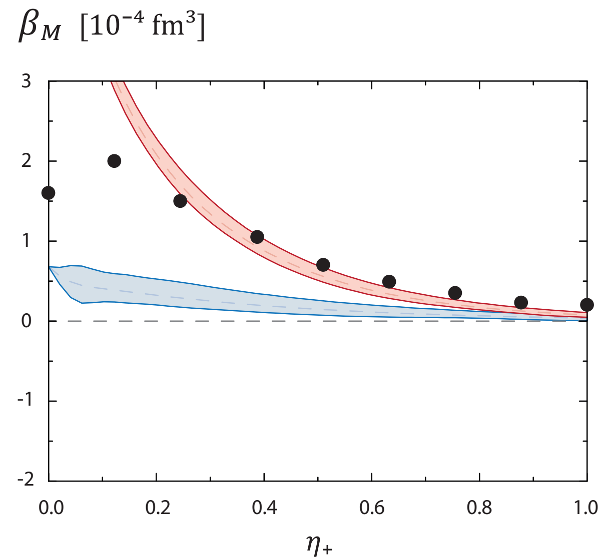
- Quark Compton vertex (Born + 1PI) calculated, added *Δ* exchange
- compared to DRs
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E dominated by handbag, β_M by *Δ* contribution

⇒ **large "QCD background"!**

In total: polarizabilities ≈

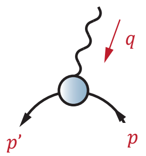
Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly *Δ*)
+ pion cloud (at low η_+)?





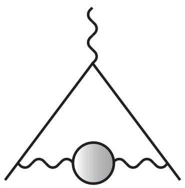
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

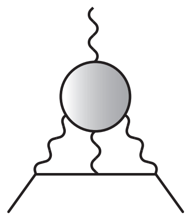


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



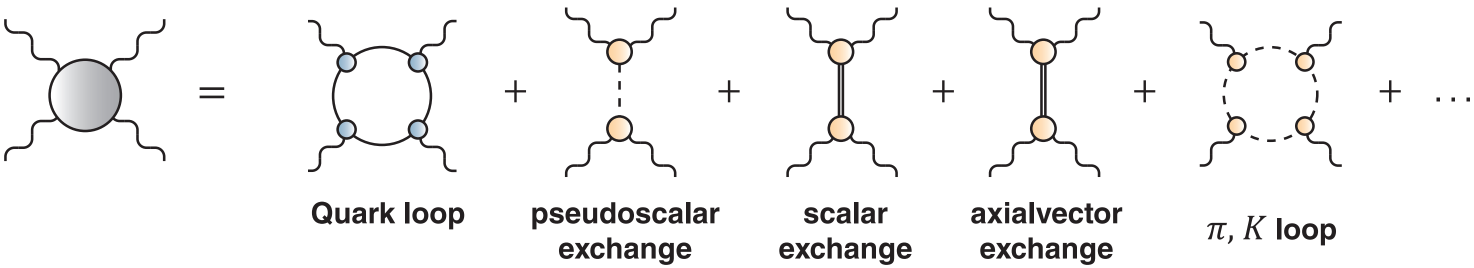
Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	−1	2	−2	$(\times 10^{-10})$

Muon g-2

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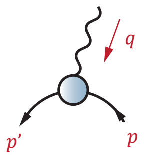
  total SM prediction deviates from exp. by ~3σ



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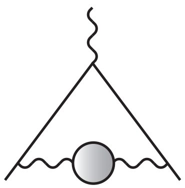

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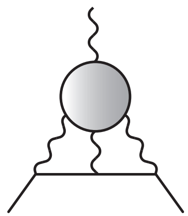

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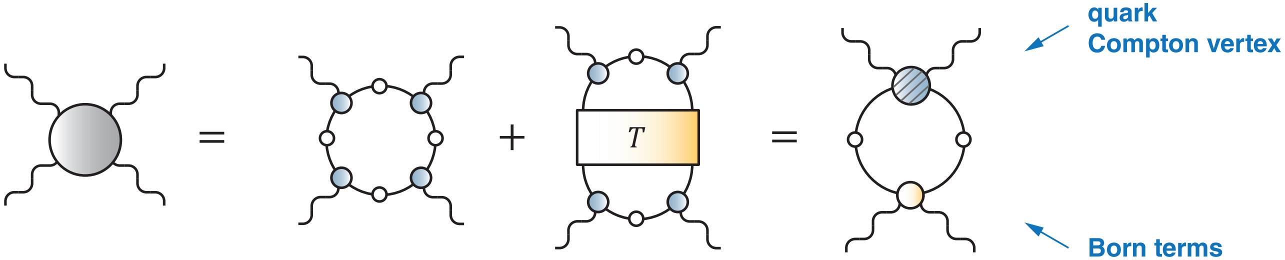
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)